

EE3-27: Principles of Classical and Modern Radar

MIMO Radar

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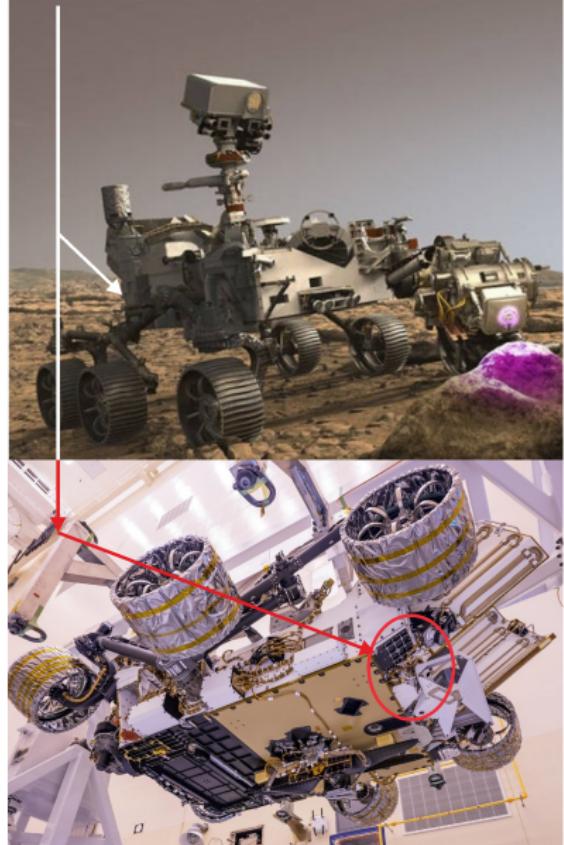
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RIMFAX is a ground penetrating Radar on NASA's Perseverance Rover

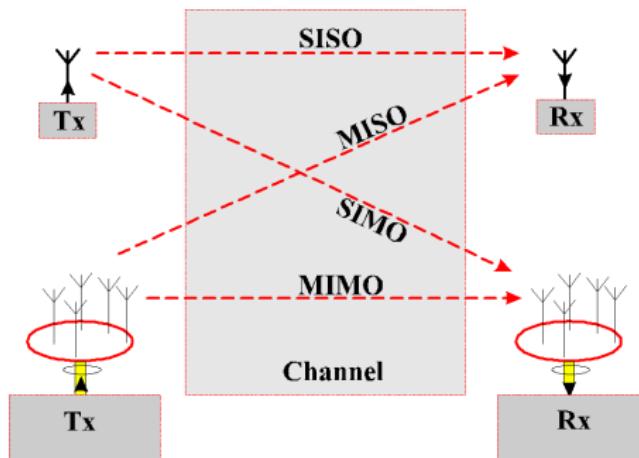


Introduction

- MIMO radar is a family of modern and very powerful radar systems which incorporates
 - ▶ antenna arrays at both the radar's Tx and Rx,
 - ▶ suitable vector-signal baseband generators at the radar's Tx
 - ▶ sophisticated array processing algorithms at the radar's baseband Rx
- MIMO radar
 - ▶ improve the spatial radar resolution,
 - ▶ provide a substantially improved immunity to interference.
 - ▶ provide considerable improvement of the signal-to-noise ratio and, consequently, the probability of detection of the targets.
- MIMO radar taxonomy:
 - ▶ with collocated antennas (so-called "Monostatic" MIMO)
 - ▶ with widely separated Tx and Rx antennas (so-called "Bistatic" MIMO).
 - ▶ with distributed antennas ("Large Aperture Arrays" MIMO) where the target is regarded by each antenna from another aspect angle.
- In the collocated MIMO radar case, the Tx and Rx antennas are close enough (small aperture arrays) such that the target radar cross-sections (RCS) observed by the Tx and Rx antenna elements are identical, i.e.
$$(\bar{\theta}, \bar{\phi})_{DOD} = (\theta, \phi)_{DOA}$$
- In this topic we focus on "monostatic" MIMO radar.

Wireless Systems and Radar Classification

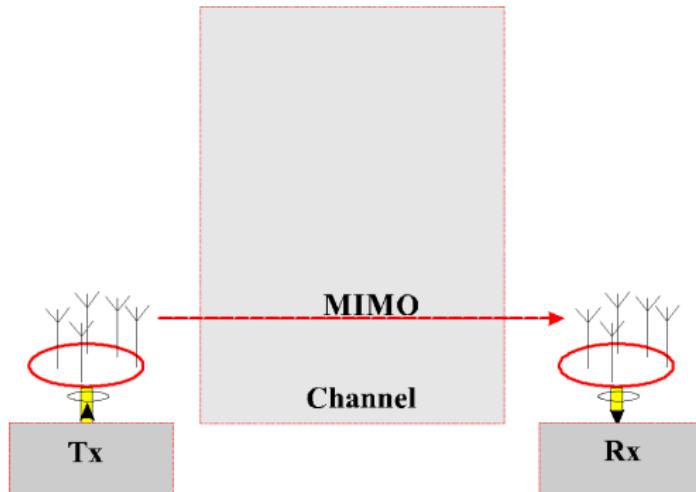
- \exists a **New** classification of radar (and, in general, of wireless systems): according to the **number of antennas/sensors** used in both Tx and Rx



Terminology

S:	Single	M:	Multiple
I:	Input	O:	Output

MIMO Radar: Backscatter Modelling



- Consider a single path from a Tx-array of \bar{N} antennas to an Rx-array of N antennas with locations given by the matrices

$$\text{Tx-array: } \underline{\bar{r}} = [\bar{r}_1, \bar{r}_2, \dots, \bar{r}_{\bar{N}}] = [\bar{r}_x, \bar{r}_y, \bar{r}_z]^T \quad (3 \times \bar{N}) \quad (1)$$

$$\text{Rx-array: } \underline{r} = [r_1, r_2, \dots, r_N] = [r_x, r_y, r_z]^T \quad (3 \times N) \quad (2)$$

Single Target MIMO Radar: Modelling of the Rx Vector-Signal $x(t)$

- we have seen before that for a monostatic radar (MIMO or not) the direction-of-departure $(\bar{\theta}, \bar{\phi})$ and the direction-of-arrival is (θ, ϕ) are the same, i.e.

$$(\bar{\theta}, \bar{\phi}) = (\theta, \phi)$$

and the impulse response for a single target is

MIMO radar
or
Phased-array

single target: $\underline{h}(t) = \beta \underline{S}(\theta, \phi) \cdot \underbrace{\overline{S}^H(\theta, \phi)}_{\text{scalar}} \cdot \underline{\delta}(t - t_{\text{echo}})$ (3)

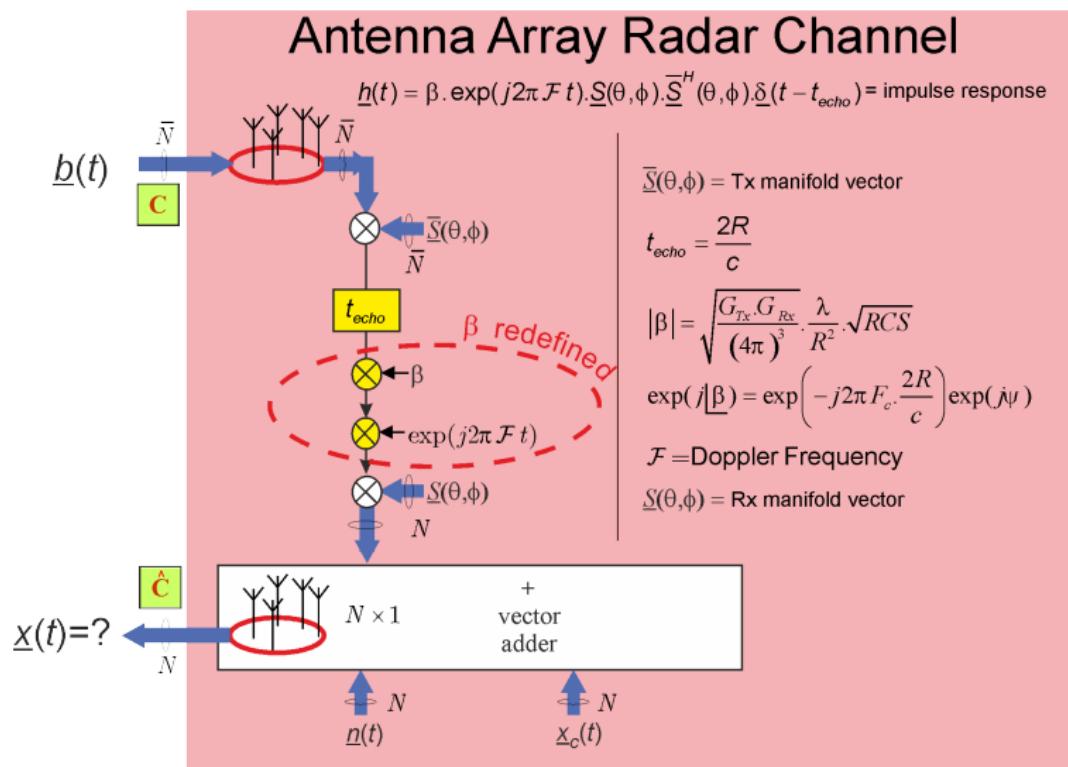
where β has been re-defined to include also the Doppler term, and

$$\text{Tx: } \overline{S} = \overline{S}(\theta, \phi) = \exp \left(+j \underline{\mathbf{r}}^T \underline{k}(\theta, \phi) \right) \quad (4)$$

$$\text{Rx: } \underline{S} = \underline{S}(\theta, \phi) = \exp \left(-j \underline{\mathbf{r}}^T \underline{k}(\theta, \phi) \right) \quad (5)$$

with $\underline{k}(\theta, \phi)$ denoting the wavenumber vectors of the Tx-array and Rx-array respectively

Single Target MIMO Radar: Modelling of the Rx Vector-Signal $x(t)$ (cont.)



