

EE3-27: Principles of Classical and Modern Radar

Target Detectability

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January 2020 (v.21a)

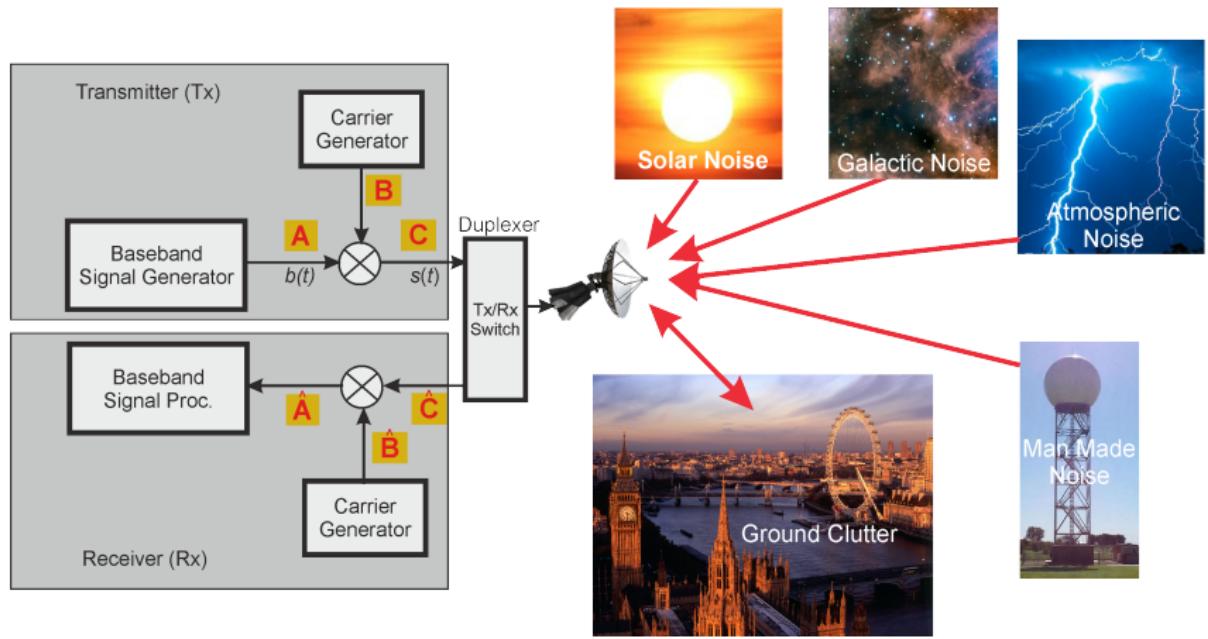
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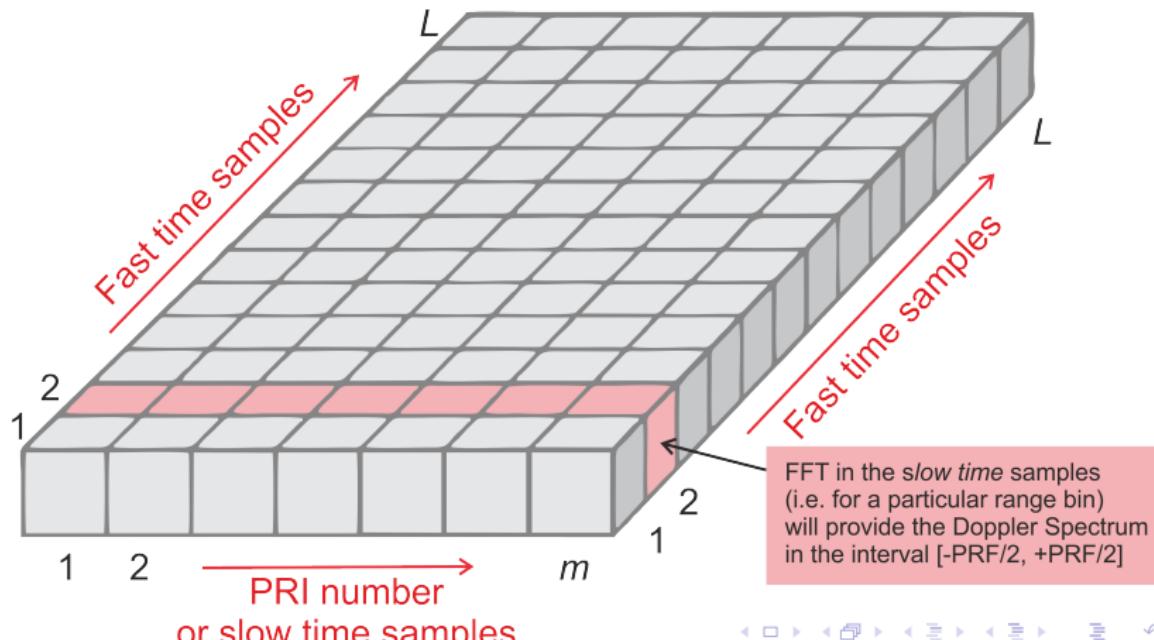
Sources of Noise Received by Radar

- Noise from many sources competes with the target echo and the radar range equation



Doppler Spectrum in One Range-Bin

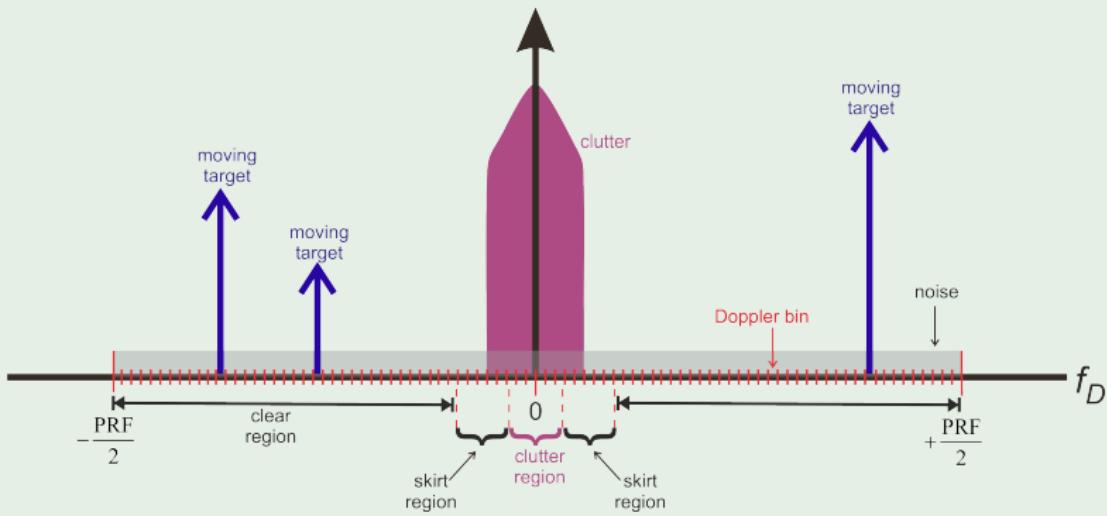
- We have seen in Topic 2 that the Doppler Spectrum is contained in the interval $[-\frac{\text{PRF}}{2}, +\frac{\text{PRF}}{3}]$
- We form the Doppler Spectrum by using the DFT in the slow time samples in the data matrix or 3D data cube



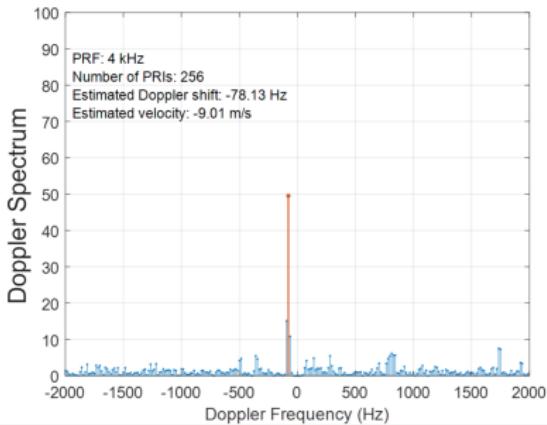
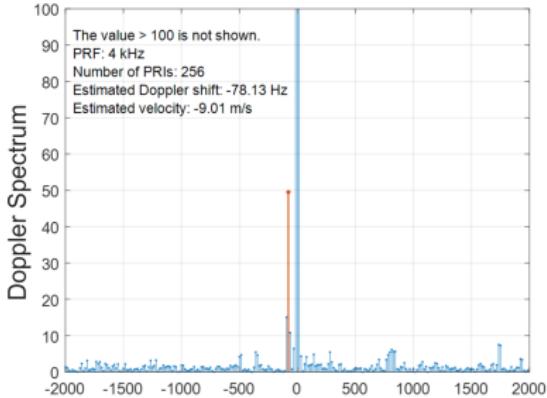
• In the Doppler Spectrum

- ▶ clutter and stationary targets are centered at $f_D = 0$;
- ▶ moving targets and noise appear throughout the spectrum.

