EE3-27: Principles of Classical and Modern Radar

Bistatic Radar

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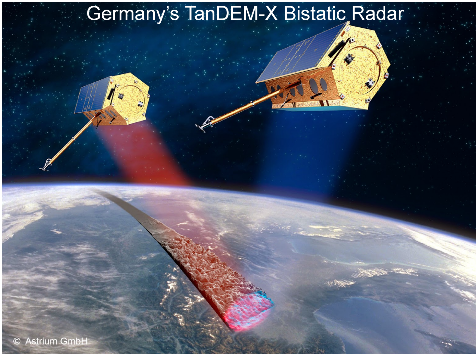
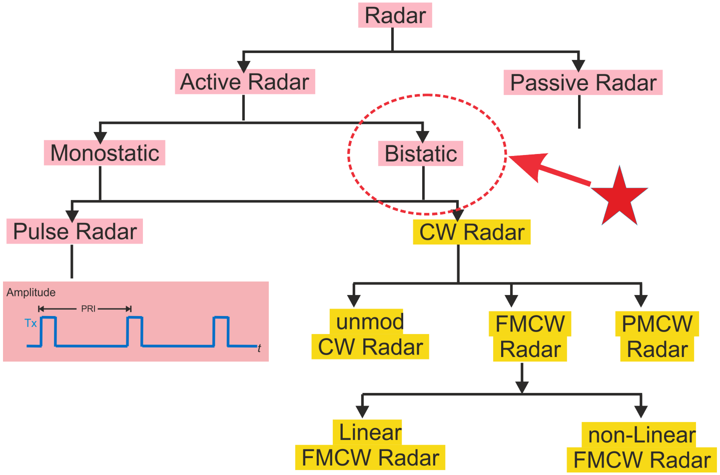


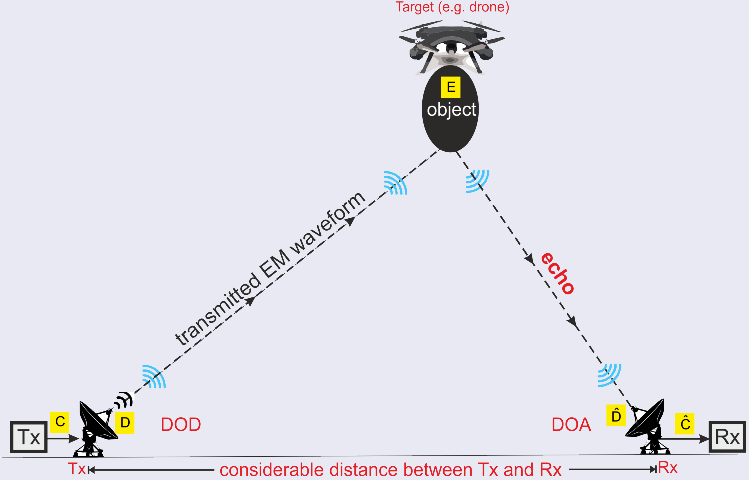
How Bistatic Radar fits into the overall Radar Classification



Basic Concepts

Definition

- A radar in which the Rx is in a different location from the transmitter is known as "bistatic radar".



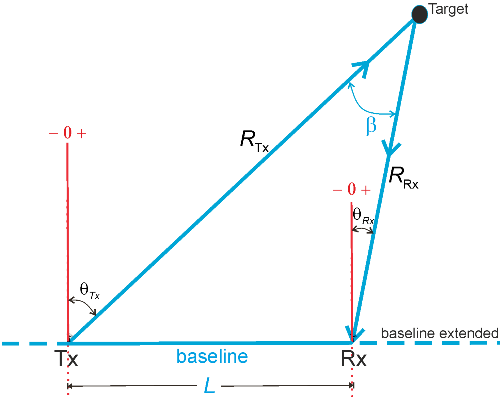
Bistatic Radar Advantages and Disadvantages

