EE401: Advanced Communication Theory

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Localisation of Wireless Signals

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Notation

$\underline{a}$, $\underline{A}$	denotes a column vector
$\mathbb{A}$, $\mathbf{A}$, $\mathbf{a}$	denotes a matrix
$\mathbb{I}_N$	$N \times N$ identity matrix
$\underline{1}_N$	vector of N ones
$\underline{0}_N$	vector of N zeros
$\mathbb{O}_{N,M}$	$N \times M$ matrix of zeros
$(.)^T$	transpose
$(.)^H$	Hermitian transpose
$\mathbb{A}^\#$	pseudo-inverse of $\mathbb{A}$
$\odot$, $\oslash$	Hadamard product, Hadamard division
$\otimes$	Kronecker product
$\boxtimes$	Khatri-Rao product (column by column Kronecker product)
$\exp(\underline{a}), \exp(\mathbb{A})$	element by element exponential
$\mathcal{L}[\mathbb{A}]$	linear space/subspace spanned by the columns of $\mathbb{A}$
$\mathcal{L}[\mathbb{A}]^\perp$	complement subspace to $\mathcal{L}[\mathbb{A}]$
$\mathcal{P}[\mathbb{A}]$ (or $\mathbb{P}_\mathbb{A}$)	projection operator on to $\mathcal{L}[\mathbb{A}]$
$\mathcal{P}[\mathbb{A}]^\perp$ (or $\mathbb{P}_\mathbb{A}^\perp$)	projection operator on to $\mathcal{L}[\mathbb{A}]^\perp$

References

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- A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 24-40, 2005.
- A. Manikas, Y. Kamil, and M. Willerton, "Source Localization Using Sparse Large Aperture Arrays," *IEEE Trans. Signal Processing*, vol. 60, no. 12, pp. 6617–6629, 2012.

Abbreviations

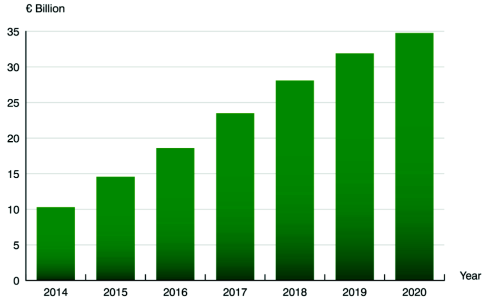
BTS (BS)	Base Transceiver Station (or simply Base Station)
Note-B	BS terminology used in 3G+
MS, UE	Mobile Station, User Equipment (terminology used in 3G+)
AP	Access Point
E911, E112	Enhanced 911, E112 (Emergency Service)
FCC	Federal Communications Commission (USA)
GPS	Global Positioning System
LBS	Location Based Services
ML	Maximum Likelihood
NLOS	Non-Line-of-Sight
PSAP	Public Safety Answering Point
SNIR	Signal-to-Noise-plus-Interference Ratio
SNR	Signal-to-Noise Ratio
RTD	Real Time Difference
OTD	Observed Time Difference
GTD	Geometric Time Difference
LMU	Location Measuring Unit
AOA, DOA	Angle of Arrival, Direction of Arrival
TOA	Time of Arrival
TDOA	Time Difference of Arrival
RSSI	Received Signal Strength Indicator
LAA	Large Aperture Array
WSN	Wireless Sensor Networks

Aim

- This course is concerned with the problem of **Locating and Tracking** **energy emitter** s or **reflecting sources** with emphasis given to applications in the area of wireless communications.
- **Applications** :
  - Radar,
  - Sonar,
  - Navigation,
  - Biomedicine,
  - Seismology,
  - Pollution Monitoring,
  - Monitoring wildlife,
  - **Location Based Services** ,
  - etc.

Location Based Services (LBSs)

- Asset Tracking,
- Fleet Management,
- Location Based Wireless Access Security,
- Location Sensitive Billing,
- Location based Advertising,
- Etc.



Mobile LBS revenue forecast, € billion (World 2014–2020)

Localisation in Cellular Networks

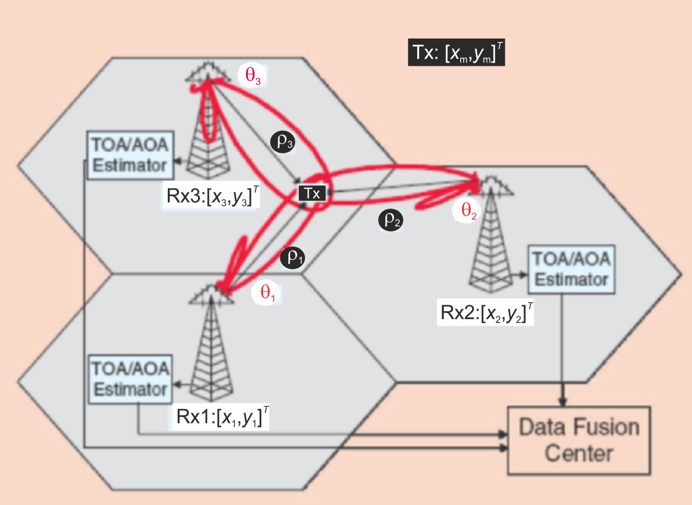
- Some of the most interesting positioning application areas have emerged in Wireless Communications. **The most prominent** :
  - **FCC** (Federal Communications Commission) which requires that the precise location of all enhanced 911 (E911) emergency calls be automatically determined.
    - ★ FCC Mandate: all handsets sold be location compatible.
  - **European Recommendation E112**
  - Both E911 and E112 require that wireless providers should be able **to locate within tens of meters users of emergency calls** .
- **Other applications** (besides E911 and E112 services)
  - Vehicle navigation
  - Network optimisation
  - Resource allocation
  - Automatic billing
  - Ubiquitous computing (e.g. for accessing personal info, corporate data, share resources, anywhere)
  - location-aware computing

Classification of Localisation Systems/Architectures

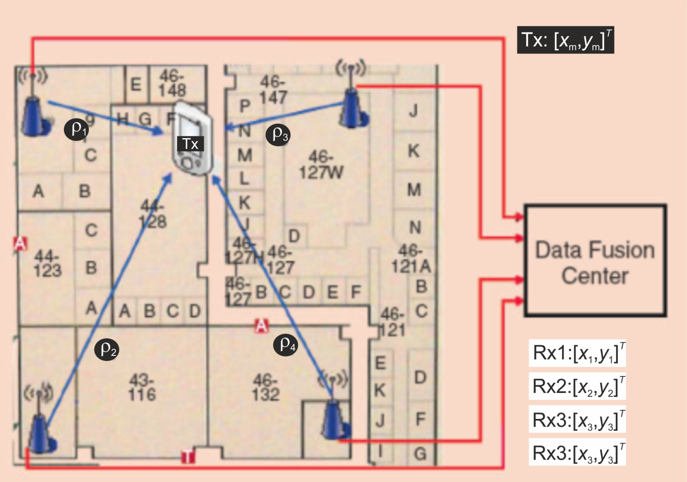
Many Classifications

- **Classification-1**
  - Indoor, e.g.:
    - WLAN
  - Outdoor, e.g.:
    - GPS
    - Cellular network-aided wireless location finding
- **Classification-2** :
  - Cellular Network-aided,
  - Sensor Network-aided,
  - GPS,
  - assisted GPS (combination of GPS and Cellular Network-aided)
- **Classification-3** :
  based on the localisation algorithms (to be discussed later)

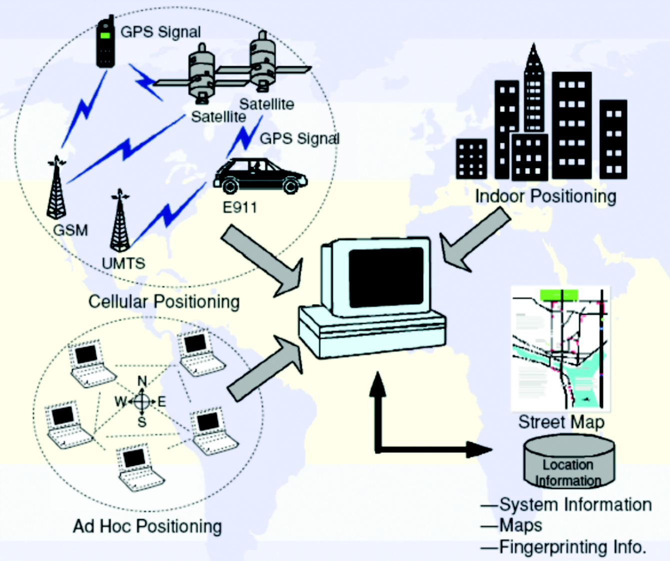
Classification-1: Outdoors



Classification-1: Indoors



Classification-2



Classification-2: GPS and A-GPS

GPS	Assisted-GPS (A-GPS)
requires minimal obstruction	Can be used even for indoor and can be much more accurate (10-50m)
Long acquisition times (30sec-15min)	Improves acquistion time ($<$10sec)
Has to be synchronous	Synchronous or Asynchronous
High power consumption	More cost effective than GPS
High cost	Little (or no) hardware changes required in Base Stations

