

# Channel Shaping Using Reconfigurable Intelligent Surfaces: From Diagonal to Beyond

Yang Zhao, *Member, IEEE*, Hongyu Li, *Graduate Student Member, IEEE*,  
Yijie Mao, *Member, IEEE*, Shanpu Shen, *Member, IEEE*, and Bruno Clerckx, *Fellow, IEEE*

## I. ASSUMPTION

We introduce Beyond-Diagonal (BD) Reconfigurable Intelligent Surface (RIS) in Multiple-Input Multiple-Output (MIMO) Point-to-point Channel (PC) and Interference Channel (IC). All proposals are based on assumption of *asymmetric* passive BD RIS, i.e., symmetry constraint  $\Theta_g = \Theta_g^T$  is relaxed. This is feasible when asymmetric passive components (e.g., ring hybrids and branch-line hybrids) [1] are available. This assumption was also made in Hongyu's papers [2], [3]. For quadratic problems, the proposed algorithms may be extended to symmetric BD RIS by replacing singular value decomposition with Takagi factorization [4].

## II. MIMO-PC

### A. Channel Power Maximization

Consider a BD RIS with  $N^S$  elements, which is divided into  $G$  groups of equal  $L$  elements.

$$\max_{\Theta} \left\| \mathbf{H}^D + \sum_g \mathbf{H}_g^B \Theta_g \mathbf{H}_g^F \right\|_F^2 \quad (1a)$$

$$\text{s.t.} \quad \Theta_g^H \Theta_g = \mathbf{I}, \quad \forall g \in \mathcal{G} \triangleq \{1, \dots, G\}. \quad (1b)$$

For *symmetric* BD-RIS, the problem has been solved in

- Matteo's paper [5]: SISO and equivalent<sup>1</sup>;
- Ignacio's paper [6]: SISO and directless MISO/SIMO.

**Remark 1.** The difficulty of (1) is that the RIS needs to balance the additive (direct-indirect) and multiplicative (forward-backward) eigenspace alignment. Interestingly, it has the same form as the weighted orthogonal Procrustes problem [7]:

$$\min_{\Theta} \left\| \mathbf{C} - \mathbf{A} \Theta \mathbf{B} \right\|_F^2 \quad (2a)$$

$$\text{s.t.} \quad \Theta^H \Theta = \mathbf{I}. \quad (2b)$$

There exists no trivial solution to (2). One lossy transformation, by moving  $\Theta$  to one side [8], formulates a standard orthogonal Procrustes problem:

$$\min_{\Theta} \left\| \mathbf{A}^\dagger \mathbf{C} - \Theta \mathbf{B} \right\|_F^2 \quad (3a)$$

$$\text{s.t.} \quad \Theta^H \Theta = \mathbf{I}. \quad (3b)$$

(3) has a global optimal solution  $\Theta^* = \mathbf{U} \mathbf{V}^H$ , where  $\mathbf{U}$  and  $\mathbf{V}$  are left and right singular matrix of  $\mathbf{A}^\dagger \mathbf{C} \mathbf{B}^H$  [9]. This low-complexity solution will be compared with the one proposed later.

<sup>1</sup>Single-stream MIMO with given precoder and combiner.

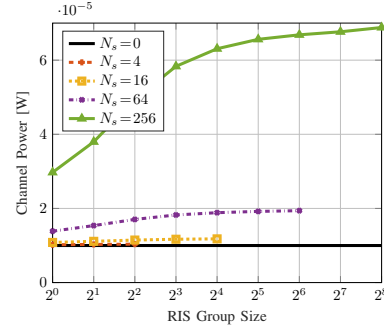


Fig. 1. Average channel power versus RIS elements  $N^S$  and group size  $L$  for  $(N^T, N^R) = (8, 4)$ ,  $(\Lambda^D, \Lambda^F, \Lambda^B) = (65, 54, 46)$  dB.

