PROBLEM SHEET with Answers

EE3-27: Principles of Classical and Modern Radar

Contents

1 Pulse radar, round trip time

A radar sends a short pulse of microwave electromagnetic energy directed towards the moon. Some of the energy scatters off of the moon's surface and returns to the radar. What is the round trip time? If the target was an aircraft 150 nmi. distant, what is the round trip time?

Answer:

(a) $t_{echo} = \frac{2R}{c} = \frac{2\times384400\times10^3}{3\times10^8} = 2.5627\text{s}$

(b) $t_{echo} = \frac{2R}{c} = \frac{2\times150\times1852}{3\times10^8} = 1.85\text{ms}$

2 Pulse radar, resolution

A radar transmits a pulse of width of 2 microseconds. What is the closest 2 targets can be and still be resolved?

Answer:

$\Delta R = \frac{cT_p}{2} = \frac{3\times10^8\times2\times10^{-6}}{2} = 300\text{m}$

3 EM intensity

The intensity of a transmitted EM wave at a range of 500 m from the radar is 0.04 W/m^2. What is the intensity at 2 km?

Answer:

$\frac{P_{Tx}}{4\pi R^2} = 0.04\text{W/m}^2$

$\Longrightarrow P_{Tx} = 4\pi R^2 \times 0.04\text{W/m}^2 = 4\pi \times 500^2 \times 0.04 = 4\pi \times 10^4\text{W/m}^2$

[intensity at 2km]$=\frac{4\pi\times10^4}{4\pi(2\times10^3)^2} = \frac{10^4}{(2\times10^3)^2} = 2.5\text{mW/m}^2$

4 Distance to radar and round-trip

Find the distance to a radar target (in meters) for the following round-trip delay times:

(a) 12 μs
(b) 120 μs
(c) 1.258 ms
(d) 650 μs

Answer:

(a) $R = \frac{t_{echo}\times c}{2} = \frac{12\times10^{-6}\times3\times10^8}{2} = 1.8\text{km}$

(b) $R = \frac{t_{echo}\times c}{2} = \frac{120\times10^{-6}\times3\times10^8}{2} = 18\text{km}$

(c) $R = \frac{t_{echo}\times c}{2} = \frac{1.258\times10^{-3}\times3\times10^8}{2} = 188.7\text{km}$

(d) $R = \frac{t_{echo}\times c}{2} = \frac{650\times10^{-6}\times3\times10^8}{2} = 97.5\text{km}$

5 Distance to radar and round-trip

Find the delay times associated with the following target distances:

(a) 1 statute mile
(b) 1 km
(c) 100 km
(d) 250 statute miles
(e) 20 feet

Answer:

(a) $t_{echo} = \frac{2R}{c} = \frac{2\times1609.34}{3\times10^8} = 10.73\text{µs}$

(b) $t_{echo} = \frac{2R}{c} = \frac{2\times10^3}{3\times10^8} = 6.67\text{µs}$

(c) $t_{echo} = \frac{2R}{c} = \frac{2\times10^5}{3\times10^8} = 666.67\text{µs}$

(d) $t_{echo} = \frac{2R}{c} = \frac{2\times250\times1609.34}{3\times10^8} = 2.68\text{ms}$

(e) $t_{echo} = \frac{2R}{c} = \frac{2\times6.096}{3\times10^8} = 40.64\text{ns}$

6 Range resolution

What is the range resolution of a pulse radar system having the following characteristics?

	Pulse duration	frequency
(a)	1.0 μs	9.4 GHz
(b)	1.0 μs	34.4 GHz
(c)	0.1 μs	9.4 GHz
(d)	0.01 μs	9.4 GHz

Answer:

(a) $\Delta R = \frac{cT_p}{2} = \frac{3\times10^8\times1\times10^{-6}}{2} = 150\text{m}$

(b) $\Delta R = \frac{cT_p}{2} = \frac{3\times10^8\times1\times10^{-6}}{2} = 150\text{m}$

(c) $\Delta R = \frac{cT_p}{2} = \frac{3\times10^8\times0.1\times10^{-6}}{2} = 15\text{m}$

