IMPERIAL

Channel Shaping Using Beyond Diagonal RIS: Analysis, Optimization, and Enhanced Flexibility

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Overview

- **What does this paper investigate?**
  The capability of Diagonal (D)- and Beyond Diagonal (BD)-Reconfigurable Intelligent Surface (RIS) to redistribute the singular values of MIMO channels.
- **How does it differ from previous work?**
  It derives analytical singular value bounds for specific channel conditions. And proposes a novel BD-RIS optimization framework for general problems.
- **What are the benefits?**
  BD-RIS improves the dynamic range of individual channel singular values and the trade-off in manipulating them. This boosts channel power gain and capacity.

BD-RIS model

Consider an $N_\mathrm{T} \times N_\mathrm{S} \times N_\mathrm{R}$ setup with BD-RIS divided into G groups of L elements each. Define $N = \min(N_\mathrm{T}, N_\mathrm{R})$ and $\mathbf{H}_{\mathrm{B/F}} \overset{\text{svd}}{=} \mathbf{U}_{\mathrm{B/F}}\boldsymbol{\Sigma}_{\mathrm{B/F}}\mathbf{V}_{\mathrm{B/F}}^\mathsf{H}$.

$$\boldsymbol{\Theta} = \mathrm{diag}(\boldsymbol{\Theta}_1, \dots, \boldsymbol{\Theta}_G), \quad \boldsymbol{\Theta}_g^\mathsf{H}\boldsymbol{\Theta}_g = \mathbf{I}_L \ \forall g, \quad \mathbf{H} = \mathbf{H}_\mathrm{D} + \overbrace{\sum\nolimits_g \underbrace{\mathbf{H}_{\mathrm{B},g}\boldsymbol{\Theta}_g\mathbf{H}_{\mathrm{F},g}}_{\substack{\text{backward-forward:}\\ \text{intra-group, multiplicative}}}}^{\substack{\text{direct-indirect:}\\ \text{inter-group, additive}}}.$$

- Branch matching: Pairing and combining the entries of $\mathbf{H}_{\mathrm{B},g}$ and $\mathbf{H}_{\mathrm{F},g}$ through unitary transformation $\boldsymbol{\Theta}_g$.
- Mode alignment: Aligning and ordering the singular vectors of $\{\mathbf{H}_g\}$ with those of $\mathbf{H}_\mathrm{D}$ through unitary transformations $\{\boldsymbol{\Theta}_g\}$.

Example 1: SISO channel gain maximization

SISO mode alignment reduces to phase matching and any L (incl. D-RIS) suffices by

$$\boldsymbol{\Theta}_g^{\mathsf{SISO}} = \frac{h_\mathrm{D}}{|h_\mathrm{D}|}\mathbf{V}_{\mathrm{B},g}\mathbf{U}_{\mathrm{F},g}^\mathsf{H} \ \forall g,$$

where $\mathbf{V}_{\mathrm{B},g} = [\mathbf{h}_{\mathrm{B},g}/\|\mathbf{h}_{\mathrm{B},g}\|, \mathbf{N}_{\mathrm{B},g}]$, $\mathbf{U}_{\mathrm{F},g} = [\mathbf{h}_{\mathrm{F},g}/\|\mathbf{h}_{\mathrm{F},g}\|, \mathbf{N}_{\mathrm{F},g}]$, and $\mathbf{N}_{\mathrm{B},g}, \mathbf{N}_{\mathrm{F},g}$ are orthonormal bases of null spaces of $\mathbf{h}_{\mathrm{B},g}, \mathbf{h}_{\mathrm{F},g}$. The channel gain is a function of L

$$\max\nolimits_{\boldsymbol{\Theta}_\mathrm{BD}} |h| = |h_\mathrm{D}| + \sum\nolimits_g \sum\nolimits_l |h_{\mathrm{B},g,\pi_{\mathrm{B},g}(l)}||h_{\mathrm{F},g,\pi_{\mathrm{F},g}(l)}|,$$

