EE3-27: Principles of Classical and Modern Radar

Continuous Wave (CW) Radar

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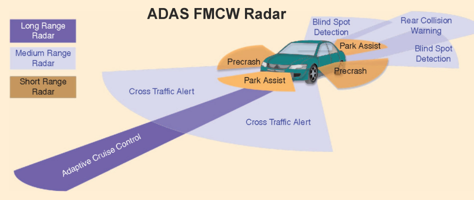
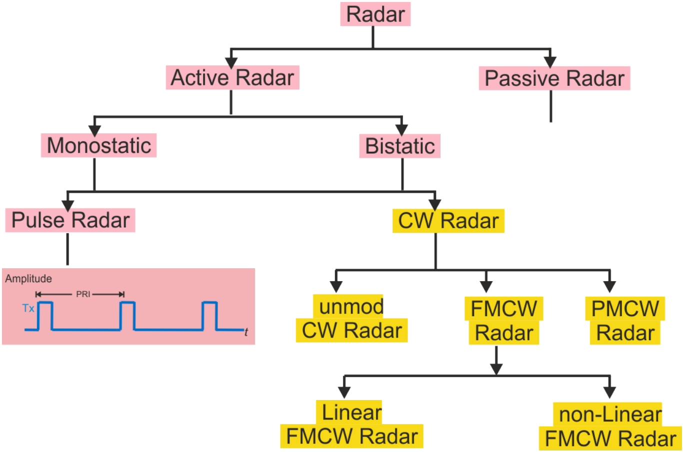


CW Radar Classification



- Continuous wave (CW) radar continuously transmit a signal and simultaneously continuously receive echo reflections scattered from objects

CW Radar Applications

- CW radar are generally used in compact, short-range, low-cost applications.
- CW radar have been used in a wide variety of applications such as
  - the measurement of liquid levels
  - in industrial storage tanks,
  - vehicular speed determination for police speed guns,
  - short-range navigation,
  - missile seekers,
  - battlefield surveillance,
  - aircraft detection,
  - automobile cruise control.
- Indeed, as automobile radar systems are set to become standard for every new car manufactured, CW radar systems may shortly become the most commonplace of any radar variant.

Advantages and Disadvantages of CW Radar

- CW radars can utilize any part of the RF electromagnetic spectrum just like their pulsed counterparts, and examples exist all the way through from HF to W band. *ubiquitous in frequency band*
- The principal advantages of CW radars include
  - simplicity,
  - low cost, and
  - small volume

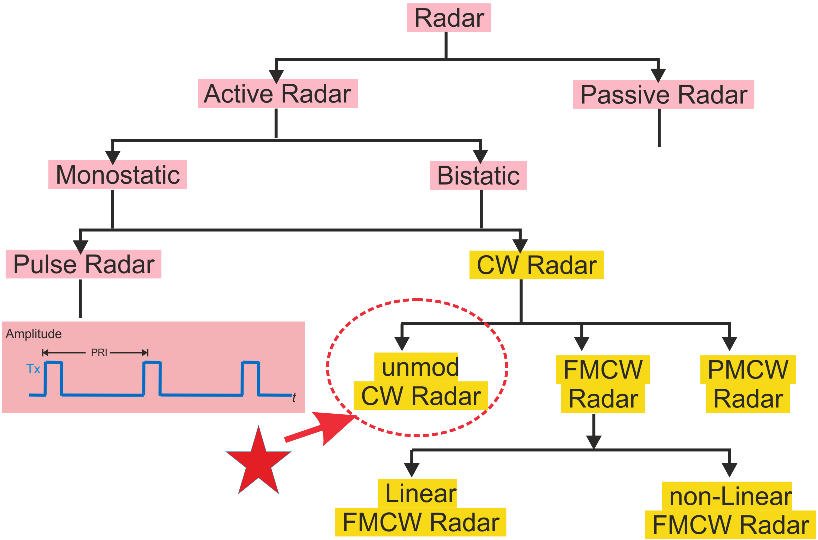
- A comparative example of CW radar with a pulse radar of the same power is given below

Example

- Consider a pulsed radar with a typical pulse length of 1 microsecond and a pulse repetition frequency (PRF) of 1kHz.
- A pulse of amplitude A=1000Volts (i.e. a pulse of power 1KW, known also as peak-power) is required to transmit an average power of just 1 W.
- A transmitter with a pulse power of 1 kW makes a complex and potentially costly system.
- A CW radar with a peak or average output power of 1W is straightforward using compact, relatively simple solid-state technology that may cost just a few pounds.