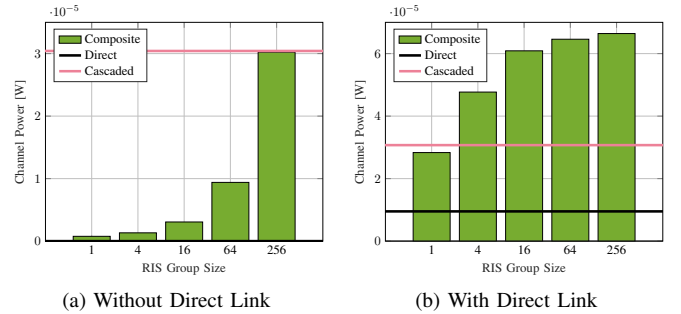


Fig. 2. Average channel power versus RIS group size  $L$  for  $(N^T, N^S, N^R) = (8, 256, 4)$ ,  $(\Lambda^D, \Lambda^F, \Lambda^B) = (65, 54, 46)$  dB.

Inspired by [10], we propose an iterative algorithm to solve (1). The idea is to successively approximate the quadratic objective with a sequence of affine functions and solve the resulting subproblems in closed form.

**Proposition 1.** Start from any  $\Theta^{(0)}$ , the sequence

$$\Theta_g^{(r+1)} = \mathbf{U}_g^{(r)} \mathbf{V}_g^{(r)}, \quad \forall g \quad (4)$$

converges to a stationary point of (1), where  $\mathbf{U}_g^{(r)}$  and  $\mathbf{V}_g^{(r)}$  are left and right singular matrix of

$$\begin{aligned} \mathbf{M}_g^{(r)} = & \mathbf{H}_g^B \mathbf{H}^D \mathbf{H}_g^H + \sum_{g' < g} \mathbf{H}_{g'}^B \mathbf{H}_{g'}^H \Theta_{g'}^{(r+1)} \mathbf{H}_{g'}^F \mathbf{H}_{g'}^H \\ & + \sum_{g' \geq g} \mathbf{H}_{g'}^B \mathbf{H}_{g'}^H \Theta_{g'}^{(r)} \mathbf{H}_{g'}^F \mathbf{H}_{g'}^H. \end{aligned} \quad (5)$$

*Proof.* To be added.  $\square$

Fig. 1 shows that, apart from adding reflecting elements  $N^S$ , increasing the group size  $L$  also improves the channel

power. This behavior is more pronounced for a large RIS. For example, the gain of pairwise connection is 2.8% for  $N^S = 16$  and 28% for  $N^S = 256$ . It implies that the channel shaping capability of BD RIS scales with group size  $L$ .

Fig. 2b and 2a compare the average channel power without and with direct link. ‘‘Cascaded’’ means the *power product* of the forward and backward channels. We observe that diagonal RIS wastes substantial cascaded power and struggles to align the direct-indirect eigenspace. When the direct link is absent, only 2.6% of available power is utilized by diagonal RIS while 100% power is recycled by fully-connected RIS. When the direct link is present, the proposed BD RIS design can balance the direct-indirect and forward-backward eigenspace alignment for an optimal channel boost. It is worth noting that, when  $L$  is sufficiently large, the composite channel power surpasses the power sum of direct and cascaded channels, thanks to the constructive *amplitude superposition* of direct and cascaded channels. This again emphasizes the advantage of in-group connection of BD RIS.

### B. Rate Maximization

The problem is formulated w.r.t. precoder (instead of transmit covariance matrix) for reference:

$$\max_{\mathbf{W}, \Theta} \log \det \left( \mathbf{I} + \frac{\mathbf{W}^H \mathbf{H}^H \mathbf{H} \mathbf{W}}{\sigma_n^2} \right) \quad (6a)$$

$$\text{s.t.} \quad \|\mathbf{W}\|_F^2 \leq P, \quad (6b)$$

$$\Theta_g^H \Theta_g = \mathbf{I}, \quad \forall g. \quad (6c)$$

(6) is jointly non-convex and solved by Alternating Optimization (AO). For a given  $\Theta$ , the optimal precoder is given by

$$\mathbf{W}^* = \mathbf{V} \mathbf{S}^{*1/2}, \quad (7)$$

where  $\mathbf{V}$  is right singular matrix of  $\mathbf{H}$  and  $\mathbf{S}^*$  is a diagonal matrix of the water-filling power allocation. For a given  $\mathbf{W}$ , we update  $\Theta$  by Riemannian Conjugate Gradient (RCG) method along the geodesics [11].

