EE3-27: Principles of Classical and Modern Radar

Radar Cross Section (RCS) & Radar Clutter

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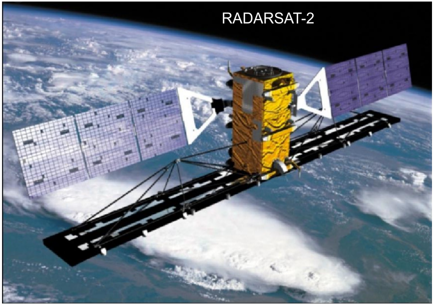


"Targets" as a Secondary Antenna

- "Targets" act as a secondary antenna
  - Electric currents are induced in a conductive object (a target) when it is illuminated with radio energy from a transmitting antenna.
  - These currents cause the object itself to act like a secondary antenna and re-radiate energy. Some of this energy is directed back towards the transmitter and is said to be reflected. A receiving antenna can then collect it, and the measured time delay of this "echo" can be used to calculate distance based on the fact that radio waves travel at the speed of light.
- Targets acting as a secondary antenna have also the three "field regions", i.e.
  - reactive near field,
  - near-far field,
  - far-field. (i.e. Radar $\in$ Fraunhofer regions of the this "secondary antenna")

Types of Targets

- Types of targets divide broadly into
  - volume targets
  - area targets
  - point targets

  the echo from each having a different range dependency.

- volume targets:
  - these are three-dimensional regions of echo, large in relation to the resolution cell dimensions, and with some radar transparency, such as rain clouds, and
  - their strength of echo varies like the square of the range

$$P_{RX} \propto \frac{1}{R^2} \tag{1}$$

- area targets:
  - these are extended two-dimensional surfaces such as large areas of surface terrain
  - the strength of returned echo varies like the cube of the distance

$$P_{RX} \propto \frac{1}{R^3} \tag{2}$$

- Point targets:
  - are the type of targets of interest to most radar operators,
  - these are discrete objects with echoing areas that are not large relative to the resolution cell dimensions, such as aircraft, birds etc.,
  - the strength of echo from these targets varies like the 4th power of range,

$$P_{RX} \propto \frac{1}{R^4} \tag{3}$$

Revisiting the Radar Range Equation

- Point targets: the simplest form of the primary radar range equation is

$$P_{RX} = \underbrace{P_{TX}}_{\text{Tx-power}} \cdot \overbrace{\frac{1}{4\pi R^2}}^{\text{spread factor}} \cdot \underbrace{G_{TX}}_{\text{Tx-antenna-Gain}} \cdot \underbrace{RCS}_{\text{target RCS}} \cdot \overbrace{\frac{1}{4\pi R^2}}^{\text{spread factor}} \cdot \underbrace{\frac{G_{RX}\lambda^2}{4\pi}}_{\text{Rx-antenna aperture}}$$

i.e.

$$\boxed{P_{RX} = \frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R^4}.RCS} \quad (4)$$

- For monostatic radar with a single antenna used to transmit and receive:

$$G_{TX} = G_{RX} = G \quad (5)$$

- Several factors have been neglected in Equ 4 that have a significant impact on radar performance. These are[1]:
  - noise
  - system losses
  - propagation behavior
  - clutter
  - waveform limitations
  - RCS and target fluctuations, etc.

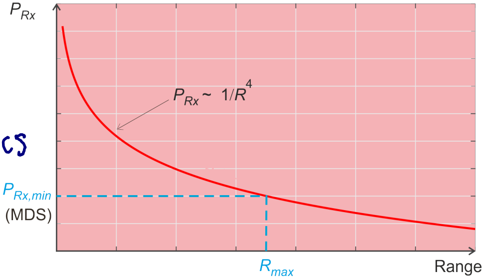
- The form of the Radar Range Equation given by Equ 4, although no complete, it does give some insight into the trade-offs involved in radar design:
  - The dominant feature is the $1/R^4$ factor.
  - Even for targets with relatively large RCS, high transmit powers must be used to overcome the $1/R^4$ when the range becomes large.

[1]We will discuss most of these in some depth throughout the course

Revisiting Maximum Detection Range

- In Topic 2 (Slide 19) we have seen that the minimum received power that the radar receiver can "sense" is referred to as the minimum detectable signal (MDS) and is denoted $P_{RX,min}$.

$$P_{RX} = \frac{P_{TX} G_{TX} G_{RX} \lambda^2}{(4\pi)^3 R^4} \cdot RCS$$



- Given the MDS, the maximum detection range R_{max} can be obtained:

$$\Rightarrow \boxed{R_{\max} = \sqrt[4]{\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3 .P_{RX,\min}}.RCS}}$$

Radar Cross Section (RCS)

- *RCS* describes the ability of the target to capture energy from the radar and reradiate it back toward the radar.

Definition (The IEEE Definition of *RCS*)

$$RCS \triangleq \lim_{R\to\infty} 4\pi R^2 \frac{\left\|\underline{E}_s\right\|^2}{\left\|\underline{E}_I\right\|^2} \tag{6}$$

where

$$\begin{aligned} \underline{E}_I &\triangleq \underline{E}_I(\underline{r}, t) = \text{Electric Field Intensity incident on the target} \\ \underline{E}_s &\triangleq \text{Scattered Electric Field Intensity from target returned to radar} \end{aligned}$$

- *RCS* Units: m^2 or dBsm (decibels relative to one square meter),.i.e.

$$RCS_{\text{dBsm}} = 10\log_{10} RCS \tag{7}$$

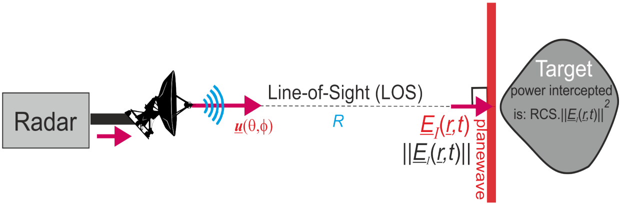
- The key idea is that the *RCS* parameter attempts to capture, in a single number, the ability of the target to capture energy from the radar and to reradiate it back toward the radar.

