EE3-27: Principles of Classical and Modern Radar

Target Detectability

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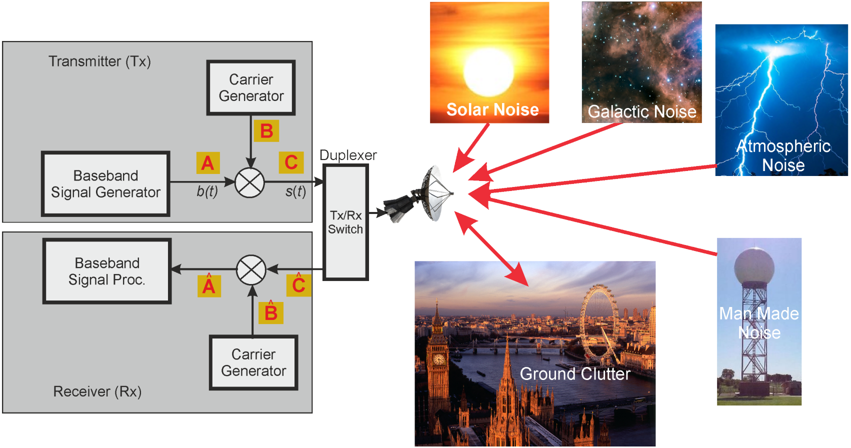
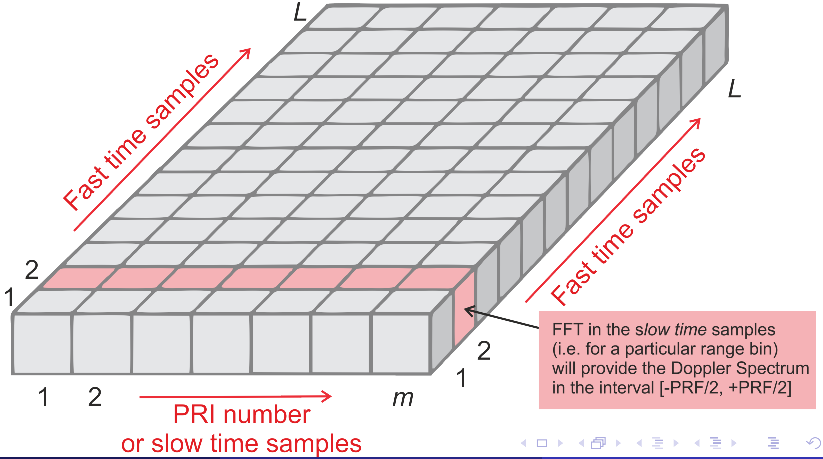
Sources of Noise Received by Radar

- Noise from many sources competes with the target echo and the radar range equation



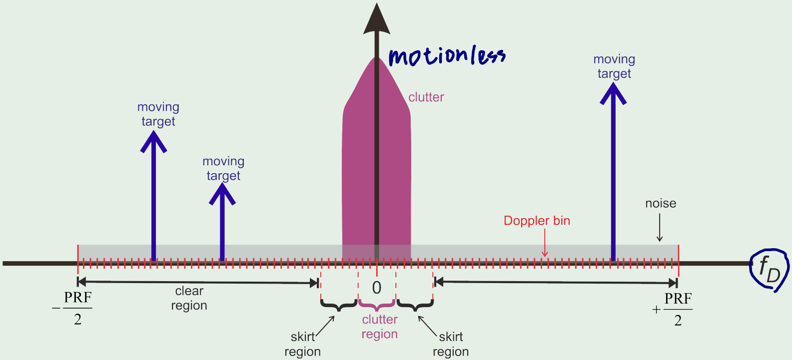
Doppler Spectrum in One Range-Bin

- We have seen in Topic 2 that the Doppler Spectrum is contained in the interval $\left[-\frac{\mathrm{PRF}}{2}, +\frac{\mathrm{PRF}}{2}\right]$
- We form the Doppler Spectrum by using the DFT in the slow time samples in the data matrix or 3D data cube

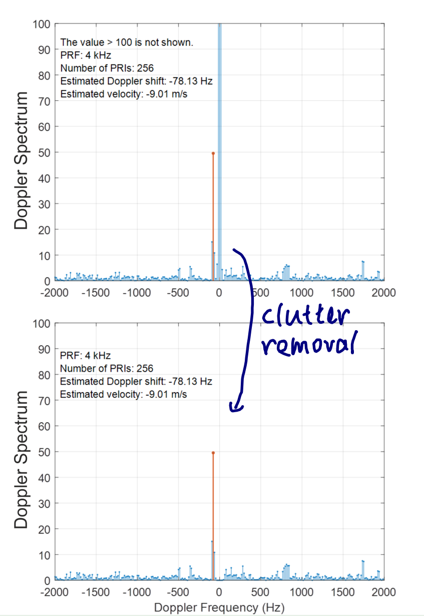


- In the Doppler Spectrum
  - clutter and stationary targets are centered at $f_D = 0$;
  - moving targets and noise appear throughout the spectrum.

Example (Doppler Spectrum in One Range-Bin)



Example (Using real radar data from a moving UAV)



Clutter

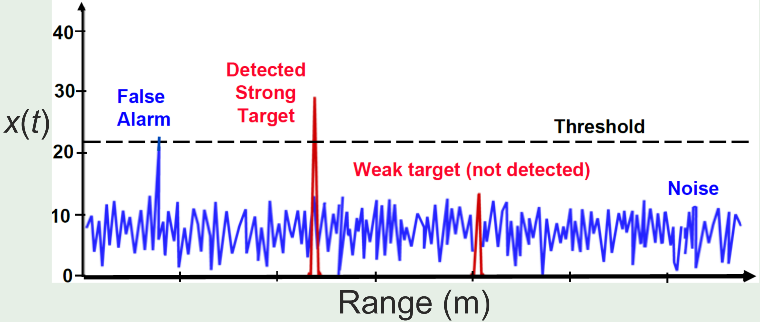
clutter can be resolved as long as its delay or Doppler profile is different from the target.