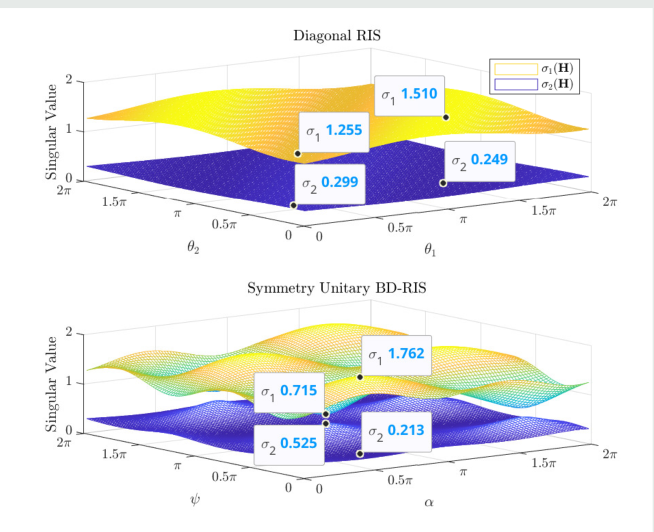
which generalizes $\max_{\boldsymbol{\Theta}_\mathrm{D}} |h| = |h_\mathrm{D}| + \sum_{n=1}^{N_\mathrm{S}} |h_{\mathrm{B},n}||h_{\mathrm{F},n}|$ using permutations $\pi_{\mathrm{B},g}, \pi_{\mathrm{F},g}$ that pair the l-th strongest backward and forward branches.

Example 2: $2 \times 2 \times 2$ shaping

D-RIS and fully-connected BD-RIS can be modeled by 2 and 4 angular parameters:

$$\boldsymbol{\Theta}_\mathrm{D} = \mathrm{diag}(e^{j\theta_1}, e^{j\theta_2}), \quad \boldsymbol{\Theta}_\mathrm{BD} = e^{j\phi}\begin{bmatrix} e^{j\alpha}\cos\psi & e^{j\beta}\sin\psi \\ -e^{-j\beta}\sin\psi & e^{-j\alpha}\cos\psi \end{bmatrix}.$$

Assume the BD-RIS is symmetric (i.e., $\beta = \pi/2$) and the direct channel is negligible (i.e., $\mathrm{sv}(e^{j\phi}\mathbf{A}) = \mathrm{sv}(\mathbf{A})$). For one channel realization, we can reveal channel singular values achieved by D- and BD-RIS by grid search over (θ_1, θ_2) and (α, ψ).



Here, both singular values are manipulated up to $\pm 9\%$ by D-RIS and $\pm 42\%$ by symmetric fully-connected BD-RIS, using 2 and 3 circuit components respectively.

Proposition 1: Degrees of Freedom (DoF)

In point-to-point MIMO, BD-RIS cannot achieve a larger number of DoF than D-RIS.

Proposition 2: Rank-deficient channels

If the minimum rank of backward and forward channels is k, then for D-RIS or BD-RIS of arbitrary number of elements, the n-th channel singular value is bounded by

$$\sigma_n(\mathbf{H}) \le \sigma_{n-k}(\mathbf{T}), \qquad \text{if } n > k,$$
$$\sigma_n(\mathbf{H}) \ge \sigma_n(\mathbf{T}), \qquad \text{if } n < N - k + 1,$$

where $\mathbf{T}$ is arbitrary auxiliary matrix satisfying

$$\mathbf{T}\mathbf{T}^\mathsf{H} = \begin{cases} \mathbf{H}_\mathrm{D}(\mathbf{I} - \mathbf{V}_\mathrm{F}\mathbf{V}_\mathrm{F}^\mathsf{H})\mathbf{H}_\mathrm{D}^\mathsf{H}, & \text{if } \mathrm{rank}(\mathbf{H}_\mathrm{F}) = k, \\ \mathbf{H}_\mathrm{D}^\mathsf{H}(\mathbf{I} - \mathbf{U}_\mathrm{B}\mathbf{U}_\mathrm{B}^\mathsf{H})\mathbf{H}_\mathrm{D}, & \text{if } \mathrm{rank}(\mathbf{H}_\mathrm{B}) = k. \end{cases}$$

Corollary 2.1: Line-of-Sight (LoS) channel

If one of backward and forward channels is LoS, then a D-RIS or BD-RIS can only manipulate the channel singular values up to

$$\sigma_1(\mathbf{H}) \ge \sigma_1(\mathbf{T}) \ge \sigma_2(\mathbf{H}) \ge \ldots \ge \sigma_{N-1}(\mathbf{T}) \ge \sigma_N(\mathbf{H}) \ge \sigma_N(\mathbf{T}).$$

As $N_\mathrm{S} \to \infty$, N out of those $2N$ bounds can be simultaneously tight.

Proposition 3: Negligible direct channel

If the direct channel is negligible, then a fully-connected BD-RIS can manipulate the channel singular values up to

$$\mathrm{sv}(\mathbf{H}) = \mathrm{sv}(\mathbf{B}\mathbf{F}),$$

where $\mathbf{B}$ and $\mathbf{F}$ are arbitrary matrices with $\mathrm{sv}(\mathbf{B}) = \mathrm{sv}(\mathbf{H}_\mathrm{B})$ and $\mathrm{sv}(\mathbf{F}) = \mathrm{sv}(\mathbf{H}_\mathrm{F})$.

