RIScatter: Unifying Backscatter Communication and Reconfigurable Intelligent Surface

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Abstract-Backscatter Communication (BackCom) nodes harvest energy from and modulate information over external electromagnetic waves. Reconfigurable Intelligent Surface (RIS) adapts its phase shift response to alter channel strength in specific directions. In this paper, we show how those two seemingly different technologies (and their derivatives) can be unified into one architecture called RIScatter. RIScatter consists of dispersed or co-located scatter nodes, whose reflection states are adapted to partially modulate their information and partially engineer the wireless channel. The key is to render the probability distribution of reflection states as a joint function of the information source, Channel State Information (CSI), and relative priority of coexisting links. This enables RIScatter to softly bridge BackCom and RIS; reduce to either under specific distribution; or evolve in a mixed form for heterogeneous traffic control and universal hardware design. We also propose a low-complexity Successive Interference Cancellation (SIC)-free receiver that exploits the properties of RIScatter. For a single-user multi-node network, we characterize the achievable primary-(total-)backscatter rate region by optimizing the input distribution at scatter nodes, the active beamforming at the Access Point (AP), and the energy decision regions at the user. Simulations demonstrate RIScatter nodes can recycle surrounding radios for backscatter modulation and passive beamforming.

Index Terms—Backscatter communication, reconfigurable intelligent surface, active-passive coexisting network, input distribution design, SIC-free receiver.

I. Introduction

TUTURE wireless network is envisioned to provide high throughout uniform covers throughput, uniform coverage, pervasive connectivity, heterogeneous control, and cognitive intelligence for trillions of low-power devices. Backscatter Communication (BackCom) separates a transmitter into a Radio-Frequency (RF) carrier emitter with power-hungry elements (e.g., synthesizer and amplifier) and an information-bearing node with power-efficient components (e.g., harvester and modulator) [1]. The receiver (reader) can be either co-located or separated with the carrier emitter, known as Monostatic BackCom (MBC) and Bistatic BackCom (BBC) in Fig. 1(a) and 1(b), respectively. Relevant applications such as Radio-Frequency Identification (RFID) [2], [3] and passive sensor network [4], [5] have been extensively researched, standardized, and commercialized to embrace the Internet of Everything (IoE). However, conventional backscatter nodes only respond when externally inquired by a nearby reader. Ambient Backscatter Communication (AmBC) in Fig. 1(c) was proposed a decade ago where battery-free nodes recycle ambient signals (e.g., radio, television and Wi-Fi) to harvest

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energy and establish connections [6]. It does not require dedicated power source, carrier emitter, or frequency spectrum, but the backscatter decoding is subject to the strong interference from the primary (legacy) link. To tackle this, cooperative AmBC [7] employs a co-located receiver to decode both coexisting links, and the concept was further refined as Symbiotic Radio (SR) in Fig. 1(d) [8]. Specifically, the active transmitter generates RF wave carrying primary information, the passive node creates a rich-scattering environment and rides its own information, and the co-located receiver cooperatively decodes both links. In those BackCom applications, the scatter node is considered as an information source and the reflection pattern depends exclusively on the information symbol. On the other hand, Reconfigurable Intelligent Surface (RIS) in Fig. 1(e) is a smart signal reflector with numerous passive elements of adjustable phase shifts. It customizes the wireless environment for signal enhancement, interference suppression, scattering enrichment, and/or non-line-of-sight bypassing [9]. Each RIS element is considered as a channel shaper and the reflection pattern depends exclusively on the Channel State Information (CSI).

As a special case of Cognitive Radio (CR), active and passive transmissions coexist and interplay in AmBC and SR. Such a coexistence is classified into commensal (overlay), parasitic (underlay), and competitive (interfering) paradigms, and their achievable rate and outage performance were investigated in [10], [11]. The achievable rate and optimal input distribution for binary-input AmBC are investigated in [12], but its impact on the primary link is not considered. In [13], the authors analyzed the energy efficiency and achievable rate region for an AmBC-aided multi-user downlink Non-Orthogonal Multiple Access (NOMA) system. However, they assumed equal symbol duration and perfect synchronization for the coexisting links. Importantly, active-passive coexisting networks have three special and important properties:

- 1) Primary and backscatter symbols are superimposed by *double modulation* (i.e., multiplication coding);
- 2) Backscatter signal strength is much weaker than primary due to the *double fading* effect;
- The spreading factor (i.e., backscatter symbol duration over primary) is usually large¹.