Single Target MIMO Radar: Modelling of the Rx Vector-Signal $\underline{x}(t)$ (cont.)

- Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ in the presence of a single target (MIMO radar).
- The received ($N \times 1$) baseband vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned}\underline{x}(t) &= \underline{h}(t) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\ &= \left(\beta \cdot \underline{S} \cdot \underline{S}^H \underline{\delta}(t - \tau_{echo,\ell}) \right) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\ \Rightarrow \underline{x}(t) &= \underbrace{\beta \cdot \underline{S} \cdot \underline{S}^H \underline{b}(t - \tau_{echo})}_{\triangleq \text{single target echo}} + \underline{n}(t) + \underline{x}_c(t)\end{aligned}\quad (6)$$

where

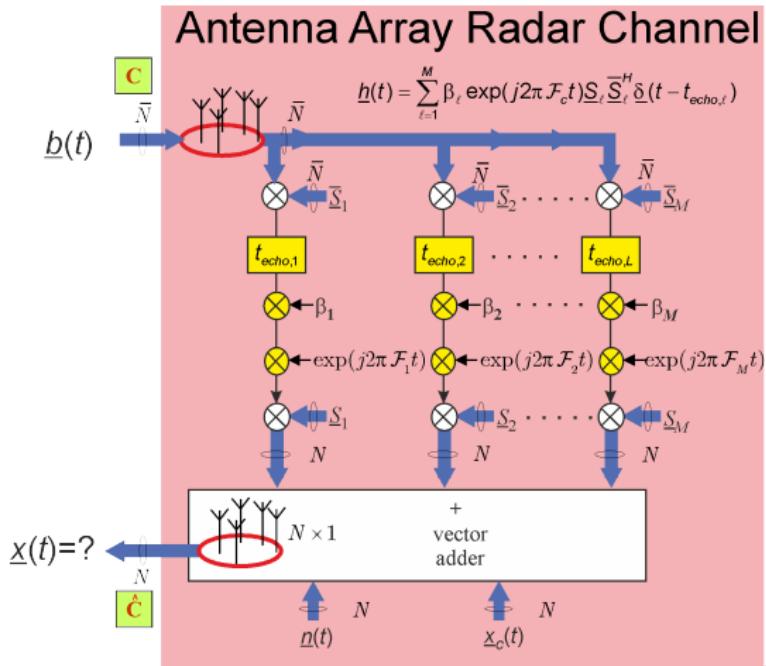
$$\underline{n}(t) = [n_1(t), n_2(t), \dots, n_N(t)]^T \quad (7)$$

$$\underline{x}_c(t) = [x_{c1}(t), x_{c2}(t), \dots, x_{cN}(t)]^T \quad (8)$$

Multi Target MIMO Radar: Modelling of the Rx Vector-Signal $\underline{x}(t)$

- In the following figure the vectors $\bar{\underline{S}}_\ell = \bar{\underline{S}}(\theta_\ell, \phi_\ell) \in \mathbb{C}^{\bar{N}}$ and $\underline{S}_\ell = \underline{S}(\theta_\ell, \phi_\ell) \in \mathbb{C}^N$ are the Tx- and Rx- array manifold vectors of the ℓ -th target.

- Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ using an antenna-array in the presence of M targets (MIMO radar).
- we want to model the vector-signal at the output of the Rx-array.



Multi Target MIMO Radar: Modelling of the Rx Vector-Signal $\underline{x}(t)$ (cont.)

- The impulse response of the MIMO multi-target backscatter is

$$\text{MIMO Radar, multi-target: } \underline{h}(t) = \sum_{\ell=1}^M \beta_{\ell} \cdot \underline{S}_{\ell} \cdot \underline{S}_{\ell}^H \delta(t - \tau_{echo,\ell}) \quad (9)$$

where β_{ℓ} has been re-defined to include also the Doppler term of the ℓ target.

- The received ($N \times 1$) vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned} \underline{x}(t) &= \underline{h}(t) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \\ &= \left(\sum_{\ell=1}^M \beta_{\ell} \cdot \underline{S}_{\ell} \cdot \underline{S}_{\ell}^H \delta(t - \tau_{echo,\ell}) \right) * \underline{b}(t) + \underline{n}(t) + \underline{x}_c(t) \end{aligned}$$

$$\Rightarrow \underline{x}(t) = \underbrace{\sum_{\ell=1}^M \beta_{\ell} \cdot \underline{S}_{\ell} \cdot \underline{S}_{\ell}^H}_{\triangleq \text{multi target echo}} \underline{b}(t - \tau_{echo,\ell}) + \underline{n}(t) + \underline{x}_c(t) \quad (10)$$

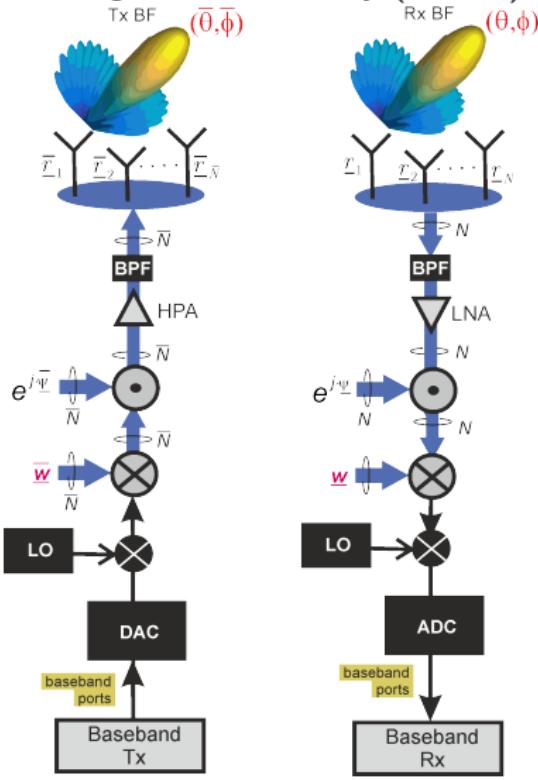
where $\underline{n}(t)$ and $\underline{x}_c(t)$ are the noise and clutter terms (see Equ.7 and 8).

Monostatic MIMO and Phased-Array Radar

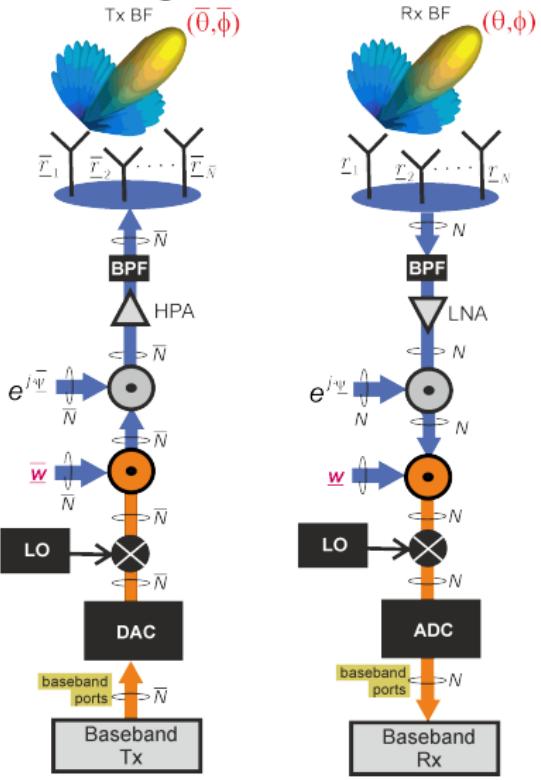
- Phased-Array radar may be SIMO or MIMO.
- Phased-Array radar: if both Tx and Rx employ an antenna array then this belongs to the family of MIMO radar.
- In a similar fashion to a digital phased array radar in a digital MIMO radar each antenna has its own transceiver module and its own A/D converter.
- In a Phased-Array radar, the baseband signal generator produces a signal (scalar signal) which is copied to each Tx-antenna.
- In a MIMO radar, the baseband signal generator produces a vector-signal and each Tx antenna has its own arbitrary baseband waveform generator. This individual waveforms is also the basis for an assignment of the echo signals to their source.
- Note that some people wrongly believe that Phased-Array radar and MIMO radar are two different systems. This is reflected in some old publications where MIMO radar and phased-array radar are compared but the phased array radar in these comparisons is often a SIMO and not a MIMO phased-array radar.