- **Ground Clutter**
  - Can be intense and discrete
  - can be 50-60dB $>$ than target
  - Doppler velocity zero for ground based radar
    - ★ Doppler spread small
- **Rain Clutter**
  - Diffused and windblown
  - can be 30^{+}dB $>$ than target
  - Doppler zero for ground based radar
    - ★ Doppler spread small
- **Sea Clutter**
  - Less intense than ground clutter by 20-30dB
    - ★ often more diffuse than ground clutter
  - Doppler velocity varies for sea radar (ship speed and wind speed)
    - ★ Doppler spread moderate
- **Bird Clutter (flocks of birds)**
  - 100s to 10000s to point targets
  - Doppler velocity - 0 to 60 knots
    - ★ Doppler of single bird has little change
    - ★ big issue for small targets

Decision Theory for Target Detection

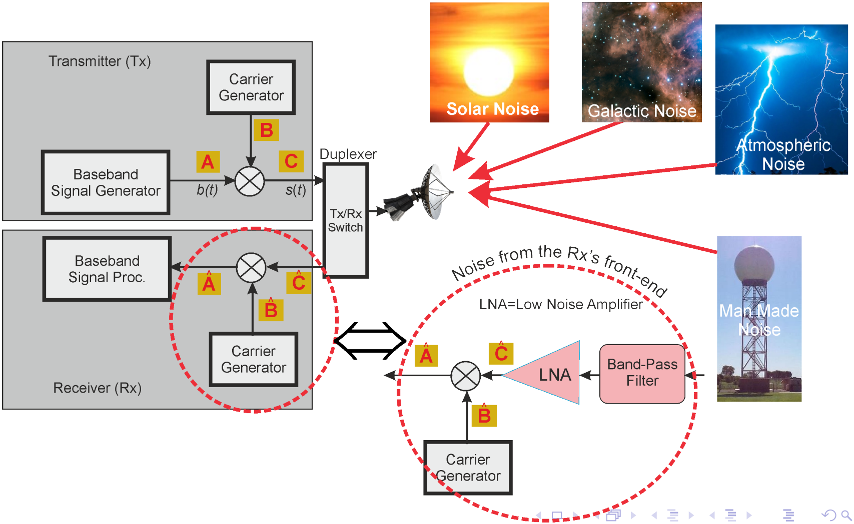
- To decide if a target is present, at a given range, we need to set a threshold (constant or variable) using "decision theory".
- Detection performance (Probability of Detection) depends on
  - the strength of the target relative to that of the noise and
  - the threshold setting (design variable)

Example (Using real radar data from a moving UAV)



Rx's front-end Radar Noise

- The sky-noise (solar+galactic+cosmic+atmospheric) plus thermal noise will appear the Rx's front-end



Total Radar Noise

- The total effect of different noise sources is represented by a single noise source at the Rx-antenna with power

$$\boxed{\sigma_n^2 = \underbrace{k_B.T_s}_{N_0}.B} \tag{1}$$

where

$$\begin{aligned}
k_B &= \text{Boltzmann constant} = 1.28 \times 10^{-23} \ Joules/^\circ K \\
T_s &= \text{system noise temperature (see Appendix-A)} \\
&\approx T_o.F_n \quad \text{(without any loss of generality, this approximation will be used here)} \\
T_0 &= 290^\circ K \ \text{(temperature)} \\
F_n &= \text{Noise Figure of the Rx-subsystem (see Appendix-B), unitless} \\
B &= \text{bandwidth Hz}
\end{aligned}$$

- Note: $k_B.T_s = N_0$

Radar Range Equation and SNR

- Signal Power reflected by the target and received by the radar:

$$\boxed{P_{RX} = \frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R^4}.RCS} \tag{2}$$

- Noise Power:

$$\boxed{\sigma_n^2 = k_B.T_o.F_n.B} \tag{3}$$

- Signal-to-Noise Ratio at the radar's input, SNR_{in},which is the standard measure of the radar's ability to detect a given target at a given range from the radar:

$$\boxed{SNR_{in} = \frac{P_{RX}}{\sigma_n^2} = \frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R^4.k_B.T_o.F_n.B}.RCS} \tag{4}$$

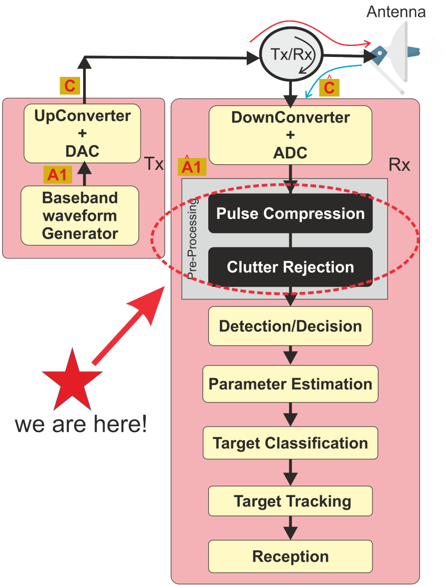
Signal-to-Noise plus Clutter plus Jammer Power Ratio (SNCIR)

$$SNCIR_{in} = \frac{P_{RX}}{\sigma_n^2 + \sigma_{clutter}^2 + \sigma_{jammer}^2}$$
$$= \frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R^4}.RCS\frac{1}{k_B T_o F_n B + \sigma_{clutter}^2 + \sigma_{jammer}^2} \quad (5)$$

N.B.:

- often only one of the noise, clutter and jammer is dominating

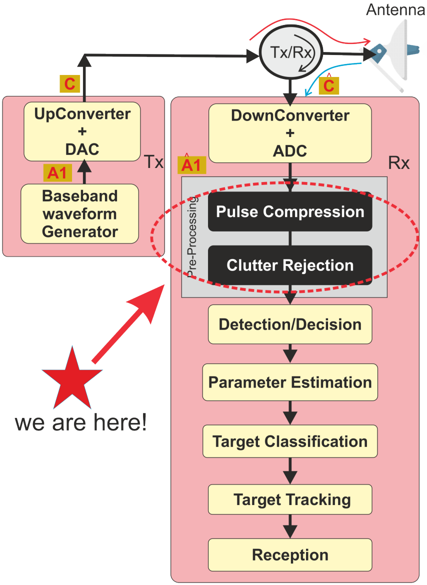
Pulse Compression and Clutter Filtering: Introduction

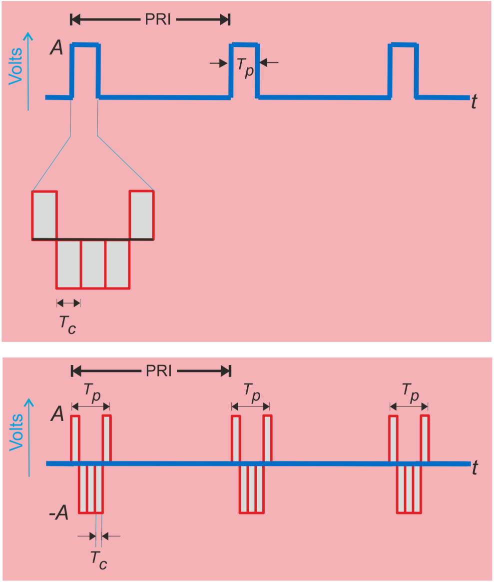


- we have seen that pulse compression, allows a radar to **simultaneously achieve**
  - the energy of a long pulse and
  - the resolution of a short pulse.

It also improves the SNR.