Example (Doppler Spectrum in One Range-Bin)



Example (Using real radar data from a moving UAV)



Clutter

● Ground Clutter

- ▶ Can be intense and discrete
- ▶ can be 50-60dB > than target
- ▶ Doppler velocity zero for ground based radar
 - ★ Doppler spread small

● Rain Clutter

- ▶ Diffused and windblown
- ▶ can be 30^+dB > than target
- ▶ Doppler zero for ground based radar
 - ★ Doppler spread small

● Sea Clutter

- ▶ Less intense than ground clutter by 20-30dB
 - ★ often more diffuse than ground clutter
- ▶ Doppler velocity varies for sea radar (ship speed and wind speed)
 - ★ Doppler spread moderate

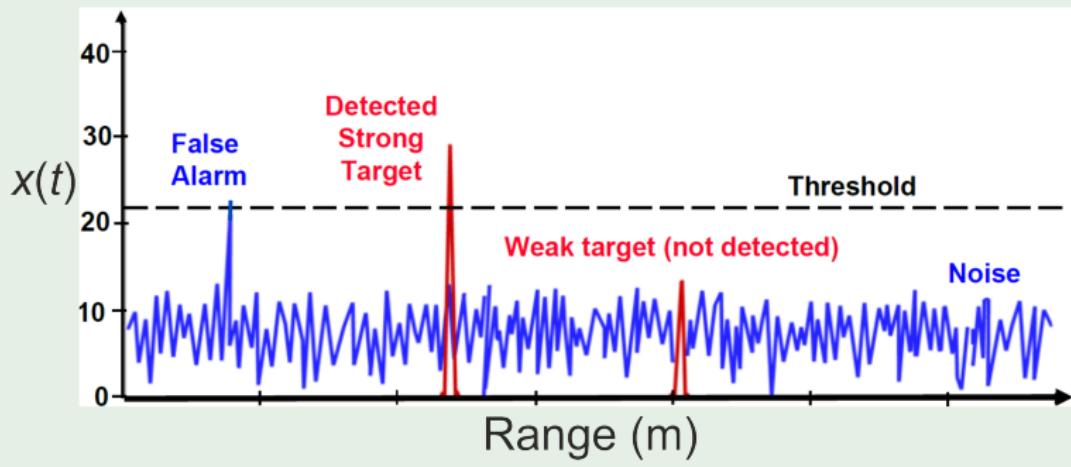
● Bird Clutter (flocks of birds)

- ▶ 100s to 10000s to point targets
- ▶ Doppler velocity - 0 to 60 knots
 - ★ Doppler of single bird has little change
 - ★ big issue for small targets

Decision Theory for Target Detection

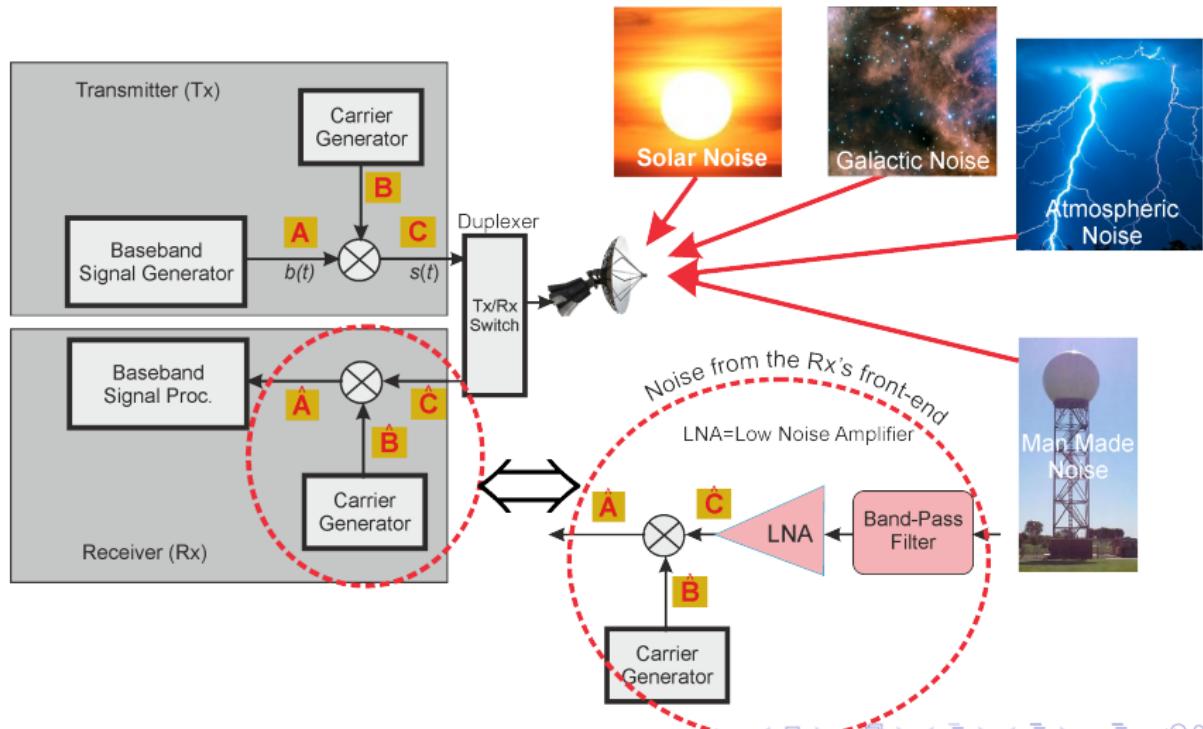
- To decide if a target is present, at a given range, we need to **set a threshold** (constant or variable) using "decision theory".
- Detection performance (**Probability of Detection**) depends on
 - the **strength of the target** relative to that of the noise and
 - the **threshold setting**

Example (Using real radar data from a moving UAV)



Rx's front-end Radar Noise

- The sky-noise (solar+galactic+cosmic+atmospheric) plus thermal noise will appear the Rx's front-end



Total Radar Noise

- The total effect of different noise sources is represented by a single noise source at the Rx-antenna with power

$$\sigma_n^2 = k_B \cdot T_s \cdot B \quad (1)$$

where

k_B = Boltzmann constant = 1.28×10^{-23} Joules/ ${}^\circ K$

T_s = system noise temperature (see Appendix-A)

$\approx T_o \cdot F_n$ (without any loss of generality, this approximation will be used here)

T_0 = $290 {}^\circ K$ (temperature)

F_n = Noise Figure of the Rx-subsystem (see Appendix-B), unitless

B = bandwidth Hz

- Note: $k_B \cdot T_s = N_0$

Radar Range Equation and SNR

- Signal Power reflected by the target and received by the radar:

$$P_{RX} = \frac{P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \lambda^2}{(4\pi)^3 \cdot R^4} \cdot RCS \quad (2)$$

- Noise Power:

$$\sigma_n^2 = k_B \cdot T_o \cdot F_n \cdot B \quad (3)$$

- Signal-to-Noise Ratio at the radar's input, SNR_{in} , which is the standard measure of the radar's ability to detect a given target at a given range from the radar:

$$SNR_{in} = \frac{P_{RX}}{\sigma_n^2} = \frac{P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \lambda^2}{(4\pi)^3 \cdot R^4 \cdot k_B \cdot T_o \cdot F_n \cdot B} \cdot RCS \quad (4)$$

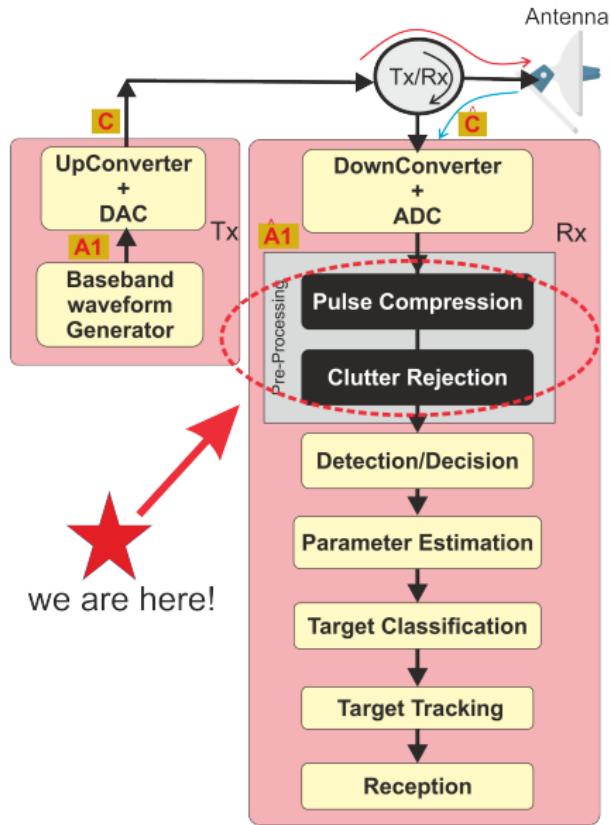
Signal-to-Noise plus Clutter plus Jammer Power Ratio (SNCIR)

$$\begin{aligned} SNCIR_{in} &= \frac{P_{RX}}{\sigma_n^2 + \sigma_{clutter}^2 + \sigma_{jammer}^2} \\ &= \frac{P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \lambda^2}{(4\pi)^3 \cdot R^4} \cdot RCS \frac{1}{k_B T_o F_n B + \sigma_{clutter}^2 + \sigma_{jammer}^2} \quad (5) \end{aligned}$$

N.B.:

- often only one of the noise, clutter and jammer is dominating

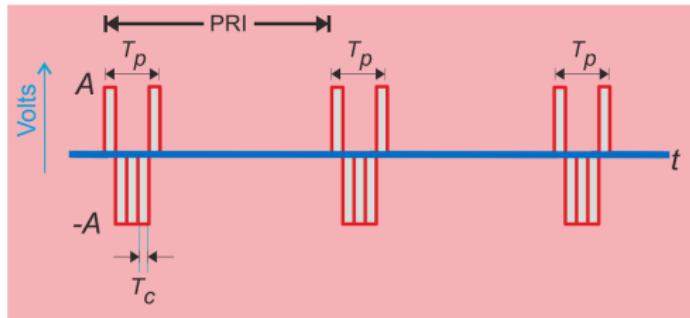
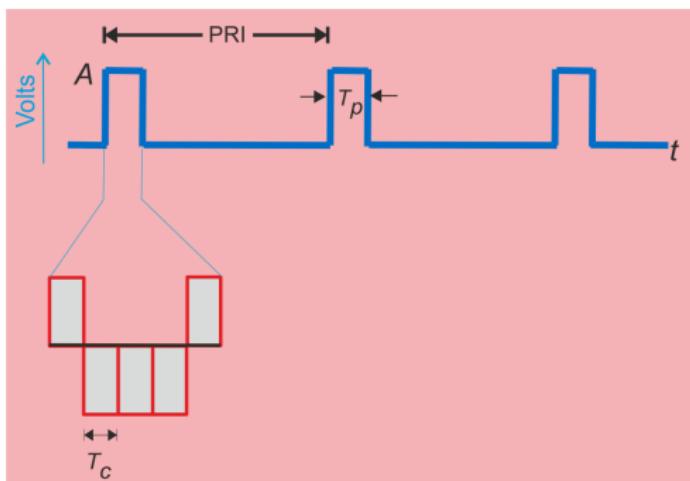
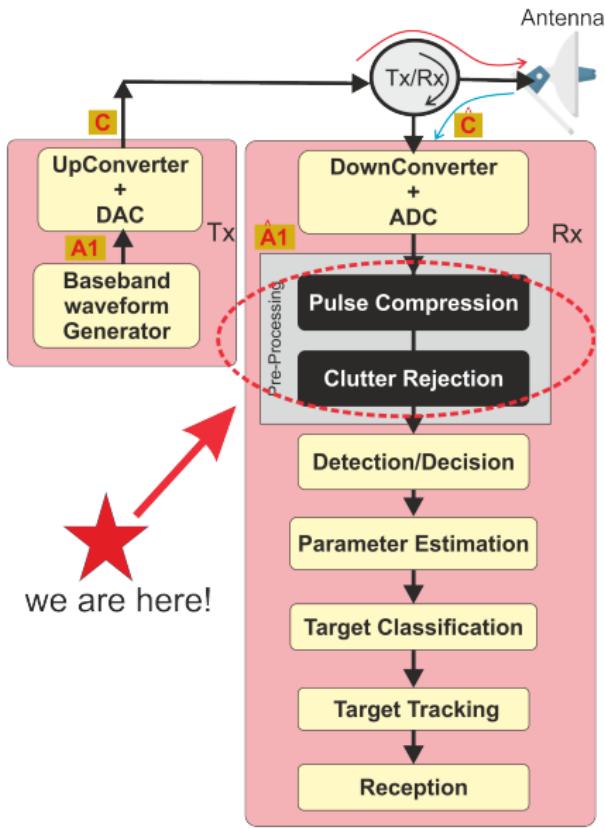
Pulse Compression and Clutter Filtering: Introduction



- we have seen that **pulse compression**, allows a radar to **simultaneously achieve**
 - ▶ the **energy of a long pulse** and
 - ▶ the **resolution of a short pulse**.

It also improves the SNR.

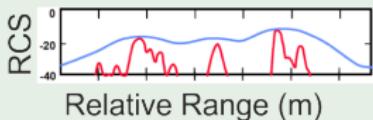
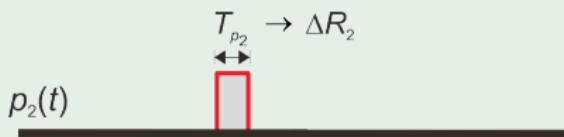
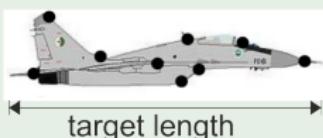
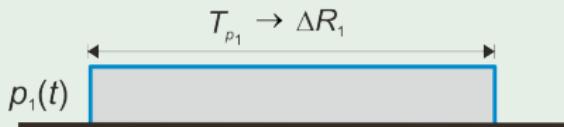
- Remember, at point-A : Pulse Compression Waveform



Important: Resolution and Target Length

- ① $\Delta R = \frac{cT_p}{2} = \frac{c}{2B} > \text{target length}$ \Rightarrow cannot resolve features along the target
- ② $\Delta R = \frac{cT_p}{2} = \frac{c}{2B} < \text{target length}$ \Rightarrow can resolve features along the target

Example



- using $p_1(t)$:

$$\begin{aligned}\Delta R_1 &= \frac{cT_{p_1}}{2}; \\ B_1 &= \frac{1}{T_{p_1}}\end{aligned}$$

- using $p_2(t)$:

$$\begin{aligned}T_{p_2} &= T_{p_1} \times 0.1 \\ \Delta R_2 &= \frac{cT_{p_2}}{2} = \frac{c \times T_{p_1} \times 0.1}{2} \\ &= \Delta R_1 \times 0.1 \\ B_2 &= B_1 \times 10\end{aligned}$$

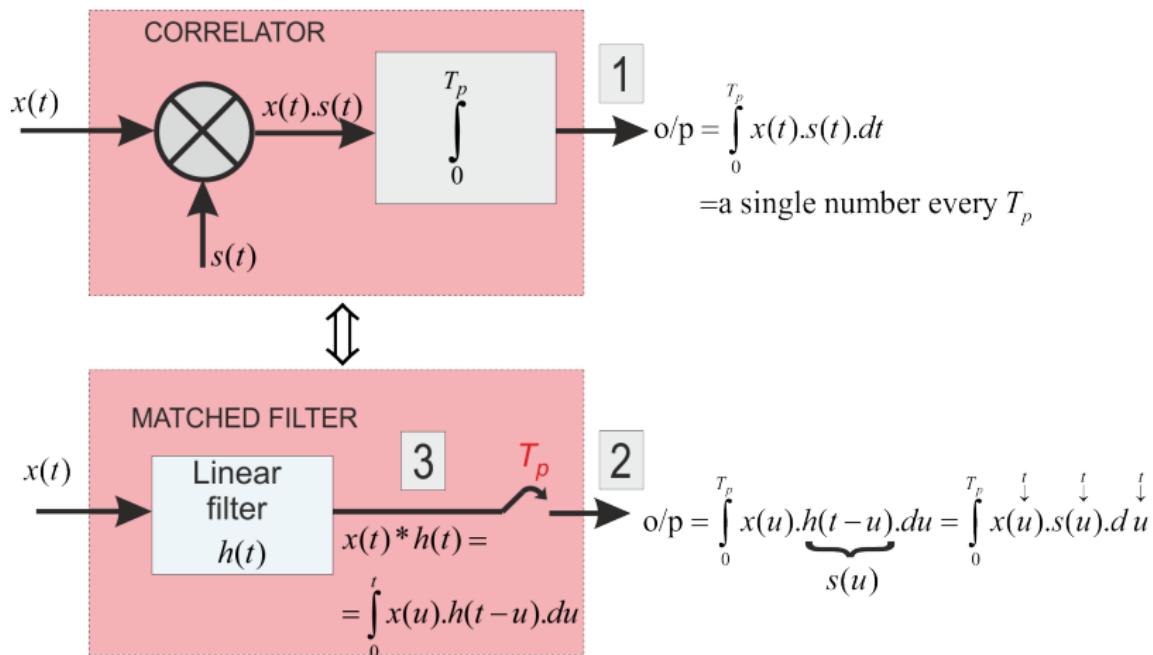
N.B.:

- Shorter pulses have higher bandwidth and better resolution
(we see more "details" of the target)

Matched Filters: Basics - Clutter Filtering

Known Signals in White Noise

- A **correlator** is equivalent to a **linear filter known as Matched Filter**



N.B.: compare correlator's o/p (point-1) with matched filter's o/p (point-2).

Definition

- If we choose the impulse response of the linear filter to be equal to the signal reversal in time, i.e.

$$h(t) = s(T_p - t), \quad 0 \leq t \leq T_p \quad (6)$$

then that linear filter is defined as "Matched Filter" (MF).

N.B.:

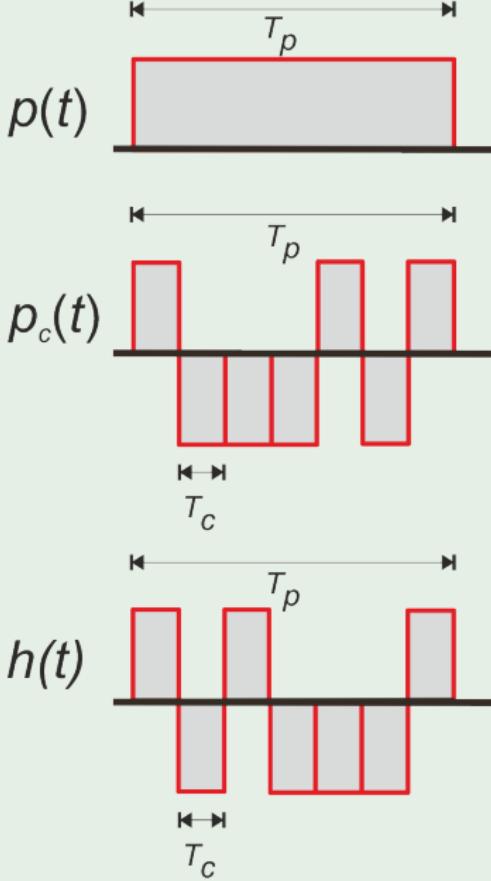
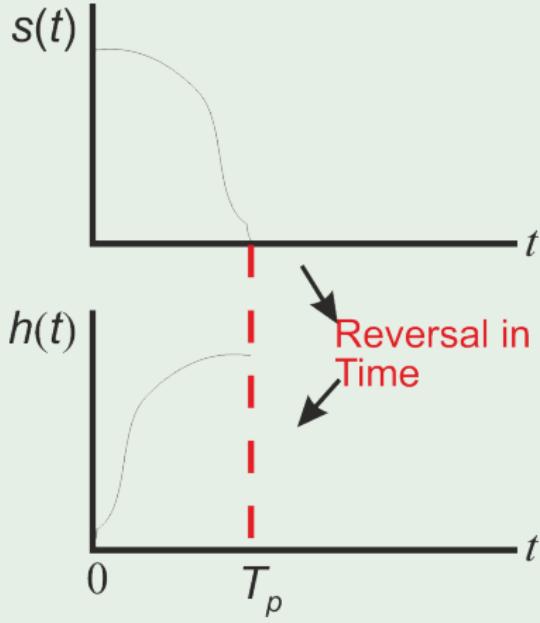
- Output Of Correlator $\stackrel{\text{only at time } t=T_p}{=} \text{Output Of Matched Filter}$
- It is also popular to normalise the MF given by Equation 6 with $\frac{2}{N_0}$. That is,

$$h(t) = \frac{2}{N_0} s(T_p - t), \quad 0 \leq t \leq T_p \quad (7)$$

This normalisation can be seen by using Equation 10 for $\text{PSD}_n(f) = N_o/2$. This normalisation does not change the system performance as it is applied to both the input signal and the input noise. Thus the MF's SNR-output remains the same