(d) $\Delta R = \frac{cT_p}{2} = \frac{3\times10^8\times0.01\times10^{-6}}{2} = 1.5\text{m}$

7 Planewave and distance from radar

How far from an antenna must one be positioned such that the wavefront whose source is at your position is estimated to be planewave at the antenna, for the following conditions:

	f	λ (m)	D (m)
(a)	10 GHz	-	1.0
(b)	-	0.1	1.0
(c)	10 GHz	-	0.1
(d)	3 GHz	-	1.0
(e)	3 GHz	-	7.5

where f is the radar carrier frequency, λ is the wavelength in meters, and D is the radar's antenna dimension (antenna aperture) in meters.

Answer:

(a) $R > \frac{2D^2F_c}{c} = \frac{2\times1^2\times10\times10^9}{3\times10^8} = 66.67\text{m}$

(b) $R > \frac{2D^2}{\lambda} = \frac{2\times1^2}{0.1} = 20\text{m}$

(c) $R > \frac{2D^2F_c}{c} = \frac{2\times0.1^2\times10\times10^9}{3\times10^8} = 0.67\text{m}$

(d) $R > \frac{2D^2F_c}{c} = \frac{2\times1^2\times3\times10^9}{3\times10^8} = 20\text{m}$

(e) $R > \frac{2D^2F_c}{c} = \frac{2\times7.5^2\times3\times10^9}{3\times10^8} = 1.125\text{km}$

8 CW radar, Doppler

You are traveling 75 mph in your new bright red Ferrari. A nearby policeman, using his hand held X-Band (frequency = 9,200 MHz) speed radar, transmits a CW signal from his radar, which then detects the Doppler shift of the echo from your car. Assuming that you are speeding directly towards his speed trap, how many Hz is the frequency of the received signal shifted by the Doppler effect? Is the Doppler shift positive or negative?

Answer:

$\mathfrak{f}_D = \frac{2v_r}{\lambda} = \frac{2v_rF_c}{c} = \frac{2\times33.528\times9200\times10^6}{3\times10^8} = 2.056\text{kHz}$

$\mathfrak{f}_D$ is positive

9 Unambiguous Doppler Shift

What is the maximum unambiguous Doppler shift that can be measured with a radar with a PRI of 0.25 milliseconds?

Answer:

$-\frac{\text{PRF}}{2} \le \mathfrak{f}_D \le +\frac{\text{PRF}}{2} \implies |\mathfrak{f}_D| = \frac{\text{PRF}}{2} = \frac{1}{2\,\text{PRI}} = \frac{1}{2\times0.25\times10^{-3}} = 2\text{kHz}$

10 Unambiguous Range

Find an expression for a radar's maximum unambiguous range in kilometers ifthe radar's PRF is x kHz.

Answer:

$R_{\max,u} = \frac{c\,\mathrm{PRI}}{2} = \frac{c}{2\,\mathrm{PRF}} = \frac{3\times 10^8}{2\cdot x\cdot 10^3} = \frac{1.5\times 10^5}{x}\mathrm{m}$

11 Unambiguous Range

A high-PRF radar has a pulse width of 1.0 μs and a duty factor of 20%. What is this radar's maximum unambiguous range?

Answer:

$\mathrm{PRI} = \frac{T_p}{20\%} = 5\mu\mathrm{s}$

$R_{\max,u} = \frac{c\,\mathrm{PRI}}{2} = \frac{3\times 10^8\times 5\times 10^{-6}}{2} = 750\mathrm{m}$

12 PSD of a Monostatic Pulse Radar

Consider a monostatic pulse radar where the Power Spectral Density of the bandpass transmitted signal $s(t)$ is given by the following equation

$$\mathrm{PSD}_s(f) = 10^{-6}.\mathrm{comb}_{10^3}\left\{\mathrm{sinc}^2\left(10^{-6}f - 10^3\right) + \mathrm{sinc}^2\left(10^{-6}f + 10^3\right)\right\}$$

Find

(a) the pulse duration T_p

(b) the pulse amplitude A

(c) the carrier frequency F_c

(d) the pulse repetition interval (PRI)

Answer:

$\mathrm{PSD}_s(f) = \frac{A^2.T_p^2.\,\mathrm{PRF}^2}{4}.\mathrm{comb}_{\mathrm{PRF}}\left\{\mathrm{sinc}^2\left(T_p\left(f - F_c\right)\right) + \mathrm{sinc}^2\left(T_p\left(f + F_c\right)\right)\right\}$

$\mathrm{PRF} = 10^3\mathrm{Hz} \Longrightarrow \mathrm{PRI} = \frac{1}{\mathrm{PRF}} = 1\mathrm{ms}$

$T_p = 10^{-6} = 1\mu\mathrm{s} \Longrightarrow F_c = \frac{10^3}{10^{-6}} = 1\mathrm{GHz}$

$\frac{A^2.T_p^2.\,\mathrm{PRF}^2}{4} = 10^{-6} \Longrightarrow A = \frac{\sqrt{4\times 10^{-6}}}{\mathrm{PRF}\times T_p} = \frac{\sqrt{4\times 10^{-6}}}{10^3\times 10^{-6}} = 2\mathrm{V}$

13 Complex Target

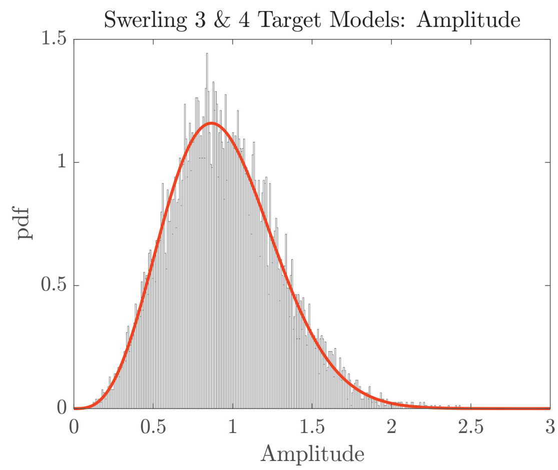
With reference to the Swerling target amplitude models, consider a monostatic pulse radar operating in the presence of a target having an average RCS of 1m^2. If this target has one scatterer which is much greater than other smaller ones,

(a) plot in MATLAB this Swerling probability density function versus the amplitude (in volts) of the backscatter from this target;

(b) generate 10000 samples corresponding to this target;

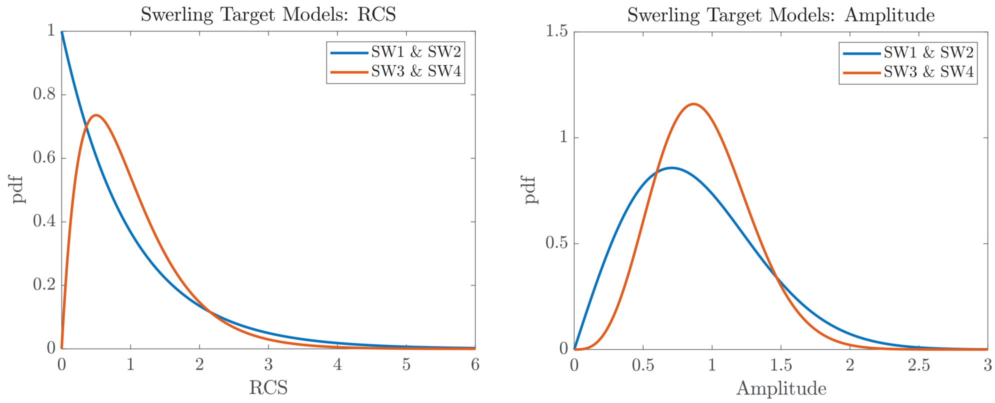
(c) plot the pdf of the 10000 samples generated in part-(b);