Corollary 3.3: Individual singular value

If the direct channel is negligible, then the n-th channel singular value is bounded by

$$\max_{i+j=n+N_\mathrm{S}} \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}) \le \sigma_n(\mathbf{H}) \le \min_{i+j=n+1} \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}),$$

which are attained respectively at

$$\boldsymbol{\Theta}^{\textit{MIMO-ND}}_{\textit{sv-}n\textit{-max}} = \mathbf{V}_\mathrm{B}\mathbf{P}\mathbf{U}_\mathrm{F}^\mathsf{H}, \quad \boldsymbol{\Theta}^{\textit{MIMO-ND}}_{\textit{sv-}n\textit{-min}} = \mathbf{V}_\mathrm{B}\mathbf{Q}\mathbf{U}_\mathrm{F}^\mathsf{H},$$

where $\mathbf{P}, \mathbf{Q}$ are permutation matrices of dimension N_S satisfying:

- *The (i,j)-th entry is 1, where*

$$(i,j) = \begin{cases} \underset{i+j=n+1}{\arg\min}\, \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}) & \text{for } \mathbf{P}, \\ \underset{i+j=n+N_\mathrm{S}}{\arg\max}\, \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}) & \text{for } \mathbf{Q}, \end{cases}$$

  and ties may be broken arbitrarily;
- *After deleting the i-th row and j-th column, the resulting submatrix $\mathbf{Y}$ is arbitrary permutation matrix of dimension $N_\mathrm{S} - 1$ satisfying*

$$\sigma_{n-1}(\hat{\boldsymbol{\Sigma}}_\mathrm{B}\mathbf{Y}\hat{\boldsymbol{\Sigma}}_\mathrm{F}) \ge \min_{i+j=n+1} \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}) \quad \text{for } \mathbf{P},$$
$$\sigma_{n+1}(\hat{\boldsymbol{\Sigma}}_\mathrm{B}\mathbf{Y}\hat{\boldsymbol{\Sigma}}_\mathrm{F}) \le \max_{i+j=n+N_\mathrm{S}} \sigma_i(\mathbf{H}_\mathrm{B})\sigma_j(\mathbf{H}_\mathrm{F}) \quad \text{for } \mathbf{Q},$$

where $\hat{\boldsymbol{\Sigma}}_\mathrm{B}, \hat{\boldsymbol{\Sigma}}_\mathrm{F}$ are $\boldsymbol{\Sigma}_\mathrm{B}, \boldsymbol{\Sigma}_\mathrm{F}$ with both i-th row and j-th column deleted.

Corollary 3.4: Channel power gain

If the direct channel is negligible, then the channel power gain is bounded by

$$\sum\nolimits_{n=1}^N \sigma_n^2(\mathbf{H}_\mathrm{B})\sigma_{N_\mathrm{S}-n+1}^2(\mathbf{H}_\mathrm{F}) \le \|\mathbf{H}\|_\mathrm{F}^2 \le \sum\nolimits_{n=1}^N \sigma_n^2(\mathbf{H}_\mathrm{B})\sigma_n^2(\mathbf{H}_\mathrm{F}),$$

which are attained respectively at

$$\boldsymbol{\Theta}^{\textit{MIMO-ND}}_{\textit{P-max}} = \mathbf{V}_\mathrm{B}\mathbf{U}_\mathrm{F}^\mathsf{H}, \quad \boldsymbol{\Theta}^{\textit{MIMO-ND}}_{\textit{P-min}} = \mathbf{V}_\mathrm{B}\mathbf{J}\mathbf{U}_\mathrm{F}^\mathsf{H},$$

where $\mathbf{J}$ is the backward identity matrix.

Corollary 3.5: Channel capacity at very low and high SNR

If the direct channel is negligible, then the channel capacity at very low and high SNR are approximately bounded from above by

$$C_{\rho_\downarrow} \lessapprox \sigma_1^2(\mathbf{H}_\mathrm{B})\sigma_1^2(\mathbf{H}_\mathrm{F}),$$
$$C_{\rho_\uparrow} \lessapprox N \log\frac{\rho}{N} + 2\log\prod\nolimits_{n=1}^N \sigma_n(\mathbf{H}_\mathrm{B})\sigma_n(\mathbf{H}_\mathrm{F}).$$

Their upper bounds can be attained at, for example, $\boldsymbol{\Theta}^{\textit{MIMO-ND}}_{\textit{P-max}}$.