Although such an arrangement introduces a number of technical complications, particularly in synchronization between Tx and Rx, and may be significantly costlier, there are several potential advantages:

- Bistatic radar has potential advantages in detection of stealthy targets which are shaped to scatter energy in directions away from the monostatic
- The receiver is covert and therefore safer in many situations. That is, it is impossible to locate via electronic support (ES) methods the Rx of a bistatic radar.
- Countermeasures are difficult to deploy against bistatic radar. That is. it is difficult to deploy countermeasures against bistatic receivers because their location is not known. Therefore any jamming has to be spread over a range of angles, diluting its effectiveness. In the same way, a bistatic receiver will not be vulnerable to attack by anti-radiation missiles (ARMs).
- Increasing use of systems based on unmanned air vehicles (UAVs) makes bistatic radar attractive. For instance UAVs can carry just the receiver, and the heavy, complex, and power-hungry transmitter can be located elsewhere.
- Many of the synchronisation and geolocation problems that were previously very difficult are now readily soluble using GPS, and
- The extra degrees of freedom may make it easier to extract information from bistatic clutter for remote sensing applications

Bistatic Range and Geometry

- Many of the properties of bistatic radar depend on the bistatic geometry – and in particular, the bistatic triangle formed by the Tx, target and Rx

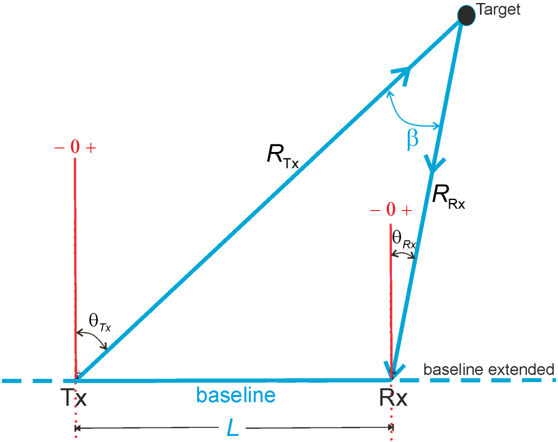


Definitions

1. The distance of the receiver from the transmitter is known as the baseline, L.
2. The angle at the target subtended by the transmitter and receiver is the bistatic angle, β.
3. $R_{Tx} \triangleq$ the Tx-to-target range;
4. $R_{Rx} \triangleq$ the target-to-Rx range
5. Bistatic Plane: defined by the position of Tx, target, Rx
6. Bistatic Range $\triangleq R_{Tx} + R_{Rx} - L$

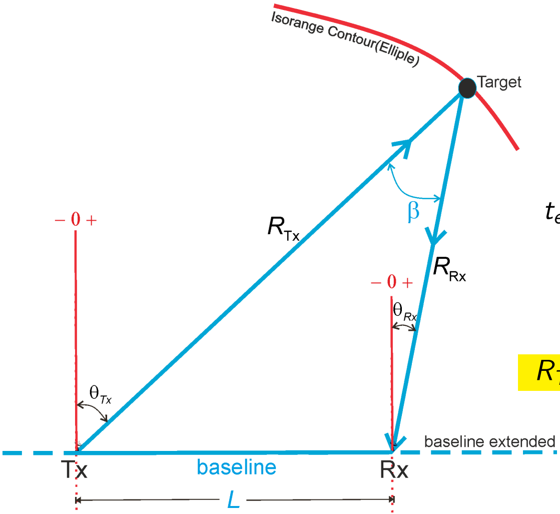
- In most arrangements, the bistatic receiver measures the difference in delay between the direct pulse from the transmitter and the target echo, which, if L is known, gives the range $R_{Tx} + R_{Rx}$.

  Such a measurement defines an ellipse, with the transmitter and receiver as the two focal points (foci).



