Midterm Mock Exam

Midterm Mock

Chapters covered: 12.1 $\rightarrow$ 15.2.

For each question, please provide a short answer in the answer sheet on the next page.

Disclaimer: This is a mock test made by a student and has not been verified by any instructor of the MATH1020 course.

Midterm Mock Exam

Answer Sheet

Problem	Your answer	Score
1		------ / 10
2		------ / 9
3		------ / 8
4		------ / 12
5		------ / 8
6		------ / 8
7		------ / 14
8		------ / 9
9		------ / 10
10		------ / 12
Total		------ / 100

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Problem 1 - 10m

Let $a, b, c \geq 0$. It can be shown that if $a + b + c > \sigma$, the limit

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^a y^b z^c}{x^2 + y^2 + z^2}$$

always exists. Find σ.

Problem 2 - 9m

Over most of the earth, a magnetic compass does not point to true (geographic) north; instead, it points at some angle east or west of true north. The angle D between magnetic north and true north is called the magnetic declination. Use Figure 1 to determine which of the following statements is true:

1. $$\frac{\partial D}{\partial x}\Big|_C > 0$$
2. $$\frac{\partial D}{\partial y}\Big|_A < 0$$
3. $$\frac{\partial D}{\partial y}\Big|_A < \frac{\partial D}{\partial y}\Big|_B$$

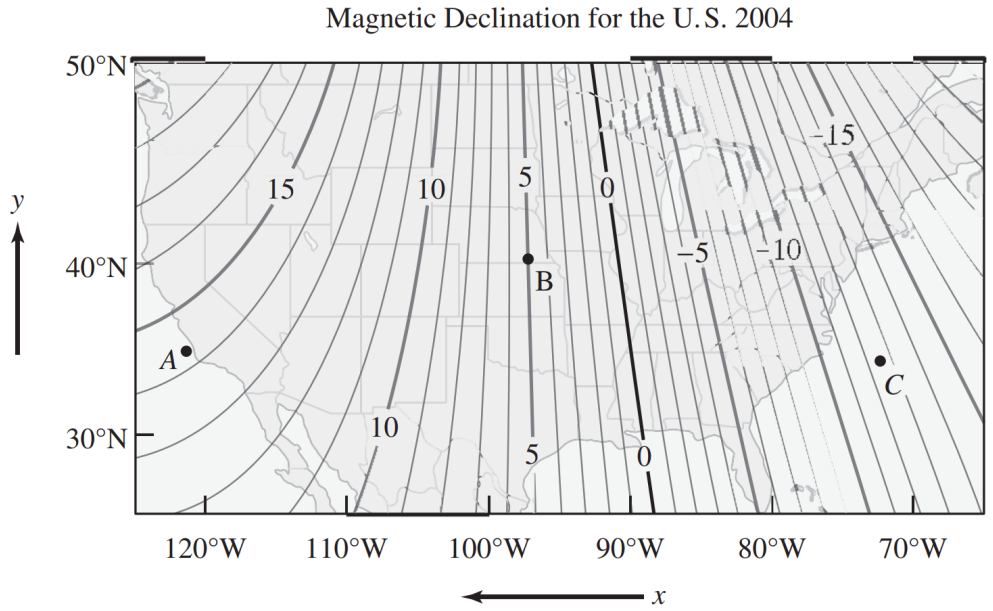

Figure 1: Contour interval 1 degree

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Problem 3 - 8m

Find a vector of length 13 that is normal to the surface $-x^3 + yz^2 = -17$ at the point P(-1, -2, -3).

Problem 4 - 12m

For all $x > 0$, there is a unique value $y = \sigma(x)$ that solves the equation $4xy + y^3 - 16 = 0$. Let $g(x) = f(x, \sigma(x))$. Calculate $g_x(1)$, knowing that $f_x(1, 2) = 9$ and $f_y(1, 2) = 10$.

For Problem 5, 6, let $f(x, y, z) = xy + 2z$ be the function that we are interested to optimize. Constraint g is given by the set of points in the Oxyz space that are of distance 6 from the origin.

Problem 5 - 8m

How many critical points are there in this optimization problem?

Problem 6 - 8m

What is the largest value of $f(x, y, z)$ under the given constraints?

Problem 7 - 14m

The number of stay-at-home fathers in Canada in recent years is given below.

Year	**1976**	**1984**	**1991**	**2000**	**2010**
Number of stay-at-home fathers	20610	28725	43530	47665	53555

This data can be assumed to follow a linear growth model. The model can be represented by the following formula:

$$f(x) = mx + b$$

where m, b are constants, and x is the year of interest. One method to evaluate the accuracy of such models is to use the best-square-fit method, where the "error" of prediction is determined by

$$E(m, b) = \frac{1}{N} \sum_{i=1}^{N} (f(x_i) - y_i)^2$$

By this criterion, the smaller the prediction error, the better the model. Determine a model that best fit the given data. Give your answer in the form $y = mx + b$. Round your m and b to 7 significant digits.

Problem 8 - 9m

Parametrize the intersection of the surfaces $y^2 - z^2 = x - 2$ and $y^2 + z^2 = 9$ using trigonometric identities.

Problem 9 - 10m

Evaluate

$$I_9 = \iint_M x \cos y dA$$

where M is bounded by $y = 0, y = x^2, x = 1$.

Problem 10 - 12m

Suppose you are hiking on a terrain modeled by $z = xy + y^3 - x^2$. You are at the point (2, 1, -1). Determine the steepest slope you could encounter from your position, and the compass direction measured in degrees from East that you would head to realize this steepest slope.

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Key

Problem	Your answer	Score
1	2	____ / 10
2	1. True; 2. False; 3. False	____ / 9
3	$\langle -3, 4, 12 \rangle$	____ / 8
4	9	____ / 12
5	10	____ / 8
6	20	____ / 8
7	y=992.3857x-1938214	____ / 14
8	$\mathbf{r}(t) = \langle 2 + 9\cos 2t, 3\cos t, 3\sin t \rangle$	____ / 9
9	$\frac{1}{2}(1 - \cos 1)$	____ / 10
10	Slope $= \sqrt{34}$ Angle = 2.1112 rad = 120.96 degrees	____ / 12