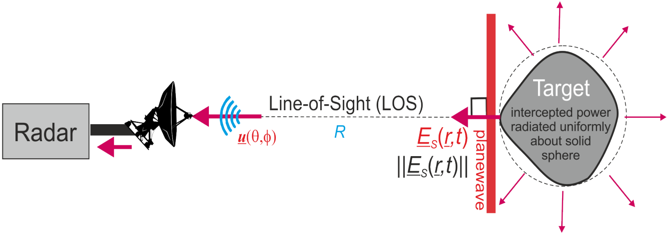
RCS Interpretation

- It is used to describe a target's scattering properties just as "gain" G (or directivity) is used for an antenna. An "isotropic scatterer" will scatter equally in all directions (just like an isotropic antenna).
- RCS is the hypothetical area effective scattering area (in m^2), that would intercept the incident power at the target, which if scattered isotropically, would produce the same echo power at the radar, as the actual target. That is: Rcs is more comprehensive and models reflection coefficient Γ.
  - Incident power density [units: V^2/m^2, i.e. W/m^2]: $\|\underline{E}_I\|^2$
  - intercepted power [units: V^2, i.e. W]: $RCS[\mathrm{m}^2] \times \|\underline{E}_I\|^2$
  - isotropic scattered power density [units: V^2/m^2]:

$$\|\underline{E}_s\|^2 = \frac{RCS \times \|\underline{E}_I\|^2}{4\pi R^2} \Rightarrow \boxed{RCS = 4\pi R^2 . \frac{\|\underline{E}_s\|^2}{\|\underline{E}_I\|^2}} \tag{8}$$

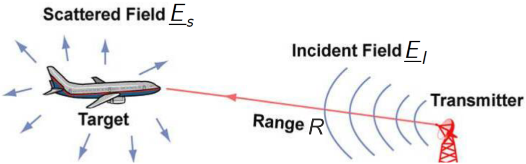
and the limit $\left[\lim_{R\to\infty}\right]$ implies "far field" (i.e. planewave propagation of the electromagnetic wave).



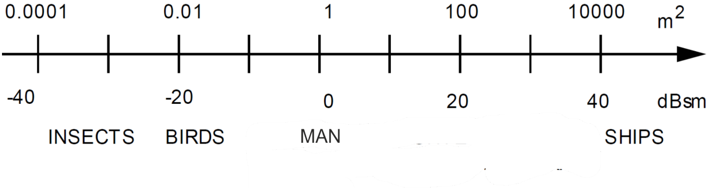


$$\|E_s(r,t)\|^2 = \frac{RCS.\,\|E_I(r,t)\|^2}{4\pi R^2} \tag{9}$$

- Equ 9 indicates that if the incident electric field $\underline{E}_i$ that impinges upon a target is known and the scattered electric field $\underline{E}_s$ is measured, then the "radar cross section" (effective area) RCS of the target located in the far-field may by calculated based on the IEEE's definition (Equ 6).



- Typical RCS values



Examples (RCS)

	TARGETS	RCS (m^2)
in the air	Conventional winged missile	0.1
	Small, single engine aircraft, or jet fighter	1
	Four passenger jet	2
	Large fighter	6
	Medium jet airliner	40
	Jumbo jet	100
	Helicopter	3
on the sea	Small open boat	0.02
	Small pleasure boat (20-30 ft)	2
	Cabin cruiser (40-50 ft)	10
	Ship (5,000 tons displacement, L Band)	10000
on the road	Automobile / Small truck	100-200
	Bicycle	2
live targets	Man	1
	Birds (large to medium)	10^{-2}-10^{-3}
	Insects (locust to fly)	10^{-4}-10^{-5}

RCS of Simple Shapes

- In general, the RCS of a target depends upon

  1. its physical size only; (this is a common case);
  2. the wavelength of the radar's carrier λ only;
  3. both physical size and wavelength.

  and its computation is very complicated.

- In fact, except for some very simple surfaces, RCS can only be approximately computed.

RCS of Simple Shapes (cont.)

- Note that for a specific and known target of complex shape the exact or approximate computation of RCS is very complicated and this is beyond the objectives of this course

- Below are some examples of targets of very simple shapes which are presented to illustrate that the RCS depends on:

  - target's physical size only (example 1, sphere)

  - the wavelength only (example 2, cone)

  - both physical size and wavelength (examples 3 and 4: imperfectly sharp cone and flat plate)

- After these example, in the rest of this topic we will be concerned with arbitrary targets of any complex shape with "characteristic dimension" ℓ.

