Study Group

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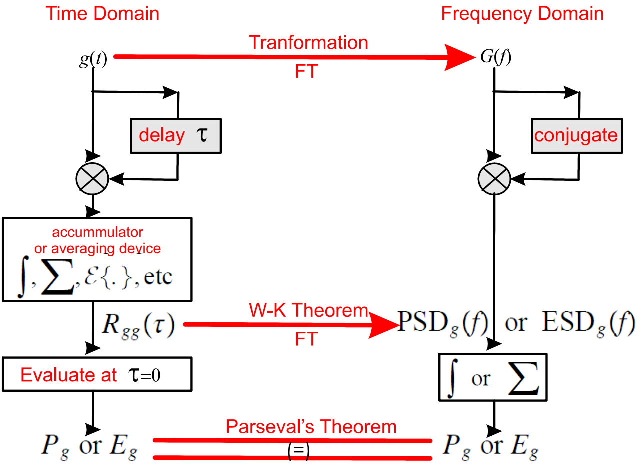
Comms-1

* Energy, Power, Autocorrelation,
* Power Spectral Density, Energy Spectral Density
* W-K theorem, Parseval's theorem


Time Domain
Frequency Domain
Tranformation
FT
g(t)
G(f)
delay τ
conjugate
accummulator
or averaging device
∫, Σ, ℰ{.}, etc
R_gg(τ)
W-K Theorem
FT
PSD_g(f) or ESD_g(f)
Evaluate at τ=0
∫ or Σ
P_g or E_g
Parseval's Theorem
(=)
P_g or E_g

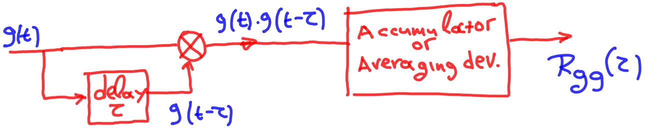
TIME DOMAN FREQ. DOMAIN

time delay

$$
\begin{tikzcd}[column sep=huge]
g(t) \arrow[r, "FT"] \arrow[d] & G(f) \arrow[d] \\
R_{gg}(\tau) \arrow[r, "FT"', "\text{W-K Theorem}"'] \arrow[d, "\tau=0"] & PSD_g(f) \text{ or } ESD_g(f) \arrow[d] \\
P_g \text{ or } E_g \arrow[r, equal, "\text{PARSEVAL'S THEOREM}"'] & P_g \text{ or } E_g
\end{tikzcd}
$$

Note: "W-K" denotes "Wiener-Khinchin"

BASIC THEORY



Accumulator: $\int_{t_1}^{t_2}$ or $\sum_{l=1}^{M}$

Averaging device: $\frac{1}{t_2-t_1}\int_{t_1}^{t_2}$ or $\frac{1}{M}\sum_{l=1}^{M}$ (i.e. normalised Accumulator)

corresponds to $ESD_g(f)$

$$R_{gg}(\tau) \triangleq \int_{t_1}^{t_2} g(t)\, g(t-\tau)\, dt \Rightarrow E_g = R_{gg}(0) = \int_{t_1}^{t_2} g^2(t)\, dt$$

Energy

or

$$R_{gg}(\tau) \triangleq \frac{1}{t_2-t_1}\int_{t_1}^{t_2} g(t).g(t-\tau)\, dt \Rightarrow P_g = R_{gg}(0) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} g^2(t)\, dt$$

corresponds to $PSD_g(f)$

Power

* Notation: The bar at the top of a time-function (i.e. signal) denotes time average.