Localisation Algorithms (Classification-3)

- Time-Of-Arrival (**TOA**)
- Time-Difference-Of-Arrival (**TDOA**)
- Received Signal Strength Indicator (**RSSI**)
- Direction-of-Arrival **(DOA) Localisation** also known as Angle-of-Arrival **(AOA) Localisation**
- Large-Aperture-Array (**LAA**) Localisation

  Association:
  - measurement
  - equations
- Hybrid Localisation algorithms

  Metric fusion stage:
  - solution
- N.B: In all the above algorithms there are two stages/phases:
  1. Association stage
  2. Metric Fusion stage

Other Classifications

- **Classification-4:**
  - Trilateration localisation algorithms $\ni$ TOA, TDOA, RSS
  - Triangulation localisation algorithms $\ni$ DOA
- **Classification-5:**
  - non-arrayed localisation algorithms $\ni$ TOA, TDOA, RSS
  - arrayed localisation algorithms $\ni$ DOA localisation, LAA Localisation
    - single array array $\ni$ LAA Localisation
    - multiple arrays $\ni$ DOA Localisation
- **Classification-6** : In **WSNs the above localisation algorithms** can be classified as
  - **Cooperative localisation algorithms**
    - Centralised Algorithms
    - Distributed Algorithms
  - **Non-cooperative algorithms**

TOA Localisation

ρ_i : distance between Tx and receiver i.

$\underline{r}_m$: coordinate of Tx.

TOA:

sync of Tx and Rx
$(t^0 . t_1 \cdots t_i)$

$\Downarrow$

distance to receivers

$\rho_i = (t_i - t^0) \cdot c$

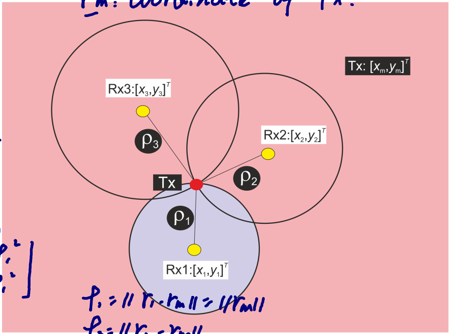
$\Downarrow$

equations $(\underline{H}\, \underline{r}_m = \underline{b})$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|r_2\|^2 - \rho_2^2 + \rho_1^2 \\ \|r_3\|^2 - \rho_3^2 + \rho_1^2 \end{bmatrix}$$

solution

$\underline{r}_m = \underline{H}^{\#}\, \underline{b}$.



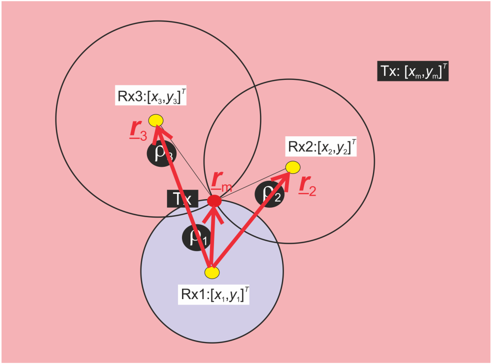
$\rho_1 = \| r_1 - r_m \| = \| r_m \|$

$\rho_2 = \| r_2 - r_m \|$

$\rho_3 = \| r_3 - r_m \|$

- **Association Stage**
  - t^0 = *known* (Tx or MS)
  - t_1, t_2, t_3 = measured-TOA (Rx1, Rx2, Rx3 or BS1, BS2, BS3)
  - ρ_1, ρ_2, ρ_3 = estimated using the following equation

$$\rho_i = \left(t_i - t^0\right).c \qquad (2)$$
$$\text{for } i = 1, 2, 3$$



- **Note-1:**
  - Rx1, Rx2, Rx3 coordinates **are known**

$$\underline{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \underline{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}; \underline{r}_1 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

  - Tx coordinates **are unknown**

$$\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \text{unknown}$$

- **Metric Fusion Stage**
  solve the following Equation

$$\mathbb{H}\underline{r}_m = \underline{b} \Rightarrow \underline{r}_m = \mathbb{H}^{\#}\underline{b} \tag{3}$$

where

$$\mathbb{H} = \begin{bmatrix}\underline{r}_2, & \underline{r}_3\end{bmatrix}^T = \begin{bmatrix} x_2, & y_2 \\ x_3, & y_3 \end{bmatrix} \tag{4}$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \left\|\underline{r}_2\right\|^2 - \rho_2^2 + \rho_1^2 \\ \left\|\underline{r}_3\right\|^2 - \rho_3^2 + \rho_1^2 \end{bmatrix} \tag{5}$$

- **Note-2** :

$$\begin{aligned} \rho_1 &= \left\|\underline{r}_1 - \underline{r}_m\right\| = \left\|\underline{r}_m\right\|; \\ \rho_2 &= \left\|\underline{r}_2 - \underline{r}_m\right\|; \\ \rho_3 &= \left\|\underline{r}_3 - \underline{r}_m\right\| \end{aligned}$$

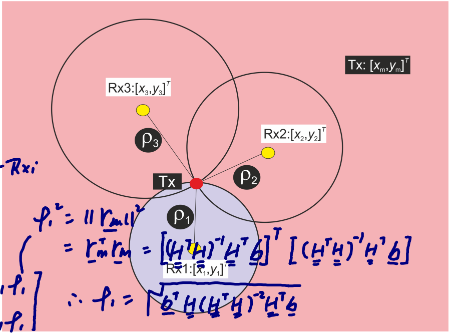
- **Note-3: TOA Requirements**
  - TOA approach **requires accurate synchronisation** between the **Rx1, Rx2, Rx3 and Tx clocks** .
    - ★ This requirement ensures that the estimated ρ_1, ρ_2, ρ_3 are good approximations of the actual distances.
  - Many of the current wireless system standards only **require tight timing synchronisation amongst BSs** .
    - ★ The **MS clock itself might have a drift** that can be even a few microseconds.
    - ★ This drift directly produces errors in ρ_1, ρ_2, ρ_3 and, consequently, errors in the location estimate of the TOA method.

TDOA (Time Difference of Arrival) Localisation

TDOA:
sync between Rx
$(t_1 \cdots t_i)$
$\Downarrow$
distance difference
between Tx - Rx1 and Tx - Rxi
$\rho_{i1} = (t_i - t_1)c$
$\Downarrow$
equations

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|r_2\| - \rho_{21}^2 - \rho_{21}\rho_1 \\ \|r_3\| - \rho_{31}^2 - \rho_{31}\rho_1 \end{bmatrix}$$

solution
$\underline{r}_m = \underline{H}^{\#}\underline{b}$

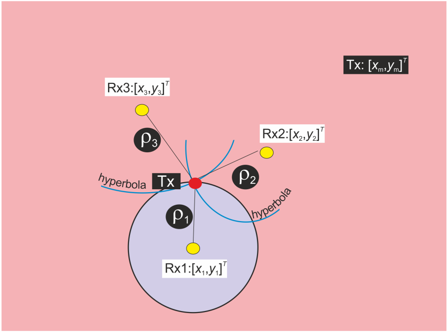


$$\rho_1^2 = \|\underline{r}_m\|^2 = \underline{r}_m^T\underline{r}_m = [(\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{b}]^T\,[(\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{b}]$$

$$\therefore \rho_1 = \sqrt{\underline{b}^T\underline{H}(\underline{H}^T\underline{H})^{-2}\underline{H}^T\underline{b}}$$

- TDOA method **does not suffer** from MS clock **synchr. errors**
- **Association Stage**
  - t^0 = *unknown* (Tx, MS)
  - t_1, t_2, t_3 = measured-TOA (Rx1, Rx2, Rx3 or BS1, BS2, BS3)
  - ρ_1, ρ_2, ρ_3 = **cannot** be estimated using the following equation

$$\rho_i = \left(t_i - t^0\right).c; \text{ for } i = 1,2,3 \qquad (6)$$



- **Note-1** : However ρ_{21}, ρ_{31} can be estimated using

$$\rho_{i1} = \rho_i - \rho_1 = \left(t_i - t^0\right) c - \left(t_1 - t^0\right) c \tag{7}$$

$$\Rightarrow \rho_{i1} = \left(t_i - t_1\right) c \tag{8}$$

- **Note-2** : The TDOA associated with the Rx$_i$ (BS$_i$) is $t_i - t_1$
  That is, the difference between the TOA of the Tx (MS) signal at the Rx$_i$ (BS$_i$) and the Rx$_1$ (BS$_1$)