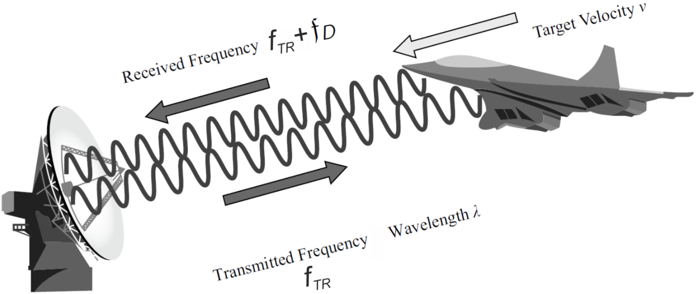
- A disadvantage of CW radar is
  - reduced dynamic range because of simultaneous transmission and reception. ↳ the ratio between largest and smallest signals that radar can process accurately.
- **Note**:
  - Transmission is continuous and therefore competes with the weak reflected echo signal, which it can easily swamp, thus preventing detection of objects. Of course, the Doppler imparted by a moving target helps mitigate this as the transmission is, in effect, at zero Doppler. This improves the isolation between the transmit and receive signals.
  - To further improve matters, it is usual to use separate antennas for transmission and reception arranged to keep the transmit signal from "leaking" across into the receive antenna.

Unmodulated CW Radar



Unmodulated CW Radar

- The operation of an unmodulated CW radar system is shown schematically below



- Unmodulated continuous wave (CW) radar systems continuously transmit a pure tone (i.e. a sinewave, the carrier) and simultaneously continuously receive echo reflections scattered from objects.
- That is, in unmodulated CW radar system a pure tone is used to measure the Doppler shift from a moving object.

- If a target/object is static, the frequency of the echo signal is unchanged from that transmitted. However, if an object is moving, then the frequency of the echo signal is altered due to the Doppler effect. By detecting this Doppler frequency, the object's motion can be determined.
- The faster the object moves in a given direction, the larger the Doppler frequency.
- The velocity of an object in the radial direction with respect to the radar is related to the Doppler frequency shift, $\mathfrak{f}_D$ as follows:

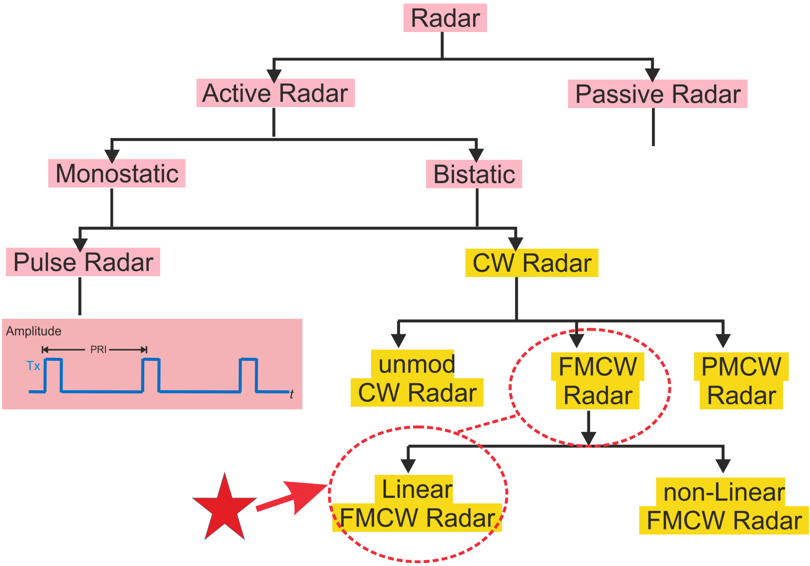
$$\mathfrak{f}_D = \frac{2v_r}{\lambda} \iff v_r = \frac{\lambda \mathfrak{f}_D}{2} \qquad (1)$$

where

$$\begin{aligned} v_r &= \text{radial velocity of the object (m/s)} \\ \lambda &= \text{wavelength of the CW signal (m)} \\ \mathfrak{f}_D &= \text{Doppler frequency (Hz)} \end{aligned}$$

i.e., the Doppler frequency is scaled by the wavelength of the Tx-signal to convert it to a measurement of velocity, and the factor 2 represents the two-way path traveled in transmission and reception.

