SPEECH PROCESSING
Speech Production Modelling

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Imperial College London

Learning objectives – Module 2

- Derive a theoretical model of how sound waves are affected by the vocal tract
- Describe a model for lip radiation
- Describe a model for the pulsating glottal waveform during voiced speech
- Assemble the components of a simple speech synthesizer

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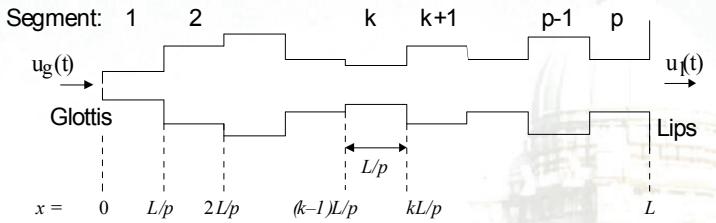
Tube model of the vocal tract

- We model the vocal tract as a tube that has p segments:

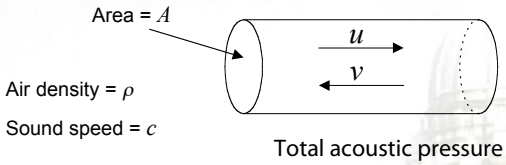


  - u_g and u_l are the volume flows of air at the glottis and lips respectively (measured in litres per second).

- Vocal tract is of length L (typically 15-17 cm in adults)
- Length of each segment is the distance sound travels in half a sample period $= 0.5cT$: 1.5 cm @ 11 kHz
  - c = speed of sound in air
    $\approx 20\sqrt{\text{absolute temperature}} \approx 340 \text{ m/s}$
  - T = sample period = $1/f_{samp}$
- Number of tube segments needed = $2L/cT \approx 0.001\, f_{samp}$

Sound Waves in a Tube

- Acoustic signal is the superposition of two waves: u in the forward direction and v in the reverse direction:



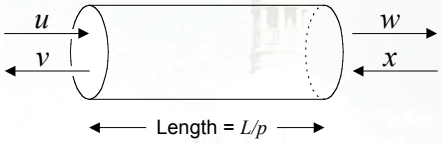
Total acoustic pressure = $(u+v) \times \rho c/A$

Total volume flow = $u–v$

- Exactly analogous to transmission lines:
  - Volume flow ≈ Current, Pressure ≈ Voltage
  - Acoustic Impedance of tube = $\rho c/A$
- Assumptions:
  - Sound waves are 1-dimensional: true for frequencies < 3 kHz whose wavelengths are long compared to the tube width
  - No frictional or wall-vibration energy losses

Segment Delays

- Time taken for sound to travel along one segment = L/cp



- Hence:

$$v(t) = x\left(t - \frac{L}{cp}\right) \quad \text{and} \quad u(t) = w\left(t + \frac{L}{cp}\right)$$

- Segment length chosen to correspond to half a sample period. If we take z-transforms, this time delay corresponds to multiplying by $z^{-½}$:

$$V(z) = z^{-1/2}X(z) \quad \text{and} \quad U(z) = z^{+1/2}W(z)$$

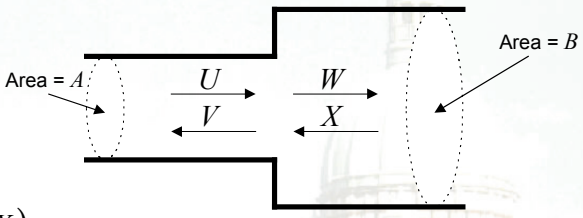
- In matrix form:

$$\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} z^{+1/2} & 0 \\ 0 & z^{-1/2} \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix} = z^{+1/2} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix}$$

Segment Junctions

- Flow continuity:

$$(U - V) = (W - X)$$



- Pressure continuity:

$$\frac{\rho c}{A}(U + V) = \frac{\rho c}{B}(W + X)$$

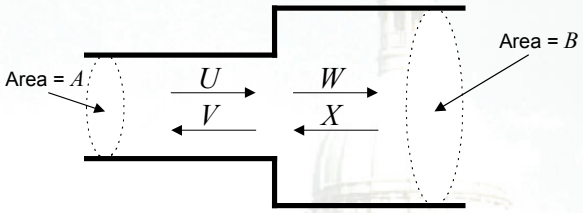
- Combine in matrix form:

$$\begin{pmatrix} 1 & -1 \\ B & B \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ A & A \end{pmatrix}\begin{pmatrix} W \\ X \end{pmatrix}$$

$$\begin{pmatrix} U \\ V \end{pmatrix} = \frac{1}{2B}\begin{pmatrix} B & 1 \\ -B & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ A & A \end{pmatrix}\begin{pmatrix} W \\ X \end{pmatrix}$$

$$= \frac{1}{2B}\begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix}\begin{pmatrix} W \\ X \end{pmatrix}$$

