# EE3-27: Principles of Classical and Modern Radar

# Radar Fundamentals

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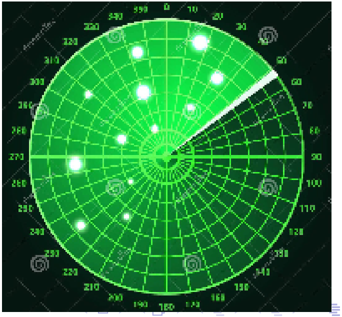
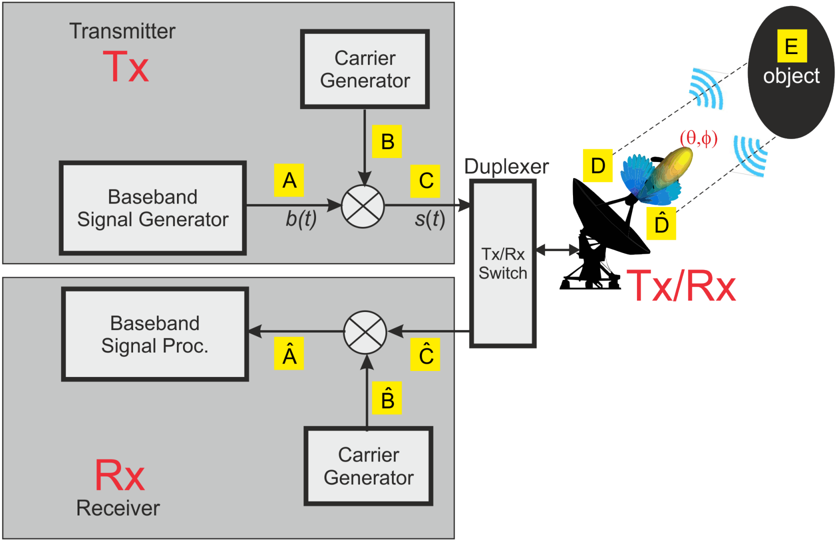
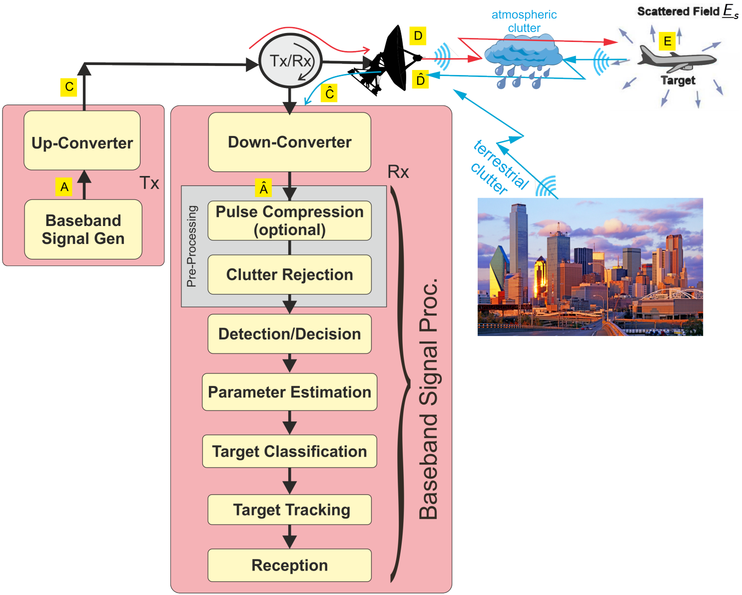


# Introduction



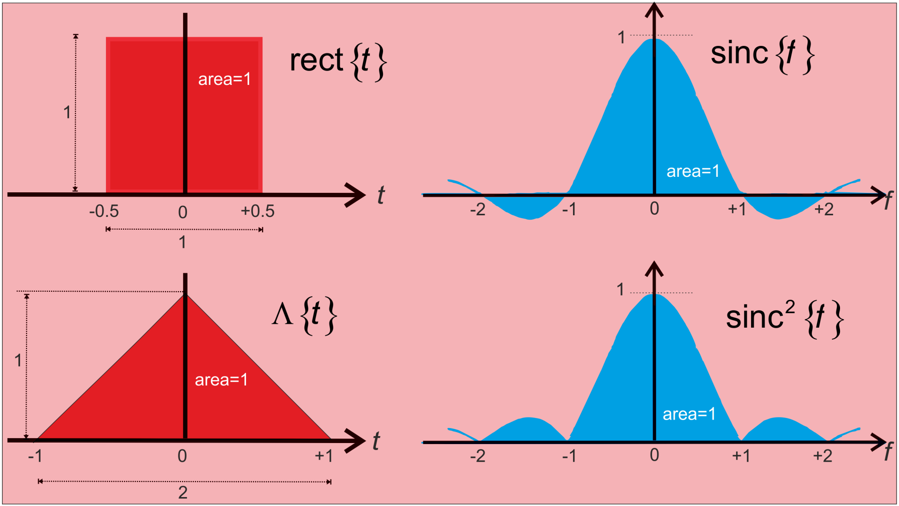
# Equivalent Diagram:



In this topic we will

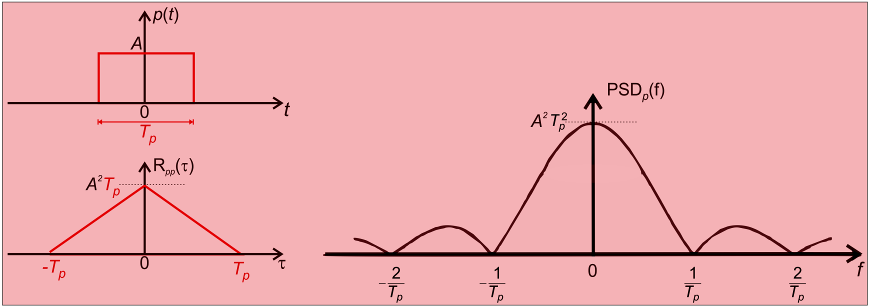
- focus on the "canonical form of a radar", i.e. pulse radar, where at Points A and Â of the previous diagrams, we have "pulses". Thus, initially in this topic, we have to summarise the basics of "pulses" and their "spectrum".
- study the fundamentals of both
  - uncompressed pulses, and
  - compressed pulses
- define various other fundamental radar parameters and concepts.

The following definitions[1] are very well known




[1] to be remembered for ever!

# Single Rectangular Pulse




- Pulse $p(t)$:

$$p(t) \triangleq A.\text{rect}\left(\frac{t}{T_p}\right) \tag{1}$$

- Autocorrelation function $\mathrm{R}_{pp}(\tau)$ of $p(t)$:

$$\mathrm{R}_{pp}(\tau) = A^2.T_p.\Lambda\left\{\frac{\tau}{T_p}\right\} \tag{2}$$

- Power Spectral Density $\mathrm{PSD}_p(f)$ of $p(t)$:

$$\begin{aligned}\mathrm{PSD}_p(f) &= \left|\mathrm{FT}\left[p(t)\right]\right|^2 & (3)\\ &= \left|A.T_p.\mathrm{sinc}\left\{fT_p\right\}\right|^2 \\ &= A^2.T_p^2.\mathrm{sinc}^2\left\{fT_p\right\} & (4)\end{aligned}$$

i.e.

$$\boxed{\mathrm{PSD}_p(f) = A^2.T_p^2.\mathrm{sinc}^2\left\{fT_p\right\}} = A^2.T_p^2.\mathrm{sinc}^2\left\{\frac{f}{1/T_p}\right\} \quad (5)$$

- or, alternatively (using Wiener Khinchin Theorem):

$$\begin{aligned}\mathrm{PSD}_p(f) &= \mathrm{FT}\left[\mathrm{R}_{pp}(\tau)\right] & (6)\\ &= \mathrm{FT}\left[A^2.T_p.\Lambda\left\{\frac{\tau}{T_p}\right\}\right] \\ &= A^2.T_p.\mathrm{FT}\left[\Lambda\left\{\frac{\tau}{T_p}\right\}\right] = A^2.T_p.T_p.\mathrm{sinc}^2\left\{fT_p\right\}\end{aligned}$$

i.e. we get again Equation 5

$$\boxed{\mathrm{PSD}_p(f) = A^2.T_p^2.\mathrm{sinc}^2\left\{fT_p\right\}} = A^2.T_p^2.\mathrm{sinc}^2\left\{\frac{f}{1/T_p}\right\} \quad (7)$$

- However, a radar does not simply transmit a single pulse $p(t)$ of duration $T_p$, i.e.

$$p(t) = \text{rect}\left\{\frac{t}{T_p}\right\} \tag{8}$$

but a pulse train $b(t)$ with pulse-repetition-interval $PRI$. That is

$$\boxed{b(t) = A.\text{rep}_{PRI}\left\{p(t)\right\} = A.\text{rep}_{PRI}\left\{\text{rect}\left\{\frac{t}{T_p}\right\}\right\}} \tag{9}$$

- Remember that $PRI$ is
  - the inverse of the pulse repetition frequency (pulses per second), $PRF$. That is,

