# EE3-27: Principles of Classical and Modern Radar

# MIMO Radar

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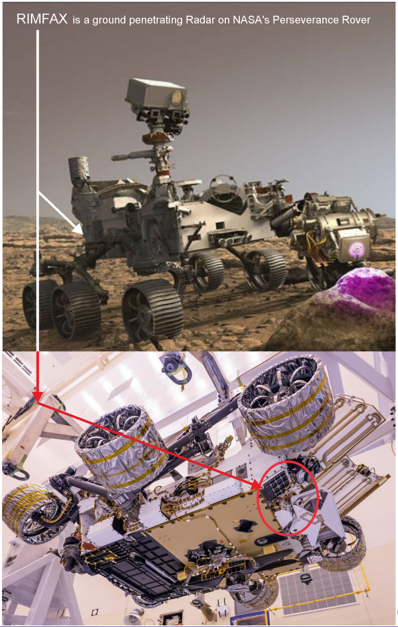
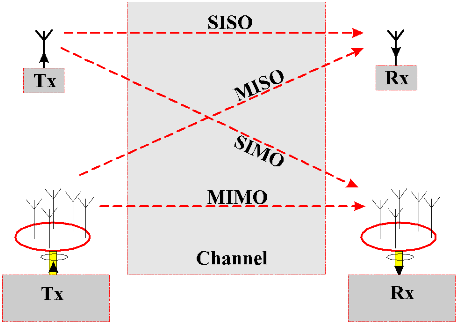


# Introduction

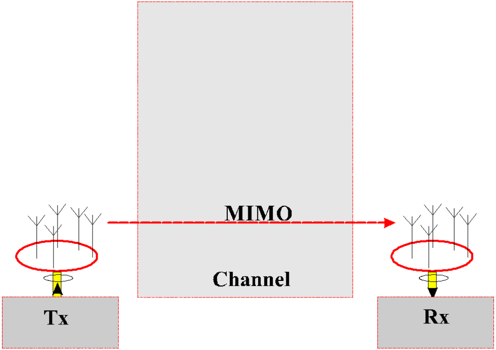
- MIMO radar is a family of modern and very powerful radar systems which incorporates
  - antenna arrays at both the radar's Tx and Rx,
  - suitable vector-signal baseband generators at the radar's Tx
  - sophisticated array processing algorithms at the radar's baseband Rx
- MIMO radar
  - improve the spatial radar resolution,
  - provide a substantially improved immunity to interference.
  - provide considerable improvement of the signal-to-noise ratio and, consequently, the probability of detection of the targets.
- MIMO radar taxonomy:
  - with collocated antennas (so-called "Monostatic" MIMO)
  - with widely separated Tx and Rx antennas (so-called "Bistatic" MIMO).
  - with distributed antennas ("Large Aperture Arrays" MIMO) where the target is regarded by each antenna from another aspect angle.
- In the collocated MIMO radar case, the Tx and Rx antennas are close enough (small aperture arrays) such that the target radar cross-sections (RCS) observed by the Tx and Rx antenna elements are identical, i.e. $(\bar{\theta}, \bar{\phi})_{DOD} = (\theta, \phi)_{DOA}$
- In this topic we focus on "monostatic" MIMO radar.

- ∃ a **New** classification of radar (and, in general, of wireless systems): according to the **number of antennas/sensors** used in both Tx and Rx




| **Terminology** | | | | | |
|---|---|---|---|---|---|
| S: | Single | | M: | Multiple |
| I: | Input | | O: | Output |

# MIMO Radar: Backscatter Modelling




- Consider a single path from a Tx-array of $\overline{N}$ antennas to an Rx-array of $N$ antennas with locations given by the matrices

$$\text{Tx-array:} \qquad \underline{\underline{\bar{\mathbf{r}}}} = [\underline{\bar{r}}_1, \underline{\bar{r}}_2, \ldots, \underline{\bar{r}}_{\overline{N}}] = [\underline{\bar{r}}_{\mathbf{x}}, \underline{\bar{r}}_{\mathbf{y}}, \underline{\bar{r}}_{\mathbf{z}}]^T \quad (3 \times \overline{N}) \qquad (1)$$

$$\text{Rx-array:} \qquad \underline{\underline{\mathbf{r}}} = [\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N] = [\underline{r}_{\mathbf{x}}, \underline{r}_{\mathbf{y}}, \underline{r}_{\mathbf{z}}]^T \quad (3 \times N) \qquad (2)$$

# Single Target MIMO Radar: Modelling of the Rx Vector-Signal x(t)

- we have seen before that for a monostatic radar (MIMO or not) the direction-of-departure $(\overline{\theta}, \overline{\phi})$ and the direction-of-arrival is $(\theta, \phi)$ are the same, i.e.

$$(\overline{\theta}, \overline{\phi}) = (\theta, \phi)$$

and the impulse response for a single target is

$$\left.\begin{array}{c}\text{MIMO radar}\\ \text{or}\\ \text{Phased-array}\end{array}\right\rangle\text{- single target: } \underbrace{\underline{h}(t)}_{\text{colinear}} = \beta \underline{S}(\theta, \phi).\underbrace{\overline{\underline{S}}^H(\theta, \phi).\underline{\delta}(t - t_{echo})}_{\text{scalar}} \quad (3)$$

lose Tx diversity!

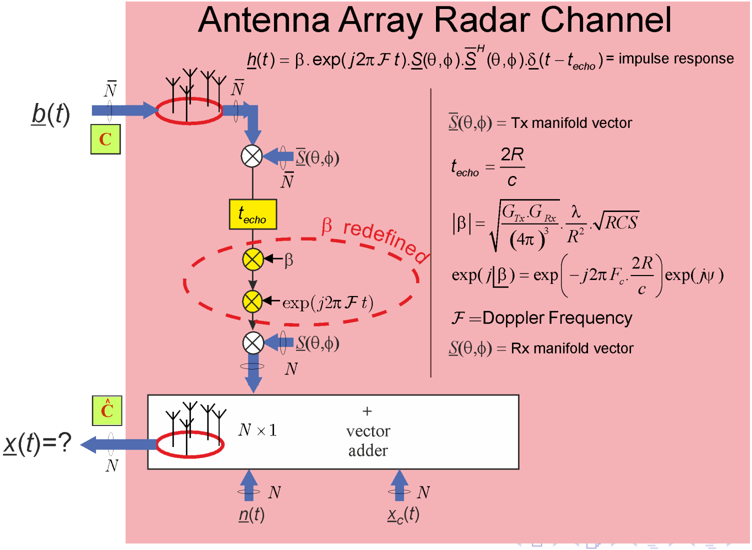
where $\beta$ has been re-defined to include also the Doppler term, and

$$\text{Tx:} \qquad \overline{\underline{S}} = \overline{\underline{S}}(\theta, \phi) = \exp\left(+j\underline{\underline{\mathbf{r}}}^T \underline{k}(\theta, \phi)\right) \quad (4)$$

$$\text{Rx:} \qquad \underline{S} = \underline{S}(\theta, \phi) = \exp\left(-j\underline{\underline{\mathbf{r}}}^T \underline{k}(\theta, \phi)\right) \quad (5)$$

with $\underline{k}(\theta, \phi)$ denoting the wavenumber vectors of the Tx-array and Rx-array respectively