MIMO Analogue Representation vs Phased-Arrays

Analogue Phased Array (MIMO)

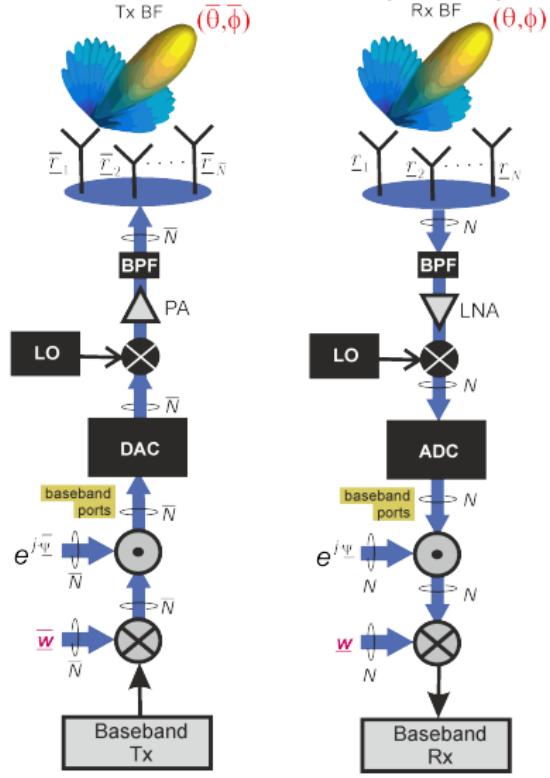


Analogue Classical MIMO

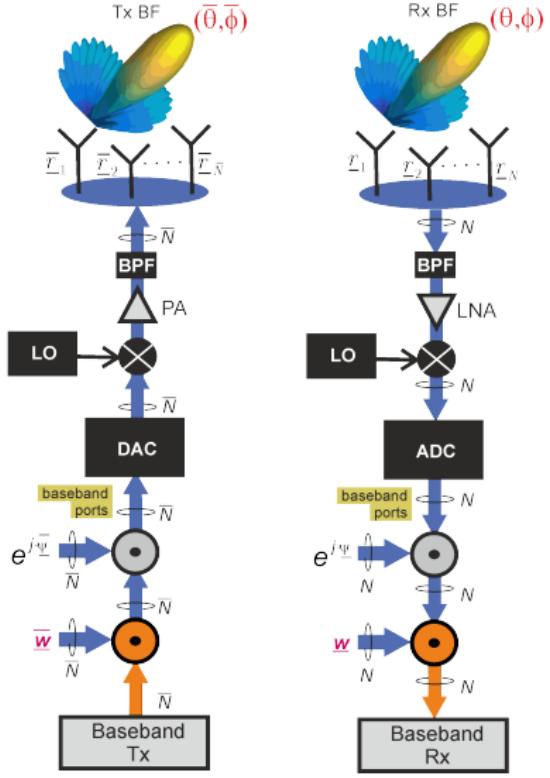


MIMO Digital Representation vs Phased-Arrays

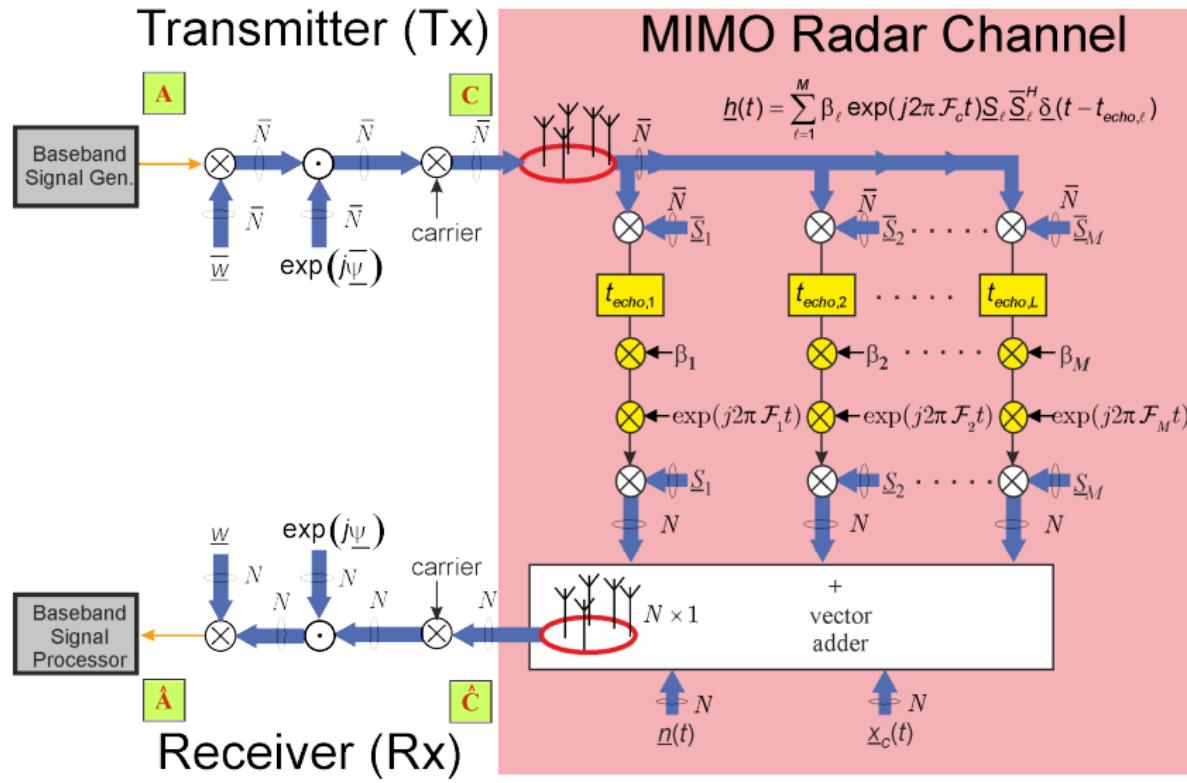
Digital Phased Array (MIMO)



