E401: Advanced Communication Theory

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Multi-Antenna Wireless Communications

Spatiotemporal Wireless Communications, mmWave & Massive MIMO
- Part-I -

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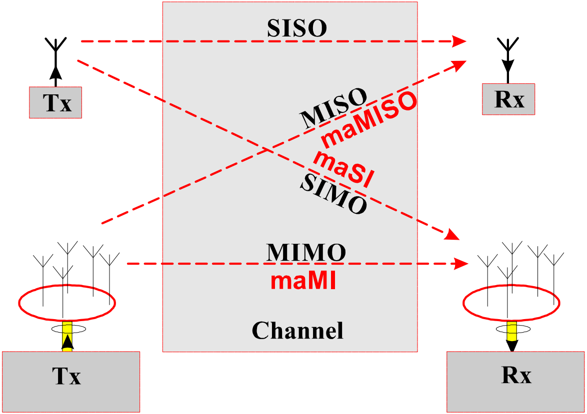
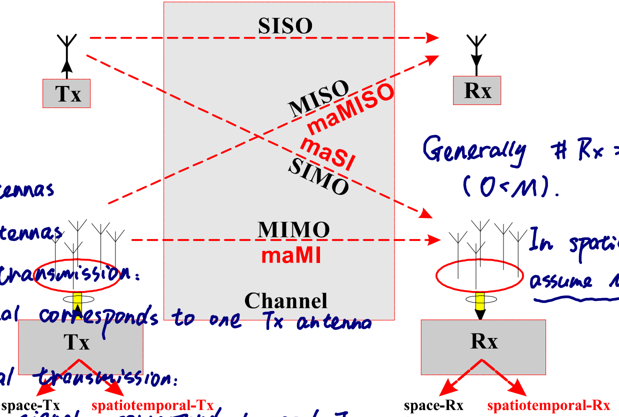
Increasing the Degrees-of-Freedom

- There is an increased interest for beamformer-based communications in 5G. Issues:
  - high path loss,
  - very dense co-channel interference environment, and
  - multipath effects in frequency selective channels.
- **Solution** : to employ **massive MIMO** , i.e. to increase the "degrees-of-freedom" by **increasing** N (remember: $M < N$)
  - Multiple-antenna (MIMO): the technology is becoming mature for wireless communications
  - It has been incorporated into wireless broadband standards like LTE and Wi-Fi.
  - *Non-parametric* massive: problematic (N=↑ ⇒number-of-unknowns=↑)
  - *Parametric* massive: OKAY (N=↑ ⇒number-of-unknowns≠↑),
- **Alternatively** : use array processing (keep number of antennas fixed & extend the array manifold) ⇒ e.g. **spatiotemporal approaches**

add more params. to manifold

array manifold
geometry
DOA
elements (fixed)
⋮


SISO
Tx
Rx
MISO
maMISO
maSI
SIMO
MIMO
maMI
Channel
Tx
Rx



M : # signals

N : # Rx antennas

O : # Tx antennas

- space-only transmission:
  - one signal corresponds to one Tx antenna ($M = O$)
- spatiotemporal transmission:
  - multiple signals correspond to each Tx antenna ($M > O$: users share antennas)

Generally # Rx > # Tx ants. ($O < M$).

In spatiotemporal Rx assume $M > N$.

- space-only RX: $M < N$
- spatiotemporal Rx: M can be greater than N ($M > N$)

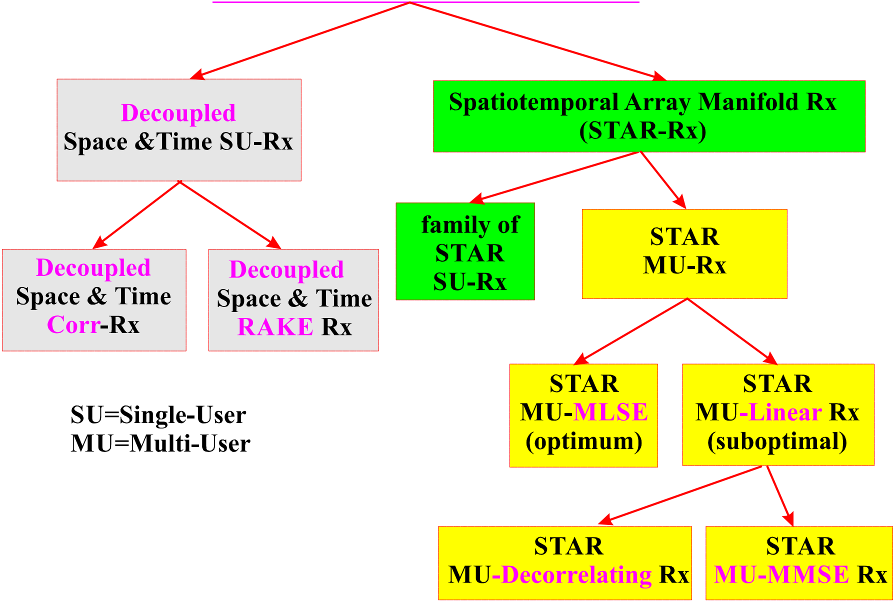
Space-Time Communications

- Space-Time Communications can be employed in both **TDMA/FDMA** and **CDMA** type systems

- In **TDMA/FDMA** the signal is not spread and only few (mainly one or two - i.e. $M < N$) strong cochannel interferences (CCI) are present when the system employs channel reuse between cells.
  The array can be used to null (remove/reduce) these few interferences TDMA/FDMA: few strong CCI
  CDMA: numerous weak MAI
  $\Big\}$ array solution

- In **CDMA** all active users use the same bandwidth and are separated by employing different PN-codes to reduce/remove the MAI interference from other user.
  i.e. in a **CDMA** environment **the array has to deal with a very large number of weak interferences** $(M > N)$.