- **TDOA Metric Fusion Stage:**
solve the following equation

$$\mathbb{H}\underline{r}_m = \underline{b} \Rightarrow \underline{r}_m = \mathbb{H}^{\#}\underline{b} \tag{9}$$

where

$$\mathbb{H} = \left[\underline{r}_2, \quad \underline{r}_3\right]^T = \begin{bmatrix} x_2, & y_2 \\ x_3, & y_3 \end{bmatrix} \tag{10}$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \left\|\underline{r}_2\right\|^2 - \rho_{21}^2 - 2\rho_{21}.\rho_1 \\ \left\|\underline{r}_3\right\|^2 - \rho_{31}^2 - 2\rho_{31}.\rho_1 \end{bmatrix} \tag{11}$$

with ρ_1 is estimated firstly as the **positive root** of the following quadratic equation:

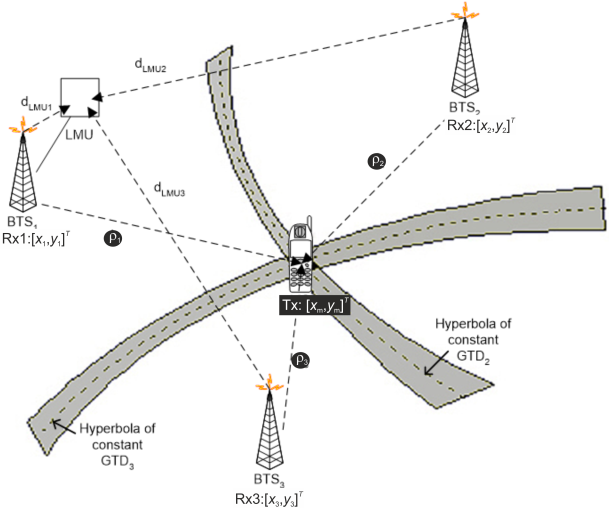
$$\rho_1^2 = \left\|\underline{r}_m\right\|^2 = \underline{r}_m^T\underline{r}_m = \left(\left(\mathbb{H}^T\mathbb{H}\right)^{-1}\mathbb{H}^T\underline{b}\right)^T\left(\mathbb{H}^T\mathbb{H}\right)^{-1}\mathbb{H}^T\underline{b}$$

$$\Rightarrow \rho_1 = \sqrt{\underline{b}^T\mathbb{H}\left(\mathbb{H}^T\mathbb{H}\right)^{-2}\mathbb{H}^T\underline{b}} \tag{12}$$

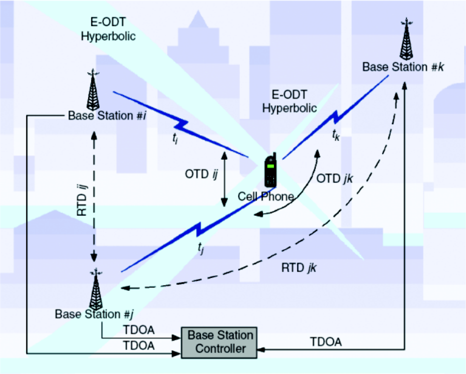
- **Note-3: TDOA Requirements**
  - TDOA approach requires **accurate synchronisation only amongst the BSs clocks** . i.e it requires a "synchronous" network
    - ★ This requirement ensures that the estimated r_{12}, r_{23}, r_{13} are good approximations of the actual distances.
  - A **cellular network** is not a "synchronous" network.
    - ★ Therefore, TDOA can not be used directly (it suffers from synchronisation errors amongst BSs)
    - ★ GSM employs a quite expensive solution known as **E-OTD** (Enhanced Observed Time Difference)

Enhanced Observed Time Difference (E-OTD)

- The real time differences (RTD) between pairs of BSs are measured by an LMU (Location Measuring Unit) device



- LMU computes the clock difference between BSs and send this information to the corresponding BSs.
- In addition the **Observed Time Differences** (OTD) are measured between pairs of BSs (The measured time difference between one pair of BS, is referred to as OTD)



$$\text{E-OTD localisation: } \mathrm{TDOA}_{ij} = \mathrm{OTD}_{ij} - \mathrm{RTD}_{ij} \qquad (13)$$

Received Signal Strength Indicator (RSSI)

- Tx signal decays with distance
- many devices measure signal strength with "received signal strength indicator (RSSI)"
  - vendor-specific interpretation and representation
  - typical RSSI values are in range of 0..RSSI_Max
  - common values for RSSI_Max: 100, 128, 256
- in free space, RSS degrades with square of distance expressed by **Friis transmission equation**

$$P_{Rx} = \underbrace{P_{Tx}.G_{Tx}.G_{Rx}\left(\frac{\lambda}{4\pi}\right)^2}_{\text{constant for system with given frequency.}} \frac{1}{\rho^2} \qquad (14)$$

or

$$P_{Rx\text{[dBm]}} = P_{Tx\text{[dBm]}} + G_{Tx\text{[dBi]}} + G_{Rx\text{[dBi]}} + 20\log_{10}\left(\frac{\lambda}{4\pi}\frac{1}{\rho}\right) \qquad (15)$$

- in practice, the actual attenuation depends on multipath propagation effects, reflections, noise, etc
- realistic models replace ρ^2 with ρ^α (α=2,..,5)

- **Association Stage** :
  - Measure powers and calculate ρ_1, ρ_2, ρ_3 using the following

$P_{Rx} = P_{Tx}\, G_{Tx} G_{Rx} \left(\frac{\lambda}{4\pi}\right)^2 \frac{1}{\rho^2}$

$\rho_i = \frac{\lambda}{4\pi}\sqrt{\frac{P_{Tx}}{P_{Rxi}} G_{Tx} G_{Rx}}$

estimate distance by received power.

$$\rho_i = \sqrt{\frac{P_{Tx}}{P_{Rx_i}}.G_{Tx}.G_{Rx}}\frac{\lambda}{4\pi}$$

- **Metric Fusion Stage** :
  solve the following equation (as in TOA)

RSSI vs. TOA:
different in distance estimation
(Rx power vs. timing)

$$\mathbb{H}\underline{r}_m = \underline{b} \Rightarrow \underline{r}_m = \mathbb{H}^{\#}\underline{b} \quad (16)$$

where

same in fusion

$\underline{b} = \frac{1}{2}\begin{bmatrix} \|r_2\|^2 - \rho_2^2 + \rho_1^2 \\ \|r_3\|^2 - \rho_3^2 + \rho_1^2 \end{bmatrix}$

$\underline{r}_m = \underline{\underline{H}}^{\#}\underline{b}$.

$$\mathbb{H} = \left[\underline{r}_2, \quad \underline{r}_3\right]^T = \begin{bmatrix} x_2, & y_2 \\ x_3, & y_3 \end{bmatrix} \quad (17)$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|\underline{r}_2\|^2 - \rho_2^2 + \rho_1^2 \\ \|\underline{r}_3\|^2 - \rho_3^2 + \rho_1^2 \end{bmatrix} \quad (18)$$

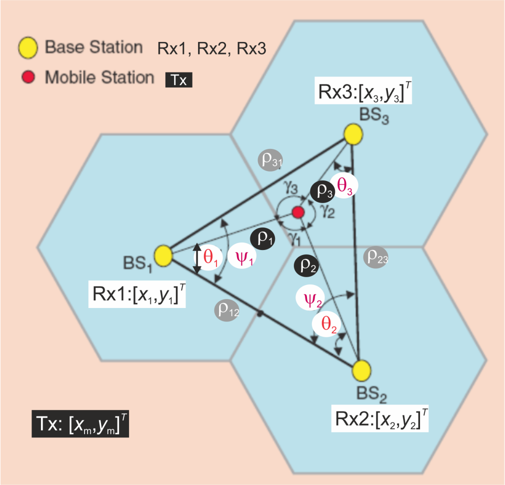
DOA Localisation

DOA:

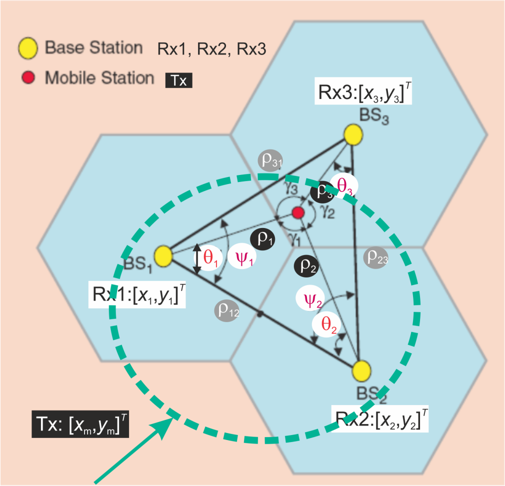
- no need for synchronisation
- need antenna array
- Rx estimate DOA (MUSIC)

- This is a better algorithm than TOA, TDOA and RSSI
  - redundancy. or diversity.
- It does not require BS or BS/MS clock synchronisation
- Note that antenna array structures did not exist in 2G but recently are used in current cellular systems 3G/4G