# Single Target MIMO Radar: Modelling of the Rx Vector-Signal x(t) (cont.)



# Single Target MIMO Radar: Modelling of the Rx Vector-Signal x(t) (cont.)

- Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ in the presence of a single target (MIMO radar).
- The received $(N \times 1)$ baseband vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned}
\underline{x}(t) &= \underline{h}(t) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\
&= \left(\beta . \underline{S} . \overline{\underline{S}}^H \underline{\delta}(t - \tau_{echo,\ell})\right) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\
\Rightarrow \underline{x}(t) &= \underbrace{\beta . \underline{S} . \overline{\underline{S}}^H \underline{b}(t - \tau_{echo})}_{\triangleq \text{single target echo}} + \underline{n}(t) + \underline{x}_c(t) \qquad (6)
\end{aligned}$$

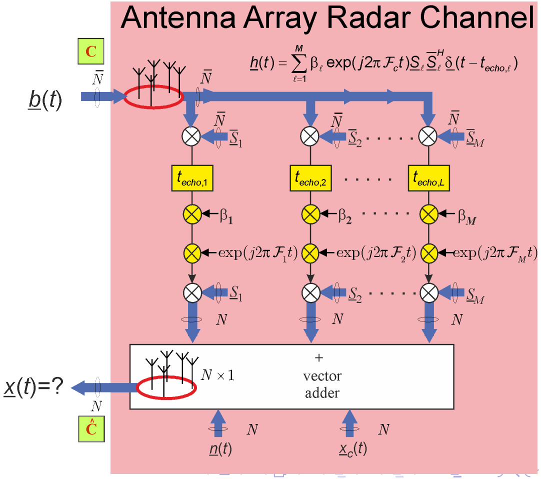
where

$$\begin{aligned}
\underline{n}(t) &= \left[ n_1(t), \quad n_2(t), \quad \cdots, \quad n_N(t) \right]^T \qquad (7) \\
\underline{x}_c(t) &= \left[ x_{c1}(t), \quad x_{c2}(t), \quad \cdots, \quad x_{cN}(t) \right]^T \qquad (8)
\end{aligned}$$

# Multi Target MIMO Radar: Modelling of the Rx Vector-Signal x(t)

- In the following figure the vectors $\underline{\overline{S}}_\ell = \underline{\overline{S}}(\theta_\ell, \phi_\ell) \in C^{\overline{N}}$ and $\underline{S}_\ell = \underline{S}(\theta_\ell, \phi_\ell) \in C^N$ are the Tx- and Rx- array manifold vectors of the $\ell$-th target.

  - Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ using and antenna-array in the presence of $M$ targets (MIMO radar).
  - we want to model the vector-signal at the output of the Rx-array.



# Multi Target MIMO Radar: Modelling of the Rx Vector-Signal x(t) (cont.)

- The impulse response of the MIMO multi-target backscatter is

$$\boxed{\text{MIMO Radar, multi-target: } \underline{h}(t) = \sum_{\ell=1}^{M} \beta_\ell . \underline{S}_\ell . \overline{\underline{S}}_\ell^H \underline{\delta}(t - \tau_{echo,\ell})} \qquad (9)$$

$\tau_{echo,\ell}$

where $\beta_\ell$ has been re-defined to include also the Doppler term of the $\ell$ target.

$$\beta_\ell = \sqrt{\frac{G_{Tx} G_{Rx}}{(4\pi)^3}} \frac{\lambda}{R_\ell^2} \sqrt{RCS} \cdot \exp\left(-2\pi f_c \frac{2R_\ell}{c}\right) \exp(j\psi)$$

- The received $(N \times 1)$ vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned} \underline{x}(t) &= \underline{h}(t) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\ &= \left( \sum_{\ell=1}^{M} \beta_\ell . \underline{S}_\ell . \overline{\underline{S}}_\ell^H \underline{\delta}(t - \tau_{echo,\ell}) \right) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \end{aligned}$$

$$\boxed{\Rightarrow \underline{x}(t) = \underbrace{\sum_{\ell=1}^{m} \beta_\ell . \underline{S}_\ell . \overline{\underline{S}}_\ell^H \underline{b}(t - \tau_{echo,\ell})}_{\triangleq \text{multi target echo}} + \underline{n}(t) + \underline{x}_c(t)} \qquad (10)$$

where $\underline{n}(t)$ and $\underline{x}_c(t)$ are the noise and clutter terms (see Equ.7 and 8).