(d) compare the pdf in part-(a) with the pdf in part-(c)



		pdf	random number	Comments
1	Gaussian Noise	`normpdf(x,mu,sigma)`	`normrnd(mu,sigma)`	
2	Chi-Square	`chi2pdf(x,DOF)`	`chi2rnd(DOF)`	
3	Log Normal	`lognpdf(x,mu,sigma)`	`lognrnd(mu,sigma)`	
4	Weibull	`wblpdf(x,A,B)`	`wblrnd(A,B)`	A: scale; B: shape.
5	SW1&2: RCS σ	`exppdf(x,mu)`	`exprnd(mu)`	$\mathtt{x} = \sigma$; $\mathtt{mu} = \overline{\sigma}$
6	SW1&2: Amp a	`raylpdf(x,B)`	`raylrnd(B)`	$\mathtt{x} = \text{a}$; $\mathtt{B} = \sqrt{\frac{\overline{\sigma}}{2}}$
7	SW3&4: RCS σ	$\frac{4}{\mathtt{mu}} \cdot \mathtt{chi2pdf}\left(\frac{4}{\mathtt{mu}} \cdot \mathtt{x}, \mathtt{DOF}\right)$	$\frac{\mathtt{mu}}{4} \cdot \mathtt{chi2rnd(DOF)}$	$\mathtt{x} = \sigma$; $\mathtt{mu} = \overline{\sigma}$; $\mathtt{DOF} = 4$
8	SW3&4: Amp a	$\frac{4}{\mathtt{mu}^4} \cdot \mathtt{x}^2 \cdot \mathtt{raylpdf}\left(\frac{2}{\sqrt{\mathtt{mu}}} \cdot \mathtt{x}, \mathtt{B}\right)$		$\mathtt{x} = \text{a}$; $\mathtt{mu} = \overline{\sigma}$; $\mathtt{B} = 1$

`x`: variable; `mu`: mean; `sigma`: standard deviation



14 Linear FM CW Radar: Beat Frequency and Target Range

Consider a CW radar (Linear FM), with modulation bandwidth 64 MHz and 2 ms modulation period, operating in the presence of a target. If the beat frequency is 4MHz find the target's range.

Answer:

$\Delta F = 64\text{MHz}; T_m = 2\text{ms}; f_{beat} = 4\text{MHz}$

$\frac{t_{echo}}{T_m} = \frac{f_{beat}}{\Delta F} \implies t_{echo} = T_m \times \frac{f_{beat}}{\Delta F} \implies \frac{2R}{c} = T_m \times \frac{f_{beat}}{\Delta F} \implies$

$R = \frac{c}{2} T_m \times \frac{f_{beat}}{\Delta F} = \frac{3\times10^8\times2\times10^{-3}}{2} \times \frac{4\times10^6}{64\times10^6} = 18.750\text{km}$

15 Sawtooth FM CW Radar: Beat Frequ and Target Range

The bandwidth and modulation period of a saw-tooth FM CW radar are 70-MHz and 1 ms respectively. If a target at 3km from the radar is detected find the beat-frequency.

Answer:

$\Delta F = 70\text{MHz}; T_m = 1\text{ms}; R = 3\text{km}$

$\frac{t_{echo}}{T_m} = \frac{f_{beat}}{\Delta F} \implies$

$f_{beat} = \Delta F \times \frac{t_{echo}}{T_m} = \Delta F \times \frac{2R}{cT_m} = 70 \times 10^6 \times \frac{2\times3\times10^3}{3\times10^8\times1\times10^{-3}} = 1.4\text{MHz}$

16 Bistatic Radar Equation

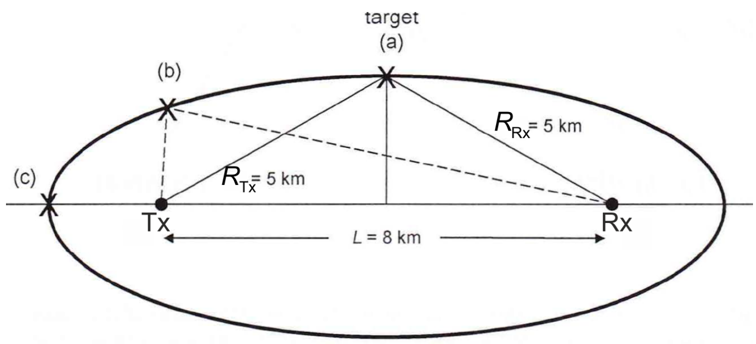
A target lies equidistant between transmitter and receiver such that $R_{Tx} + R_{Rx} =$ 10km. The baseline is equal to 8km (i.e. $L = 8$ km). Assuming that the Tx and Rx antennas are omnidirectional, and that all other parameters remain unchanged, what is the power of the received echo signal, relative to that

(a) when the target is at point (a)

(b) when it is instead at point (b), and

(c) when the target is at point (c)?