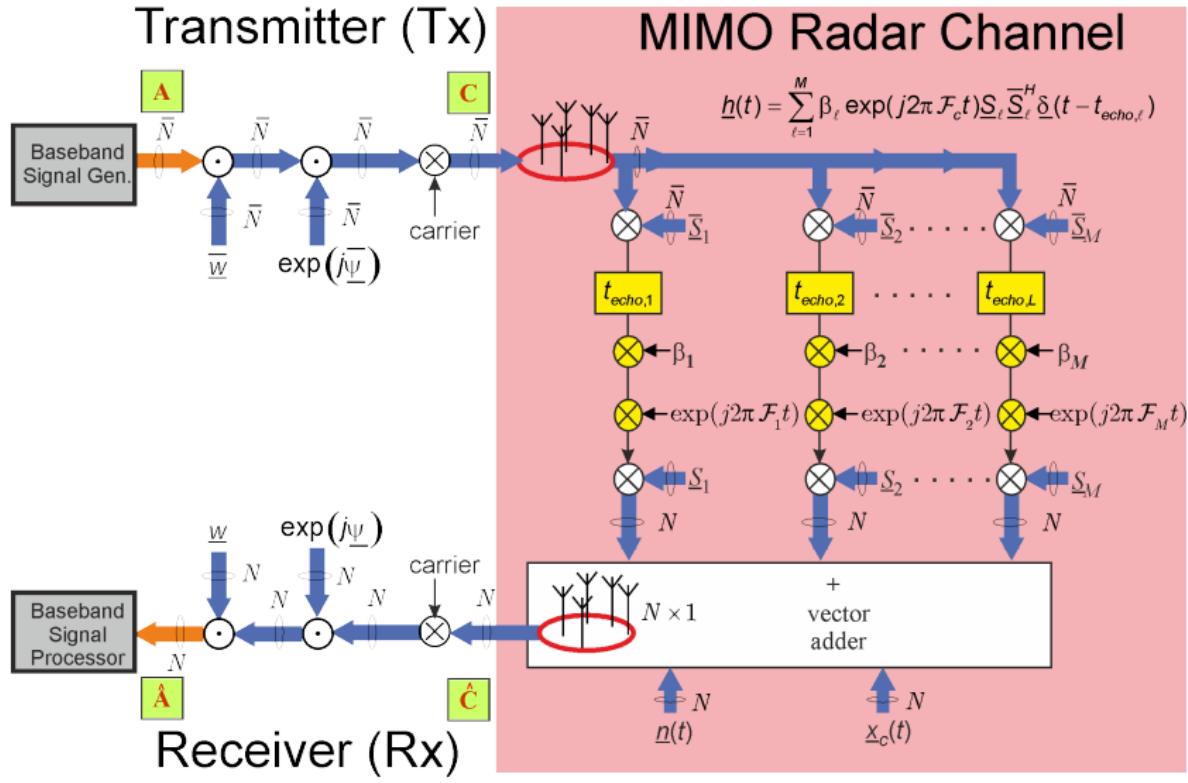
Digital Classical MIMO



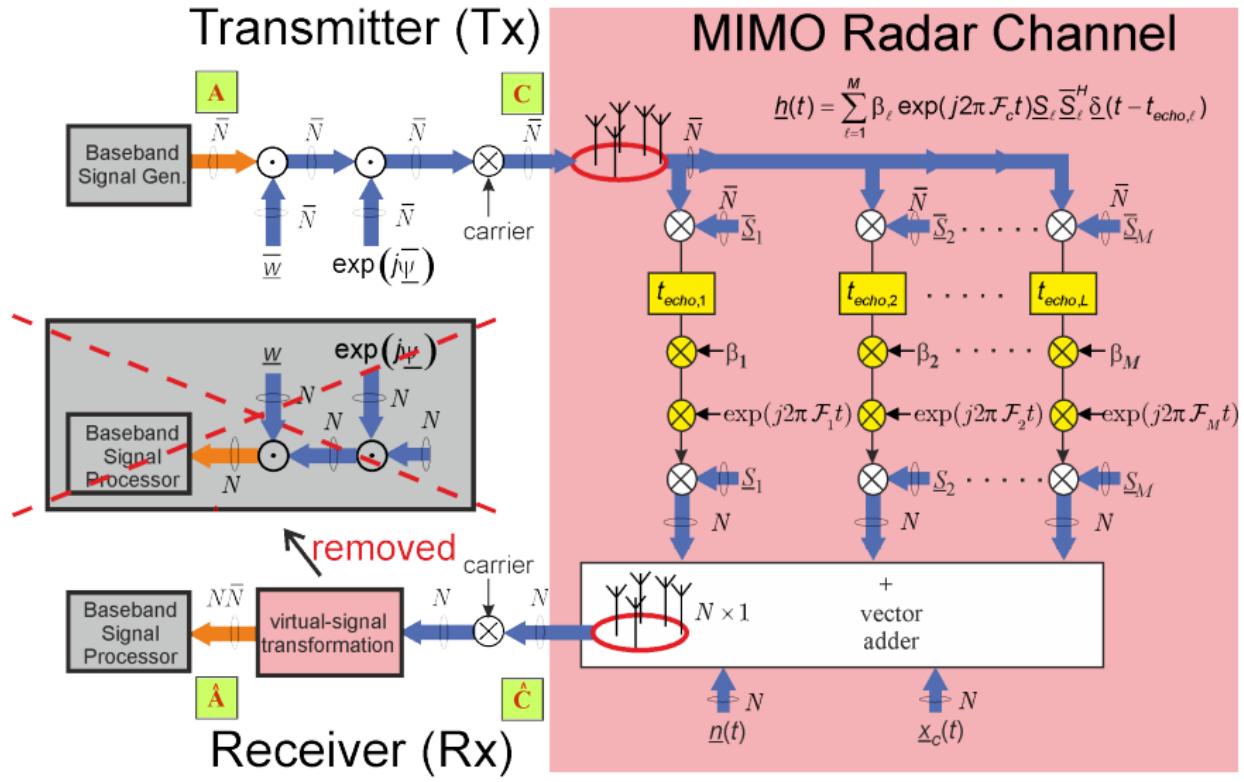
Modelling: Phased-Array Radar



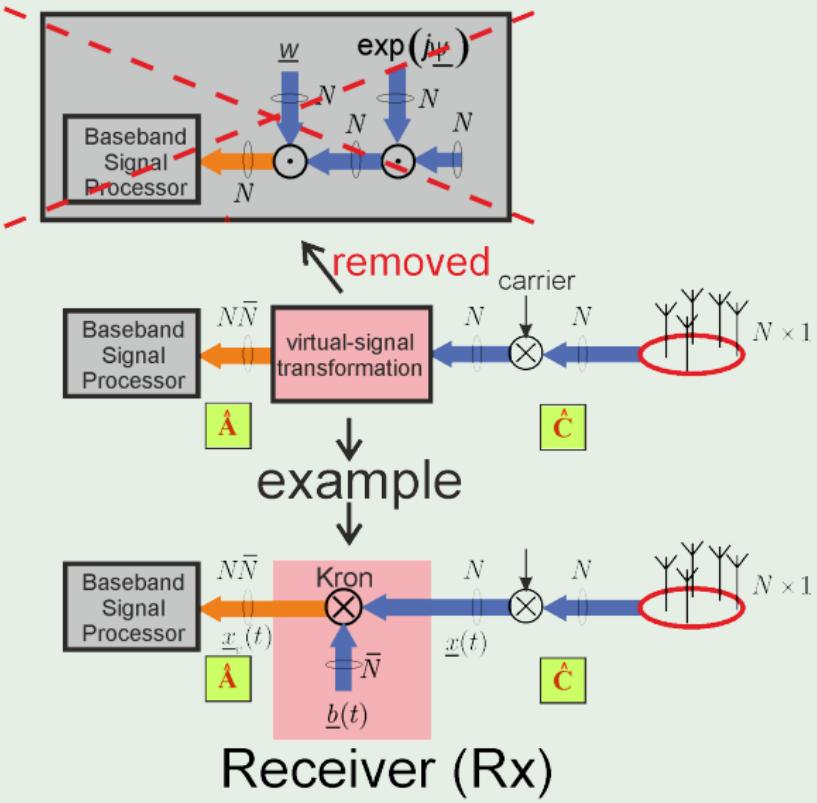
Modelling: Classical MIMO Radar



Virtual-MIMO Radar Transformation



Example (Example of a virtual signal transformation)



Kronecker Product

- the symbol \otimes denotes the Kronecker product of two matrices or vectors
- for two matrices \mathbb{A} and \mathbb{B}

$$\text{if } \mathbb{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ then } \mathbb{A} \otimes \mathbb{B} = \begin{bmatrix} A_{11}\mathbb{B} & A_{12}\mathbb{B} \\ A_{21}\mathbb{B} & A_{22}\mathbb{B} \end{bmatrix} \quad (11)$$

- for two vectors \underline{A} and \underline{B}

$$\text{if } \underline{A} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \text{ then } \underline{A} \otimes \underline{B} = \begin{bmatrix} A_1 \underline{B} \\ A_2 \underline{B} \end{bmatrix} \quad (12)$$

Three Linear Properties used in Virtual MIMO

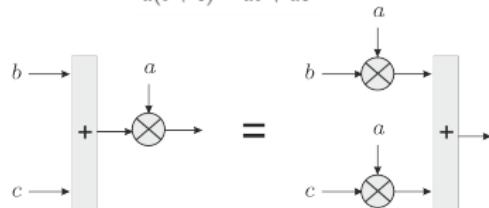
The **Commutative Property** (commutative comes from "commute" or "move around") which is the rule that refers to moving stuff around.

for addition, the rule is
 $a+b=b+a$

for multiplication, the rule is
 $ab=ba$

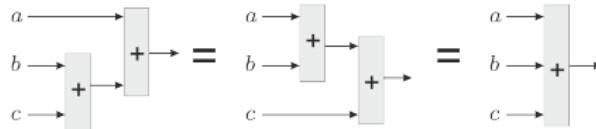
The **Distributive Property** which is the rule that states that "multiplication distributes over addition".

$$a(b+c) = ab + ac$$

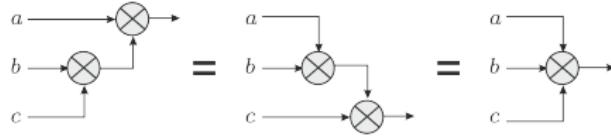


The **Associative Property** (associative comes from "associate" or "group"), which is the rule that refers to grouping.