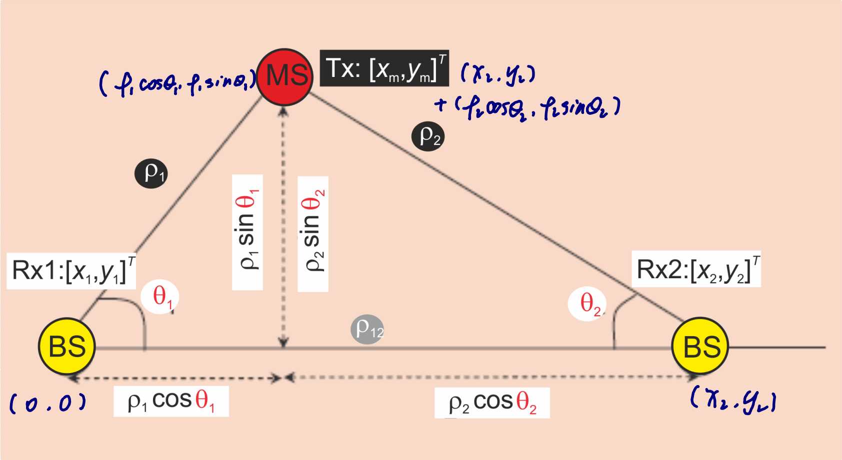
- **Association Stage**
  Consider that each Rx has an antenna array and estimates the Direction-of-Arrival of the Tx (MS) signal, e.g using MUSIC algorithm (for 3 BS's: estimate the directions $\theta_1, \theta_2, \theta_3$ say)



- By combining the DOA estimates from two different BSs (e.g. θ_1, θ_2 say) an estimate of the MS location can be obtained



let us focus on this triangle


MS
Tx: $[x_m, y_m]^T$
(x_2, y_2)
$+(\rho_2\cos\theta_2, \rho_2\sin\theta_2)$
$(\rho_1\cos\theta_1, \rho_1\sin\theta_1)$
ρ_1
ρ_2
$\rho_1 \sin\theta_1$
$\rho_2 \sin\theta_2$
Rx1: $[x_1, y_1]^T$
Rx2: $[x_2, y_2]^T$
θ_1
θ_2
ρ_{12}
BS
BS
$(0,0)$
$\rho_1 \cos\theta_1$
$\rho_2 \cos\theta_2$
(x_2, y_2)

- **Note-1** :
  - Mobile Location: $\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$ =unknown
  - However:

$$\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \end{bmatrix}$$

and

$$\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{=\underline{r}_2} + \begin{bmatrix} \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{bmatrix}$$

  - Thus, for any Rx (the i^{th} say)

$$\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \underbrace{\begin{bmatrix} x_i \\ y_i \end{bmatrix}}_{=\underline{r}_i} + \begin{bmatrix} \rho_i \cos\theta_i \\ \rho_i \sin\theta_i \end{bmatrix}$$

- **Metric Fusion Stage** :
  - solve the following equation

DOA:

Δ estimate ρ with other methods, say RSSI or spherical array manifold vector.

$$\mathbb{H}\underline{r}_m = \underline{b} \Rightarrow \underline{r}_m = \mathbb{H}^{\#}\underline{b} \tag{19}$$

where

estimate DOA with algorithms. say MUSIC.

$$r_m = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} \rho_i \cos\theta_i \\ \rho_i \sin\theta_i \end{bmatrix}$$

equations

$$\underline{\underline{H}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} \rho_1\cos\theta_1 \\ \rho_1\sin\theta_1 \\ x_2+\rho_2\cos\theta_2 \\ y_2+\rho_2\sin\theta_2 \\ \vdots \\ x_n+\rho_n\cos\theta_n \\ y_n+\rho_n\sin\theta_n \end{bmatrix}$$

solution

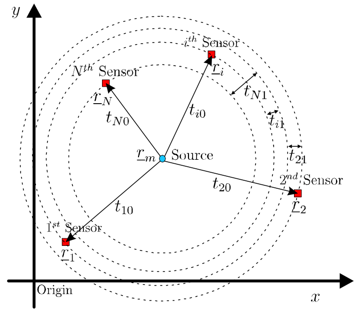
$$r_m = \underline{\underline{H}}^{\#}\underline{b}$$

$$\mathbb{H} = \begin{bmatrix} 1, & 0 \\ 0, & 1 \\ 1, & 0 \\ 0, & 1 \\ \vdots & \vdots \\ 1, & 0 \\ 0, & 1 \end{bmatrix} = \begin{bmatrix} \mathbb{I}_2 \\ \mathbb{I}_2 \\ \vdots \\ \mathbb{I}_2 \end{bmatrix} = \underline{1}_N \otimes \mathbb{I}_2; \tag{20}$$

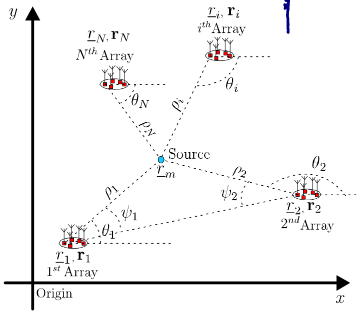
$$\underline{b} = \begin{bmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ x_2 + \rho_2 \cos\theta_2 \\ y_2 + \rho_2 \sin\theta_2 \\ \vdots \\ x_N + \rho_N \cos\theta_N \\ y_N + \rho_N \sin\theta_N \end{bmatrix} \tag{21}$$

Large Aperture Array (LAA) Localisation

LAA: sensors distribute separately → regard as spherical propagation.
DOA: arrays with elements together → regard as planar propagation.



(a) TOA/TDOA (N sensors at locations $\underline{r}_1, \underline{r}_2, \cdots, \underline{r}_N$)



(b) DOA (N arrays with reference points at $\underline{r}_1, \underline{r}_2, \cdots, \underline{r}_N$ and local geometries at $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_N$)

Propagation Model

- Consider a single spherical wave

$$\sqrt{P}.\exp(j2\pi F_c t)$$

propagating from the MS to the 1st receiver Rx1 through our 3D real space.

- This spherical wave arrives at the Rx1 receiver's antenna and produces a constant-amplitude voltage

$$\begin{aligned} & \sqrt{P}.\left(\frac{\mathrm{k}}{\rho_1}\right)^{\overset{\text{path loss exponent 2-5.}}{\alpha}} \exp(j2\pi F_c\,(t-\tau_1)) \\ = \quad & \underbrace{\sqrt{P}.\exp(j2\pi F_c t)}_{\text{Tx wave}} \underbrace{\left(\frac{\mathrm{k}}{\rho_1}\right)^{\alpha} \exp\left(-j2\pi F_c \frac{\rho_1}{c}\right)}_{\text{propagation effect}} \end{aligned}$$

- Furthermore, this spherical wave arrives at the i-th Rx's antenna ($i = 2, 3$,etc.) and produces a constant-amplitude voltage

$$
\begin{aligned}
& \sqrt{P}.\left(\frac{\mathrm{k}}{\rho_i}\right)^{\alpha} \exp\left(j2\pi F_c\left(t-\tau_i\right)\right) \\
= \; & \sqrt{P}.\exp(j2\pi F_c t)\left(\frac{\mathrm{k}}{\rho_i}\right)^{\alpha} \exp\left(-j2\pi F_c \frac{\overset{\overset{\rho_1+\Delta\rho_{i1}}{\uparrow}}{\rho_i}}{c}\right) \\
= \; & \sqrt{P}.\exp(j2\pi F_c t)\left(\frac{\rho_1}{\rho_1}\frac{\mathrm{k}}{\rho_i}\right)^{\alpha} \underbrace{\exp\left(-j2\pi F_c \frac{\rho_1+\Delta\rho_{i1}}{c}\right)}_{\substack{\downarrow \\ \exp\left(-j2\pi F_c \frac{\rho_1}{c}\right)\exp\left(-j2\pi F_c \frac{\Delta\rho_{i1}}{c}\right)}} \\
= \; & \overbrace{\underbrace{\sqrt{P}.\exp(j2\pi F_c t)}_{\text{Tx-wave}}\underbrace{\left(\frac{\mathrm{k}}{\rho_1}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_1}{c}\right)}_{\text{complex path-gain}}}^{\text{signal at Rx1 (array reference point)}}\overbrace{\left(\frac{\rho_1}{\rho_i}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{i1}}{c}\right)}^{i^{th}\text{ element of the manifold vector}}
\end{aligned}
\tag{22}
$$

$\sqrt{P}\, e^{j2\pi F_c t}$

$\downarrow$ Tx → Rx 1

$\sqrt{P}\left(\frac{k}{\rho_1}\right)^{\alpha} e^{j2\pi F_c(t-\tau_1)}$

$\downarrow$ TX → Rx i

$\sqrt{P}\left(\frac{k}{i}\right)^{\alpha} e^{j2\pi F_c(t-\tau_i)}$

$= \sqrt{P}\left(\frac{k}{\rho_1}\frac{\rho_1}{\rho_i}\right)^{\alpha} e^{j2\pi F_c(t-\tau_1)}\, e^{j2\pi F_c(\tau_1-\tau_i)}$