The second property motivated [7], [8], [10], [11], [15]–[22] to view SR as a multiplicative NOMA and perform Successive Interference Cancellation (SIC) from primary to backscatter link. During primary decoding, the backscatter signal can be

 $^1 The load-switching interval of low-power backscatter modulators is usually 0.1 to <math display="inline">10\,\mu s$ [14], accounting for a typical spreading factor between 10 and 10^3 .

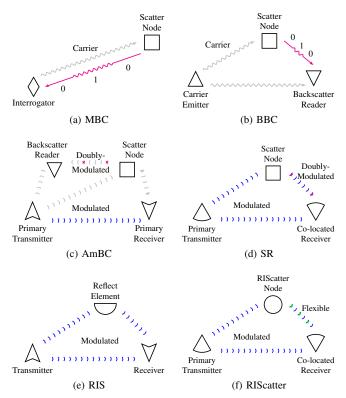


Fig. 1. Illustration of scattering applications. The blue flow(s) constitutes the primary link while the magenta/green flow denotes the backscatter link.

modelled as channel uncertainty or multiplicative interference when the spreading factor is large or small, respectively. Decoding each backscatter symbol also requires multiple SIC followed by a Maximal Ratio Combining (MRC) over primary blocks, which is operation-intensive and CSI-sensitive. Under those assumptions, the achievable rate region of cell-free SR is characterized in [22]. When the spreading factor is sufficiently large, the primary achievable rate under semi-coherent detection² asymptotically approaches its coherent counterpart and both links are decoded interference-free [15]. However, this severely limits the backscatter throughput and requires numerous SIC per backscatter symbol.

On the other hand, static RIS that employs fixed reflection pattern per channel block has been extensively studied in wireless communication, sensing, and power literature [23]–[28]. Dynamic RIS performs time sharing between different phase shifts and introduces artificial channel diversity within each channel block. The idea was first proposed to fine-tune the Orthogonal Frequency-Division Multiplexing (OFDM) resource blocks [29], then extended to the downlink power and uplink information phases of Wireless Powered Communication Network (WPCN) [30]–[32]. However, dynamic RIS carries no information because the reflection state at a specific time is known to the receiver. RIS can also be used as an information source, and prototypes have been developed for Phase Shift Keying (PSK) [33] and Quadrature Amplitude Modulation (QAM) [34]. From an information-theoretic perspective, the authors of [35]

reported that joint transmitter-RIS encoding achieves the capacity of RIS-aided finite-input channel, and using RIS as a naive passive beamformer to maximize the receive Signal-to-Noise Ratio (SNR) is generally suboptimal. This inspired [36]–[45] to combine passive beamforming and backscatter modulation in the overall RIS design. In particular, symbol level precoding maps the information symbols to the optimized RIS coefficient sets [36], [37], overlay modulation superposes the information symbols over a common auxiliary matrix [38]-[41], spatial modulation switches between the reflection coefficient sets that maximize SNR at different receive antennas [42]-[44], and index modulation employs dedicated reflection elements (resp. information elements) for passive beamforming (resp. backscatter modulation) [45]. Those RIS-based backscatter modulation schemes incur advanced hardware architecture and high optimization complexity. In contrast, [46] exploits commodity RFID tags, powered and controlled by a software-defined radio reader, to perform passive beamforming (but no backscatter modulation) towards a legacy user at a different frequency. Most relevant literature considers either Gaussian codebook [10], [11], [15]–[19], [40] that is impractical for low-power nodes, or finite equiprobable inputs [7], [8], [20], [21], [36]-[39], [41]-[45] that does not fully exploit the CSI and properties of activepassive coexisting networks. Those problems are addressed in this paper and the contributions are summarized below.