# Monostatic MIMO and Phased-Array Radar

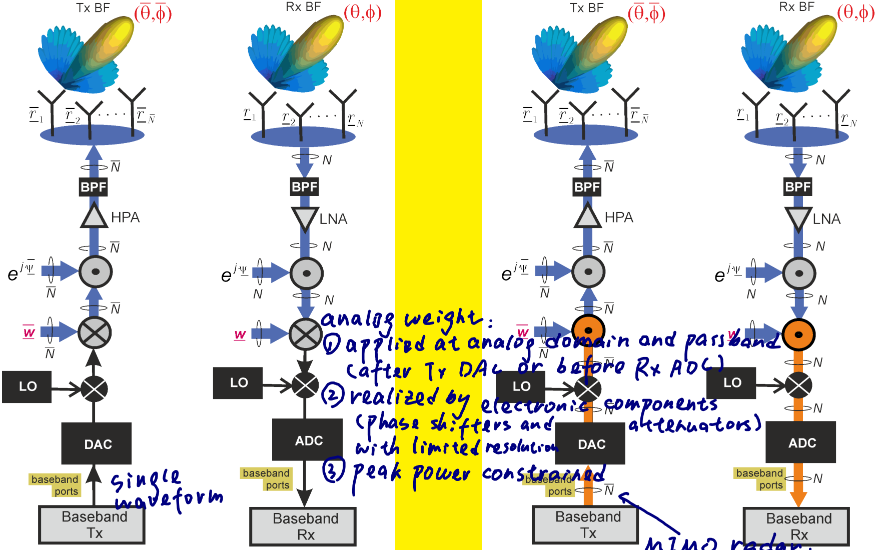
- Phased-Array radar may be SIMO or MIMO.
- Phased-Array radar: if both Tx and Rx employ an antenna array then this belongs to the family of MIMO radar.
- In a similar fashion to a digital phased array radar in a digital MIMO radar each antenna has its own transceiver module and its own A/D converter.
- In a Phased-Array radar, the baseband signal generator produces a signal (scalar signal) which is copied to each Tx-antenna.
- In a MIMO radar, the baseband signal generator produces a vector-signal and each Tx antenna has its own arbitrary baseband waveform generator. This individual waveforms is also the basis for an assignment of the echo signals to their source.

  phased array: same waveform on all Tx elements.
  MIMO radar: independent waveform (ideally orthogonal) for diversity.
- Note that some people wrongly believe that Phased-Array radar and MIMO radar are two different systems. This is reflected in some old publications where MIMO radar and phased-array radar are compared but the phased array radar in these comparisons is often a SIMO and not a MIMO phased-array radar.

# MIMO Analogue Representation vs Phased-Arrays

## Analogue Phased Array (MIMO)

## Analogue Classical MIMO



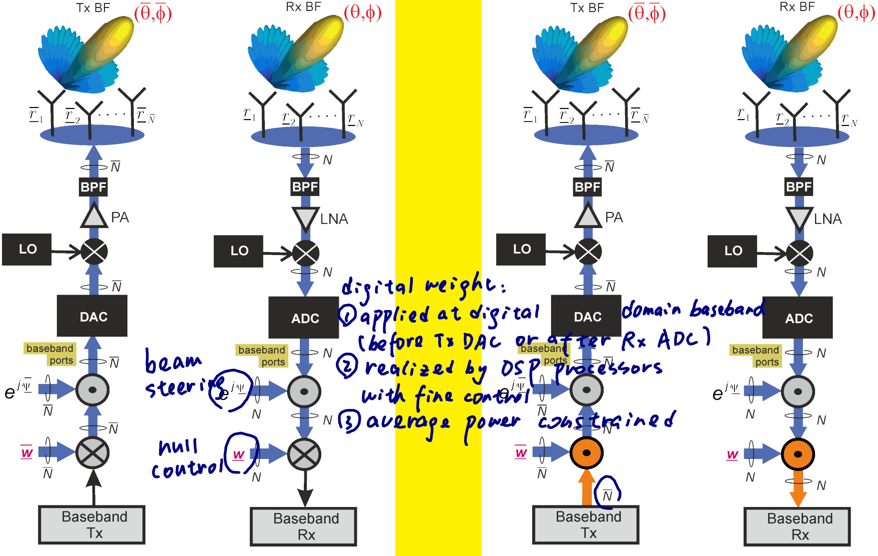

single waveform

analog weight:
① applied at analog domain and passband (after Tx DAC or before Rx ADC)
② realized by electronic components (phase shifters and attenuators) with limited resolution
③ peak power constrained

MIMO radar: N waveform

# MIMO Digital Representation vs Phased-Arrays

## Digital Phased Array (MIMO) / Digital Classical MIMO

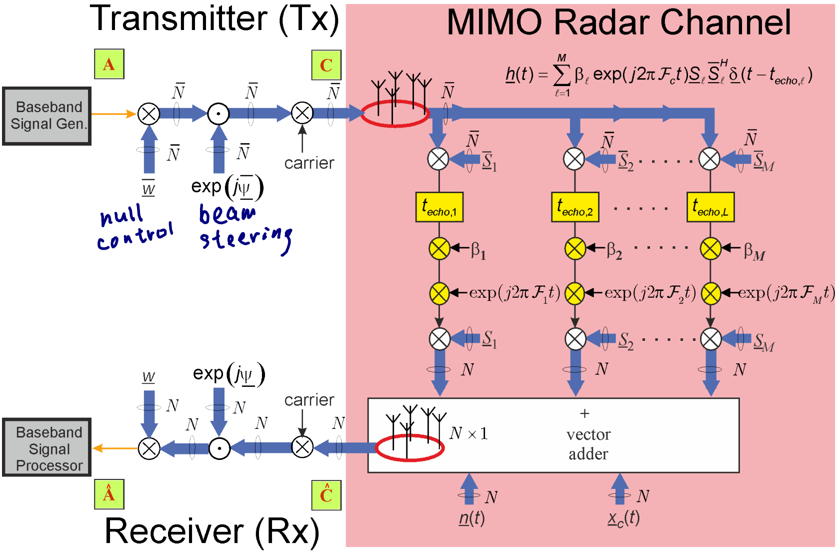


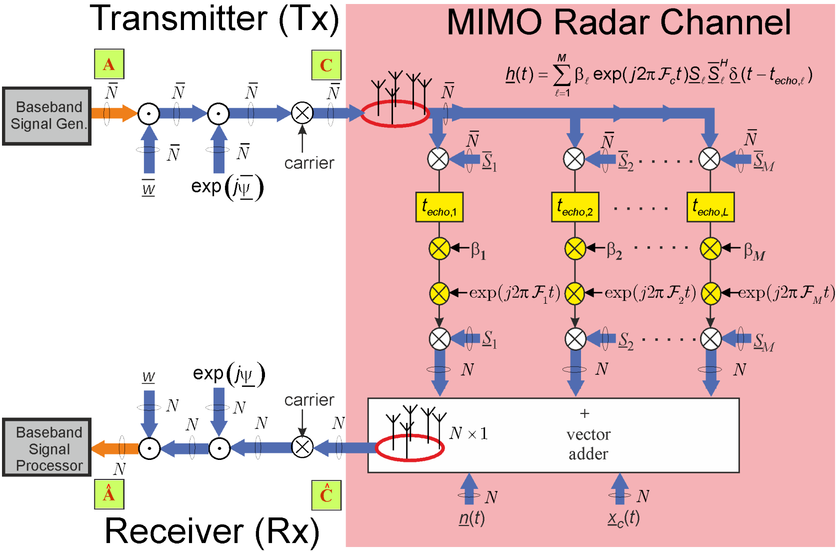

beam steering

null control

digital weight:
① applied at digital domain baseband (before Tx DAC or after Rx ADC)
② realized by DSP processors with fine control
③ average power constrained

