EE3-27: Principles of Classical and Modern Radar

Target Reflectivity and EM-Waves Refresher

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Target as a Mathematical/Wave Transformer

- Any target can be seen as a "transformer" which transforms the incident electromagnetic wave (EM wave) to a somewhat different backscatter EM wave as shown below

- The difference between the incident wave and the backscatter wave depends on the following target features
  1. Shape of the target
  2. Aspect of the target
  3. Movement of the target, including the movement of the moving parts of the target
  4. Material of the target, including conductivity, dielectric constant, permeability, and even semiconductor nonlinearity in the junction of metal parts

Electrical Properties of a Target and EM field Intensities

1. Electrical properties of a medium (or radar target) are specified by its constitutive parameters:
   - permeability, $\mu = \mu_0 . \mu_r$
     (for free space, $\mu = \mu_0 = 4\pi \times 10^{-7}$H/m [Henries/m])
   - permittivity, $\varepsilon = \varepsilon_0 . \varepsilon_r$
     (for free space, $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m [Farads/m])
   - conductivity, σ (for a metal, $\sigma \approx 10^7$ S/m [Siemens/m])
2. Electrical properties of the boundary between two media
   - reflection coefficient, $\Gamma = \dfrac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$
   - transmission (or emission) coefficient, $\mathcal{E} = \dfrac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = 1 + \Gamma$
3. Electric and magnetic field intensities: $\underline{E}(\underline{r}, t)$ V/m, $\underline{H}(\underline{r}, t)$ A/m
   - these are vector functions of location in space $\underline{r} \in R^{3\times 1}$ and time t,
   - the fields arise from current $\underline{J}$ and charge ρ_V on the source
     ($\underline{J}$ is the volume current density in A/m^2 and ρ_V is volume charge density in C/m^3)

Electric and Magnetic Field Intensities:

- $\underline{E}(\underline{r}, t)$ V/m, and $\underline{H}(\underline{r}, t)$ A/m are the (3×1) column vector which are functions of location in space $\underline{r}$ and time t, where

$$\underline{r} = [x, y, z]^T$$

are the Cartesian coordinates, with $\|\underline{r}\| = R =$range.

- An alternative but equivalent representation to Cartesian coordinates of vector $\underline{r}$ is based on the "spherical coordinates". That is,

$$\begin{aligned} \text{Cartesian} \quad &: \quad \underline{r} = [x, y, z]^T && (1) \\ \text{Spherical} \quad &: \quad \underline{r} = [\|\underline{r}\|, \theta, \phi]^T && (2) \end{aligned}$$

where

$$(\theta, \phi) \triangleq (\text{azimuth,elevation}) \text{ angles} \tag{3}$$

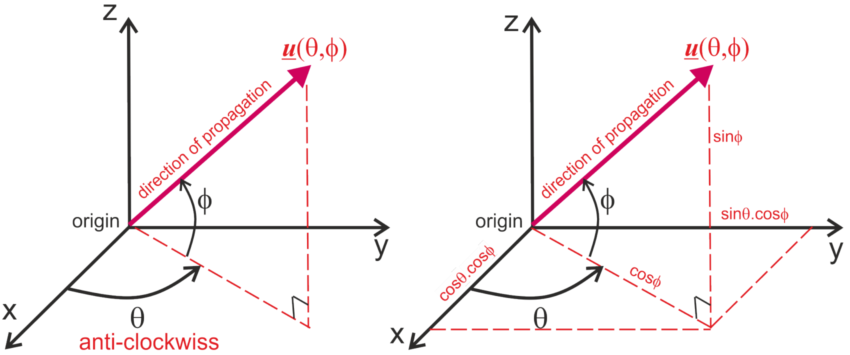
- That is,

$$\text{Cartesian} \quad : \quad \underline{r} = [x, y, z]^T \tag{4}$$

$$\text{Spherical} \quad : \quad \underline{r} = [\|\underline{r}\|, \theta, \phi]^T \tag{5}$$

where

$$(\theta, \phi) \triangleq \text{direction of propagation} \tag{6}$$



- Relationship between Cartesian & Spherical coordinates:

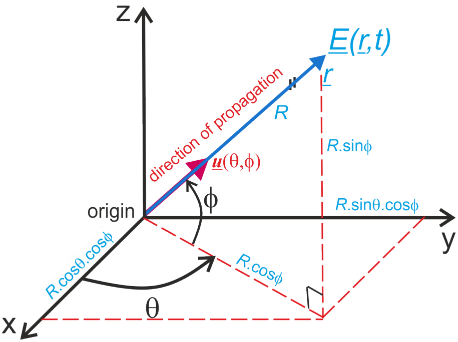
$$\underline{r} = [x, y, z]^T = [\overbrace{R\cos\theta\cos\phi}^{x}, \overbrace{R\sin\theta\cos\phi}^{y}, \overbrace{R\sin\phi}^{z}] \quad (7)$$

$$= \underbrace{R[\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi]^T}_{\underline{u} \triangleq \underline{u}(\theta,\phi)} \quad (8)$$

where

$$\underline{u}(\theta,\phi) = \text{unity norm vector} \quad (9)$$

$$R = \|\underline{r}\| \quad (10)$$



Maxwell Equations Refresher

Electromagnetic fields are completely described by Maxwell's equations which are given below.