- There are essentially three parameters that the bistatic Rx may measure:

  1. the difference in range $R_{Tx} + R_{Rx} - L$ between the direct signal and the Tx–target–Rx path,
  2. the angle of arrival θ_{Rx} of the received echo, and
  3. the Doppler shift f_D of the received echo.



$$t_{echo,1} = \frac{L}{c} \quad (1)$$

$$t_{echo2} = \frac{R_{Tx} + R_{Rx}}{c} \quad (2)$$

$$t_{echo} = t_{echo,2} - t_{echo,1} \quad (3)$$

$$= \frac{R_{Tx} + R_{Rx} - L}{c} \quad (4)$$

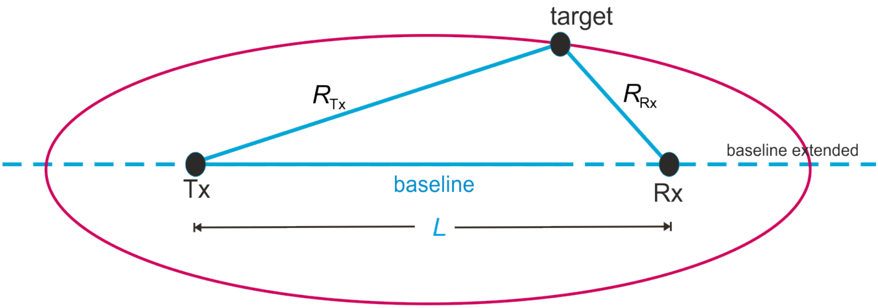
$$\Downarrow$$

$$R_{Tx} + R_{Rx} = t_{echo}.c + L \quad (5)$$

- Equ 5 defines an ellipse, with the Tx and Rx as the two focal points.

Bistatic Radar Contours

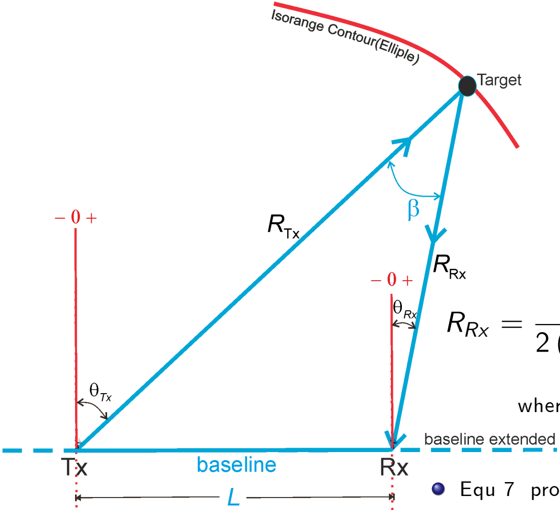
- Contours of constant bistatic range are ellipses, with Tx and Rx as the two foci



$$R_{Tx} + R_{Rx} = const.$$

- Targets lying on the Tx-Rx baseline have zero bistatic range

Bistatic Range



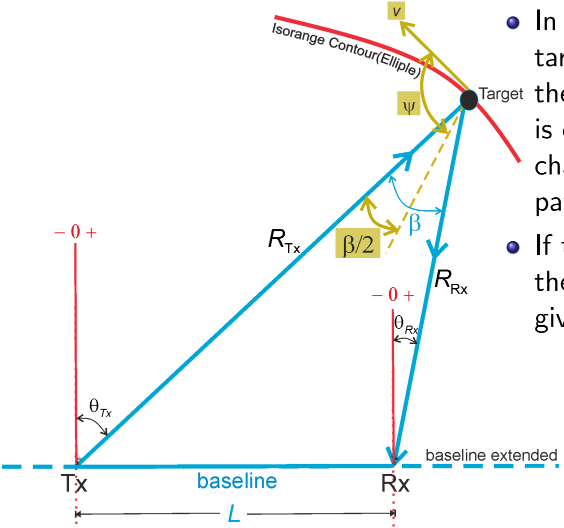
$$\beta = \theta_{Tx} - \theta_{Rx} \tag{6}$$

$$R_{Rx} = \frac{(R_{Tx} + R_{Rx})^2 - L^2}{2\,(R_{Tx} + R_{Rx} + L \sin\theta_{Rx})} \tag{7}$$

where θ_{Rx} is positive or negative

- Equ 7 provides a conversion from the measured $R_{Tx} + R_{Rx}$ to the target range from RX.

