EE3-27: Principles of Classical and Modern Radar

Phased-Array Radar

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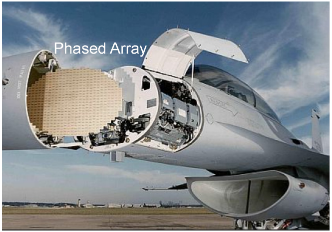
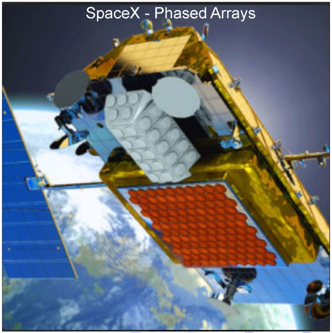
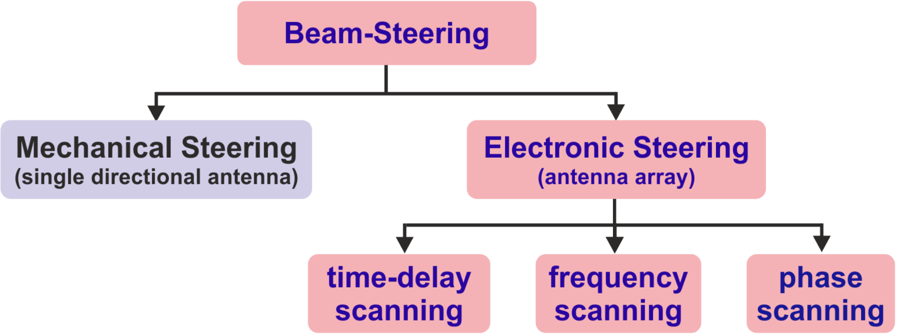




Beam-Steering Classification



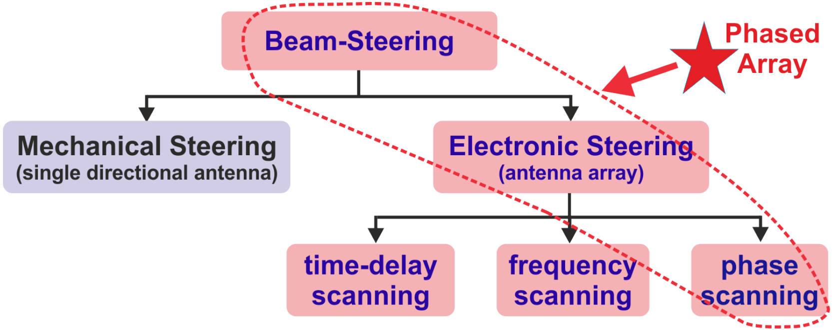
Mechanical Beam-Steering Disadvantages

- positioning the antenna is slow
- reduced reaction time
- blind sided
- mechanical errors

Example (Mechanical Beam Steering)

- Note: Radome = Radar dome (fiberglass)

Electronic Beam-Steering Advantages



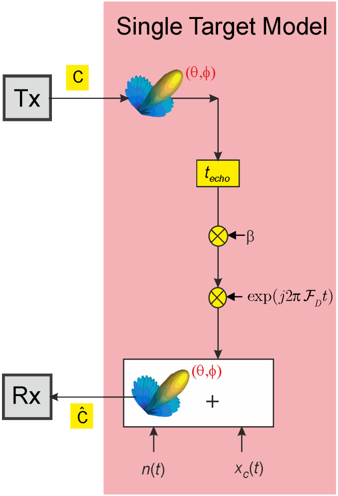
Mechanical Beam-Steering Disadvantages

- positioning the antenna is slow
- reduced reaction time
- blind sided
- mechanical errors

Electronic Beam-Steering Advantages

+ Instantaneous/rapid beam-steering
+ multimode (multi-beam) operation
+ multi-target capability
+ Elimination of mechanical errors

Single Antenna, Single Target Modelling



$$h(t) = \beta.\exp(j2\pi\mathcal{F}_D t).\delta(t - t_{echo})$$
$$= \text{impulse response}$$

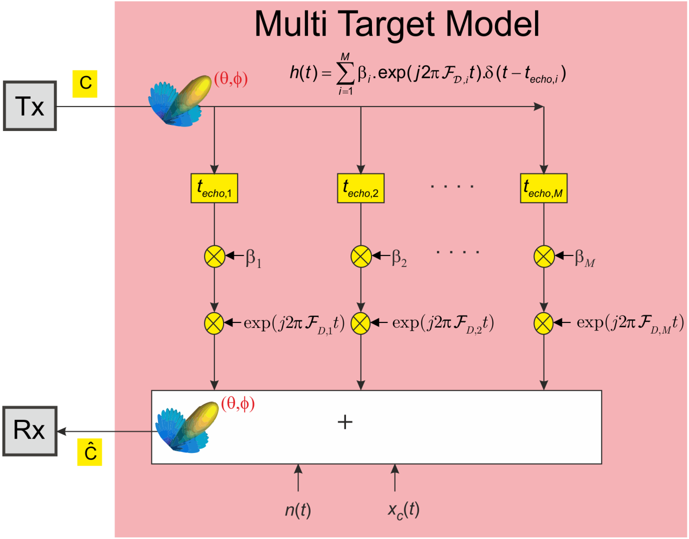
$$t_{echo} = \frac{2R}{c}$$

$$\beta = \sqrt{\frac{G_{Tx}.G_{Rx}}{(4\pi)^3}}.\frac{\lambda}{R^2}\sqrt{RCS}.\exp\left(-j2\pi F_c\frac{2R}{c}\right)\exp(j\psi)$$

$$\mathcal{F}_D = \text{Doppler frequency}$$

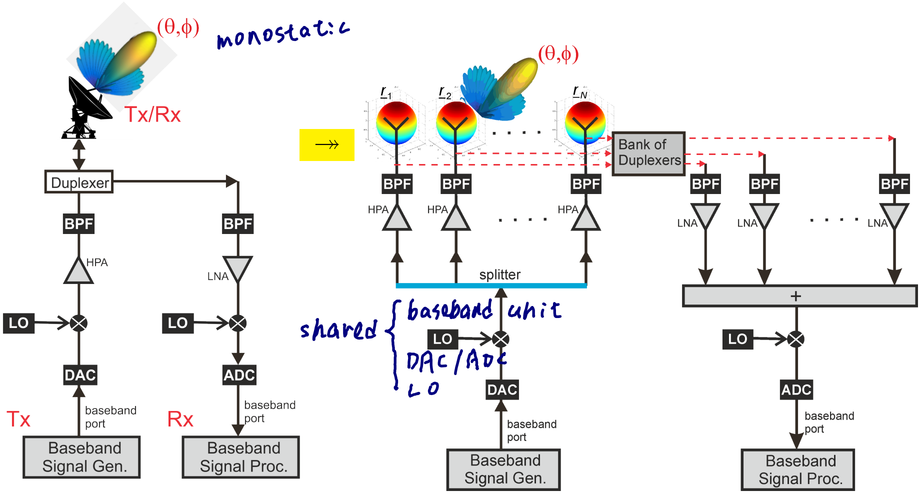
$$\psi = \text{random phase}$$

Single Antenna, Multi-target Modelling



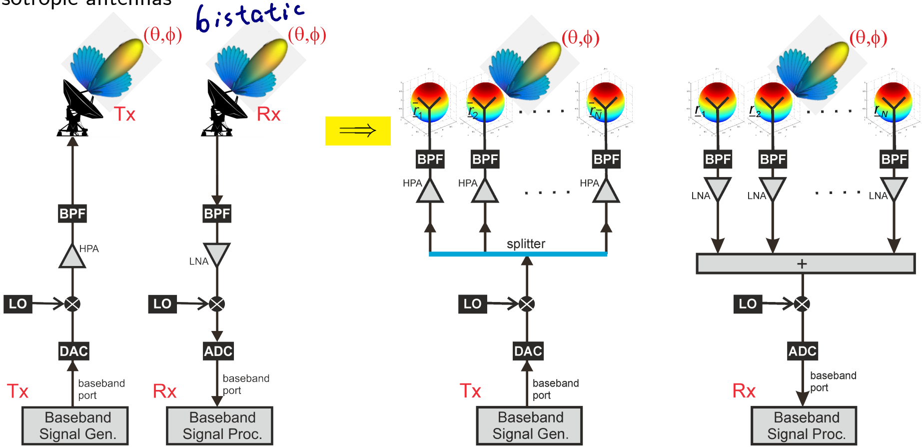
Replacing a Single Antenna with an Antenna Array

Replace the single antenna (directional; mechanical steering) with 2 or more (N, say) isotropic antennas

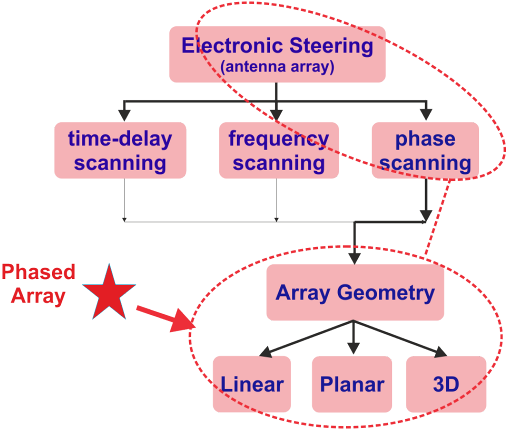


Replacing two Single Antennas with Antenna Arrays

Replace the single antenna (directional; mechanical steering) with 2 or more (N, say) isotropic antennas



