Prof. Wei Yi
Associate Editor
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Response to Decision on Manuscript T-SP-33360-2025.R1

Dear Editor and Reviewers,

Thank you for giving us an opportunity to revise the paper "MIMO Channel Shaping and Rate Maximization Using Beyond-Diagonal Reconfigurable Intelligent Surface (RIS)". Your feedback and suggestions have been invaluable in helping us improve the quality of the manuscript. Below we prepare a point-to-point response and highlight the corresponding changes in text, where labels have been matched to the submission for your convenience. We hope the revisions and clarifications make the manuscript meet the TSP publication standards.

Yours sincerely,

Yang Zhao, Hongyu Li, Bruno Clerckx, and Massimo Franceschetti

Editorial Decision

In this round of review, while two reviewers have acknowledged the improvements made in the revised paper, the third reviewer has raised critical concerns regarding the practical usefulness and overall contribution of the work. Specifically, the reviewer points out that the bounds on the singular values of Beyond-Diagonal (BD)-RIS may not be easily exploitable in practice, and that the number of derived inequalities appears excessive without demonstrating clear practical value. Furthermore, the reviewer notes that the proposed method is limited to single-user scenarios without QoS constraints, thereby limiting its novelty and relevance when compared to existing literature.

Response Please refer to our response to Reviewer 3.

Reviewer 1

(There are no comments.)

Reviewer 2

This version has addressed all my comments.

Response Thank you for your positive feedback and continued support.

Reviewer 3

The reviewer sincerely appreciates the authors' efforts in revising the manuscript and preparing the response. However, from the reviewer's viewpoint, the major issue remains unresolved — the major results in the manuscript seem not quite useful.

3.1 *The reviewer does not think the bounds on singular values of BD-RIS (Corollary 3.1 - 3.5) are useful. In fact, as reflected in (45a) - (45c), these bounds are actually not easily exploitable. Especially, recalling BD-RIS practically has a number of elements no smaller than several tens, the inequalities seem to be too many to be useful.*

Response It is a pity that the reviewer found our results not quite useful. We believe there could a misunderstanding on the main gist and logic flow of the paper. And we will try to explain this as follows.

(a) We are not dealing with "bounds on singular values of BD-RIS". The research question is using BD-RIS to shape Multiple-Input Multiple-Output (MIMO) in terms of *channel* singular values and their functions. We believe a comprehensive answer serves as a theoretical support/reference for the vast number of RIS research papers and real-world applications.

(b) The reviewer may have reasoned that "(45a) - (45c) are not easily exploitable" →"Corollary 3.1 - 3.5 are not useful" →"the major results in the manuscript seem not quite useful." However, the message we were trying to convey is that Corollary 3.1 can be hard to exploit so that Corollaries 3.2 – 3.6 are introduced to show its profound implications and (45a) - (45c) are introduced to visualize its complications. Then we develop optimization methods to validate those bounds and handle more general singular value functions. We believe the most important results are the propositions, the closed-form BD-RIS solutions for different shaping objectives, and the geodesic BD-RIS design algorithm.

(c) We believe these theoretical results are useful in the sense that they provide strong mathematical supports to *understand exactly* to what extent passive RIS can shape the MIMO channel. The research question is answered from different perspectives in Propositions 1 – 3. It turns out that Proposition 3 induces Corollary 3.1 with exceeding number of inequalities as indicated by the reviewer; but they indeed apply to arbitrary number of RIS elements. We then carefully pick *subsets* of those in Corollaries 3.2 – 3.6 to provide (i) ready-to-use performance bounds for typical wireless applications and (ii) closed-form optimal RIS solutions. In the context of BD-RIS-aided MIMO, those corollaries translate to:

- closed-form transmit precoder for spatial multiplexing with a limited number of Radio Frequency (RF) chains;
- closed-form transmit precoder for MIMO wireless power transfer with RF combining;
- maximum and minimum channel power gains, which further motivates low-complexity transmit precoder design for wireless communication and power transfer in Section IV;
- channel capacity at general and extreme Signal-to-Noise Ratios (SNRs) with water-filling precoder.

The above non-exhaustive list demonstrates the outcomes of understanding channel shaping limits; we really hope our ingenious readers can discover more results specific to their research. In fact, that is the reason we presented Proposition 3 and Corollary 3.1 as general as possible.

The manuscript has been updated as follows.

> Unlike existing works that focus on specific performance metrics or deployment scenarios, we aim for an understanding of the theoretical shaping limits (via analysis) and the achievable shaping results (via optimization) that are broadly applicable across diverse wireless applications. We believe a comprehensive shaping answer can serve as a theoretical support/reference for the vast number of RIS research papers and real-world applications. Without a framework that identifies the fundamental shaping limits of RIS, the design of truly optimal and efficient architectures will remain elusive.