**Remark 2.** A geodesic refers to the shortest path between two points in a Riemannian manifold. Unitary constraint (6c) translates to a Stiefel manifold where the geodesics have simple expressions described by the exponential map [12].

For general optimization problems with block unitary constraint, the adapted RCG method at iteration  $r$  for block  $g$  is summarized below, where  $f^{(r)}$  is evaluated over  $\{\{\Theta_{g'}^{(r+1)}\}_{g'=1}^{g-1}, \{\Theta_{g'}^{(r)}\}_{g'=g}^G\}$ .

1) Compute the Euclidean gradient

$$\nabla_g^E f^{(r)} = \frac{\partial f^{(r)}}{\partial \Theta_g^*}; \quad (8)$$

2) Translate to the Riemannian gradient

$$\nabla_g^R f^{(r)} = \nabla_g^E f^{(r)} \Theta_g^{(r)H} - \Theta_g^{(r)} \nabla_g^E f^{(r)H}; \quad (9)$$

3) Determine the weight factor

$$\gamma_g^{(r)} = \frac{\text{tr} \left( (\nabla_g^R f^{(r)} - \nabla_g^R f^{(r-1)}) \nabla_g^R f^{(r)H} \right)}{\text{tr} \left( \nabla_g^R f^{(r-1)} \nabla_g^R f^{(r-1)H} \right)}; \quad (10)$$

### Algorithm 1: RCG Method for RIS MIMO-PC Rate Maximization

**Input:**  $\mathbf{H}^D, \mathbf{H}^F, \mathbf{H}^B, \mathbf{W}, L, \sigma_n^2$

**Output:**  $\Theta^*$

```

1:  $r \leftarrow 0, \Theta^{(0)}$ 
2: Repeat
3:    $r \leftarrow r + 1$ 
4:   For  $g \leftarrow 1$  to  $G$ 
5:      $\Theta_g^{(r)} \leftarrow (14), (9)-(13)$ 
6:   End For
7: Until  $|R^{(r)} - R^{(r-1)}|/R^{(r-1)} \leq \epsilon$ 

```

4) Compute the conjugate direction

$$\mathbf{D}_g^{(r)} = \nabla_g^R f^{(r)} + \gamma_g^{(r)} \mathbf{D}_g^{(r-1)}; \quad (11)$$

5) Determine the Armijo step size<sup>2</sup>

$$\mu_g^{(r)} = \arg \max_{\mu_g} f^{(r)} \left( \exp(\mu_g \mathbf{D}_g^{(r)}) \Theta_g^{(r)} \right); \quad (12)$$

6) Perform rotational update along local geodesics

$$\Theta_g^{(r+1)} = \exp \left( \mu_g^{(r)} \mathbf{D}_g^{(r)} \right) \Theta_g^{(r)}. \quad (13)$$

**Remark 3.** The adapted RCG method leverages the fact that block unitary matrices are closed under multiplication (but not necessarily under addition). Its advantage over universal manifold optimization [13], [14] is trifold:

- No retraction is involved;
- Lower computational complexity per iteration [12];
- Faster convergence thanks appropriate operational space.

The complex derivative of (6a) w.r.t. RIS block  $g$  is

$$\frac{\partial R}{\partial \Theta_g^*} = \frac{1}{\sigma_n^2} \mathbf{H}_g^B \mathbf{H}_g^H \mathbf{W} \left( \mathbf{I} + \frac{\mathbf{W}^H \mathbf{H}^H \mathbf{H} \mathbf{W}}{\sigma_n^2} \right)^{-1} \mathbf{W}^H \mathbf{H}_g^F \mathbf{H}. \quad (14)$$

The adapted RCG method for the RIS rate maximization subproblem is summarized in Algorithm 1.