Phased Array Geometries



Linear Arrays: uniform (ULA) or non-uniform Linear Arrays
Planar Arrays: circular, rectangular, 2D-grid, etc
3D Arrays: cube, spherical, 3D-grid, etc.,

Common Symbols and Antenna Array Definitions

- Remember:

N	number of Rx array elements	c	velocity of light
ϕ	elevation angle	F_c	carrier frequency
θ	azimuth angle	λ	wavelength
$\underline{u}$	$[\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi]^T$	k	wavenumber
	$\underline{u}^T\underline{u} = 1$	$\underline{k}$	wavevector

Definition (Array System)

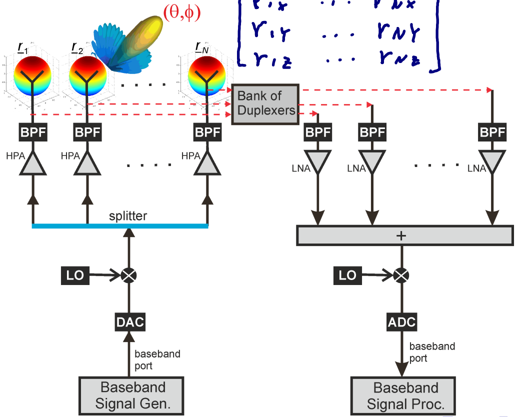
- A wireless system is known as an array system if it employs an array of $N > 1$ sensors (transducing elements, receivers, antennas, etc) distributed in our 3-dimensional real space with a common reference point (common origin).

Monostatic Antenna Array Radar

- Consider a monostatic radar with an antenna array of N elements having Cartesian coordinates

$$[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N] = [\underline{r}_x, \underline{r}_y, \underline{r}_z]^T \quad (3 \times N) \qquad (1)$$

$$\begin{bmatrix} r_{1x} & \cdots & r_{Nx} \\ r_{1y} & \cdots & r_{Ny} \\ r_{1z} & \cdots & r_{Nz} \end{bmatrix}$$



Definition (Antenna Array)

- An antenna-array is a collection of $N > 1$ antennas, working together, with locations given by the matrix

$$[\underline{r}_1, \underline{r}_2, ..., \underline{r}_N] = \left[\underline{r}_x, \underline{r}_y, \underline{r}_z\right]^T \ \ (3 \times N) \tag{2}$$

where

  - $\underline{r}_k$, $\forall k = 1, 2, ..., N$, is a 3×1 real vector denoting the location of the k^{th} antenna, and
  - $\underline{r}_x$, $\underline{r}_y$ and $\underline{r}_z$ are $N \times 1$ vectors with elements the x, y and z coordinates of the N antennas

Definition (Array Aperture)

- The region over which the array elements are distributed is called the aperture of the array. In particular the array aperture is defined as follows

$$\text{array aperture} \doteq \max_{\forall ij} \left\| \underline{r}_i - \underline{r}_j \right\| \tag{3}$$

max elementwise distance

Revisiting the Electric and Magnetic Fields

- We have seen in Topic 3 (EM Waves Refresher) that in the far field the Electric and Magnetic Fields $\underline{E}(\underline{r}, t)$ and $\underline{H}(\underline{r}, t)$, respectively, are represented as:

$$\underline{E}(\underline{r}, t) = \boxed{\underline{E}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \cos\left(2\pi F_c t - \frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right)} \quad (4)$$

$$\underline{H}(\underline{r}, t) = \underline{H}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \cos\left(2\pi F_c t - \frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right) \quad (5)$$

where

$$\underline{E}_o, \underline{H}_o \; : \; \text{are vector-amplitudes} \quad (6)$$

$$\alpha = \text{attenuation constant (Neper/m)} \quad (7)$$

$$\lambda = \text{wavelength} \quad (8)$$

with $\underline{r} = [x, y, z]^T$ (with $R = \|\underline{r}\|$) denoting a point in 3D and t representing time.

- We have seen also that this is, equivalently, in phasor format, as follows:

$$\underline{E}(\underline{r}, t) = \text{Re}\left\{\underline{E}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \exp\left(j 2\pi F_c t - j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right)\right\} \quad (9)$$

$$\underline{H}(\underline{r}, t) = \text{Re}\left\{\underline{H}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \exp\left(2\pi F_c t - \frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right)\right\} \quad (10)$$

- Next we want to remove the Real operator (Re) from Equations 9 and 10.

- We can remove the Re operator by using a single-carrier oscillator in the Tx/Rx of the system with a phase shifter of 90°, producing so two parallel carrier signals: the "I" and "Q" signals.
  - The I (inphase) signal is the cosine of the carrier frequency F_c, i.e. $\cos(2\pi F_c t)$, and
  - the Q (Quadrature) signal is the sine of the carrier frequency, that is 90° wrt cosine, i.e. $\sin(2\pi F_c t)$,

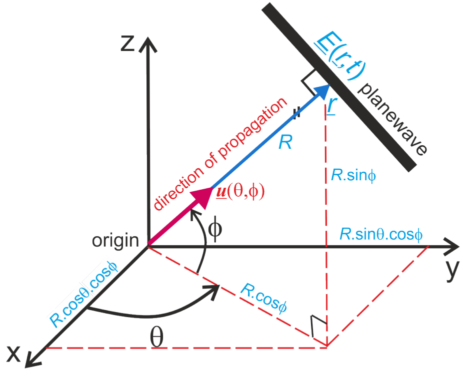
  The two parallel carrier signals are common in all modern wireless systems.
- Although these two (I and Q) signals are "real", we represent them as a single "complex signal".
- In this case, at a point $\underline{r} = [x, y, z]^T$ (with $R = \|\underline{r}\|$) in space and at time t, the Electric and Magnetic Fields $\underline{E}(\underline{r}, t)$ and $\underline{H}(\underline{r}, t)$, respectively, can be represented in pure phasor (complex) format as follows:

$$\underline{E}(\underline{r}, t) = \boxed{\underline{E}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \exp\left(j2\pi F_c t - j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right)} \quad (11)$$

$$\underline{H}(\underline{r}, t) = \underline{H}_o . \underbrace{\exp\left(-\alpha . \underline{u}^T \underline{r}\right)}_{\text{decay}} . \exp\left(j2\pi F_c t - j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right) \quad (12)$$

where $\underline{u} \triangleq u(\theta, \phi)$.

- Remember:



- If we assume α=0 (i.e. free space propagation/lossless media) then the Electric and Magnetic Fields of Equations 11 and 12 are simplified as follows:

$$\underline{E}(\underline{r}, t) = \boxed{\underline{E}_o . \exp\left(j2\pi F_c t - j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right)} \tag{13}$$

$$\underline{H}(\underline{r}, t) = \underline{H}_o . \exp\left(j2\pi F_c t - j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}\right) \tag{14}$$

From EM-Waves to Array Manifold Vector

$E(r,t) = E_o \exp(-\alpha \cdot u^T r) \cdot \exp(j2\pi f_c t - j\frac{2\pi}{\lambda} u^T r)$

- At the array's reference point (Cartesian origin $[0,0,0]^T$) the electromagnetic wave equation (far field), given by Equ 13, becomes:

$$\underline{E}(\underline{r},t)|_{\underline{r}=\underline{0}_3} = \underline{E}(\underline{0}_3,t) = \underline{E}_o . \exp\left(j2\pi F_c t\right) \tag{15}$$

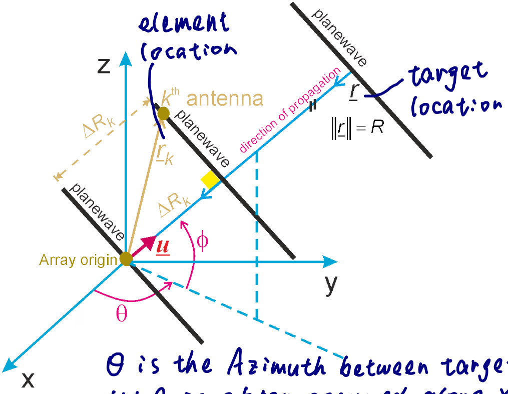
- Note that:
    - at distance $R=0$, the EM-wave is transmitted (Tx). That is,

$$\|\underline{E}(\underline{0}_3,t)\|\,|_{R=0} = \sqrt{P_{Tx}} \tag{16}$$

    - at distance $2R = c.t_{echo}$,the EM-wave has reached the Rx antenna and $\|\underline{E}(\underline{0}_3,t)\|$ is the square-root of the radar equation given by Equ 29. That is,

$$\|\underline{E}(\underline{0}_3,t)\|\,|_{2R=c.t_{echo}} = \sqrt{P_{Rx}} = \sqrt{\frac{P_{TX}}{(4\pi)^3}.\frac{\lambda}{R^2}\sqrt{RCS}} \tag{17}$$

- The array manifold vector $\underline{S}(\theta, \phi)$ is one of the most important antenna array and target parameters. In this section we will try to introduce and define this vector by using the previous EM-wave discussion.
- Consider that the k-th antenna of an array of N elements is not at the reference point $\underline{0}_3$ but it is in the vicinity of the reference point, which is represented by a small displacement $\underline{r}_k$ - as shown below:



θ is the Azimuth between target and x-axis.
ULA is often assumed along x-axis.

where the k-th antenna is located at the point $\underline{r}_k = [x_k, y_k, z_k]^T$

- In this case, and with reference to the previous figure, the electromagnetic wave will travel with the velocity of light c for a small time $\Delta\tau_k$ (+ve or -ve) which corresponds to the small distance $\Delta R_k = \underline{u}(\theta,\phi)^T \underline{r}_k$ between the ref. point and the k-th antenna's location. That is,

$$\Delta\tau_k = \frac{\Delta R_k}{c} = \frac{\underline{u}(\theta,\phi)^T \underline{r}_k}{c} \tag{18}$$

wave traveling time : ref → element k

- Thus, the Electric Field at the k**-th antenna**, $\underline{E}(\underline{r}_k, t)$, can be expressed as a function of the Electric Field at the origin, $\underline{E}(\underline{0}_3, t)$, as follows:

$$\begin{aligned}
\underline{E}(\underline{r}, t)|_{\underline{r}=\underline{r}_k} &= \underline{E}(\underline{r}_k, t) = \underline{E}(\underline{0}_3, t - \Delta\tau_k) = [\text{using Equ 15}] \\
&= \underline{E}_o . \exp\left(j2\pi F_c\left(t - \frac{\Delta R_k}{c}\right)\right) = \underline{E}_o . \exp\left(j2\pi F_c t - j\frac{2\pi}{\lambda}.\underline{u}^T \underline{r}_k\right) \\
&= \underline{E}(\underline{0}_3, t) . \exp\left(-j\frac{2\pi}{\lambda}.\underline{u}^T \underline{r}_k\right)
\end{aligned} \tag{19}$$

i.e. by using "scalar" rather than "vector" format,

$$\|\underline{E}(\underline{r}_k, t)\| = \|\underline{E}(\underline{0}_3, t)\| . \exp\left(-j\frac{2\pi}{\lambda}.\underline{u}^T \underline{r}_k\right) \tag{20}$$

- Equation 20 indicates that the Electric Field at the k-th antenna can be represented as

$$\left\| \text{E-Field at } k\text{-th antenna} \right\| = \left\| \underline{E}(\underline{0}_3, t) \right\| . \exp\left(-j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}_k \right)$$

- If we assume $\underline{E}(\underline{0}_3, t) = 1$ then we define the k-th antenna response

$$k\text{-th antenna response} \triangleq \left\| \text{E-Field at } k\text{-th antenna} \right\| \Big|_{\| \underline{E}(\underline{0}_3, t) \| = 1} = \boxed{\exp\left(-j\frac{2\pi}{\lambda} . \underline{u}^T \underline{r}_k \right)} \tag{21}$$

- Using Equ 21 for every array element, i.e. $\forall k : 1, 2, ..., N$, we can form the following vector:

$K(\theta,\phi) = \frac{2\pi}{\lambda} u(\theta,\phi)$

$$\begin{bmatrix} 1^{st} \text{ antenna response} \\ 2^{nd} \text{ antenna response} \\ \vdots \\ k^{\text{th}} \text{ antenna response} \\ \vdots \\ N^{\text{th}} \text{ antenna response} \end{bmatrix} = . \begin{bmatrix} \exp\left(-j.\underline{r}_1^T \underline{k}(\theta, \phi) \right) \\ \exp\left(-j.\underline{r}_2^T \underline{k}(\theta, \phi) \right) \\ \vdots \\ \exp\left(-j.\underline{r}_k^T \underline{k}(\theta, \phi) \right) \\ \vdots \\ \exp\left(-j.\underline{r}_N^T \underline{k}(\theta, \phi) \right) \end{bmatrix} = \underbrace{.\exp\left(-j[\overset{N\times 3}{\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N}]^T \overset{3\times 1}{\underline{k}(\theta, \phi)} \right)}_{\triangleq \underline{S}(\theta,\phi)}$$

relative geometry of elements

- The column vector $\underline{S}(\theta, \phi) \in C^{N \times 1}$ is the array manifold vector which, as stated before, is one of the most important parameters in antenna array systems.

Definition (Manifold Vector)

- The $(N \times 1)$ complex vector $\underline{S}$, i.e. $\underline{S} \in C^{N\times 1}$,

$$\underline{S} \triangleq \underline{S}(\theta,\phi) = \exp(-j\underbrace{[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N]}_{\triangleq \underline{\underline{\mathbf{r}}}}{}^T \underline{k}(\theta,\phi)) \tag{22}$$

$N \times 3$ $\quad$ 3×1

is defined as the array manifold vector and represents the response of the array for an electromagnetic signal of unity power (i.e. $\|\underline{E}(\underline{0}_3, t)\|^2 = 1$) and direction of propagation (θ, ϕ).

- Note that Equation 22 can be rewritten as follows:

$$\underline{S}(\theta,\phi) = \exp(-j\underbrace{[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N]}_{\triangleq \underline{\underline{\mathbf{r}}}}{}^T \underline{k}(\theta,\phi)) = \exp(-j[\underline{r}_x, \underline{r}_y, \underline{r}_z]\underline{k}(\theta,\phi)) = \exp(-j\underline{\underline{\mathbf{r}}}^T \underline{k}(\theta,\phi))$$

where

$$\underline{k}(\theta,\phi) = \frac{2\pi}{\lambda}.\underline{u}(\theta,\phi) = \text{wavenumber vector}$$

with

$$\begin{aligned} \underline{u}(\theta,\phi) &= [\cos\theta\cos\phi,\ \ \sin\theta\cos\phi,\ \ \sin\phi]^T && (23)\\ &= (3\times 1) \text{ real unit-vector in the direction of propagation } (\theta,\phi) \\ \|\underline{u}(\theta,\phi)\| &= 1 && (24) \end{aligned}$$

- In many cases the signals are assumed to be on the (x,y) plane (i.e. $\phi = 0^\circ$). In this case the manifold vector is simplified to

$$\begin{aligned} \underline{S}(\theta) &= \exp(-j\overbrace{[\underline{r}_x, \underline{r}_y, \underline{r}_z]}^{\triangleq \underline{\underline{\mathbf{r}}}}{}^T \underline{k}(\theta, 0^\circ)) \\ &= \exp(-j\frac{2\pi}{\lambda}(\underline{r}_x \cos\theta + \underline{r}_y \sin\theta)) \end{aligned} \quad (25)$$

2D target

- A popular class of arrays is that of linear arrays - where

$$\underline{r}_y = \underline{r}_z = \underline{0}_N$$

In this case, Equation 22 is simplified to

$$\underline{S}(\theta) = \exp(-j\frac{2\pi}{\lambda}\underline{r}_x \cos\theta) \quad (26)$$

linear array
on x-axis

The Structure of the Array Manifold Vector

- Remember: The vector $\underline{S}$ is known as
  - Array Manifold Vector, or
  - Array Response Vector, or
  - Source Position Vector (SPV)
- The vector $\underline{S}$ has a **profound mathematical structure** and is a function of a number of parameters such as Directions, carrier,etc

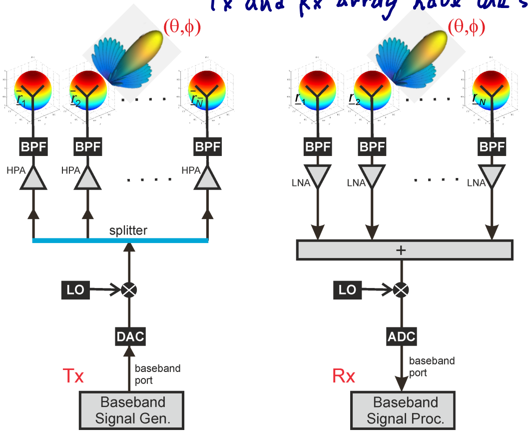
$k = \frac{2\pi}{\lambda} u$

$$\underline{S} \triangleq \underline{S}(\underbrace{\theta, \phi}_{uknown}, \underbrace{F_c, c, \underline{r}_1, \underline{r}_2, \underline{r}_3, \ldots, \underline{r}_N}_{known})$$

Theorem (Antenna Reciprocity Theorem)

- *Antenna characteristics are independent of the direction of energy flow.*
  - *The impedance & radiation pattern are the same when the antenna radiates a signal and when it receives it.*

Tx and Rx array have the same response.



Monostatic and Bistatic Radar

- Notation for Tx arrays:

$\overline{(.)}$	the bar at the top of a symbol denotes a Tx-parameter
$\bar{N}$	number of Tx array elements
$\bar{\phi}$	elevation angle (Direction-of-Departure)
$\bar{\theta}$	azimuth angle (Direction-of-Departure)

- The Tx-array is an array of $\overline{N}$ antennas with locations $\underline{\underline{\bar{\mathbf{r}}}}$

$$\underline{\underline{\bar{\mathbf{r}}}} = [\underline{\bar{r}}_1, \underline{\bar{r}}_2, ..., \underline{\bar{r}}_m, ..., \underline{\bar{r}}_{\overline{N}}] = [\underline{\bar{r}}_x, \underline{\bar{r}}_y, \underline{\bar{r}}_z]^T \ (3 \times \overline{N})$$

with $\underline{\bar{r}}_m$ denoting the location of the m^{th} Tx-antenna $\forall m = 1, 2, ..., \overline{N}$

- The theory presented in this topic is valid for both monostatic and bistatic radar.
  - monostatic radar: $(\theta, \phi) = (\bar{\theta}, \bar{\phi})$. However, $\underline{\underline{\bar{\mathbf{r}}}} \overset{?}{=} \underline{\underline{\mathbf{r}}}$ or $\underline{\underline{\bar{\mathbf{r}}}} \neq \underline{\underline{\mathbf{r}}}$
  - bistatic radar: $(\theta, \phi) \neq (\bar{\theta}, \bar{\phi})$. However, again, $\underline{\underline{\bar{\mathbf{r}}}} = \underline{\underline{\mathbf{r}}}$ or $\underline{\underline{\bar{\mathbf{r}}}} \neq \underline{\underline{\mathbf{r}}}$

Rx and Tx Manifold Vectors

- As we have $\bar{N}$ Tx antennas, this will produce a vector of $\overline{N}$ electromagnetic waves (one per antenna) which will propagate through our 3D physical space. The various "displacements" of these $\overline{N}$ electromagnetic waves at the Tx antennas are a function of
  - the Tx-array geometry, and
  - their direction of propagation.