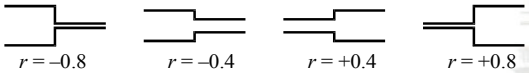
Reflection Coefficients



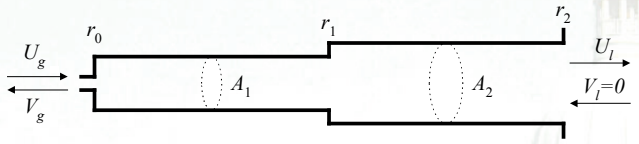
- Reflection coefficient $r = \dfrac{B - A}{B + A}$

$$\begin{pmatrix} U \\ V \end{pmatrix} = \frac{1}{2B}\begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix}\begin{pmatrix} W \\ X \end{pmatrix} = \frac{1}{1+r}\begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix}\begin{pmatrix} W \\ X \end{pmatrix}$$

- Reflection coefficients always lie in the range ±1:



2-segment Vocal Tract Tube Model



$$r_k = \frac{A_{k+1} - A_k}{A_{k+1} + A_k}$$

$$\frac{1}{1+r_2}\begin{pmatrix} 1 & -r_2 \\ -r_2 & 1 \end{pmatrix}\begin{pmatrix} U_l \\ 0 \end{pmatrix}$$

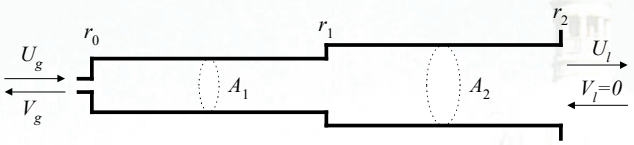
$$\frac{1}{1+r_1}\begin{pmatrix} 1 & -r_1 \\ -r_1 & 1 \end{pmatrix} \times z^{1/2}\begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \times$$

$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{1}{1+r_0}\begin{pmatrix} 1 & -r_0 \\ -r_0 & 1 \end{pmatrix} \times z^{1/2}\begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \times$$

Computing the 2-segement Model

- Assume $V_l = 0$: no sound reflected back into mouth
- Work backwards from lips towards glottis:
  - Junction: use the reflection matrix
  - Tube segment: use the delay matrix
- A_3 is large but not infinite: assumption of narrow tube breaks down at this point
- A_0 is approximately zero: area of glottis opening

Vocal Tract Transfer Function



Multiplying out the matrices gives:

$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{z^{+1}}{\prod_{k=0}^{2}(1+r_k)} \begin{pmatrix} 1+(r_0r_1+r_1r_2)z^{-1}+r_0r_2z^{-2} \\ -r_0-(r_1+r_0r_1r_2)z^{-1}-r_2z^{-2} \end{pmatrix} U_l$$

We can ignore V_g as it gets absorbed in the lungs.

The vocal tract transfer function is given by the ratio of U_l to U_g:

$$\frac{U_l}{U_g} = \frac{\prod_{k=0}^{2}(1+r_k)\times z^{-1}}{1+(r_0r_1+r_1r_2)z^{-1}+r_0r_2z^{-2}} = \frac{Gz^{-1}}{1+(r_0r_1+r_1r_2)z^{-1}+r_0r_2z^{-2}} = \frac{Gz^{-1}}{1-a_1z^{-1}-a_2z^{-2}}$$

Multi-segment Vocal Tract Model

- Using:

$$\frac{1}{1+r}\begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix} \times z^{1/2}\begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} = \frac{z^{1/2}}{1+r}\begin{pmatrix} 1 & -rz^{-1} \\ -r & z^{-1} \end{pmatrix}$$

- Multiplying together all the matrices for a p-segment vocal tract gives:

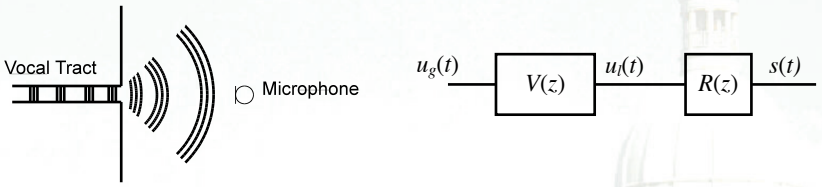
$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{z^{1/2\,p}}{\prod_{k=0}^{p}(1+r_k)} \prod_{k=0}^{p-1}\begin{pmatrix} 1 & -r_kz^{-1} \\ -r_k & z^{-1} \end{pmatrix} \times \begin{pmatrix} 1 \\ -r_p \end{pmatrix} U_l$$

- This results in a transfer function of the form:

$$V(z) = \frac{U_l}{U_g} = \frac{Gz^{-1/2\,p}}{1-a_1z^{-1}-a_2z^{-2}-\ldots-a_pz^{-p}}$$

  - G is a gain term
  - $z^{-½p}$ is the acoustic time delay along the vocal tract
  - The denominator represents a p^{th} order all-pole filter

Lip Radiation



- $R(z)$ is the transfer function between airflow at the lips and pressure at the microphone.
- For a lip-opening area of A, acoustic theory predicts a 1st-order high-pass response with a corner frequency of:

$$\frac{c}{\sqrt{4A}}\ \text{Hz} \approx 5\ \text{kHz}$$

- For f_{samp} < 20 kHz, a good approximation is:

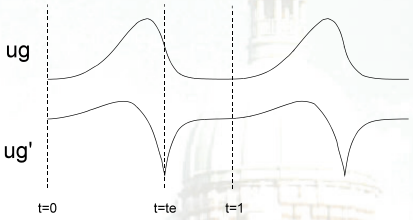
$$R(z) = \frac{S(z)}{U_l(z)} = 1 - z^{-1} \quad \Rightarrow \quad |R(z)| = 2\sin\left(\frac{\omega T}{2}\right)$$

Spectrum of the Glottal Waveform

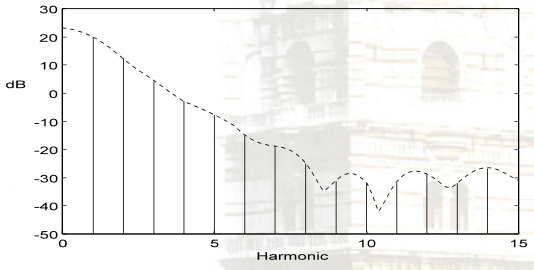
- "LF Model" (Liljencrants & Fant)

$$u_g'(t) = \begin{cases} e^{at}\sin(bt) & 0 \le t < t_e \\ c + de^{-ft} & t_e \le t < 1 \end{cases}$$

with $u_g(0) = u_g(1) = 0; u_g(t)$
and $u_g'(t)$ continuous at t_e

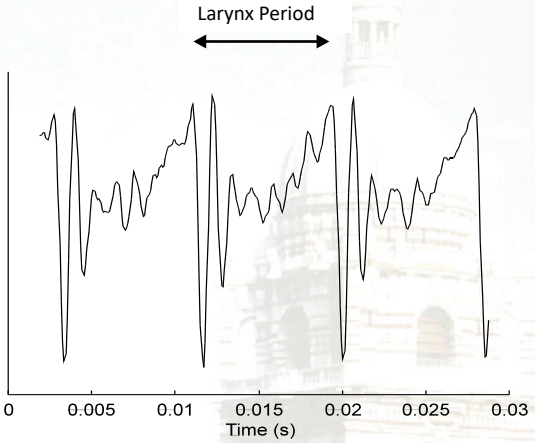


- Line Spectrum of u_g (approx –12 dB/octave):



Vowel Waveform

- Vowel /a/ from "part"



- Larynx frequency ~ 130 Hz
- First formant ~ 1 kHz
- There is not necessarily any relation between the larynx frequency and the vocal tract resonances.
- Resonances at a multiple of the larynx frequency will be louder (good for singers).

Vocal Tract Shape and Response

- Example: /a/ vowel ("part")