FMCW Radar



Radar
Active Radar
Passive Radar
Monostatic
Bistatic
Pulse Radar
CW Radar
Amplitude
PRI
Tx
t
unmod CW Radar
FMCW Radar
PMCW Radar
Linear FMCW Radar
non-Linear FMCW Radar

Angle Modulation (FM and PM) Refresher

- The general angle-modulation equation (FM and PM), is defined as:

$$s(t) = \cos\Theta(t) = \begin{cases} \cos(\underbrace{2\pi F_c t + k_p.b(t)}_{\Theta(t)}) & \text{for PM} \\ \cos(\overbrace{2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} b(u)du}^{\Theta(t)}) & \text{for FM} \end{cases} \tag{2}$$

where

- $s(t)$ is the bandpass FM signal;
- $b(t)$ is the baseband signal to be modulated;
- k_p is a PM constant (known as the phase-deviation constant) in rads/V;
- k_f is an FM constant (known as the frequency-deviation constant) in Hz/V;
- u is the integration dummy variable.

- Based on Equation 2, the 1st derivative of $\Theta(t)$ will provide the instantaneous frequency $f(t)$, as follows

$$\Theta'(t) = \frac{d\Theta(t)}{dt} = 2\pi f(t) \tag{3}$$

$$\Rightarrow \boxed{f(t) = \frac{1}{2\pi}\frac{d\Theta(t)}{dt} = \begin{cases} F_c + \frac{k_p}{2\pi}.\frac{db(t)}{dt} & \text{for PM} \\ F_c + k_f.b(t) & \text{for FM} \end{cases}} \tag{4}$$

- Finally, the instantaneous frequency chirpiness, $c_p(t)$, is defined to be the first derivative of instantaneous frequency, $f'(t)$. That is:

$$c_p(t) \triangleq f'(t) = \begin{cases} \frac{k_p}{2\pi}.b''(t) & \text{for PM} \\ k_f.b'(t) & \text{for FM} \end{cases} \tag{5}$$

- Thus chirpiness is the rate of change of the instantaneous frequency.
- Note: The prime $'$ in a symbol, i.e. $(.)'$, denotes the first derivative wrt time t.

Linear FM - Signal Modelling

- In a linear-frequency chirp or simply linear chirp, the instantaneous frequency $f(t)$ varies exactly linearly with time, i.e.

$$b(t) = \underbrace{\mathrm{c_r}}_{\text{chirp rate}}.t + f_0 \tag{6}$$

- In this case, assuming $k_f = 1$, Equation 5 (FM part) becomes

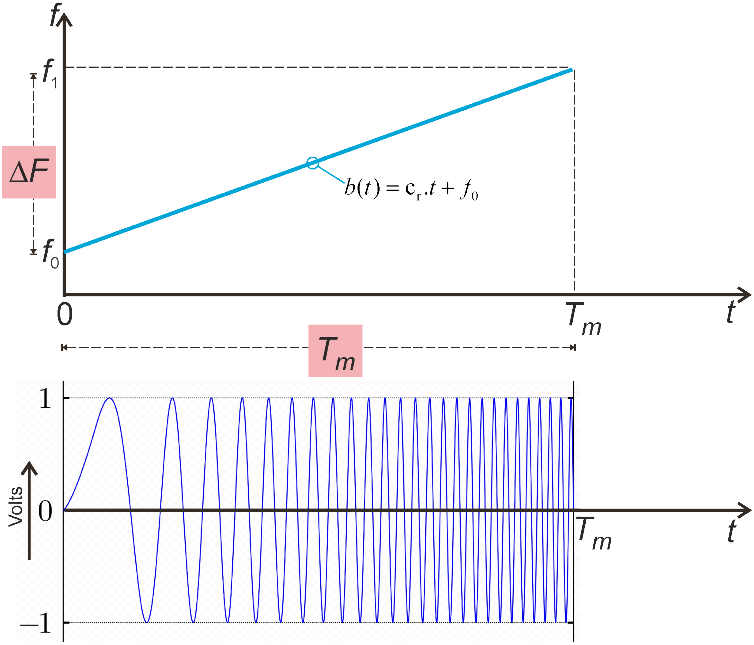
$$c_p(t) \triangleq f'(t) = \mathrm{c_r} \overset{(8)}{=} \frac{f_1 - f_0}{T_m} \tag{7}$$

where

  - f_0 is the starting frequency at time $t = 0$,
  - $\mathrm{c_r}$ is the chirp rate (constant),
  - f_1 is the final frequency, at time T_m, i.e.

$$f_1 \triangleq b(t)|_{t=T_m} = \mathrm{c_r} T_m + f_0 \tag{8}$$

  - T_m is the time it takes to sweep from f_0 to f_1, (known as sweep time)
  - $\Delta f \triangleq f_1 - f_0$ is known as sweep (modulation) bandwidth


f
f1
ΔF
f0
b(t) = c_r.t + f_0
0
T_m
t
T_m
1
0
−1
Volts
T_m
t

- Using Equation 6 in conjunction with the FM part of Equation 2, the Tx signal at Point-C, the linear-FMCW radar signal can be written as follows:

$$
\begin{aligned}
s(t) &= \text{linear FMCW} \\
&= \cos\overbrace{\left(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} b(u)du\right)}^{\Theta(t)} \qquad (9) \\
&= \cos\left(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} (\mathrm{c_r}.u + f_o)du\right) \\
&= \cos\underbrace{\left(2\pi F_c t + 2\pi k_f (\mathrm{c_r}.\frac{t^2}{2} + f_o t)\right)}_{\Theta(t)} \qquad (10)
\end{aligned}
$$

- The angle signal $\Theta(t)$ is known as quadratic phase signal.