First, we propose RIScatter as a novel protocol that unifies BackCom and RIS by adaptive reflection state (backscatter input) distribution design. The concept is shown in Fig. 1(f) where one or more RIScatter nodes ride over an active transmission, simultaneously modulating their information and engineering the wireless channel. A co-located receiver cooperatively decodes both coexisting links. Each reflection state is simultaneously an information codeword and a passive beamforming codeword. The reflection pattern over time is semi-random and guided by the input probability assigned to each state. This probability distribution is carefully designed to incorporate the backscatter information, CSI, and Quality of Service (OoS)³. Such an adaptive channel coding boils down to the degenerate distribution of RIS when the primary link is prioritized, and outperforms the uniform distribution of BackCom (by accounting the CSI) when the backscatter link is prioritized. Table I compares RIScatter to BackCom and RIS. However, two major challenges for RIScatter are the practical receiver and input distribution design. This is the first paper to unify BackCom and RIS from the perspective of input distribution.

Second, we address the first challenge and propose a low-complexity SIC-free receiver. It semi-coherently decodes the weak backscatter signal using an energy detector, re-encodes for the exact reflection pattern, then coherently decodes the primary link. Thanks to double modulation, the backscatter detection becomes part of channel training, and the impact of backscatter modulation can be modelled as dynamic passive beamforming. The proposed receiver may be built over legacy receivers with minor hardware upgrade, since it only requires one additional energy comparison and re-encoding per backscatter symbol (instead of primary symbol). The energy detector can also be

²In this paper, semi-coherent detection refers to the primary/backscatter decoding with known CSI and unknown backscatter/primary symbols.

³QoS refers to the relative priority of the primary link.

TABLE I
COMPARISON OF SCATTERING APPLICATIONS

	MBC/BBC	AmBC	SR (large spreading factor)	RIS	RIScatter
Information link(s)	Backscatter	Coexisting	Coexisting	Primary	Coexisting
Primary signal on backscatter decoding	Carrier	Multiplicative interference	Spreading code	_	Energy uncertainty
Backscatter signal on primary decoding	_	Multiplicative interference	CSI uncertainty	Passive beamforming	Dynamic passive beamforming
Cooperative devices	_	No	Primary transmitter and co-located receiver	_	Primary transmitter, scatter nodes, and co-located receiver
Sequential decoding	_	No	Primary-to-backscatter, SIC and MRC	_	Backscatter-to- primary, no SIC/MRC
Reflection pattern depends on	Information source	Information source	Information source	CSI	Information source, CSI, and QoS
Reflection state distribution	Equiprobable	Equiprobable	Equiprobable or Gaussian	Degenerate	Flexible
Load-switching speed	Fast	Slow	Slow	Quasi-static	Arbitrary

tailored for arbitrary input distribution and spreading factor to increase backscatter throughput. This is the first paper to propose a SIC-free receiver for active-passive coexisting networks.

Third, we address the second challenge in a single-user multi-node Multiple-Input Single-Output (MISO) scenario. We characterize the achievable primary-(total-)backscatter rate region by optimizing the input distribution at RIScatter nodes, the active beamforming at the Access Point (AP), and the energy decision regions at the user under different QoS. A Block Coordinate Descent (BCD) algorithm is proposed where the Karush-Kuhn-Tucker (KKT) input distribution is numerically evaluated by limit of sequences, the active beamforming is optimized by Projected Gradient Ascent (PGA), and the decision regions are refined by existing sequential quantizer for Discrete Memoryless Thresholding Channel (DMTC). Uniquely, our optimization problem takes into account the CSI, QoS, and backscatter constellation, and the resulting input distribution is applicable to any detection scheme. This is also the first paper to reveal the importance of backscatter input distribution and decision region designs in active-passive coexisting networks.