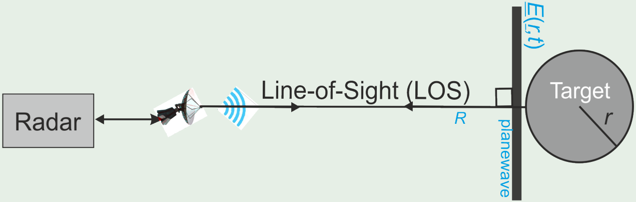
Example ([1] RCS of a Sphere: only size)

- An example of the case where RCS depends upon physical size is a sphere. For a sphere of radius r

$$\text{RCS} = \pi r^2 \tag{10}$$

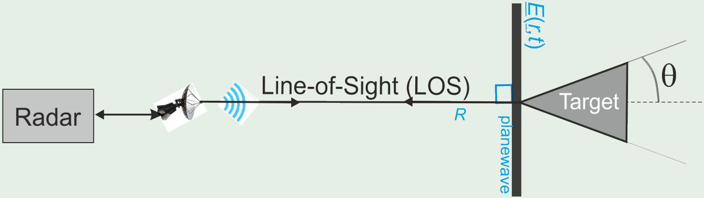
provided

$$r \ggg \lambda \tag{11}$$



Example ([2] RCS of a Cone: only λ)

- A case where RCS does not depend upon physical size is a cone where the nose of the cone is facing toward the radar, as shown below. For this case the RCS is given by

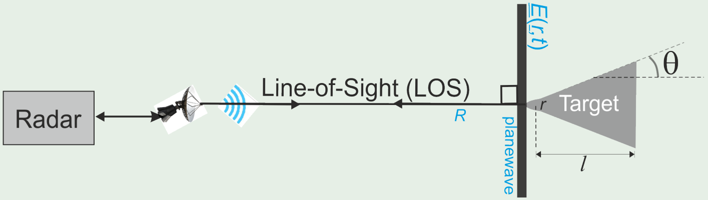


$$\text{RCS} = \frac{\lambda^2}{16\pi}\tan^4\theta \tag{12}$$

- In this case it will be noted that the RCS is proportional to wavelength but is not dependent on the overall size of the cone.
- N.B.: If the cone had any other orientation relative to the line-of-sight (LOS) to the radar, its RCS would depend upon the length of the cone and the diameter of the base

Example ([3] RCS of Imperfectly Sharp Cone: both size and λ)

- if the point of the cone is not perfectly sharp the RCS will depend upon the size of the nose.



$$\text{RCS} = \pi r^2 + \frac{\lambda^2}{16\pi}\tan^4\theta \qquad (13)$$

- This is the shape of an ideal Reentry Vehicle RCS (Nose-on Aspect), which is the part of a space vehicle designed to re-enter the Earth's atmosphere in the terminal portion of its trajectory (also called RV)

Example ([4] RCS of a Flat Plate: both size and λ)

- RCS of an object depends upon the orientation of the object relative to the LOS. Assume a signal coming from direction, azimuth θ and elevation $\phi = 90 - \psi$, impinges on a flat plate with length d and width w:

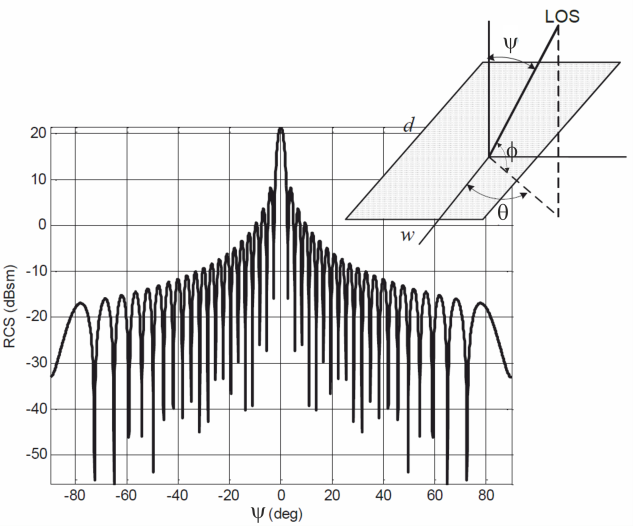
$$\begin{aligned} \text{RCS} &= (k.d.w)2\pi\text{sinc}^2(k.d\sin\psi\cos\theta)\text{sinc}^2(k.w\sin\psi\sin\theta) \\ &\quad \times 12\sqrt{(1-\sin 2\psi\cos 2\theta + 1 - \sin 2\psi \sin 2\theta)} \end{aligned} \tag{14}$$

where

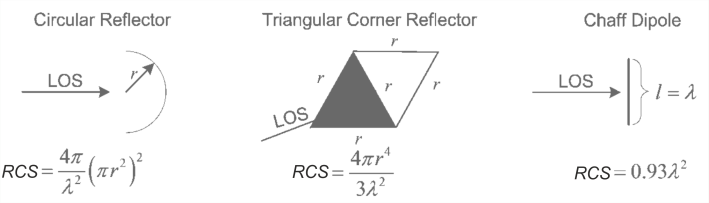
$$\begin{aligned} k &= \frac{2\pi}{\lambda} = wavenumber \\ k.w &\ggg 1 \\ k.d &\ggg 1 \end{aligned}$$

Example ([4 cont.]: RCS of a Flat Plate $d = w = 1$m)

- For a flat plate with $d = w = 1$m and θ=0 and λ=0.3m (L-band), the RCS varies significantly as the angle of the LOS changes.



Examples (Some Other Simple Shapes)



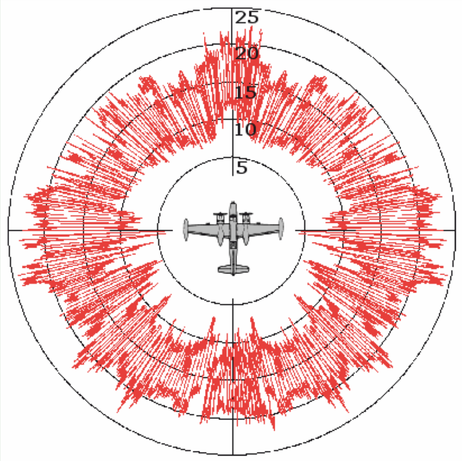
RCS of Complex Targets

- Most targets of interest are not the simple shapes indicated thus far.
- In fact, many targets consist of many different shapes that are in different orientations.