Integral Form		**Differential Form**	Radar (carrier $f \triangleq F_c$)
$\iint \underline{E}.d\underline{S} = \frac{1}{\varepsilon}\iiint \rho_V.dV$	$\Leftrightarrow$	$\nabla.\underline{E} = \frac{\rho_V}{\varepsilon}$	
$\iint \underline{H}.d\underline{S} = 0$	$\Leftrightarrow$	$\nabla.\underline{H} = 0$	
$\oint \underline{E}.d\underline{s} = -\mu \iint \frac{\partial \underline{H}}{\partial t}.d\underline{S}$	$\Leftrightarrow$	$\underline{\nabla} \times \underline{E} = -\mu\frac{\partial \underline{H}}{\partial t}$	$= -2\pi f \mu \underline{H}$
$\oint \underline{H}.d\underline{s} = \iint (\varepsilon\frac{\partial \underline{E}}{\partial t} + \underline{J}).d\underline{S}$	$\Leftrightarrow$	$\underline{\nabla} \times \underline{H} = -\varepsilon\frac{\partial \underline{E}}{\partial t} + \underline{J}$	$= -2\pi f \varepsilon \underline{E} + \underline{J}$

(11)

James Clerk Maxwell

The Wave Equations

- The wave equations are derived from Maxwell's equations:

$$\underline{\nabla}^2 \underline{E} - \frac{1}{v_p^2}\frac{\partial^2 \underline{E}}{\partial t^2} = 0 \quad (12)$$

$$\underline{\nabla}^2 \underline{H} - \frac{1}{v_p^2}\frac{\partial^2 \underline{H}}{\partial t^2} = 0 \quad (13)$$

$$\text{where} \quad v_p = 1/\sqrt{\mu\varepsilon} \quad (14)$$

- The phase velocity v_p in free space:

$$v_p = c = 2.998 \times 10^8 \text{ m/s} \quad (15)$$

$$\approx 3 \times 10^8 \text{ m/s} \quad (16)$$

- The simplest solutions to the wave equations are planewaves.

Planewave Solution: Electric and Magnetic Fields

- At a point $\underline{r} = [x, y, z]^T$ (with $R = \|\underline{r}\|$) on the plane at distance R and at time t, the Electric and Magnetic Fields can be represented as $\underline{E}(\underline{r}, t)$ and $\underline{H}(\underline{r}, t)$, respectively, and expressed as follows:

$$\begin{aligned}
\underline{E}(\underline{r}, t) &= \mathrm{Re}\left\{\underline{E}_o . \exp\left(j2\pi F_c t - (\alpha + j\beta)\underline{u}^T \underline{r}\right)\right\} && (17)\\
&= \underline{E}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \cos\left(2\pi F_c t - \beta . \underline{u}^T \underline{r}\right) && (18)\\
\underline{H}(\underline{r}, t) &= \mathrm{Re}\left\{\underline{H}_o . \exp\left(j2\pi F_c t - (\alpha + j\beta)\underline{u}^T \underline{r}\right)\right\} && (19)\\
& \underline{H}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \cos\left(2\pi F_c t - \beta . \underline{u}^T \underline{r}\right) && (20)
\end{aligned}$$

where $\underline{E}_o$, $\underline{H}_o$ are vector-amplitudes and

$$\begin{aligned}
\alpha + j\beta &= \text{complex propagation constant} && (21)\\
\alpha &= \text{attenuation constant (Nepers/m)} && (22)\\
\beta &= \text{phase constant}=\frac{2\pi}{\lambda} \text{ (rads/m)} && (23)\\
\lambda &= \text{wavelength} && (24)
\end{aligned}$$

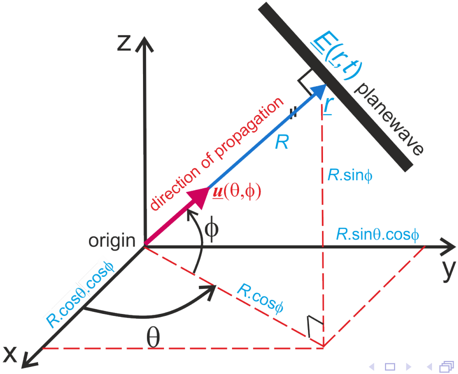
- Note that here, the vectors $\underline{u}$ and $\underline{r}$ are colinear and thus $\underline{u}^T \underline{r} = R$.

- The main result of the previous slide is Equation 18, which is also repeated below:

$$\underline{E}(\underline{r}, t) = \underline{E}_o . \exp\left(-\alpha . \underline{u}^T \underline{r}\right) . \cos\left(2\pi F_c t - \beta . \underline{u}^T \underline{r}\right) \tag{25}$$

and the direction of propagation, the Electric Field Intensity vector

$\underline{E}(\underline{r}, t)$ and the planewave approximation are shown in the following figure:

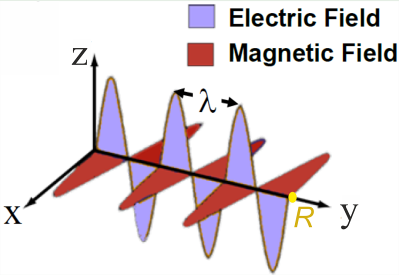


Example (1)

- At a point $\underline{r} = \underbrace{[0, \overset{R}{\overset{\downarrow}{y}}, 0]^T}_{Cartesian} = \underbrace{R.\underline{u}(\overset{\theta}{\overset{\downarrow}{90^\circ}}, \overset{\phi}{\overset{\downarrow}{0^\circ}})}_{spherical}$ and time t

  the Electric Field Intensity is

$$\underline{E}(\underline{r}, t) = \underline{E}_o.\exp(-\alpha.\overset{R}{\overset{\downarrow}{y}}).\cos(2\pi F_c t - \beta.\overset{R}{\overset{\downarrow}{y}})$$

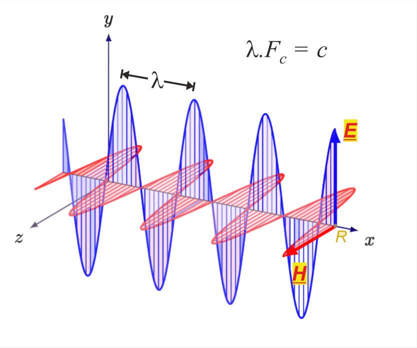


Example (2)

