Prof. Wei Yi
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Response to Decision on Manuscript T-SP-33360-2025

Dear Editor and Reviewers,

Thank you for giving us an opportunity to revise the paper "MIMO Channel Shaping and Rate Maximization Using Beyond Diagonal RIS". Your feedback and suggestions have been invaluable in helping us improve the quality of the manuscript. Below we prepare a point-to-point response and highlight the corresponding changes in text, where labels have been matched to the submission for your convenience. We hope the revisions and clarifications make the manuscript meet the TSP publication standards.

Yours sincerely,

Yang Zhao, Hongyu Li, Bruno Clerckx, and Massimo Franceschetti

Editorial Decision

The paper has been improved after this round of revision. However, reviewers still pointed out several critical issues regarding the novelty, practical merit, performance assessment of the paper. For example, one reviewer criticized that the rate maximization problem is actually extensively studied in existing papers and solved by different methods, by different algorithms, e.g., [36]. The application of the derived bounds on Singular Values (SVs) on Beyond Diagonal (BD)-Reconfigurable Intelligent Surface (RIS) channel has been well established in early papers, e.g., [45]. I am deciding on the major revision decision. However, in the revised version, the authors need to clarify the above-mentioned very fundamental issues, otherwise the paper can still be rejected.

Response We appreciate your feedback and summary again.

In [36], the authors considered an energy efficiency maximization problem for *multi-user Multiple-Input Single-Output (MISO)* downlink systems. A Penalty Dual Decompistion (PDD) method was proposed for symmetric BD-RIS design, which is a two-layer iterative procedure alternating between the primal variables in the inner layer and the penalty coefficient in the outer layer. The method is often used to tackle hard-to-satisfy constraints (e.g., Signal-to-Interference Noise Ratio (SINR) threshold) and is notoriously tricky to implement. Invoking PDD for Multiple-Input Multiple-Output (MIMO) rate maximization problem (36) is theoretically feasible but computationally very inefficient. For example, PDD requires $O(N_\mathrm{s}^2)$ and $O(N_\mathrm{s}^4)$ flops to obtain a stationary solution for Diagonal (D)-RIS and fully-connected BD-RIS, respectively [36, Table I], whereas our proposed geodesic Riemannian Conjugate Gradient (RCG) Algorithm 1 achieves the same with respectively $O(N_\mathrm{s})$ and $O(N_\mathrm{s}^3)$ flops. The latter exploits the structure of the block-unitary constraint for acceleration and can accommodate for the symmetric constraint when necessary (as discussed in Section V-D1).

We would also like to point out that [45] investigated the optimal beamforming design for multi-hop *Amplify-and-Forward (AF) relays* with *no channel SV manipulation bounds* taken into account. The AF relays are active devices that consume power to amplify the signal in a noisy manner, and the channel SVs are effectively unbounded. The optimal structure of AF relays [45, (17)] comes in the form

$$\mathbf{F} = \mathbf{V}_\mathrm{B}\mathbf{\Lambda}\mathbf{U}_\mathrm{F}^\mathsf{H}, \tag{i}$$

where $\mathbf{\Lambda}$ is a diagonal matrix that models the effect of relay power allocation. The aim of [45] is to show "The optimal source and relay matrices jointly diagonalize the multi-hop MIMO relay system into a set of parallel scalar channels". On the other hand, our paper aims to characterize the fundamental limits of BD-RIS in MIMO channel SVs shaping. BD-RIS can only manipulate the phase and amplitude of the signal by passive scattering without amplification, which is fundamentally different from AF relays in terms of hardware architecture, power consumption, and noise characteristics. It just happens that the mathematical modeling of AF relays of unit power coincide with that of fully-connected BD-RIS, and thus the rate-optimal solutions. The underlying physical constraints and the resulting practical implications are clearly distinct. We also supplied extensive case studies of BD-RIS in Propositions 1 – 3 and the resulting Corollaries, most of which have not been established in the literature.

Reviewer 1

The authors have addressed my concerns.

Response Thank you for your positive feedback and continued support.

Reviewer 2

This paper explores potentials of a new type of RIS, specifically adopting a BD scattering model, in shaping point-to-point MIMO channels for improved wireless performance. The authors derive analytical bounds under specific scenarios and propose a numerical optimization method for broader shaping problems, both of which are verified by simulation results. The paper tackles an important and timely question regarding the channel shaping capabilities of passive RIS, particularly moving beyond the conventional D-RIS model.