  These are complex targets that have RCS that changes significantly with very small changes in frequency and/or viewing angle. That is, as the targets move relative to the radar LOS, the relative orientations of the various shapes change significantly.
- Examples: Aircraft, Missiles, Ships.

[1] LOS = Line-of-Sight

Example (Complex Target)



Example (Stovepipe Aircraft)

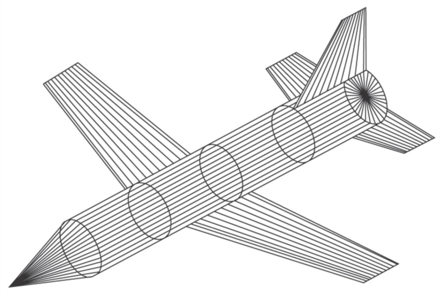


Fig 6-32, [1]: aircraft shape

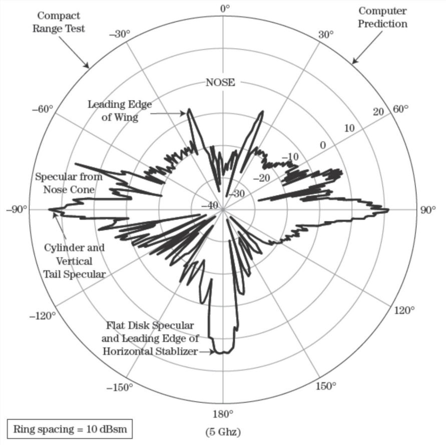


Fig 6-33, [1]: RCS for this aircraft at C-band

Spheres as Targets for RCS Calibration

- The RCS of a sphere is in most cases especially simple[2]
- If $2r > \lambda$, the RCS of a smooth reflective sphere is just its projected geometrical area, or $\text{RCS} = \pi r^2$, and this is constant with regard to wavelength and with regard to any linear polarisation
- Thanks to spherical symmetry everything also stays constant regardless of aspect
- For these reasons spheres are often used as radar calibration targets
- For large and complex targets the phase differences between wavefronts coming from different parts of the target can add or subtract in complicated ways to shift the phase centroid away from the geometric centre of the target - even outside the physical target volume. This is called a 'glint' error, which can be significant for tactical tracking radars in particular. But in the case of a smooth sphere there is no glint or tracking error, as the phase centroid always coincides with the geometric centre.

[2] Martin L. Shough, NARCAP Research Associate (U.K.), Radar Detection

RCS Wavelength Regions

- The RCS has three distinct wavelength regions:

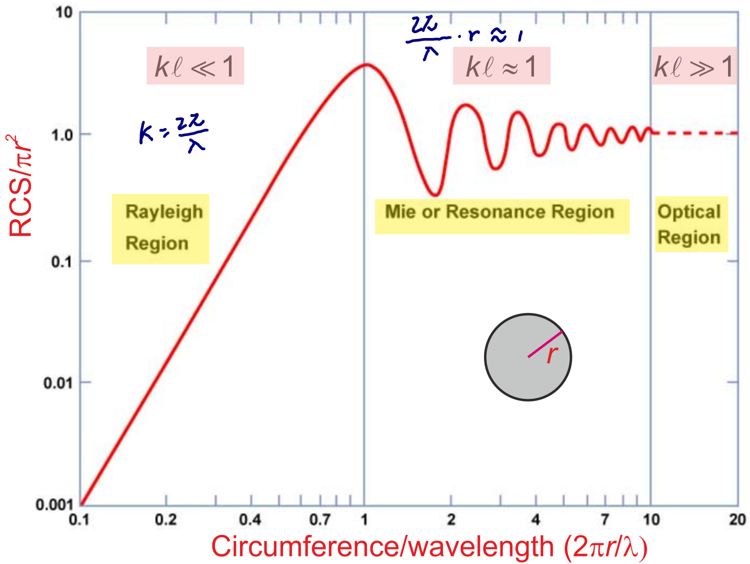
  1. the Rayleigh Region
  2. the Mie, or Resonance, Region
  3. the Optical Region

- These three regions
  - are related to the scattering mechanisms discussed in Topic 3 (Slides 28, 29 and 30)
  - will be discussed next by considering an arbitrary target with a "characteristic dimension", ℓ.

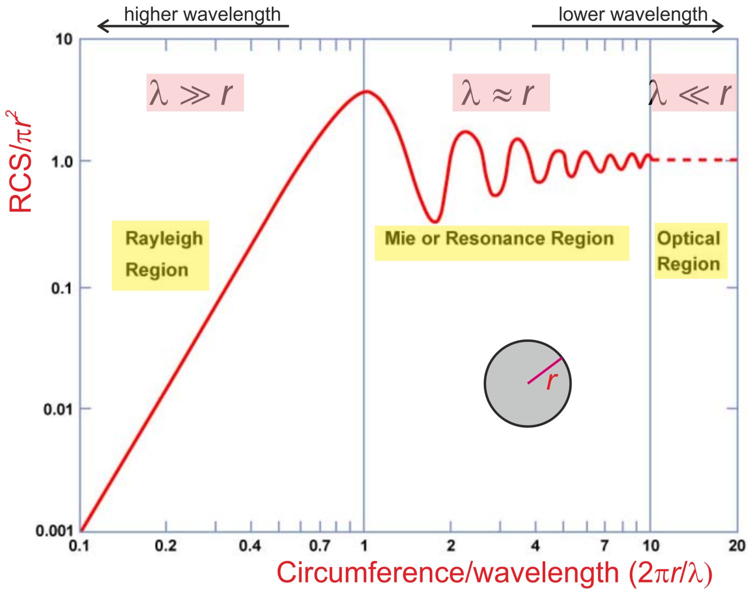
- Note: $k.\ell$ is the length ℓ in "wavelengths" where k denotes the wavenumber, i.e. $k = \frac{2\pi}{\lambda}$.