- Remember, at point-A : Pulse Compression Waveform

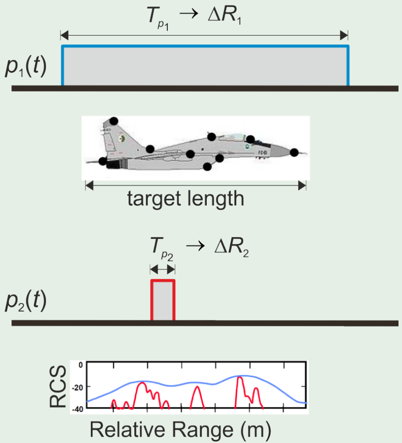




Important: Resolution and Target Length

1. $\Delta R = \frac{cT_p}{2} = \frac{c}{2B} >$ target length $\Rightarrow$**cannot** resolve features along the target
2. $\Delta R = \frac{cT_p}{2} = \frac{c}{2B} <$ target length $\Rightarrow$**can** resolve features along the target

Example



- using $p_1(t)$:

$$\begin{aligned} \Delta R_1 &= \frac{cT_{p_1}}{2}; \\ B_1 &= \frac{1}{T_{p_1}} \end{aligned}$$

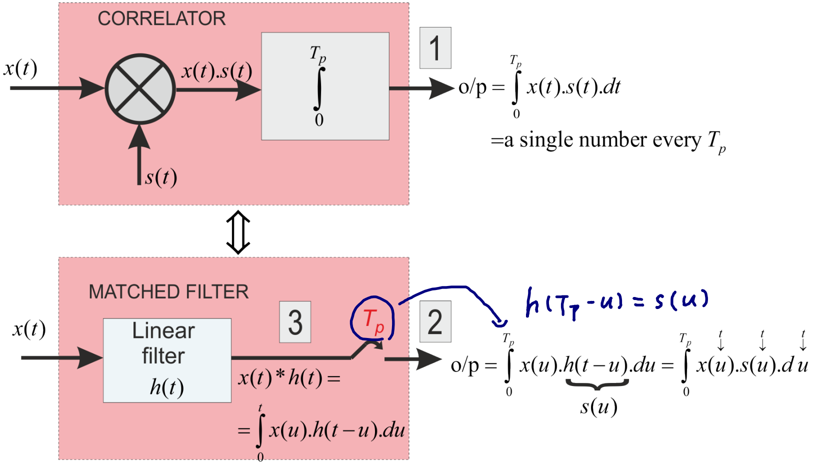
- using $p_2(t)$:

$$\begin{aligned} T_{p_2} &= T_{p_1} \times 0.1 \\ \Delta R_2 &= \frac{cT_{p_2}}{2} = \frac{c \times T_{p_1} \times 0.1}{2} \\ &= \Delta R_1 \times 0.1 \\ B_2 &= B_1 \times 10 \end{aligned}$$

N.B.:

- Shorter pulses have higher bandwidth and better resolution (we see more "details" of the target)

- A **correlator** is equivalent to a **linear filter known as Matched Filter**



N.B.:compare correlator's o/p (point-1) with matched filter's o/p (point-2).

Definition

- If we choose the impulse response of the linear filter to be equal to the signal reversal in time, i.e.

$$\boxed{h(t) = s(T_p - t),\ \ 0 \leq t \leq T_p} \qquad (6)$$

then that linear filter is defined as "Matched Filter" (MF).

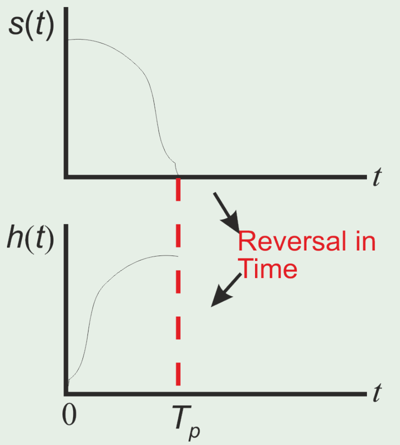
N.B.:

- $\boxed{\text{Output Of Correlator}} \underset{\text{only at time } t=T_p}{\underset{\uparrow}{=}} \boxed{\text{Output Of Matched Filter}}$
- It is also popular to normalise the MF given by Equation 6 with $\frac{2}{N_0}$. That is,

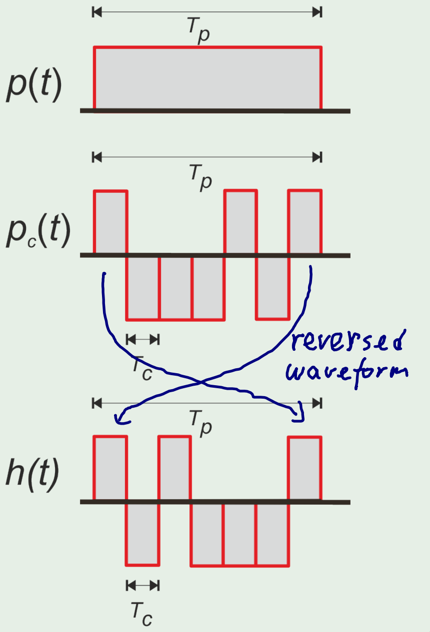
$$\boxed{h(t) = \frac{2}{N_0} s(T_p - t),\ \ 0 \leq t \leq T_p} \qquad (7)$$

This normalisation can be seen by using Equation 10 for $\text{PSD}_n(f) = N_o/2$. This normalisation does not change the system performance as it is applied to both the input signal and the input noise. Thus the MF's SNR-output remains the same

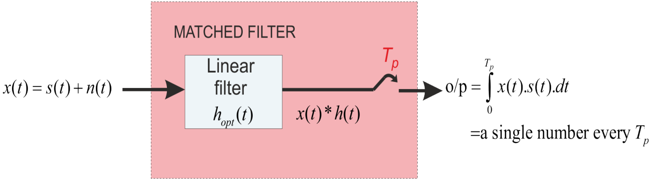
Example (general MF)



Example (Radar Compression MF)



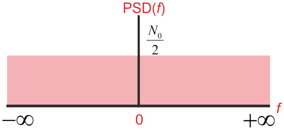
- A matched filter (matched to a pulse signal $s(t)$ of Energy E) is an optimum filter with impulse response $h_{opt}(t) = s(T_p - t)$)



- The SNR at the output of a matched filter is maximum and is given by the following expression:

$$\boxed{\mathrm{SNR}_{\underset{\max}{out}} = 2\frac{E}{N_0}} \quad (8)$$

where N_0 is the single-sided PSD(f) of the noise (AWGN)



Matched Filters in Freq. Domain

- Let $h(t) = \begin{cases} s(T_p - t) & 0 \leq t \leq T_p \\ 0 & \text{elsewhere} \end{cases}$
- $\boxed{\text{Time Domain}} \xmapsto{\text{FT}} \boxed{\text{Freq. Domain}}$

$$\begin{aligned} s(t) \xmapsto{\text{FT}} S(f) = \text{FT}\{s(t)\} &= \int_0^{T_p} s(t) e^{-j2\pi ft} dt \\ h(t) \xmapsto{\text{FT}} H(f) = \text{FT}\{h(t)\} &= \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \\ &= \int_o^{T_p} s(T_p - t) e^{-j2\pi ft} dt \\ &= \int_0^{T_p} s(u) e^{-j2\pi f(T_p - u)} du \\ &= e^{-j2\pi f T_p} \int_0^{T_p} s(u) e^{+j2\pi fu} du \\ &= e^{-j2\pi f T_p} \cdot S^*(f) \end{aligned}$$