- At a point $\underline{r} = \underbrace{[\overset{R}{\downarrow}{x}, 0, 0]^T}_{Cartesian} = \underbrace{R.\underline{u}(\overset{\theta}{\downarrow}{0^\circ}, \overset{\phi}{\downarrow}{0^\circ})}_{spherical}$. and time t

  the Electric Field Intensity is

$$\underline{E}(\underline{r}, t) = \underline{E}_o.\exp(-\alpha.\overset{R}{\downarrow}{x}).\cos(2\pi F_c t - \beta.\overset{R}{\downarrow}{x})$$



Wave Properties

- Plane and spherical waves belong to a class called *transverse electromagnetic* (TEM) waves and have the following features:
  1. $\underline{E}(\underline{r},t)$, $\underline{H}(\underline{r},t)$ and the direction of propagation $\underline{u}$ are mutually orthogonal, i.e.
  $$\underline{E}(\underline{r},t) \perp \underline{H}(\underline{r},t) \perp \underline{u}(\theta,\phi) \tag{26}$$
  2. $\underline{E}(\underline{r},t)$ and $\underline{H}(\underline{r},t)$ are related by the intrinsic impedance of the medium
  $$Z_0 = \begin{cases} \sqrt{\dfrac{\mu}{\varepsilon - j\frac{\sigma}{2\pi f}}} \\ \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} \approx 377\Omega & \text{for free space} \end{cases} \tag{27}$$

- The above two relationships can be expressed in a vector format as follows:
$$\underline{H}(\underline{r},t) = \frac{\underline{u} \times \underline{E}(\underline{r},t)}{Z_0} \tag{28}$$

Conductivity and Attenuation Propagation Constants

- A material's conductivity σ causes attenuation of a wave as it propagates through the medium.
  - Energy is extracted from the wave and dissipated as heat (ohmic loss).
- The attenuation constant α determines the rate of decay of the wave.
- In general the attenuation and phase constants are given by the following equations (are functions of carrier frequency f (i.e. F_c) and the material parameters μ, ε and σ as follows:

$$\alpha = 2\pi f \sqrt{\frac{\mu\varepsilon}{2}\left(-1+\sqrt{1+\left(\frac{\sigma}{2\pi f\varepsilon}\right)^2}\right)} \tag{29}$$

$$\beta = 2\pi f \sqrt{\frac{\mu\varepsilon}{2}\left(1+\sqrt{1+\left(\frac{\sigma}{2\pi f\varepsilon}\right)^2}\right)} \tag{30}$$

N.B.:

- For lossless media (i.e. $\sigma = 0$) Equations 29 and 30 are simplified

$$\boxed{\sigma = 0 \Rightarrow \begin{cases} \alpha = 0 \\ \beta = 2\pi f \sqrt{\mu\varepsilon} \end{cases}} \tag{31}$$

- Traditionally, for lossless media, k (i.e. the wavenumber) is used rather than β

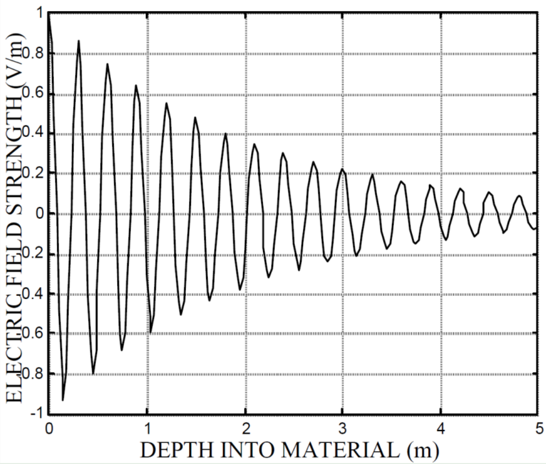
$$k = \frac{2\pi}{\lambda} = \beta = 2\pi f \underbrace{\sqrt{\mu\varepsilon}}_{=c^{-1}} \tag{32}$$

- For good conductors,

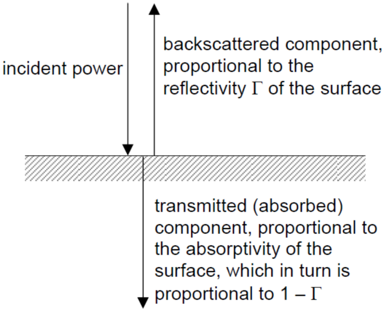
$$\frac{\sigma}{2\pi f \varepsilon} >> 1 \stackrel{(29)\wedge(30)}{\Longrightarrow} \alpha \approx \sqrt{\pi\mu f \sigma} \approx \beta \tag{33}$$

and the wave (see Equation 25) decays rapidly with distance into the material (see the example below).

Example (Electric Field vs Distance)



Absorptivity and Reflectivity



- At thermal equilibrium, emitted power must be equal to absorbed power, or

$$\mathcal{E} = 1 - \Gamma$$

where Γ is the reflectivity (power measure). Note that a highly reflective surface ($\Gamma \approx 1$) has low emissivity, ($\mathcal{E} = 0$). Hence, metal surfaces appear as black in passive sensing.