> Corollary 3.1 applies to arbitrary number of RIS elements as inherited from Proposition 3. The set (12), also recognized as a variation of Horn's inequality [42], provides a comprehensive analytical answer to the shaping question – it can be interpreted as the *outer bounds* of the achievable singular value region of the BD-RIS-aided MIMO channel. An example is given by (45) and their visualization in Fig. 2. Remarkably, the number of inequalities in (12) increases exponentially with N_{S}. At a first glance the results may seem excessive to be useful; but they are given in this form to be general and one can pick any *subset* of them for specific applications. Below we showcase how to induce some ready-to-use wireless performance bounds with closed-form BD-RIS solutions from Corollary 3.1. The applications mentioned therein are non-exhaustive; we really hope our ingenious readers can discover more results specific to their research.

> Proposition 1 – 3 and the resulting Corollaries provide a partial answer to the channel shaping question. These analyses provide a theoretical foundation for understanding exactly *how* and *to what extent* a BD-RIS can shape the MIMO channel regarding typical singular value functions under specific channel conditions. Extending the results to more general setups is challenging due to the limited branch matching capability and the inherent tradeoff in mode alignment.

3.2 *The rest main results are Alg. 1 and rate optimization in Sec. IV. However, these algorithms only apply to simple scenarios, e.g., single user case, without additional constraints (QoS, interference, CRB). In fact, rate maximization for single user have been extensively studied by existing literature. There exist a huge body of works on BD-RIS beamforming design considering much more complicated scenarios, e.g., multi-user context and scenarios with more complicated constraints. See the survey [i].*

Response We appreciate and fully respect the reviewer's feedback and suggestions. In fact, the survey paper [i] made its debut 10 months after this paper and has heavily cited our algorithms. We choose not to do the other way around. It is also clear from [i, Table V] that other MIMO rate maximization designs are quite limited even in the single-user scenario – they considered either fully-connected BD-RIS or line-of-sight direct channel, both of which fall into special cases of our channel shaping analysis in Section III-A. That is, it remains unclear in the literature how to design a symmetric/asymmetric BD-RIS of arbitrary group size for rate maximization under general MIMO channel conditions. The proposed geodesic manifold optimizer not only fills in this gap but also features higher computational efficiency than the canonical off-the-shelf optimizer, as analyzed in Section III-B and simulated in Section V. Besides, this paper is a pioneer to demonstrate how channel shaping can be applied to rate maximization and other joint beamforming problems, which offers a low-complexity candidate to the de facto alternating optimization.

On the other hand, we would also like to emphasize that our focus on the single-user case is a deliberate and methodical choice. We believe in a "walk-before-run" approach. A thorough understanding of the fundamental limits of point-to-point MIMO channel shaping via RIS is a prerequisite for properly tackling the more complex, resource-limited multi-user scenarios. Our paper aims to provide this foundational baseline, which we see as critical and necessary for future works in the area. While Algorithm 1 and the two solutions in Section IV are readily extendable to weighted sum-rate maximization and leakage interference minimization in MIMO interference channel, we have decided not to include the results in this paper to avoid blurring the focus and confusing our readers. We have updated the conclusion to address this concern.

> This paper investigated the capability of BD-RIS to shape a MIMO channel in terms of singular values and their functions. We started from a toy example and derived some analytical bounds (with closed-form solutions) on the channel singular values, power gain, and capacity. An efficient framework was then proposed to optimize the BD-RIS for a broader class of singular value functions. We also presented two beamforming designs for the rate maximization problem, one for optimal performance and the other exploits shaping implications for much lower complexity while remaining close-to-optimal. Extensive simulation show that the significant power and rate gains of BD-RIS over Diagonal (D)-RIS stems from its superior MIMO branch matching and mode alignment potentials, which scales with the number of elements, group size, and MIMO dimensions.
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> The analysis and optimization methods in this paper have been tailored for group-connected BD-RIS. Extension to other RIS architectures remains a promising area for future research. One straightforward extension to the multi-sector model [33] is to retrieve the optimal scattering matrix for each sector individually by methods in this paper and then play with the power splitting factors. Meanwhile, transitioning from single- to multi-layer RIS models [60] mirrors that from single- to multi-hop Amplify-and-Forward (AF) relays; interested readers may be inspired by [44].
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> Finally, we remark that the principle of channel shaping is not limited to point-to-point MIMO. Algorithm 1 and the two solutions in Section IV are readily extendable to weighted sum-rate maximization and leakage interference minimization in MIMO interference channel.

References

[i] H. Li, M. Nerini, S. Shen, and B. Clerckx, "A tutorial on beyond-Diagonal Reconfigurable Intelligent Surfaces: Modeling, architectures, system design and optimization, and applications," *arXiv [eess.SP]*, 22 May 2025.