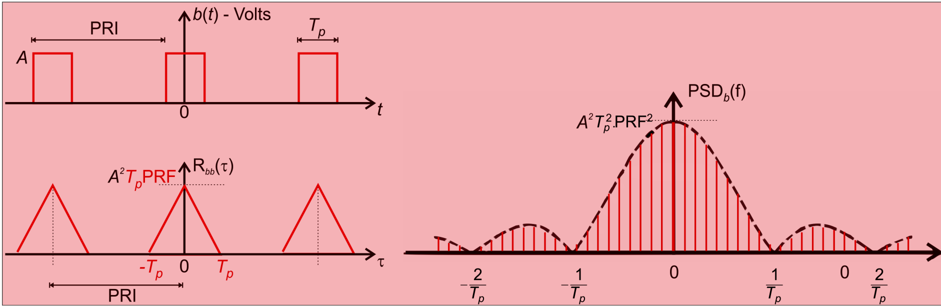
$$\boxed{PRI = \frac{1}{PRF}} \tag{10}$$

  - the time of entire cycle. That is

$$\boxed{PRI = [\text{time of entire cycle}] = [\text{Tx-time}] + [\text{Tx-rest-time}]} \tag{11}$$

# Spectrum of Pulse-Radar Tx Signal

At point-A




$$b(t) = A.\text{rep}_{PRI}\left\{\text{rect}\left(\frac{t}{T_p}\right)\right\} \quad (12)$$

$$\text{FT}\{b(t)\} = A.T_p.PRF.\text{comb}_{PRF}\{\text{sinc}(fT_p)\} \quad (13)$$

$$\text{PSD}_b(f) = |\text{FT}\{b(t)\}|^2 = A^2.T_p^2.PRF^2.\text{comb}_{PRF}\{\text{sinc}^2(fT_p)\} \quad (14)$$

- or, alternatively (using Wiener Khinchin Theorem):

$$b(t) = A.\mathrm{rep}_{PRI}\left\{\mathrm{rect}\left(\frac{t}{T_p}\right)\right\} \tag{15}$$

$$\Downarrow$$

$$\mathrm{R}_{bb}(\tau) = A^2.T_p.PRF.\mathrm{rep}_{PRI}\left\{\Lambda\left\{\frac{\tau}{T_p}\right\}\right\} \tag{16}$$

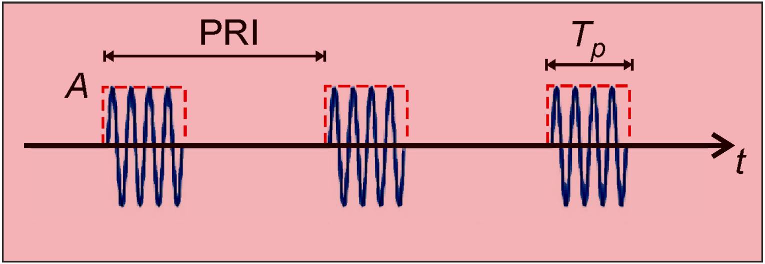
$$\Downarrow$$

$$\begin{aligned} \mathrm{PSD}_b(f) &= \mathrm{FT}\left[\mathrm{R}_{bb}(\tau)\right] \qquad (17)\\ &= \mathrm{FT}\left[A^2.T_p.PRF.\mathrm{rep}_{PRI}\left\{\Lambda\left\{\frac{\tau}{T_p}\right\}\right\}\right]\\ &= A^2.T_p.PRF.\mathrm{FT}\left[\mathrm{rep}_{PRI}\left\{\Lambda\left\{\frac{\tau}{T_p}\right\}\right\}\right]\\ &= A^2.T_p.PRF.PRF.T_p.\mathrm{comb}_{PRF}\left\{\mathrm{sinc}^2\left\{fT_p\right\}\right\}\\ &= A^2.T_p^2.PRF^2.\mathrm{comb}_{PRF}\left\{\mathrm{sinc}^2\left\{fT_p\right\}\right\} \qquad (18) \end{aligned}$$

i.e.

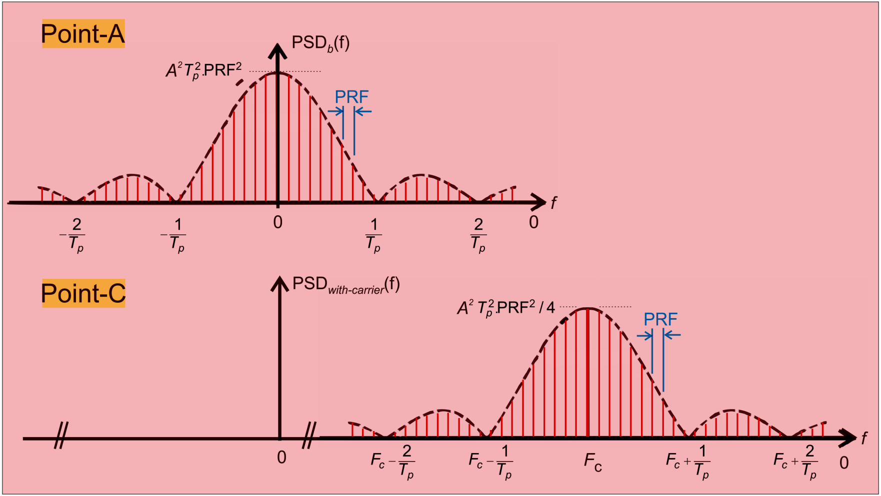
$$\boxed{\mathrm{PSD}_b(f) = \mathrm{FT}\left[\mathrm{R}_{bb}(\tau)\right] = A^2.T_p^2.PRF^2.\mathrm{comb}_{PRF}\left\{\mathrm{sinc}^2\left\{fT_p\right\}\right\}}$$

at point-C:




$$
\begin{aligned}
s(t) &= b(t).\cos(2\pi F_c t) \\
&= A.\text{rep}_{PRI}\left\{\text{rect}\left(\frac{t}{T_p}\right)\right\}.\cos(2\pi F_c t) \qquad (19)\\
\text{FT}\{s(t)\} &= \frac{1}{2}A.T_p.PRF.\text{comb}_{PRF}\left\{\text{sinc}\left(T_p\left(f-F_c\right)\right)+\text{sinc}\left(T_p\left(f+F_c\right)\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{PSD}_s(f) = |\text{FT}\{s(t)\}|^2 \\
&= \frac{A^2.T_p^2.PRF^2}{4}.\text{comb}_{PRF}\left\{\text{sinc}^2\left(T_p\left(f-F_c\right)\right)+\text{sinc}^2\left(T_p\left(f+F_c\right)\right)\right\}
\end{aligned} \qquad (20)
$$




- Bandwidth at Point-C (centered at $F_c$):

$$B = 1/T_p \qquad (21)$$

- Doppler Bandwidth (centered at $F_c$):

$$B_{Dop} = PRF \qquad (22)$$

# Radar Bandwidth

- In this course the term "bandwidth" will refer to the "Nyquist Bandwidth", i.e. the minimum bandwidth (see Equation 21). For various bandwidth definitions please see Appendix-A.

- It is clear from the previous slide that, in the frequency domain, the Tx and Rx signals (i.e. the signals at Points C and Ĉ) have spectral components centered on the radar's carrier frequency $F_c$ and a $\mathrm{PSD}(f)$ of $\mathrm{sinc}^2(.)$.shape.

- Indeed the minimum limits of frequency response is $F_c \pm \frac{1}{2T_p}$, and therefore the bandwidth should be

$$B \geq \frac{1}{T_p}$$

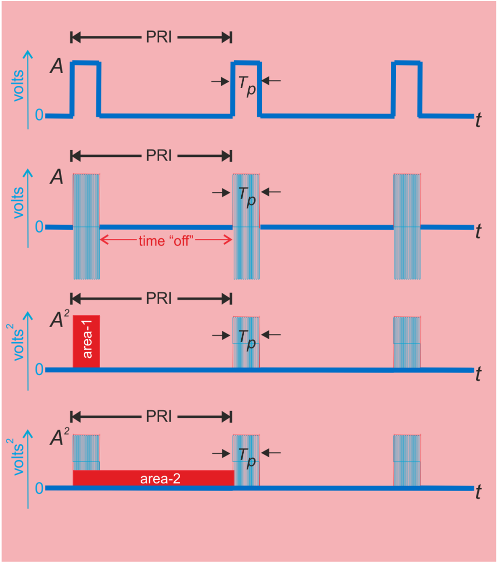
- The Doppler Bandwidth $B_{Dop}$ given by Equation 22 will be discussed later on in this topic.