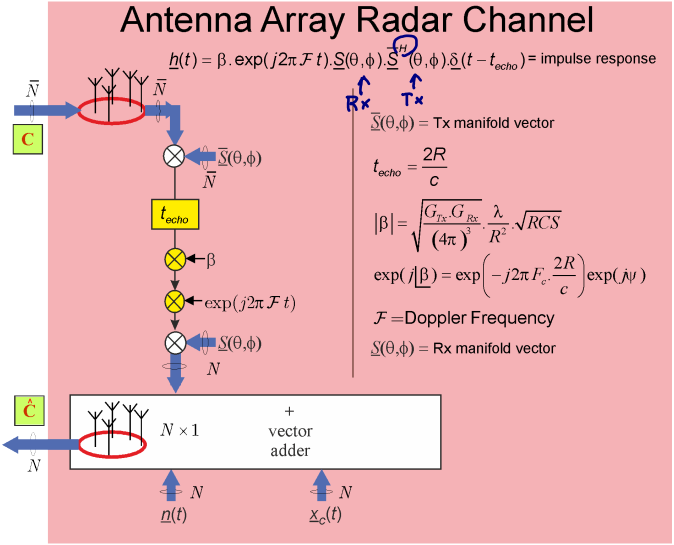
  These electromagnetic "displacements" for the target's direction $(\bar{\theta}, \bar{\phi})$ are modelled by the Tx manifold vector $\underline{\overline{S}}(\bar{\theta}, \bar{\phi}) \in \mathcal{C}^{\overline{N} \times 1}$ which is defined below:

$$\underline{\overline{S}} = \underline{\overline{S}}(\bar{\theta}, \bar{\phi}) = \exp\left(+j\underbrace{[\underline{\overline{r}}_1, \underline{\overline{r}}_2, \ldots, \underline{\overline{r}}_{\overline{N}}]}_{\triangleq \underline{\underline{\overline{\mathbf{r}}}}}^T \underline{k}(\bar{\theta}, \bar{\phi})\right) = \exp\left(+j[\underline{\overline{r}}_x, \underline{\overline{r}}_y, \underline{\overline{r}}_z]\underline{k}(\bar{\theta}, \bar{\phi})\right) \qquad (27)$$

$S = S_{Rx} = S_{Tx}^*$

- The transmitted electromagnetic waves will be reflected by the target and the echo will be received by the radar's Rx antenna array after a propagation time t_{echo}. Furthermore, the various "displacements" of the electromagnetic waves arriving at the Rx antenna array are modelled by the Rx manifold vector $\underline{S}(\theta, \phi) \in \mathcal{C}^{N \times 1}$, i.e.

$$\underline{S} = \underline{S}(\theta, \phi) = \exp\left(-j\underbrace{[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N]}_{\triangleq \underline{\underline{\mathbf{r}}}}^T \underline{k}(\theta, \phi)\right) = \exp\left(-j[\underline{r}_x, \underline{r}_y, \underline{r}_z]\underline{k}(\theta, \phi)\right) \qquad (28)$$



Antenna Array Radar Channel
h(t) = β.exp(j2π𝓕 t).S(θ,φ).S̄^H(θ,φ).δ(t − t_echo) = impulse response
Rx
Tx
S̄(θ,φ) = Tx manifold vector
t_echo = 2R/c
|β| = sqrt(G_Tx.G_Rx/(4π)^3) . λ/R^2 . sqrt(RCS)
exp(j∠β) = exp(−j2πF_c.2R/c) exp(jψ)
𝓕 = Doppler Frequency
S(θ,φ) = Rx manifold vector
N̄
C
N̄
S̄(θ,φ)
N̄
t_echo
β
exp(j2π𝓕 t)
S(θ,φ)
N
Ĉ
N × 1
+
vector
adder
N
N
N
n(t)
x_c(t)

The figure in the previous slide the delay t_{echo}, the path weight β and the effects of the target's Doppler frequency $\mathcal{F}$.

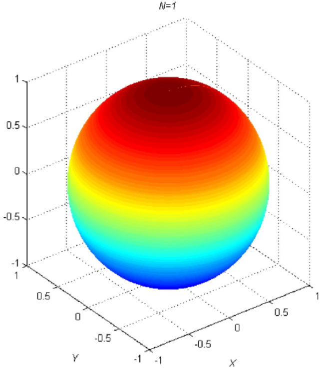
Note that the parameter β in this figure is expressed (with the help of the radar's equation) as a function of

$$\beta = \sqrt{\frac{G_{Tx}G_{Rx}}{(4\pi)^3}} \frac{\lambda}{R^2} \sqrt{RCS}$$

- the target's radar cross section (RCS),
- the antenna gains of the Tx and Rx, i.e. G_{Tx} and G_{Rx} respectively,
- the target's range R, and
- the power of the transmitted signal P_{Tx}.

Antenna Array: Space Response

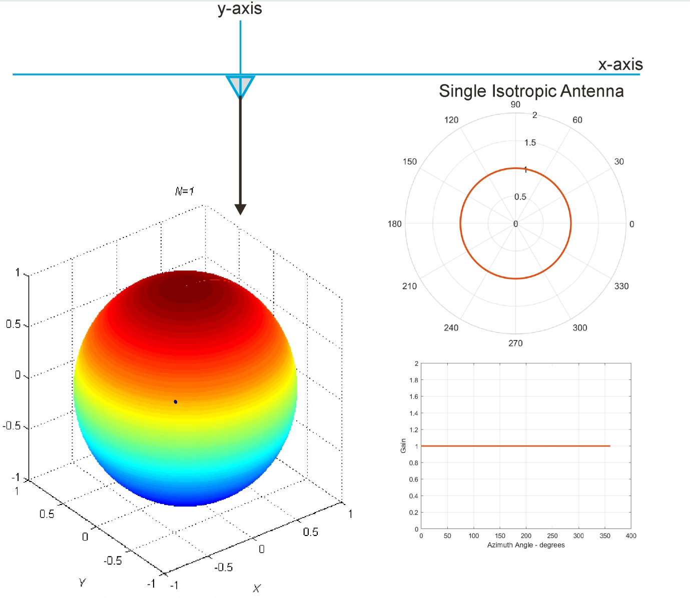
- Isotropic Antenna:



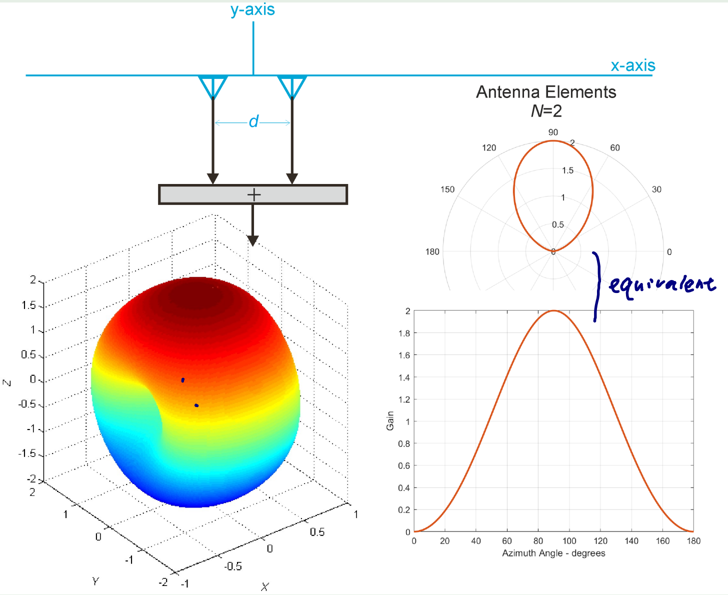
- Radar Equation of an Isotropic Antenna:

$$P_{Rx} = \frac{P_{TX}.\overset{1}{\overset{\downarrow}{G_{Tx}}}.\overset{1}{\overset{\downarrow}{G_{Rx}}}.\lambda^2}{(4\pi)^3.R^4}.RCS \tag{29}$$