Notations: Italic, bold lower-case, and bold upper-case letters denote scalars, vectors and matrices, respectively. $\mathbf{0}$ and $\mathbf{1}$ denote zero and one array of appropriate size, respectively. $\mathbb{I}^{x \times y}$, $\mathbb{R}_+^{x \times y}$, and $\mathbb{C}^{x \times y}$ denote the unit, real nonnegative, and complex spaces of dimension $x \times y$, respectively. j denotes the imaginary unit. $\operatorname{diag}(\cdot)$ returns a square matrix with the input vector on its main diagonal and zeros elsewhere. $\operatorname{card}(\cdot)$ returns the cardinality of a set. $\log(\cdot)$ denotes logarithm of base $e.\ (\cdot)^*$, $(\cdot)^\mathsf{T},\ (\cdot)^\mathsf{H},\ |\cdot|$, and $\|\cdot\|$ denote the conjugate, transpose, conjugate transpose (Hermitian), absolute value, and Euclidean norm operators, respectively. $(\cdot)^{(r)}$ and $(\cdot)^*$ denote the r-th iterated and optimal results, respectively. The distribution of a Circularly Symmetric Complex Gaussian (CSCG) random variable with zero mean and variance σ^2 is denoted by $\mathcal{CN}(0,\sigma^2)$, and \sim means "distributed as".

II. RISCATTER

A. Principles

RF wave scattering or reflecting are often manipulated by passive antennas or programmable metamaterial [47]. The former receives the impinging signals and reradiates some back to the space, while the latter reflects at the space-cell boundary and mainly applies a phase shift. In the scattered signal, the structural mode component depends on the scatterer geometry and material. Its impact is usually modelled as part of environment multipath [8], [48], or simply a Direct Current (DC) offset in the baseband signal when the impinging signal is a Continuous Waveform (CW) [1]. On the other hand, the antenna mode component depends on the impedance mismatch and is widely exploited in scattering applications. For an antenna (resp. metamaterial) scatterer with M reflection states, the reflection coefficient at state $m \in \mathcal{M} \triangleq \{1,...,M\}$ is [47], [49]

$$\Gamma_m = \frac{Z_m - Z^*}{Z_m + Z},\tag{1}$$

where Z_m is the antenna load (resp. metamaterial cell) impedance at state m, and Z is the antenna input (resp. medium characteristic) impedance. Specifically,

 BackCom: The scatterer is an information source with random reflection pattern over time. The reflection coefficient is used merely an information codeword [50]

$$\Gamma_m = \alpha_m \frac{c_m}{\max_{m'} |c_{m'}|},\tag{2}$$

where $\alpha_m \in \mathbb{I}$ is the amplitude scattering ratio and c_m is the constellation point.

 RIS: The scatterer is a channel shaper with deterministic reflection pattern over time. The reflection coefficient is used merely a passive beamforming codeword [23]

$$\Gamma_m = \alpha_m \exp(j\theta_m),\tag{3}$$

where θ_m is the phase shift.⁴

 $^{^4}$ It is commonly assumed $\alpha_m \ll 1$ in BackCom and $\alpha_m = 1$ in RIS.

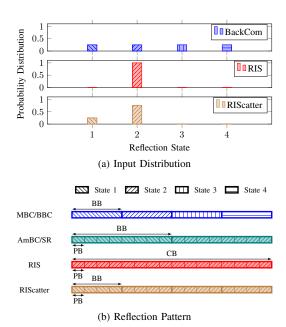


Fig. 2. Input distribution and reflection pattern of scattering applications. "PB", "BB", and "CB" refer to primary symbol block, backscatter symbol block, and channel block, respectively. Shadowing means presence of primary link. In this example, the optimal passive beamformer corresponds to state 2. The spreading factor is 4 for RIScatter and 8 for AmBC/SR. BackCom and RIS can be viewed as extreme cases of RIScatter, where the input distribution boils down to uniform and degenerate, respectively.

RIScatter generalizes BackCom and RIS from a probabilistic perspective. Each reflection coefficient simultaneously acts as an information codeword and a passive beamforming codeword. As shown in Fig. 2, the reflection pattern of each RIScatter node over time is semi-random and guided by the input probability assigned to each state. This probability distribution is carefully designed to incorporate the backscatter information, CSI, and QoS, in order to strike a balance between backscatter modulation and passive beamforming.

Remark 1. Unlike dynamic RIS that simply performs a time sharing between reflection states, RIScatter conveys additional information by randomizing the reflection pattern over time while still guaranteeing the probability of occurrence of each state. Upon successful backscatter detection, the impact of RIScatter nodes on the primary link can be modelled as dynamic passive beamforming.