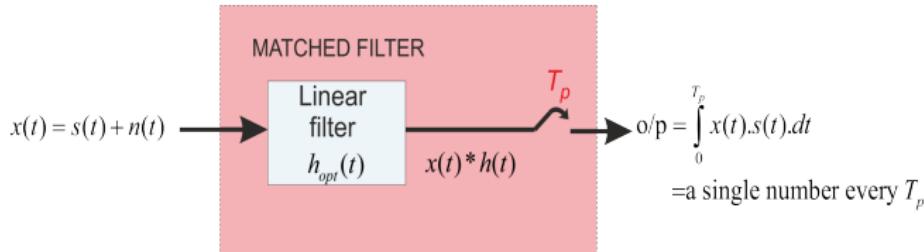
Example (Radar Compression MF)

Example (general MF)



Matched Filter: Output SNR

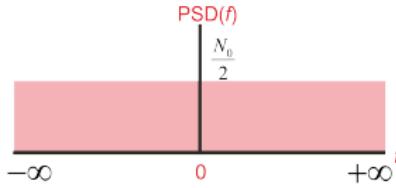
- A matched filter (matched to a pulse signal $s(t)$ of Energy E) is an optimum filter with impulse response $h_{opt}(t) = s(T_p - t)$



- The SNR at the output of a matched filter is maximum and is given by the following expression:

$$\text{SNR}_{\max} = 2 \frac{E}{N_0} \quad (8)$$

where N_0 is the single-sided PSD(f) of the noise (AWGN)



Matched Filters in Freq. Domain

- Let $h(t) = \begin{cases} s(T_p - t) & 0 \leq t \leq T_p \\ 0 & \text{elsewhere} \end{cases}$



$$\begin{aligned}s(t) &\xrightarrow{\text{FT}} S(f) = \text{FT}\{s(t)\} = \int_0^{T_p} s(t) e^{-j2\pi ft} dt \\ h(t) &\xrightarrow{\text{FT}} H(f) = \text{FT}\{h(t)\} = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \\ &= \int_0^{T_p} s(T_p - t) e^{-j2\pi ft} dt \\ &= \int_0^{T_p} s(u) e^{-j2\pi f(T_p - u)} du \\ &= e^{-j2\pi fT_p} \int_0^{T_p} s(u) e^{+j2\pi fu} du \\ &= e^{-j2\pi fT_p} \cdot S^*(f)\end{aligned}$$

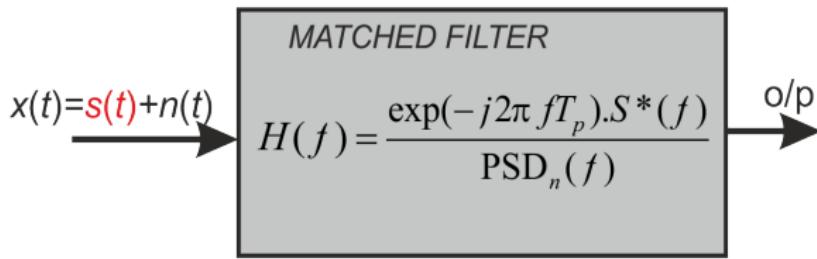
therefore

$$H(f) = e^{-j2\pi fT_p} \cdot S^*(f) \quad (9)$$

Matched Filters: Known Signals in Non-White Noise

- It can be proven that for non-white noise $n(t)$ the matched filter's transfer function $H(f)$ is given as follows:

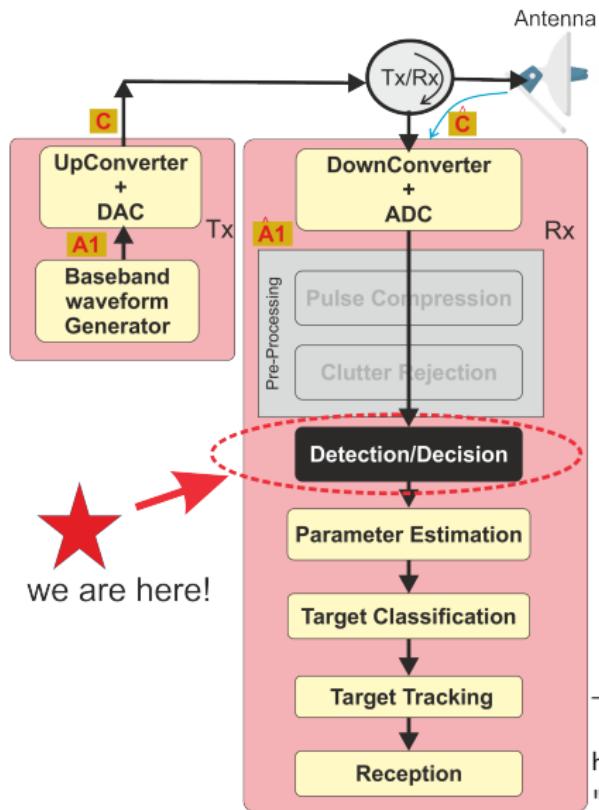
$$H(f) \approx \frac{e^{-j2\pi f T_p} \cdot S^*(f)}{\text{PSD}_n(f)} \quad (10)$$



N.B.:

- the noise at the output of the matched filter is "white"
- If $\text{PSD}_n(f) = \frac{N_0}{2}$, then Equ 10 gives $h(t) = \frac{2}{N_0} s(T_p - t)$

Target Detection: Basic Detection Theory



- The signal $x(t)$ at the input of the detection/decision block includes
 - ▶ the received background noise^a which fluctuates randomly (this is a random signal)
 - ▶ the target echo which also fluctuates randomly (this is also a random signal)
- Thus, overall, the signal $x(t)$ is a random signal
- Based on the observed signal $x(t)$, the aim of this block is to decide if a target is present at a given range (range-bin)

^aIf we have not filtered the "clutter" then we will have both noise and clutter at the input of the "decision" block.

The ‘Detection’ Problem: Basic Detection Theory

Problem

- *To determine/detect the presence of a target in noise*

Solution

- *To use optimum Detection Theory which is based on the following decision criteria*
 - ① **BAYES Criterion**
 - ② **Minimum Probability of Error** ($\min(p_e)$) Criterion
 - ③ **MAP Criterion**
 - ④ **MINIMAX Criterion**
 - ⑤ **Newman-Pearson (N-P) Criterion**
 - ⑥ **Maximum Likelihood (ML) Criterion**

Terminology and Definitions

Definition (Hypothesis)

A Hypothesis \triangleq a statement of a possible condition

Definition (Hypothesis Testing)

To choose one from a number (two or more) hypotheses

Definition (Target Detection Hypotheses)

- we have an observed signal $x(t)$ and we define two hypotheses H_0 and H_1 as follows:

$$H_0 : x(t) = n(t)$$

i.e. the target is not present

$$H_1 : x(t) = s(t) + n(t)$$

i.e. the target is present

Definition (A priori probabilities)

- These are the probabilities which can be calculated before the radar is used (i.e. before any signal is observed and before the target detection is performed)

$$\Pr(H_0), \Pr(H_1)$$

(i.e. these are calculated **BEFORE** the experiment is performed)

Definition (A posterior probabilities)

- These are the conditional probabilities

$$\Pr(H_0/x), \Pr(H_1/x)$$

which can be calculated after the signal x is received/observed (i.e. these are calculated **AFTER** the experiment is performed).

- $\Pr(H_0/x)$ and $\Pr(H_1/x)$: difficult to find. A more natural approach is to find

$$\Pr(x/H_0), \Pr(x/H_1) \quad (11)$$

since in general pdf_{x/H_0} and pdf_{x/H_1}

- ▶ are known or
- ▶ can be found

Definition (Likelihood Functions)

The two conditional probability density functions $\text{pdf}_{x/H_0}(x), \text{pdf}_{x/H_1}(x)$, i.e.

$$\text{pdf}_{x/H_0}(x), \text{pdf}_{x/H_1}(r) \quad (12)$$

are known as "Likelihood Functions" (LF)

Definition (Likelihood Ratio)

The ratio

$$\frac{\text{pdf}_{x/H_0}(x)}{\text{pdf}_{x/H_1}(x)}, \frac{\text{pdf}_{x/H_1}(x)}{\text{pdf}_{x/H_0}(x)} \quad (13)$$

is known as "Likelihood Ratio"

Hypothesis Testing re-Defined

- Note that the **statistics** of the observed signal $x(t)$ are different for different hypotheses.