### REFERENCES

- [1] H.-R. Ahn, *Asymmetric Passive Components in Microwave Integrated Circuits*. Wiley, 2006. [Online]. Available: <https://books.google.co.uk/books?id=X6WdLbOuSNQC>
- [2] H. Li, S. Shen, and B. Clerckx, ‘‘Beyond diagonal reconfigurable intelligent surfaces: From transmitting and reflecting modes to single-, group-, and fully-connected architectures,’’ *IEEE Transactions on Wireless Communications*, vol. 22, pp. 2311–2324, 4 2023.
- [3] —, ‘‘Beyond diagonal reconfigurable intelligent surfaces: A multi-sector mode enabling highly directional full-space wireless coverage,’’ *IEEE Journal on Selected Areas in Communications*, vol. 41, pp. 2446–2460, 8 2023.
- [4] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 2012. [Online]. Available: <https://books.google.co.uk/books?id=O7sgAwAAQBAJ>
- [5] M. Nerini, S. Shen, and B. Clerckx, ‘‘Closed-form global optimization of beyond diagonal reconfigurable intelligent surfaces,’’ *IEEE Transactions on Wireless Communications*, pp. 1–1, 2023. [Online]. Available: <https://ieeexplore.ieee.org/document/10155675/>
- [6] I. Santamaria, M. Soleymani, E. Jorswieck, and J. Guti  rrez, ‘‘Snr maximization in beyond diagonal ris-assisted single and multiple antenna links,’’ *IEEE Signal Processing Letters*, vol. 30, pp. 923–926, 2023. [Online]. Available: <https://ieeexplore.ieee.org/document/10187688/>
- [7] J. C. Gower and G. B. Dijkstra, *Procrustes Problems*. OUP Oxford, 2004. [Online]. Available: <https://books.google.co.uk/books?id=kRRREAAQBAJ>

<sup>2</sup>To double the step size, simply square the argument instead of recomputing the matrix exponential, i.e.,  $\exp(2\mu_g \mathbf{D}_g) = \exp^2(\mu_g \mathbf{D}_g)$ .

- [8] T. Bell, "Global positioning system-based attitude determination and the orthogonal procrustes problem," *Journal of Guidance, Control, and Dynamics*, vol. 26, pp. 820–822, 9 2003. [Online]. Available: <https://arc.aiaa.org/doi/10.2514/2.5117>
- [9] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Johns Hopkins University Press, 2013. [Online]. Available: <https://jhupbooks.press.jhu.edu/title/matrix-computations>
- [10] F. Nie, R. Zhang, and X. Li, "A generalized power iteration method for solving quadratic problem on the stiefel manifold," *Science China Information Sciences*, vol. 60, p. 112101, 11 2017. [Online]. Available: <http://link.springer.com/10.1007/s11432-016-9021-9>
- [11] T. Abrudan, J. Eriksson, and V. Koivunen, "Conjugate gradient algorithm for optimization under unitary matrix constraint," *Signal Processing*, vol. 89, pp. 1704–1714, 9 2009. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0165168409000814>
- [12] T. E. Abrudan, J. Eriksson, and V. Koivunen, "Steepest descent algorithms for optimization under unitary matrix constraint," *IEEE Transactions on Signal Processing*, vol. 56, pp. 1134–1147, 3 2008. [Online]. Available: <http://ieeexplore.ieee.org/document/4436033/>
- [13] P.-A. Absil, R. Mahony, and R. Sepulchre, *Optimization Algorithms on Matrix Manifolds*. Princeton University Press, 2009. [Online]. Available: <https://books.google.co.uk/books?id=NSQGQeLN3NcC>
- [14] C. Pan, G. Zhou, K. Zhi, S. Hong, T. Wu, Y. Pan, H. Ren, M. D. Renzo, A. L. Swindlehurst, R. Zhang, and A. Y. Zhang, "An overview of signal processing techniques for ris/firs-aided wireless systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 16, pp. 883–917, 8 2022. [Online]. Available: <https://ieeexplore.ieee.org/document/9847080/>