$= \underbrace{\boxed{\sqrt{P}\, e^{j2\pi F_c t}}}_{\text{TX}}\;\underbrace{\boxed{\left(\frac{k}{\rho_1}\right)^{\alpha} e^{-j2\pi F_c\frac{\rho_1}{c}}}}_{\text{path gain } \beta_1}\;\underbrace{\boxed{\left(\frac{\rho_1}{\rho_i}\right)^{\alpha} e^{-j2\pi F_c\frac{\Delta\rho_i}{c}}}}_{\text{ith manifold vector}}$

- or, the baseband signal (i.e. Equ.22 $\times \exp(j2\pi F_c t)$),is

$$\underbrace{\overbrace{\sqrt{P}.\left(\frac{\mathrm{k}}{\rho_1}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_1}{c}\right)}^{\text{signal at Rx1 (array reference point)}}}_{\text{complex path-gain }=\beta_1}\overbrace{\left(\frac{\rho_1}{\rho_i}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{i1}}{c}\right)}^{i^{th}\text{ element of the manifold vector}}$$

- Using all the baseband signals from all N Receivers, the baseband vector-signal is:

$$\sqrt{P}\beta_1\begin{bmatrix}\left(\frac{\rho_1}{\rho_1}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{11}}{c}\right)\\ \left(\frac{\rho_1}{\rho_2}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{21}}{c}\right)\\ \vdots\\ \left(\frac{\rho_1}{\rho_i}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{i1}}{c}\right)\\ \vdots\\ \left(\frac{\rho_1}{\rho_N}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\Delta\rho_{N1}}{c}\right)\end{bmatrix}=\sqrt{P}\begin{bmatrix}\left(\frac{\mathrm{k}}{\rho_1}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_1}{c}\right)\\ \left(\frac{\mathrm{k}}{\rho_2}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_2}{c}\right)\\ \vdots\\ \left(\frac{\mathrm{k}}{\rho_i}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_i}{c}\right)\\ \vdots\\ \left(\frac{\mathrm{k}}{\rho_N}\right)^{\alpha}\exp\left(-j2\pi F_c\frac{\rho_N}{c}\right)\end{bmatrix}$$

Spherical Array Manifold Vector $\rho_i = \rho_1 + \Delta\rho_{i1}$

- manifold vector:

$$\underline{S} = \underbrace{\rho^{\alpha}}_{\rho_1} \cdot \underbrace{\underline{d}^{-\alpha}}_{\rho_i} \odot \exp\left(-j\frac{2\pi F_c}{c}\left(\underbrace{\rho \cdot \underline{1}_N - \underline{d}}_{\rho_1 - \rho_i = \Delta\rho_{1i} = -\Delta\rho_{i1}}\right)\right) \in \mathcal{C}^{N\times 1}, \quad (23)$$

$$\begin{aligned} \underline{d} &= \underline{d}(\theta, \phi, \rho, \mathbf{r}) \\ &= \sqrt{\rho^2 \cdot \underline{1}_N + \underline{r}_x^2 + \underline{r}_y^2 + \underline{r}_z^2 - \frac{\rho c}{\pi F_c}\left[\underline{r}_x, \underline{r}_y, \underline{r}_z\right]\underline{k}(\theta, \phi)}. \\ &= \sqrt{\rho^2 \cdot \underline{1}_N + diag(\mathbf{r}^T\mathbf{r}) - \frac{\rho c}{\pi F_c}\mathbf{r}^T\underline{k}(\theta, \phi)} \end{aligned} \quad (24)$$

- It can be proven also that $\underline{d}$ is the unknown vector of ranges from the source to each of the array elements, i.e.

$$\underline{d} = \left[\rho_1, \rho_{2,} \cdots, \rho_N\right]^T \in \mathcal{R}^{N\times 1}. \quad (25)$$

where in Equation 24 the ranges $\rho_1, \rho_{2,} \cdots, \rho_N$ have been expressed as a function of a single set of unknown parameters (θ, ϕ, ρ).

- remember that:
  - α = path loss exponent
  - the matrix $\mathbf{r} \in \mathcal{R}^{3\times N}$ has as columns the Cartesian coordinates of the array elements with respect to the array reference point. That is,

$$\mathbf{r} = [\underline{r}_1, \underline{r}_2, \cdots, \underline{r}_N] = \left[\ \underline{r}_x, \quad \underline{r}_y, \quad \underline{r}_z\ \right]^T. \tag{26}$$

  - the vector, $\underline{k}(\theta, \phi) \in \mathcal{R}^{3\times 1}$ is the wavenumber vector which points towards the direction (θ, ϕ) and is defined by,

$$\underline{k}(\theta, \phi) = \frac{2\pi F_c}{c} \left[\begin{array}{c} \cos(\theta)\cos(\phi) \\ \sin(\theta)\cos(\phi) \\ \sin(\phi) \end{array} \right]. \tag{27}$$

Changing the array reference point

- The manifold vector depends on the array reference point relative to which the bearings and the range of the source as well as the Cartesian coordinates of the array configuration are measured. Thus consider 2 different reference points 1 and 2

- By denoting the manifold vector wrt the 1st and 2nd reference point using a subscript, i.e.

$$\begin{aligned}\underline{S}_1 &\triangleq \underline{S}(\theta_1, \phi_1, \rho_1, \mathbf{r}_1, F_c), \quad \text{and} \\ \underline{S}_2 &\triangleq \underline{S}(\theta_2, \phi_2, \rho_2, \mathbf{r}_2, F_c)\end{aligned}$$

  then it can be easily seen that

$$d_1 = d_2 = \underline{d} = \left[\rho_1, \rho_{2,} \cdots, \rho_N\right]^T \in \mathcal{R}^{N\times 1}. \tag{28}$$

- Furthermore, it can be proved that the array manifold vectors $\underline{S}_1$ and $\underline{S}_2$ are inter-related as follows:

$$\underline{S}_2 = const \cdot \underline{S}_1, \tag{29}$$

where

$$const = \left(\frac{\rho_2}{\rho_1}\right)^{\alpha} \exp\left(-j\frac{2\pi F_c}{c}(\rho_2 - \rho_1)\right). \tag{30}$$

- The above implies that the effect of a change in the reference point on the manifold vector is simply a change in the norm of the vector, (which depends only on the range of the mobile with respect to the two reference points).
- Consequently,
  - the signal subspace spanned by the manifold vector remains unchanged, but
  - the signal eigenvalues will change

- **Association Stage**
  - Four measurement (minimum) for four different reference points

$$
\begin{aligned}
\mathbb{R}_{xx}^{(1)} \quad &\Rightarrow \quad \gamma_1 = \max\left(\text{eig }\mathbb{R}_{xx}^{(1)}\right) \\
\mathbb{R}_{xx}^{(2)} \quad &\Rightarrow \quad \gamma_2 = \max\left(\text{eig }\mathbb{R}_{xx}^{(2)}\right) \\
\mathbb{R}_{xx}^{(3)} \quad &\Rightarrow \quad \gamma_4 = \max\left(\text{eig }\mathbb{R}_{xx}^{(3)}\right) \\
\mathbb{R}_{xx}^{(4)} \quad &\Rightarrow \quad \gamma_4 = \max\left(\text{eig }\mathbb{R}_{xx}^{(4)}\right)
\end{aligned}
$$

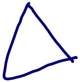
etc.

- Estimate noise power $\widehat{\sigma}_n$, and then

$$\begin{aligned}
\lambda_1 &= \gamma_1 - \widehat{\sigma}_n \\
\lambda_2 &= \gamma_2 - \widehat{\sigma}_n \Rightarrow \text{form } \mathcal{K}_2 \triangleq \tfrac{\lambda_2}{\lambda_1} \text{ which is equal to } \left(\tfrac{\rho_2}{\rho_1}\right)^{2\alpha} \\
\lambda_3 &= \gamma_3 - \widehat{\sigma}_n \Rightarrow \text{form } \mathcal{K}_3 \triangleq \tfrac{\lambda_3}{\lambda_1} \text{ which is equal to } \left(\tfrac{\rho_3}{\rho_1}\right)^{2\alpha} \\
\lambda_4 &= \gamma_4 - \widehat{\sigma}_n \Rightarrow \text{form } \mathcal{K}_4 \triangleq \tfrac{\lambda_4}{\lambda_1} \text{ which is equal to } \left(\tfrac{\rho_4}{\rho_1}\right)^{2\alpha} \\
&\quad \text{etc.}
\end{aligned}$$

That is:

signal eigenvalue
↓
distance.



$$\frac{\lambda_2}{\lambda_1} = \left(\frac{\rho_2}{\rho_1}\right)^{2\alpha} \Rightarrow \mathcal{K}_2 \triangleq \frac{\rho_2}{\rho_1} = \sqrt[\alpha]{\frac{\lambda_2}{\lambda_1}} \quad (31)$$

$$\frac{\lambda_3}{\lambda_1} = \left(\frac{\rho_3}{\rho_1}\right)^{2\alpha} \Rightarrow \mathcal{K}_3 \triangleq \frac{\rho_3}{\rho_1} = \sqrt[\alpha]{\frac{\lambda_3}{\lambda_1}} \quad (32)$$

$$\frac{\lambda_4}{\lambda_1} = \left(\frac{\rho_4}{\rho_1}\right)^{2\alpha} \Rightarrow \mathcal{K}_4 \triangleq \frac{\rho_4}{\rho_1} = \sqrt[\alpha]{\frac{\lambda_4}{\lambda_1}} \quad (33)$$

etc.