FMCW Radar

- FMCW Radar is a low cost radar system, often used in shorter range applications.
- The system is simple to fabricate but requires care to obtain high accuracy.
- FMCW has the same conceptual basis as pulse compression
- Although there are other forms of frequency modulation, such as sinusoidal and nonlinear FM, we concentrate on the linear case (linear FMCW) as it enables the key principles to be introduced and is the most widespread form of modulation in use in CW radar systems today.

Linear FMCW Radar

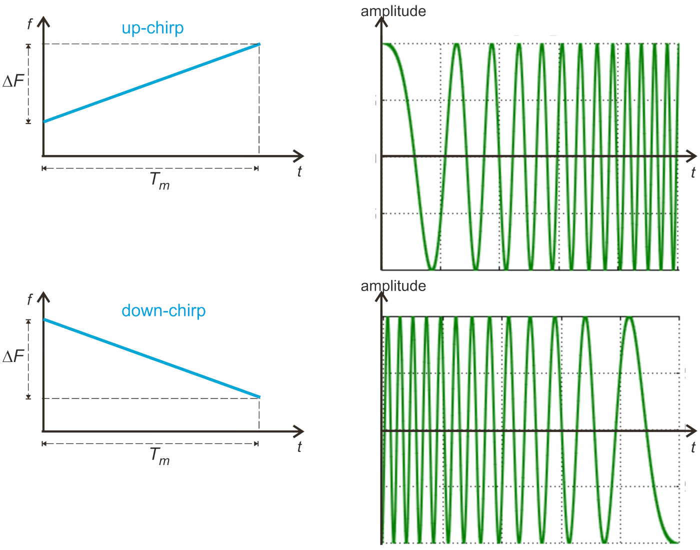
- A drawback of an unmodulated CW radar is that it is unable to detect static objects or measure the range to an object (because range is ambiguous to a wavelength).

$s(t) = \cos\left(2\pi f_c t + 2\pi k_f \left(c_r \frac{t^2}{2} + f_0 t\right)\right)$

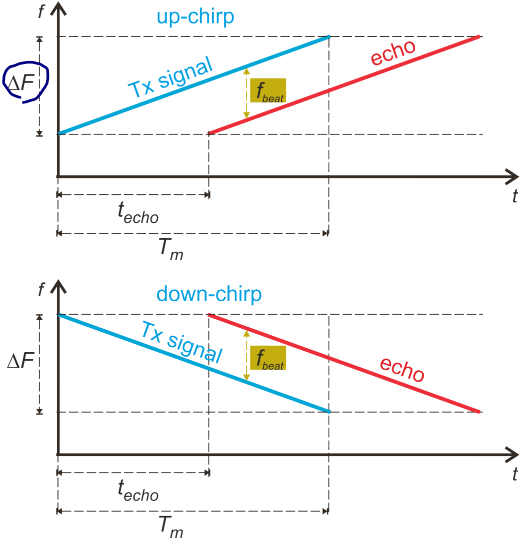
- Measuring range requires a timing reference encoded onto the transmit waveform. In CW systems, this is applied by modulating the frequency or phase of Tx signal (and this overcome the limitations of unmodulated CW).

- For example, the frequency of the transmission can be linearly changed as a function of time. In this way a particular value of frequency represents a particular time delay and hence can be associated with a particular range. In fact, linear FM modulation is probably the most common form used.