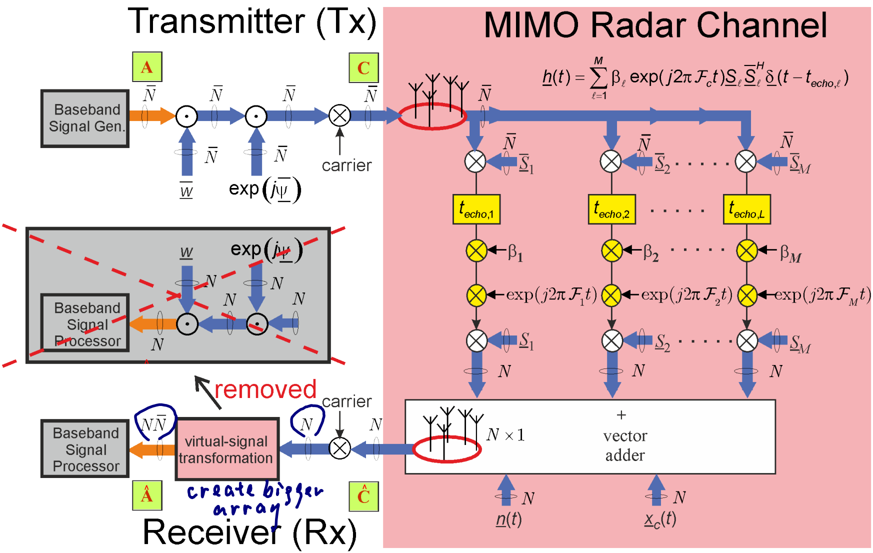
# Modelling: Phased-Array Radar



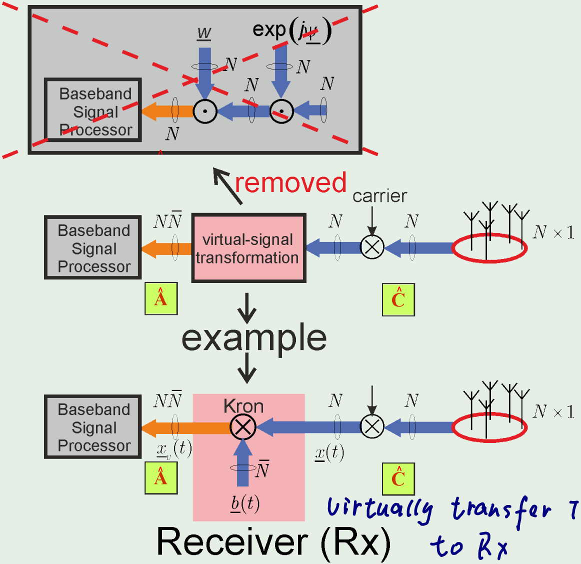


Transmitter (Tx)
MIMO Radar Channel
A
C
Baseband Signal Gen.
carrier
exp(jψ)
$\underline{h}(t)=\sum_{\ell=1}^{M}\beta_\ell \exp(j2\pi\mathcal{F}_c t)\underline{S}_\ell\underline{\bar{S}}_\ell^H\delta(t-t_{echo,\ell})$
$t_{echo,1}$
$t_{echo,2}$
$t_{echo,L}$
$\beta_1$
$\beta_2$
$\beta_M$
$\exp(j2\pi\mathcal{F}_1 t)$
$\exp(j2\pi\mathcal{F}_2 t)$
$\exp(j2\pi\mathcal{F}_M t)$
$N\times 1$
+
vector adder
$\underline{n}(t)$
$\underline{x}_c(t)$
Baseband Signal Processor
Â
Ĉ
Receiver (Rx)

# Virtual-MIMO Radar Transformation



# Example (Example of a virtual signal transformation)




Virtually transfer Tx antennas to Rx

## Kronecker Product

- the symbol $\otimes$ denotes the Kronecker product of two matrices or vectors

- for two matrices $\mathbb{A}$ and $\mathbb{B}$

$$\text{if } \mathbb{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ then } \mathbb{A} \otimes \mathbb{B} = \begin{bmatrix} A_{11}\mathbb{B} & A_{12}\mathbb{B} \\ A_{21}\mathbb{B} & A_{22}\mathbb{B} \end{bmatrix} \tag{11}$$

- for two vectors $\underline{A}$ and $\underline{B}$

$$\text{if } \underline{A} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \text{ then } \underline{A} \otimes \underline{B} = \begin{bmatrix} A_1\underline{B} \\ A_2\underline{B} \end{bmatrix} \tag{12}$$

# Three Linear Properties used in Virtual MIMO

The ***Commutative Property*** (commutative comes from “commute” or “move around”) which is the rule that refers to moving stuff around.

for addition, the rule is

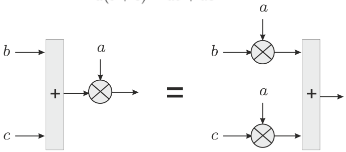
$$a+b=b+a$$

for multiplication, the rule is

$$ab=ba$$

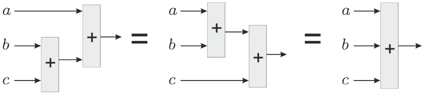
The ***Distributive Property*** which is the rule that states that "multiplication distributes over addition".

$$a(b+c)=ab+ac$$

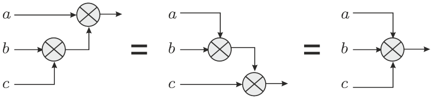



The ***Associative Property*** (associative comes from "associate" or "group"), which is the rule that refers to grouping.