Note that all three positions are lying on the constant range sum ellipse.



Answer:

Radar Equation:

$P_{Rx} = \frac{P_{Tx}G_{Tx}G_{Rx}\lambda^2\text{RCS}}{(4\pi)^3} \times \frac{1}{R_{Tx}^2 R_{Rx}^2} = \text{const} \times \frac{1}{R_{Tx}^2 R_{Rx}^2}$; $R_{Tx} + R_{Rx} = 10\text{km}; L = 8\text{km}$

(a) $R_{Tx} = R_{Rx} = 5\text{km} \Longrightarrow$

$P_{Rx}^{(a)} = \text{const} \times \frac{1}{(5\text{k})^2(5\text{k})^2}$

(b) $L^2 + R_{Tx}^2 = R_{Rx}^2 \Longrightarrow R_{Tx} = 1.8\text{km}; R_{Rx} = 8.2\text{km} \Longrightarrow$

$P_{Rx}^{(b)} = \text{const} \times \frac{1}{(1.8\text{k})^2(8.2\text{k})^2}$

(c) $R_{Tx} = 1\text{km}; R_{Rx} = 9\text{km} \Longrightarrow$

$P_{Rx}^{(c)} = \text{const} \times \frac{1}{(1\text{k})^2(9\text{k})^2}$

$P_{Rx}^{(a)} < P_{Rx}^{(b)} < P_{Rx}^{(c)} \Longrightarrow P_{Rx}^{(a)} = 0.3486 P_{Rx}^{(b)} = 0.1296 P_{Rx}^{(c)}$

17 Bistatic Radar: Doppler Frequency

Consider a bistatic radar with 8km baseline, omindirection antennas at both Tx and Rx, and wavelength of $\lambda = 10\text{cm}$. If there is a target which lies on an isorange contour with $R_{Tx} + R_{Rx} = 10\text{km}$ and moves with velocity (v) of 40km/h, estimate the Doppler frequency for the following cases:

Answer:

$v_{Tx} = v_{Rx} = 0; v \neq 0$

using Equation (9) (slide 14) $\mathfrak{f}_D = \frac{2v}{\lambda}\cos\psi\cos\frac{\beta}{2}$; or table (slide 14)

(a) $v \perp$bisector $\psi = \pm 90°$

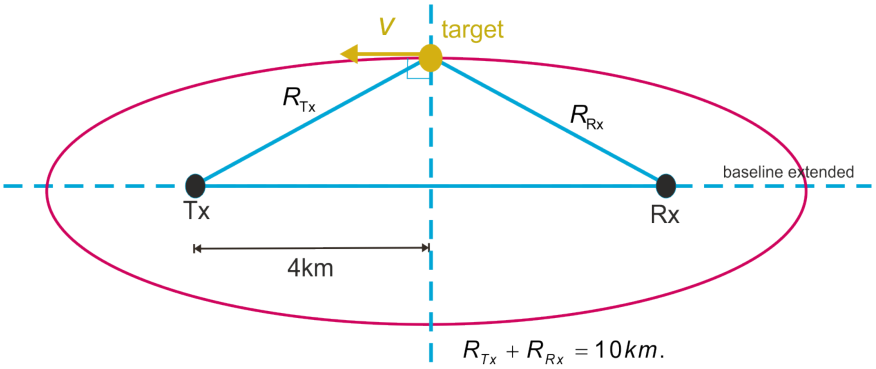
$\mathfrak{f}_D = \frac{2v}{\lambda}\cos\psi\cos\frac{\beta}{2} = \frac{2v}{\lambda}\cos(\pm 90°)\cos\frac{\beta}{2} = 0$

(b) $v \Rightarrow$bisector $\frac{\beta}{2} = \arcsin\frac{4\times 10^3}{5\times 10^3} = 0.9273$ rad