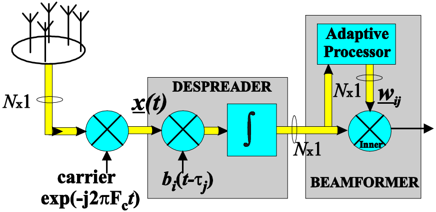
CLASSIFICATION of ST-CDMA Rx



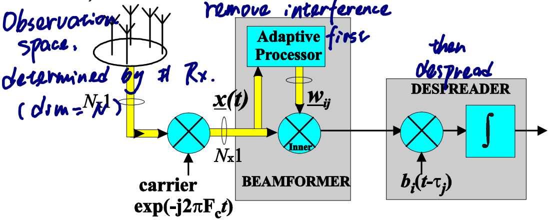
Decoupled Space &Time CDMA Rx

- Two representative examples of decoupled 'Space-Time CDMA Base Station Receiver' architecture are shown below (to receive the j-th path of the i-th user)

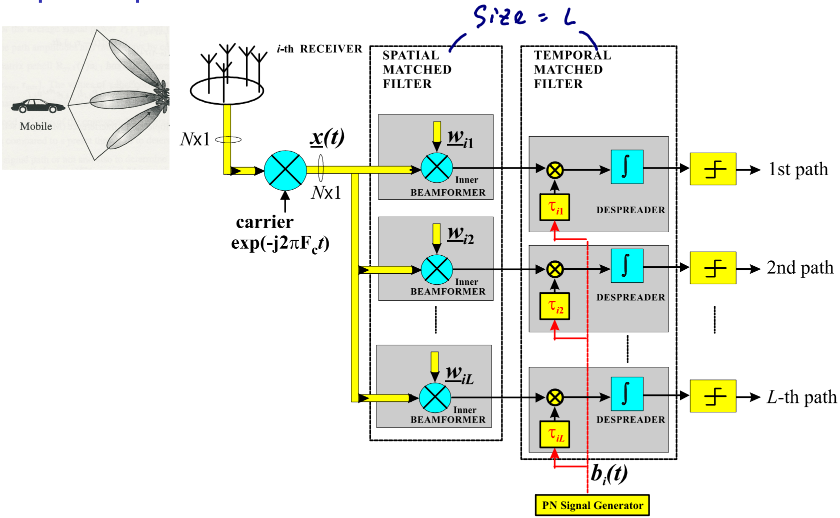
1) "**Time** " and then "**Space** "



2) "**Space** " and then "**Time** "

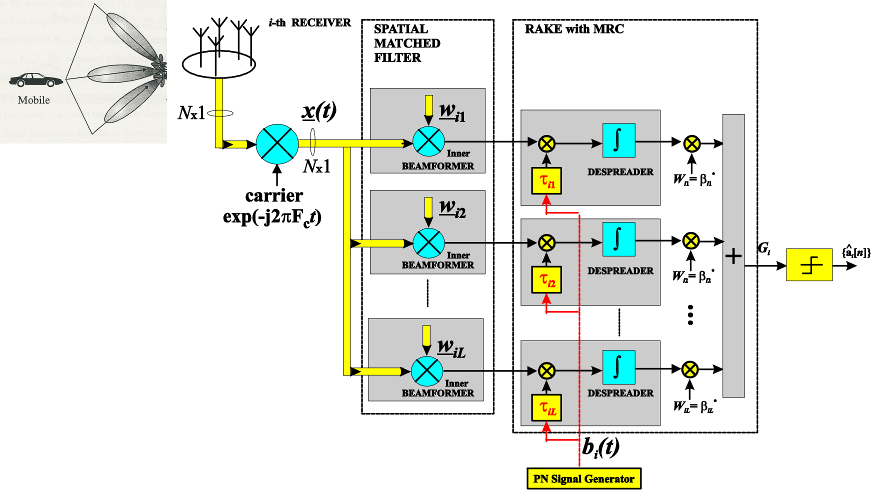


Decoupled Space & Time CDMA Receiver



- each path is received/isolated (i.e. no diversity)

Decoupled Space & Time RAKE CDMA Receiver



- Multipath diversity (all multipaths are separated and combined)

Spatiotemporal Wireless Approaches

- array manifold: we can add more wireless parameters from the Tx, Rx and channel

- For instance

DOA carrier freq.

position of Rx array

$\underline{S}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z)$ → params. of a standard manifold vector

pseudorandom sequ, delay, polarisation parameters,

No.of subcarriers/carriers, bandwidth, Doppler frequency).

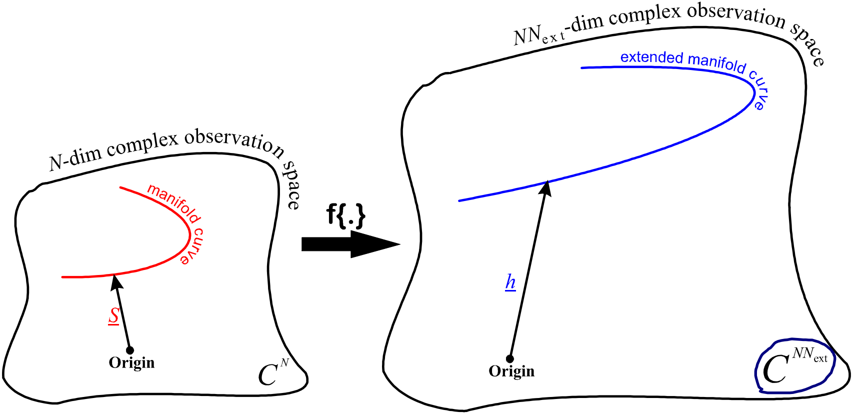
$\Downarrow$ $\underline{h}$: extended version of original $\underline{S}(\theta.\phi.F_c.c.\underline{r}_x.\underline{r}_y.\underline{r}_t)$

$\underline{h}$ in a larger space than $\underline{S}$: more freedom.