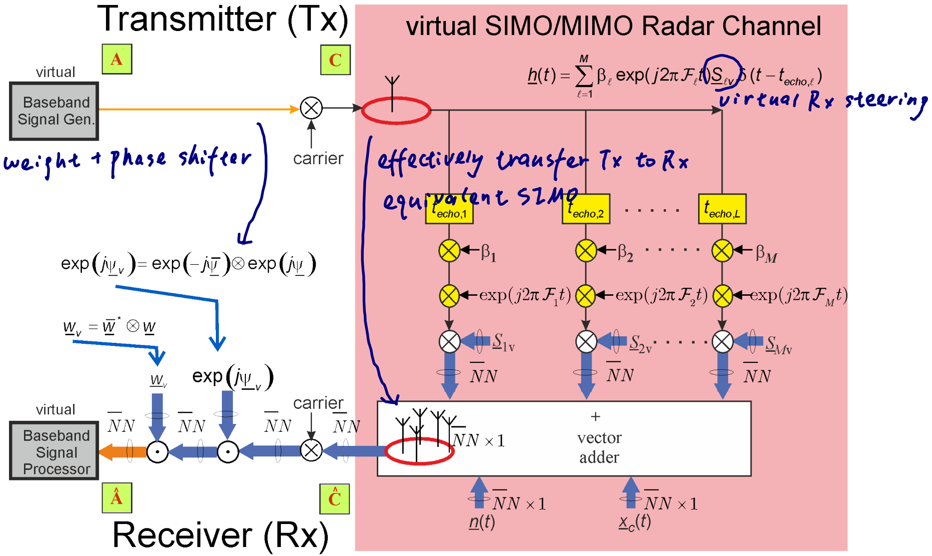
$$a+(b+c)=(a+b)+c=a+b+c$$




$$a(bc)=(ab)c=abc$$



# Equivalent Virtual MIMO Modelling



# Spatial Convolution and Virtual Antenna Array

Tx-array: $\underline{\underline{\overline{\mathbf{r}}}} \triangleq [\underline{\overline{r}}_1, \underline{\overline{r}}_2, ..., \underline{\overline{r}}_{\overline{N}}] = [\underline{\overline{r}}_{\mathbf{x}}, \underline{\overline{r}}_{\mathbf{y}}, \underline{\overline{r}}_{\mathbf{z}}]^T \quad (3 \times \overline{N})$

Rx-array: $\underline{\underline{\mathbf{r}}} \triangleq [\underline{r}_1, \underline{r}_2, ..., \underline{r}_N] = [\underline{r}_{\mathbf{x}}, \underline{r}_{\mathbf{y}}, \underline{r}_{\mathbf{z}}]^T \quad (3 \times N)$

virtual-array:

$$\underline{\underline{\mathbf{r}}}_{\text{virtual}} \triangleq \underline{\underline{\overline{\mathbf{r}}}} \otimes \underline{1}_N^T + \underline{1}_{\overline{N}}^T \otimes \underline{\underline{\mathbf{r}}} \qquad (13)$$

$$\Updownarrow$$

$$\underline{S}_{\text{virtual}} \triangleq \underline{\overline{S}}^* \otimes \underline{S} \qquad (14)$$

$\bar{N}N \times 1$

recall that $h(t)$ and $S_i(\theta,\phi)$ are colinear.

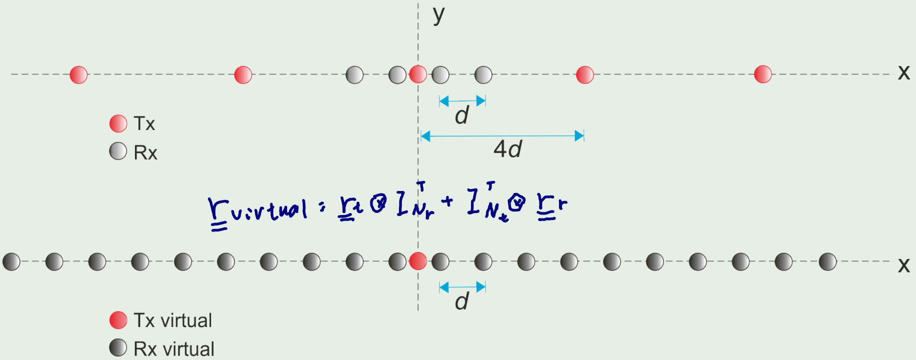
virtualization allows to process more objects without physical adjustment

$(M \leq N_r \Rightarrow M \leq N_t N_r)$

## N.B.:

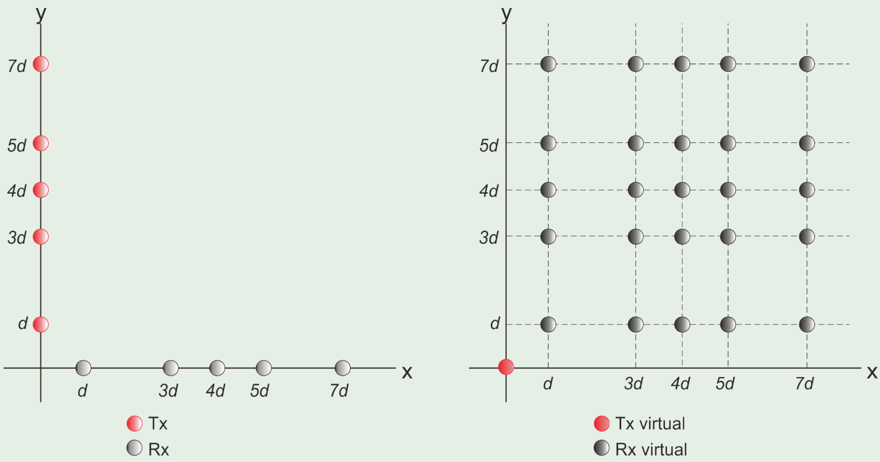
1. Equation 13 is known as "spatial convolution".
2. The concept of the "virtual array", is also applicable to MISO.
3. A MIMO is equivalent to a "virtual-MISO" (by virtually transferring the Tx antenna array to the Rx) or to a "virtual-SIMO" (by virtually transferring the Rx antenna array to the Tx).