RIScatter nodes can be implemented, for example, by adding an integrated receiver⁵ [51] and adaptive encoder [52] to off-the-shelf passive RFID tags. The block diagram, equivalent circuit, and scatter model are illustrated in Fig. 3.

B. System Model

As shown in Fig. 4, we consider a RIScatter network where a Q-antenna AP serves a single-antenna user and K nearby dispersed or co-located RIScatter nodes. All nodes have M available reflection states. In the primary point-to-point system,

the AP transmits information to the user over a multipath channel⁶ enhanced by RIScatter nodes. In the backscatter multiple access system, the AP acts as carrier emitter, the RIScatter nodes modulate over scattered signal, and the user jointly decodes all node messages.⁷ For simplicity, we consider a quasi-static block fading model and focus on a specific channel block where the CSI remains constant. Denote the AP-user direct channel as $h_D^H \in \mathbb{C}^{1 \times Q}$, the AP-node $k \in \mathcal{K} \triangleq \{1,...,K\}$ forward channel as $h_{F,k}^H \in \mathbb{C}^{1 \times Q}$, the node k-user backward channel as $h_{B,k}^H$, and the cascaded AP-node k-user channel as $h_{C,k}^H \triangleq h_{B,k} h_{F,k}^H \in \mathbb{C}^{1 \times Q}$. We assume the direct and cascaded CSI are available at the AP and user⁸.

Let $\alpha_k \in \mathbb{I}$ be the amplitude scattering ratio of node k, $x_k \in \mathcal{X} \triangleq \{c_1, ..., c_M\}$ be the coded backscatter symbol of node k, and $x_K \triangleq (x_1, ..., x_K)$ be the backscatter symbol tuple of all nodes. As a sequence of double modulation, the composite channel is a function of backscatter symbol tuple⁹

$$\boldsymbol{h}^{\mathsf{H}}(x_{\mathcal{K}}) \triangleq \boldsymbol{h}_{\mathsf{D}}^{\mathsf{H}} + \sum_{k} \alpha_{k} \boldsymbol{h}_{\mathsf{C},k}^{\mathsf{H}} x_{k}$$
 (4a)

$$= \boldsymbol{h}_{\mathrm{D}}^{\mathsf{H}} + \boldsymbol{x}^{\mathsf{H}} \mathrm{diag}(\boldsymbol{\alpha}) \boldsymbol{H}_{\mathrm{C}}, \tag{4b}$$

where $\alpha \triangleq [\alpha_1, ..., \alpha_K]^\mathsf{T} \in \mathbb{I}^K$, $\mathbf{x} \triangleq [x_1, ..., x_K]^\mathsf{H} \in \mathcal{X}^K$, and $\mathbf{H}_C \triangleq [\mathbf{h}_{C,1}, ..., \mathbf{h}_{C,K}]^\mathsf{H} \in \mathbb{C}^{K \times Q}$. Without loss of generality, we assume the spreading factor N is a positive integer. Within one backscatter block, the signal received by the user at primary block $n \in \mathcal{N} \triangleq \{1, ..., N\}$ is

$$y[n] = \mathbf{h}^{\mathsf{H}}(x_{\mathcal{K}})\mathbf{w}s[n] + v[n], \tag{5}$$

where $\boldsymbol{w} \in \mathbb{C}^Q$ is the active beamformer satisfying $\|\boldsymbol{w}\|^2 \leq P$, P is the average transmit power budget, $s \sim \mathcal{CN}(0,1)$ is the primary symbol, and $v \sim \mathcal{CN}(0,\sigma_v^2)$ is the Additive White Gaussian Noise (AWGN) with variance σ_v^2 .

Let $m_k \in \mathcal{M} \triangleq \{1,...,M\}$ be the reflection state index of node k, and $m_{\mathcal{K}} \triangleq (m_1,...,m_K)$ be the state index tuple of all nodes. The backscatter symbol x_k (resp. symbol tuple $x_{\mathcal{K}}$) is a random variable that takes value x_{m_k} (resp. value tuple $x_{m_{\mathcal{K}}}$) with probability $p(x_{m_k})$ (resp. $p(x_{m_{\mathcal{K}}})$).