The singular value analysis and optimization is a fresh and relevant perspective in the field. However, there are areas which need further improvement:

2.1 *Implementation of BD-RIS: The paper could benefit from more thorough discussions on practical implementation challenges of BD-RIS, including manufacturing complexity, calibration, and hardware imperfections. Addressing these would enhance the paper's practical relevance and applicability.*

2.2 *Comparison between RCG methods: The link could be highlighted by mentioning that the non-geodesic update (additive + retraction) effectively employs a first-order Taylor approximation of the geodesic update (multiplicative via matrix exponential), thus necessitating the retraction step to ensure iterates remain on the manifold.*

2.3 *Applicability to other BD-RIS models: The focus is strongly on the group-connected BD-RIS versus D-RIS. The authors could clarify how the proposed methods and results might extend to other BD-RIS models, such as multi-sector or multi-layer configurations. This would broaden the applicability of the findings and provide a more comprehensive understanding of the RIS landscape.*

2.4 *Readability of the Technical Parts: The readability of the paper could be improved by having more intuitions and/or explanations after important theorems/lemmas/corollaries.*

Reviewer 3

The reviewer appreciates the authors' efforts to prepare the revision. The reviewer has the following concerns:

3.1 *The authors provide a bunch of bounds (Prop. 2) on SVs of BD-RIS channels. The significance of these bounds is unclear. What can these bounds be used for? One possible solution lies in the achievable maximal channel capacity (Corollary 3.5). However, this result (the well-known forward and backward channel alignment) has long been established, e.g., [45]. Especially, the bounds developed in corollaries 3.1-3.3 are a great number of inequalities coupling with each other. How could these bounds be used?*

Response We appreciate the reviewer for the clinical questions and aim to answer them one-by-one below.

- The SV bounds in Proposition 2 complements the Degrees of Freedom (DoF) result in Proposition 1 by quantifying the dynamic range of extreme singular values in low-multipath scenarios. They reveal a saturation effect of increasing the number of BD-RIS elements and group size in enhancing channel shaping capability. Therefore, the bounds can be used to guide practical RIS configurations, especially in millimeter-wave and terahertz systems with sparse propagation environment, for a balanced performance gain and hardware complexity. Another use case is strategic network planning where the bounds can help compare the shaping capabilities of a single massive RIS versus multiple smaller distributed ones.
- The bounds in Corollary 3.1 sketch the entire achievable channel SV region under the assumption of negligible direct channel and fully-connected BD-RIS. While direct application of every inequality within is not straightforward, they indeed provide a rich theoretical foundation and ultimate performance limit for MIMO channel shaping using BD-RIS. One may choose any subset of those bounds for specific wireless applications and metrics, and some examples are given in Corollaries 3.2 and 3.3.
- The bounds in Corollary 3.2 reveal the upper (resp. lower) bound on the product of some largest (resp. smallest) channel SVs. These bounds can be applied, for instance, as a shortcut to establish the upper bound of BD-RIS-aided MIMO channel capacity at extreme Signal-to-Noise Ratios (SNRs).
- The bounds in Corollary 3.3 reveal the shaping limits of the n-th largest channel SV. These bounds can be applied, for instance, to provide a closed-form passive (and thus active) beamformer for spatial multiplexing with a limited number n of Radio Frequency (RF) chains. Another use case is to enhance the harvested power for MIMO wireless power transfer with RF combining by maximizing the dominant

channel SV. One may also improve the channel condition number for numerically stable signal processing by maximizing the smallest channel SV.

- While [45] discussed the optimal beamforming structure for multi-hop AF relays, its focuses of channel diagonalization and relay power allocation are fundamentally different from this paper. The literature provides neither channel SV bounds nor channel capacity bounds. Please kindly refer to our response to the editor on this matter.

3.2 *Prop. 4 imposes the assumption that $f(\cdot)$ must be symmetric gauge function in SVs of $\mathbf{H}$. Note gauge function is strong assumption since it should be homogeneous and convex (e.g., [62, Sec 3-I]) in SVs of $\mathbf{H}$, which indeed severely restricts the range of its application. When $f(\cdot)$ is nonconvex, subdifferential ($\mathbf{D}$ in eq. (26)) generally does not exist. In fact, any commonly adopted performance metrics, e.g., spectral efficiency, energy efficiency, receiving power are not gauge functions (they are inhomogeneous and nonconvex). On the other hand, to obtain partial differential with respect to (w.r.t.) RIS, (i.e., the aim of Prop. 4), the assumption of gauge function is unnecessary. Directly taking the derivative of $f(\cdot)$ w.r.t. RIS is more preferable.*

3.3 *The authors propose Riemannian Conjugate Gradient (RCG) algorithm (Algorithm 1) to optimize RIS configuration. Note CG descent on manifold is indeed well established and is widely used for optimization on manifolds, e.g., [i]. It can be implemented conveniently by off-the-shelf solvers, e.g., Manopt [ii]. Then, what is the novelty or significance of Sec. III-B?*

References

[i] N. Boumal, *An Introduction to Optimization on Smooth Manifolds*. Cambridge, UK: Cambridge University Press, Mar 2023.

[ii] N. Boumal, B. Mishra, P.-A. Absil, and R. Sepulchre, "Manopt, a Matlab toolbox for optimization on manifolds," *Journal of Machine Learning Research*, vol. 15, no. 42, pp. 1455–1459, 2014. [Online]. Available: https://www.manopt.org