## Example (MIMO: Tx=linear; Rx=linear)

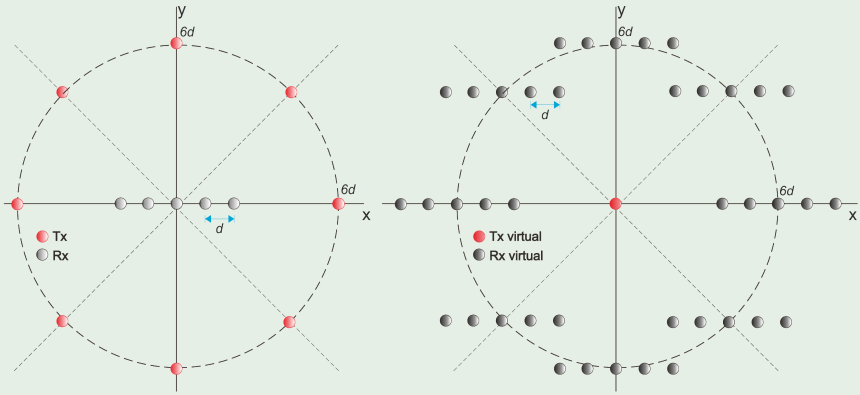



$$\underline{r}_{virtual} = \underline{r}_t \otimes 1_{N_r}^T + 1_{N_t}^T \otimes \underline{r}_r$$

## Example (MIMO: Tx=linear, y-axis; Rx=linear, x-axis.)



## Example (MIMO: Tx=circular; Rx=Linear)



# MIMO Radar: Classical MIMO Estimation Algorithms

- Classical MIMO algorithms: $\exists$ various algorithms for estimating $\beta$ and directions $(\theta, \phi)$. This is done by expressing $\beta$ as a function of direction $(\theta, \phi)$ and then searching for the maximum.
  - Least Squares (LS) algorithm
  - Capon's algorithm
  - Amplitude and Phase Estimation (APES) algorithm
  - Approximate Maximum Likelihood (AML)
  - Capon-and-AML (CAML) algorithm

- LS equation

$$\boxed{\beta(\theta,\phi) = \frac{\underline{S}(\theta,\phi)^H.\mathbb{R}_{xb}.\overline{\underline{S}}(\theta,\phi)}{N.\overline{N}}} ; \tag{15}$$

- Capon's equation:

$$\boxed{\beta(\theta,\phi) = \frac{\underline{S}(\theta,\phi)^H.\mathbb{R}_{xx}^{-1}.\mathbb{R}_{xb}.\overline{\underline{S}}(\theta,\phi)}{\overline{N}.\underline{S}(\theta,\phi)^H.\mathbb{R}_{xx}^{-1}.\underline{S}(\theta,\phi)}} ; \tag{16}$$

- In Equs 15 and 16, $\mathbb{R}_{xm}$ and $\mathbb{R}_{xx}$ are defined as follows:

$$\mathbb{R}_{xb} = \mathcal{E}\left\{\underline{x}(t).\underline{b}^H(t)\right\} \simeq \frac{1}{L}\mathbb{X}\mathbb{B}^H;\ \mathbb{R}_{xb}\in C^{N\times\overline{N}} \tag{17}$$

$$\mathbb{R}_{xx} = \mathcal{E}\left\{\underline{x}(t).\underline{x}^H(t)\right\} \simeq \frac{1}{L}\mathbb{X}\mathbb{X}^H;\ \mathbb{R}_{xx}\in C^{N\times N} \tag{18}$$

$$L = \text{number of samples/snapshots} \tag{19}$$

$$\mathbb{X} = L \text{ snapshots of } \underline{x}(t);\ \mathbb{X}\in C^{N\times L} \tag{20}$$

$$\mathbb{B} = L \text{ snapshots of } \underline{b}(t);\ \mathbb{B}\in C^{\overline{N}\times L} \tag{21}$$

$$\boxed{(\theta,\phi) = \arg\max_{\forall\theta,\phi}\ \beta(\theta,\phi)} \tag{22}$$

# MIMO Radar: Virtual MIMO Estimation Algorithms

- Virtual MIMO algorithms:
  - Virtual Steering Vector algorithms
  - Virtual Capon's algorithm
  - Virtual Multiple-Signal-Classification (virtual-MUSIC) algorithm
- Virtual Steering Vector equation

$$\xi(\theta, \phi) = \frac{\underline{S}_v^H(\theta,\phi)\mathbb{R}_{x_v x_v}\underline{S}_v(\theta,\phi)}{(\overline{N}N)^2} \tag{23}$$

- Virtual Capon's equation

$$\xi(\theta, \phi) = \frac{1}{\underline{S}_v^H(\theta,\phi)\mathbb{R}_{x_v x_v}^{-1}\underline{S}_v(\theta,\phi)} \tag{24}$$

- $(\theta, \phi)$ can be estimated as follows:

$$(\theta, \phi) = \arg\max_{\forall\theta,\phi} \ \xi(\theta, \phi) \tag{25}$$

- $\mathbb{R}_{x_v x_v}$ denotes the covariance matrix of the virtual signal $\underline{x}_v(t)$, i.e.

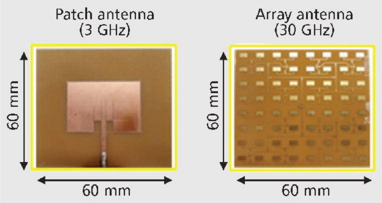
$$\mathbb{R}_{x_v x_v} = \mathcal{E}\left\{\underline{x}_v(t).\underline{x}_v\,(t)^H\,(t)\right\} \simeq \frac{1}{L}\mathbb{X}_v\mathbb{X}_v^H; \mathbb{R}_{x_v x_v} \in C^{N\overline{N}\times N\overline{N}} \tag{26}$$

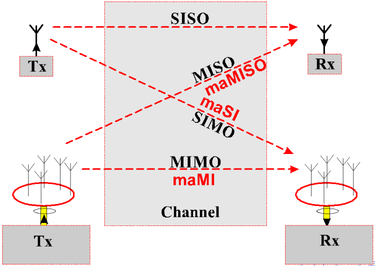
# MIMO Advantages

- More degrees of freedom due to the potential of using the concept of "virtual" antennas
- Higher angular resolution.
- Higher number of targets/clutter in a given range-Doppler cell, which can be detected and localized.
- Lower sidelobes by virtual spatial windowing.
- Digital beamforming and beam-steering at both the Tx and the Rx arrays, and therefore avoid beam shape loss in cases that the target is not in the center of the beam.
- Decrease the spatial power density of the Tx signal – spatial spread spectrum (spatial-SSS) which is critical for Low Probability of Intercept Radar (LPI-Radar).