Remark 2. Dispersed RIScatter nodes encode independently such that

$$p(x_{m_{\mathcal{K}}}) = \prod_{k} p(x_{m_k}). \tag{6}$$

When multiple nodes are co-located, they can jointly encode by designing the joint probability $p(x_{m_K})$ directly.

Let $z = \sum_n |y[n]|^2$ be the receive energy per backscatter block. When x_{m_K} is transmitted, the receive signal y follows CSCG distribution $\mathcal{CN}(0,\sigma_{m_K}^2)$ with variance

$$\sigma_{m_{\mathcal{K}}}^2 = |\boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}})\boldsymbol{w}|^2 + \sigma_v^2, \tag{7}$$

⁶It is assumed the primary symbol duration is much longer than multipath delay spread (i.e., no inter-symbol interference).

⁷It is assumed the signal going through two or more RIScatter nodes is too weak to be received by the user.

⁸The cascaded CSI can be estimated by sequential [54]–[56] or parallel [57] approaches for dispersed nodes, or group-based [58] or hierarchical [59] approaches for co-located nodes.

⁹(4a) and (4b) are often used in BackCom and RIS literature, respectively.

⁵The aim is to coordinate the node with the active source and to acquire the optimized input distribution, instead of decoding the primary information dedicated for the user. The node receiver can be implemented using very simple circuits or even integrated with the rectifier [51] to reduce cost and complexity.

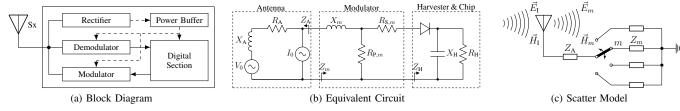


Fig. 3. Block diagram, equivalent circuit, and scatter model of a RIScatter node. The solid and dashed vectors represent signal and energy flows. The scatter antenna behaves as a constant power source, where the voltage V_0 and current I_0 are introduced by incident electric field \vec{E}_1 and magnetic field \vec{H}_1 [53].

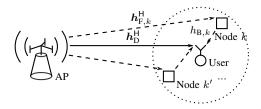


Fig. 4. A single-user multi-node RIScatter network.

and z follows Gamma distribution with conditional Probability Density Function (PDF)

$$f(z|x_{m_K}) = \frac{z^{N-1} \exp(-z/\sigma_{m_K}^2)}{\sigma_{m_K}^{2N}(N-1)!}.$$
 (8)

Remark 3. We have assumed Gaussian codebook for the primary source and finite support for the backscatter nodes, since they are relatively practical and widely adopted in relevant literatures. The proposed framework is extendable to non-Gaussian primary source, and the conditional PDF (8) can be approximated using Central Limit Theorem (CLT) for large N.

The user first jointly decodes the backscatter message of all nodes using a low-complexity energy detector. Then, the user decodes the primary message using the Maximum-Likelihood (ML) detector. The energy detector formulates a DMTC of size $M^K \times M^K$.

Remark 4. The capacity-achieving decision region design for DMTC with non-binary inputs in arbitrary distribution remains an open issue. It was proved deterministic detectors can be rate-optimal, but non-convex regions (comprise non-adjacent partitions) are generally required and the optimal number of thresholds is unknown [61], [62]. Next, we restrict the energy detector to convex deterministic decision regions and consider sequential threshold design.

Let $L = M^K$ be the number of decision regions. Sort $\{\sigma_{m_K}^2\}$ in ascending order and denote the result sequence as $\sigma_1^2, \dots, \sigma_L^2$. With sequential thresholding, the decision region

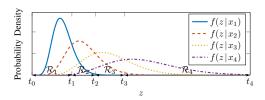


Fig. 5. PDF of the receive energy per backscatter block conditioned on different reflection state.

of backscatter symbol tuple $l \in \mathcal{L} = \{1, ..., L\}$ is 11

$$\mathcal{R}_l \triangleq [t_{l-1}, t_l), \quad 0 \le t_{l-1} \le t_l, \tag{9}$$

where t_l is the decision threshold between hypotheses x_l and x_{l+1} . An example is shown in Fig. 5.