1. Rayleigh Region: the object size is less than a wavelength. In this region, the RCS of the object is a function of the size of the object relative to a wavelength. wavelength matters for small targets.
   - $k\ell \ll 1$, $RCS \propto \frac{1}{\lambda^4}$,
   - RCS vs $k\ell$ is smooth, and $RCS \propto (\text{volume})^2$.
   - the sphere RCS in the Rayleigh region: $RCS = 80\pi r^4 (\frac{4}{3}\pi^2)^2 (\frac{r}{\lambda})^4$
   - e.g., rain and clouds
2. Mie or Resonance Region: the object size is on the order of a wavelength and the RCS is transitioning from being dependent upon both object size and wavelength to being dependent mainly on object size.
   - $k\ell \approx 1$,
   - RCS vs $k\ell$ oscillates dimension matters for large targets.
   - e.g., birds, bullets, artillery shells, some missiles and very small aircraft
3. Optical Region: the object is much larger than a wavelength. The RCS is (or can be) a strong function of the size of the object.
   - $k\ell \gg 1$
   - RCS vs $k\ell$ is smooth and may be independent of λ

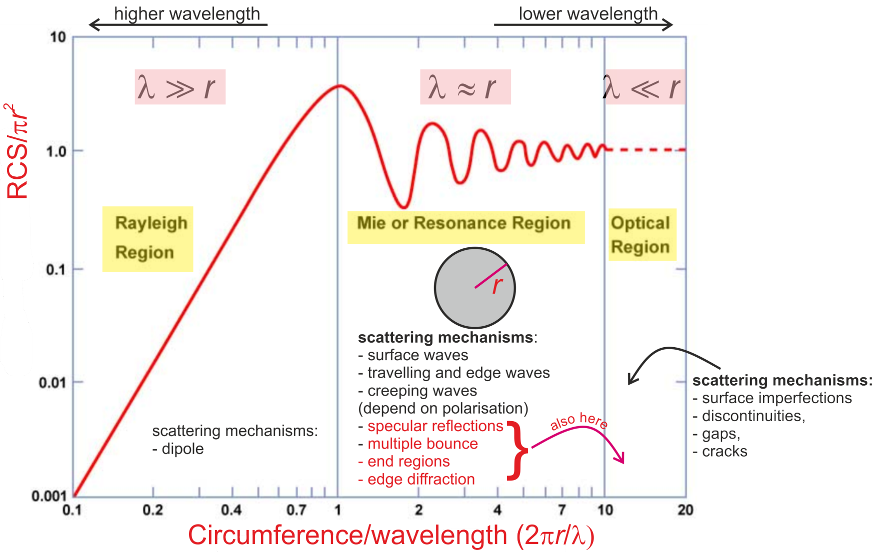
RCS Rayleigh, Mie & Optical Regions



RCS Rayleigh, Mie & Optical Regions (cont.)



Scattering Mechanisms per RCS Region



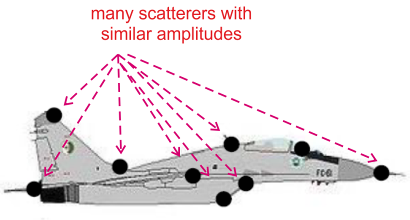
Flunctuating Targets - Statistical Representation of RCS

- The target returns appear also to vary with time due to sources other than a change in range:
  1. meteorological conditions and path variations
  2. radar system instabilities (platform motion and equipment instabilities)
  3. target aspect changes

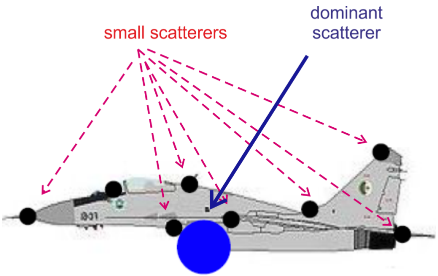
- For systems analysis purposes we only need to know the "gross" behavior of a target, not the detailed physics behind the scattering. In this case we treat the RCS as a random variable with a probability density function (pdf) that depends on the factors above.

- There are two classes of fluctuating targets with respect to the nature of the scattering mechanism
  - class-1: These are Rayleigh targets which consist of many independent scattering elements of which no single one (or few) predominate.
  - class-2: These targets have one main scattering element that dominates, together with smaller independent scattering sources.

- Class-1 example:



- Class-2 example:



Swerling RCS Definitions[3]

- Peter Swerling developed statistical representations of RCS that are commonly referred to as the Swerling RCS models.
- There are four Swerling models termed
  - Swerling 1,
  - Swerling 2,
  - Swerling 3, and
  - Swerling 4

  complex targets

  predominant + misc
- Swerling 1 and 2 models are generally associated with complex targets that have a large number of surfaces and joints with different orientations,
  - e.g., aircraft, tanks, ships, cruise missiles, etc.
- Swerling 3 and 4 models apply to somewhat simple targets that consist of a predominant scatterer and several small scatterers,
  - e.g., bullets, artillery shells reentry vehicles, etc.