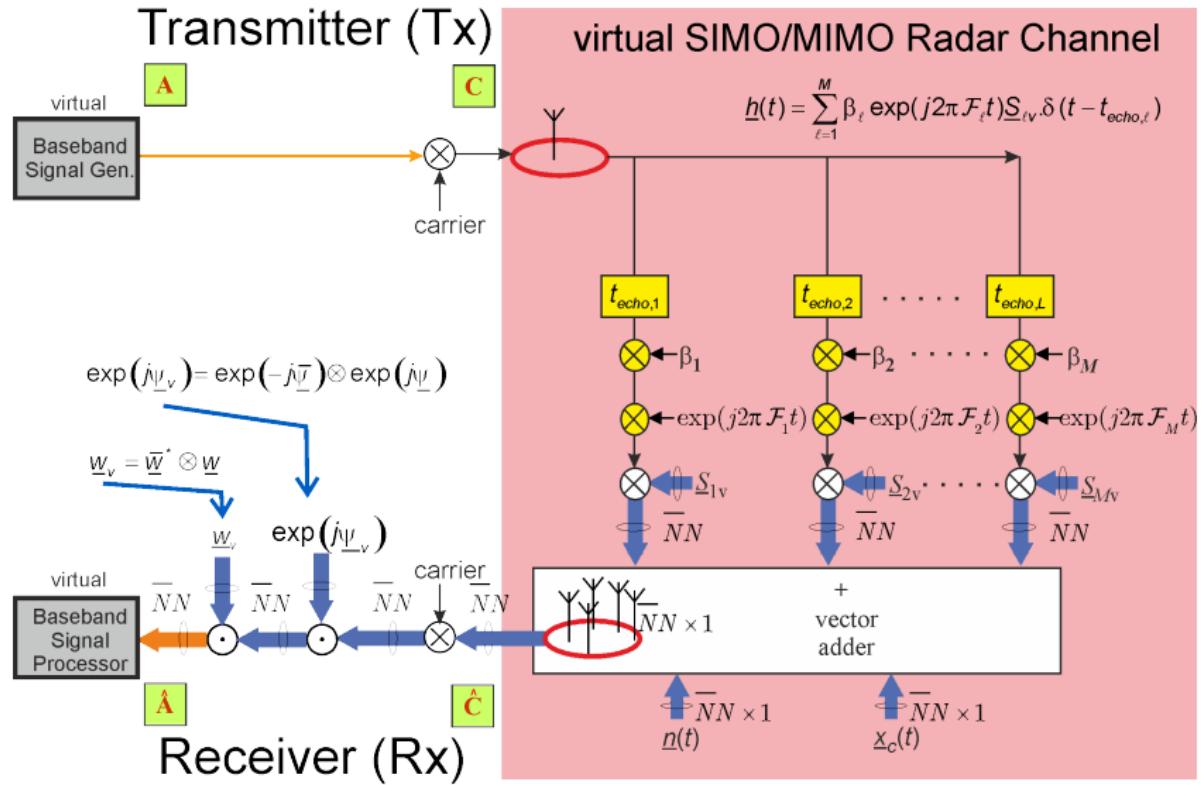
$$a + (b + c) = (a + b) + c = a + b + c$$



$$a(bc) = (ab)c = abc$$



Equivalent Virtual MIMO Modelling



Spatial Convolution and Virtual Antenna Array

Tx-array: $\underline{\underline{r}} \triangleq [\bar{r}_1, \bar{r}_2, \dots, \bar{r}_N] = [\bar{r}_x, \bar{r}_y, \bar{r}_z]^T \quad (3 \times \bar{N})$

Rx-array: $\underline{\underline{r}} \triangleq [r_1, r_2, \dots, r_N] = [r_x, r_y, r_z]^T \quad (3 \times N)$

virtual-array: $\underline{\underline{r}}_{\text{virtual}} \triangleq \underline{\underline{r}} \otimes \underline{1}_N^T + \underline{1}_N^T \otimes \underline{\underline{r}}$ (13)



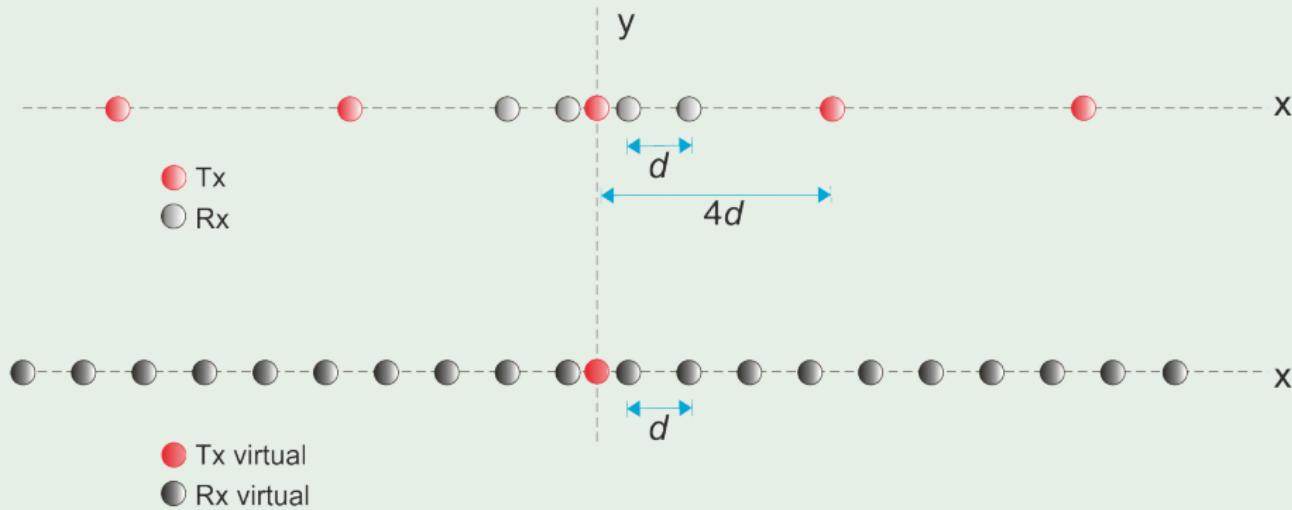
$$\underline{\underline{S}}_{\text{virtual}} \triangleq \underline{\underline{S}}^* \otimes \underline{\underline{S}} \quad (14)$$

N.B.:

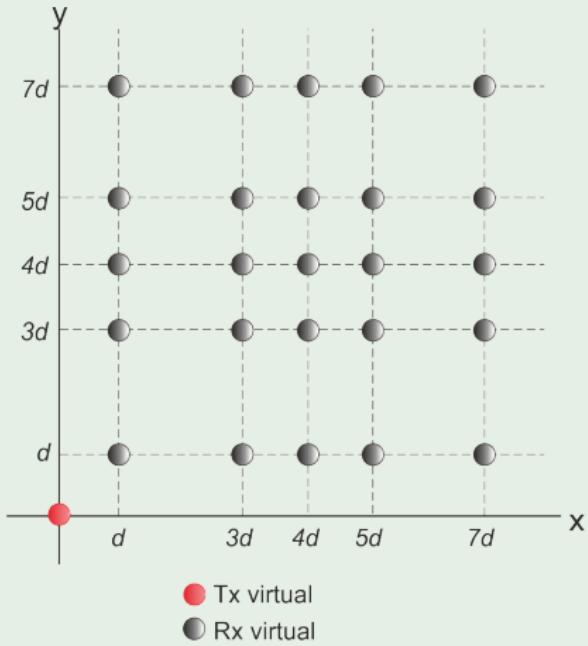
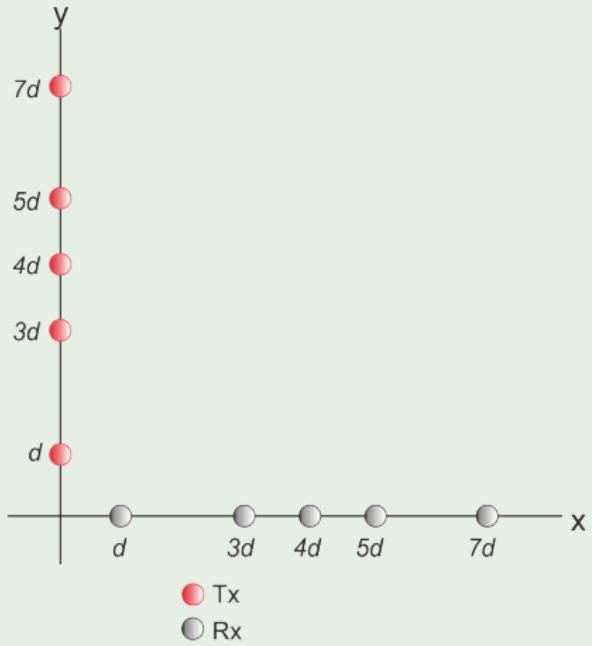
- ① Equation 13 is known as "spatial convolution".
- ② The concept of the "virtual array", is also applicable to MISO.
- ③ A MIMO is equivalent to a "virtual-MISO" (by virtually transferring the Tx antenna array to the Rx) or to a "virtual-SIMO" (by virtually transferring the Rx antenna array to the Tx).