# Pulse Radar Fundamental Parameters

Some fundamental radar parameters are as follows:

- peak Tx power
- average Tx power
- radar duty-cycle
- maximum detection range
- range bins
- range resolution
- range ambiguity and unambiguous range
- various Doppler shift parameters

# Tx-Power




Tx-Energy: area-1 = area-2

- Peak power (sometimes known as max power) transmitted:
  - It is the signal power of a single pulse:

$$P_{Tx,peak} = \frac{A^2 T_p}{T_p} \qquad (23)$$

i.e.

$$\boxed{P_{Tx,peak} = A^2} \qquad (24)$$

  - It affects the max range $R_{max}$ of radar.
- Average power transmitted (or simply Tx-power):

$$\boxed{P_{Tx} = \frac{A^2 T_p}{PRI}} \qquad (25)$$

# Tx-Power (cont.)

- Since a pulse radar only transmits for a small portion of the time, the average power of the radar is quite low. This is clear by rewriting Equation 25 as follows:

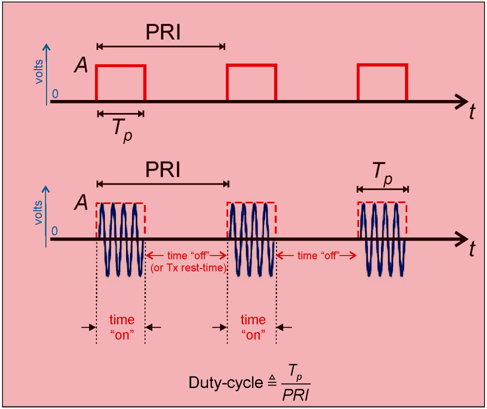
$$P_{Tx} = P_{Tx,peak} \cdot \frac{T_p}{PRI} \tag{26}$$

- Note: $T_p << PRI$

## Example

- Consider a pulse radar with a 1 $\mu$sec pulse width ($T_p$) and a PRF of 4 kHz. If the radar transmits at a peak power of 10kW then the average Tx power is only 40W. Indeed:

$$\begin{aligned} P_{Tx} = P_{Tx,peak} \cdot \frac{T_p}{PRI} &= P_{Tx,peak} . T_p . PRF \\ &= 10k \times 1\mu \times 4k \\ &= 40W \end{aligned}$$

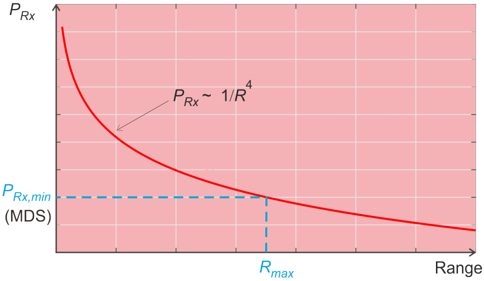



$$\text{Duty-Cycle} \triangleq \frac{T_p}{PRI} \quad (27)$$

$$= T_p \times PRF \quad (28)$$

$$= \frac{P_{Tx}}{P_{Tx,peak}} \quad (29)$$

# Maximum Detection Range




- The received signal with the minimum received power ($P_{Rx,\text{min}}$) that the radar receiver (Rx) can "sense" a target is referred to as the "minimum detectable signal" (MDS)

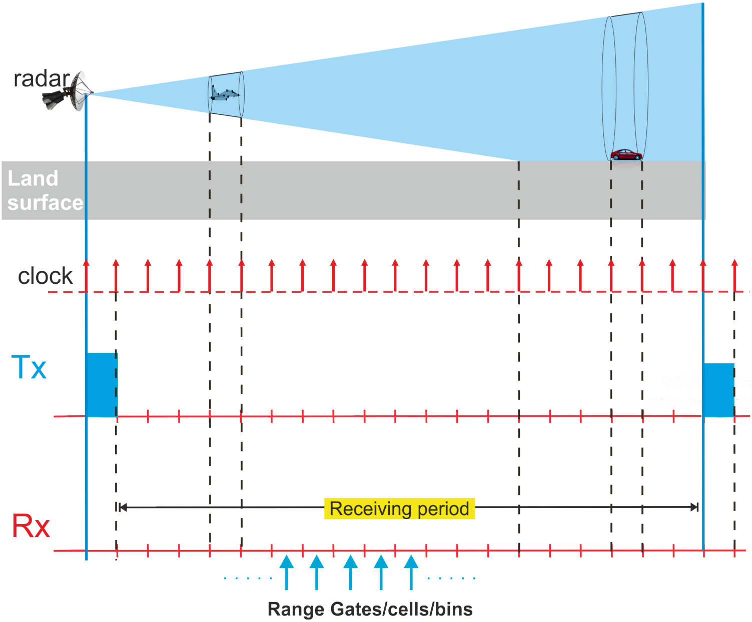
- Given the MDS, i.e. $P_{Rx,\text{min}}$, the maximum detection range can be obtained from the radar equation (see Topic-1) as follows:

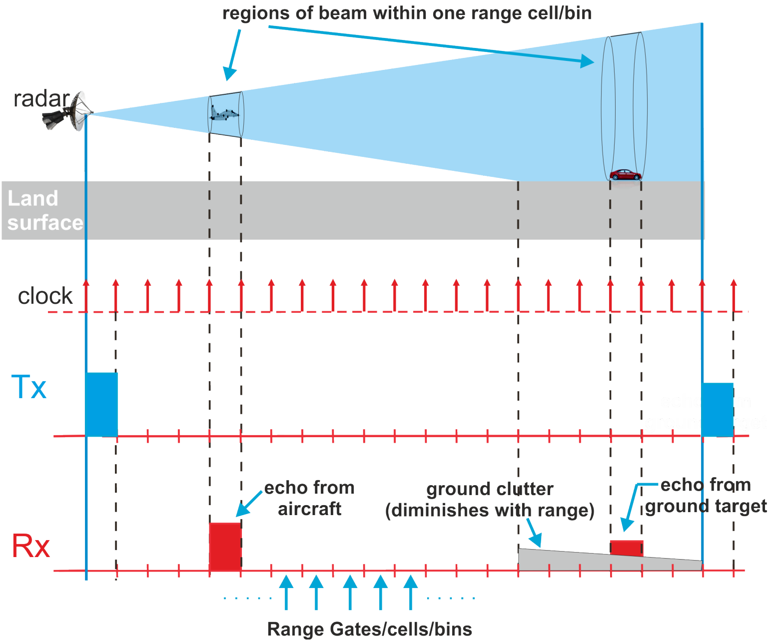
$$P_{Rx} = \left.\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R^4}.RCS\right|_{P_{Rx}=P_{Rx,\text{min}}} \tag{30}$$

$$\Rightarrow \boxed{R_{\max}=\sqrt[4]{\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.P_{Rx,\text{min}}}.RCS}} \tag{31}$$

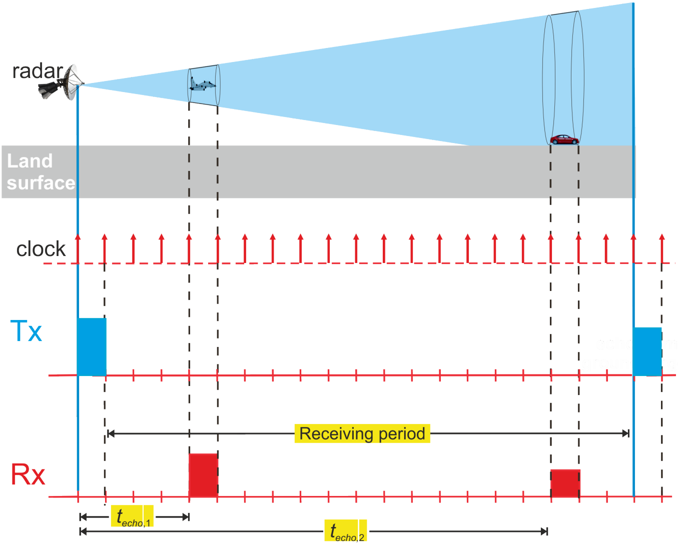
# Radar Range Bins

- Next the "range bins" will be illustratively described/defined in conjunction with other radar terms such as:
  - radar clock
  - receiving period
  - echoes from targets
  - ground clutter
  - Radar's minimum range = if the Tx is still transmitting when the echo from a target is received then the target "cannot be seen" by the radar's Rx.


radar
Land surface
clock
Tx
Rx
Receiving period
Range Gates/cells/bins


regions of beam within one range cell/bin
radar
Land surface
clock
Tx
Rx
echo from aircraft
ground clutter (diminishes with range)
echo from ground target
Range Gates/cells/bins

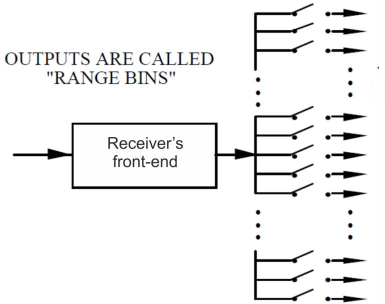
# Range measurement are performed after removing the clutter




we measure $t_{echo} = \frac{2R}{c}$ and consequently we estimate $R = \frac{ct_{echo}}{2}$

# Range-Bins Conceptual Implementation

- Analogue conceptual implementation of range gates (bins) at the Rx




  - Gates are opened and closed sequentially
  - The time each gate is closed corresponds to a range increment
  - Gates must cover the entire interpulse period or the ranges of interest
  - For tracking a target: a single gate can remain closed until the target leaves the bin

# Range Bins and Range Resolution

- Clock period = smallest pulse duration$=T_p$
- Clock frequency$=1/T_p$
  Note: clock frequency = Bandwidth $(B) = \frac{1}{T_p}$
- The "clock" quantises the time into intervals of time matched to the pulse width $T_p$. This defines the range resolution of the system.
- Range resolution
  - is the radar's ability to distinguish two closely spaced targets along the same line of sight (LOS).