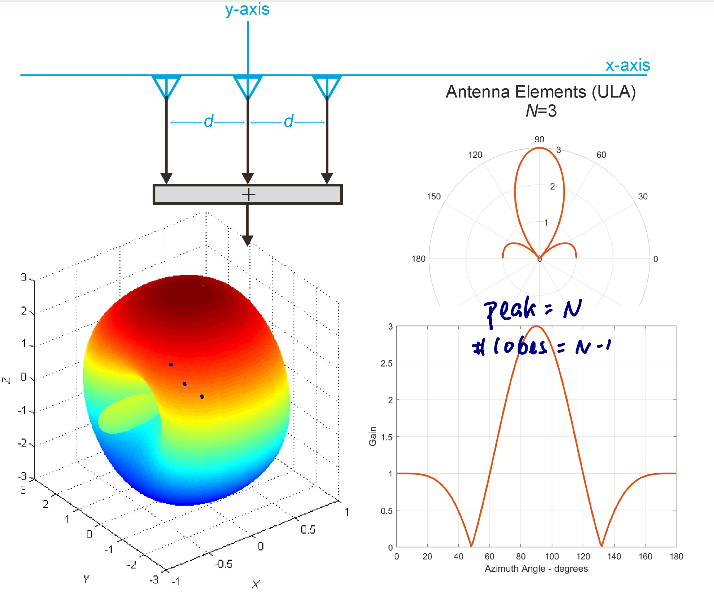
Example (Single Antenna: $N = 1$)



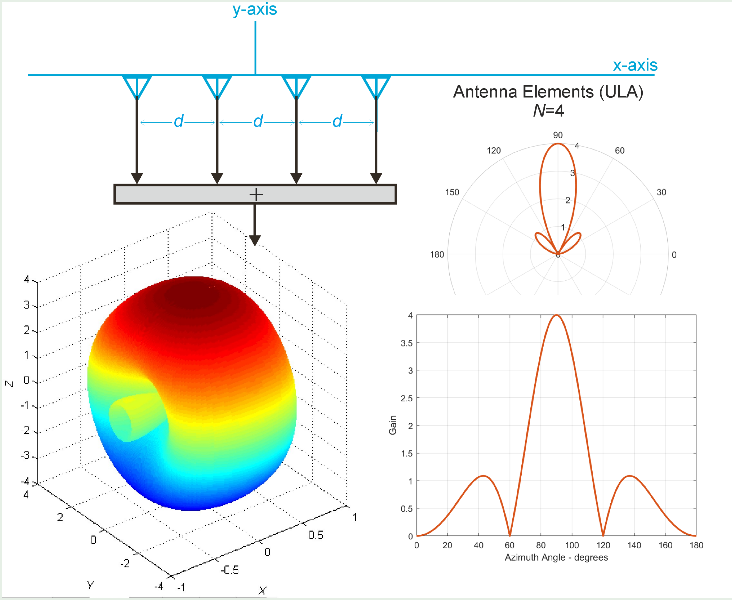
Example (Antenna Array of 2 Elements (always linear): $N = 2, d = \lambda/2$)



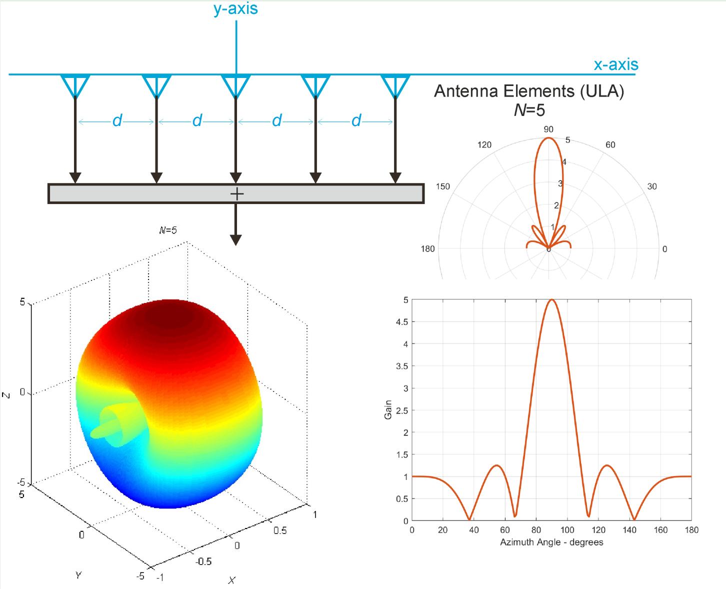
Example (Linear Antenna Array of 3 Elements: $N = 3, d = \lambda/2$)



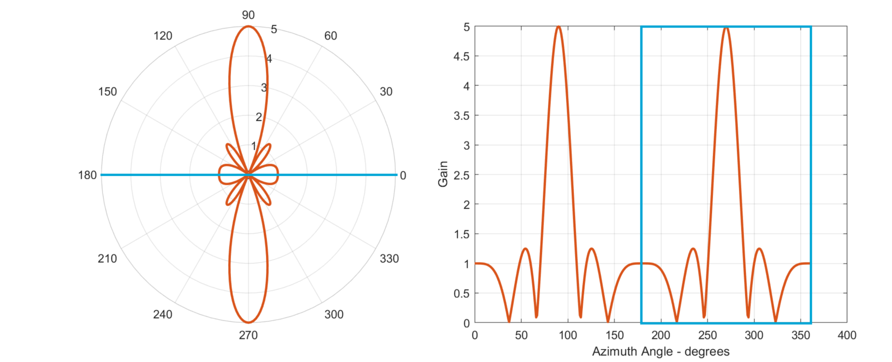
Example (Linear Antenna Array of 4 Elements: $N = 4$, $d = \lambda/2$)



Example (Linear Antenna Arrays of 5 elements: $N = 5, d = \lambda/2$)



Example (Linear Antenna Arrays of 5 elements: $N = 5, d = \lambda/2$)



Array Trivial Ambiguities

- Linear Array Geometries: only sees Azimuth (xy-plane)
  - "trivial" ambiguous directions: azimuth [0-180] and [180-360]

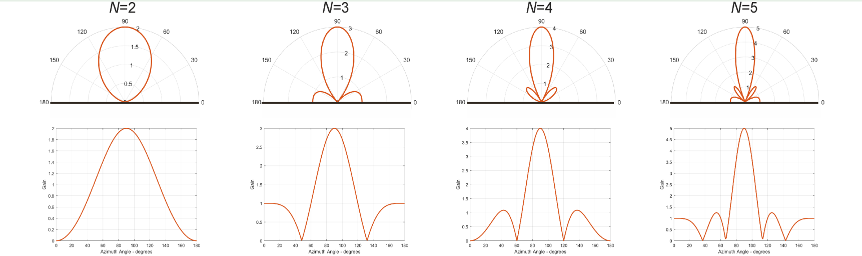
    same response

    (symmetric w.r.t. x-axis)
- Planar Array Geometries: also sees elevation (xyz-cube)
  - "trivial" ambiguous directions: elevation [0,+90] and [0,-90]

    (symmetric w.r.t. xy-plane)
- 3D-Array Geometries:
  - don't suffer from "trivial" ambiguities

Example (Summary - Array Pattern)

- ARRAY PATTERN: for arrays of $N = 1, 2, 3, 4$ and 5 sensors (Mainlobe at 90°, $d = \lambda/2$)



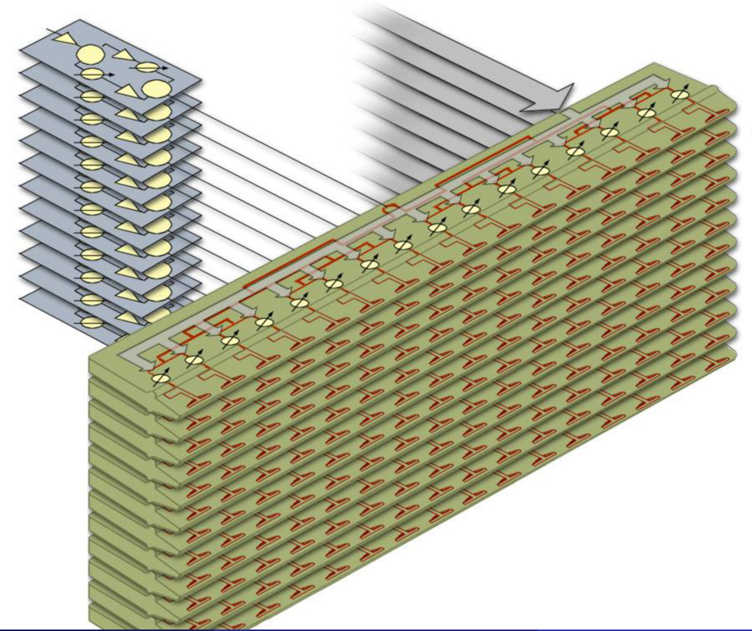
$$\underline{r}_x = \begin{bmatrix} -0.5d \\ +0.5d \end{bmatrix}^T = \begin{bmatrix} -1d \\ 0 \\ +1d \end{bmatrix}^T = \begin{bmatrix} -1.5d \\ -0.5d \\ +0.5d \\ +1.5d \end{bmatrix}^T = \begin{bmatrix} -1.5d \\ -0.5d \\ 0 \\ +0.5d \\ +1.5d \end{bmatrix}^T$$

Phased Array Radar Geometries

1. Linear Arrays
   - Advantage: simple arrangement;
   - Disadvantage: beam steering only in a single plane (the most popular is the [x,y]-plane);

2. Planar Array
   - These phased array antennas consist completely of single elements with a phase shifter per element. In general, the elements are arranged like a matrix, the flat arrangement of all elements forms the entire antenna.
   - Advantage: beam deflection possible in two planes
   - Disadvantage: a large number of phase shifters

Example (Linear and Planar Phased Antenna Array)



Array Pattern

Definition (Array Pattern)

- If the elements of an array of N antennas are weighted by a weight-vector $\underline{w}$

$$\underline{w} = [w_1, w_2, ..., w_N]^T \quad \text{beamformer} \tag{30}$$

then, the **array pattern** is defined as the function $g(\theta, \phi)$ such as

$$g(\theta, \phi) \triangleq \underline{w}^H \underline{S}(\theta, \phi); \forall\theta, \forall\phi \tag{31}$$

- N.B.:
  - The array pattern function, for a particular direction (θ, ϕ), provides the gain of the array for a signal with that direction.
  - The array pattern is a function of the manifold vector $\underline{S}(\theta, \phi)$ (and consequently of the array geometry and target's direction (θ, ϕ)) and any weight vector $\underline{w}$ (including any phase-shifters).