Definition (Hypothesis Testing re-defined)

- If we have an observed signal $x(t)$ and have defined two hypotheses H_0 and H_1 as follows:

$$H_0 : x(t) = n(t), \quad \text{LF}^{(0)} = \text{pdf}_{x|H_0}$$

i.e. the target is not present

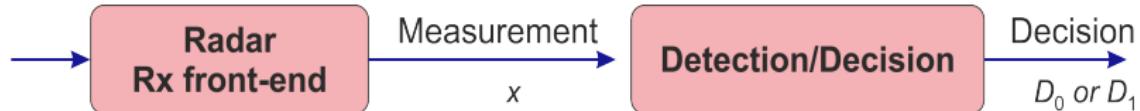
$$H_1 : x(t) = s(t) + n(t), \quad \text{LF}^{(1)} = \text{pdf}_{x|H_1}$$

i.e. the target is present

where the distributions $\text{LF}^{(0)}$ and $\text{LF}^{(1)}$ are **known**

then the problem of choosing one of the two hypotheses after having observed $x(t)$, **is known as 'Hypothesis Testing'**

Hypothesis Testing: Radar Detection Problem



- For each measurement there are 2 possibilities

<i>possibilities</i>	Measurement	LF
Target absent hypothesis, H_0 (noise only)	$x = n$	pdf_{x/H_0}
Target present hypothesis, H_1 (signal + noise)	$x = s + n$	pdf_{x/H_1}

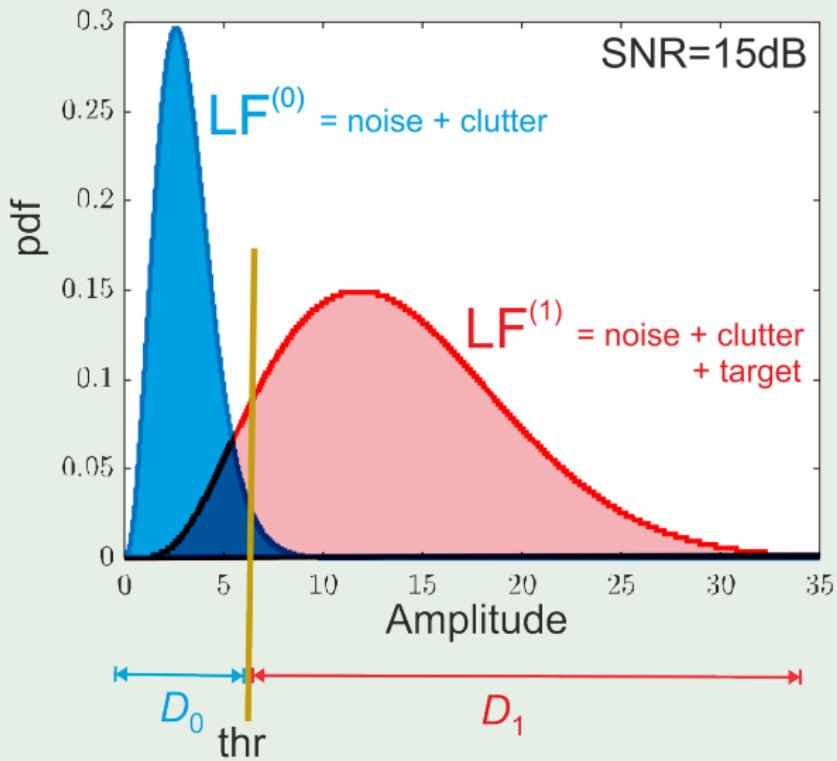
- For each measurement there are 4 decisions:

	D_0	D_1
H_0	Don't Report	False Alarm
H_1	Missed Detection	Detection

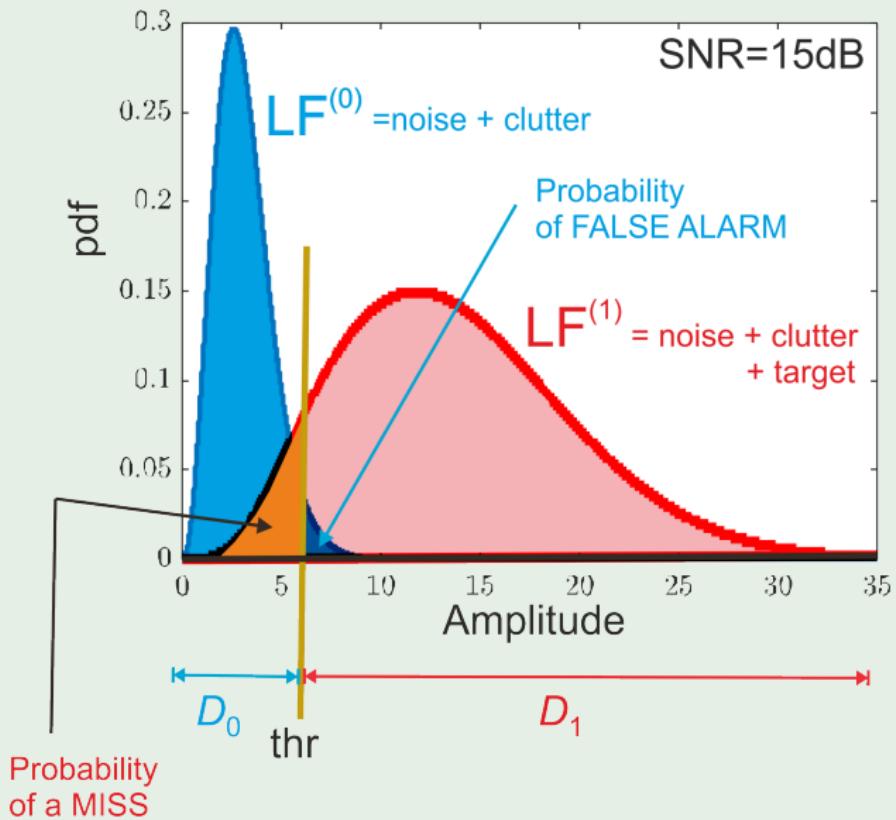
Basic Steps

- ① identify the likelihood functions $LF^{(0)}$ and $LF^{(1)}$
- ② place the two likelihood functions together on the same graph/plot
(don't add them together!!)
- ③ set a threshold (thr) for making optimum decisions. This threshold should be set by using one of the optimum decision criteria (see next section)

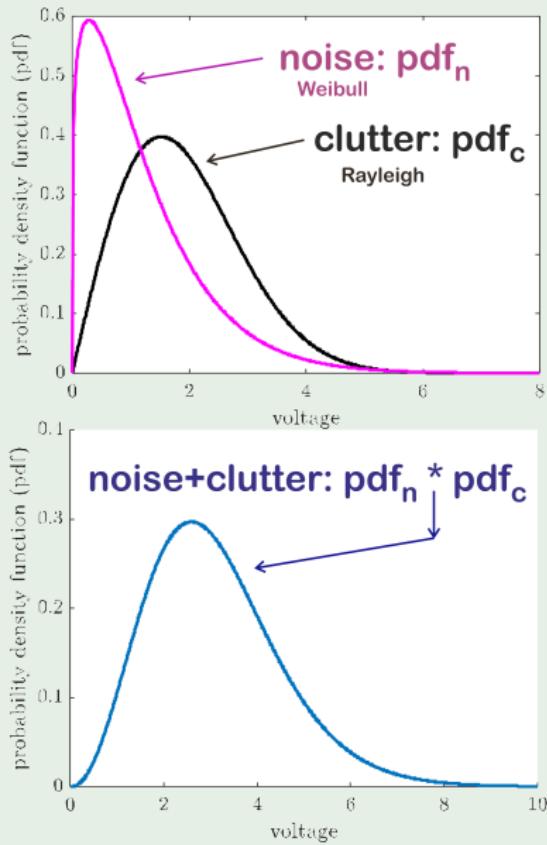
Example



Example (cont.)



Example (cont.)



Detection Theory: Main Decision Criteria

Decision Criteria

Assumptions: Consider an observed signal $x(t)$ and 2 hypotheses H_0, H_1 .
For these two hypotheses, estimate/identify the likelihood functions

$$\begin{aligned} \text{LF}^{(0)} &= \text{pdf}_{x|H_0}(x), \\ \text{LF}^{(1)} &= \text{pdf}_{x|H_1}(x) \end{aligned}$$

- Let us define the sets of parameters \mathcal{P}_1 and \mathcal{P}_2 where:

\mathcal{P}_1 denotes the set of parameters $\Pr(H_0), \Pr(H_1)$

\mathcal{P}_2 represents the set of costs/weights $C_{00}, C_{11}, C_{01}, C_{10}$

associated with the probabilities $\Pr(D_i|H_j)$,

(where D_i indicates "decision" by choosing hypothesis H_i)

Main Decision Criteria

Decision: choose the hypothesis H_i (i.e. D_i) having the maximum $G_i(x)$ where $G_i(x)$ depends on the chosen criterion as follows:

- **BAYES** Criterion
- **Minimum Probability of Error** ($\min(p_e)$) Criterion
- **MAP** Criterion
- **MINIMAX** Criterion
- Newman-Pearson (**N-P**) Criterion
- Maximum Likelihood (**ML**) Criterion

	\mathcal{P}_1	\mathcal{P}_2	choose Hypothesis with $\max_{\forall i} (G_i(x))$
Bayes	known	known	$G_i(x) \triangleq \text{weight}_i \times \Pr(H_i) \times \text{pdf}_{x H_i}$
min(p_e) or MAP	known	unknown	$G_i(x) \triangleq \Pr(H_i) \times \text{pdf}_{x H_i}$
Minimax	unknown	known	$G_i(x) \triangleq \text{weight}_i \times \text{pdf}_{x H_i}$
N-P	unknown	unknown	find threshold to satisfy: Prob of FA = given
ML	don't care	don't care	$G_i(x) \triangleq \text{pdf}_{x H_i}$

where

$$\text{weight}_0 = C_{10} - C_{00} \quad (14)$$

$$\text{weight}_1 = C_{01} - C_{11} \quad (15)$$

- N.B.:

- ▶ Note-1:
if an approximate/initial solution is required then any information about the sets of parameters \mathcal{P}_1 and/or \mathcal{P}_2 can be ignored. In this case the Maximum Likelihood (ML) Criterion should be used.
- ▶ Note-2:
Sometimes, for convenience, G_i will be used (i.e. $G_i \triangleq G_i(x)$) - i.e. the argument will be ignored.
- ▶ Note-3:
In radar the Newman Pearson criterion is quite popular: the threshold level is set based on a given probability of False Alarm

Decision Theory - ROC curve

- We normally design the "decision" threshold in accordance with some set of criteria, and this will determine the value of threshold.
- However, in many applications the receiver **performance is examined as a function of the threshold setting**.
- Since

$$\begin{aligned}\overbrace{\Pr(D_1 / H_0)}^{\triangleq p_{FA}} &= f\{\text{threshold setting}\} \\ \overbrace{\Pr(D_0 / H_1)}^{\triangleq p_{miss}} &= f\{\text{threshold setting}\}\end{aligned}$$

by plotting for a fixed threshold the following expression,

$$\underbrace{1 - \Pr(D_0 / H_1)}_{\triangleq p_{\text{detection}}} = f\{\Pr(D_1 / H_0)\}$$

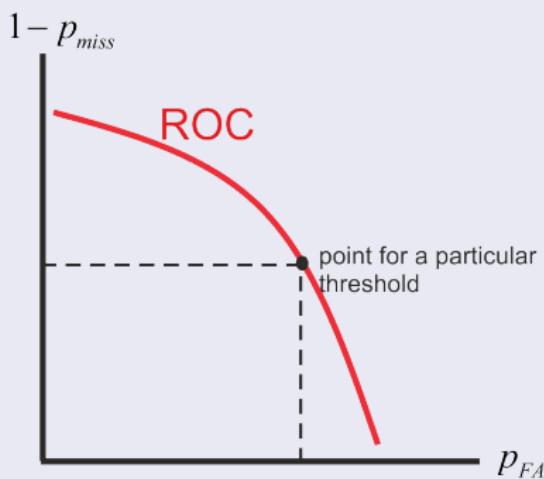
we get one point.

Definition (ROC)

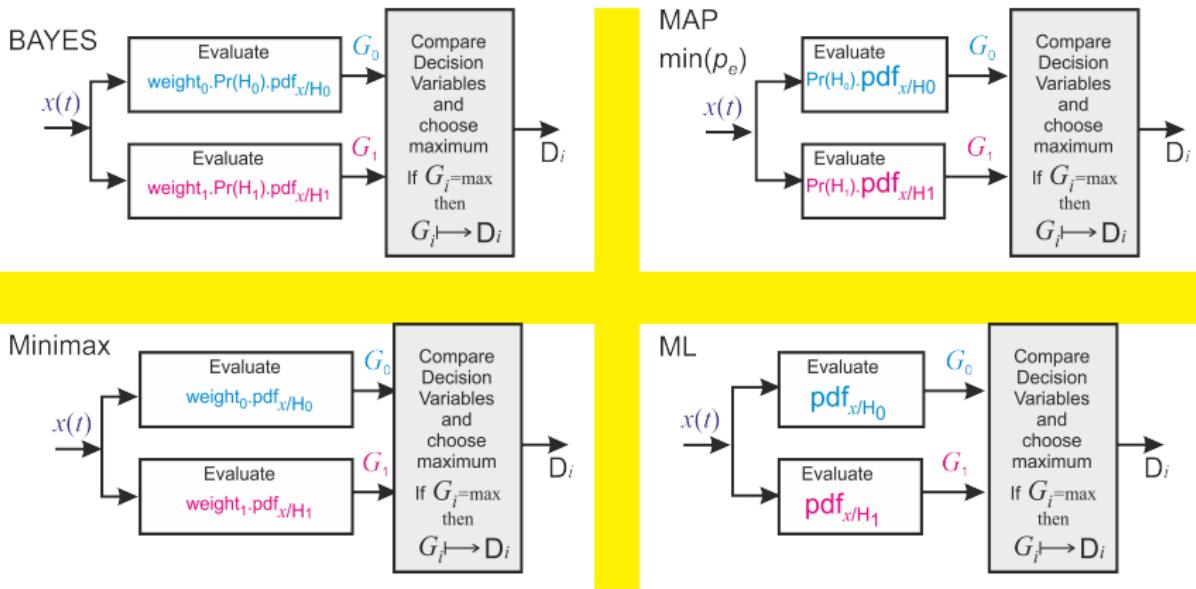
- ROC (Receiver Operating Characteristics) is defined as the decision function/curve of the probability of detection $p_{\text{detection}}$ versus the probability of false alarm p_{FA} , i.e.

$$1 - \Pr(D_0 / H_1) = f\{\Pr(D_1 / H_0)\}, \quad (16)$$

for various thresholds (one point on the curve for each threshold).



Decision Criteria: Mathematical Architectures



Optimum Detection/Rx Architecture

It can be easily proven that $G_i(x)$, for $i = 0, 1$, [i.e. $G_0(x)$ and $G_1(x)$], for all the optimum decision criteria, i.e.

$$D_i = \begin{cases} \arg \max_i \left\{ \underbrace{\text{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \right\} & \text{ML} \\ \arg \max_i \left\{ \underbrace{\Pr(H_i) \times \text{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \right\} & \text{MAP, min(pe)} \\ \arg \max_i \left\{ \underbrace{\text{weight}_i \times \text{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \right\} & \text{minmax} \\ \arg \max_i \left\{ \underbrace{\text{weight}_i \times \Pr(H_i) \times \text{pdf}_{x/H_i}(x(t))}_{=G_i(x)} \right\} & \text{Bayes} \end{cases} \quad (17)$$

are simplified, as follows:

$$G_0(x) = DC_0 , \quad (18)$$

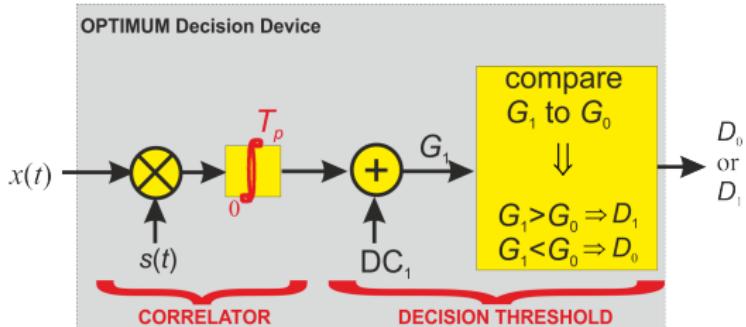
$$G_1(x) = \int_0^{T_{cs}} x(t)s(t)dt + DC_1 \quad (19)$$

- where DC_i (for $i = 0, 1$) depends on the decision criterion and is given by

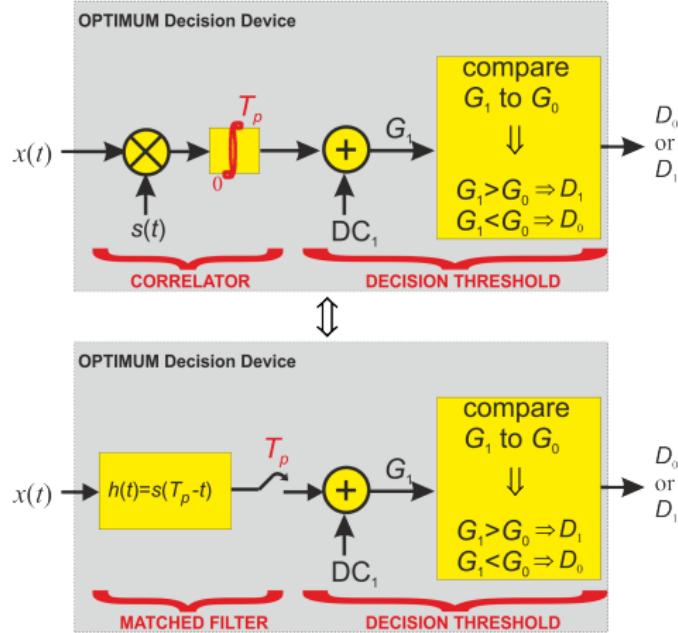
$$DC_i \triangleq \begin{cases} -\frac{1}{2}E_i; & \text{ML} \\ \frac{N_0}{2} \ln(\Pr(H_i)) - \frac{1}{2}E_i; & \text{MAP} \\ \frac{N_0}{2} \ln(\text{weight}_i) - \frac{1}{2}E_i; & \text{minmax} \\ \frac{N_0}{2} \ln(\text{weight}_i \cdot \Pr(H_i)) - \frac{1}{2}E_i; & \text{Bayes} \end{cases} \quad (20)$$

$$\text{with } E_i \triangleq \begin{cases} 0; & i = 0 \\ \text{energy of } s(t); & i = 1 \end{cases} \quad (21)$$

- Equations 17, 18 and 19 are implemented by the following optimum architecture/device:

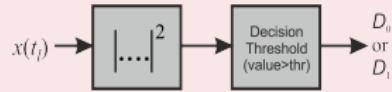


Summary: Optimum Decision Devices

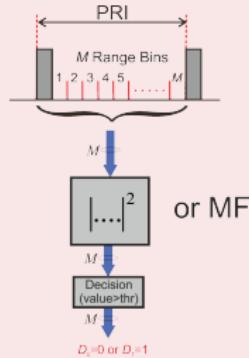


N.B.:

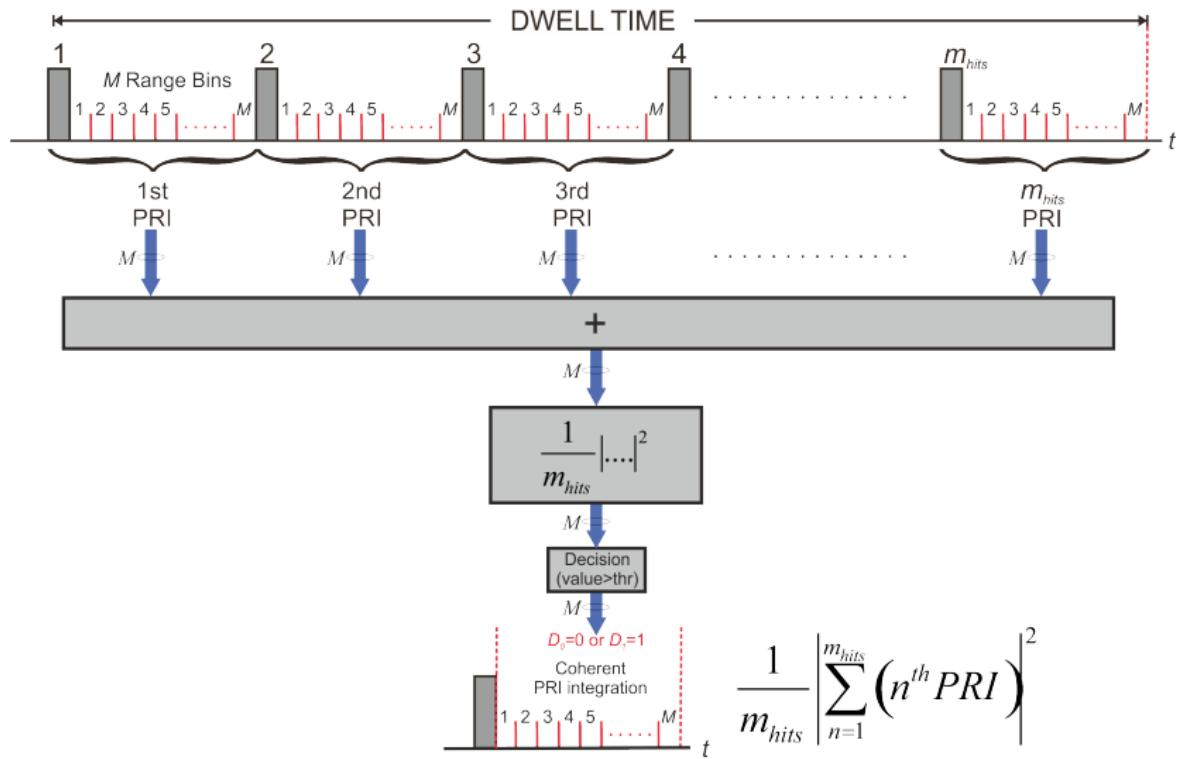
- If $x(t)$ is a single sample at time t_l , then the MF and the correlator are equivalent to $|...|^2$ and the decision device is simplified:



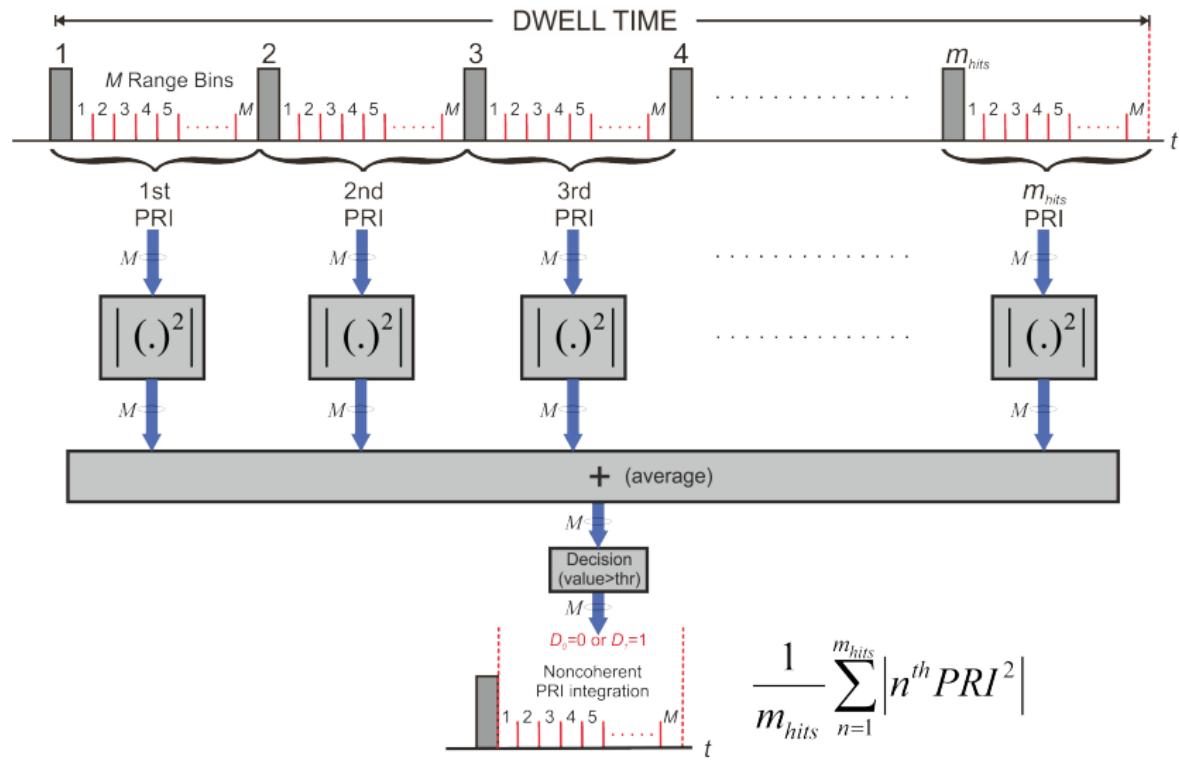
- Single PRI processing:



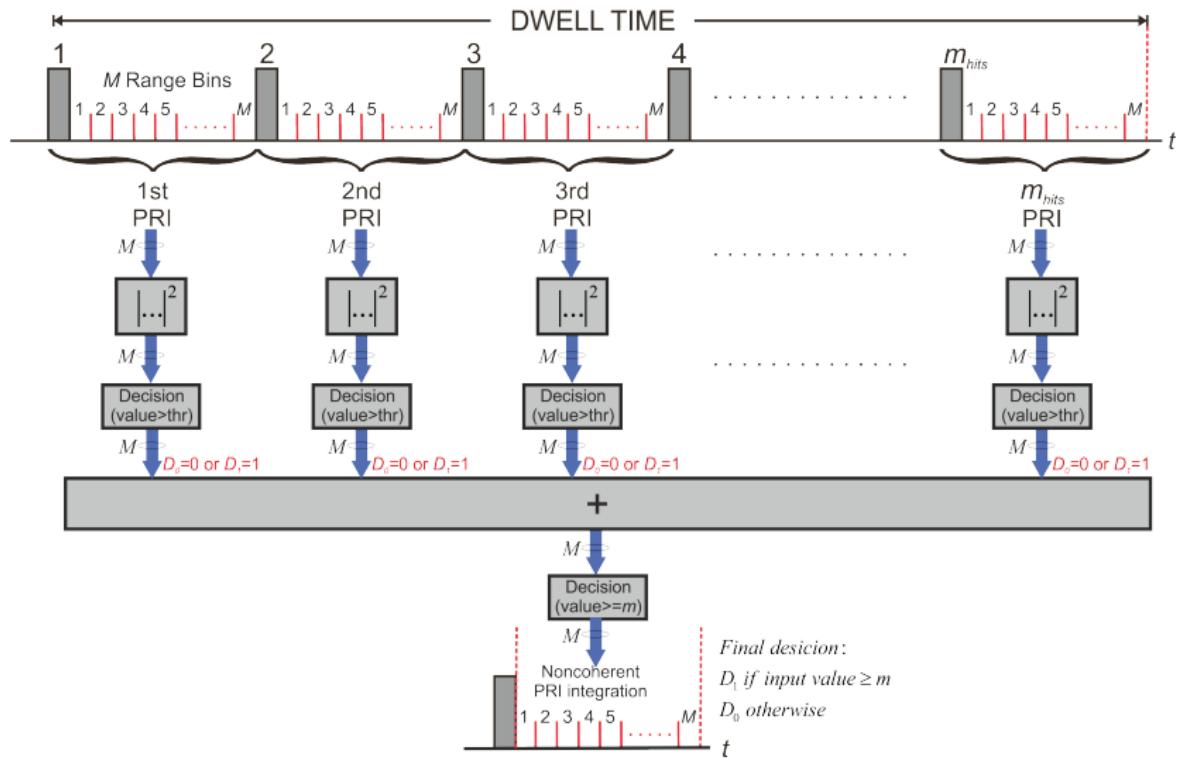
Integration of Radar Pulses - Coherent PRI Integration



Integration of Radar Pulses - Noncoherent PRI Integration



Binary PRI Integration



Summary of Types of Noncoherent PRI Integration

- **Non-Coherent Integration — Also called ("video integration")**

- ▶ Generate magnitude for each of m_{hits} pulses (PRI's)
- ▶ Add magnitudes and then threshold (decision)

- **Binary Integration (m -of- m_{hits} Detection)**

- ▶ Separately threshold each pulse (each PRI)

$$\text{Decision} = \begin{cases} D_1 = 1, & \text{if signal value} > \text{threshold} \\ D_0 = 0, & \text{otherwise} \end{cases} \quad (22)$$

- ▶ Count number of threshold crossings (the number of 1's)
- ▶ Threshold this sum of threshold crossings
- ▶ Simpler to implement than coherent and non-coherent (video integration)

- **Cumulative Detection (1-of- m_{hits} Detection)**

- ▶ Similar to Binary Integration
- ▶ Require at least 1 threshold crossing for m_{hits} PRI's