$\Downarrow$ more params depend on applications.

$\underline{h} \triangleq$ spatiotemporal manifold [it is a function of the original $\underline{S}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z)$]

Extended Manifolds



Extension of the spatial manifold to the spatiotemporal domain

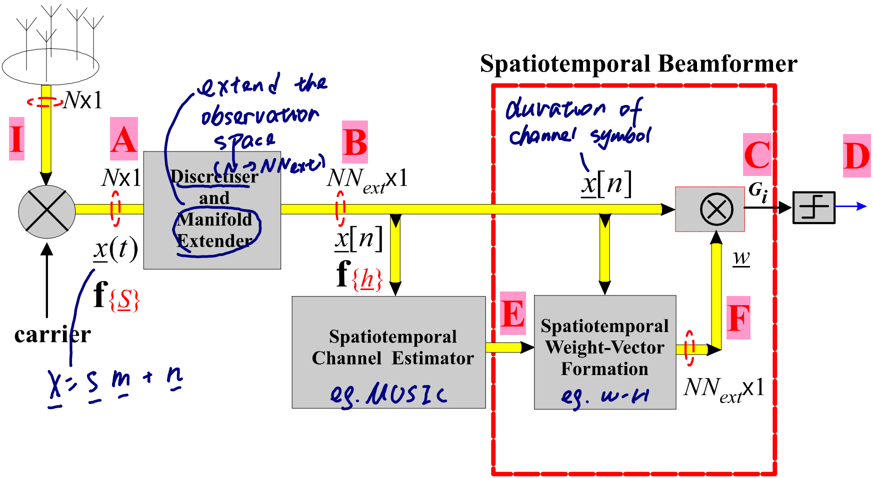
- Extended array manifold vectors can be re-expressed as

define new manifold based on the standard one.

$$\underline{h} = \mathbb{A}\ \underline{S} \tag{1}$$

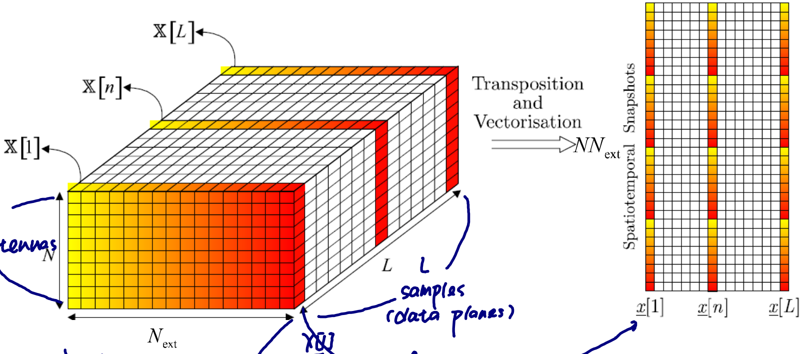
- Each vector $\underline{h}$ is produced by a different $\mathbb{A}$
- spatiotemporal manifold: $\mathcal{H} \triangleq$ locus of all vectors $\underline{h}$
- No need to perform the same analysis multiple times
- Easier to evaluate the effect of changing the system architecture

A Generic Spatiotemporal Rx-Structure



- at point $\boxed{A}$: $\underline{x}(t)$ =function$\{\underline{S}_{ij}\}+\underline{n}(t)$
- at point $\boxed{B}$: $\underline{x}[n]$ =function$\{\underline{h}_{ij}\}+\underline{n}[n]$

3D-data Cube



↳ N_{ext} requirements

frame vectorisation

3D-data Cube

of the problem specifications.

↳ func. of actual systems

↳ PN code with delays.

Vectorisation of 3D-data Cube

ensure antenna data in columns.

$$\underline{x}[n] = \text{vec}\left\{\mathbb{X}[n]^T\right\} \quad (2)$$

Spatiotemporal Wireless Approaches

Introduction

- In this course we will focus on an extended manifold which is:

$$\underline{S}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z, \text{pseudorandom sequ}, \textit{delay}, {\scriptstyle \text{polarisation parameters}}, {\scriptstyle \text{No.of subcarriers/carriers}}, {\scriptstyle \text{bandwidth}}, {\scriptstyle \text{Doppler frequency}}).$$

$$\Downarrow$$

$$\underline{h} \Rightarrow \underline{\text{s}}\text{patio}\underline{\text{t}}\text{emporal } \underline{\text{ar}}\text{ray (STAR) manifolds}$$

- The above is suitable, for instance, for spatiotemporal CDMA systems

Spatiotemporal Manifold Rx

- For the j^{th} path of the i^{th} user the **array manifold vector** is

$$\underline{S}(\theta_{ij}, \phi_{ij}) = \exp(-j[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N]^T \underline{k}(\theta_{ij}, \phi_{ij}) \in \mathcal{C}^{N\times 1}$$

  where $[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N]$ represents the array geometry and $\underline{k}(\theta_{ij}, \phi_{ij})$ is the wavenumber vector.

  array manifold + (PN-code, multipath delay) = STAR manifold

- By taking the **PN-code** and **multipath delay** into consideration, we extend the concept of the array manifold vector to

  SPATIO−**T**EMPORAL **AR**RAY (STAR) manifold vector

  defined as follows:

  shifting-matrix (delay l_{ij})

$$\underline{h}_{ij} \triangleq \underline{S}_{ij} \otimes \mathbb{J}^{l_{ij}} \underline{c}_i \in \mathcal{C}^{2\mathcal{N}_c N\times 1} \quad (3)$$