$$K_i = \frac{\rho_i}{\rho_1} = \left(\frac{\lambda_i}{\lambda_1}\right)^{\frac{1}{2\alpha}}$$

- **Conclusion of the Association Phase**

  the ratio of the two signal eigenvalues, estimated when the array reference point is at the i^{th} and 1^{st} array sensor, is related to the ratio of ranges of the source with respect to the i^{th} and 1^{st} array sensors via

$$\frac{\lambda_i}{\lambda_1} = \left(\frac{\rho_i}{\rho_1}\right)^{2\alpha} \Rightarrow \mathcal{K}_i \triangleq \frac{\rho_i}{\rho_1} = \sqrt[\alpha]{\frac{\lambda_i}{\lambda_1}}; \quad i = 2, \cdots, N. \tag{34}$$

  Assuming the propagation constant a is known or has been previously estimated these signal eigenvalues can all be used to construct $N-1$ ratio of ranges of the source with respect to each of the array sensors and the arbitrarily chosen 1^{st} sensor in the array. This creates the vector $\underline{\mathcal{K}} \in \mathcal{R}^{(N-1)\times 1}$ which is used in the Metric-Fusion phase of the localization approach where

$$\underline{\mathcal{K}} \triangleq \begin{bmatrix} \mathcal{K}_2 \\ \mathcal{K}_3 \\ \vdots \\ \mathcal{K}_N \end{bmatrix} = \begin{bmatrix} \frac{\lambda_2}{\lambda_1} \\ \frac{\lambda_3}{\lambda_1} \\ \vdots \\ \frac{\lambda_N}{\lambda_1} \end{bmatrix}^{\frac{1}{2\alpha}}. \tag{35}$$

- **Metric Fusion Stage**

  solve the following set of linear equations: $\rho_1^2 = \|\underline{r}_m\|^2 = \underline{b}^T \underline{\mathbb{H}}^{\#T} \underline{\mathbb{H}}^{\#} \underline{b}$

LAA.

$r_i = \max(\mathrm{eig}\, \mathbb{R}_{xx}^{(i)})$

$$\mathbb{H}\underline{r}_m = \underline{b} \Rightarrow \underline{r}_m = \mathbb{H}^{\#}\underline{b}$$

$$\underline{b} = \begin{bmatrix} \|\underline{r}_2\|^2 + \left[1 - \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2\alpha}}\right]\rho_1^2 \\ \vdots \\ \|\underline{r}_N\|^2 + \left[1 - \left(\frac{\lambda_N}{\lambda_1}\right)^{\frac{1}{2\alpha}}\right]\rho_1^2 \end{bmatrix}$$

where,

$\lambda_i = r_i - \hat{\sigma}_n$

$\left(\frac{\rho_i}{\rho_1}\right)^{2\alpha} = \frac{\lambda_i}{\lambda_1}$

$$\mathbb{H} = \left[\underline{r}_2, \quad \underline{r}_3, \quad \underline{r}_4, \quad \cdots \quad \underline{r}_N\right]^T = \begin{bmatrix} x_2, & x_3, & x_4, & \cdots & x_N \\ y_2, & y_3, & y_4, & \cdots & y_N \end{bmatrix}^T$$

$$\underline{b} = \begin{bmatrix} \|\underline{r}_2\|^2 + \left(1 - \sqrt[\alpha]{\frac{\lambda_2}{\lambda_1}}\right)\rho_1^2 \\ \|\underline{r}_3\|^2 + \left(1 - \sqrt[\alpha]{\frac{\lambda_3}{\lambda_1}}\right)\rho_1^2 \\ \|\underline{r}_4\|^2 + \left(1 - \sqrt[\alpha]{\frac{\lambda_4}{\lambda_1}}\right)\rho_1^2 \\ \vdots \\ \|\underline{r}_N\|^2 + \left(1 - \sqrt[\alpha]{\frac{\lambda_N}{\lambda_1}}\right)\rho_1^2 \end{bmatrix} \in \mathcal{R}^{(N-1)\times 1} \tag{36}$$

where ρ_1^2 is estimated firstly as the positive root of quadr. equ

$$\rho_1^2 = \|\underline{r}_m\|^2 = \underline{b}^T \mathbb{H}^{\#T} \mathbb{H}^{\#} \underline{b}$$

- **Alternative Metric Fusion Equations**
  solve the following set of linear equations:

$$\mathbb{H}\begin{bmatrix} \underline{r}_m \\ \rho_1^2 \end{bmatrix} = \underline{b} \Rightarrow \begin{bmatrix} \underline{r}_m \\ \rho_1^2 \end{bmatrix} = \mathbb{H}^{\#}\underline{b}$$

where, in the general $\mathcal{R}^3$ case,

$$\mathbb{H} = \left[2\left(\underline{1}_{N-1}\underline{r}_1^T - \overline{\mathbf{r}}^T\right), \left(\underline{1}_{N-1} - \underline{\mathcal{K}}^2\right)\right] \in \mathcal{R}^{(N-1)\times 4} \tag{37}$$

$$\underline{b} = \left[\left\|\underline{r}_1\right\|^2 \underline{1}_{N-1} - \overline{\underline{r}}_x^2 - \overline{\underline{r}}_y^2 - \overline{\underline{r}}_z^2\right] \in \mathcal{R}^{(N-1)\times 1} \tag{38}$$

with

$$\overline{\mathbf{r}} = \left[\underline{r}_2, \underline{r}_3, \cdots, \underline{r}_N\right] = \left[\overline{\underline{r}}_x, \overline{\underline{r}}_y, \overline{\underline{r}}_z\right]^T \in \mathcal{R}^{3\times(N-1)} \tag{39}$$

denoting the array sensor locations excluding the first sensor.

Hybrid Localisation

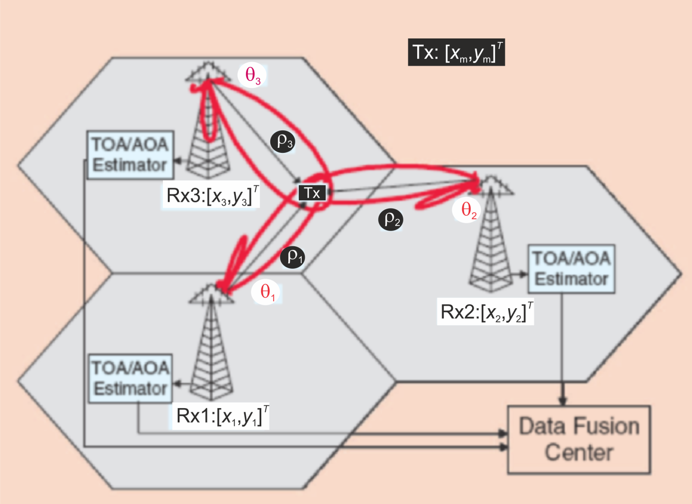
- In TOA, TDOA and DOA approaches two or more BS are employed in the MS location estimation.

- In some situations, for example.
  - MS much closer to one BS,
  - BS antenna array is surrounded by many scatterers

  an alternative (hybrid) procedure may be used

- Hybrid Localisation combines, for instance, TOA and DOA estimates using a 3-step procedure.

TOA/DOA


Tx: $[x_m, y_m]^T$
θ_3
ρ_3
TOA/AOA Estimator
Rx3:$[x_3, y_3]^T$
Tx
ρ_2
θ_2
ρ_1
θ_1
TOA/AOA Estimator
Rx2:$[x_2, y_2]^T$
TOA/AOA Estimator
Rx1:$[x_1, y_1]^T$
Data Fusion Center

- **3-step procedure** :
  - **Step-1** : using TOA measurements, the least-squares estimate of $\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$ is:

$$\mathbb{H}_{\text{TOA}} \underline{r}_m^{(\text{TOA})} = \underline{b}_{\text{TOA}} \Rightarrow \underline{r}_m^{(\text{TOA})} = \mathbb{H}_{\text{TOA}}^{\#} \underline{b}_{\text{TOA}} \tag{40}$$

  - **Step-2** : using DOA measurements the least-squares estimate of $\underline{r}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$ is also