- The range resolution is a function of the pulse-length, where

$$\boxed{\text{pulse-length (in m)} = cT_p} \quad (32)$$

- Two targets with ranges $R_{echo,1}$ and $R_{echo,2}$ with $R_{echo,2} > R_{echo,1}$ can be resolved if:

$$\boxed{\underbrace{R_{echo,2} - R_{echo,1}}_{\triangleq \Delta R} \geqslant \frac{cT_p}{2}} \quad (33)$$

## Example (Pulse-length)

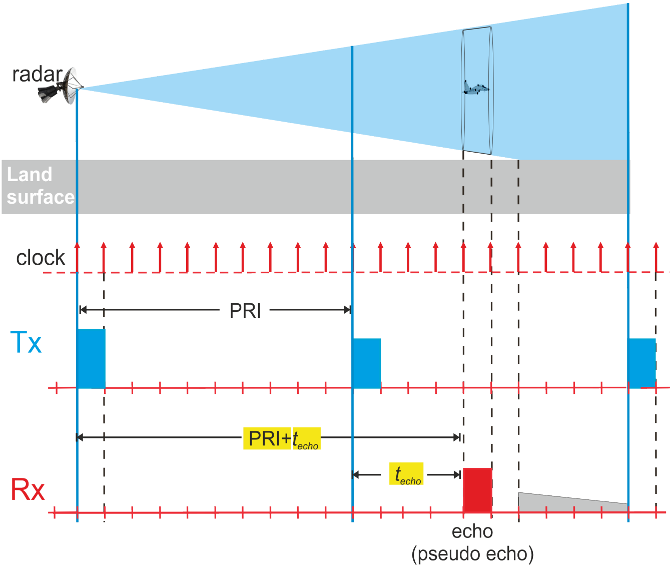
- A radar with pulse width 1 $\mu$sec yields a pulse length

$$\text{pulse-length} = cT_p = 3 \times 10^8 \times 10^{-6} = 300\text{m}$$

Furthermore, this radar can resolve two targets if

$$\Delta R > \frac{300}{2} = 150\text{m}$$

# Range Ambiguity



# Range Ambiguity (cont.)

- The PRF is another key radar parameter and is arguably one of the most difficult design decisions.
- The range of a target becomes ambiguous as a function of $PRI$. In particular:

$$2R_{amb} = c.PRI \tag{34}$$

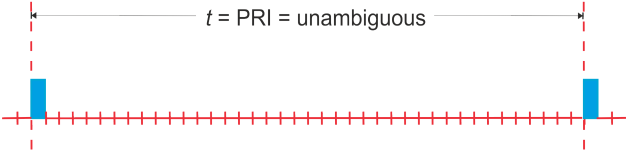
i.e.

$$R_{amb} = \frac{c.PRI}{2} = \frac{c}{2.PRF} \tag{35}$$

- This implies that targets which are further than PRI/2 yield ambiguous range results.

$$\boxed{\text{ambiguous-ranges} > \underbrace{\frac{c.PRI}{2}}_{R_{amb}}} \tag{36}$$

# Unambiguous Range




- Unambiguous ranges:

$$\text{unambiguous-ranges} \leqslant \underbrace{\frac{c}{2.PRF}}_{R_{amb}} \tag{37}$$

- Equation 37 does not imply that **all** ranges are unambiguous - as some ranges satisfy this equation and other ranges satisfy Equation 36.

## Unambiguous Range (cont.)

- To avoid range ambiguity, one must ensure that the radar's maximum range $R_{\max}$ satisfies the following condition:

$$R_{\max} < \underbrace{\frac{c}{2.PRF}}_{R_{amb}}$$

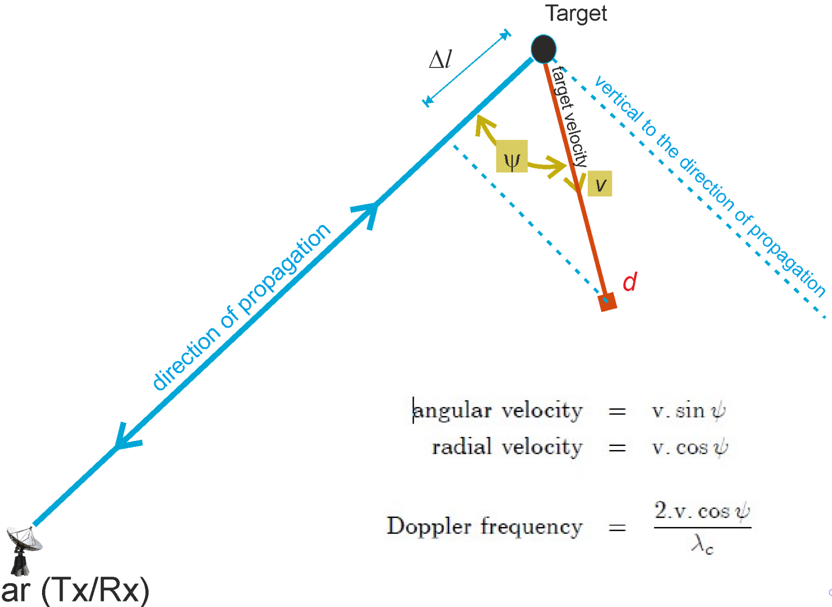
- Note that a long $R_{\max}$ requires a LOW PRF

### Definition (Low PRF)

- Low PRF is simply defined as a PRF low enough to avoid range ambiguity.

- Range ambiguity is also a feature of Medium PRF (and, probably, High PRF, too).

# Doppler Frequency




$$\begin{aligned}\text{angular velocity} &= \text{v}.\sin\psi \\ \text{radial velocity} &= \text{v}.\cos\psi \\ \text{Doppler frequency} &= \frac{2.\text{v}.\cos\psi}{\lambda_c}\end{aligned}$$

# Proof

- With reference to the previous figure the difference in path lengths from radar to a moving target with velocity $v$ is

$$\Delta l = d\cos\psi = \underbrace{v.\Delta t}_{=d}.\cos\psi \tag{38}$$

- If $\lambda_c$ denotes the wavelength of the carrier, then the phase change in Rx-signal due to difference in path length is:

$$\Delta\varphi = 2\pi F_c \underset{\frac{2\Delta l}{c}}{\underset{\updownarrow}{\Delta t}} = 2\pi.\frac{2\Delta l}{\lambda_c} \overset{(38)}{=} 2\pi\frac{2v.\Delta t.\cos\psi}{\lambda_c} \tag{39}$$

- Doppler frequency (in Hz): it is defined as the rate of phase change due to target's motion

$$\mathfrak{f}_D = \frac{1}{2\pi}.\frac{\Delta\varphi}{\Delta t} = \frac{2v\cos\psi}{\lambda_c} \tag{40}$$

$$\text{where} \quad \mathfrak{f}_{\max} \triangleq \frac{2v}{\lambda_c} \tag{41}$$

# Doppler Bandwidth

- We have seen that the Doppler frequency $\mathfrak{f}_D$ and the target's relative (radial) velocity $v_r$ are given as follows:

$$\mathfrak{f}_D = \pm \frac{2 \overset{v\cos\psi}{\overset{\uparrow}{v_r}}}{\lambda} \Longleftrightarrow v_r = \pm \frac{\lambda . \mathfrak{f}_D}{2} \tag{42}$$

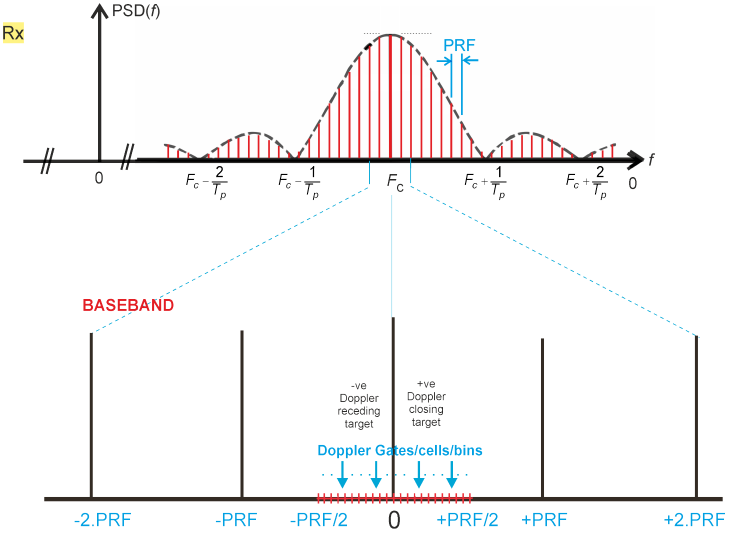
  where the "minus" sign indicates that the target is moving away from the radar platform.

- Doppler Bandwidth:

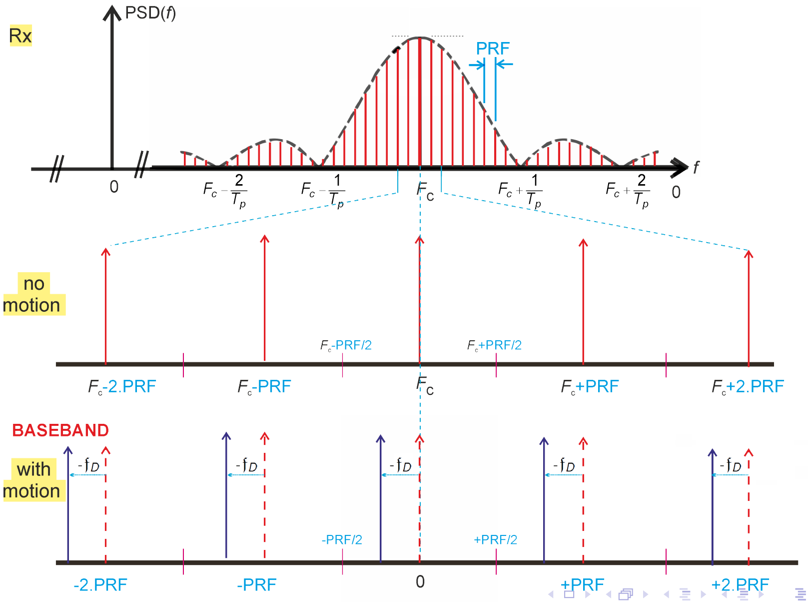
$$B_{Dop} = PRF \tag{43}$$

- N.B.: The FFT processing subdivides the Doppler bandwidth using a set of $N_{D,cells}$ filters into Doppler cells/gates/bins