# Massive MIMO Radar

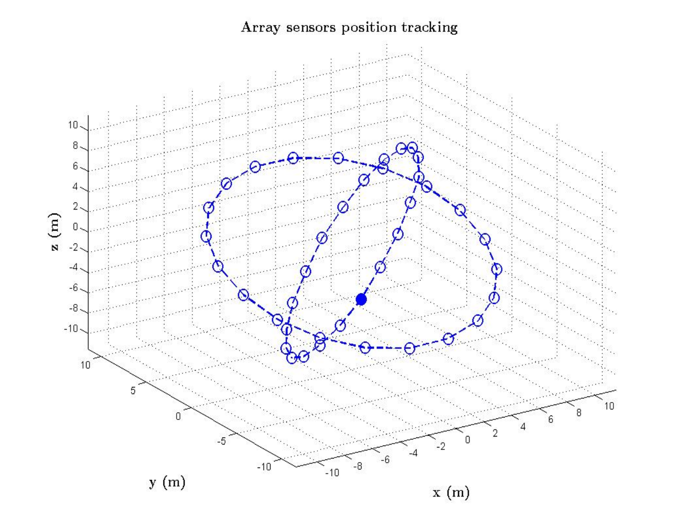
- There is an increased interest in "**massive" MIMO**, i.e. to increase the "degrees-of-freedom" by **increasing the number of antennas to 100s or 1000s**.

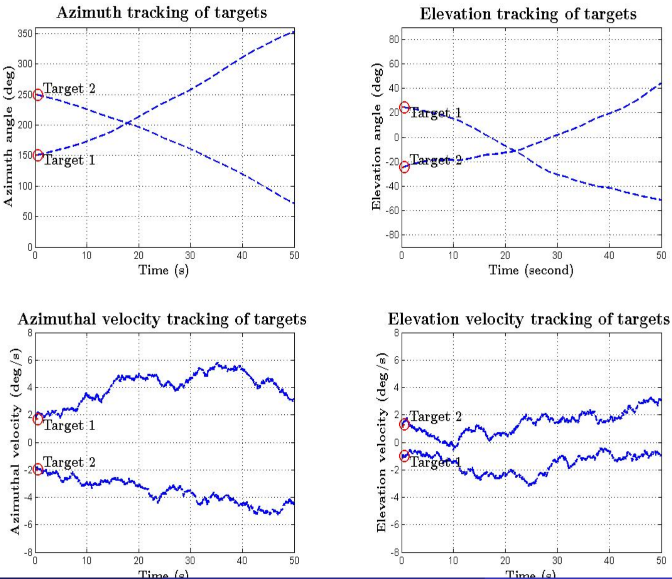


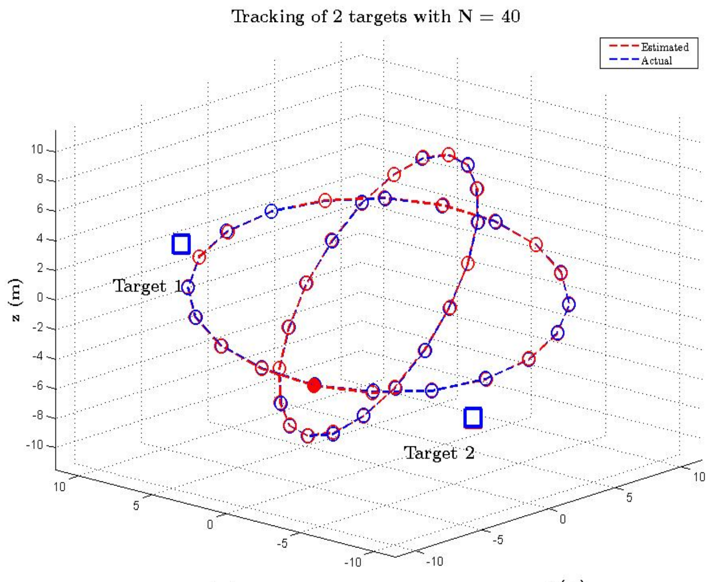
# MIMO Radar: Representative Example

## Example (MIMO Radar - Cluster of UAVs)




Azimuth tracking of targets
Azimuth angle (deg)
Time (s)
Target 2
Target 1
Elevation tracking of targets
Elevation angle (deg)
Time (second)
Target 1
Target 2
Azimuthal velocity tracking of targets
Azimuthal velocity (deg/s)
Target 1
Target 2
Elevation velocity tracking of targets
Elevation velocity (deg/s)
Target 2
Target 1

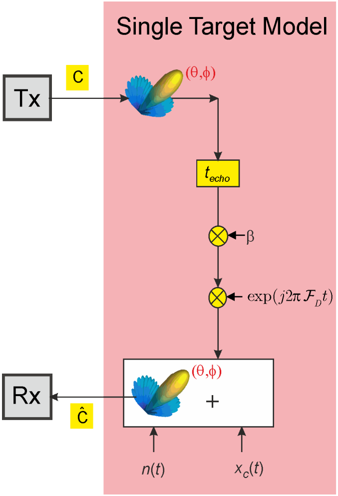
# Example (MIMO Radar: Tracking the UAV Geometry)



# Appendices

- SIMO Radar: quite popular
- SIMO architectures: very powerful; more popular in bistatic radar (the Rx antenna can easily estimate, amongst others, the target direction $\theta_{Rx}$).
- MISO architectures: Not so popular (limited radar applications)
- MIMO Radar architectures: more powerful than other radar architectures; more complex

# Appendix-A: Monostatic SISO Radar




$$
\begin{aligned}
h(t) &= \beta.\exp(j2\pi\mathcal{F}_D t).\delta(t-t_{echo}) \\
&= \text{impulse response}
\end{aligned}
$$

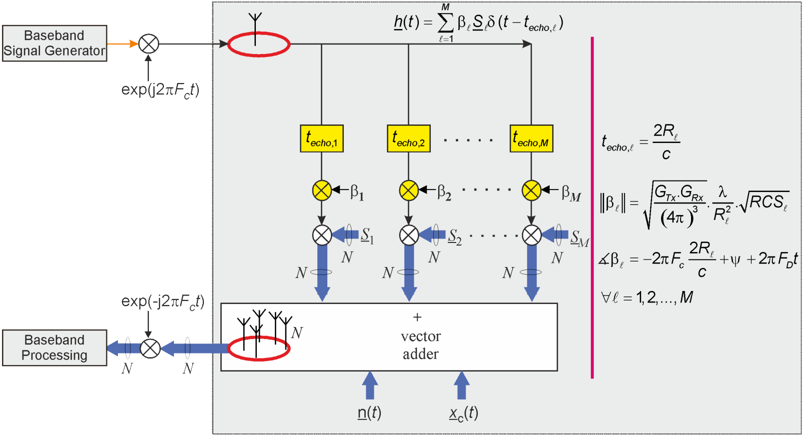
$$t_{echo} = \frac{2R}{c}$$

$$\beta = \sqrt{\frac{G_{Tx}.G_{Rx}}{(4\pi)^3}}.\frac{\lambda}{R^2}\sqrt{RCS}.\exp\left(-j2\pi F_c\frac{2R}{c}\right)\exp(j\psi)$$

$$\mathcal{F}_D = \text{Doppler frequency}$$

$$\psi = \text{random phase}$$

# Appendix-B: Monostatic SIMO Radar




- The vector $\underline{S}_\ell = \underline{S}(\theta_\ell, \phi_\ell) \in \mathcal{C}^N$, is the Rx array manifold vector of the $\ell$-th target.