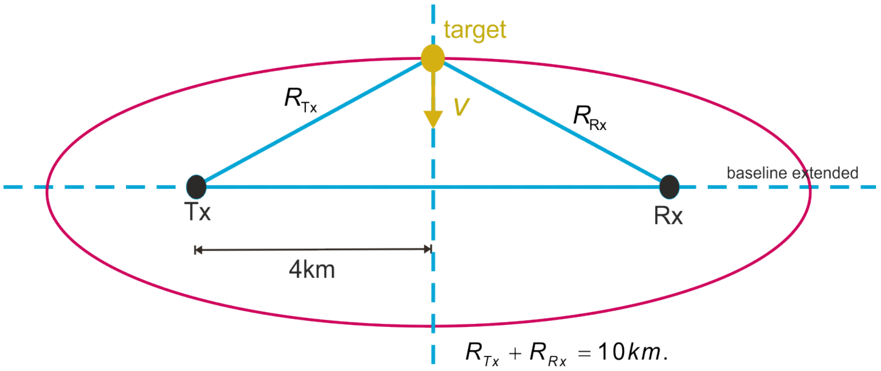
$\mathfrak{f}_D = \frac{2v}{\lambda}\cos\frac{\beta}{2} = \frac{2\times 40\text{km/h}}{0.1} = \frac{2\times 11.11\text{m/s}}{0.1} \times 0.6 = 133.32\text{Hz}$

(c) $\beta = 0°; \psi = \pm 90° \Longrightarrow \mathfrak{f}_D = 0$

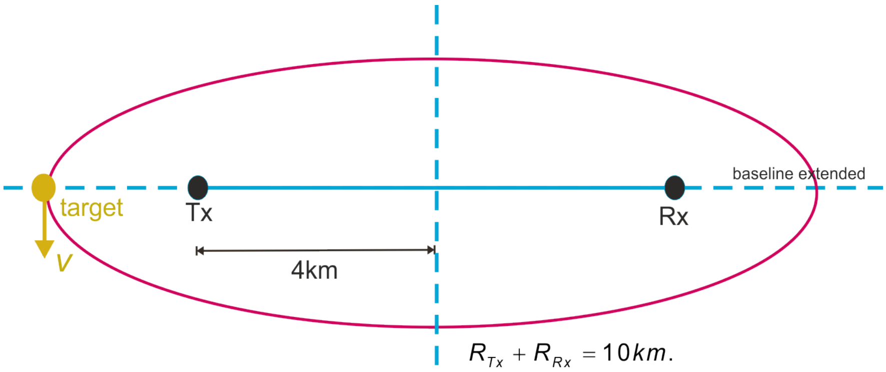
(d) $\beta = 0°; \psi = 0° \Longrightarrow \mathfrak{f}_D = \frac{2v}{\lambda} = \frac{2\times 11.11\text{m/s}}{0.1} = 222.20\text{Hz}$


V
target
R_{Tx}
R_{Rx}
baseline extended
Tx
Rx
4km
$R_{Tx} + R_{Rx} = 10km.$

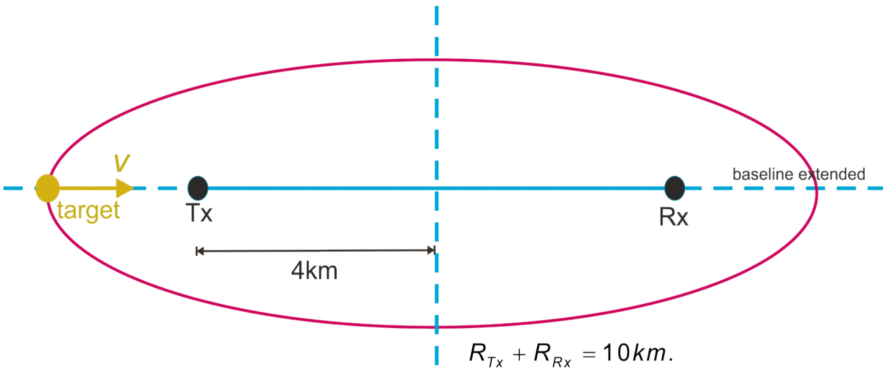
(a)


target
R_{Tx}
V
R_{Rx}
baseline extended
Tx
Rx
4km
$R_{Tx} + R_{Rx} = 10km.$

(b)


baseline extended
target
Tx
Rx
V
4km
$R_{Tx} + R_{Rx} = 10km.$

(c)


V
baseline extended
target
Tx
Rx
4km
$R_{Tx} + R_{Rx} = 10km.$

(d)

18 Phased-Array Radar: ULA, Beamsteering 50°

Consider a phased-array radar that operates at a wavelength of $\lambda = 10$cm and employs 7 antennas with an inter-antenna spacing $d = 15$ cm.