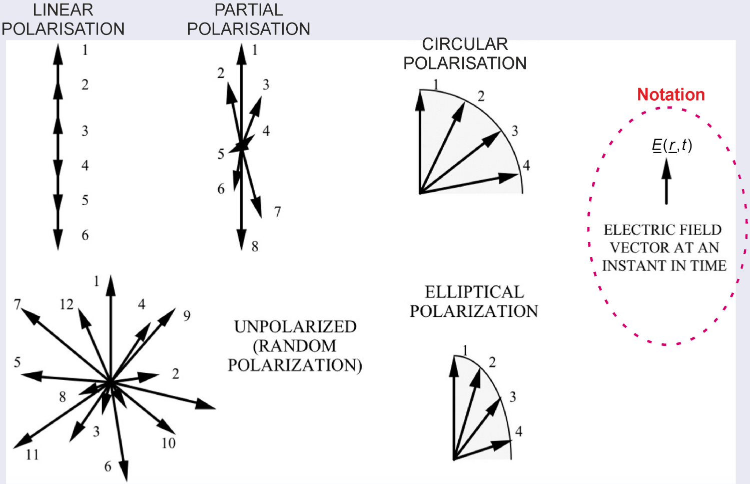
Penetration Depth of some Surface Material

TABLE 10-5 ■ Penetration Depth for Various Surface Materials [45]

Material	Approximate Penetration Depth
Soil, moist (0.3 g/cm^3 water)	~λ/8 to ~λ/3
Soil, dry (0.02 g/cm^3 water)	~1 to 3λ
Sand, dry	to ~10λ
Sea ice, first year	~1 to 3λ
Sea ice, multiyear	~4 to 9λ
Snow, wet (4% liquid water)	~1 to 2λ
Snow, dry (0.2% liquid water)	~30 to 100λ

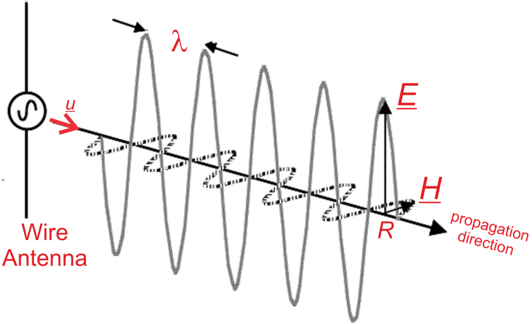
Definition (Polarisation)

Polarisation is the property of EM wave radiations in which the direction of the electric field vector $\underline{E}(\underline{r}, t)$ and its magnitude $\|\underline{E}(\underline{r}, t)\|$ are related as a function of time t in a specified way.



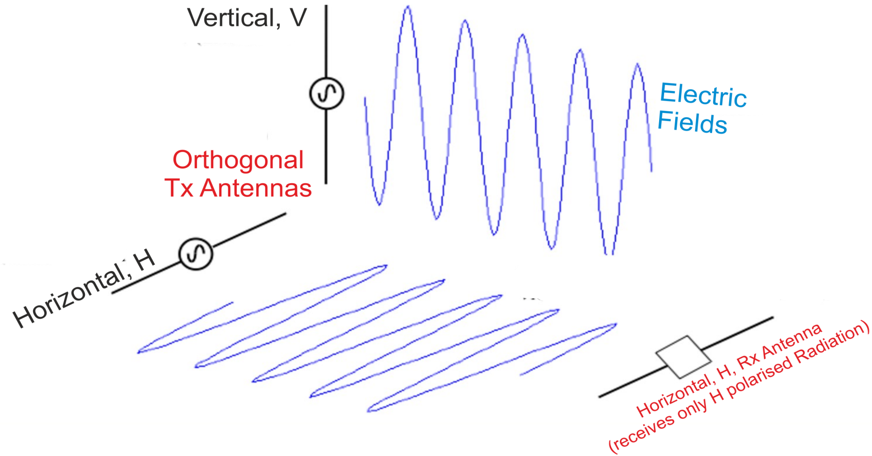
Wire Antenna Polarisation

Far field EM-wave (planewave) generated by a wire antenna



- Note that the antenna generates spherical waves (near far region) which in the far field are approximated locally by planewaves
- The Electric and Magnetic fields are orthogonal to each other and to the direction of propagation $\underline{u}(\theta, \phi)$. Their magnitudes are related by the intrinsic impedance of the medium (see Equ. 27), i.e. TEM-wave.

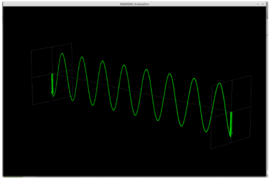
- Vertical (V) and Horizontal (H) Polarisations are defined with respect to the earth's surface



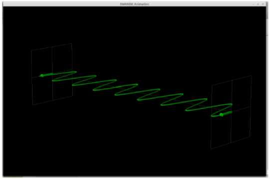
Polarisation Examples

Examples (Polarisation Examples)

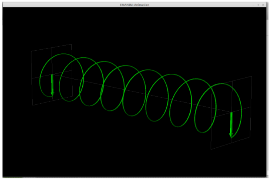
- polarisations: https://emanim.szialab.org/index.html
  - linear (Vertical and Horizontal)
  - circular (Right-Hand and Left-Hand)



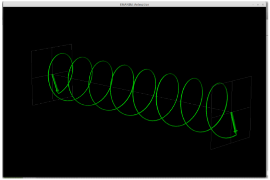
Vertical



Horizontal



Right hand

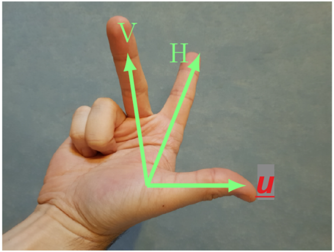


Left hand

Right-Hand Rule

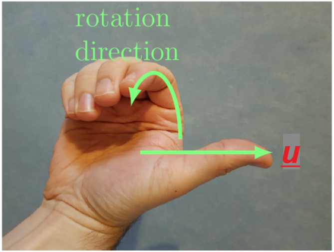
Let the propagation direction $\underline{\boldsymbol{u}}$ be along the thumb. At any time, $\boldsymbol{E}$, $\boldsymbol{H}$, and $\underline{\boldsymbol{u}}$ are orthogonal to each other.

Linear polarization



$\boldsymbol{E}$ oscillating along Horizontal or Vertical direction, $\boldsymbol{H}$ along the other.