Examples of Spatial Convolution and Virtual Arrays

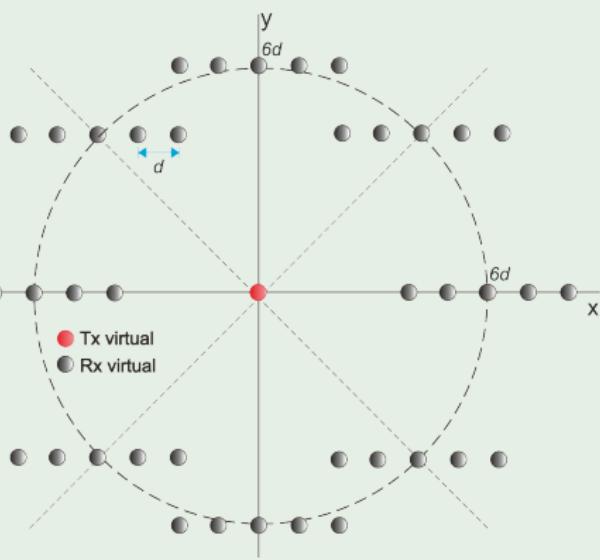
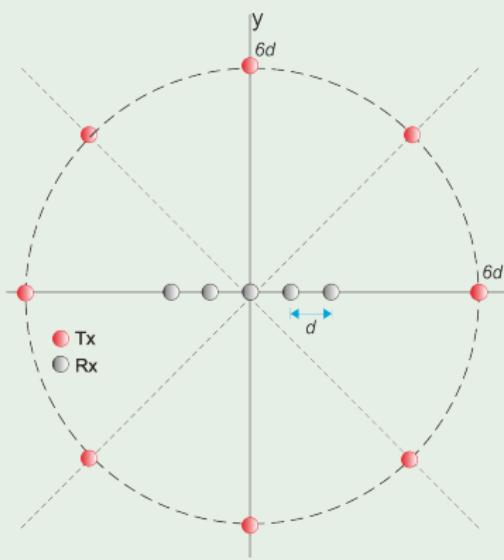
Example (MIMO: Tx=linear; Rx=linear)



Example (MIMO: Tx=linear, y-axis; Rx=linear, x-axis.)



Example (MIMO: Tx=circular; Rx=Linear)



- Classical MIMO algorithms: \exists various algorithms for estimating β and directions (θ, ϕ) . This is done by expressing β as a function of direction (θ, ϕ) and then searching for the maximum.
 - ▶ Least Squares (LS) algorithm
 - ▶ Capon's algorithm
 - ▶ Amplitude and Phase Estimation (APES) algorithm
 - ▶ Approximate Maximum Likelihood (AML)
 - ▶ Capon-and-AML (CAML) algorithm

- LS equation

$$\beta(\theta, \phi) = \frac{\underline{S}(\theta, \phi)^H \cdot \mathbb{R}_{xb} \cdot \overline{S}(\theta, \phi)}{N \cdot \bar{N}} ; \quad (15)$$

- Capon's equation:

$$\beta(\theta, \phi) = \frac{\underline{S}(\theta, \phi)^H \cdot \mathbb{R}_{xx}^{-1} \cdot \mathbb{R}_{xb} \cdot \overline{S}(\theta, \phi)}{\bar{N} \cdot \underline{S}(\theta, \phi)^H \cdot \mathbb{R}_{xx}^{-1} \cdot \underline{S}(\theta, \phi)} ; \quad (16)$$

- In Equations 15 and 16, \mathbb{R}_{xm} and \mathbb{R}_{xx} are defined as follows:

$$\mathbb{R}_{xb} = \mathcal{E} \left\{ \underline{x}(t) \cdot \underline{b}^H(t) \right\} \simeq \frac{1}{L} \mathbb{X} \mathbb{B}^H; \quad \mathbb{R}_{xb} \in C^{N \times \bar{N}} \quad (17)$$

$$\mathbb{R}_{xx} = \mathcal{E} \left\{ \underline{x}(t) \cdot \underline{x}^H(t) \right\} \simeq \frac{1}{L} \mathbb{X} \mathbb{X}^H; \quad \mathbb{R}_{xx} \in C^{N \times N} \quad (18)$$

$$L = \text{number of samples/snapshots} \quad (19)$$

$$\mathbb{X} = L \text{ snapshots of } \underline{x}(t); \quad \mathbb{X} \in C^{N \times L} \quad (20)$$

$$\mathbb{B} = L \text{ snapshots of } \underline{b}(t); \quad \mathbb{B} \in C^{\bar{N} \times L} \quad (21)$$

$$(\theta, \phi) = \arg \max_{\forall \theta, \phi} \beta(\theta, \phi) \quad (22)$$

MIMO Radar: Virtual MIMO Estimation Algorithms

- Virtual MIMO algorithms:
 - ▶ Virtual Steering Vector algorithms
 - ▶ Virtual Capon's algorithm
 - ▶ Virtual Multiple-Signal-Classification (virtual-MUSIC) algorithm
- Virtual Steering Vector equation

$$\xi(\theta, \phi) = \frac{\underline{S}_v^H(\theta, \phi) \mathbb{R}_{x_v x_v} \underline{S}_v(\theta, \phi)}{(\bar{N}N)^2} \quad (23)$$

- Virtual Capon's equation

$$\xi(\theta, \phi) = \frac{1}{\underline{S}_v^H(\theta, \phi) \mathbb{R}_{x_v x_v}^{-1} \underline{S}_v(\theta, \phi)} \quad (24)$$

- (θ, ϕ) can be estimated as follows:

$$(\theta, \phi) = \arg \max_{\forall \theta, \phi} \xi(\theta, \phi) \quad (25)$$

- $\mathbb{R}_{x_v x_v}$ denotes the covariance matrix of the virtual signal $\underline{x}_v(t)$, i.e.

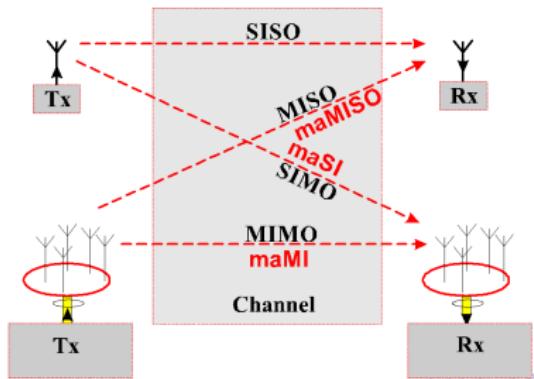
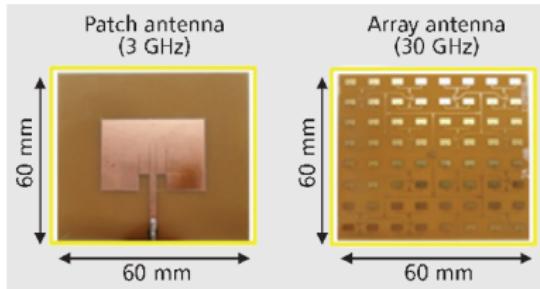
$$\mathbb{R}_{x_v x_v} = \mathcal{E} \left\{ \underline{x}_v(t) \cdot \underline{x}_v(t)^H \right\} \simeq \frac{1}{L} \mathbb{X}_v \mathbb{X}_v^H; \mathbb{R}_{x_v x_v} \in C^{N\bar{N} \times N\bar{N}} \quad (26)$$

MIMO Advantages

- More degrees of freedom due to the potential of using the concept of "virtual" antennas
- Higher angular resolution.
- Higher number of targets/clutter in a given range-Doppler cell, which can be detected and localized.
- Lower sidelobes by virtual spatial windowing.
- Digital beamforming and beam-steering at both the Tx and the Rx arrays, and therefore avoid beam shape loss in cases that the target is not in the center of the beam.
- Decrease the spatial power density of the Tx signal – spatial spread spectrum (spatial-SSS) which is critical for Low Probability of Intercept Radar (LPI-Radar).