Appendix-A: Radar System Noise Temperature

- The radar system noise temperature T_s is divided into 3 components

$$T_s = T_a + T_r + L_r \cdot T_e \quad (23)$$

where

T_a = contribution from antenna

T_r = contribution from the RF components
(between antenna and Rx)

T_e = temperature of the Receiver

L_r = loss of the input RF components (natural units)

- The 3 temperature components can be broken down even further:

$$T_a = \frac{0.88 T_{sky} - 254}{L_a + 290} \quad (24)$$

$$T_r = T_{tr}(L_r - 1) \quad (25)$$

$$T_e = T_o(F_n - 1) \quad (26)$$

where

T_{sky} = the apparent temperature of the sky (from graph)

L_a = the dissipative loss within the antenna (natural units)

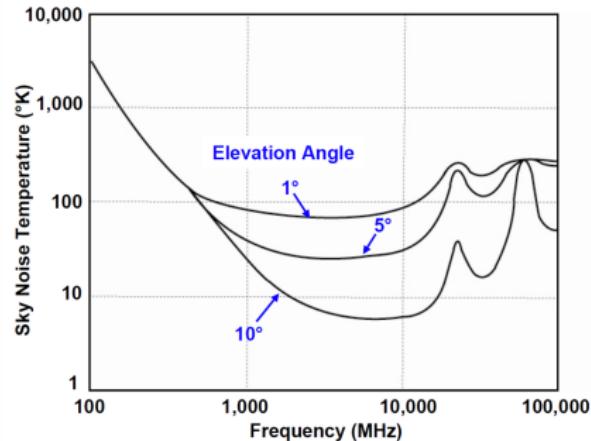
T_{tr} = the physical temperature of the RF components

F_n = the noise factor of the Rx (natural units)

T_o = the noise reference temperature of 290 K

- Note: all the temperature quantities are in units of $^{\circ}\text{K}$.

Sky Noise Graph



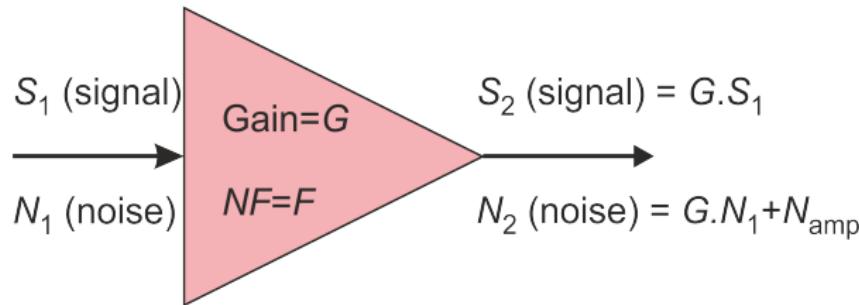
- Note: The above graph takes into account:

- galactic noise,
- cosmic blackbody radiation,
- solar noise and
- atmospheric noise (due to the troposphere)

Appendix-B: Noise Factor (or Noise Figure)

- An amplifier will introduce noise signal. We characterise this increase in noise by defining the *Noise Factor* for a stage

$$NF \triangleq \frac{\text{signal to noise ratio at i/p}}{\text{signal to noise ration at o/p}} = \frac{S_1 / N_1}{S_2 / N_2} \quad (27)$$



- That is

$$\left. \begin{array}{l} NF = \frac{S_1/N_1}{S_2/N_2} \\ S_2 = GS_1 \\ N_2 = GN_1 + N_{amp} \end{array} \right\} \Rightarrow NF = \frac{N_2}{GN_1} \quad \left. \right\} \Rightarrow NF = \frac{GN_1 + N_{amp}}{GN_1} \quad (28)$$

- Note: If no noise is introduced by the amplifier the $NF = 1$. That is

noiseless ampl $\Rightarrow NF = 1$

(29)

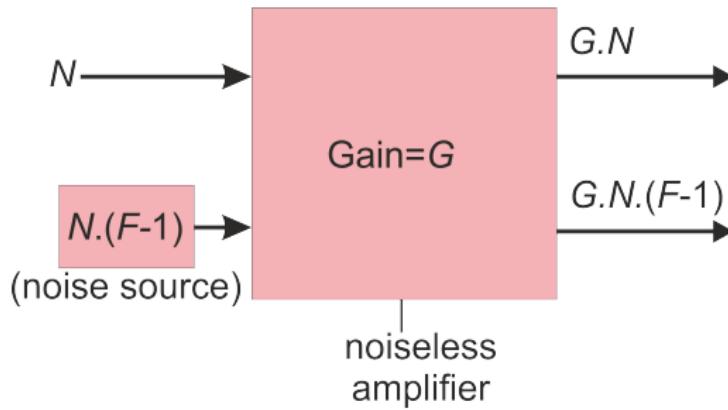
- NF is defined for a particular noise input. This implies added noise is constant but the input noise is amplified.

Appendix-C: Noise Factor of Cascaded Stages

- Assume that NF is defined for an i/p noise level N
- Let us define

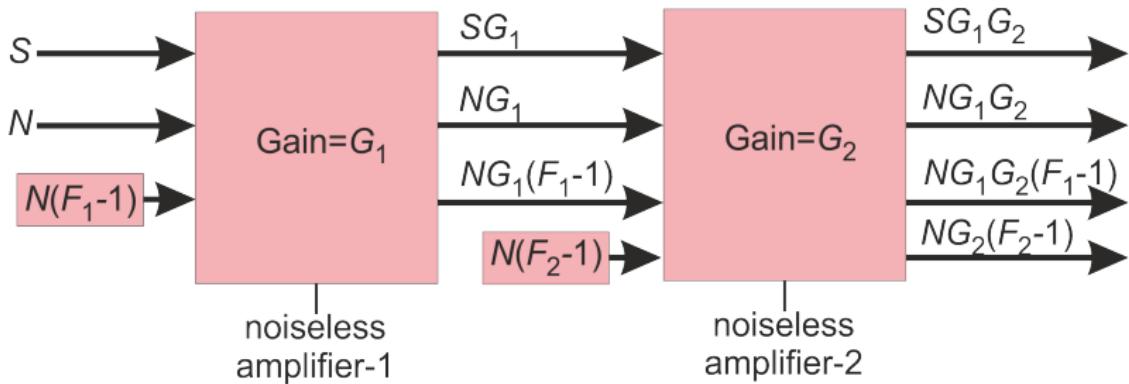
$$F \triangleq NF \text{ of amplifier} \quad (30)$$

- Replace each stage by a noiseless amplifier with an additive noise source at i/p. That is,



- For two stages, let us define

$$\text{1st amplifier: } F_1 \triangleq NF; \text{ 2nd amplifier: } F_2 \triangleq NF \quad (31)$$



- overall NF:

$$\begin{aligned}
 NF &= \frac{\text{signal to noise ratio at I/P}}{\text{signal to noise ratio at O/P}} = \frac{S/N}{\frac{SG_1G_2}{NG_1G_2 + NG_1G_2(F_1-1) + NG_2(F_2-1)}} \\
 &= \frac{G_1G_2 + G_1G_2(F_1 - 1) + G_2(F_2 - 1)}{G_1G_2} \\
 \Rightarrow & NF = F_1 + \frac{F_2 - 1}{G_1} \quad (32)
 \end{aligned}$$

Appendix-D: Clutter Distributions

- Detection Statistics Calculations
 - ▶ Steady Targets and Swerling 1,2,3,4 Targets in Gaussian Noise
 - ▶ Chi- Square Targets in Gaussian Noise
 - ▶ Log Normal Targets in Gaussian Noise
 - ▶ Steady Targets in Log Normal Noise
 - ▶ Log Normal Targets in Log Normal Noise
 - ▶ Weibull Targets in Gaussian Noise
- N.B: Chi Square, Log Normal and Weibull Distributions have long tails
- When used
 - ▶ Ground clutter: Weibull
 - ▶ Sea Clutter: Log Normal
 - ▶ HF noise: Log Normal
 - ▶ Birds: Log Normal

Matlab Functions of Useful pdf's

- see also "Classes" in OneNote and talk to GTAs:

		pdf	random number	Comments
1	Gaussian Noise	<code>normpdf(x,mu,sigma)</code>	<code>normrnd(mu,sigma)</code>	
2	Chi-Square	<code>chi2pdf(x,DOF)</code>	<code>chi2rnd(DOF)</code>	
3	Log Normal	<code>lognpdf(x,mu,sigma)</code>	<code>lognrnd(mu,sigma)</code>	
4	Weibull	<code>wblpdf(x,A,B)</code>	<code>wblrnd(A,B)</code>	A: scale param.; B: shape param.
5	Swerling 1 & 2: RCS σ	<code>expPDF(x,mu)</code>	<code>exprnd(mu)</code>	$x = \sigma$; $mu = \bar{\sigma}$
6	Swerling 1 & 2: Amp a	<code>raylpdf(x,B)</code>	<code>raylrnd(B)</code>	$x = a$; $B = \sqrt{\frac{\sigma}{2}}$
7	Swerling 3 & 4: RCS σ	$\frac{4}{\mu} \cdot \text{chi2pdf}\left(\frac{4}{\mu} \cdot x, \text{DOF}\right)$	$\frac{\mu}{4} \cdot \text{chi2rnd}(\text{DOF})$	$x = \sigma$; $mu = \bar{\sigma}$; DOF = 4
8	Swerling 3 & 4: Amp a	$\frac{4}{\mu^4} \cdot x^2 \cdot \text{raylpdf}\left(\frac{2}{\sqrt{\mu}} \cdot x, B\right)$		$x = a$; $mu = \bar{\sigma}$; $B = 1$

Note:

- x : variable
- μ : mean
- σ : standard deviation

To find the pdf from a number of random numbers, use:

- `[N,edges] = histcounts(data,NumBins,'normalization','pdf')`
- `histogram(data,NumBins,'normalization','pdf')`