  $l = \left\lceil \frac{\tau}{T_c} \right\rceil \bmod N_c$

  delay

  T_c - chip period

  N_c - code period

  with $\underline{S}_{ij} \triangleq \underline{S}(\theta_{ij}, \phi_{ij})$ and $\underline{h}_{ij} \triangleq \underline{h}(\theta_{ij}, \phi_{ij}, \ell_{ij})$

- **To achieve this extension,** the array received signal vector $\underline{x}(t) \in \mathcal{C}^{N\times 1}$ is transformed to a discretised signal $\underline{x}[n] \in \mathcal{C}^{2\mathcal{N}_c N\times 1}$

The "Shifting Matrix"

$\underline{\underline{J}}$: down-shift by 1

$\underline{\underline{J}}^T$: up-shift by 1

- The matrix $\mathbb{J}$ is known as a **shifting matrix** (a $2\mathcal{N}_c \times 2\mathcal{N}_c$ matrix) defined as follows

$$\begin{bmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0 \end{bmatrix}\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=\begin{bmatrix} 0\\ a\\ b\\ c \end{bmatrix}$$

$$\begin{bmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=\begin{bmatrix} b\\ c\\ d\\ 0 \end{bmatrix}$$

$$\mathbb{J}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} = \begin{bmatrix} \underline{0}^T_{2\mathcal{N}_c-1} & 0 \\ \mathbb{I}_{2\mathcal{N}_c-1} & \underline{0}_{2\mathcal{N}_c-1} \end{bmatrix} \quad (4)$$

having the property that every time the matrix $\mathbb{J}$ (or $\mathbb{J}^T$) operates on a column vector it down-shifts (or up-shifts) the elements of the vector by one.

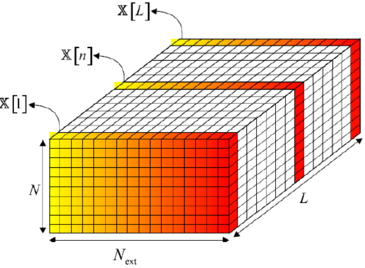
$$\begin{cases} \underline{\underline{J}}^{l}: \text{ down-shift by } l \\ (\underline{\underline{J}}^{T})^{l}: \text{ up-shift by } l. \end{cases} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

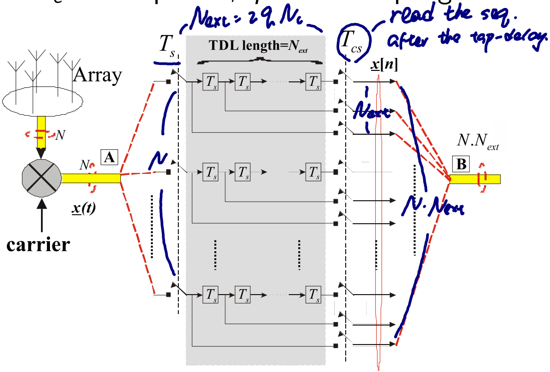
- For instance, $\mathbb{J}^{\ell}\underline{x}$ is a version of $\underline{x}$ down-shifted by ℓ elements, while $\left(\mathbb{J}^{T}\right)^{\ell}\underline{x}$ is a version of $\underline{x}$ up-shifted by ℓ elements.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_K \end{bmatrix}; \quad \mathbb{J}^{3}\underline{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_1 \\ x_2 \\ \vdots \\ x_{K-3} \end{bmatrix}; \quad \left(\mathbb{J}^{T}\right)^{2}\underline{x} = \begin{bmatrix} x_3 \\ x_4 \\ \vdots \\ x_K \\ 0 \\ 0 \end{bmatrix}$$

Example: Spatiotemporal Rx Architecture

- $\boxed{N_{ext} = 2 \times q \times \mathcal{N}_c}$, where: $\mathcal{N}_c$=code period; q =oversampling factor.





- Note: In this course: $q = 1$.
  That is, the array received signal vector $\underline{x}(t)$ is discretised by a chip rate sampler, i.e. $T_s = T_c$.

- The discrete samples are then passed through a tapped-delay line of length equal to $2\mathcal{N}_c$.
  This is to ensure that one whole data symbol of the desired user and the corresponding multipath components are captured within this $2\mathcal{N}_c$ interval.
- As shown in preprocessor's figure the received space-time signal vector $\underline{x}[n]$ is formed by concatenating the contents of the tapped-delay lines of all the antennas,

$$\begin{aligned} \text{i.e. } \underline{x}[n] &= \left[\underline{x}_1[n]^T, \underline{x}_2[n]^T, \ldots, \underline{x}_N[n]^T\right]^T \in \mathcal{C}^{2\mathcal{N}_c N \times 1} \quad (5) \\ &= \text{vec}(\mathbb{X}[n]^T) \quad (6) \end{aligned}$$

  where $\underline{x}_k[n]$ represents the contents of the tapped-delay line at the k^{th} antenna associated with the n^{th} data symbol period, and

$$\mathbb{R}_{xx} = \mathcal{E}\left\{\underline{x}[n].\underline{x}[n]^H\right\} \in \mathcal{C}^{2\mathcal{N}_c N \times 2\mathcal{N}_c N} \quad (7)$$

- Note that the whole theory of PART C can be applied using the above $\mathbb{R}_{xx}$

Estimation Problem M>N

Spatiotemporal Channel Estimation

- The receiver initially estimates, over an observation time, the spatio-temporal manifold parameters of the desired signal(s), which are then employed to remove the MAI and ISI terms
- The estimation, for instance, can be carried out using the following 2D. 'STAR' subspace cost function $\left(n\text{-}th\right.$ interval):

$$\xi(\theta,\ell)=\frac{1}{\underline{h}(\theta,\ell)^{H}\mathbb{P}_{\mathrm{n}}\,\underline{h}(\theta,\ell)} \tag{8}$$

- In Equation 8 the matrix $\mathbb{P}_{\mathrm{n}}$ is the projection operator associated with the "noise subspace" of

$$\mathbb{R}_{xx}=\mathcal{E}\left\{\underline{x}\left[n\right].\underline{x}\left[n\right]^{H}\right\}$$

Main Properties of STAR subspace-type CDMA Receivers:

1. blind (estimation of channel parameters without pilots)
2. separates/estimates all the paths of the desired user in the presence of MAI
3. The number of multipaths that can be resolved is not constrained by the number of array elements (antennas), i.e.

$$M > N$$

4. near-far resistant (i.e. in CDMA there is no need for power control)
5. superresolution capabilities.