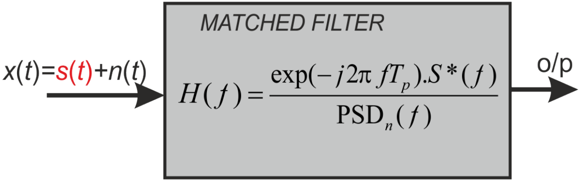
therefore

$$\boxed{H(f) = e^{-j2\pi f T_p} \cdot S^*(f)} \tag{9}$$

Matched Filters: Known Signals in Non-White Noise

- It can be proven that for non-white noise $n(t)$ the matched filter's transfer function $H(f)$ is given as follows:

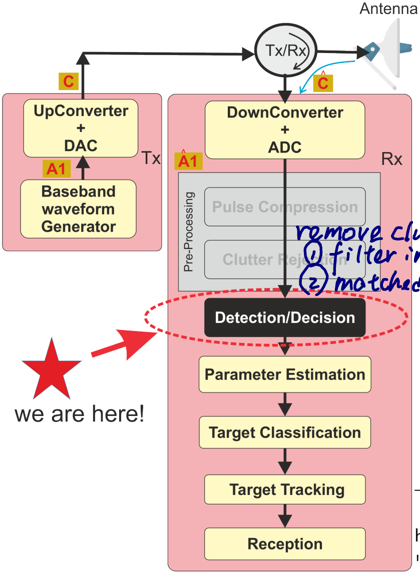
$$\boxed{H(f) \approx \frac{e^{-j2\pi f T_p} \cdot S^*(f)}{\mathrm{PSD}_n(f)}} \tag{10}$$



N.B.:

- the noise at the output of the matched filter is "white"
- If $\mathrm{PSD}_n(f) = \frac{No}{2}$, then Equ 10 gives $h(t) = \frac{2}{N_0} s(T_p - t)$

Target Detection: Basic Detection Theory



remove clutters:
(1) filter in delay-Doppler domain
(2) matched filter to the pulse code

- The signal $x(t)$ at the input of the detection/decision block includes
  - the received background noise[a] which fluctuates randomly (this is a random signal)
  - the target echo which also fluctuates randomly (this is also a random signal)
- Thus, overall, the signal $x(t)$ is a random signal
- Based on the observed signal $x(t)$, the aim of this block is to decide if a target is present at a given range (range-bin)

[a] If we have not filtered the "clutter" then we will have both noise and clutter at the input of the "decision" block.

The 'Detection' Problem: Basic Detection Theory

Problem

- *To determine/detect the presence of a target in noise*

Solution

- *To use optimum Detection Theory which is based on the following decision criteria*
  1. **BAYES** *Criterion*
  2. **Minimum Probability of Error** $(\min(p_e))$ *Criterion*
  3. **MAP** *Criterion*
  4. **MINIMAX** *Criterion*
  5. *Newman-Pearson* (**N-P**) *Criterion*
  6. *Maximum Likelihood* (**ML**) *Criterion*

Terminology and Definitions

Definition (Hypothesis)

A Hypothesis $\triangleq$ a statement of a possible condition

Definition (Hypethesis Testing)

To choose one from a number (two or more) hypotheses

Definition (Target Detection Hypotheses)

- we have an observed signal $x(t)$ and we define two hypotheses H_0 and H_1 as follows:

$$\boxed{\begin{array}{ll} H_0: & x(t) = n(t) \\ & \text{i.e. the target is not present} \\ \\ H_1: & x(t) = s(t) + n(t) \\ & \text{i.e. the target is present} \end{array}}$$

Definition (A priori probabilities)

- These are the probabilities which can be calculated before the radar is used (i.e. before any signal is observed and before the target detection is performed)

$$\Pr(H_0), \Pr(H_1)$$

(i.e. these are calculated **BEFORE** the experiment is performed)

Definition (A posterior probabilities)

- These are the conditional probabilities

$$\Pr(H_0/x), \Pr(H_1/x)$$

which can be calculated after the signal x is received/observed (i.e. these are calculated **AFTER** the experiment is performed).

- $\Pr(H_0/x)$ and $\Pr(H_1/x)$: difficult to find. A more natural approach is to find

$$\Pr(x/H_0), \Pr(x/H_1) \tag{11}$$

since in general $\text{pdf}_{x/H_0}$and pdf_{x/H_1}
  - are known or
  - can be found

Definition (Likelihood Functions)

The *two* conditional probability density functions $\text{pdf}_{x/H_0}(x)$,$\text{pdf}_{x/H_1}(x)$, i.e.

$$\text{pdf}_{x/H_0}(x), \text{pdf}_{x/H_1}(r) \tag{12}$$

are known as "Likelihood Functions" (LF)

Definition (Likelihood Ratio)

The ratio

$$\frac{\text{pdf}_{x/H_0}(x)}{\text{pdf}_{x/H_1}(x)}, \frac{\text{pdf}_{x/H_1}(x)}{\text{pdf}_{x/H_0}(x)} \tag{13}$$

is known as "Likelihood Ratio"

Hypothesis Testing re-Defined

- Note that the statistics of the observed signal $x(t)$ are different for different hypotheses.

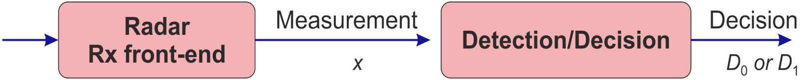
Definition (Hypothesis Testing re-defined)

- If we have an observed signal $x(t)$ and have defined two hypotheses H_0 and H_1 as follows:

$$\boxed{\begin{array}{lll} H_0: & x(t) = n(t), & \mathrm{LF}^{(0)} = \mathrm{pdf}_{x|H_0} \\ & \text{i.e. the target is not present} & \\ & & \\ H_1: & x(t) = s(t) + n(t), & \mathrm{LF}^{(1)} = \mathrm{pdf}_{x|H_1} \\ & \text{i.e. the target is present} & \end{array}}$$

where the distributions $\mathrm{LF}^{(0)}$ and $\mathrm{LF}^{(1)}$ are **known**
then the problem of choosing one of the two hypotheses after having observed $x(t)$, **is known as 'Hypothesis Testing'**

Hypothesis Testing: Radar Detection Problem



- For each measurement there are 2 possibilities

possibilities	Measurement	LF
Target absent hypothesis, H_0 (noise only)	$x = n$	pdf_{x/H_0}
Target present hypothesis, H_1 (signal + noise)	$x = s + n$	pdf_{x/H_1}