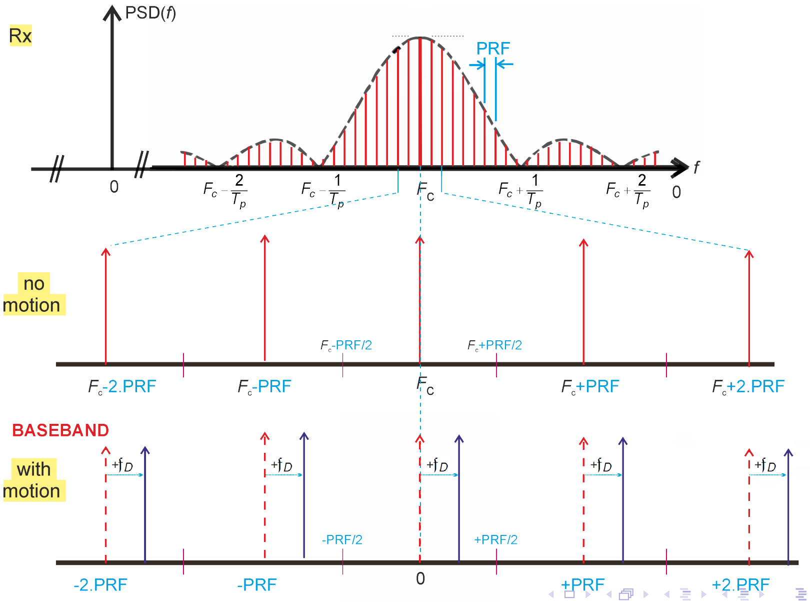
# Doppler Gates/Cells/Bins



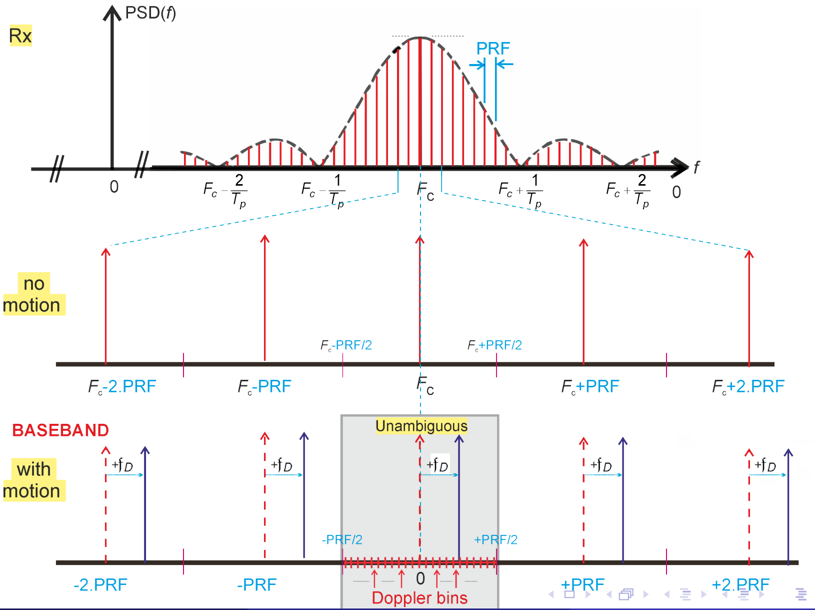
# Doppler -ve Frequency/Offset



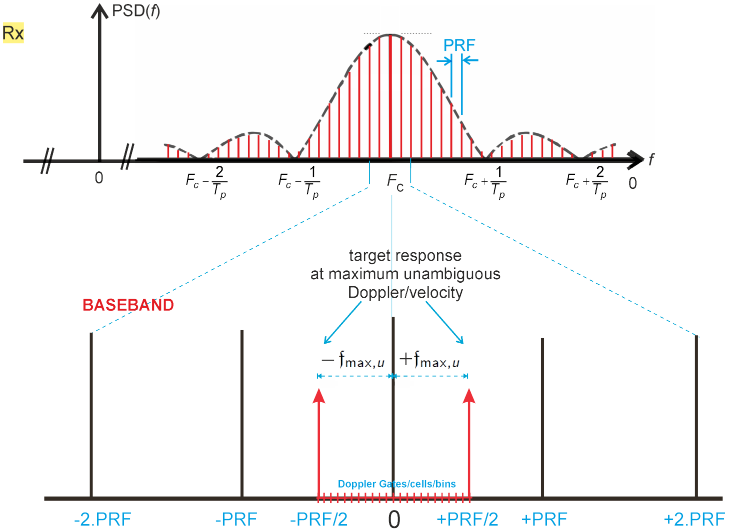
# Doppler +ve Frequency/Offset



# Doppler Bins



# Maximum Unambiguous Doppler/Velocity



- maximum unambiguous Doppler shift $\mathfrak{f}_{\max,u}$:

$$\mathfrak{f}_{\max,u} = \pm\frac{PRF}{2} \tag{44}$$

- maximum unambiguous velocity $v_{\max,u}$:

$$v_{\max,u} = \pm\frac{\lambda.PRF}{4} \tag{45}$$

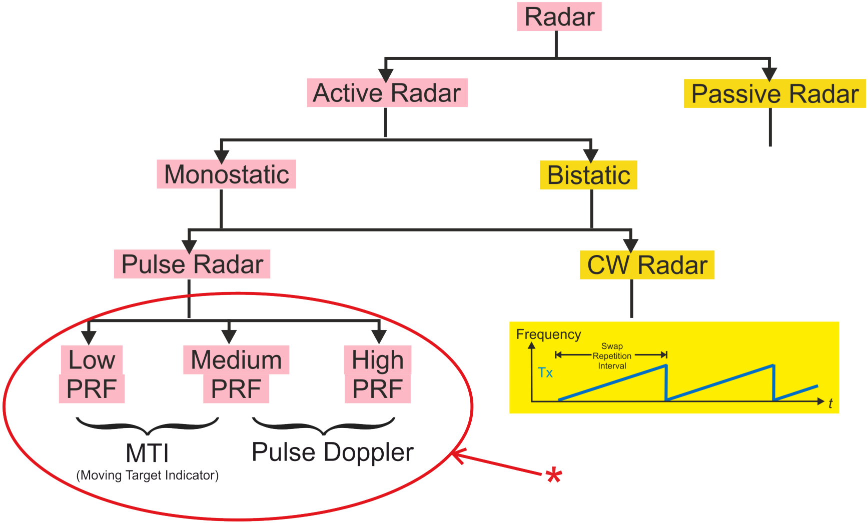
**Definition (High PRF)**

- High PRF is simply defined as a PRF high enough to avoid velocity (Doppler) ambiguity

# Summary: Ambiguities and PRF

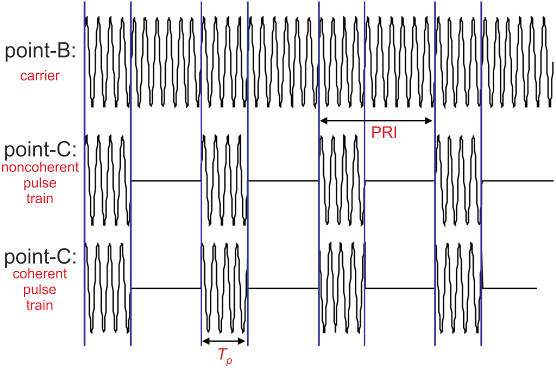
In summary we have seen

- PRF selection falls into three different regimes defined by their ambiguity characteristics.
  - Low PRF avoids range ambiguity,
  - high PRF avoids velocity (Doppler) ambiguity,
  - medium PRF incurs ambiguity in both range and velocity
    - which has, however, good all-round performance even in the face of testing clutter conditions because it cycles its operation over several medium PRFs.



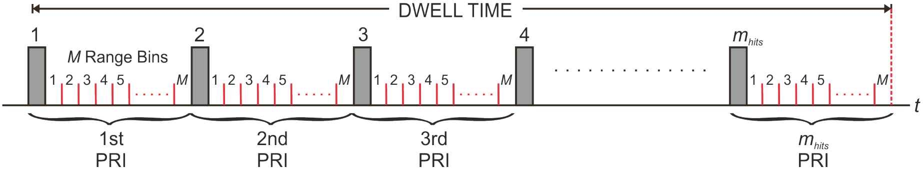
Radar
Active Radar
Passive Radar
Monostatic
Bistatic
Pulse Radar
CW Radar
Frequency
Swap Repetition Interval
Tx
t
Low PRF
Medium PRF
High PRF
MTI
(Moving Target Indicator)
Pulse Doppler
*

- Noncoherent pulse train
  If the initial value of the sinewave at point C of the radar's Tx, for each pulse, is essentially random.
- Coherent pulse train
  if the waveform (carrier) at point B of the radar's Tx overlap exactly with the waveform at point C (same initial value).