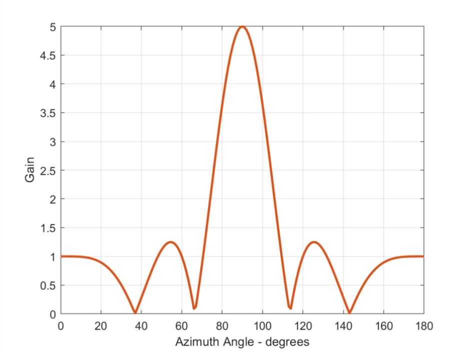
N.B.:

- No weights is equivalent to a weight vector of ones and the array pattern is

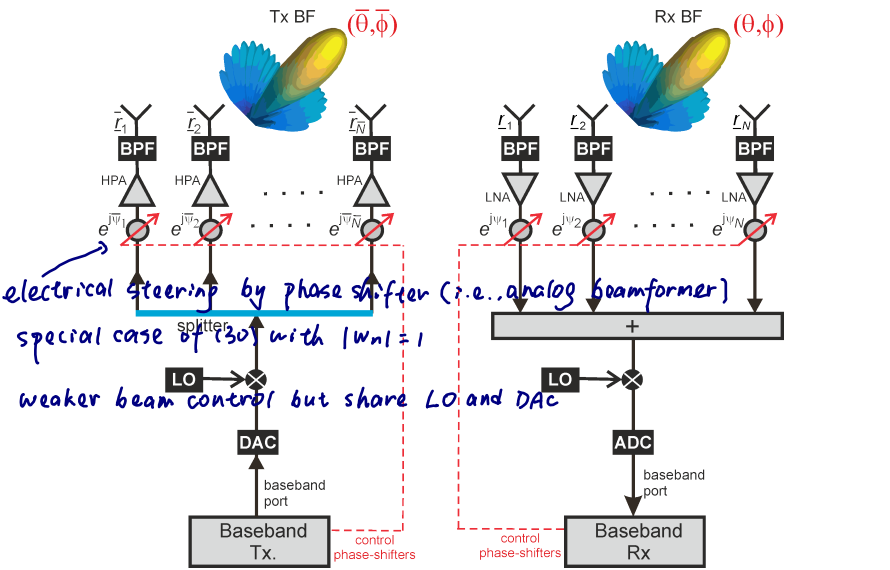
$$g(\theta,\phi) = \underline{1}_N^T \underline{S}(\theta,\phi)\,;\forall\theta,\forall\phi \qquad (32)$$

i.e. $\underline{w} = \underline{1}_N$

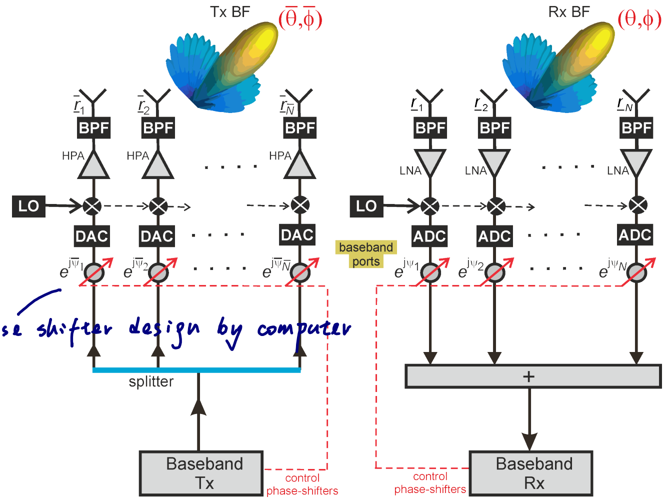
Example (Array Pattern, ULA of 5 elements, no weights.)



- no weights $\Longleftrightarrow$ $\underline{w} = \underline{1}_5$ (Note: this is known as the default Array Pattern)

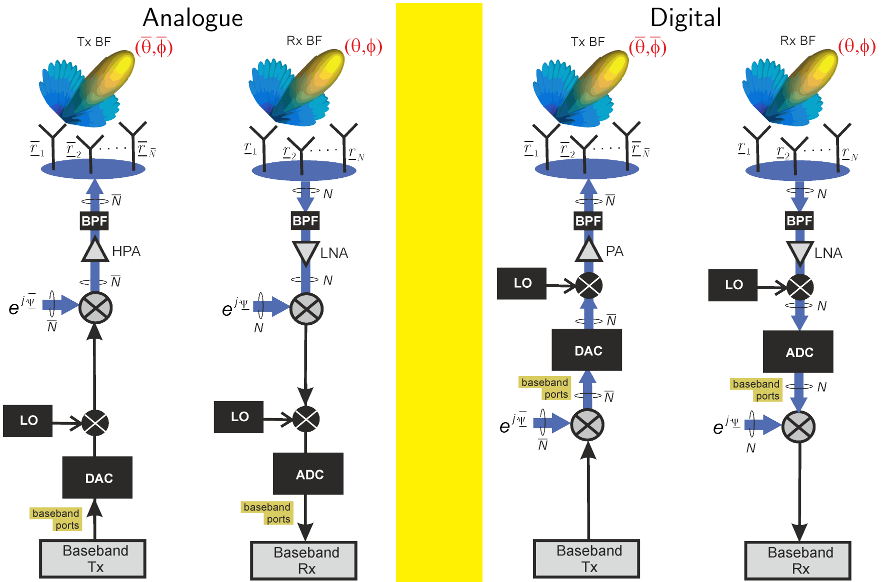

Tx BF
$(\bar{\theta},\bar{\phi})$
Rx BF
(θ,ϕ)
$\bar{r}_1$
$\bar{r}_2$
$\bar{r}_N$
r_1
r_2
r_N
BPF
HPA
LNA
$e^{j\bar{\psi}_1}$
$e^{j\bar{\psi}_2}$
$e^{j\bar{\psi}_N}$
$e^{j\psi_1}$
$e^{j\psi_2}$
$e^{j\psi_N}$
splitter
+
LO
DAC
ADC
baseband port
Baseband Tx.
Baseband Rx
control phase-shifters
electrical steering by phase shifter (i.e., analog beamformer)
special case of (30) with $|w_n| = 1$
weaker beam control but share LO and DAC

Monostatic Phased-Array Radar Architecture: Digital



phase shifter design by computer

Phased Array Radar - Compact Representation



Calculation of the Phase Shift in a Phased Array Radar

- For a general antenna array geometry with Cartesian coordinates $\underline{\underline{\mathbf{r}}} = [\underline{r}_1, \underline{r}_2, ..., \underline{r}_N] \in R^{3\times N}$: to **steer the main lobe** towards a specific (known) direction $(\theta_{\text{main-lobe}}, \phi_{\text{main-lobe}})$, a **vector of phase-shifters** should be used which should be equal to

$$\underline{\psi} \triangleq [\psi_1, \psi_2, ..., \psi_N]^T = \underline{\underline{\mathbf{r}}}^T \underline{k}_{\text{main}} \tag{33}$$

$$\underline{k}_{\text{main}} \triangleq \underline{k}(\theta_{\text{main}}, \phi_{\text{main}}) \tag{34}$$

- This implies that

$$\boxed{\underline{S}(\theta_{\text{main}}, \phi_{\text{main}})^* = \exp(j\underline{\psi})}$$

$$w^* = S(\theta_{main}, \phi_{main})$$

- In this case the array pattern will be

$$g(\theta, \phi) = w^H S(\theta, \phi)$$

$$g(\theta, \phi) = S(\theta_{\text{main}}, \phi_{\text{main}})^H \underline{S}(\theta, \phi), \forall\theta, \forall\phi \tag{35}$$

and if Equ 35 is evaluated at $\theta = \theta_{\text{main}}$, and $\phi = \phi_{\text{main}}$ then

$$g(\theta_{\text{main}}, \phi_{\text{main}}) = N = \text{max at main lobe direction}$$

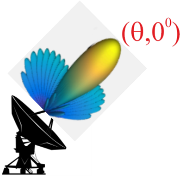
$S(\theta) = \exp(-j\frac{2\pi}{\lambda} r_x \cos\theta)$ by (26), pp 22

- For a linear array to **steer the main lobe** towards a specific (known) direction θ, the **vector of phase-shifters** should be:

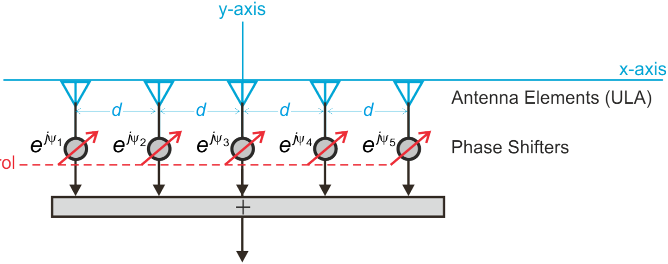
$\exp(j\psi) = S^*(\theta_{main}) = \exp(j\frac{2\pi}{\lambda} r_x \cos\theta)$

$$\underline{\psi}_{\text{main-}} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{bmatrix} = \frac{2\pi}{\lambda} \underline{r}_x \cos\theta \qquad (36)$$

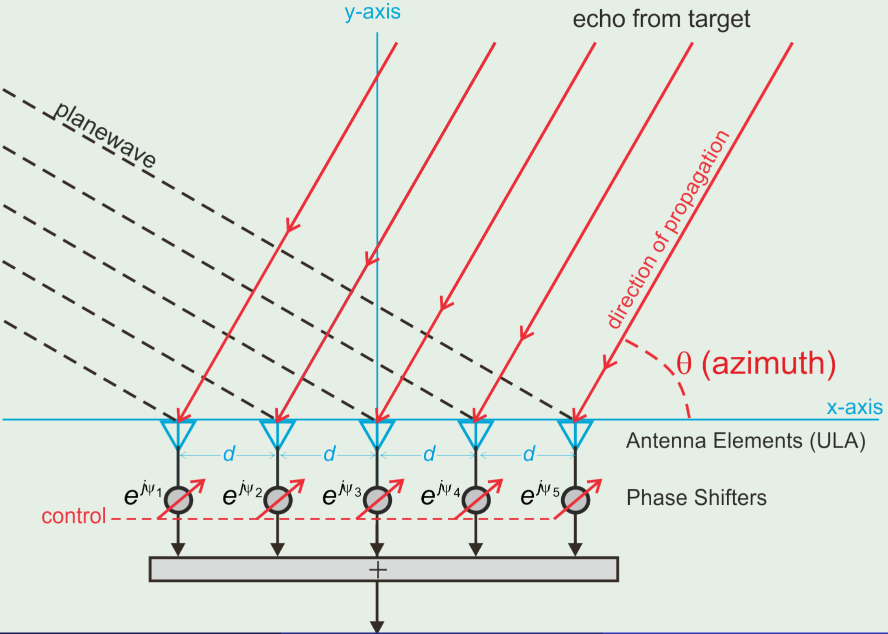
mechanical steering



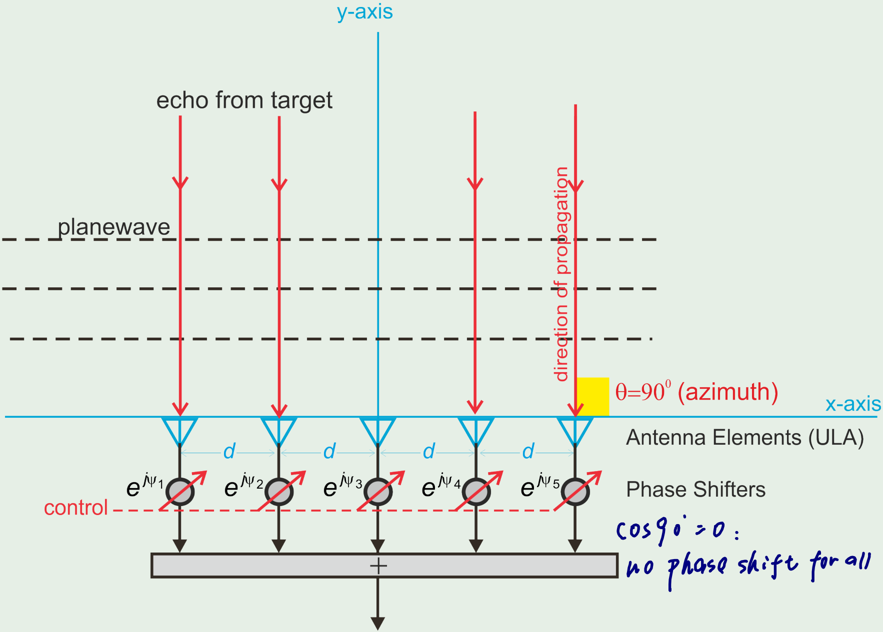
electrical steering (Rx phased-array)



Example (Equ 36 - general θ)

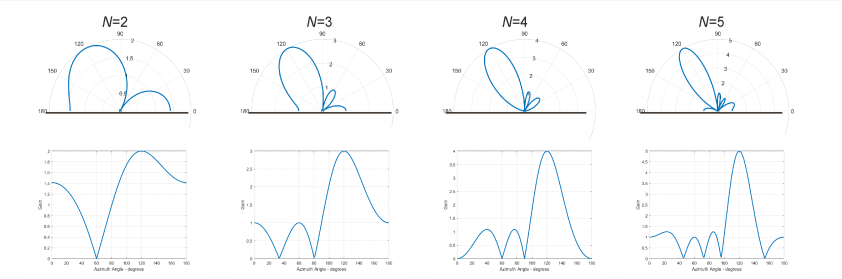


Example (Equ 36 - $\theta = 90^\circ$)



$\cos 90^\circ = 0$:
no phase shift for all

- ARRAY PATTERN: for arrays of $N = 1, 2, 3, 4$ and 5 sensors (Mainlobe at $120°$, $d = \lambda/2$)

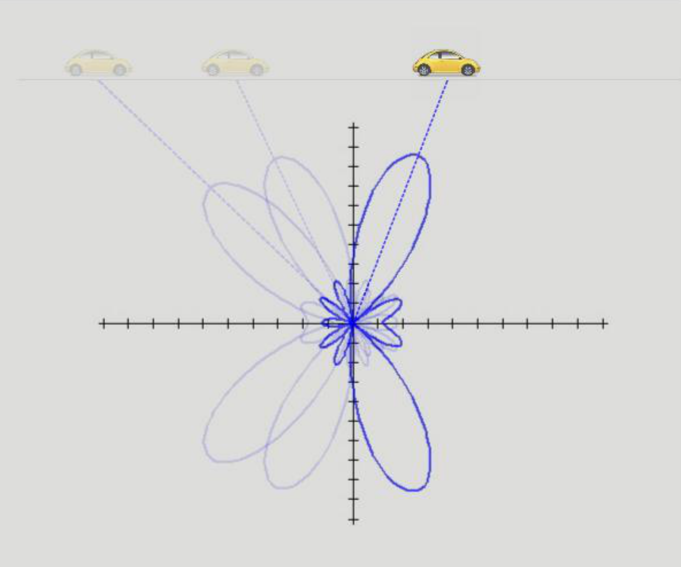


$$\underline{r}_x = \begin{bmatrix} -0.5d \\ 0.5d \end{bmatrix}^T = \begin{bmatrix} -1d \\ 0 \\ +1d \end{bmatrix}^T = \begin{bmatrix} -1.5d \\ -0.5d \\ 0.5d \\ 1.5d \end{bmatrix}^T = \begin{bmatrix} -1.5d \\ -0.5d \\ 0 \\ 0.5d \\ 1.5d \end{bmatrix}^T$$

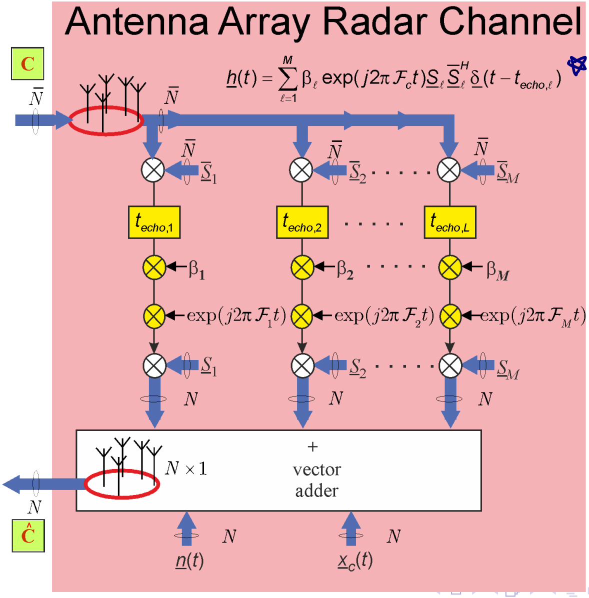
$$\exp(j\underline{\psi}) = \underline{S}^*(120°, 0) = \exp(+j\underline{r}^T\underline{k}(120°, 0°)) \tag{37}$$

$$(\text{simplified to}) = \exp(+j\pi\underline{r}_x \cos(120°)) \tag{38}$$

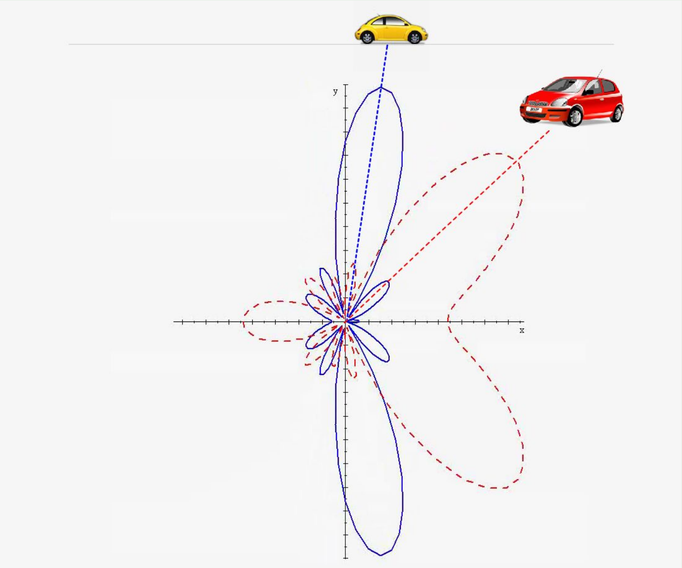
Example (Phased Array Radar - Tracking (ULA, N=5, d=1))