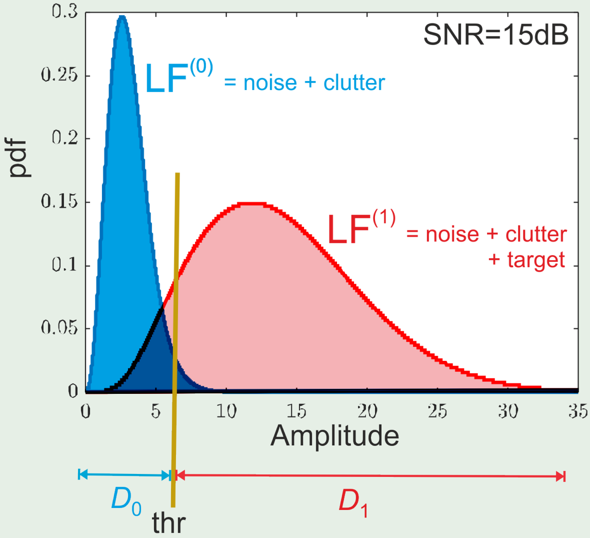
- For each measurement there are 4 decisions:

	D_0	D_1
H_0	Don't Report	False Alarm
H_1	Missed Detection	Detection

Basic Steps

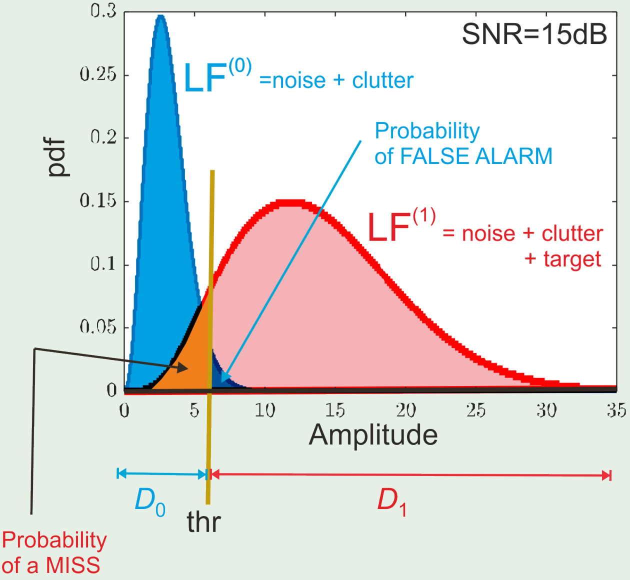
1. identify the likelihood functions $\mathrm{LF}^{(0)}$ and $\mathrm{LF}^{(1)}$
2. place the two likelihood functions together on the same graph/plot (don't add them together!!)
3. set a threshold (thr) for making optimum decisions. This threshold should be set by using one of the optimum decision criteria (see next section)

Example

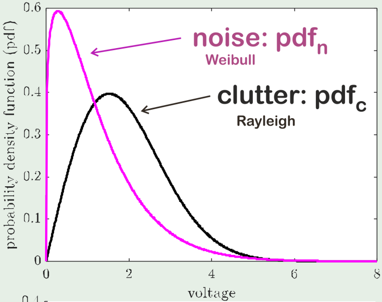


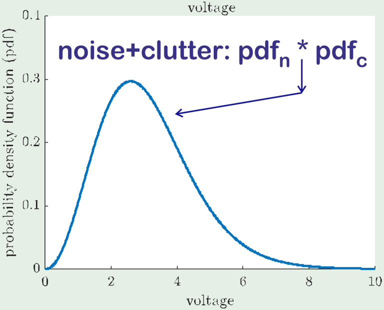
SNR=15dB
LF(0) = noise + clutter
LF(1) = noise + clutter + target
pdf
Amplitude
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
35
D0
thr
D1

Example (cont.)



Example (cont.)





Assumptions: Consider an observed signal $x(t)$ and 2 hypotheses H_0, H_1. For these two hypotheses, estimate/identify the likelihood functions

$$\begin{aligned} \mathrm{LF}^{(0)} &= \mathrm{pdf}_{x|H_0}(x), \\ \mathrm{LF}^{(1)} &= \mathrm{pdf}_{x|H_1}(x) \end{aligned}$$

- Let us define the sets of parameters $\mathcal{P}_1$ and $\mathcal{P}_2$ where:

  $\mathcal{P}_1$ denotes the set of parameters $\Pr(H_0)$,$\Pr(H_1)$
  $\mathcal{P}_2$ represents the set of costs/weights $C_{00,} C_{11}, C_{01}, C_{10}$ associated with the probabilities $\Pr(D_i|H_j)$, (where D_i indicates "decision" by choosing hypothesis H_i)

Main Decision Criteria

Decision: choose the hypothesis H_i (i.e. D_i) having the maximum $G_i(x)$ where $G_i(x)$ depends on the chosen criterion as follows:

- **BAYES** Criterion
- **Minimum Probability of Error** ($\min(p_e)$) Criterion
- **MAP** Criterion
- **MINIMAX** Criterion
- Newman-Pearson (**N-P**) Criterion
- Maximum Likelihood (**ML**) Criterion

$\mathcal{P}_1$: a priori $\Pr(H_0)$, $\Pr(H_1)$

$\mathcal{P}_2$: cost of i/o transfer

$C_{ij} \leftrightarrow P(D_i | H_j)$

	$\mathcal{P}_1$	$\mathcal{P}_2$	choose Hypothesis with $\max_{\forall i}(G_i(x))$
Bayes	**known**	**known**	$G_i(x) \triangleq \text{weight}_i \times \Pr(H_i) \times \text{pdf}_{x\|H_i}$
$\min(p_e)$ or MAP	**known**	unknown	$G_i(x) \triangleq \Pr(H_i) \times \text{pdf}_{x\|H_i}$
Minimax	unknown	**known**	$G_i(x) \triangleq \text{weight}_i \times \text{pdf}_{x\|H_i}$
N-P	unknown	unknown	find threshold to satisfy: Prob of FA = given
ML	don't care	don't care	$G_i(x) \triangleq \text{pdf}_{x\|H_i}$

special case

special case

where

mistake penalty

$$\text{weight}_0 = C_{10} - C_{00} \qquad (14)$$

$$\text{weight}_1 = C_{01} - C_{11} \qquad (15)$$

correct decision

fine-tunable

- N.B.:
  - Note-1:
    if an approximate/initial solution is required then any information about the sets of parameters $\mathcal{P}_1$ and/or $\mathcal{P}_2$ can be ignored. In this case the Maximum Likelihood (ML) Criterion should be used.
  - Note-2:
    Sometimes, for convenience, G_i will be used (i.e. $G_i \triangleq G_i(x)$) - i.e. the argument will be ignored.
  - Note-3:
    In radar the Newman Pearson criterion is quite popular: the threshold level is set based on a given probability of False Alarm

- We normally design the "decision" threshold in accordance with some set of criteria, and this will determine the value of threshold.
- However, in many applications the receiver performance is examined as a function of the threshold setting.
- Since

$$\begin{aligned} \overbrace{\Pr(D_1/H_0)}^{\triangleq p_{FA}} &= \text{f}\{\text{threshold setting}\} \\ \underbrace{\Pr(D_0/H_1)}_{\triangleq p_{miss}} &= \text{f}\{\text{threshold setting}\} \end{aligned}$$

by plotting for a fixed threshold the following expression,

$$\underbrace{1-\Pr(D_0/H_1)}_{\triangleq p_{\text{detection}}} = \text{f}\{\Pr(D_1/H_0)\}$$

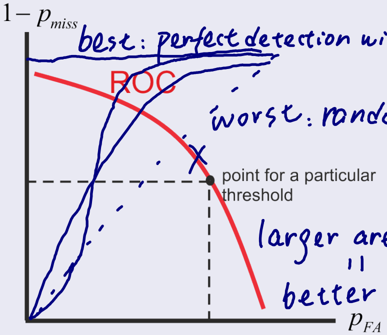
we get one point.