- The following figure illustrates the relationship between frequency and time for the Linear FM form of modulation (Note: $\frac{\Delta F}{T_m} = \frac{\text{sweep-BW}}{\text{sweep-time}}$)



Linear FMCW Radar: Beat Frequency

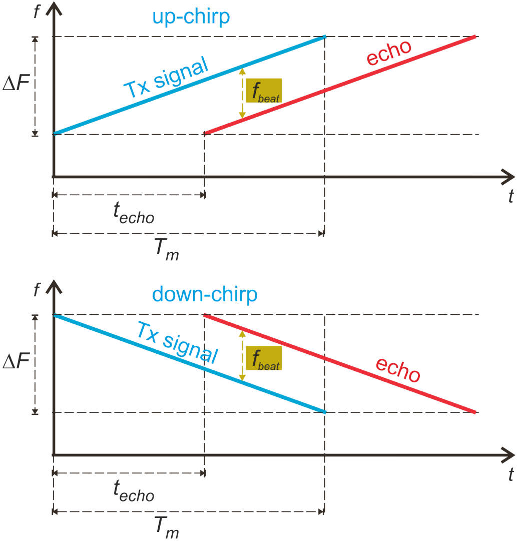


Definition (f_{beat})

- The instantaneous difference in frequency between the transmit and receive waveforms is known as beat frequency, f_{beat}

Definition (ΔF)

- the total peak-to-peak frequency deviation ΔF is termed the modulation bandwidth.



- The time delay, t_{echo}, between the Tx-frequency and the Rx-frequency (i.e. the time taken for an echo to be received), at any instant in time, is given by

$$t_{echo} = \frac{2R}{c} \qquad (11)$$

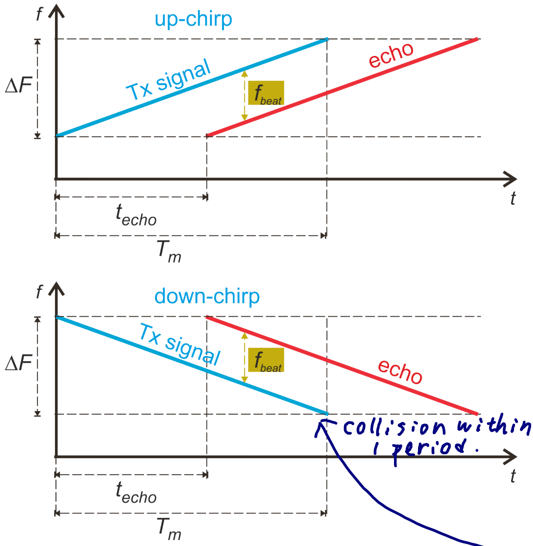
where

R = target range (m)
c = velocity of light (m/s)

N.B.:

- The problem is that t_{echo} cannot be directly measured. However, it can be estimated based on $f_{beat} \propto t_{echo}$. Measure frequency instead.

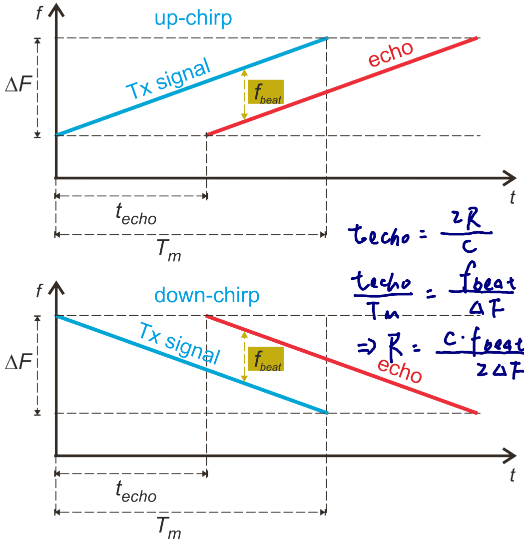
Linear FMCW Radar: Static & Moving Targets



collision within 1 period.

Static Target

- Measurement of the beat frequency allows us to determine the range R to a detected target because it is directly related to target delay (t_{echo}).
- The duration of the linear modulation T_m is set so that it lasts longer than the round-trip transit time for the most distant target to be observed, thus avoiding ambiguities.



$t_{echo} = \frac{2R}{c}$

$\frac{t_{echo}}{T_m} = \frac{f_{beat}}{\Delta F}$

$\Rightarrow R = \frac{c \cdot f_{beat} \cdot T_m}{2 \Delta F}$

N.B.:

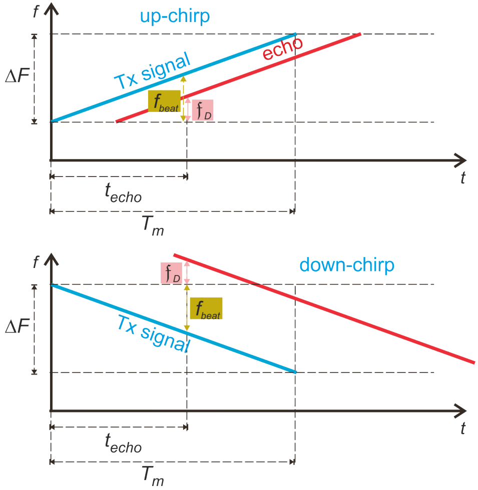
- The two quantities T_m and ΔF together (along with any repetition) form a triangle similar but offset to that triangle formed by the t_{echo} and the beat frequency, f_{beat}.
- That is,