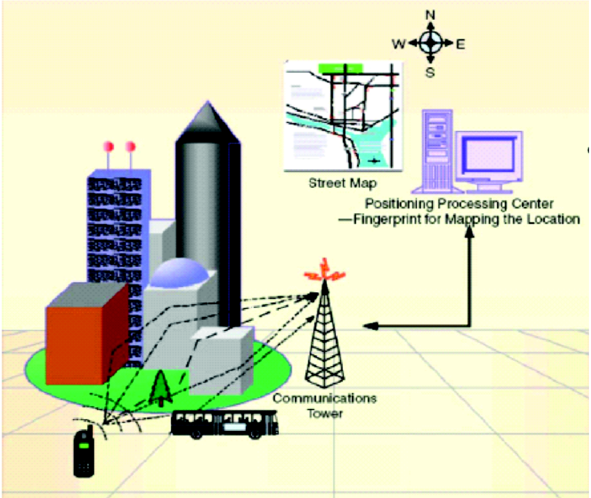
$$\mathbb{H}_{\text{DOA}} \underline{r}_m^{(\text{DOA})} = \underline{b}_{\text{DOA}} \Rightarrow \underline{r}_m^{(\text{DOA})} = \mathbb{H}_{\text{DOA}}^{\#} \underline{b}_{\text{DOA}} \tag{41}$$

  - **Step-3** : the location could be taken as the linear combination of the two estimates/measurements, e.g.

$$\underline{r}_m = \eta . \underline{r}_m^{(\text{TOA})} + (1-\eta) . \underline{r}_m^{(\text{DOA})} \tag{42}$$

where $0 < \eta < 1$ is chosen depending on the relative accuracy of the TOA and DOA measurements

Fingerprinting Localisation

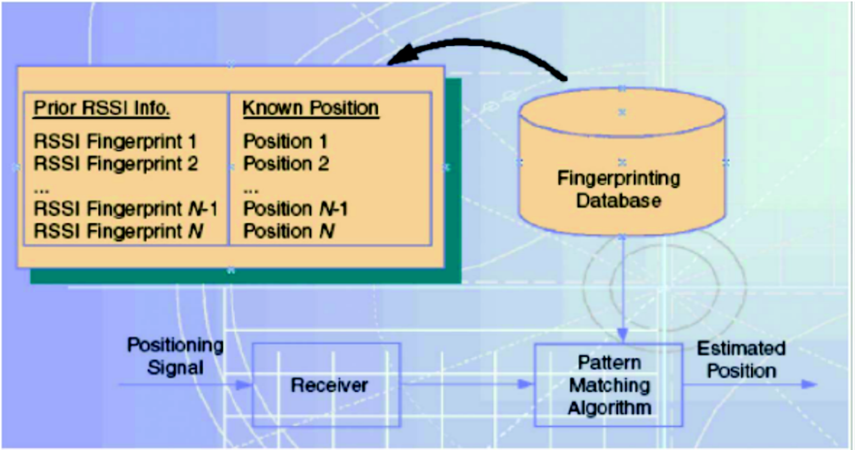


fingerprinting: match received patterns in database.

- One way of solving the multipath problem is to consider **multipath signal characteristics as "fingerprintings"** of MS

- **Comparing** received signal patterns/characteristics by a BS **with** signal characteristics stored in a database (*using pattern matching algorithms known as fingerprintings*)
  - e.g. signal levels, time delays, multipath delay profile

- Needs **one BS/Rx** with several multipath copies.


Prior RSSI Info.
RSSI Fingerprint 1
RSSI Fingerprint 2
...
RSSI Fingerprint N-1
RSSI Fingerprint N
Known Position
Position 1
Position 2
...
Position N-1
Position N
Fingerprinting Database
Positioning Signal
Receiver
Pattern Matching Algorithm
Estimated Position

Cellular Network-aided Positioning Architectures

- There are two main classifications of Cellular Network-aided Positioning Systems/Architectures
  - "standard"
  - "non-standard"
- standard
  - 2G: GSM with E-OTD (Existing Observed Time Difference)
    - ★ Accuracy 50-125m
    - ★ slow (~5sec)
    - ★ software change is needed
  - 2.5G: CDMA/GPRS with A-GPS
    - ★ MS-assisted GPS (MS needs an A-GPS Rx; accurcay <10m)
    - ★ BS-based GPS
  - 3G: WCDMA with OTDOA (Observed TDOA)
  - 4G: Hybrid TOA/DOA localisation
  - Cellular ID
    - ★ No Air-interface needed
    - ★ accuracy depends on sector size
    - ★ accuracy can be improved by hybridisation with other algorithms

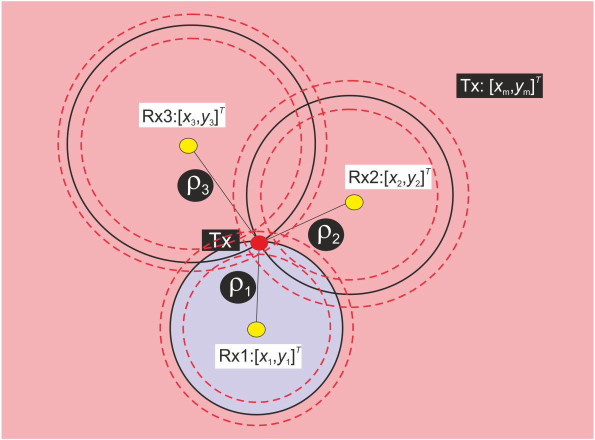
- "non-standard"
  - Architectures based on Antenna Arrays
    - ★ very accurate
    - ★ No changes in the handset (MS)
  - Hybrid Positioning Using Data Fusion
    - ★ Hybrid TOA/TDOA/DoA can improve accuracy
    - ★ GPS+CDMA can also improve accuracy and coverage
  - Pattern Matching Positioning
    - ★ Only server BS required
    - ★ Software solution with hardware modification
  - Note - other techniques:
    - ★ IPDL: Idle Period DownLink (it is a modification of OTDOA)
    - ★ TA-IPDL: synchronised IPDL
    - ★ OTDOA-PE: OTDA Positioning Elements

Localisation Sources of Errors

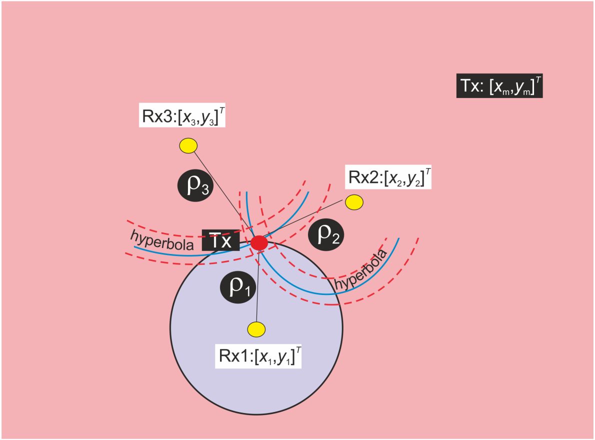
error { multipaths
NLoS
MAI / cochannel interference

- Multipaths
  - RSS: 30-40dB variation over distances in the order of one halfwavelength.
  - DOA: scattering near and around MS & BS will affect the measurements
  - TOA and TDOA: results in a shift in the peak of the correlator
- NLoS (signal takes a longer path)
  - TOA based 400-700m error
- MAI or co-channel interference

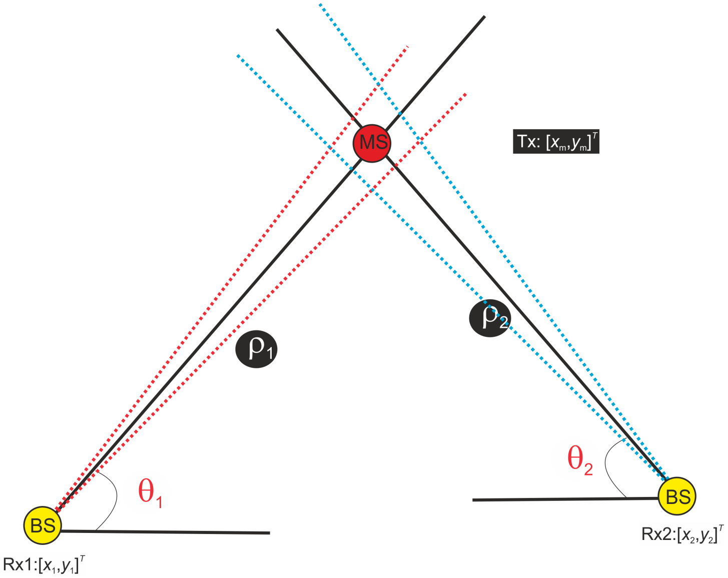
TOA Errors



TDOA Errors


Tx: $[x_m, y_m]^T$
Rx3:$[x_3, y_3]^T$
ρ_3
Rx2:$[x_2, y_2]^T$
ρ_2
hyperbola
Tx
hyperbola
ρ_1
Rx1:$[x_1, y_1]^T$

DOA Errors



Localisation in WSN

- Most localization methods in sensor networks are based on RF signals.
- Sensor networks are, in general, **asynchronous networks** . Therefore, **very difficult to estimate node-positions**.
- Localisation Taxonomy in WSNs: In terms of systems, the types of localisation solutions can generally be classified into three categories
  - beacon-based or beacon-free algorithms
  - increamental of concurrent algorithms
  - range based or range-free algorithms

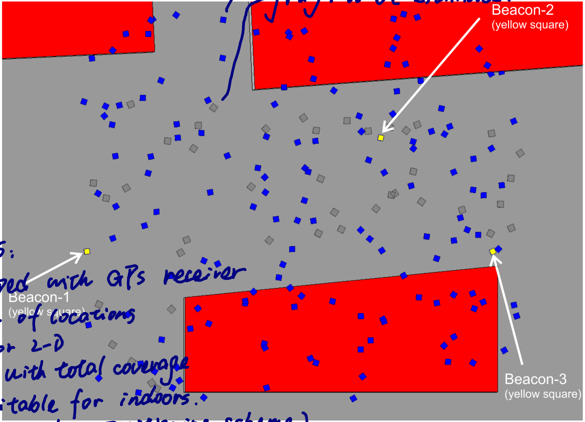
Localisation with Beacons

- Beacon-based or beacon-free
  - Localisation with static beacons
    - single-hop
    - multi-hop
  - Localisation with moving beacons
  - Beacon-free localisation

Localisation with Static Beacons

- Each Beacon provides total coverage

blue: esmitated.
gray: to be estimate.