Definition (ROC)

- ROC (Receiver Operating Characteristics) is defined as the decision function/curve of the probability of detection $p_{\text{detection}}$ versus the probability of false alarm p_{FA}, i.e.

$$\underbrace{1-\overbrace{\Pr(D_0/H_1)}^{P_{MD}}}_{\Pr(D_1|H_1).\ P_{\text{detection}}} = \mathsf{f}\{\overbrace{\Pr(D_1/H_0)}^{P_{FA}}\}, \qquad (16)$$

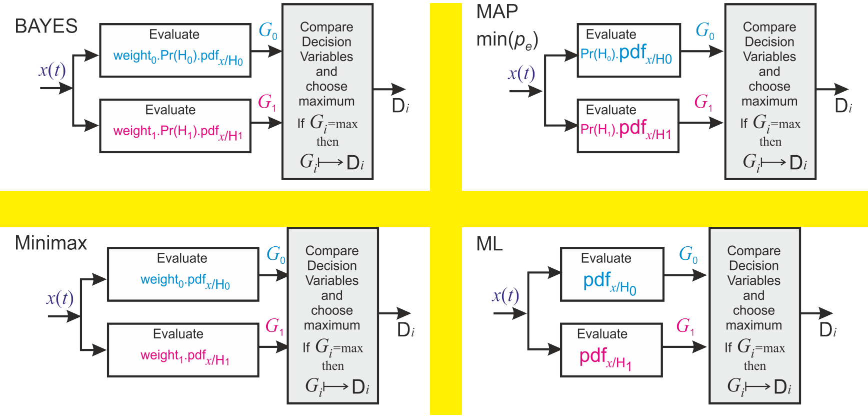
for various thresholds (one point on the curve for each threshold).



best: perfect detection with infinitesimal FA

worst: random detector

larger area under = better


BAYES
x(t)
Evaluate
weight0.Pr(H0).pdfx/H0
Evaluate
weight1.Pr(H1).pdfx/H1
G0
G1
Compare Decision Variables and choose maximum
If Gi=max then
Gi ⟼ Di
Di
MAP
min(pe)
x(t)
Evaluate
Pr(H0).pdfx/H0
Evaluate
Pr(H1).pdfx/H1
G0
G1
Compare Decision Variables and choose maximum
If Gi=max then
Gi ⟼ Di
Di
Minimax
x(t)
Evaluate
weight0.pdfx/H0
Evaluate
weight1.pdfx/H1
G0
G1
Compare Decision Variables and choose maximum
If Gi=max then
Gi ⟼ Di
Di
ML
x(t)
Evaluate
pdfx/H0
Evaluate
pdfx/H1
G0
G1
Compare Decision Variables and choose maximum
If Gi=max then
Gi ⟼ Di
Di

Optimum Detection/Rx Architecture

It can be easily proven that $G_i(x)$, for $i = 0, 1$, [i.e. $G_0(x)$ and $G_1(x)$], for all the optimum decision criteria, i.e.

$$D_i = \begin{cases} \arg\max_i \{ \underbrace{\mathrm{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \} & \text{ML} \\ \arg\max_i \{ \underbrace{\Pr(H_i) \times \mathrm{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \} & \text{MAP, min(pe)} \\ \arg\max_i \{ \underbrace{\mathrm{weight}_i \times \mathrm{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \} & \text{minmax} \\ \arg\max_i \{ \underbrace{\mathrm{weight}_i \times \Pr(H_i) \times \mathrm{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \} & \text{Bayes} \end{cases} \tag{17}$$

are simplified, as follows:

wildcard structure

$$G_0(x) = \mathrm{DC}_0, \tag{18}$$

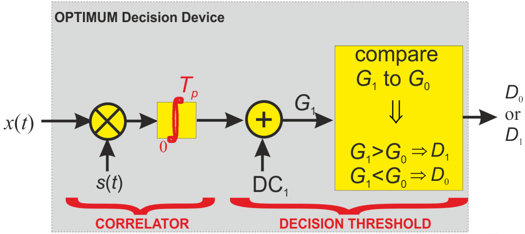
$$G_1(x) = \int_0^{T_{cs}} x(t)s(t)dt + \mathrm{DC}_1 \tag{19}$$

- where DC_i (for $i = 0, 1$) depends on the decision criterion and is given by

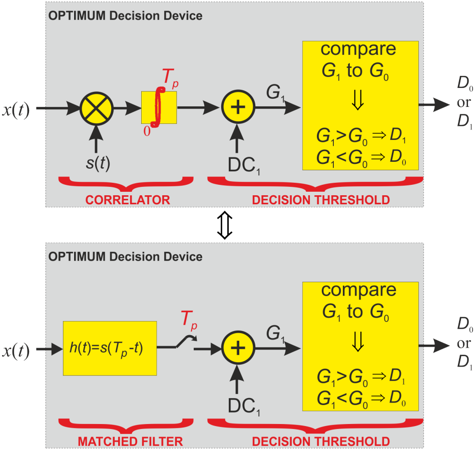
$$\text{DC}_i \triangleq \begin{cases} -\frac{1}{2}E_i; & \text{ML} \\ \frac{N_0}{2}\ln(\Pr(H_i)) - \frac{1}{2}E_i; & \text{MAP} \\ \frac{N_0}{2}\ln(\text{weight}_i) - \frac{1}{2}E_i; & \text{minmax} \\ \frac{N_0}{2}\ln(\text{weight}_i.\Pr(H_i)) - \frac{1}{2}E_i; & \text{Bayes} \end{cases} \quad (20)$$

$$\text{with } E_i \triangleq \begin{cases} 0; & i = 0 \\ \text{energy of } s(t); & i = 1 \end{cases} \quad (21)$$

- Equs 17,18 and 19 are implemented by the following optimum architecture/device:

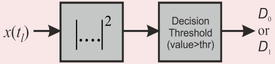


Summary: Optimum Decision Devices

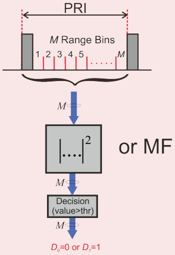


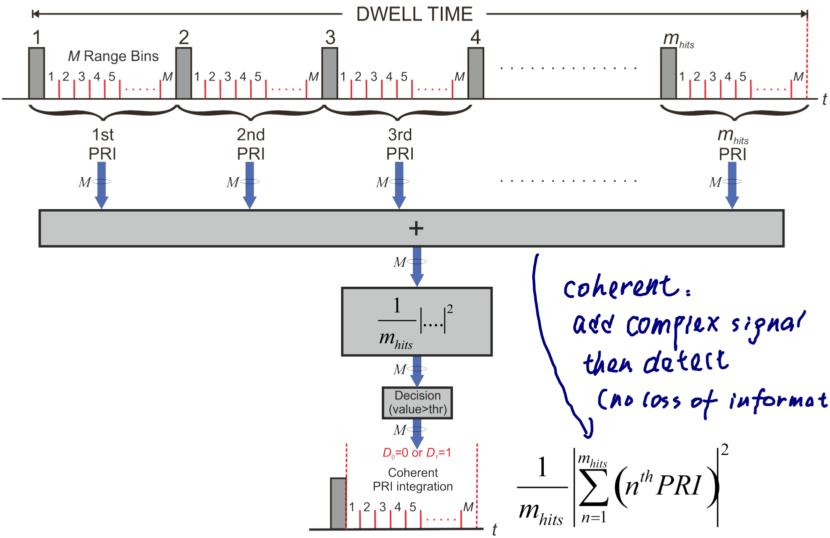
N.B.:

- If $x(t)$ is a single sample at time t_l, then the MF and the correlator are equivalent to $|\ldots|^2$ and the decision device is simplified:



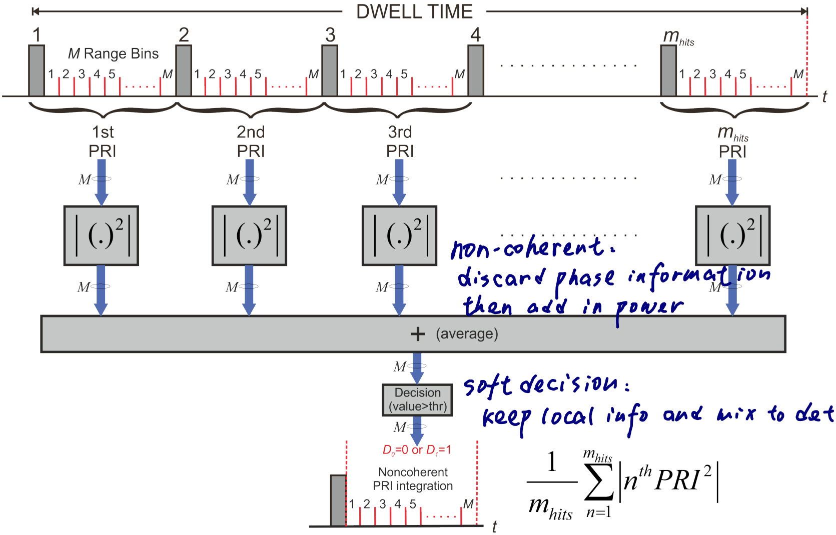
- Single PRI processing:





coherent:
add complex signal
then detect
(no loss of information)

$$\frac{1}{m_{hits}}\left|\sum_{n=1}^{m_{hits}}\left(n^{th}\,PRI\right)\right|^2$$

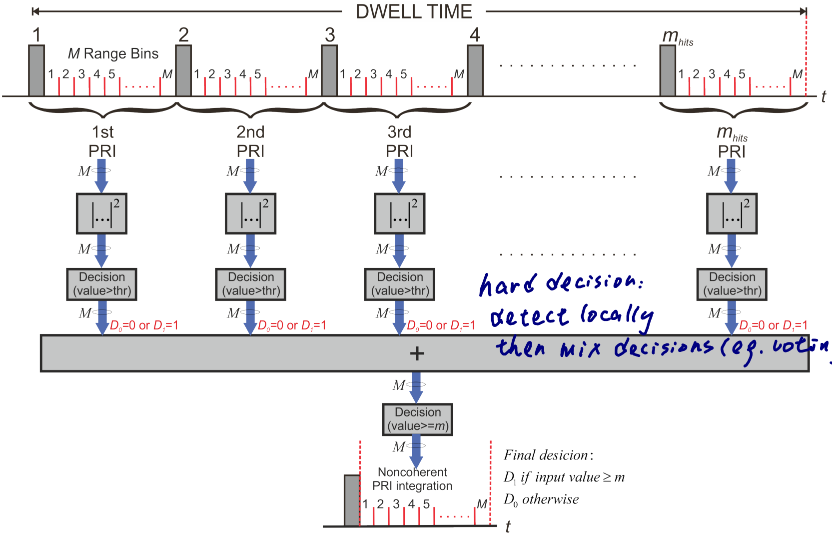


non-coherent:
discard phase information
then add in power

soft decision:
keep local info and mix to detect

$$\frac{1}{m_{hits}} \sum_{n=1}^{m_{hits}} \left| n^{th} PRI^2 \right|$$

Binary PRI Integration



hard decision: detect locally then mix decisions (eg. voting)

Summary of Types of Noncoherent PRI Integration

- **Non-Coherent Integration — Also called ("video integration")**
  - Generate magnitude for each of $\mathbf{m}_{hits}$ pulses (PRI's)
  - Add magnitudes and then threshold (decision)
- **Binary Integration (m-of-$\mathbf{m}_{hits}$ Detection)**
  - Separately threshold each pulse (each PRI)

$$\text{Decision} = \begin{cases} D_1 = 1, & \text{if signal value} > \text{threshold} \\ D_0 = 0, & \text{otherwise} \end{cases} \tag{22}$$

  - Count number of threshold crossings (the number of 1's)
  - Threshold this sum of threshold crossings
  - Simpler to implement than coherent and non-coherent (video integration)
- **Cumulative Detection (1-of-$\mathbf{m}_{hits}$ Detection)**
  - Similar to Binary Integration higher false alarm
  - Require at least 1 threshold crossing for $\mathbf{m}_{hits}$ PRI's

Appendix-A: Radar System Noise Temperature

- The radar system noise temperature T_s is divided into 3 components

$$T_s = T_a + T_r + L_r.T_e \tag{23}$$

where

$$\begin{aligned} T_a &= \text{contribution from antenna} \\ T_r &= \text{contribution from the RF components} \\ &\quad \text{(between antenna and Rx)} \\ T_e &= \text{temperature of the Receiver} \\ L_r &= \text{loss of the input RF components (natural units)} \end{aligned}$$

- The 3 temperature components can be broken down even further:

$$T_a = \frac{0.88\,T_{sky} - 254}{L_a + 290} \tag{24}$$

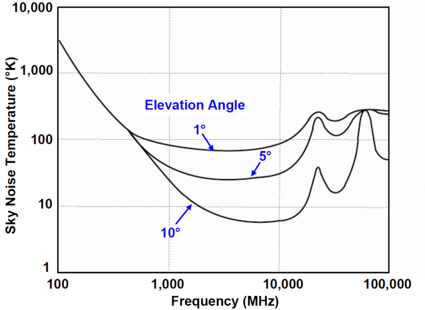
$$T_r = T_{tr}(L_r - 1) \tag{25}$$

$$T_e = T_o(F_n - 1) \tag{26}$$

where

$$\begin{aligned} T_{sky} &= \text{the apparent temperature of the sky (from graph)} \\ L_a &= \text{the dissipative loss within the antenna (natural units)} \\ T_{tr} &= \text{the physical temperature of the RF components} \\ F_n &= \text{the noise factor of the Rx (natural units)} \\ T_o &= \text{the noise reference temperature of } 290^\circ K \end{aligned}$$