[3] Swerling, P. "Probability of Detection for Fluctuating Targets", IRE Transactions, IT-6, April 1960, pp.269-308

- Swerling Statistics: Probability density functions (pdf)

$$\text{Swerling 1 \& 2: } \mathrm{pdf}_{RCS}(\sigma) = \frac{1}{\overline{\sigma}}\exp\left(-\frac{\sigma}{\overline{\sigma}}\right) \quad (15)$$

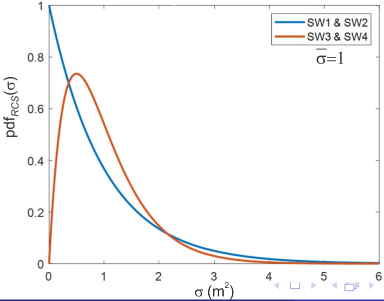
exponential

$$\text{Swerling 3 \& 4: } \mathrm{pdf}_{RCS}(\sigma) = \frac{4\sigma}{\overline{\sigma}^2}\exp\left(-\frac{2\sigma}{\overline{\sigma}}\right) \quad (16)$$

Rayleigh

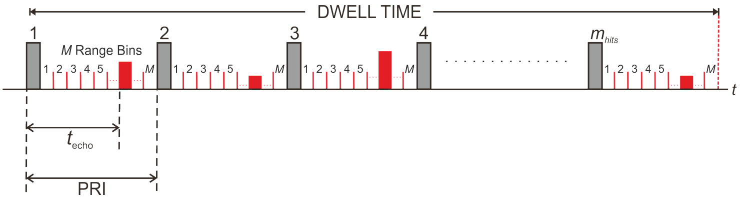
where

- σ is the *RCS* variable
- $\overline{\sigma}$ is the mean value of the target *RCS*

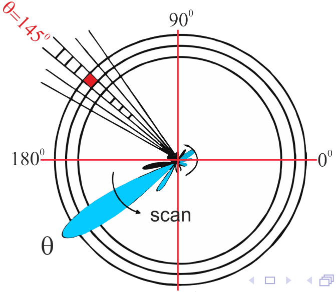


Pulse-to-Pulse and Scan-to-Scan Fluctuation

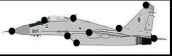
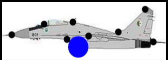
- Fluctuating targets may vary
  - from PRI-to-PRI (also known as pulse-to-pulse)



  - from scan-to-scan

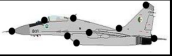
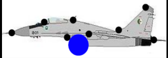


Summary of Swerling RCS Target Models in a Table

Nature of scattering	RCS Model	Flunctuation Rate: slow (scan-to-scan)	Flunctuation Rate: fast (pulse-to-pulse)
many scatterers with similar amplitudes (class-1) 	Exponential (chi-sq DOF=2) $\text{pdf}_{RCS}(\sigma) = \frac{1}{\overline{\sigma}} \exp\left(-\frac{\sigma}{\overline{\sigma}}\right)$	Swerling-1	Swerling-2
one scatterer dominates many smaller ones (class-2) 	chi-sq DOF=4 $\text{pdf}_{RCS}(\sigma) = \frac{4\sigma}{\overline{\sigma}^2} \exp\left(-\frac{2\sigma}{\overline{\sigma}}\right)$	Swerling-3	Swerling-4

$\overline{\sigma}$ =average RCS (m^2)

Swerling Target Models: Amplitude

Nature of scattering	Amplitude Model $A=\sqrt{RCS}$	Flunctuation Rate	
		slow (scan-to-scan)	fast (pulse-to-pulse)
many scatterers with similar amplitudes (class-1) 	Rayleigh $\text{pdf}_A(\text{a})=\frac{2\text{a}}{\overline{\sigma}}\exp\left(-\frac{\text{a}^2}{\overline{\sigma}}\right)$	Swerling-1	Swerling-2
one scatterer dominates many smaller ones (class-2) 	Central Rayleigh (DOF=4) $\text{pdf}_A(\text{a})=\frac{8\text{a}^3}{\overline{\sigma}^2}\exp\left(-\frac{2\text{a}^2}{\overline{\sigma}}\right)$	Swerling-3	Swerling-4

$\overline{\sigma}$ =average RCS (m^2)

Radar Channel Effects

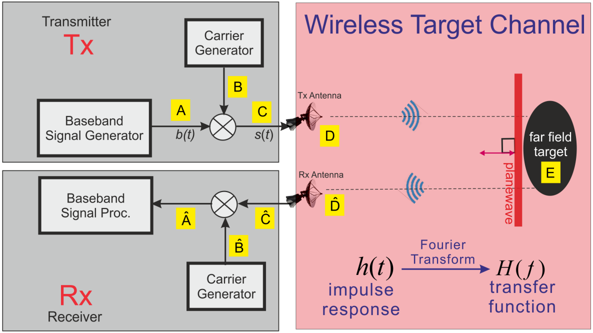
- Radar Channel Effects: The received signals at the antenna output are directly related to
  - the backscatterers from the target
  - the backscatterers from clutter in the target's environment
  - the noise effects
  - **unintentional interference** from other emitters
  - **intentional (hostile)** interference (from Jammers)
- The received signal does not care about the mechanisms of

  model as random variables
  - reflections
  - diffraction
  - refraction
  - scattering

  but all the received paths of the transmitted signal can be represented as by a **complex number** β **with magnitude** $|\beta|$ **and random phase** $\measuredangle\beta$