# Dwell Time (scanning)

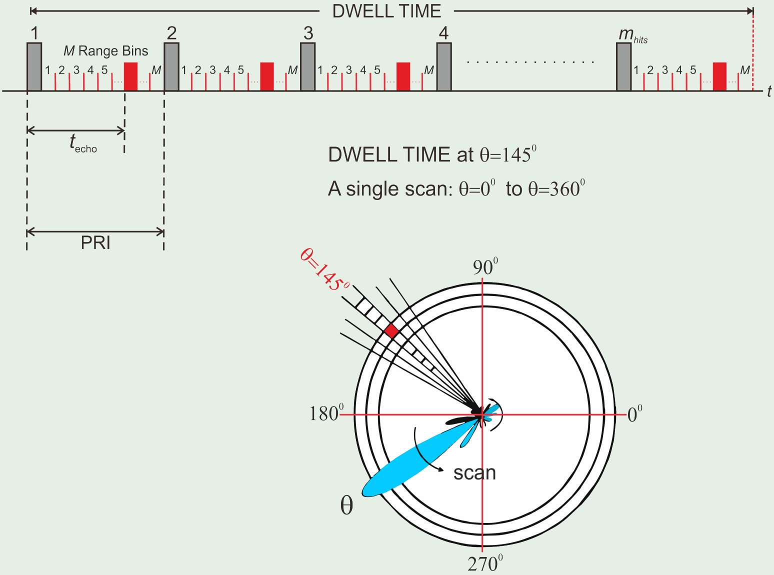
Typical Pulse train and range bins (range gates)




## Definition (Dwell Time)

- The time ($T_{dwell}$) that an antenna beam spends on a target is called dwell time.

# Example (Range Bins)



- The dwell time of a 2D–search radar depends predominantly on the antennas horizontally beamwidth $\theta_{3dB}$ and the speed of rotation $v_{rotation}$ of the antenna (rotations per minute).
- The dwell time can be calculated using the following equation:

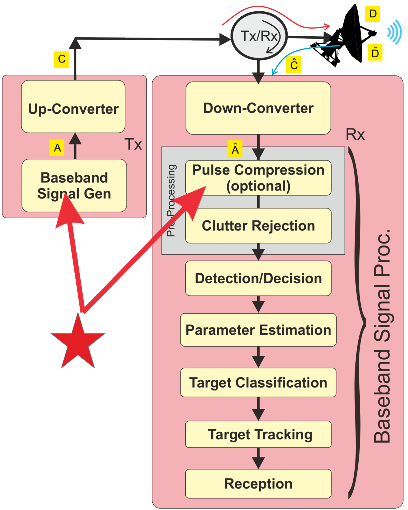
$$\boxed{T_{dwell} \triangleq \theta_{3dB} \times \frac{1}{360 \times v_{rotation}} \times 60 \text{ sec}} \tag{46}$$

- **Hits-per-scan**: The value of hits-per-scan $m_{hits}$ says how many echo signals per single target during every antenna swing are received. This can be calculated as follows:

$$\boxed{m_{hits} \triangleq \frac{T_{dwell}}{\text{PRI}} = T_{dwell} \times \text{PRF}} \tag{47}$$

  The hit number stands (e.g. for a search radar with a rotating antenna) for the number of the received echo pulses of a single target per antenna turn.
- NB: For a radar to evaluate the target information with sufficient precision, hit-numbers are between 1 and 20 as necessary, which depends on the radar set operating mode.

# Pulse Compression: Introduction



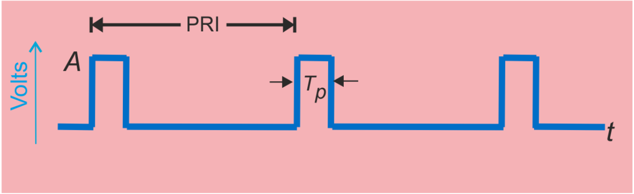

- Pulse compression, allows a radar to **simultaneously achieve**
  - the energy of a long pulse and
  - the resolution[a] of a short pulse.

  It also improves the SNR.
- Pulse compression is a method for improving the range resolution of pulse radar.

[a] The range resolution of a simple pulse radar depends on the pulse duration $T_p$ [two reflective objects located within the spatial extent of the pulse are only displayed as one target].

# Pulse Compression Waveform

at point-A : Basic Pulse Waveform (uncompressed pulse)




$$\text{resolution } (\Delta R) = \frac{cT_p}{2} = \frac{c}{2B} \quad (48)$$

$$\text{Bandwidth } (B) = \frac{1}{T_p} \Longrightarrow \boxed{T_p = \downarrow\downarrow\downarrow \Longrightarrow B = \uparrow\uparrow\uparrow} \quad (49)$$

$$\Longrightarrow \text{ we see more "details" of the target}$$

**Example**

- bandwidth $B$=1MHz$\Rightarrow$ $T_p = 1\mu s \Rightarrow 150m$ resolution range (i.e. $\Delta R = 150m$)

- Because $B = 1/T_p$, to increase the bandwidth $B$ we have to reduce the pulse duration $T_p$. That is

$$T_p = \downarrow \Leftrightarrow B = \uparrow$$

- However,

$$P_{Tx} = \frac{A^2 T_p}{PRI} \Longrightarrow \boxed{T_p = \downarrow \Leftrightarrow P_{Tx} = \downarrow} \Rightarrow SNR_{in} = \downarrow \tag{50}$$

  which means that the ability of the radar to detect a given target at a given range is reduced ($\downarrow$).

- we want to increase $B$ without to reduce $P_{Tx}$ (and consequently without reducing $SNR_{in}$)

## Definition (Pulse Compression)

- The increase of Bandwidth without reducing the $P_{Tx}$ is known as Pulse Compression

- Pulse compression[2] allows a radar system to transmit a pulse of relatively long duration and low peak power to attain the range resolution and detection performance of a short-pulse, high-peak power system.

- This is accomplished by coding the RF carrier to increase the bandwidth of the transmitted waveform and then compressing the received echo waveform.

[2] also known as *intra-pulse modulation*

- Thus, a radar with pulse compression can use a Tx pulse of duration $T_p$ and yet achieve a range resolution equivalent to that of a shorter Tx pulse of duration $T_c (<< T_p)$.

- The ratio of the uncompressed pulse length $T_p$ to the compressed pulse length $T_c$ is known as the Pulse Compression Ratio (PCR)[3] and is given by

$$\text{PCR} \triangleq N_c = \frac{T_p}{T_c} \tag{51}$$

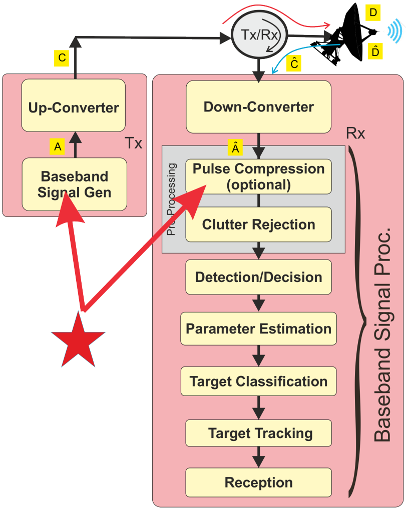
$$= T_p.B \tag{52}$$

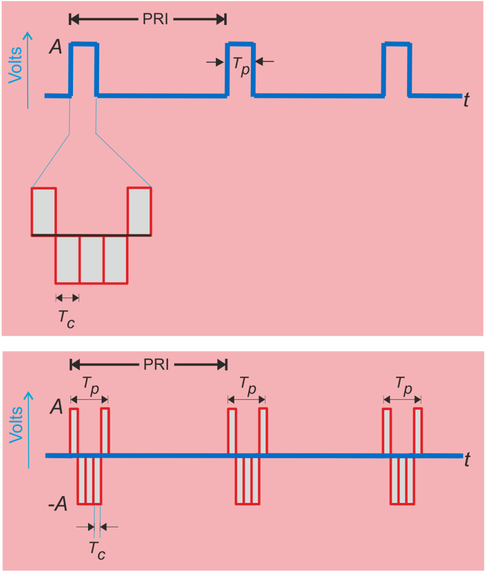
where

$$B = \frac{1}{T_c} \tag{53}$$

[3] also known as Pulse Compression Factor or Processing Gain

- at point-A: Pulse Compression Waveform



- Radar applications that require high-range resolution include
  - object detection,
  - object classification,
  - terrain mapping,
  - accurate ranging,

  and as an aid in any application in distributed clutter suppression.

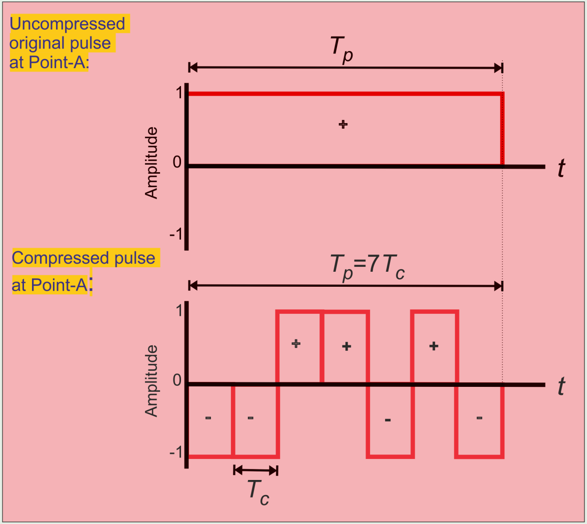
- High-range resolution can be achieved
  - either by transmitting a uncompressed short-duration pulse,
  - or by transmitting a lower-peak-power, compressed pulse of greater duration

- A radar system that transmits a compressed pulse rather than an uncompressed pulse to achieve high-range resolution provides the following potential advantages:
  - Improved detection performance.
  - Mutual interference reduction.
  - Increased system operational flexibility.