Bistatic Doppler

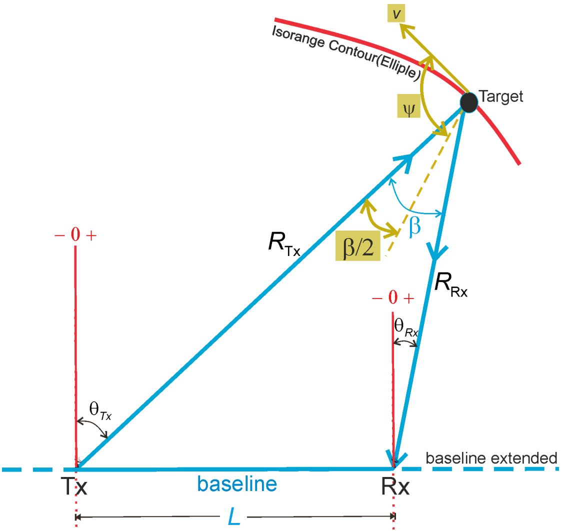


- In the general case when Tx, target and Rx are all moving, the Doppler shift on the echo $\mathfrak{f}_D$ is obtained from the rate of change of the Tx–target–Rx path.
- If the Tx and Rx are stationary, then the Doppler shift $\mathfrak{f}_D$ is given by

$$\mathfrak{f}_D = \frac{2v}{\lambda} \cos\psi \cos\left(\frac{\beta}{2}\right) \qquad (8)$$

where $v =$ target velocity

Bistatic Radar: Doppler Special Cases



- It can be seen that, if the target is crossing the bistatic baseline, then $\beta = 180^o$ and $f_D = 0$, no matter what the direction or magnitude of v.
- Physically this can be understood because at this point the transmitter-to-target range is changing in an equal and opposite way to the target-to-receiver range.

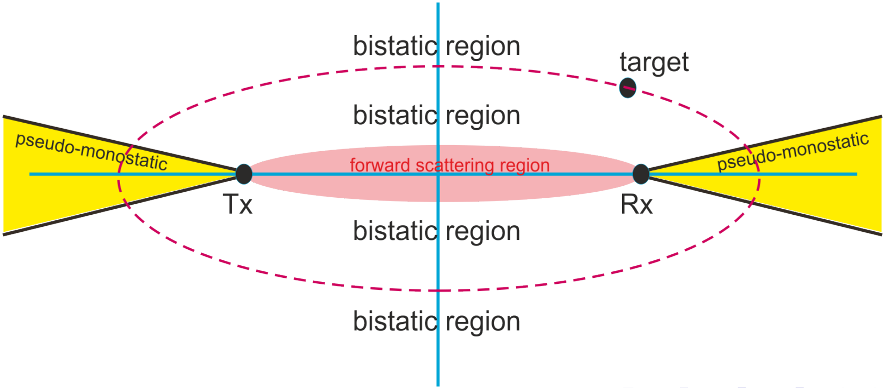
Table of Doppler Special Cases

- For
$$v_{Tx} = v_{Rx} = 0;\ v \neq 0;\ f_D = \frac{2v}{\lambda} \cos\psi \cos\frac{\beta}{2} \tag{9}$$
then for some special values of the two angles β and ψ we have the following table:

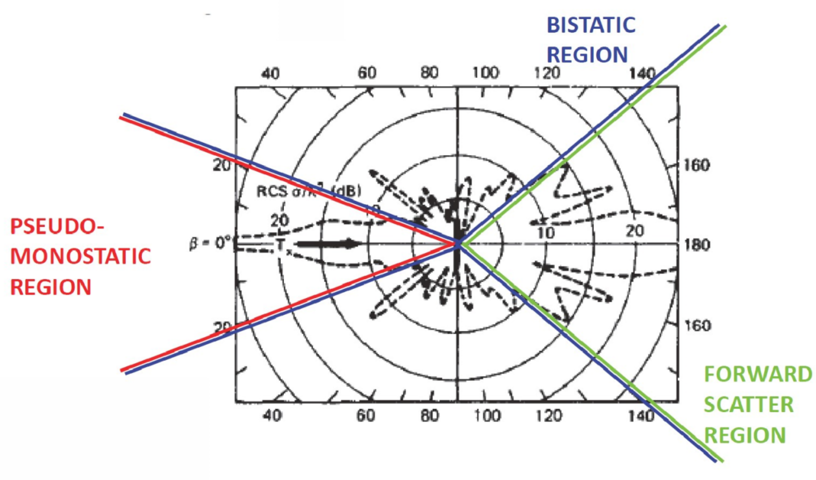
β	ψ	f_D	condition
0°	-	$\frac{2v}{\lambda}\cos\psi$	monostatic
0°	0°	$\frac{2v}{\lambda}$	monostatic
180°	-	0	forward scatterer
-	$\pm 90^\circ$	0	v $\perp$to bisector
-	$\pm\frac{\beta}{2}$	$\frac{2v}{\lambda}\cos^2\frac{\beta}{2}$	v $\Rightarrow$Tx or Rx
-	0°, 180°	$\pm\frac{2v}{\lambda}\cos\frac{\beta}{2}$	v $\Rightarrow$bisector
-	$90^\circ \pm \frac{\beta}{2}$	$\mp\frac{v}{\lambda}\sin\beta$	$v \perp$ Tx or Rx LOS

The Three Bistatic Regions

- According the the bistatic angle β, there are three regions
    1. pseudo-monostatic region: β approaching 0°
    2. Bistatic region: β
    3. Forward scattering region: β approaching 180°.
- The extend of each region is set by the target's physical characteristics



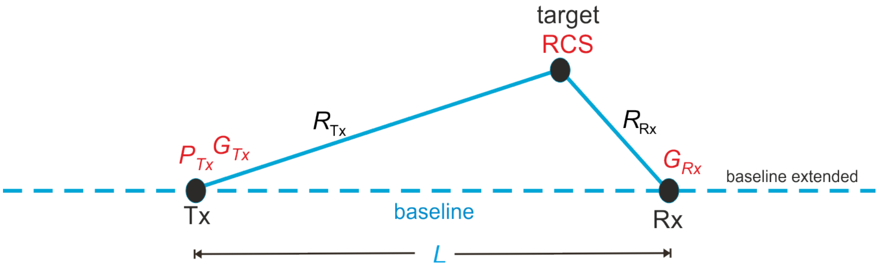
Example (for a particular target)



Skolnik, "Radar Handbook", Chapter 23, (target: cylinder 16×1.65cm; frequency: 35GHz)

Bistatic Radar Equation

- The radar equation for a bistatic radar is derived in exactly the same way as for a monostatic radar



Bistatic Radar SNR Equation

- That is,

$$
\begin{aligned}
SNR &= \frac{P_{Rx}}{\sigma_n^2} \\
&= \underbrace{P_{TX}}_{\text{Tx-power}} \cdot \overbrace{\frac{1}{4\pi R_{Tx}^2}}^{\text{spread factor}} \cdot \underbrace{G_{TX}}_{\text{Tx-antenna Gain}} \cdot \underbrace{RCS}_{\text{target RCS}} \cdot \overbrace{\frac{1}{4\pi R_{Rx}^2}}^{\text{spread factor}} \cdot \underbrace{\frac{G_{RX}\lambda^2}{4\pi}}_{\text{Rx-antenna aperture}} \cdot \frac{1}{k_B.T_0.B.F_n} \\
&= \boxed{\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R_{Tx}^2.R_{Rx}^2.k_B.T_0.B.F_n}.RCS} \qquad (10)
\end{aligned}
$$

- The dynamic range of signals to be handled is reduced, because of the defined minimum range.