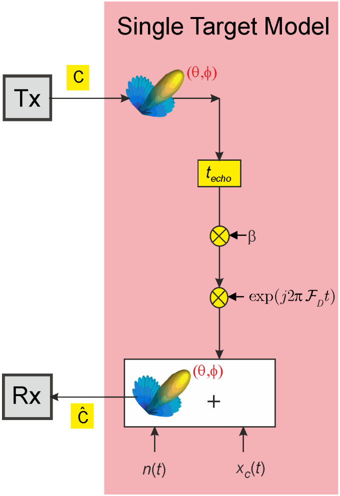
Radar Wireless Channel: Impulse Response

- Like any other system, the radar wireless channel may be represented by its impulse response $h(t)$ in the time domain and its transfer function $H(f)$ in the frequency domain.



- To get the impulse response we assume that at Point-A we have a delta line $\delta(t)$ and we observe the system response at its output (Point-Â).

Single Target Modelling



$$h(t) = \beta.\exp(j2\pi\mathcal{F}_D t).\delta(t - t_{echo})$$
$$= \text{impulse response}$$

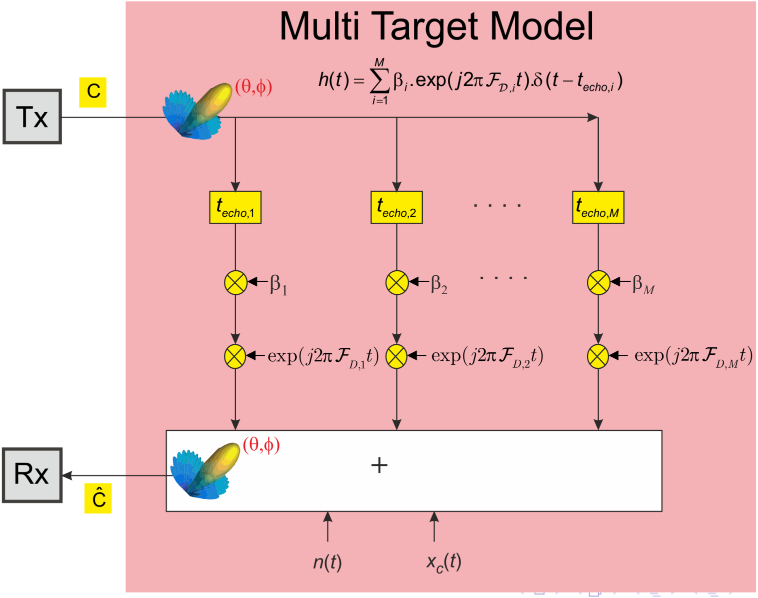
$$t_{echo} = \frac{2R}{c}$$

$$\beta = \underbrace{\sqrt{\frac{G_{Tx}.G_{Rx}}{(4\pi)^3}.\frac{\lambda}{R^2}}\sqrt{RCS}}_{\text{radar equation}}\underbrace{\exp\left(-j2\pi F_c\frac{2R}{c}\right)}_{\text{propagation phase}}\underbrace{\exp(j\psi)}_{\text{random phase}}$$

$$\mathcal{F}_D = \text{Doppler frequency}$$

$$\psi = \text{random phase}$$

where $n(t)$ is the AWGN and $x_c(t)$ denotes "clutter".


Multi Target Model
$h(t) = \sum_{i=1}^{M} \beta_i . \exp(j2\pi \mathcal{F}_{D,i} t) . \delta(t - t_{echo,i})$
Tx
c
(θ,ϕ)
$t_{echo,1}$
$t_{echo,2}$
$t_{echo,M}$
β_1
β_2
β_M
$\exp(j2\pi \mathcal{F}_{D,1} t)$
$\exp(j2\pi \mathcal{F}_{D,2} t)$
$\exp(j2\pi \mathcal{F}_{D,M} t)$
(θ,ϕ)
+
Rx
ĉ
$n(t)$
$x_c(t)$

Multi-Target Modelling (cont.)

- In the following equations the subscript "i" refers to the i-th target

$$
\begin{aligned}
h(t) &= \sum_{i=1}^{M} \beta_i . \exp(j2\pi \mathcal{F}_{D,i} t).\delta(t - t_{echo,i}) && (17)\\
&= \text{impulse response}
\end{aligned}
$$

$$
t_{echo,i} = \frac{2R_i}{c} \quad (18)
$$

$$
\beta_i = \sqrt{\frac{G_{Tx}.G_{Rx}}{(4\pi)^3}} . \frac{\lambda}{R_i^2} \sqrt{RCS_i} . \exp\left(-j2\pi F_c \frac{2R_i}{c}\right) \exp(j\psi_i) \quad (19)
$$

$$
\mathcal{F}_{D,i} = \text{Doppler frequency} \quad (20)
$$

$$
\psi_i = \text{random phase} \quad (21)
$$

Clutter and Noise

- There are two kinds of radar echoes.
  - One is the set of echoes reflected from the **targets**/**objects of interest**
  - The other is the set of echoes reflected from **unrelated targets/objects, known as clutter**.
- Radar clutter is defined:
  - as "the unwanted reflective waves from irrelevant targets", or,
  - as "unwanted echoes, typically from the ground, sea, rain or other precipitation, chaff, birds, insects, or aurora."
- Note: target of one radar is another radar's clutter
  - for example to the radar meteorologist, precipitation is the target and aircraft are clutter.