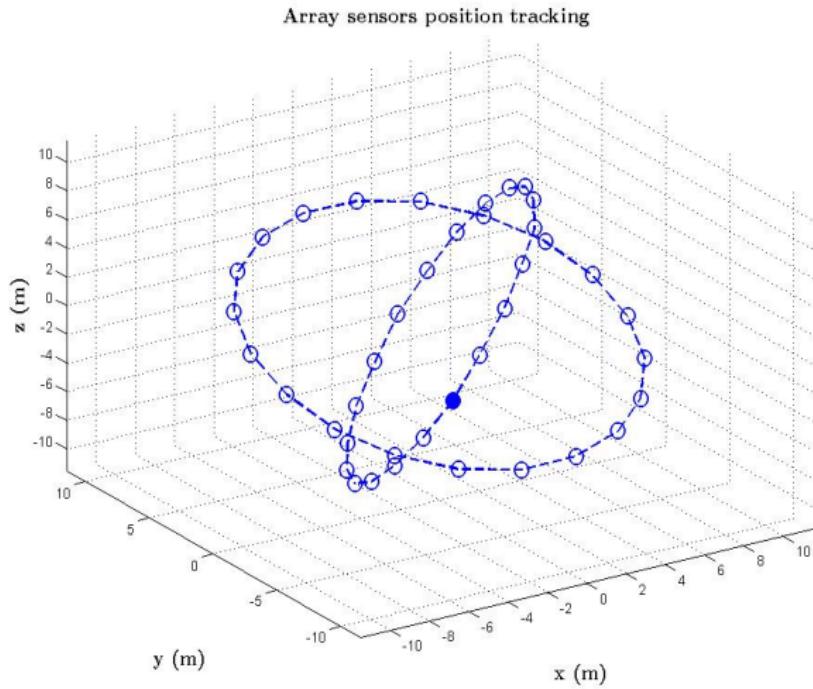
Massive MIMO Radar

- There is an increased interest in "**massive**" MIMO, i.e. to increase the "degrees-of-freedom" by **increasing the number of antennas to 100s or 1000s.**

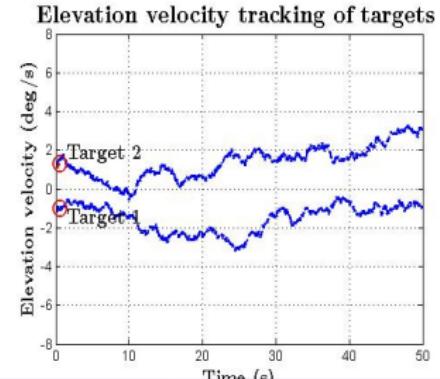
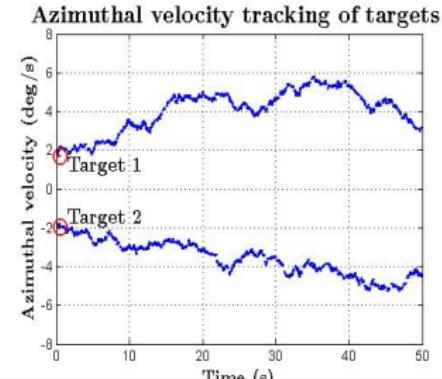
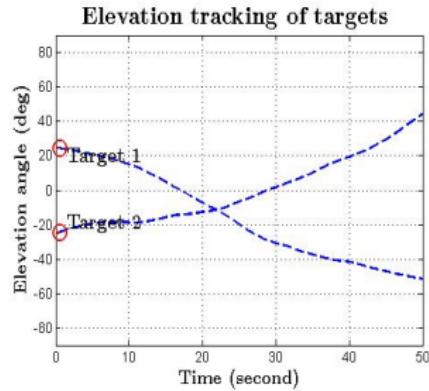
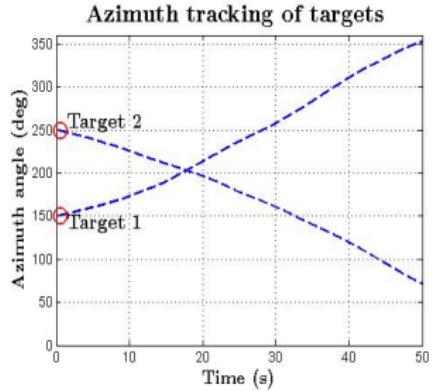


MIMO Radar: Representative Example

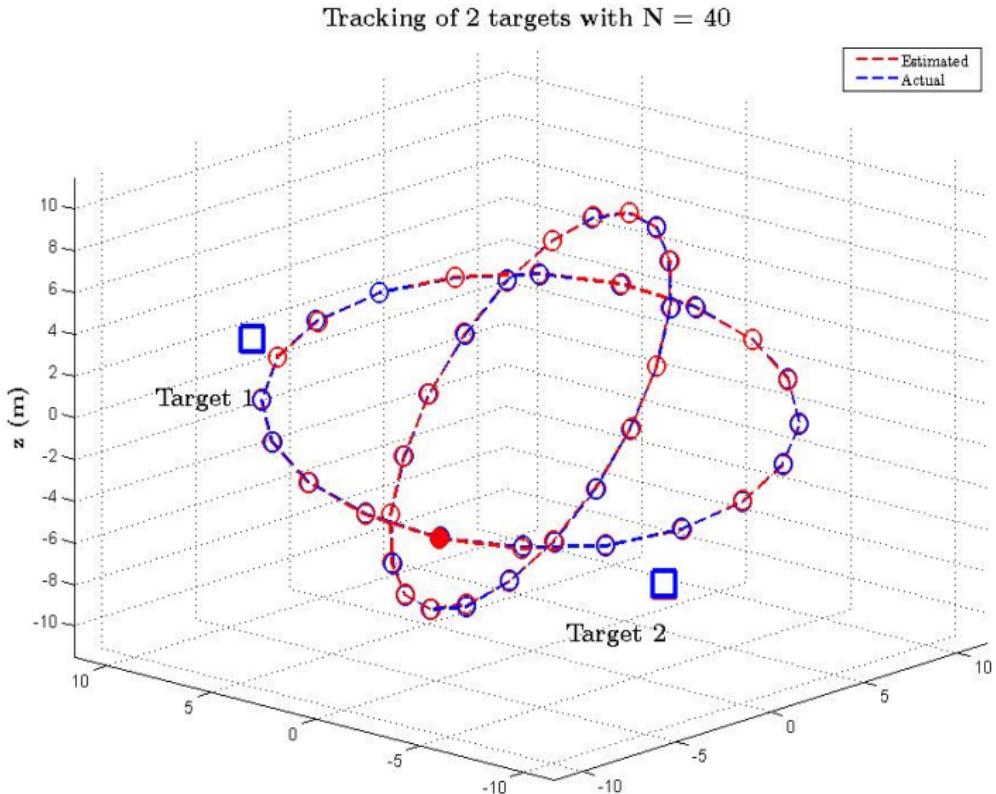
Example (MIMO Radar - Cluster of UAVs)



Example (MIMO Radar: Trajectory Tracking with a Cluster of UAVs)

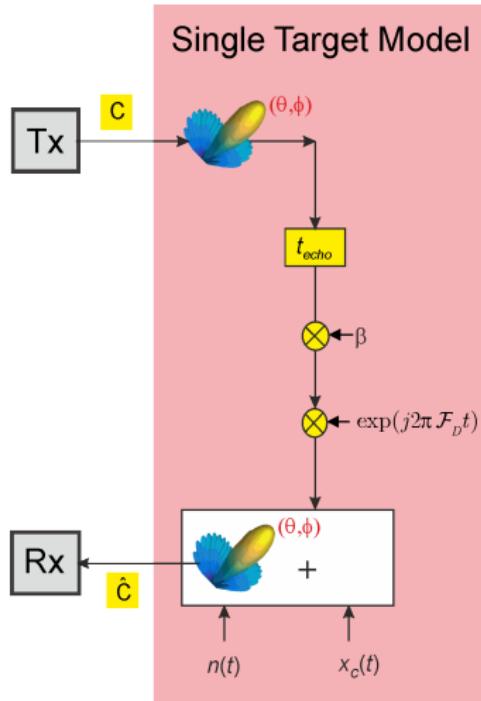


Example (MIMO Radar: Tracking the UAV Geometry)



- **SIMO Radar:** quite popular
- **SIMO architectures:** very powerful; more popular in bistatic radar (the Rx antenna can easily estimate, amongst others, the target direction θ_{Rx}).
- **MISO architectures:** Not so popular (limited radar applications)
- **MIMO Radar architectures:** more powerful than other radar architectures; more complex

