Channel Shaping Using Reconfigurable Intelligent Surfaces: From Diagonal to Beyond

Yang Zhao, *Member, IEEE,* Hongyu Li, *Graduate Student Member, IEEE,*
Yijie Mao, *Member, IEEE,* Shanpu Shen, *Member, IEEE,* and Bruno Clerckx, *Fellow, IEEE*

I. Assumption

All proposals in this paper based on assumption of *asymmetric* passive Beyond-Diagonal (BD) Reconfigurable Intelligent Surface (RIS), i.e., symmetry constraint $\mathbf{\Theta}_g=\mathbf{\Theta}_g^\mathsf{T}$ is relaxed. This is feasible when asymmetric passive components (e.g., ring hybrids and branch-line hybrids) [1] are available. This assumption was also made in Hongyu's papers [2], [3]. For quadratic problems, the proposed algorithms may be extended to symmetric BD RIS by replacing singular value decomposition with Takagi factorization [4].

II. Point-to-Point MIMO

A. Channel Power Maximization

Consider a BD RIS with N_s elements, which is divided into G groups of equal L elements.

$$\max_{\mathbf{\Theta}} \quad \left\|\mathbf{H}^\mathrm{D}+\sum\nolimits_g \mathbf{H}_g^\mathrm{B}\mathbf{\Theta}_g\mathbf{H}_g^\mathrm{F}\right\|_\mathrm{F}^2 \tag{1a}$$

$$\text{s.t.} \quad \mathbf{\Theta}_g^\mathsf{H}\mathbf{\Theta}_g=\mathbf{I}_L, \quad \forall g\in\mathcal{G}\triangleq\{1,...,G\} \tag{1b}$$

For *symmetric* BD-RIS, the problem has been solved in

- Matteo's paper [5]: SISO and equivalent[1];
- Ignacio's paper [6]: SISO and directless MISO/SIMO.

Remark 1. *The difficulty of* (1) *is that the RIS needs to balance the additive (direct-indirect) and multiplicative (backward-forward) eigenspace alignment. Interestingly, it has the same form as the* weighted orthogonal Procrustes problem *[7]:*

$$\min_{\mathbf{\Theta}} \quad \|\mathbf{C}-\mathbf{A}\mathbf{\Theta}\mathbf{B}\|_F^2 \tag{2a}$$

$$\text{s.t.} \quad \mathbf{\Theta}^\mathsf{H}\mathbf{\Theta}=\mathbf{I} \tag{2b}$$

There exists no trivial solution to (2)*. One lossy transformation, by moving* $\mathbf{\Theta}$ *to one side [8], formulates a standard orthogonal Procrustes problem:*

$$\min_{\mathbf{\Theta}} \quad \|\mathbf{A}^\dagger\mathbf{C}-\mathbf{\Theta}\mathbf{B}\|_F^2 \tag{3a}$$

$$\text{s.t.} \quad \mathbf{\Theta}^\mathsf{H}\mathbf{\Theta}=\mathbf{I} \tag{3b}$$

(3) *has a global optimal solution* $\mathbf{\Theta}^\star=\mathbf{U}\mathbf{V}^\mathsf{H}$*, where* $\mathbf{U}$ *and* $\mathbf{V}$ *are left and right singular matrix of* $\mathbf{A}^\dagger\mathbf{C}\mathbf{B}^\mathsf{H}$ *[9]. This low-complexity solution will be compared with the one proposed later.*

Inspired by [10], we propose an iterative algorithm to solve (1). The idea is to successively approximate the quadratic objective with a sequence of affine functions and solve the resulting subproblems in closed form.

[1] Single-stream MIMO with given precoder and combiner.

Proposition 1. *Start from any* $\mathbf{\Theta}^{(0)}$*, the sequence*

$$\mathbf{\Theta}_g^{(r+1)}=\mathbf{U}_g^{(r)}\mathbf{V}_g^{(r)}, \quad \forall g\in\mathcal{G} \tag{4}$$

converges to a stationary point of (1)*, where* $\mathbf{U}_g^{(r)}$ *and* $\mathbf{V}_g^{(r)}$ *are left and right singular matrix of*

$$\begin{aligned}\mathbf{M}_g^{(r)}=&\,{\mathbf{H}_g^\mathrm{B}}^\mathsf{H}\mathbf{H}^\mathrm{D}{\mathbf{H}_g^\mathrm{F}}^\mathsf{H}+\sum_{g'<g}{\mathbf{H}_{g'}^\mathrm{B}}^\mathsf{H}\mathbf{H}_{g'}^\mathrm{B}\mathbf{\Theta}_{g'}^{(r+1)}\mathbf{H}_{g'}^\mathrm{F}{\mathbf{H}_{g'}^\mathrm{F}}^\mathsf{H}\\&+\sum_{g'\geq g}{\mathbf{H}_{g'}^\mathrm{B}}^\mathsf{H}\mathbf{H}_{g'}^\mathrm{B}\mathbf{\Theta}_{g'}^{(r)}\mathbf{H}_{g'}^\mathrm{F}{\mathbf{H}_{g'}^\mathrm{F}}^\mathsf{H}.\end{aligned} \tag{5}$$

Proof. TODO □

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