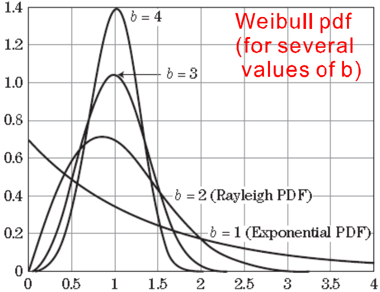
Probability Density Function (pdf) and PSD(f)

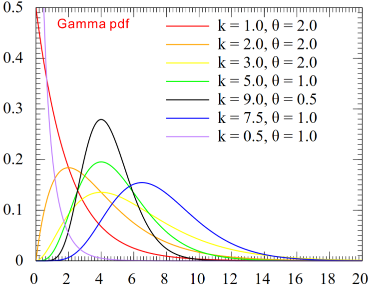
- Random signals, like "noise" and "clutter", are fully described by their
  - pdf = this is a statistical function in the time domain.
  - PSD(f) =this is a function in the frequency domain which provides the distribution of a signal's power in the frequency domain, known as "power spectral density" PSD(f) or simply power spectrum.

	pdf	PSD(f)
noise	Gaussian	White
clutter (or target)	Rayleigh, Rician Exponential log-normal Weibull K-distribution, Γ-distribution	Colour (or non-white)
target	Swerling's four Models	Colour

- Note: natural clutter, or terrestrial clutter, can be approximated by a Weibull distribution

Weibull and Gamma Clutter Distributions

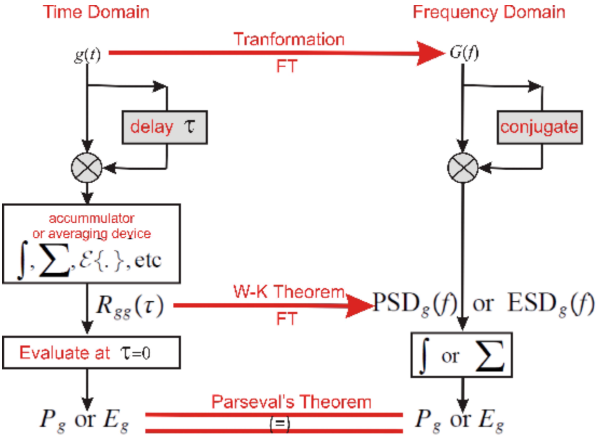




- The target echoes are often embedded in the clutter. Therefore, to **suppress clutter** is essential for any terrestrial radar system.

M. A. Richards, J. A. Scheer, and W. A. Holm, "**Principles of Modern Radar: Vol.I - Basic Principles**". SciTech Publishing Inc., 2010. (p.176)

Appendix-A: Parseval's and Wiener-Khinchin (W-K) Theorems



Appendix-B: "Accumulators" and "Averaging" Devices

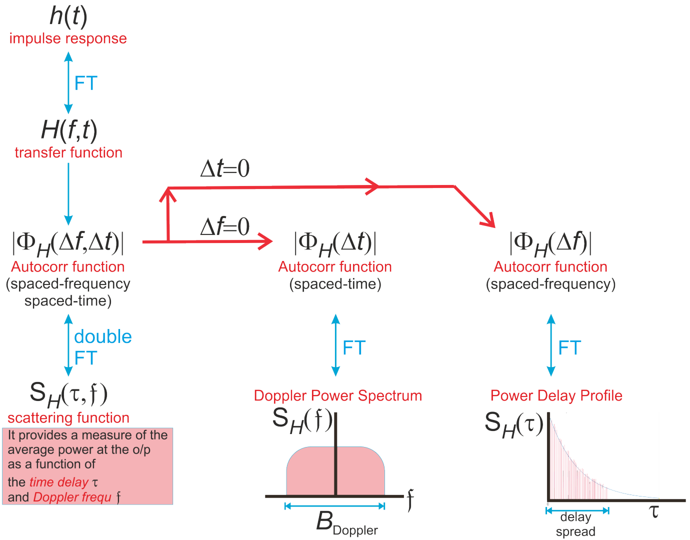
- Definitions

Accumulators:	$\int_{t_1}^{t_2}$	$\sum_{i=1}^{M}$	
Averaging:	$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}$	$\frac{1}{M}\sum_{i=1}^{M}$	$\mathcal{E}\{.\}$

- Examples

Accumulators corresponding to Energy	$R_{gg}(\tau) \triangleq \int_{t_1}^{t_2} g(t).g(t-\tau)dt$ $\mathrm{ESD}_g(f) = \mathrm{FT}\{R_{gg}(\tau)\}$
Averaging corresponding to Power	$R_{gg}(\tau) \triangleq \frac{1}{t_2-t_1}\int_{t_1}^{t_2=t_1+\tau} g(t).g(t-\tau)dt$ $R_{gg}(\tau) \triangleq \mathcal{E}\{g(t).g(t-\tau)\}$ $\mathrm{PSD}_g(f) = \mathrm{FT}\{R_{gg}(\tau)\}$

Appendix-C: Scattering Function Analysis



Appendix-D: Delay Spread "mean" and "rms"

$$T_{mean} \triangleq \frac{\int_0^{T_{max}} \tau S_h(\tau) d\tau}{\int_0^{T_{max}} S_h(\tau) d\tau} \tag{22}$$

$$T_{rms} \triangleq \sqrt{\frac{\int_0^{T_{max}} (\tau - T_{mean})^2 S_h(\tau) d\tau}{\int_0^{T_{max}} S_h(\tau) d\tau}} \tag{23}$$