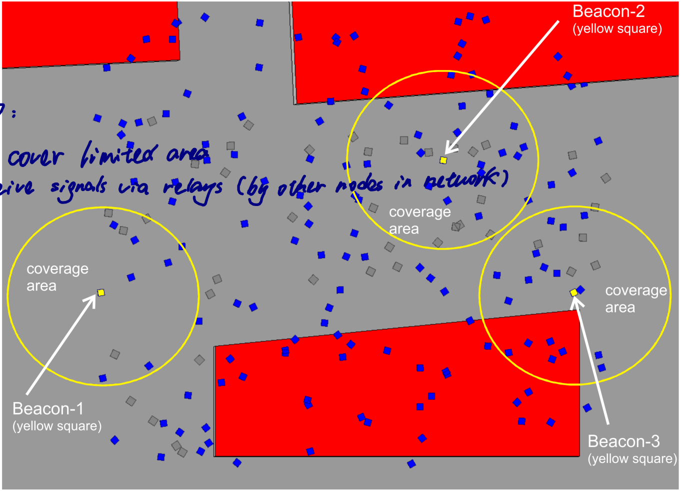
Static Beacons:
- equipped with GPs receiver
- aware of locations
- ≥ 3 for 2-D
- each with total coverage
- unsuitable for indoors.
  (no another positioning scheme)

- Some nodes of the sensor network, are equipped with special positioning devices that are aware of their locations (e.g., equipped with a GPS receiver).
  - These nodes are called beacons.
  - Other nodes that do not initially know their locations are called **unknowns** .
- When these systems perform localization, the unknowns are located using ranging or connectivity (also known as proximity). Generally, an unknown can estimate its location if three or more beacons are available in its 2-D coverage.
- Once an unknown has estimated its position, it becomes a beacon and other unknowns can use it in their position estimations.
- The major challenge in localisation with beacons is to make the algs as robust as possible using as few beacons as possible.
- The resulting design consumes little energy and few radio resources
- Limitations: they need a GPS or another positioning scheme to bootstrap the beacon node positioning algorithms
- Are unsuitable for use indoors

Multi-hop

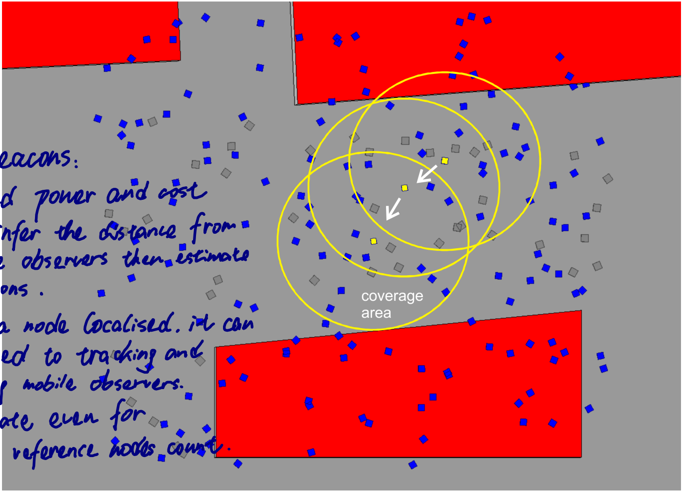
Multi-hop:

- beacons cover limited area
- nodes receive signals via relays (by other nodes in network)



- In multihop positioning systems, nodes typically do not receive beacon nodes' signals directly.
- Given the influence of single-hop positioning systems such as cellular networks and WLANs, it is not surprising that the first multihop localization algorithms tried to adapt single-hop technologies (i.e. TDOA, RSS, and/or AOA information is collected, and the position of each node is then computed using triangulation.)
- Example: It first determines the hop distance (called gradient) to the beacons (called seeds) and, as a function of the average node density, calculates the actual average hop distance to a beacon.

Localisation with Moving Beacons



Moving beacons:

- reduced power and cost
- nodes infer the distance from mobile observers then estimate positions.
- once a node localised, it can be used to tracking and guilding mobile observers.
- accurate even for small reference nodes count.

- Using moving beacons in a system design can significantly reduce power consumption and cost.
- In this type of system, nodes determine their own locations by estimating their distance from moving beacons (also referred to as mobile observers) in a coordinated fashion by applying a transform to the range estimations to determine each node's position within a global coordinate system.
- Sensor nodes receiving beacon packets infer their distance from a mobile beacon and use these measurements as constraints to construct and maintain position estimates.
- Also, once localized, network nodes can localize and track a mobile object (robot) and guide its navigation. (Note: The mobility of targets can be used to significantly enhance position estimation accuracy of nodes, even when the number of reference nodes is small.)

Beacon-Free Localization

- In non-urban outdoor environments, localization may be achieved using several beacons equipped with GPS.

  However, equipping sensors with GPS does not work in indoor or urban environments.

  In addition, the use of beacons, even assuming that sensors are scattered randomly at the start, increases the cost of building a sensor network.

- In practice, a larger network may be designed to operate without beacons, which is known as beacon-free design.
  Such a design determines the position of every node via local node-to-node communication.

Beacon-free :

- node-to-node communication
- start with random assignment
- determine coordinate by local distance estimation

- Beacon-free positioning should be a fully decentralized solution: all nodes start from a random initial coordinate assignment.

  - convert the coordinate to absolute position via reference.

  Then, they cooperate with each other using only local distance estimations to figure out a coordinate assignment.

  The resulting coordinate assignment has both translation and orientation degrees of freedom and has to be correctly scaled.

  A post-process is needed to convert the translation and orientation coordinate assignment to absolute position information based on reference information, such as information from GPS.

Some Comments on BEACON-BASED Algorithms

- Assume that a certain minimum number of nodes know their own positions through manual configuration or GPS. Individual nodes' location could then be determined by referring to a beacon's position.

- All beacon-based positioning algorithms, however, have their limitations because they need another positioning scheme to bootstrap the beacon node positions, and they cannot be easily employed in environments where other location systems are unavailable; thus, they are unsuitable for use indoors.

- In practice, a large number of beacon nodes are required to achieve an acceptable level of position error.

Some Comments on BEACON-FREE Algorithms

- use local distance information to attempt to determine each node's relative coordinates without relying on beacons that are aware of their positions.
- Of course, any algorithm that does not use beacon nodes can be easily converted to one that uses a small number of beacon nodes by adding a final step in the procedure, in which all node coordinates are transformed using three (in 2-D positioning) or four (in 3-D positioning) beacon nodes.
- With three or more beacons, the absolute coordinates of all the nodes can be determined at the same time.

Incremental and Concurrent Algorithms

1. **Incremental algorithms** begin with only three or four core nodes being aware of their own coordinates.
   - They then recursively add appropriate nodes to this set by calculating each node's coordinates using measured relative distances from nodes with previously known coordinates.
   - Limitations: they propagate measurement error, resulting in poor overall network localization. Some incremental algorithms can thus be applied in later stages of global optimization to reduce propagation error. Escaping from local minima and reaching global minima in the incremental stage continues to be a major challenge.
2. In **concurrent algorithms** , all nodes calculate and then refine their coordinates in parallel using local information.
   - Some of these algorithms use iterative optimization schemes that reduce the differences between measured distances and calculated distances based on current coordinate estimates.
   - Concurrent optimization algorithms have a better chance of avoiding local minima than incremental schemes.
   - They can also avoid error propagation by continuously reducing global errors.

RANGE-BASED AND RANGE-FREE

- **Range-based** :
  - algorithms rely on the distances between nodes: this is known as range-based localization.
    (signal strength decay, TOA, or TDOA for internode range estimation).
- **Range-Free** :
  - While range-based algorithms require absolute point-to-point distance estimation (range) or angle estimation for positioning, range free algorithms do not require this information.
  - In addition to measuring range information, range-free localization algorithms achieve position estimation by solving a convex optimization problem using a connectivity matrix of sensor nodes.