

Midterm Mock

Chapters covered: 12.1 \rightarrow 15.2.

For each question, please provide a short answer in the answer sheet on the next page.

Disclaimer: This is a mock test made by a student and has not been verified by any instructor of the MATH1020 course.

Answer Sheet

Problem	Your answer	Score
1		----- / 10
2		----- / 9
3		----- / 8
4		----- / 12
5		----- / 8
6		----- / 8
7		----- / 14
8		----- / 9
9		----- / 10
10		----- / 12
Total		----- / 100

Problem 1 - 10m

Let $a, b, c \geq 0$. It can be shown that if $a + b + c > \sigma$, the limit

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^a y^b z^c}{x^2 + y^2 + z^2}$$

always exists. Find σ .

Problem 2 - 9m

Over most of the earth, a magnetic compass does not point to true (geographic) north; instead, it points at some angle east or west of true north. The angle D between magnetic north and true north is called the magnetic declination. Use Figure 1 to determine which of the following statements is true:

1.

$$\left. \frac{\partial D}{\partial x} \right|_C > 0$$

2.

$$\left. \frac{\partial D}{\partial y} \right|_A < 0$$

3.

$$\left. \frac{\partial D}{\partial y} \right|_A < \left. \frac{\partial D}{\partial y} \right|_B$$

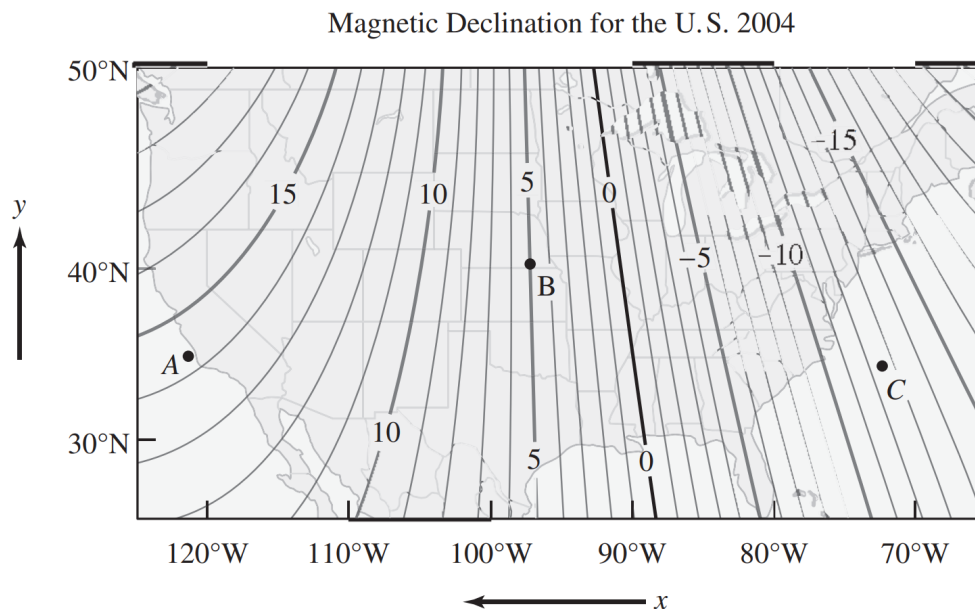


Figure 1: Contour interval 1 degree

Problem 3 - 8m

Find a vector of length 13 that is normal to the surface $-x^3 + yz^2 = -19$ at the point $P(-1, -2, -3)$.

Problem 4 - 12m

For all $x > 0$, there is a unique value $y = \sigma(x)$ that solves the equation $4xy + y^3 - 16 = 0$. Let $g(x) = f(x, \sigma(x))$. Calculate $g_x(1)$, knowing that $f_x(1, 2) = 9$ and $f_y(1, 2) = 10$.

For Problem 5, 6, let $f(x, y, z) = xy + 2z$ be the function that we are interested to optimize. Constraint g is given by the set of points in the Oxyz space that are of distance 6 from the origin.

Problem 5 - 8m

How many critical points are there in this optimization problem?

Problem 6 - 8m

What is the largest value of $f(x, y, z)$ under the given constraints?

Problem 7 - 14m

The number of stay-at-home fathers in Canada in recent years is given below.

Year	1976	1984	1991	2000	2010
Number of stay-at-home fathers	20610	28725	43530	47665	53555

This data can be assumed to follow a linear growth model. The model can be represented by the following formula:

$$f(x) = mx + b$$

where m , b are constants, and x is the year of interest. One method to evaluate the accuracy of such models is to use the best-square-fit method, where the "error" of prediction is determined by

$$E(m, b) = \frac{1}{N} \sum_{i=1}^N (f(x_i) - y_i)^2$$

By this criterion, the smaller the prediction error, the better the model. Determine a model that best fit the given data. Give your answer in the form $y = mx + b$. Round your m and b to 7 significant digits.

Problem 8 - 9m

Parametrize the intersection of the surfaces $y^2 - z^2 = x - 2$ and $y^2 + z^2 = 9$ using trigonometric identities.

Problem 9 - 10m

Evaluate

$$I_9 = \iint_M x \cos y dA$$

where M is bounded by $y = 0, y = x^2, x = 1$.

Problem 10 - 12m

Suppose you are hiking on a terrain modeled by $z = xy + y^3 - x^2$. You are at the point $(2, 1, -1)$. Determine the steepest slope you could encounter from your position, and the compass direction measured in degrees from East that you would head to realize this steepest slope.

Key

Problem	Your answer	Score
1	2	----- / 10
2	1. True; 2. False; 3. False	----- / 9
3	$\langle -3, 4, 12 \rangle$	----- / 8
4	9	----- / 12
5	10	----- / 8
6	20	----- / 8
7	$y=992.3857x-1938214$	----- / 14
8	$\mathbf{r}(t) = \langle 2 + 9 \cos 2t, 3 \cos t, 3 \sin t \rangle$	----- / 9
9	$\frac{1}{2}(1 - \cos 1)$	----- / 10
10	Slope = $\sqrt{34}$ Angle = 2.1112 rad = 120.96 degrees	----- / 12