STAR Array Pattern

- If the array elements are weighted by complex-weights then the **array pattern** provides the gain of the array as a function of DOAs

$$\text{if } (\theta,\ell) \longmapsto \underline{h}(\theta,\ell) \text{ then } g(\theta,\ell) = \underline{w}^H \underline{h}(\theta,\ell) \tag{9}$$

where $g(\theta,\ell)$ denotes the STAR gain of the array for a signal arriving from direction θ and delayed by τ

$$\text{where } \ell = \left\lceil \frac{\tau}{T_c} \right\rceil \bmod \mathcal{N}_c \tag{10}$$

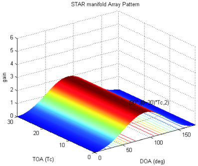
Then

The function $g(\theta,\ell), \forall\theta$ and $\forall\ell$, is known as **the STAR Array Pattern**

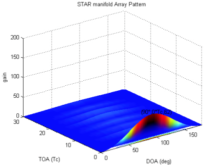
- N.B.: default pattern :
  - space only:
    $g(\theta,\ell) = \underline{1}^T_{2N\mathcal{N}_c}\underline{h}(\theta,\ell)$, i.e. $\underline{w} = \underline{1}_{2N\mathcal{N}_c}$ (i.e. no weights)
  - spatiotemporal:
    $g(\theta,\ell) = (1_N \otimes \underline{c})^T\, \underline{h}(\theta,\ell)$, i.e. $\underline{w} = \underline{h}(90°, 0T_c)$ (i.e. no weights)

filter in { dir
time

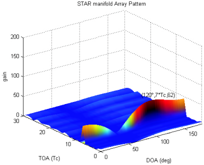
- As an example consider a Uniform Linear Array of N elements using a PN-code of length $\mathcal{N}_c = 31$. The STAR array pattern for $\underline{w} = \underline{\mathfrak{h}}\left(90^{\circ}, 15T_c\right)$ and $N = 2, 3, 4, 5$ is as follows:



$N = 2, \underline{w} = \underline{1}_{2N\mathcal{N}_c}$

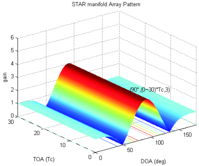


$N = 2, \underline{w} = \underline{h}(90^\circ, 0T_c)$

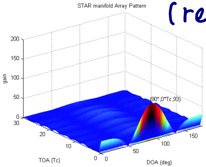


$N = 2, \underline{w} = \underline{h}(120^\circ, 7T_c)$

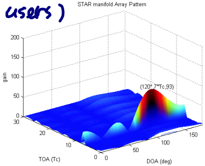
(remove other users)



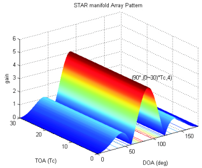
$N = 3, \underline{w} = \underline{1}_{2N\mathcal{N}_c}$



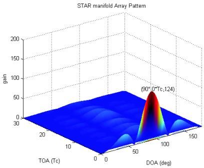
$N = 3, \underline{w} = \underline{h}(90^\circ, 0T_c)$



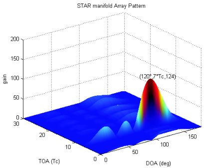
$N = 3, \underline{w} = \underline{h}(120^\circ, 7T_c)$



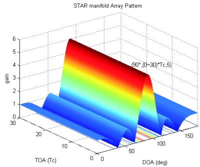
$$N = 4, \underline{w} = \underline{1}_{2N\mathcal{N}_c}$$



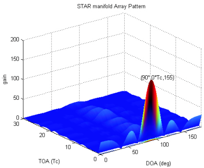
$$N = 4, \underline{w} = \underline{h}(90^\circ, 0T_c)$$



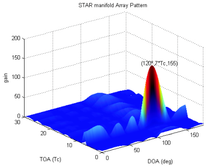
$$N = 4, \underline{w} = \underline{h}(120^\circ, 7T_c)$$



$$N = 5, \underline{w} = \underline{1}_{2N\mathcal{N}_c}$$

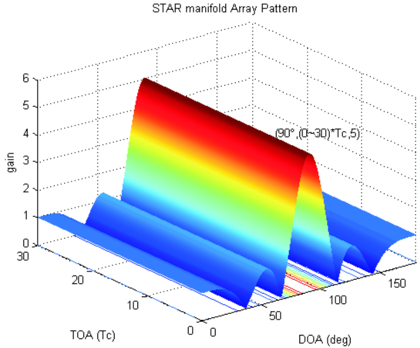


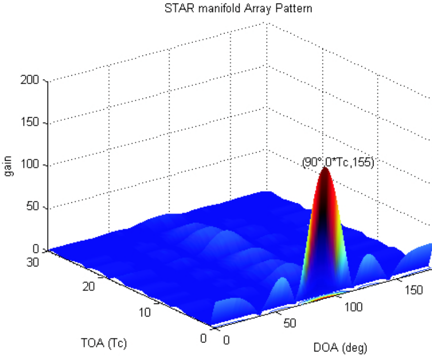
$$N = 5, \underline{w} = \underline{h}(90^\circ, 0T_c)$$