Maximum Range Product

$$SNR_{in} = \frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.R_{Tx}^2.R_{Rx}^2.k_B.T_0.B.F_n}.RCS \tag{11}$$

- From the above equation it is clear (by differentation) that $\frac{1}{R_{Tx}.R_{Rx}}$ and hence SNR_{in} has a minimum value for $R_{Tx} = R_{Rx}$

  That is, the SNR_{in} is higher for targets close to the Tx or close to the Rx

- maximum $R_{Tx}.R_{Rx}$:

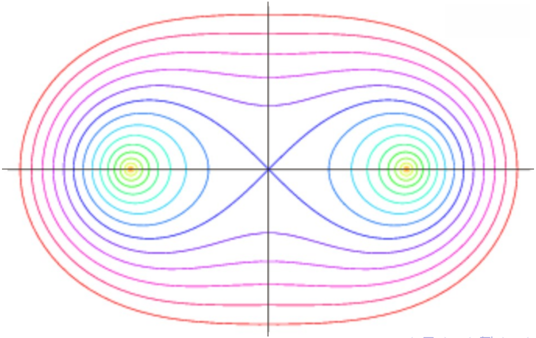
$$\begin{aligned}[R_{Tx}.R_{Rx}]_{\max} &= \sqrt{\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.SNR_{in,\min}.k_B.T_0.B.F_n}.RCS} \\ &= \sqrt{\frac{P_{TX}.G_{TX}.G_{RX}.\lambda^2}{(4\pi)^3.P_{Rx,\min}}.RCS} \qquad (12)\end{aligned}$$

Ovals of Cassini

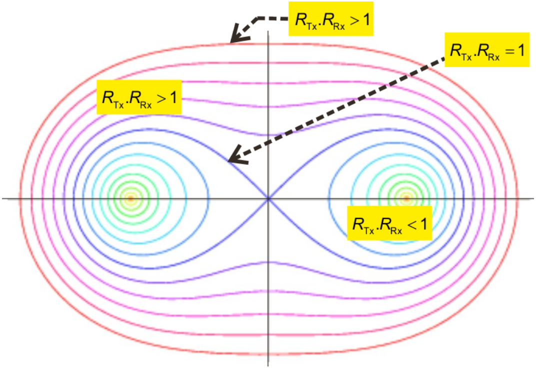
- We can see from the bistatic radar equation that contours of constant SNR are defined by constant detection range, i.e. by

$$R_{Tx}.R_{Rx} = \text{constant} \tag{13}$$

- These are: Ovals of Cassini, which are plotted (in matlab) below for various "constant" values



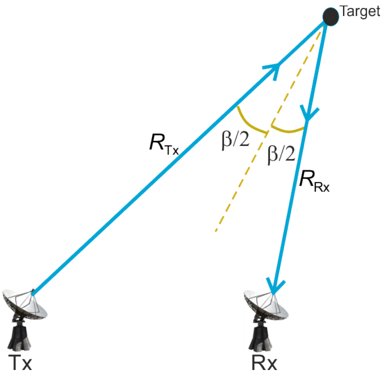
- For small values of the "constant" these tend to circular regions centered on the transmitter and receiver; for large values they tend to ellipses, and for very large values of "constant" to circles[1]



[1] These figures are appropriate only for omnidirectional transmit and receive antenna patterns; if the patterns are directional, the contours are weighted by the radiation patterns and may be completely different in shape