Circular polarization



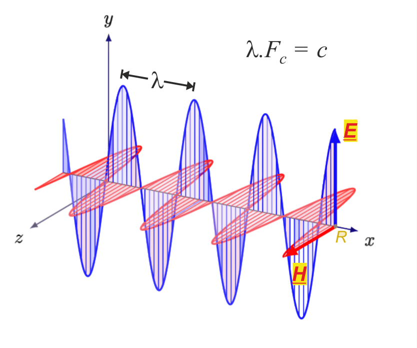
$\boldsymbol{E}$ rotating along Right or Left hand fingers, $\boldsymbol{H}$ rotating the same but at right angle.

Example (Polarisation Animation - revisiting a previous example)

- At a point $\underline{r} = [\overset{R}{\overset{\downarrow}{x}}, 0, 0]$ and time t, the Electric Field Intensity is

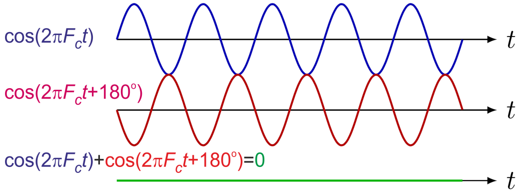
$$\underline{E}(\underline{r}, t) = \underline{E}_o . \exp(-\alpha . x) . \cos(2\pi F_c t - \beta . x)$$

  which, for lossless media (i.e. $\sigma = 0 \Rightarrow \alpha = 0$), is animated (click figure) as follows:



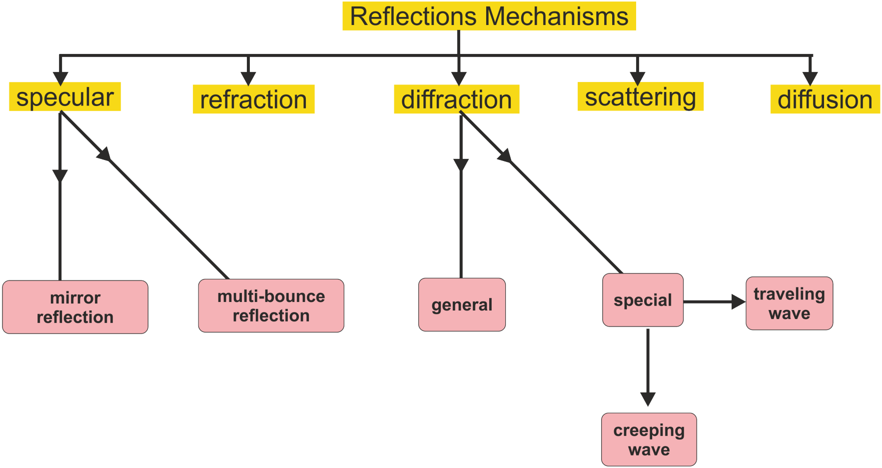
Superposition of EM-Waves

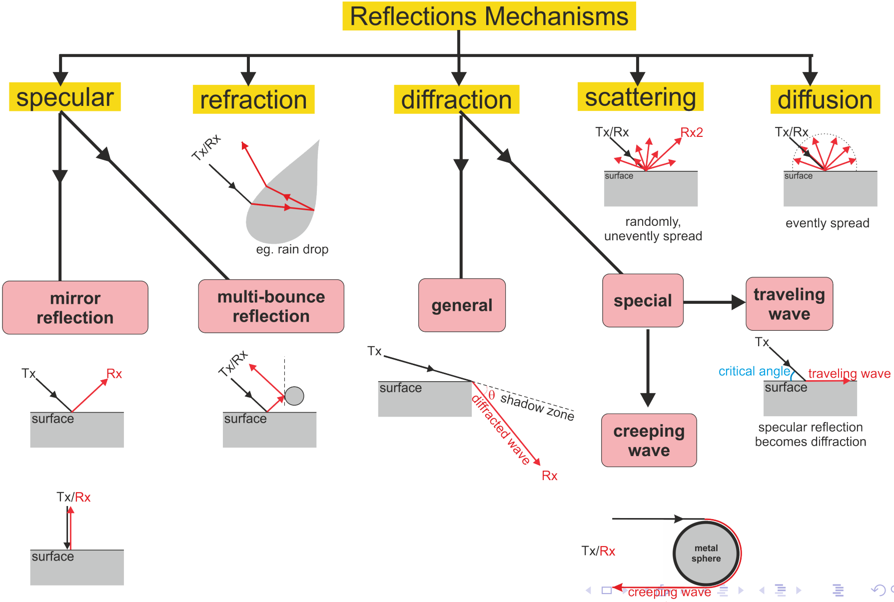
- A very important property of Maxwell's equations is that they are linear. Thus, an electromagnetic field can be treated as a superposition of two or more electromagnetic waves, meaning that the phase difference is very important
- Both constructive and destructive interference may occur. For instance, two antenna elements may add constructively in one direction, and cancel in another.
- Example of cancellation[1]:



[1] This is the origin of nulls in the antenna pattern (blind directions of the antenna)

Common Reflections/Scattering Mechanisms in Radar

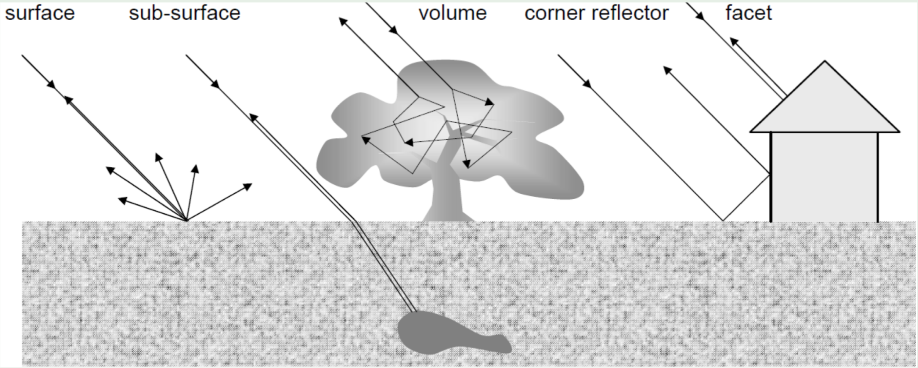




Reflections Mechanisms
specular
refraction
diffraction
scattering
diffusion
Tx/Rx
eg. rain drop
Tx/Rx
Rx2
surface
randomly, unevently spread
Tx/Rx
surface
evently spread
mirror reflection
multi-bounce reflection
general
special
traveling wave
Tx
Rx
surface
Tx/Rx
surface
Tx
surface
θ
shadow zone
diffracted wave
Rx
Tx
critical angle
traveling wave
surface
specular reflection becomes diffraction
creeping wave
Tx/Rx
surface
Tx/Rx
metal sphere
creeping wave