$$\frac{t_{echo}}{T_m} = \frac{f_{beat}}{\Delta F} \qquad (12)$$

which allows computation of the time, t_{echo}

- Indeed, Equs $11 \wedge 12 \Rightarrow \Downarrow$

$$f_{beat} = \frac{\Delta F}{T_m} \cdot \frac{2.R}{c} \Leftrightarrow R = \frac{T_m}{\Delta F} \cdot \frac{c.f_{beat}}{2} \qquad (13)$$



Moving Target

- The Doppler effect will shift the frequency of the received signals for moving targets. In this case f_{beat} for a linear FMCW waveform that sweeps up in frequency is

$$f_{beat} = \frac{\Delta F}{T_m} \frac{2R}{c} + \frac{2v_r}{\lambda} \quad (14)$$

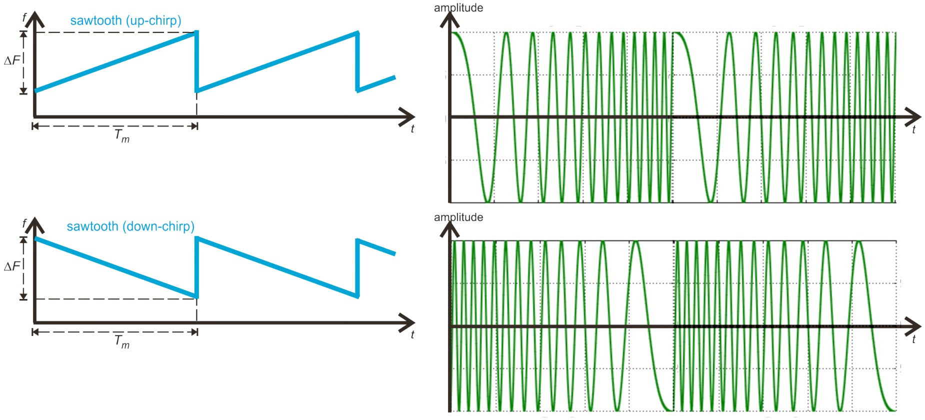
✓ ? f_D ?

1 observation, 2 unknowns

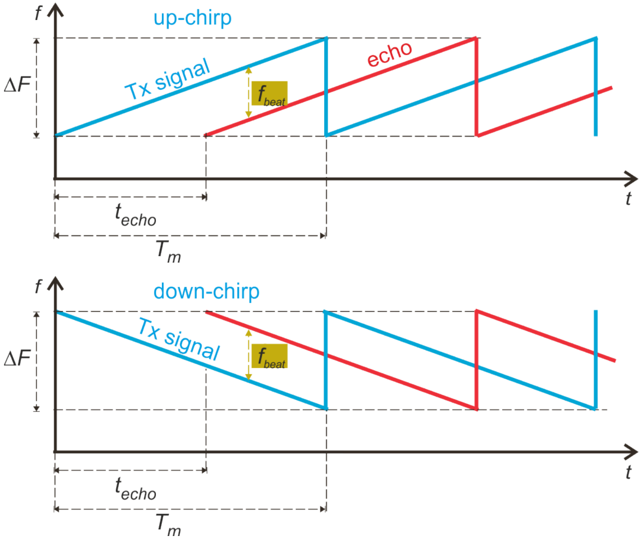
solution: sawtooth

- Equation 14 shows the measured f_{beat} depends on both target's velocity and range.
- In other words, if we use Equ 13, moving targets will appear at an incorrect range.

Sawtooth FMCW Radar



Sawtooth FMCW Radar: Beat Frequency Estimation



- Similar comments to linear FMCW. That is:
  - A relation can be formed between the modulation bandwidth ΔF, the modulation period T_m, the beat frequency f_{beat}, and the transit time t_{echo}, i.e.

$$\frac{t_{echo}}{T_m} = \frac{f_{beat}}{\Delta F} \tag{15}$$

    that leads to the determination of range to a target.