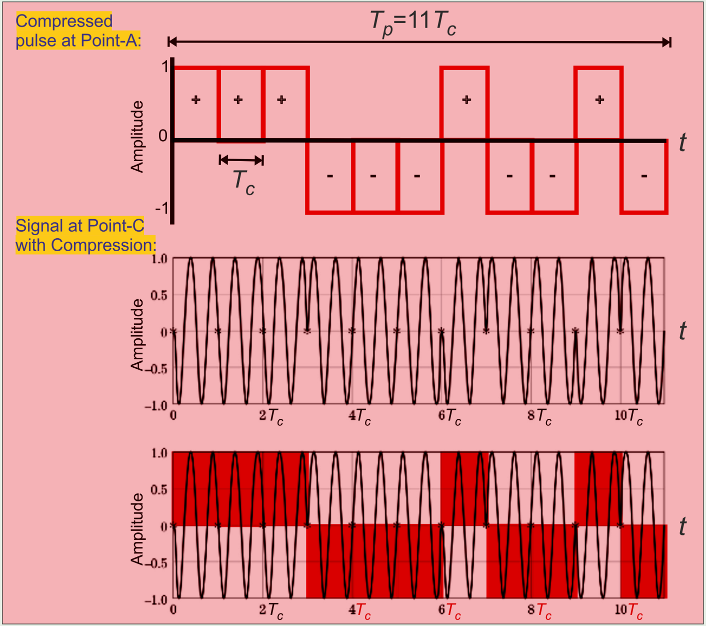
# Compression Codes

- Barker codes
- combined Barker codes
- PN-codes:
  - m-sequences (see Appendix-B)
  - gold-sequences (see Appendix-C)
- polyphase codes

# Example (m-sequence)



# Example



Compressed pulse at Point-A:
$T_p$=11$T_c$
1
0
-1
Amplitude
+ + + + +
- - - - - -
$T_c$
t
Signal at Point-C with Compression:
1.0
0.5
0
−0.5
−1.0
Amplitude
0
$2T_c$
$4T_c$
$6T_c$
$8T_c$
$10T_c$
t

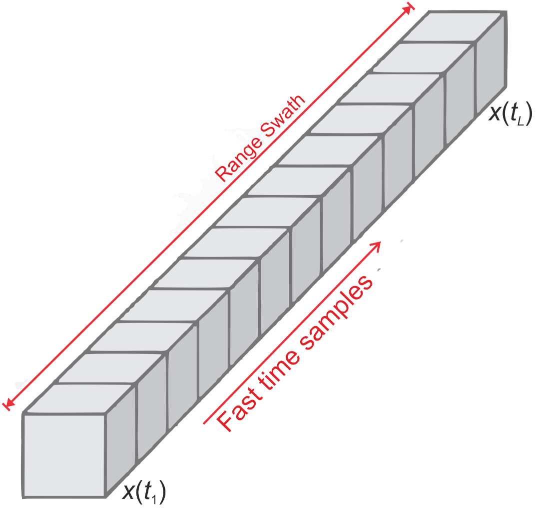
Modulation methods for pulse compression:

- frequency modulated (or called Frequency Modulation on Pulse, FMOP)
  - with linear frequency modulation,
  - with non-linear frequency modulation,
  - with time-dependent coded frequency modulation (e.g. the Costas code) and
- phase modulated (or called Phase Modulation on Pulse, PMOP)
  - with time-dependent coded pulse-phase modulation.

# Advantages

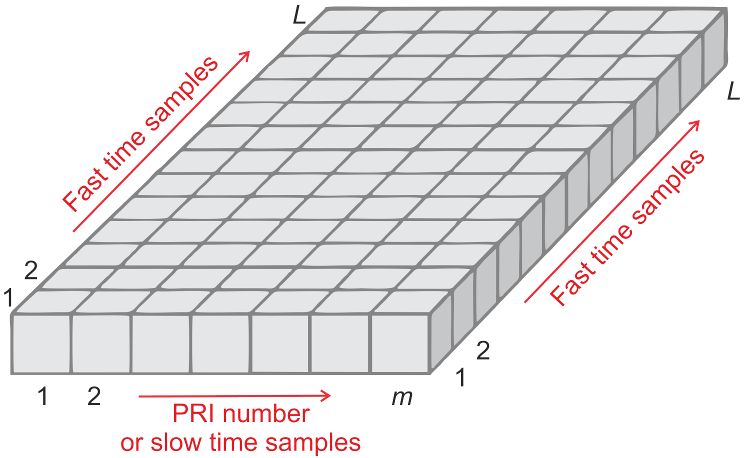
- lower pulse-power (therefore suitable for Solid-State-amplifier)
- higher maximum range
- good range resolution
- better jamming immunity

# Fast-Time Samples



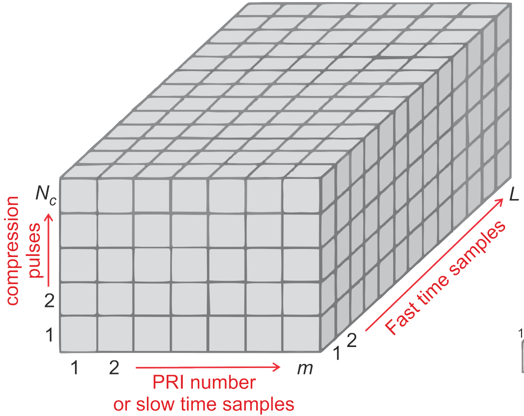

- Each cube in the figure represents a single complex sample (complex voltage measurement) at Point Â1 of a radar architecture (one sample at the Rx's ADC output)
- Each reflected pulse contributes to one sample
- Store the samples as vector, with elements being also called range bins, range gates, range cells or fast-time samples.

# Slow-Time and Fast-Time Samples

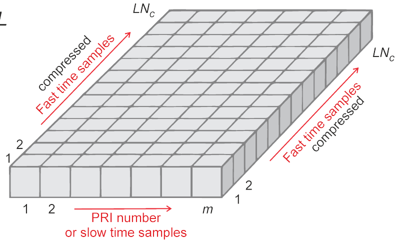



- Fast-time/slow-time CPI data matrix
- Note: CPI = Coherent Processing Interval

# 3D Data Cube

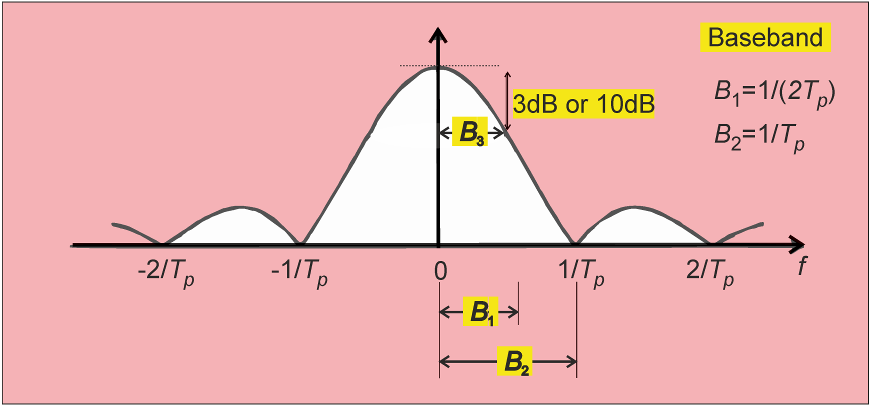


$N_c$
compression pulses
2
1
1
2
PRI number
or slow time samples
$m$
1
2
Fast time samples
$L$

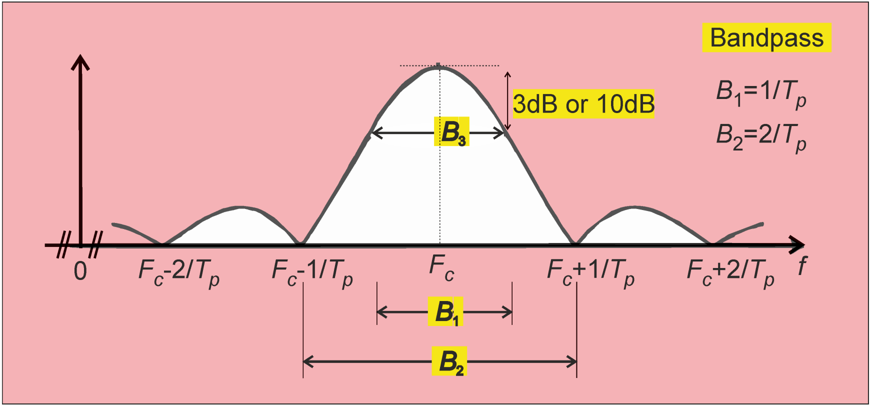



$LN_c$
compressed
Fast time samples
2
1
$LN_c$
Fast time samples
compressed
2
1
1
2
PRI number
or slow time samples
$m$

# Appendix-A: Baseband Bandwidth Definitions



# Appendix-A: Bandpass Bandwidth Definitions



# Appendix-B: m-sequences

- m-seq.: widely used in SSS because of their very good autocorrelation properties.
- PN code generator: is periodic
  - i.e. the sequence that is produced repeats itself after some period of time

**Definition (m-sequence )**

A sequence generated by a linear $m$-stages Feedback shift register is called a maximal length, a maximal sequence, or simply m-sequence, if its period is

$$N_c = 2^m - 1 \tag{54}$$

(which is the maximum period for the above shift register generator)

- The initial contents of the shift register are called initial conditions.