Appendix-A: Monostatic SISO Radar



$$h(t) = \beta \cdot \exp(j2\pi\mathcal{F}_D t) \cdot \delta(t - t_{echo})$$

= impulse response

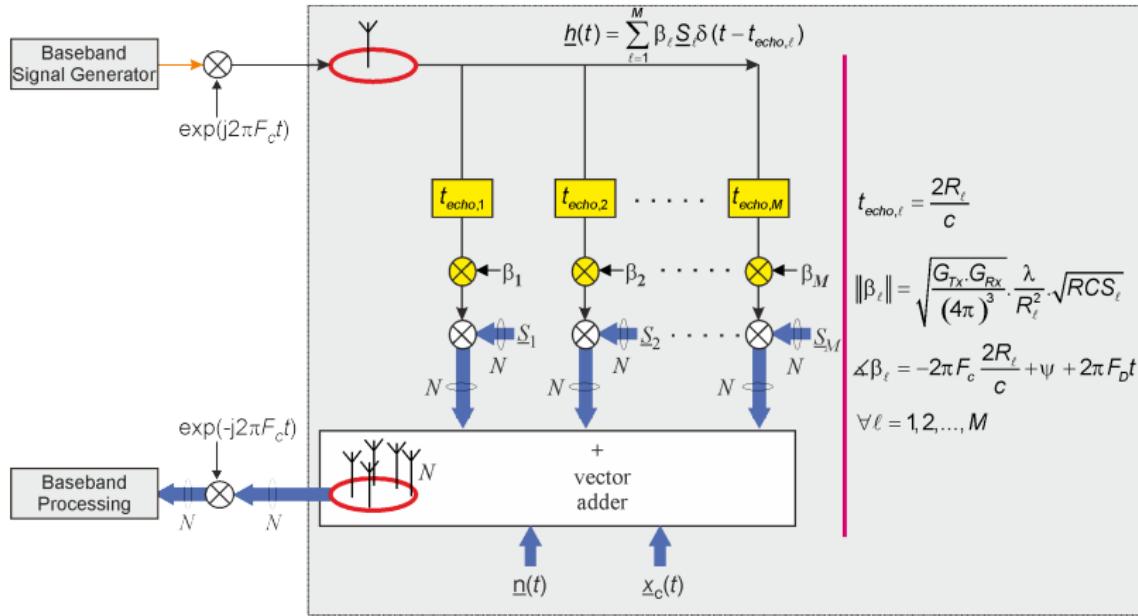
$$t_{echo} = \frac{2R}{c}$$

$$\beta = \sqrt{\frac{G_{Tx} \cdot G_{Rx}}{(4\pi)^3}} \cdot \frac{\lambda}{R^2} \sqrt{RCS} \cdot \exp(-j2\pi F_c \frac{2R}{c}) \exp(j\psi)$$

\mathcal{F}_D = Doppler frequency

ψ = random phase

Appendix-B: Monostatic SIMO Radar



- The vector $\underline{S}_\ell = \underline{S}(\theta_\ell, \phi_\ell) \in C^N$, is the Rx array manifold vector of the ℓ -th target.

- Consider the Tx transmits a scalar baseband signal $b(t)$ in the presence of M targets (SIMO radar).
- In a SIMO radar, if there are M targets, the multi-target backscatter impulse response (vector) is

$$\text{SIMO radar, multi-target: } \underline{h}(t) = \sum_{\ell=1}^M \underline{S}(\theta_\ell, \phi_\ell) \cdot \beta_\ell \cdot \delta(t - \tau_{echo,\ell}) \quad (27)$$

and the received ($N \times 1$) vector-signal $\underline{x}(t)$ can be modelled as follows:

$$\begin{aligned} \underline{x}(t) &= \underline{h}(t) * b(t) + \underline{n}(t) + \underline{x}_c(t) \\ &= \left(\sum_{\ell=1}^M \underline{S}(\theta_\ell, \phi_\ell) \cdot \beta_\ell \cdot \delta(t - \tau_{echo,\ell}) \right) * b(t) + \underline{n}(t) + \underline{x}_c(t) \end{aligned} \quad (28)$$

$$\Rightarrow \underline{x}(t) = \underbrace{\sum_{\ell=1}^M \underline{S}(\theta_\ell, \phi_\ell) \cdot \beta_\ell \cdot b(t - \tau_{echo,\ell})}_{\triangleq \text{multi-target echoes}} + \underline{n}(t) + \underline{x}_c(t) \quad (29)$$

where $\underline{n}(t)$ and $\underline{x}_c(t)$ are the noise and clutter terms (see Equ.7 and

Appendix-C: Monostatic MISO Radar

- If the radar Tx employs an array of \bar{N} antennas with locations $\underline{\bar{r}}$ then the backscatter impulse response is:

- ▶ for a single target:

$$\text{MISO-radar, single target: } h(t) = \beta \bar{\underline{S}}^H \underline{\delta}(t - t_{echo}) \quad (30)$$

where

$$\bar{\underline{S}} = \bar{\underline{S}}(\theta, \phi) = \exp(+j[\bar{r}_1, \bar{r}_2, \dots, \bar{r}_N]^T \underline{k}(\bar{\theta}, \bar{\phi})) \quad (31)$$

$$= \exp(+j[\bar{r}_x, \bar{r}_y, \bar{r}_z] \underline{k}(\bar{\theta}, \bar{\phi})) \quad (32)$$

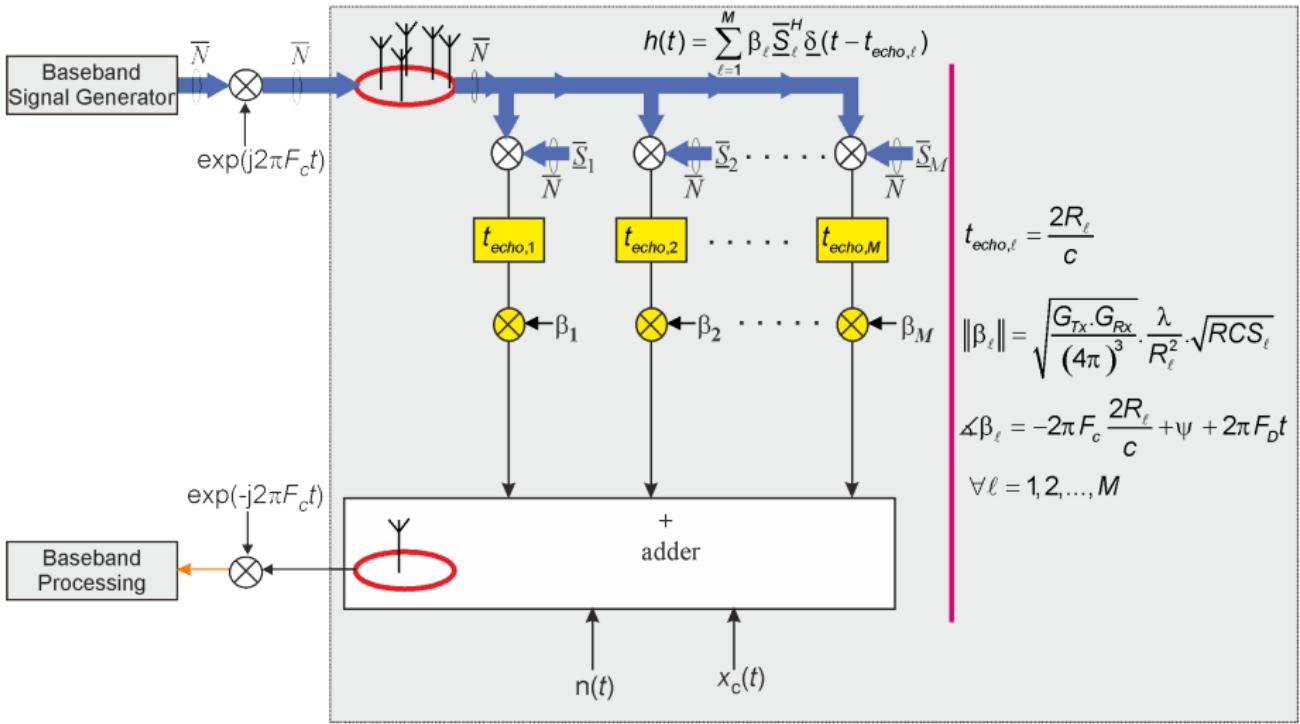
= ($N \times 1$) complex vector

$$\underline{k}(\bar{\theta}, \bar{\phi}) = \frac{2\pi F_c}{c} \cdot \underline{u}(\bar{\theta}, \bar{\phi}) = \frac{2\pi}{\lambda_c} \cdot \underline{u}(\bar{\theta}, \bar{\phi})$$

- ▶ for a multi-target:

$$\text{MISO-radar, multi-target: } h(t) = \sum_{\ell=1}^M \beta_\ell \bar{\underline{S}}_\ell^H \underline{\delta}(t - \tau_{echo,\ell}) \quad (33)$$

where the vector $\bar{\underline{S}}_\ell = \bar{\underline{S}}(\theta_\ell, \phi_\ell) \in C^N$, is the Tx array manifold vector of the ℓ -th target.



- Consider a single Tx transmitting a baseband vector-signal $\underline{b}(t)$ in the presence of M targets (MISO radar).
- The received scalar-signal $x(t)$ can be modelled as follows:

$$\begin{aligned} x(t) &= h(t) * \underline{b}(t) + n(t) + x_c(t) \\ &= \left(\sum_{\ell=1}^M \beta_{\ell} \cdot \underline{S}_{\ell}^H \delta(t - \tau_{echo,\ell}) \right) * \underline{b}(t) + n(t) + x_c(t) \end{aligned}$$

$$\implies x(t) = \underbrace{\sum_{\ell=1}^M \beta_{\ell} \cdot \underline{S}_{\ell}^H \underline{b}(t - \tau_{echo,\ell})}_{\triangleq \text{scalar echoes}} + n(t) + x_c(t)$$