  - Substituting for $t_{echo} = \frac{2R}{c}$ and rearranging terms leads to the following expression,

$$R = \frac{c \cdot f_{beat} \cdot T_m}{2\Delta F}$$

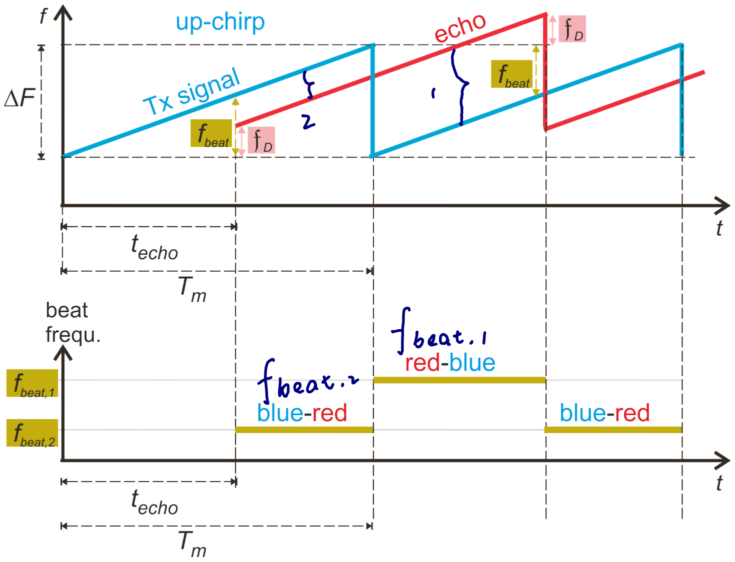
$$f_{beat} = \frac{\Delta F}{T_m} \frac{2R}{c} \tag{16}$$

    which is the same with that of linear FM and is known as the FMCW equation, relating beat frequency and range.

Example

- Assuming a 28 MHz modulation bandwidth and a 1 ms modulation period, the resulting beat frequency is 2.1 MHz, which equates to a target located at a range of 11.250 km.

Sawtooth FMCW Radar: Moving Targets



- In a similar fashion to linear-FMCW, the Doppler effect will shift the frequency of the received signals for moving targets.

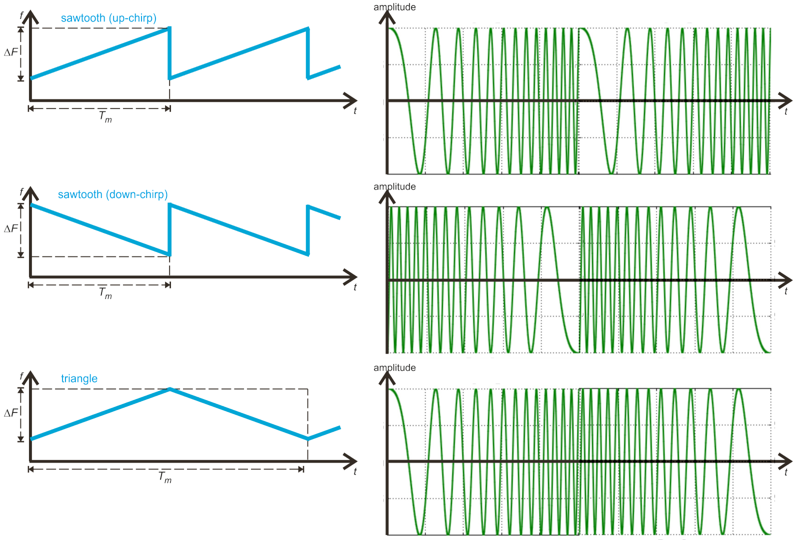
Moving Target

- The beat frequency for a linear-FMCW and sawtooth waveform that sweeps up in frequency is given by

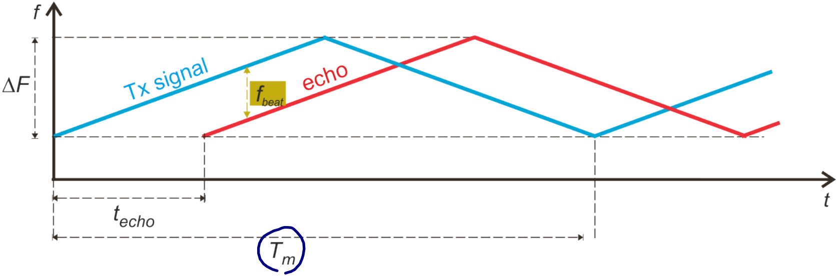
$$f_{beat} = \frac{\Delta F}{T_m}\frac{2R}{c} + \frac{2v_r}{\lambda} \quad (17)$$

- Equation 17 shows the measured beat frequency to be dependent on both Doppler velocity and target range. Again, in other words, moving targets will appear at an incorrect range.
- The ambiguity resulting from this "range-Doppler" coupling can be resolved if the waveform employs two frequency slew rates or slopes.
  - The triangle waveform with alternate up and down frequency sweeps is a common choice.
  - For the triangle waveform, range is linearly proportional to the difference in the upsweep and downsweep beat frequencies, and velocity is proportional to the sum of the beat frequencies.