- scattering occurs due to many different features

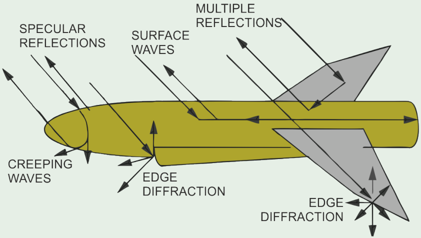
Examples



N.B.:

- Different scattering pathways may add coherently or incoherently. With a large number of randomly placed scatterers, incoherent addition (adding RCS) **is a reasonable assumption**

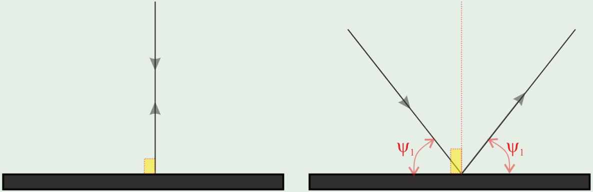
Example



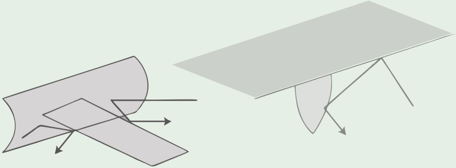
N.B.:

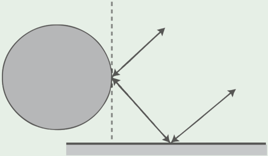
- Scattering mechanisms are used to describe wave behavior. Especially important for standard radar targets (planes, ships, etc.) at radar frequencies:
  - specular reflection= "mirror like" reflections that satisfy Snell's law
  - surface/traveling waves = the body acts like a transmission line guiding waves along its surface
  - diffraction/scattering = scattered waves that originate at abrupt discontinuities (e.g., edges)

Examples (specular reflections)



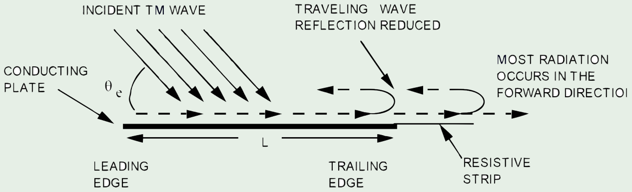
Examples (multi-bounce specular reflections)





Example (traveling wave)

- A traveling wave is a very loosely bound surface wave that occurs for gently curved or flat conducting surfaces. The surface acts as a transmission line; it "captures" the incident wave and guides it until a discontinuity is reached. The surface wave is then reflected, and radiation occurs as the wave returns to the leading edge of the surface.



Reflection from Rough Surfaces

- The trend to diffuse surface scattering as roughness increases

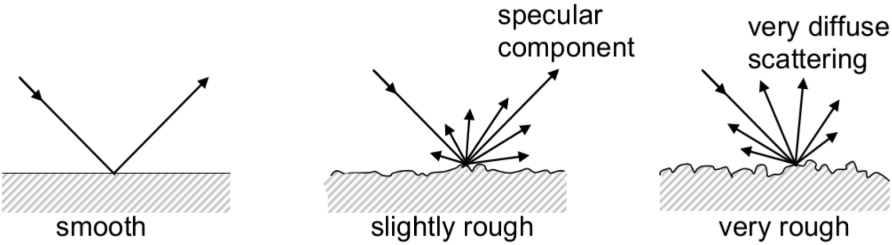
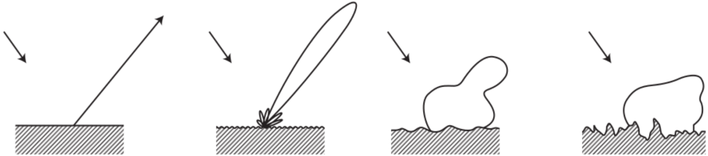


FIGURE 4-37 ■ Illustration of specular to diffuse scattering transitions with increasing surface roughness.



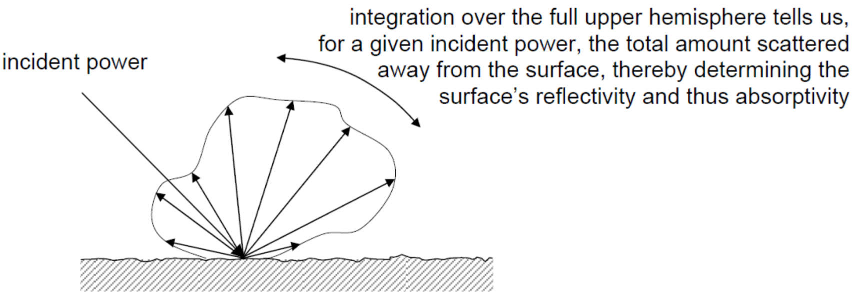


Fig. 9.3. All the power scattered into the upper part has to be found when determining reflectivity for a rough surface

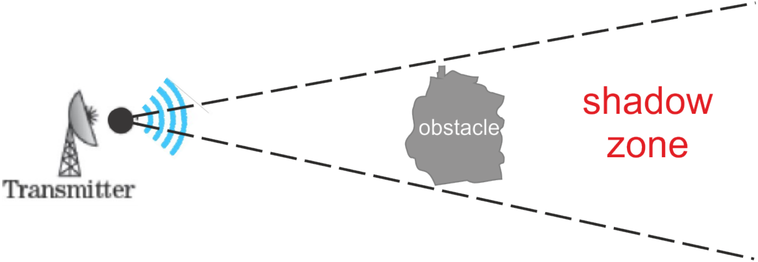
- For rough surfaces, the reflectivity Γ must correspond to power reflected in all directions.
- see https://earth.esa.int/web/guest/content/-/article/carbon-cycle (page 37, remote sensing)

Diffraction

Definition

- Diffraction is a mechanism by which waves can curve around edges and penetrate the shadow region behind an opaque obstacle.