Algorithm 1: Group-wise geodesic optimization for BD-RIS

For BD-RIS optimization problem of the form

$$\max_{\boldsymbol{\Theta}} \ f(\boldsymbol{\Theta}) \qquad \text{s.t.} \ \ \boldsymbol{\Theta}_g^\mathsf{H}\boldsymbol{\Theta}_g = \mathbf{I}, \ \ \forall g,$$

we propose a geodesic Riemannian Conjugate Gradient (RCG) algorithm below.

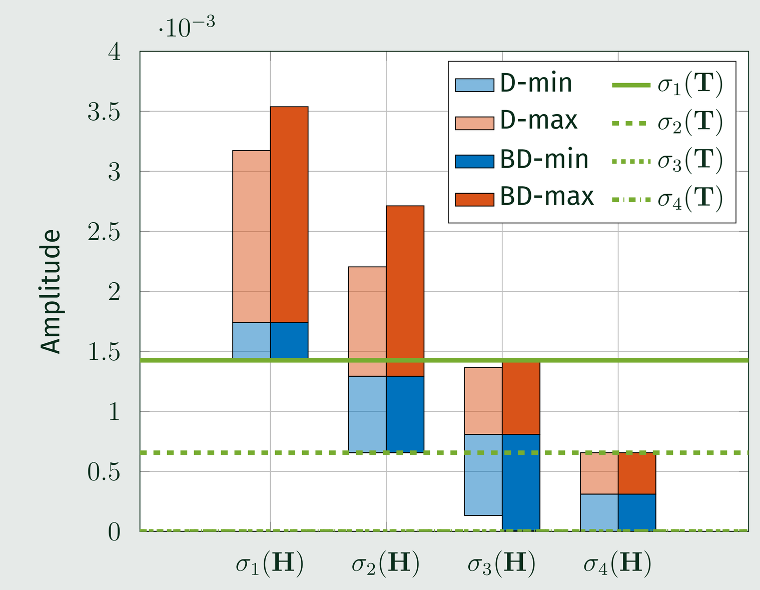
1. *Compute the Euclidean gradient:*
$$\nabla_{\mathrm{E},g}^{(r)} = \frac{\partial f(\boldsymbol{\Theta}_g^{(r)})}{\partial \boldsymbol{\Theta}_g^*};$$
2. *Translate to the Riemannian gradient evaluated at the identity:*
$$\tilde{\nabla}_{\mathrm{R},g}^{(r)} = \nabla_{\mathrm{E},g}^{(r)}\boldsymbol{\Theta}_g^{(r)\mathsf{H}} - \boldsymbol{\Theta}_g^{(r)}\nabla_{\mathrm{E},g}^{(r)\mathsf{H}};$$
3. *Determine the conjugate direction:*
$$\mathbf{D}_g^{(r)} = \tilde{\nabla}_{\mathrm{R},g}^{(r)} + \tilde{\gamma}_g^{(r)}\mathbf{D}_g^{(r-1)}, \quad \tilde{\gamma}_g^{(r)} = \frac{\mathrm{tr}\big((\tilde{\nabla}_{\mathrm{R},g}^{(r)} - \tilde{\nabla}_{\mathrm{R},g}^{(r-1)})\tilde{\nabla}_{\mathrm{R},g}^{(r)\mathsf{H}}\big)}{\mathrm{tr}\big(\tilde{\nabla}_{\mathrm{R},g}^{(r-1)}\tilde{\nabla}_{\mathrm{R},g}^{(r-1)\mathsf{H}}\big)};$$
4. *Perform multiplicative update along geodesic:*
$$\boldsymbol{\Theta}_g^{(r+1)} = \mathbf{G}_g^{(r)}(\mu) = \exp(\mu\mathbf{D}_g^{(r)})\boldsymbol{\Theta}_g^{(r)},$$

where μ is refinable by the Armijo rule. To double the step size, one can simply square the rotation matrix instead of recomputing the matrix exponential.