Multi-target Antenna Array Radar Channel Modelling

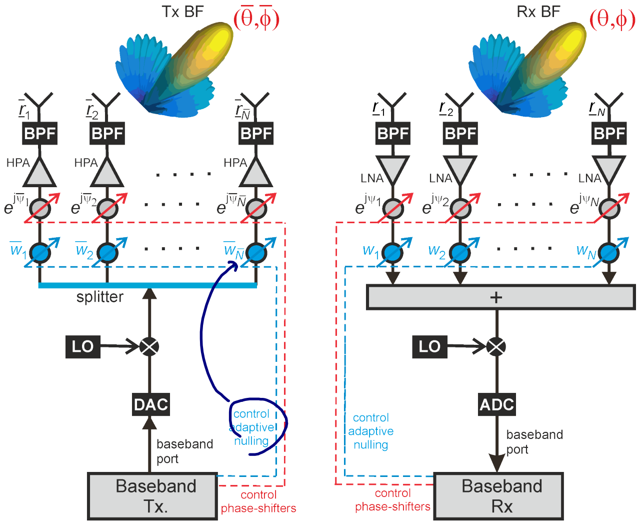


Example (Two-Beams - no nulling)

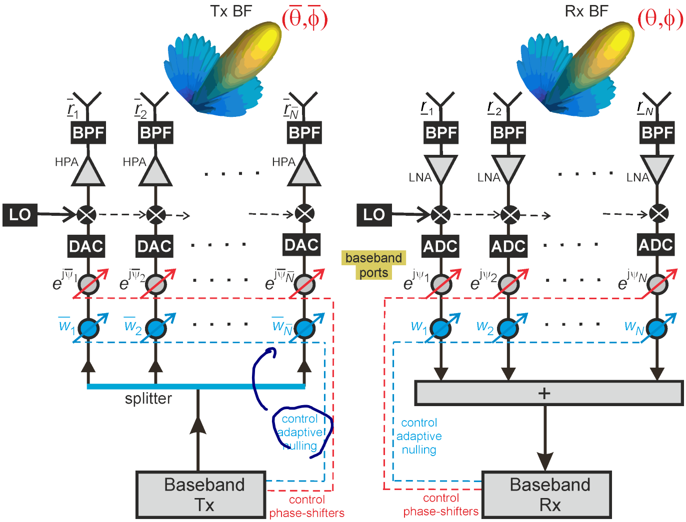


ULA
$N = 5$
$d = \frac{\lambda}{2}$

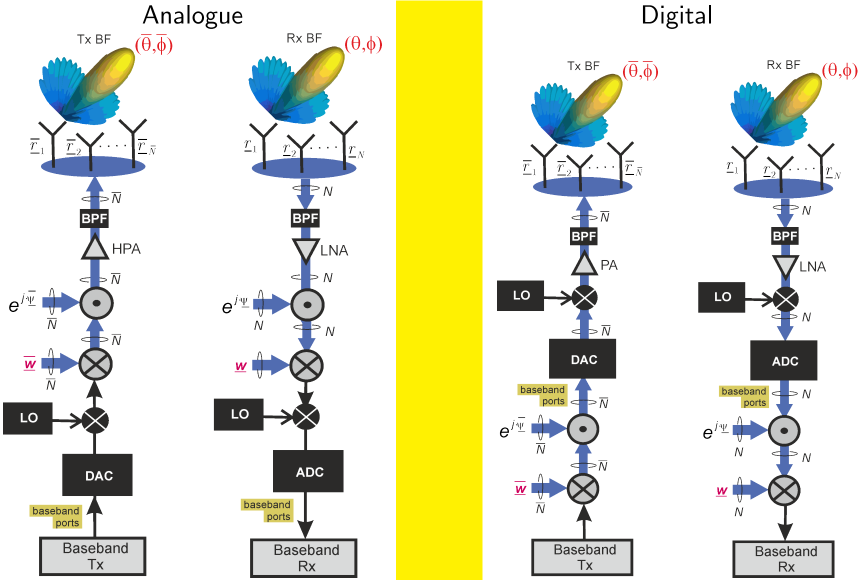
Adaptive Nulling (Inter-target Interference Suppression): Analogue



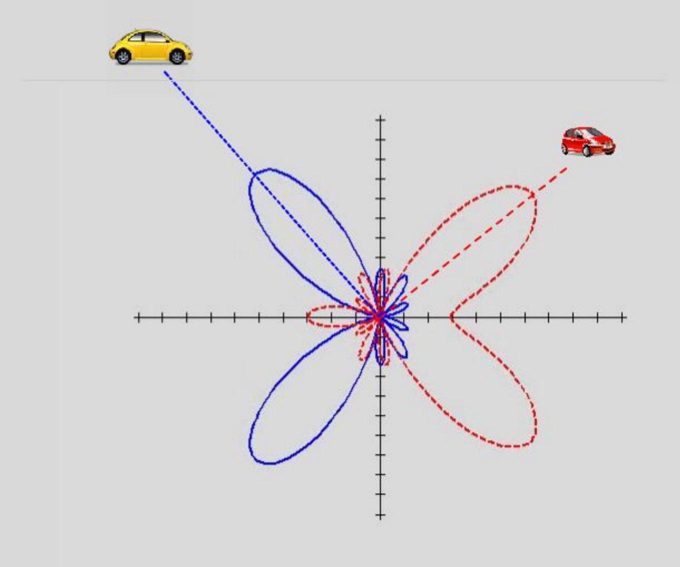
Adaptive Nulling (Inter-target Interference Suppression): Digital



Phased Array Radar: Compact Representation

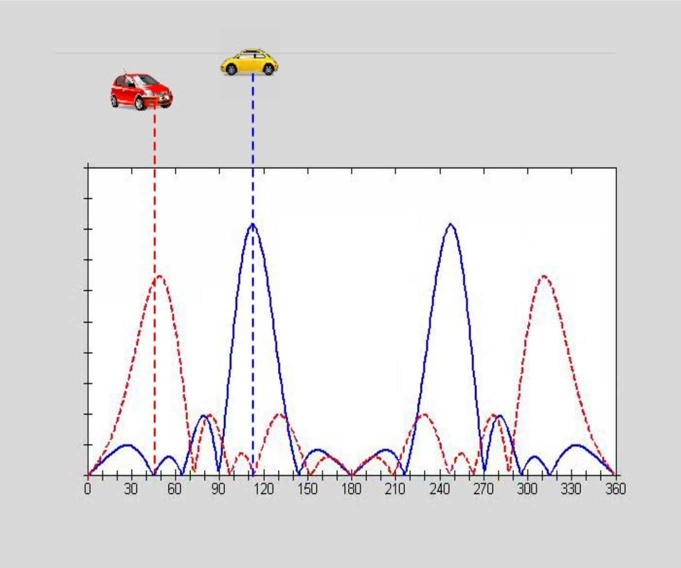


Example (Two-Beams - with nulling - Polar)



ULA
$N = 5$
$d = \frac{\lambda}{2}$

Example (Two-Beams - with nulling - Linear)



ULA
$N = 5$
$d = \frac{\lambda}{2}$

Example (ULA - Phase Shift)

- A phased array radar operates at a wavelength of $\lambda = 10$ cm.
- The antenna array geometry is ULA of 8 antennas with an inter-antenna spacing $d = 15$ cm and the first antenna is at the array reference point.
- If the angle to be steered should be $\theta_{main} = 50^\circ$, answer the following question: How large must the phase shift ψ_8 of phase shifter no. 8 (outside left) be in order to achieve this angle?

  $\psi_{main} = \frac{2\pi}{\lambda} r_x \cos\theta$
- Solution
  - We start with the determination of the phase shift x from one radiator to the next radiator.
  - Because of the angle function we need a calculator:

$$\begin{aligned}\Delta\psi_8 &= \frac{2\pi}{\lambda} \times 7d \times \cos(\theta_{main}) = \\ &= \frac{2\pi}{10} \times 7 \times 15 \times \cos(50\pi/180) \times 180/\pi \\ &= \mathrm{mod}(2429.7^\circ, 360^\circ) \\ &= 269.7^\circ\end{aligned}$$

  - Note-1: In digital implementation, no phase shifter will be able to realize this as accurately as this. With a 4-bit phase shifter, the phase shift can be made in steps of 11.25°. So in practice, a phase angle of $\psi_8 = 270^\circ$ will be used.
  - Note-2: With serial feeding, part of this phase shift is already realized by the delay time in the feed line. In practice, an individually calibrated table with the phase shift data for each desired angle exists in the computer for the antenna control (extra for each transmission frequency).