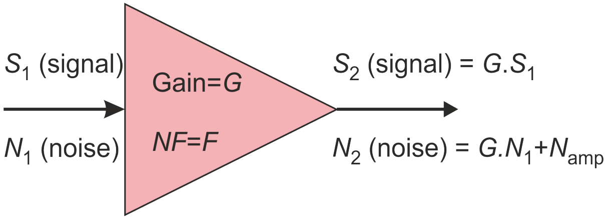
- Note: all the temperature quantities are in units of $^\circ K$.
- **Sky Noise Graph**



- Note: The above graph takes into account:
  - galactic noise,
  - cosmic blackbody radiation,
  - solar noise and
  - atmospheric noise (due to the troposphere)

- An amplifier will introduce noise signal. We characterise this increase in noise by defining the *Noise Factor* for a stage

$$NF \triangleq \frac{\text{signal to noise ratio at i/p}}{\text{signal to noise ration at o/p}} = \frac{S_1 / N_1}{S_2 / N_2} \tag{27}$$



- That is

$$\left.\begin{array}{c}\left.\begin{array}{c} NF = \frac{S_1/N_1}{S_2/N_2} \\ S_2 = GS_1 \end{array}\right\} \Rightarrow NF = \frac{N_2}{GN_1} \\ N_2 = GN_1 + N_{amp} \end{array}\right\} \Rightarrow NF = \frac{GN_1 + N_{amp}}{GN_1} \quad (28)$$

- Note: If no noise is introduced by the amplifier the $NF = 1$. That is

$$\boxed{\text{noiseless ampl} \Rightarrow NF = 1} \quad (29)$$

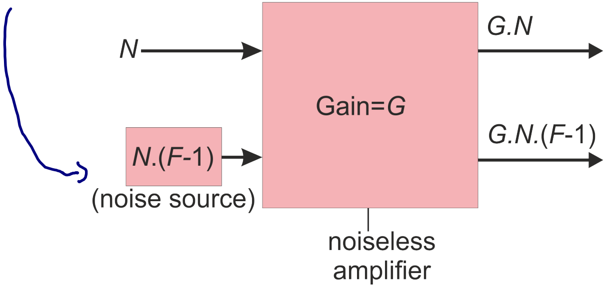
- NF is defined for a particular noise input. This implies added noise is constant but the input noise is amplified.

Appendix-C: Noise Factor of Cascaded Stages

- Assume that NF is defined for an i/p noise level N
- Let us define

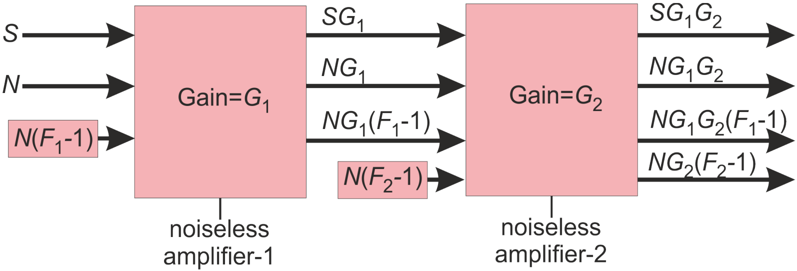
$$F \triangleq NF \text{ of amplifier} \tag{30}$$

- Replace each stage by a noiseless amplifier with an additive noise source at i/p. That is,



- For two stages, let us define

$$\text{1st amplifier: } F_1 \triangleq NF; \text{ 2nd amplifier: } F_2 \triangleq NF \tag{31}$$



- overall NF:

$$NF = \frac{\text{signal to noise ratio at I/P}}{\text{signal to noise ration at O/P}} = \frac{S/N}{\frac{SG_1G_2}{NG_1G_2+NG_1G_2(F_1-1)+NG_2(F_2-1)}}$$

$$= \frac{G_1G_2 + G_1G_2(F_1-1) + G_2(F_2-1)}{G_1G_2}$$

$$\Rightarrow \boxed{NF = F_1 + \frac{F_2-1}{G_1}} \tag{32}$$

Appendix-D: Clutter Distributions

- Detection Statistics Calculations
  - Steady Targets and Swerling 1,2,3,4 Targets in Gaussian Noise
  - Chi- Square Targets in Gaussian Noise
  - Log Normal Targets in Gaussian Noise
  - Steady Targets in Log Normal Noise
  - Log Normal Targets in Log Normal Noise
  - Weibull Targets in Gaussian Noise
- N.B: Chi Square, Log Normal and Weibull Distributions have long tails
- When used
  - Ground clutter: Weibull
  - Sea Clutter: Log Normal
  - HF noise: Log Normal
  - Birds: Log Normal

Matlab Functions of Useful pdf's

- see also "Classes" in OneNote and talk to GTAs:

		pdf	random number	Comments
1	Gaussian Noise	`normpdf(x,mu,sigma)`	`normrnd(mu,sigma)`	
2	Chi-Square	`chi2pdf(x,DOF)`	`chi2rnd(DOF)`	
3	Log Normal	`lognpdf(x,mu,sigma)`	`lognrnd(mu,sigma)`	
4	Weibull	`wblpdf(x,A,B)`	`wblrnd(A,B)`	A: scale param.; B: shape param.
5	Swerling 1 & 2: RCS σ	`exppdf(x,mu)`	`exprnd(mu)`	$\mathtt{x} = \sigma$; $\mathtt{mu} = \overline{\sigma}$
6	Swerling 1 & 2: Amp a	`raylpdf(x,B)`	`raylrnd(B)`	$\mathtt{x} = a$; $\mathtt{B} = \sqrt{\frac{\overline{\sigma}}{2}}$
7	Swerling 3 & 4: RCS σ	$\frac{4}{\mathtt{mu}} \cdot \mathtt{chi2pdf}\left(\frac{4}{\mathtt{mu}} \cdot \mathtt{x}, \mathtt{DOF}\right)$	$\frac{\mathtt{mu}}{4} \cdot \mathtt{chi2rnd(DOF)}$	$\mathtt{x} = \sigma$; $\mathtt{mu} = \overline{\sigma}$; $\mathtt{DOF} = 4$
8	Swerling 3 & 4: Amp a	$\frac{4}{\mathtt{mu}^4} \cdot \mathtt{x}^2 \cdot \mathtt{raylpdf}\left(\frac{2}{\sqrt{\mathtt{mu}}} \cdot \mathtt{x}, \mathtt{B}\right)$		$\mathtt{x} = a$; $\mathtt{mu} = \overline{\sigma}$; $\mathtt{B} = 1$

Note:

- `x`: variable
- `mu`: mean
- `sigma`: standard deviation

To find the pdf from a number of random numbers, use:

- `[N,edges] = histcounts(data,NumBins,'normalization','pdf')`
- `histogram(data,NumBins,'normalization','pdf')`