- Consider the Tx transmits a scalar baseband signal $b(t)$ in the presence of $M$ targets (SIMO radar).
- In a SIMO radar, if there are $M$ targets, the multi-target backscatter impulse response (vector) is

$$\text{SIMO radar, multi-target: } \underline{h}(t) = \sum_{\ell=1}^{M} \underline{S}(\theta_\ell, \phi_\ell).\beta_\ell.\delta(t - \tau_{echo,\ell}) \quad (27)$$

and the received $(N \times 1)$ vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned} \underline{x}(t) &= \underline{h}(t) * b(t) + \underline{n}(t) + \underline{x}_c(t) \quad (28) \\ &= \left( \sum_{\ell=1}^{M} \underline{S}(\theta_\ell, \phi_\ell).\beta_\ell.\delta(t - \tau_{echo,\ell}) \right) * b(t) + \underline{n}(t) + \underline{x}_c(t) \end{aligned}$$

$$\Rightarrow \underline{x}(t) = \underbrace{\sum_{\ell=1}^{M} \underline{S}(\theta_\ell, \phi_\ell).\beta_\ell.b(t - \tau_{echo,\ell})}_{\triangleq \text{multi-target echoes}} + \underline{n}(t) + \underline{x}_c(t) \quad (29)$$

where $\underline{n}(t)$ and $\underline{x}_c(t)$ are the noise and clutter terms (see Equ.7 and 8)

- **If** the radar Tx employs an array of $\overline{N}$ antennas with locations $\underline{\underline{\mathbf{r}}}$ **then** the backscatter impulse response is:
  - for a single target:

$$\text{MISO-radar, single target: } h(t) = \beta \underline{\overline{S}}^H \underline{\delta}(t - t_{echo}) \qquad (30)$$

where

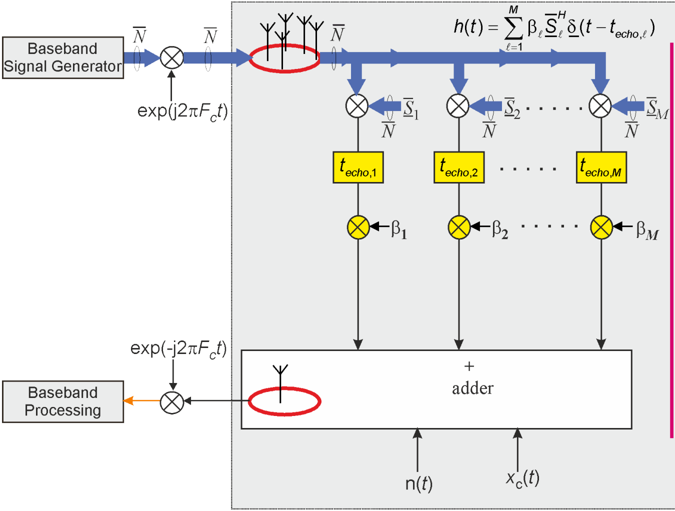
$$\begin{aligned} \underline{\overline{S}} &= \underline{\overline{S}}(\theta, \phi) = \exp(+j[\underline{\overline{r}}_1, \underline{\overline{r}}_2, \ldots, \underline{\overline{r}}_N]^T \underline{k}(\overline{\theta}, \overline{\phi})) & (31) \\ &= \exp(+j[\underline{\overline{r}}_x, \underline{\overline{r}}_y, \underline{\overline{r}}_z]\underline{k}(\overline{\theta}, \overline{\phi})) & (32) \\ &= (N \times 1) \text{ complex vector} \end{aligned}$$

$$\underline{k}(\overline{\theta}, \overline{\phi}) = \frac{2\pi F_c}{c} . \underline{u}(\overline{\theta}, \overline{\phi}) = \frac{2\pi}{\lambda_c} . \underline{u}(\overline{\theta}, \overline{\phi})$$

  - for a multi-target:

$$\text{MISO-radar, multi-target: } h(t) = \sum_{\ell=1}^{M} \beta_\ell . \underline{\overline{S}}_\ell^H \underline{\delta}(t - \tau_{echo,\ell}) \qquad (33)$$

where the vector $\underline{\overline{S}}_\ell = \underline{\overline{S}}(\theta_\ell, \phi_\ell) \in C^N$, is the Tx array manifold vector of the $\ell$-th target.




$$t_{echo,\ell} = \frac{2R_\ell}{c}$$

$$\|\beta_\ell\| = \sqrt{\frac{G_{Tx}.G_{Rx}}{(4\pi)^3}.\frac{\lambda}{R_\ell^2}}.\sqrt{RCS_\ell}$$

$$\measuredangle\beta_\ell = -2\pi F_c\frac{2R_\ell}{c} + \psi + 2\pi F_D t$$

$$\forall \ell = 1, 2, \ldots, M$$

- Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ in the presence of $M$ targets (MISO radar).
- The received *scalar*-signal $x(t)$ can be modelled as follows:

$$\begin{aligned} x(t) &= h(t) * \underline{b}(t) + \mathrm{n}(t) + \underline{x}_c(t) \\ &= \left( \sum_{\ell=1}^{M} \beta_\ell . \overline{\underline{S}}_\ell^H \underline{\delta}(t - \tau_{echo,\ell}) \right) * \underline{b}(t) + \mathrm{n}(t) + x_c(t) \end{aligned}$$

$$\Longrightarrow x(t) = \underbrace{\sum_{\ell=1}^{M} \beta_\ell . \overline{\underline{S}}_\ell^H \underline{b}(t - \tau_{echo,\ell})}_{\triangleq \text{scalar echoes}} + \mathrm{n}(t) + x_c(t)$$