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Homework 0

Final Mock

For each question, please provide a short answer. After that, capture the draft work that you have written down, paste it into a WORD / PDF file and submit it at the end of the form. Please indicate which image belongs to which question.

Calculator is allowed ONLY FOR NUMERICAL CALCULATIONS. In other words, do not use the calculator to directly evaluate a derivative or an integral. All short answers should be rounded to 3 significant digits. Specify the unit of calculation, if any.

You will immediately know the correct short answer after you submit your solution. If I have time, I will grade each question manually. Note that a correct answer at the end of your work does not guarantee a full mark for the question, since all the necessary steps are taken into account when grading manually.

Submission form: https://forms.office.com/r/KzpeAktiw6

Problem 1 - 9m

Find the escape velocity v_0 that is needed to propel a rocket of mass $m = 1.9988 \times 10^6$ kg out of the gravitational field of a planet with mass $M = 8.0254 \times 10^{24}$ kg and radius $R = 4567$ km. (Use the fact that the initial kinetic energy of $\frac{1}{2}mv_0^2$ supplies the needed work.) Given the gravitational constant $G = 6.6743 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Problem 2 - 6m (2m each)

Determine whether the following series converges or diverges. State the radii of convergence of the series, if any.

1. $$f_1(x) = \sum_{n=0}^{\infty} \frac{n^3 x^{3n}}{n^4 + 1}$$

2. $$f_2(x) = \sum_{n=1}^{\infty} \frac{n + 5}{n\sqrt{n + 3}}$$

3. $$f_3(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 1}$$

Problem 3 - 6m

1. State the conditions necessary to use the integral test to determine the convergence of the series $f_4(x) = \sum_{n=0}^{\infty} \frac{1}{e^n}$. Use the integral test to show that $f_4(x) = \sum_{n=0}^{\infty} \frac{1}{e^n}$ converges. (4m)
2. The first two terms of the series $f_3(1) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n+1}$ are used to approximate $f_3(1)$. Determine an upper bound on the error of the approximation. (2m)

Problem 4 - 6m

Find the arc length function for the curve $y = 2x^{\frac{3}{2}}$ with starting point $P_0 = (1, 2)$.
Evaluate at x = 7.

Problem 5 - 7m

The **THEOREM OF PAPPUS** is stated as follow:
Let **T** be a plane region that lies entirely on one side of a line l in the plane. If **T** is rotated about l, then the volume of the resulting solid is the product of the area of **T** and the distance traveled by the centroid of **T**.

1. Prove the Theorem of Pappus for the special case $l : y = 0$, in other words l is the x-axis. (4m)
2. Calculate the volume of solid V generated by rotating the parabola $y^2 + 1 = x$ $(1 \leq x \leq 3)$ about the line $x = \frac{-4}{5}$. (3m)

Problem 6 - 6m

One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor $(1 - y)$.

1. Write a differential equation that is satisfied by y. (2m)
2. Solve the differential equation. (4m)