The Equivalence RCS Theorem

- The bistatic RCS is equal to the monostatic RCS at the bisector of the bistatic angle β, reduced in frequency by the factor $\cos\frac{\beta}{2}$, given:
  - sufficiently smooth targets
  - no shadowing
  - retroreflectors persist



$$\boxed{RCS_B \approx RCS_M \, . \cos\frac{\beta}{2}} \qquad (14)$$

Note: B as subscript indicates "bistatic" and M denotes "monostatic"

Kell, R.E. , 'On the derivation of bistatic RCS from monostatic measurements', Proc. IEEE, Vol.53, pp983-988, 1965

Forward Scatter: Babinet's Principle

- Forward scatter can substantially enhance RCS, even for stealthy targets.
- This occurs when the target lies on or close tothe baseline, and the effect can be understood using Babinet's principle from physical optics.

Definition (Babinet's Principle)

- Imagine that an infinite screen is placed between the Tx and Rx so that the signal received is zero. Now suppose that a target-shaped hole is cut in the screen between the Tx and Rx.
- Babinet's principle states that the signal that would be diffracted through the target-shaped hole must be equal and opposite to the signal diffracted around the target, since the two contributions must add to zero

- Babinet's principle tells us that we get exactly the same scattering from a perfectly-absorbing target as we would from a target-shaped hole in an infinite perfectly-conducting sheet!

- Thus a target on the Tx-Rx baseline, even if it is completely stealthy, will scatter a significant amount of energy- in fact the RCS will be of the order of

$$RCS_B = \frac{4\pi A^2}{\lambda^2} \tag{15}$$

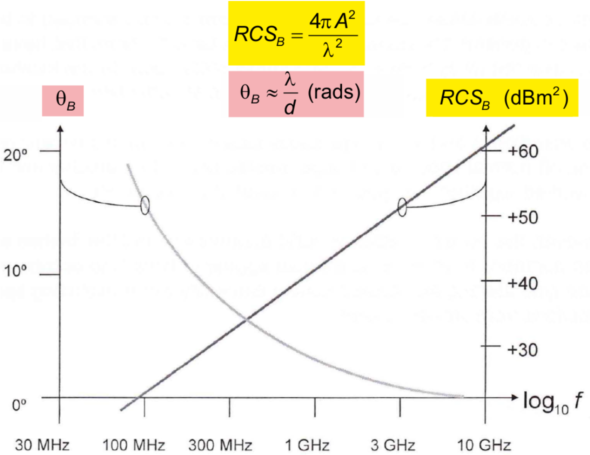
- The angular width of the scattering[2] will be of the order of

$$\theta_B = \frac{\lambda}{d} \text{ (rads)} \tag{16}$$

which tends to favour a low frequency. That is

$$\boxed{F_c = \downarrow} \Longrightarrow \boxed{\lambda = \uparrow} \Longrightarrow \boxed{\theta_B = \uparrow}$$

[2] $d \approx$ the diameter of the aperture of the scatterer



- But a target which lies exactly on the transmitter-receiver baseline will give no range information and no Doppler information, and even for a target only slightly off-baseline the range and Doppler resolution will be poor
  - So whilst a forward scatter radar will be good for target detection, location and tracking will be more difficult

Summary of Bistatic Pros and Cons

- Pros
  - Undetectable Rx and also potentially simple and cheap
  - can provides counter-stealth capabilities
  - allow the exploitation of illuminators of opportunity
- Cons
  - bistatic geometry more complicated than monostatic geometry
  - some form of synchronisation between Tx and Rx is required which is more difficult than monostatic

From Bistatic to Multistatic Radar

- Previously we saw that bistatic radar is a system where the receiver is in a different location to the transmitter
- In a distributed radar system, there will be more than one transmitter (or receiver) and some of them may be at different or the same locations
- In the example shown in the next slide there is one monostatic radar (i.e., a transmitter and a collocated receiver) and five receivers located separately
- Together, all make a single coherent radar network
- Each transmitter or receiver is called a node
- Pair of nodes will form either monostatic or bistatic configurations

Example (Multistatic Radar)