$$N = 5, \underline{w} = \underline{h}(120^\circ, 7T_c)$$

Example: Space-only & Spatiotemporal Gain Patterns





Spatiotemporal Beamformers

- If a single-user Rx (SU-Rx) is used then we only need to estimate the channel parameters of the *i*-th user only (with reference to the last figure, these are provided at the output of "Processor-2". Then, the STAR manifold matrix of the *i*-th user can be formed in the block "Processor-3" as follows

$$\mathbb{H}_i = \left[\underline{h}_{i1}, \quad \underline{h}_{i2}, \quad ..., \quad \underline{h}_{iL_p}\right] \in C^{2N\mathcal{N}c \times L_p} \qquad (11)$$

  where L_p is the number of multipaths.
- If a multi-user Rx (MU-Rx) is used then the channel estimator ("Processor-2") should provide estimates for all users and thus Equ. 11 should be formed in the block "Processor-3" for every i (i.e. $\forall i$).

- Examples of spatiotemporal weight vectors to receive the multipath signals from the i-th user:
  - spatiotemporal-RAKE (SU):

$$\underline{w}_i = \mathbb{H}_i \underline{\beta}_i \tag{12}$$

estimate β_i.
(eg. MRC)

  - spatiotemporal-subspace Rx (SU)

$$\underline{w}_i = \text{constant} \times \mathbb{P}_n \mathbb{H}_i \left(\mathbb{H}_i^H \mathbb{P}_n \mathbb{H}_i \right)^{-1} \underline{\beta}_i . \tag{13}$$

where a scalar constant is used as a normalising factor such that $\|\underline{w}_i\| = 1$.
and $\mathbb{P}_\mathrm{n}$ is the projection operator associated with the "noise subspace" of $\mathbb{R}_{xx} = \mathcal{E}\left\{ \underline{x}[n] . \underline{x}[n]^H \right\}$

Spatiotemporal Capacity

- **SISO capacity** :

$$C = B\log_2(1+\mathrm{SNIR}_{out}) \quad \text{bits/sec}$$

- **MIMO Capacity** :

$$C = B\log_2\left(\frac{\det(\mathbb{R}_{xx})}{\det(\mathbb{R}_{nn})}\right) \quad \text{bits/sec}$$

- If **bandwidth** $\longrightarrow \infty$ then $C = ?$

psd. associated with all other users.

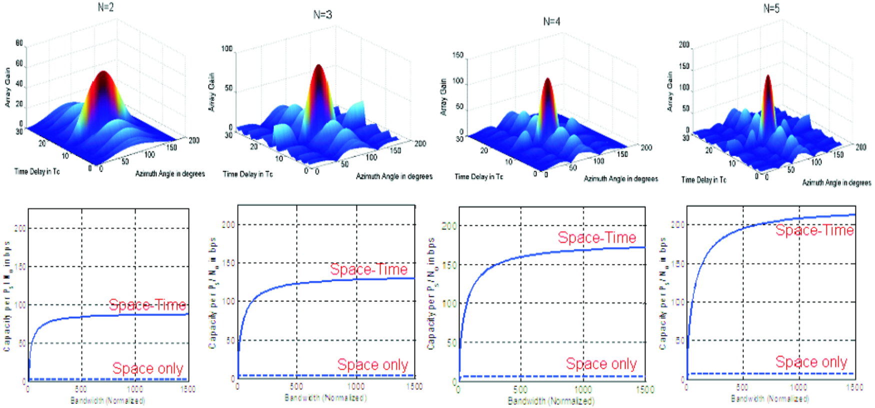
$$\text{SISO} \quad : \quad \lim_{B\to\infty} C = 1.44\frac{P_s}{N_0 + N_J}$$

beamformer reduce $N_J \to 0$ (nulls)

$$\text{space-only SIMO} \quad : \quad \lim_{B\to\infty} C = N \times 1.44\frac{P_s}{N_0 + \underset{0}{\underset{\downarrow}{N_J}}}$$

$$\text{spatiotemporal-SIMO} \quad : \quad \lim_{B\to\infty} C = N \times N_{ext} \times 1.44\frac{P_s}{N_0 + \underset{0}{\underset{\downarrow}{N_J}}}$$

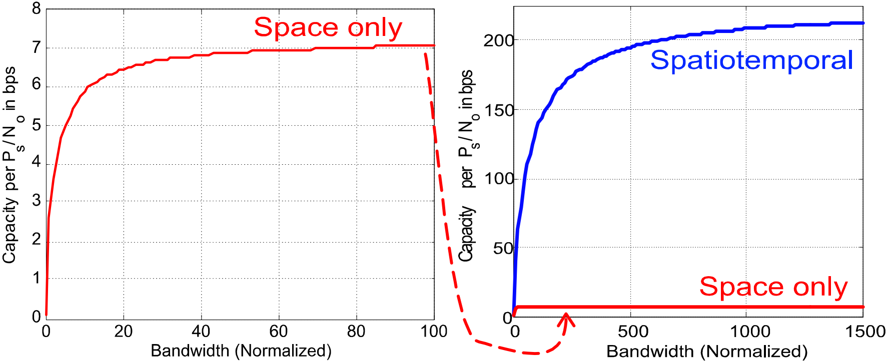
- Capacity gain of arrayed-CDMA-Rx employed joint space-time (spatiotemporal) processing:



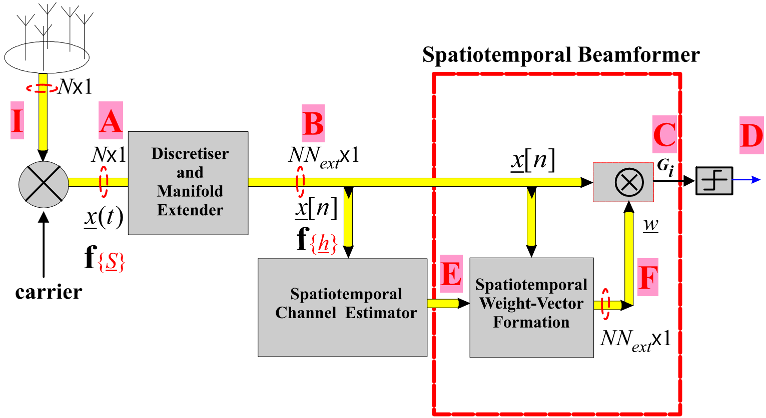
$$\lim_{B\to\infty} C = N\mathcal{N}_c \times 1.44\frac{P_s}{N_0 + N_j \downarrow}$$

Space and Spatiotemporal Capacity Curves

$N = 5$ antennas



Example-1: Multipaths



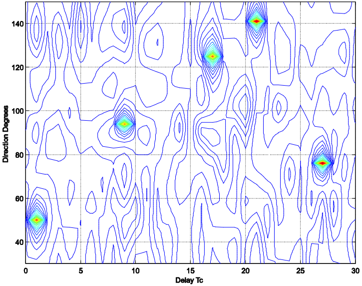
- at point $\boxed{A}$: $\underline{x}(t) = \text{function}\{\underline{S}_{ij}\} + \underline{n}(t)$
- at point $\boxed{B}$: $\underline{x}[n] = \text{function}\{\underline{h}_{ij}\} + \underline{n}[n]$

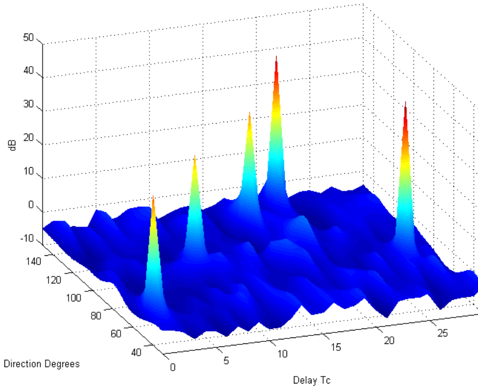
- 3 users, 5 paths per user, 1st user=desired

User 1 (Desired)	Path 1	Path 2	Path 3	Path 4	Path 5
Path Delay (T_c)	1	9	17	21	27
Path Direction (^{o})	50	94	125	141	76
Path Coefficient	-0.10 + 0.26**j**	-0.01 - 0.24**j**	-0.31 - 0.02**j**	-0.31 - 0.02**j**	0.42 - 0.35**j**

User 2 (Interfer)	Path 1	Path 2	Path 3	Path 4	Path 5
Path Delay (T_c)	4	8	17	26	27
Path Direction (^{o})	92	35	149	67	61
Path Coefficient	-0.20 + 0.56**j**	-0.41 - 0.74**j**	-0.39 - 0.92**j**	-0.91 - 0.12**j**	0.76 - 0.00**j**
User 3 (Interfer)	Path 1	Path 2	Path 3	Path 4	Path 5
Path Delay (T_c)	2	13	19	25	27
Path Direction (^{o})	103	84	80	79	116
Path Coefficient	-0.15 + 0.27**j**	-0.71 - 0.24**j**	-0.11 - 0.01**j**	-0.21 - 0.05**j**	0.45 - 0.55**j**

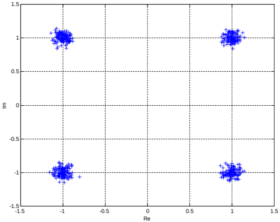
- Surface and contour plots of the cost function (Equ 8) shows that all 5 path delays and directions are correctly estimated



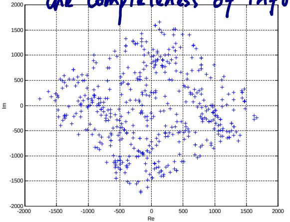


Constellation Diagram (Decision Variables):

STAR MU receiver is sentive to the completeness of info.

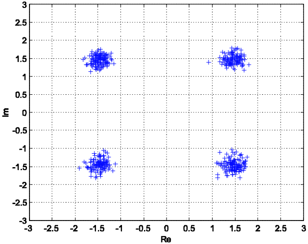


ST Decorrel. MU Rx

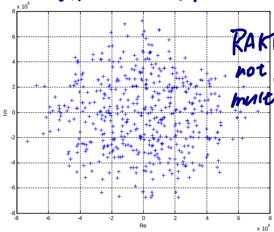


ST Decorrel. MU Rx (Incomp)

(signal from a path of certain user is lost)

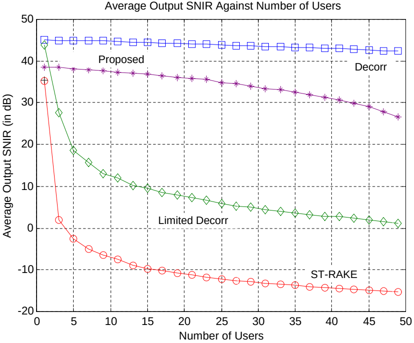


STAR manifold Rx

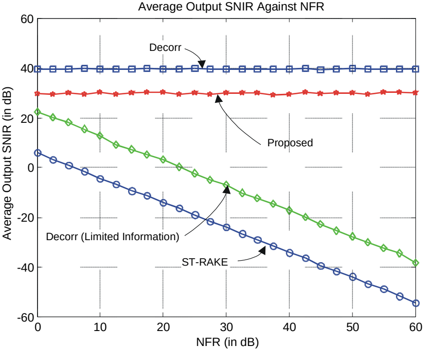


ST-RAKE SU Rx

RAKE receiver is not good for multiplex interference.