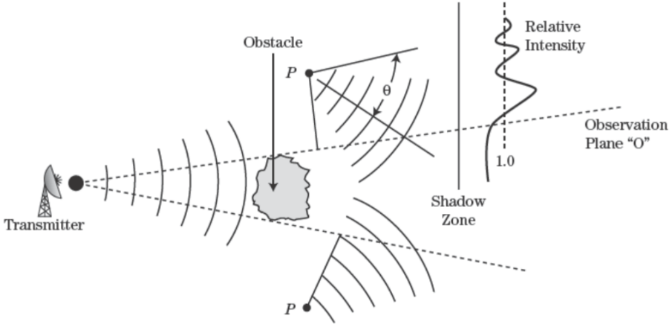
- The incident wave diffracts around the obstacle and then it will recombine with scaled replicas of itself within the observation plane.
- Even though an obstacle is blocking the path, some power can be diffracted into the shadow zone.



Diffraction (cont.)

- The interface pattern produced is that of two new waves originating from virtual phase centers at P.
- These virtual phase centers are also known as virtual sources and are an equivalent representation of the incident wave structure after diffraction has occurred.

FIGURE 4-24 ■ Illustration of virtual sources for diffraction around an obstacle.



Diffraction: Knife-edge and Rounded-tip

- Depending on the transmitted wavelength λ, the edges of the diffracting object may be appear as a smooth, curved edge or as a sharp knife edge or wedge.
- At the boundary between the interference and diffraction regions, some signal enhancement may be realized.
- In general, as the observation angle falls into the shadow zones, diffracted wave attenuations increase.

Diffraction: Knife-edge and Rounded-tip (cont.)

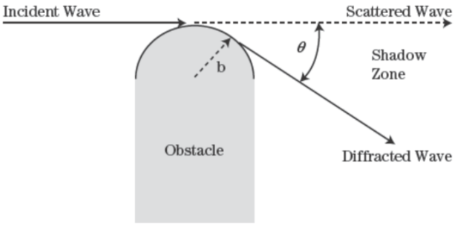
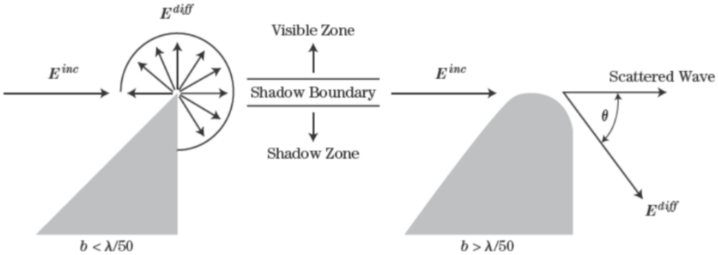


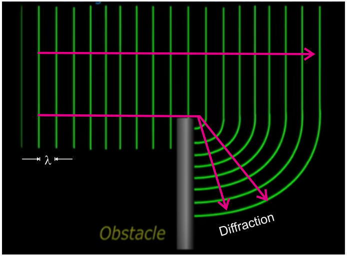
FIGURE 4-25 Geometry for diffraction into shadow zones.

FIGURE 4-26 Local diffraction coefficient, F^2, behavior for two types of edges.

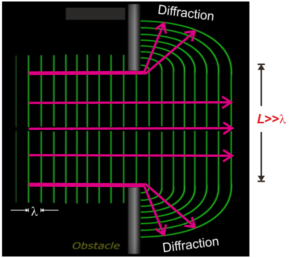


Diffraction Across an Aperture

When a wave passes through an aperture, there is interaction with the edges

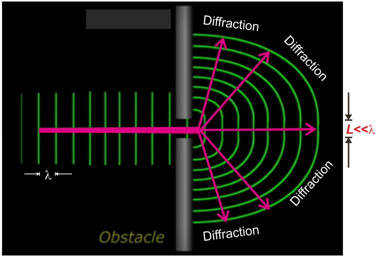


very Little Diffraction



Little Diffraction (wave passes mostly forward)

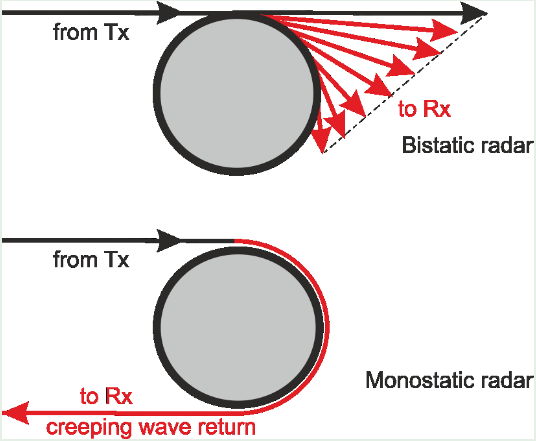
Many paths of width λ across the aperture



Significant Diffraction (wave significantly distorted)