# Shift Registers and Primitive Polynomials

- The period $N_c$ depends on the feedback connections (i.e. coefficients $c_i$) and $N_c = max$, i.e. $N_c = 2^m - 1$, when the characteristic polynomial

$$c(D) = c_m D^m + c_{m-1} D^{m-1} + .... + c_1 D + c_0 \quad \text{with} \quad c_0 = 1 \tag{55}$$

is a primitive polynomial of degree $m$.

$$\boxed{\text{rule: if } c_i = \begin{cases} 0 \Longrightarrow & \text{no connection} \\ 1 \Longrightarrow & \text{there is connection} \end{cases}} \tag{56}$$

- Definition of PRIMITIVE polynomial = very important (see Appendix C)

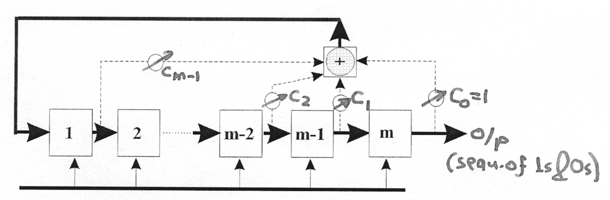
## Examples (Some Primitive Polynomials)

| degree-$m$ | polynomial |
|---|---|
| 3 | $D^3 + D + 1$ |
| 4 | $D^4 + D + 1$ |
| 5 | $D^5 + D^2 + 1$ |
| 6 | $D^6 + D + 1$ |
| 7 | $D^7 + D + 1$ |

# Implementation of an m-sequence

- use a maximal length shift register
  i.e. in order to construct a shift register generator for sequences of any permissible length, it is only necessary to know the coefficients of the primitive polynomial for the corresponding value of $m$

$$f_c = \frac{1}{T_c} = \text{chip-rate} = \text{clock-rate} \tag{57}$$

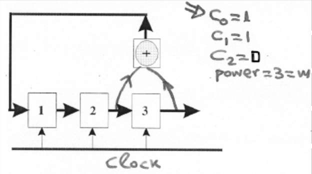



$$c(D) = c_m D^m + c_{m-1} D^{m-1} + .... + c_1 D + c_0 \tag{58}$$

$$\text{with} \quad c_0 = 1 \tag{59}$$

## Example ($c(D)= D^3+ D + 1$ = primitive$\Longrightarrow$power$= m = 3$)

- coefficients$=(1, 0, 1, 1) \Rightarrow N_c = 7 = 2^m - 1$ i.e. period$= 7T_c$

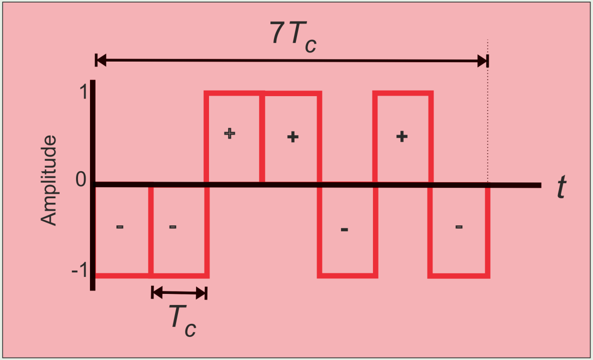



| | **1**st | **2**nd | o/p **3**rd |
|---|---|---|---|
| initial condition | 1 | 1 | 1 |
| clock pulse No.**1** | 0 | 1 | 1 |
| clock pulse No.**2** | 0 | 0 | 1 |
| clock pulse No.**3** | 1 | 0 | 0 |
| clock pulse No.**4** | 0 | 1 | 0 |
| clock pulse No.**5** | 1 | 0 | 1 |
| clock pulse No.**6** | 1 | 1 | 0 |
| clock pulse No.**7** | 1 | 1 | 1 |

- Note that the sequence of 0's and 1's is transformed to a sequence of $\pm$1s by using the following function

$$\text{o/p} = 1 - 2 \times \text{i/p} \tag{60}$$

## Example ($c(D) = D^3 + D + 1$ = primitive: one period)

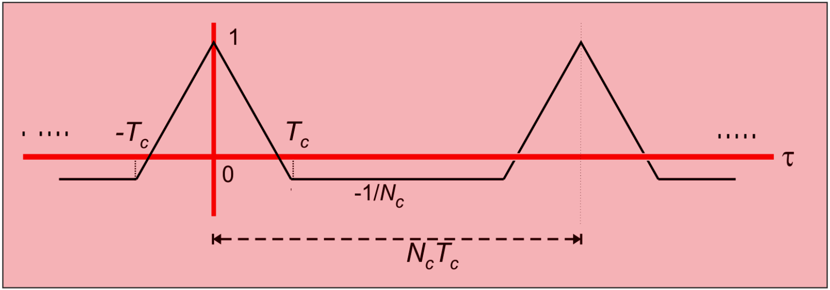


# Auto-correlation Properties

m-sequences have a two-valued autocorrelation function (normalised)

$$R_{bb}(\tau) = \begin{cases} 1 & \tau = 0 \\ -1/N_c & \tau = kT_c;\, k \bmod N_c \end{cases} \tag{61}$$

$R_{bb}(\tau)$ :



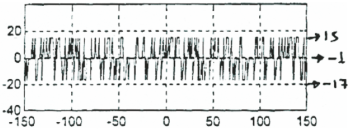
# Cross-correlation Properties

- It can be shown that the cross-correlation of **preferred sequences** takes on values (unnormalised) from the set

$$\{-1, -R_{cross}, R_{cross} - 2\} \tag{62}$$

where
$$R_{cross} = \begin{cases} 2^{\frac{m+1}{2}} + 1 & m = odd \\ 2^{\frac{m+2}{2}} + 1 & m = even \end{cases} \tag{63}$$

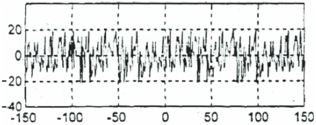
$R_{b_i b_j}(\tau)$ =preferred:




$m = 7$

delay $\tau = kT_c$

$R_{b_i b_j}(\tau)$ = *non*-preferred:

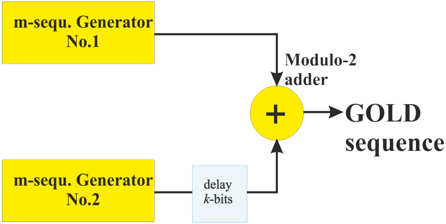



$m = 7$

delay $\tau = kT_c$

# Appendix-C: Gold Sequences - Implementation

- The Gold sequence is actually obtained by the modulo-2 sum of two $m$-sequences with different phase-shifts for the first $m$-sequence relative to the second.
- There are $N_c = 2^m - 1$ different relative phase shifts, and for every phase-shift a different Gold sequence is generated.

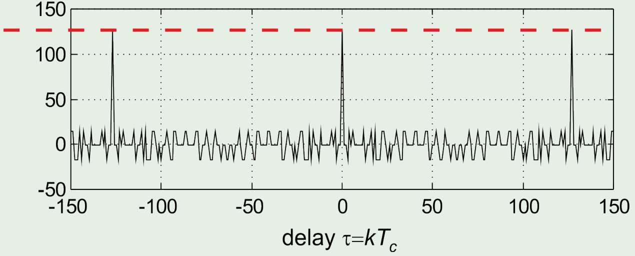


# Auto-Correlation Properties

- Gold sequences are not maximal length sequences.
- Therefore, their auto-correlation function is not a two-valued one
- The auto-correlation still has the periodic peaks, but between the peaks the auto-correlation is no longer flat (for examples see next slide).

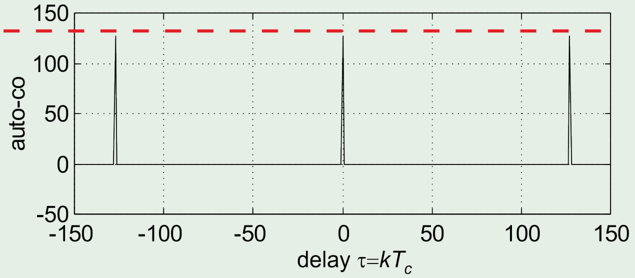
## Example (Gold Sequence of $N_c = 127 = 2^7 - 1$)

$R_{bb}(\tau)$: **127**




## Example (m-sequence of $N_c = 127 = 2^7 - 1$)

$R_{bb}(\tau)$: **127**



# Cross-Correlation Properties

- Gold-sequences have the same cross-correlation characteristics as preferred m-sequences,
  i.e. their cross-correlation is three valued.

- Gold sequences have higher $R_{auto}$ and lower $R_{cross}$ than m-sequences, and provide a trade-off between these parameters.

# Balanced Gold codes.

- Balanced Gold Sequence: The number of "-1s" in a code period exceed the number of "1s" by one as is the case for m-sequences.
- We should note that not all Gold codes (generated by modulo-2 addition of 2 m-sequences) are balanced, i.e. the number of "-1s" in a code period does not always exceed the number of "1s" by one.
- Balanced Gold codes have more desirable spectral characteristics than non-balanced.
- Balanced Gold codes are generated by appropriately selecting the relative phases of the two original m-sequences.
- SUMMARY: By selecting any preferred pair of primitive polynomials it is easy to construct a very large set of PN-sequences (Gold-sequences).