Triangle FMCW Radar



Triangle FMCW Radar: Beat Frequency Estimation

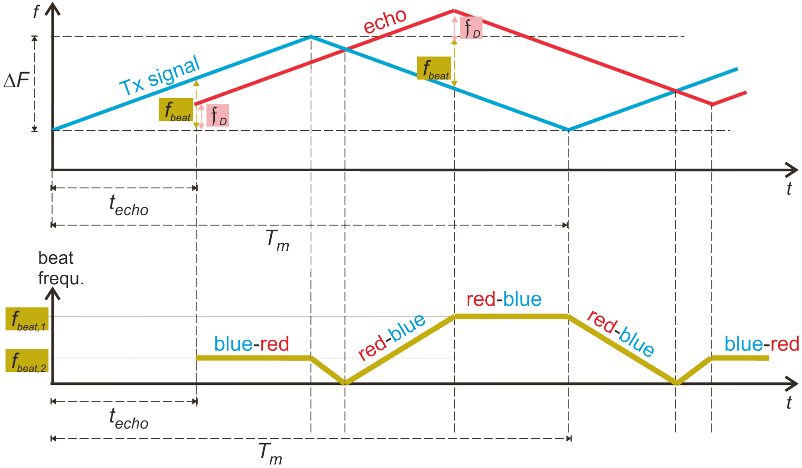


- All the three formulations of the FMCW radar equation may be constructed and thus the FMCW equation can be written as

$$f_{beat} = \begin{cases} \frac{\Delta F}{T_m} \frac{2R}{c} & \text{Linear} \\ \frac{\Delta F}{T_m} \frac{2R}{c} & \text{sawtooth} \\ \frac{\Delta F}{T_m} \frac{4R}{c} & \text{triangular} \end{cases} \quad (18)$$

T_m is doubled for triangular FMCW

Triangle FMCW Radar: Moving Targets



- Thus, for moving targets we have two f_{beat} frequencies:
  - $f_{beat,1}$ corresponding to the triangle's downsweep i.e. [red −blue], and
  - $f_{beat,2}$ corresponding to the triangle's upsweep, i.e. [blue −red]

Triangle FMCW Radar: Range and Velocity Estimation of Moving Targets

- For triangle Tx-waveforms we have:

$$\text{triangle, upsweep: } f_{beat,up} = \frac{\Delta F}{T_m}\frac{4R}{c} - \frac{2v_r}{\lambda} \tag{19}$$

$$\text{triangle, downsweep: } f_{beat,down} = \frac{\Delta F}{T_m}\frac{4R}{c} + \frac{2v_r}{\lambda} \tag{20}$$

- Thus for a single moving target there are 2 equations with two unknowns (range and velocity) and hence:
  - range, R, is given by

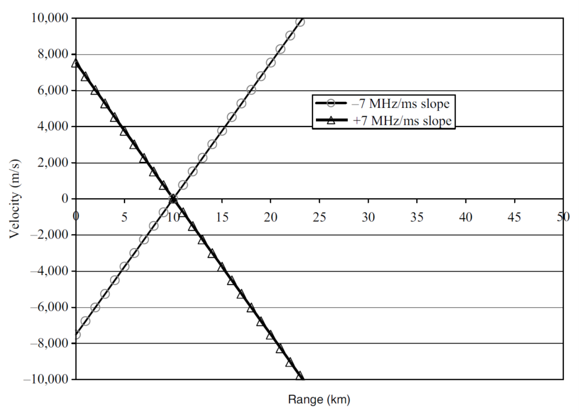
$$R = \frac{T_m c}{8\Delta F}\left(f_{beat,down} + f_{beat,up}\right) \tag{21}$$

  - and velocity, v_r, by

$$v_r = \frac{\lambda}{4}\left(f_{beat,down} - f_{beat,up}\right) \tag{22}$$

Example

- The figure below shows a graphical example where separate target range and velocity measurements are made using a triangular modulation waveform. The intersection of the two lines provides the solution to Equs 19 and 20. The two lines on the graph represent the two different beat frequencies derived from the two different slopes of the triangular modulation and hence the actual target range and velocity. In this example, the target range is 10 km and the target was static.



Appendix-A: Chirp Spread Spectrum

- Chirp spread spectrum (CSS) is a spread spectrum technique that uses wideband linear frequency modulated chirp pulses to encode information.

- A chirp is a sinusoidal signal of frequency increase or decrease over time (often with a polynomial expression for the relationship between time and frequency).

- The figures show two examples of a chirp in which the frequency increases linearly and exponentially over time.