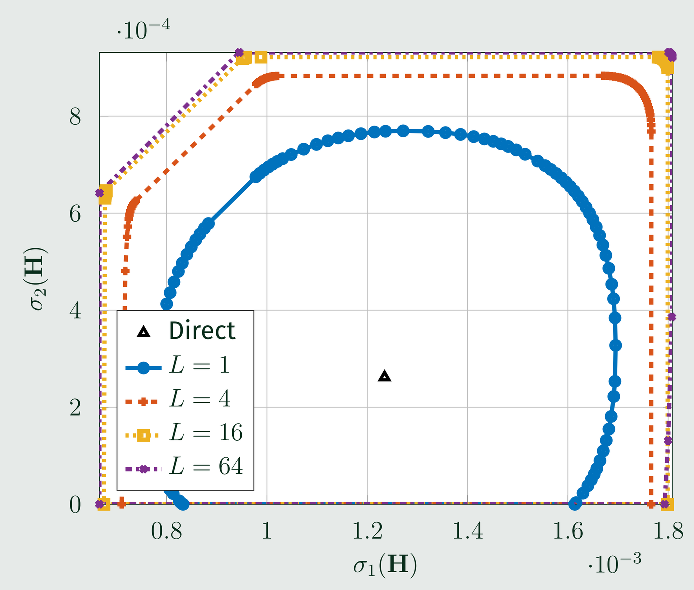
Result 1: Algorithm evaluation

RCG path	$N_\mathrm{S} = 16$			$N_\mathrm{S} = 256$		
	Objective	Iterations	Time [s]	Objective	Iterations	Time [s]
Geodesic	4.359×10^{-3}	11.59	1.839×10^{-2}	1.163×10^{-2}	25.58	3.461
Non-geodesic	4.329×10^{-3}	30.92	5.743×10^{-2}	1.116×10^{-2}	61.40	13.50

Result 2: Bounds and Pareto frontier of singular values

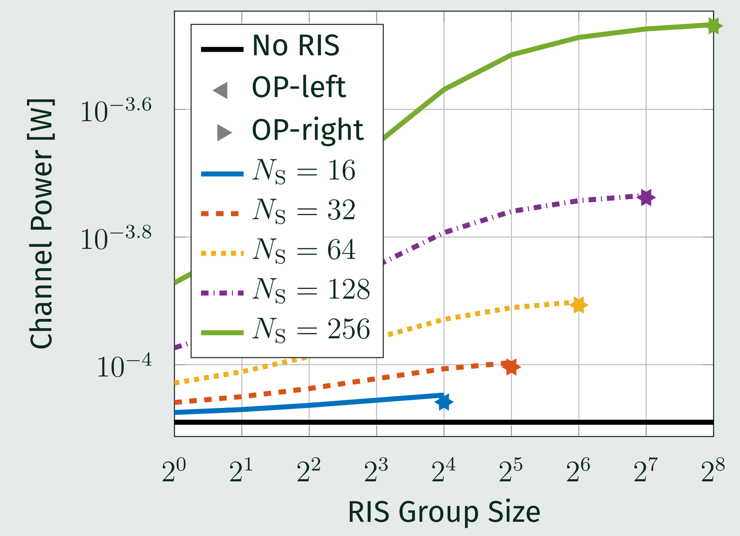


(a) Analytical vs numerical results: $4 \times 128 \times 4$, $\mathrm{rank}(\mathbf{H}_\mathrm{F}) = 2$

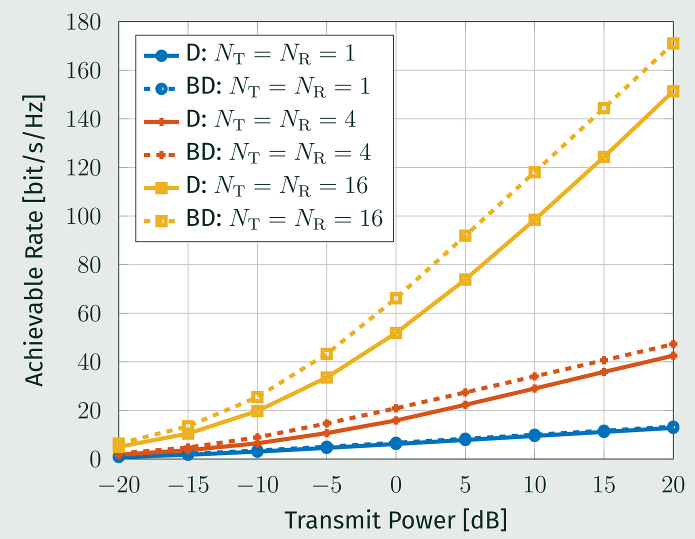


(b) Pareto frontier: $2 \times 64 \times 2$

Result 3: Channel power gain and achievable rate



(c) Channel power gain: $16 \times N_\mathrm{S} \times 16$



(d) Achievable rate: $N_\mathrm{T} \times 128 \times N_\mathrm{R}$

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