That is, if $f(t)$ = a given function

then $$\overline{f(t)} \triangleq \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f(t)\,dt$$

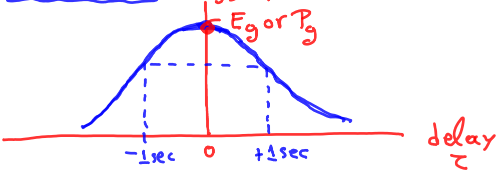
* Some very useful expressions of sin & cos of any frequency

* $\overline{\sin} = \overline{\cos} = 0$
* $\overline{\sin^2} = \overline{\cos^2} = \frac{1}{2}$
* $\overline{\cos \times \sin} = 0$

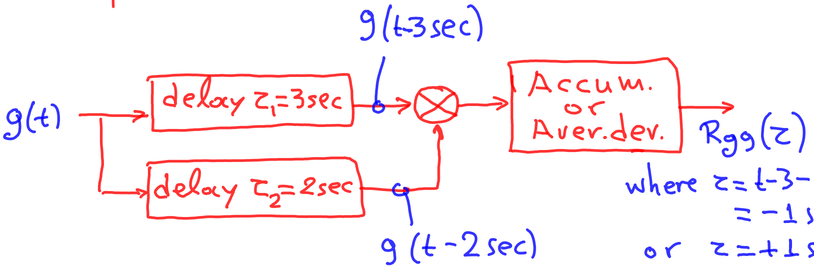
* $E_{\cos} = E_{\sin} = \infty$ $\quad P_{\cos} = P_{\sin}$
* Energy over one period T: $E_{\cos} = E_{\sin} = \frac{1}{2}T$

* Note: Autocorrelation function $R_{gg}(\tau)$ of a signal $g(t)$ is alway $\boxed{\text{symmetric}}$

i.e.



* Example:



where $\tau = t-3-t+2$
$= -1$ sec
or $\tau = +1$ sec

Q1(c):

$$g(t) = 3\cdot\sin(40t) + 4\cos(\pi 10^4 t)$$

* mean (i.e. DC level): $\mu_g = \overline{g(t)} = \overline{3\sin(40t)+4\cos(\pi 10^4 t)}$

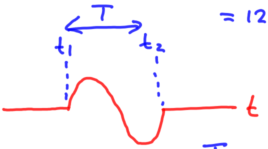
$$= 3\,\overline{\sin(40t)} + 4\,\overline{\cos(\pi 10^4 t)}$$
$$= 3\times 0 + 4\times 0$$
$$= 0$$

* Energy: $E_g = \infty$

* Power: $P_g = \overline{g^2(t)} = \overline{9\sin^2(40t)+16\cos^2(\pi 10^4 t)+2\times3\times4\sin(40t)\cdot\cos(\pi 10^4 t)}$

$$= 9\,\overline{\sin^2(40t)} + 16\,\overline{\cos^2(\pi 10^4 t)} + 24\,\overline{\sin(40t)\cos(\pi 10^4 t)}$$
$$= 9\times\frac{1}{2} + 16\times\frac{1}{2} + 24\times 0$$
$$= 12.5$$

Q2(a):

$g(t) =$ 

T = period = 2π

$(2\pi F t = t \Rightarrow F = \frac{1}{2\pi} \Rightarrow T = \frac{1}{F} = 2\pi)$

$$E_g = \int_{t_1}^{t} g^2(t)\,dt = \int_0^T \sin^2(t)\,dt = \frac{1}{2}T = \frac{1}{2}2\pi = \pi$$

Q2(b): if $g(t)$ is the signal then its power is $\overline{g^2(t)}$

$kg(t)$ - - - - - - - - - - - - - - - - > $\overline{k^2 g^2(t)} = k^2\,\overline{g^2(t)}$

$g(t-\tau)$ - - - - - - - - - - - - - - - - > $\overline{g^2(t-\tau)} = \overline{g^2(t)}$

Q3: period $T = 2$

$$P_g = \frac{1}{T}\int_0^T g^2(t)\,dt = \frac{1}{2}\int_0^2 g^2(t)\,dt = \frac{1}{2}\int_0^1 \underbrace{g^2(t)}_{=0}\,dt + \frac{1}{2}\int_1^2 \underbrace{g^2(t)}_{=1}\,dt$$

$$= \frac{1}{2}\times 0 + \frac{1}{2}\times 1 = \frac{1}{2}$$