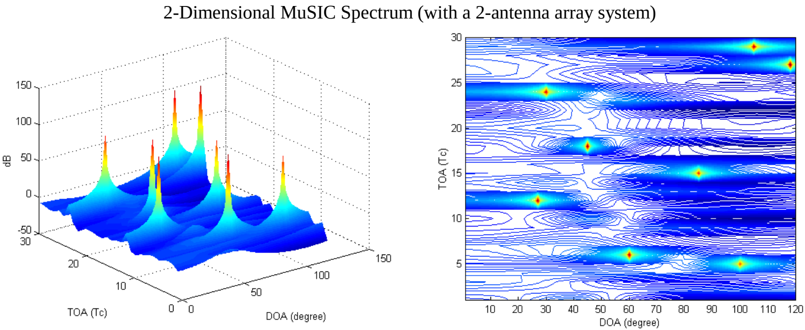


$\text{SNIR}_{out} = f\{M\}$



'Near-Far' Resistance

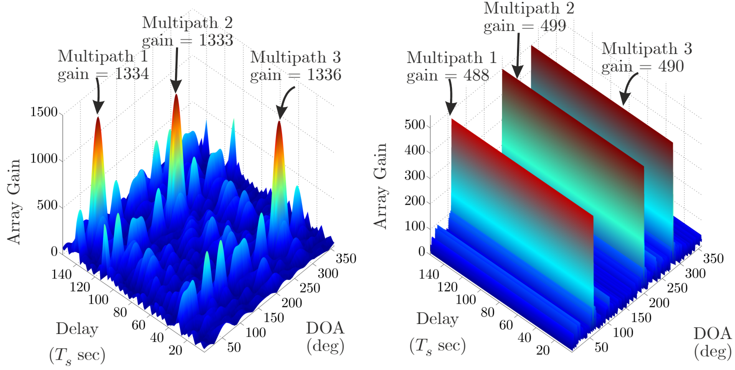
- Two antennas= good for "handsets"
- Example Desired user's parameters = 8 paths with (TOA in T_c, DOA in degrees) as follows:
  - $(5, 100°)$, $(6, 60°)$, $(12, 27°)$, $(15, 85°)$, $(18, 45°)$, $(24, 30°)$, $(27, 118°)$, $(29, 105°)$
  - The desired user's STAR MUSIC-type spectrum (and contour diagram) for an array of $N = 2$ antennas (good for a handset) operating in the presence of three CDMA users is shown below:



Example-2: Spatiotemporal Beamformer versus maMI (parametric)

- see IEEE Transactions on Wireless Comms: "*Spatiotemporal-MIMO Channel Estimator and Beamformer for 5G*" (very recently published)
- Rx Antenna arrays
  - Circular Array of N=**9** antennas: **Spatiotemporal beamformer** (16x9).
  - Circular Array of N=**500** antennas: **Massive MIMO** (maMI, 16x500)
- **4** **co-channel users** with **3** **paths per user** in a frequency selective channel
- the spatiotemporal manifold incorporates the following system parameters: 5 subcarriers and a pn-code of length 15.

N=9 antennas **N=500 antennas**



(a) Beampattern of a Doppler-STAR-subspace receiver consisting of 9 antenna elements.

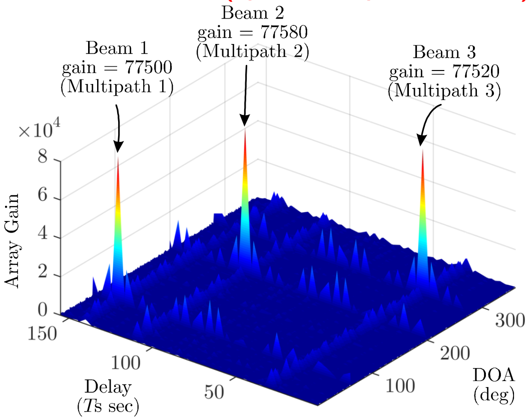
(b) Beampattern of a massive MIMO subspace receiver consisting of 500 antenna elements.

- The **9-antenna** spatiotemporal beamformer provides
  - higher gain and spatiotemporal selectivity
  - better performance ($\approx$ 15dB, see paper)

than a traditional **500-antenna** MIMO system

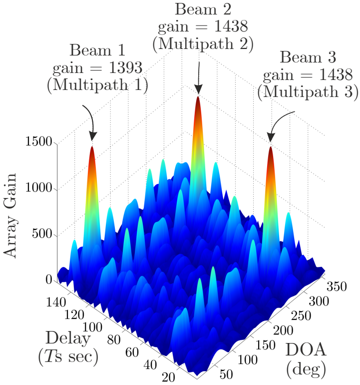
Example-2: Spatiotemporal Beamformer (cont.)

N=500 antennas (spatiotemporal maMI)



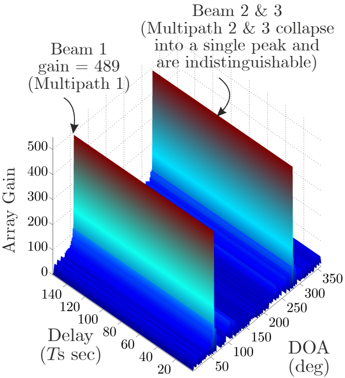
Example-2: Spatiotemporal Beamformer (Co-directional Signals)

N=9 antennas



(a) Beampattern of a Doppler-STAR-subspace receiver consisting of 9 antenna elements.

N=500 antennas



(b) Beampattern of a massive MIMO subspace receiver consisting of 500 antenna elements.