(a) If the reference point (origin) of the array is the array centroid and the angle to be steered is $\theta_{steer} = 50°$, find
- the phase shift ψ_7 of 7^{th} phase shifter
- the phase shift ψ_4 of the 4^{th} phase shifter

(b) If the reference point of the array changes to be the location of the 1st antenna and the angle to be steered is still $\theta_{steer} = 50°$, find the vector of the phase shifters $\underline{\psi}$

Answer:

$$\underline{\psi}\left(\theta_{steer}\right) = \mathbf{r}^T \underline{k}\left(\theta_{steer}\right) = \left[\underline{r}_x, \underline{0}_7, \underline{0}_7\right] \frac{2\pi}{\lambda} \underline{u}\left(\theta_{steer}, 0\right) = \underline{r}_x \frac{2\pi}{\lambda} \cos\theta_{steer}$$

(a) the reference point is the array centroid

$$\mathbf{r} = \begin{bmatrix} -3d & -2d & -d & 0 & d & 2d & 3d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \underline{r}_x^T \\ \underline{r}_y^T \\ \underline{r}_z^T \end{bmatrix}$$

- $\psi_7\left(50°\right) = 3d \times \frac{2\pi}{\lambda} \times \cos 50° = 3 \times 0.15 \times \frac{2\pi}{0.1} \times \cos 50° = 1041.3°$
  $\psi_7\left(50°\right) = \mathrm{mod}\left(1041.3°, 360°\right) = 321.32°$
- $\psi_4\left(50°\right) = 0°$

(b) the reference point is the 1st antenna

$$\mathbf{r} = \begin{bmatrix} 0 & d & 2d & 3d & 4d & 5d & 6d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \underline{r}_x^T \\ \underline{r}_y^T \\ \underline{r}_z^T \end{bmatrix}$$

$$\begin{aligned} \underline{\psi}\left(50°\right) &= \underline{r}_x \frac{2\pi}{\lambda} \cos 50° \\ &= \left[0°, 347.11°, 334.21°, 321.32°, 308.42°, 295.53°, 282.63°\right]^T \end{aligned}$$

19 Phased-Array Radar: Array Pattern

Consider a phased-array radar that operates at a wavelength of $\lambda = 10$cm, employs a ULA of 9 antennas with half-wavelength inter-antenna spacing and has its mainlobe steered at 45°. Using Matlab find the vector of its phase-shifters and plot the array pattern in both linear and polar forms when the array reference point is

(a) at the location of the first antenna

(b) at the array centroid.

Answer:

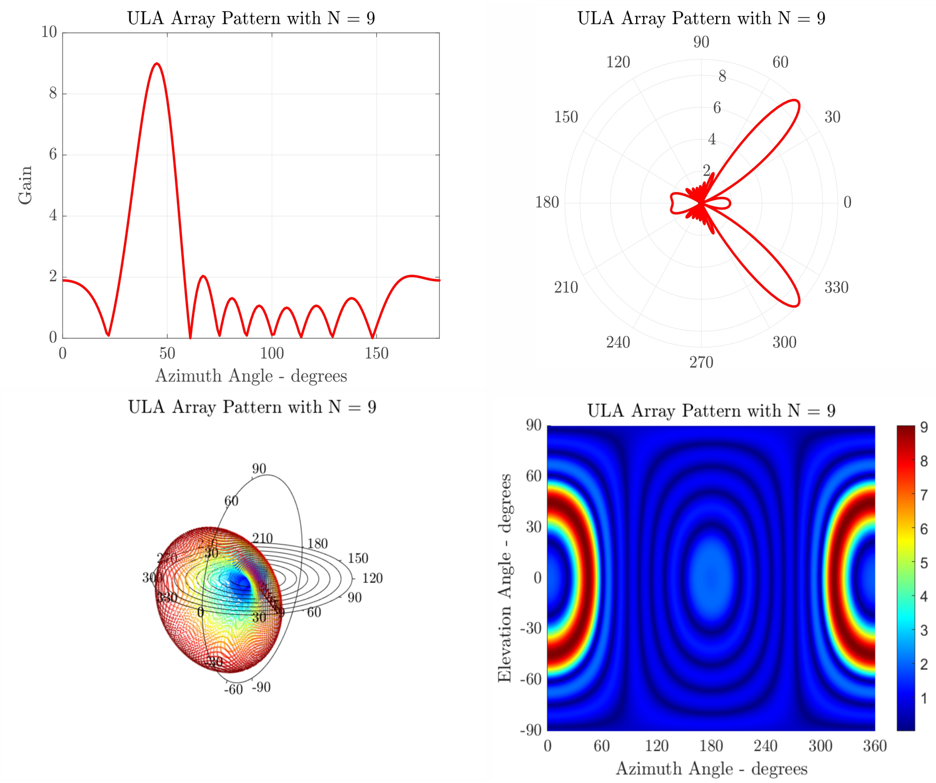
$N = 9; \lambda = 0.1\text{m}; d = \frac{\lambda}{2} = 0.05\text{m}$

(a) $\underline{r}_x = [0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4]^T$

$$\begin{aligned}\underline{\psi}(45°) &= \underline{r}_x \frac{2\pi}{\lambda} \cos 45° \\ &= [0°, 127.28°, 254.56°, 21.84°, 149.12°, 276.40°, 43.68°, 170.95°, 298.23°]^T\end{aligned}$$

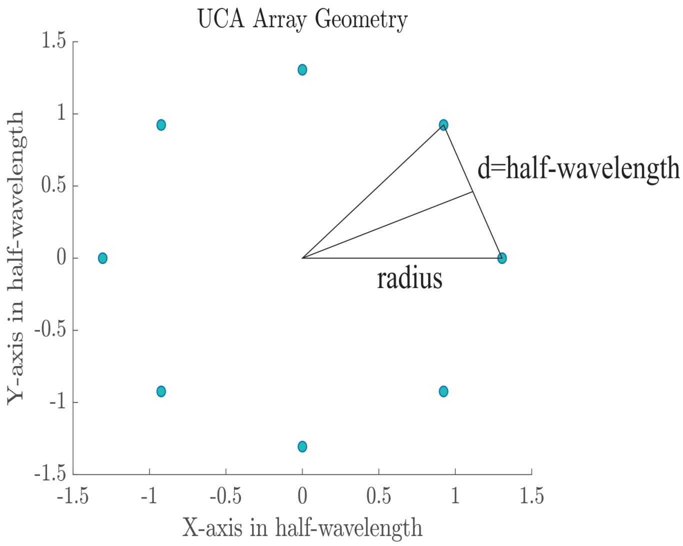
(b) $\underline{r}_x = [-0.2, -0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15, 0.2]^T$

$$\begin{aligned}\underline{\psi}(45°) &= \underline{r}_x \frac{2\pi}{\lambda} \cos 45° \\ &= [210.88°, 338.16°, 105.44°, 232.72°, 0°, 127.28°, 254.56°, 21.84°, 149.12°]^T\end{aligned}$$



20 Phased-Array Radar: UCA and Far-Field

Consider a monostatic phased-array radar that operates at a wavelength of $\lambda = 10\text{cm}$ and employs a uniform circular array (UCA) of 8 antennas with half-wavelength inter-antenna spacing. How far from the radar should be a target such as to be located in the far field of the radar?



Answer:

Array Aperture $D = 2 \times \text{radius} = 2 \times \frac{1}{2} \times \frac{\lambda}{2} / \sin\left(\frac{45^\circ}{2}\right) = 0.1307\text{m}$

Far field target range $R > \frac{2D^2}{\lambda} = \frac{2 \times 0.1307^2}{0.1} = 0.3414\text{m}$