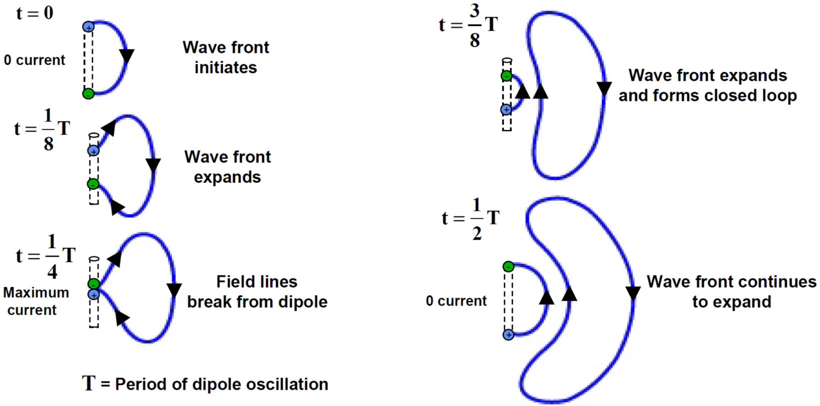
only one path of width λ across the aperture, thus diffracts on the edges, propagating spherically

Example (creeping wave)



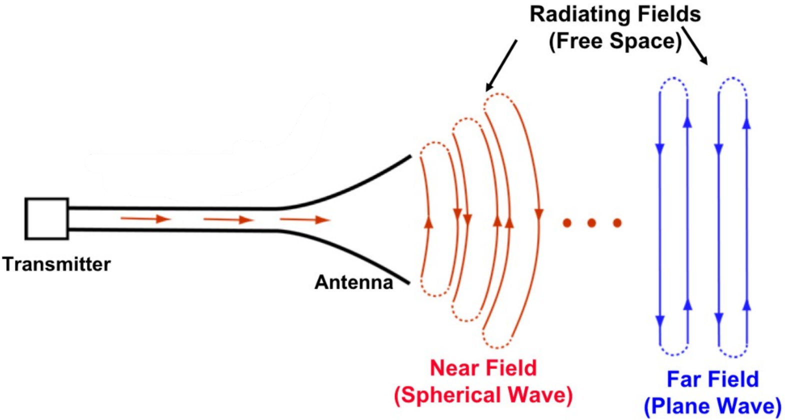
Radiation from an Oscillating Electric Dipole[2]

- Illustration of propagation and detachment of electric field lines from the dipole (two charges in simple harmonic motion)

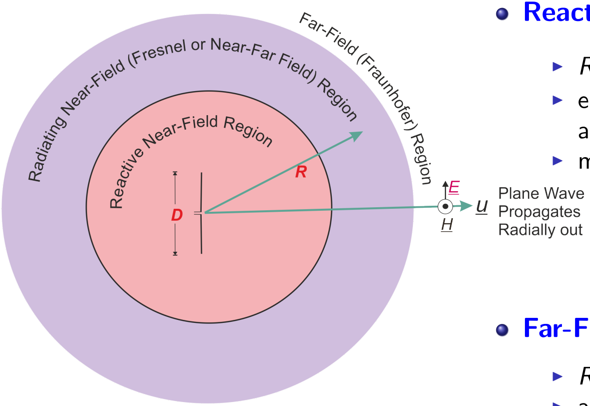


[2] Radiation is created by a time-varying current, or an acceleration (or deceleration) of charge. Example: An oscillating electric dipole where two electric charges, of opposite sign, whose separation oscillates accordingly $x(t) = A.\sin(2\pi F_c t)$; $T = 1/F_c$; F_c =radar carrier

Radiation from a Directional Antenna



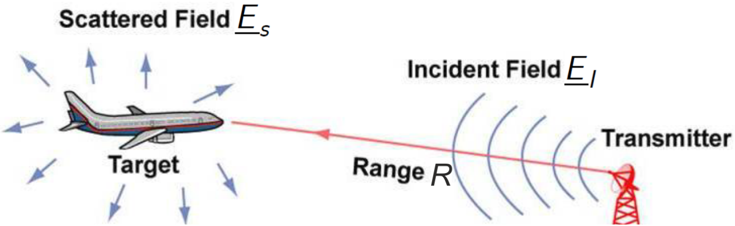
Field Regions



D denotes the aperture of the antenna

- **Reactive Near-Field region**:
  - $R < 0.62\sqrt{\frac{D^3}{\lambda}}$
  - energy is stored in vicinity of antenna
  - mutual coupling issues
- **Far-Field region**:
  - $R > 2\frac{D^2}{\lambda}$
  - all power is radiated out
  - radiated wave is a plane wave
  - **Target Radar Cross Section (RCS)**
- **Near-far Field**: radiated wave is a spherical wave

Target RCS

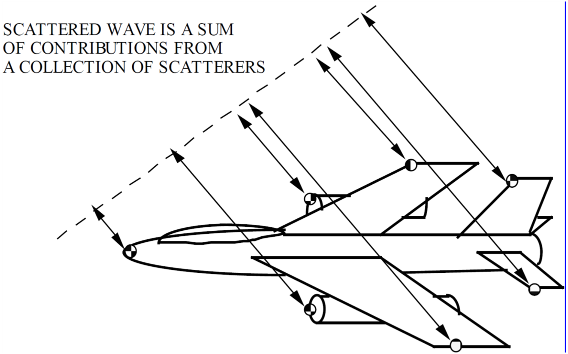


- If the incident electric field $\underline{E}_i$ that impinges upon a target is known and the scattered electric field $\underline{E}_s$ is measured, then the "radar cross section" (effective area) RCS of the target located in the far-field may by calculated.

$$RCS = \lim_{R\to\infty} 4\pi R^2 \frac{\left\|\underline{E}_s\right\|^2}{\left\|\underline{E}_I\right\|^2} \tag{34}$$

- RCS of a target is a very important parameter and it will be discussed in Topic-4 (i.e. next topic).

Scattering Sources for a Complex Target



- Typical for a target in the optical region (i.e., target large compared to wavelength)
- In some directions all scattering sources may add in phase and result in a large RCS.
- In other directions some sources may cancel other sources resulting in a very low RCS.