When the threshold vector $\mathbf{t} \triangleq [t_0,...,t_L]^\mathsf{T} \in \mathbb{R}_+^{L+1}$ is given, we can formulate a Discrete Memoryless Multiple Access Channel (DMMAC) with transition probability from input $x_{m_{\mathcal{K}}}$ to output $\hat{x}_{m_{\mathcal{K}}'}$ as

$$q(\hat{x}_{m_{\mathcal{K}}'}|x_{m_{\mathcal{K}}}) = \int_{\mathcal{R}_{m_{\mathcal{K}}'}} f(z|x_{m_{\mathcal{K}}}) dz.$$
 (10)

The backscatter mutual information is

$$I_{\mathbf{B}}(x_{\mathcal{K}}; \hat{x}_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) I_{\mathbf{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}), \tag{11}$$

where $I_{\rm B}(x_{m_{\rm K}};\hat{x}_{\rm K})$ is the backscatter information function

$$I_{\mathbf{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}) \triangleq \sum_{m_{\mathcal{K}}'} q(\hat{x}_{m_{\mathcal{K}}'} | x_{m_{\mathcal{K}}}) \log \frac{q(\hat{x}_{m_{\mathcal{K}}'} | x_{m_{\mathcal{K}}})}{p(\hat{x}_{m_{\mathcal{K}}'})}.$$
(12)

Once the backscatter information is successfully decoded, the user re-encodes to recover the exact reflection pattern and thus the composite channel by (4), then coherently decodes the primary link. The primary mutual information is

$$I_{\mathsf{P}}(s;y|x_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) I_{\mathsf{P}}(s;y|x_{m_{\mathcal{K}}}),\tag{13}$$

where $I_{\rm P}(s;y|x_{m_K})$ is the primary information function

$$I_{P}(s;y|x_{m_{\mathcal{K}}}) \triangleq \log\left(1 + \frac{|\boldsymbol{h}^{H}(x_{m_{\mathcal{K}}})\boldsymbol{w}|^{2}}{\sigma_{v}^{2}}\right).$$
 (14)

 $^{11}m_{\mathcal{K}}$ and l are one-to-one and used interchangeably in the following context.

¹⁰The reliability of the energy detector is improved by the adaptive input distribution and thresholding design. With high-order modulation or large number of scatter nodes, the reliability can be further enhanced by increasing the spreading factor or using error correction codes with lower code-rate. In practice, users can decode backscatter nodes ranging from a few to hundreds of meters in the presence of noise and interference, and the backscatter throughput can reach few Kbps to tens of Mbps [60].

III. RATE-REGION CHARACTERIZATION

With a slight abuse of notation, we define the weighed sum mutual information and information function as

$$I(x_{\mathcal{K}}) \triangleq \rho I_{\mathcal{P}}(s;y|x_{\mathcal{K}}) + (1-\rho)I_{\mathcal{B}}(x_{\mathcal{K}};\hat{x}_{\mathcal{K}}), \tag{15}$$

$$I(x_{m_{\mathcal{K}}}) \triangleq \rho I_{\mathbf{P}}(s;y|x_{m_{\mathcal{K}}}) + (1-\rho)I_{\mathbf{B}}(x_{m_{\mathcal{K}}};\hat{x}_{\mathcal{K}}), \quad (16)$$

where $\rho \in \mathbb{I}$ is the QoS. To obtain the achievable primary-(total-)backscatter rate region, we consider the weighted sum mutual information maximization problem with independent encoding at all nodes¹².

$$\max_{\{\boldsymbol{p}_k\}_{k\in\mathcal{K}},\boldsymbol{w},\boldsymbol{t}} I(x_{\mathcal{K}}) \tag{17a}$$

s.t.
$$\mathbf{1}^\mathsf{T} \boldsymbol{p}_k = 1, \quad \forall k,$$
 (17b)

$$p_k \ge 0, \quad \forall k,$$
 (17c)

$$\|\boldsymbol{w}\|^2 \leq P, \tag{17d}$$

$$t_{l-1} \le t_l, \quad \forall l,$$
 (17e)

$$t \ge 0,$$
 (17f)

where $p_k = [p(x_{1_k}),...,p(x_{M_k})]^\mathsf{T} \in \mathbb{I}^M$ is the input distribution of node k. Problem (17) generalizes BackCom by allowing CSI- and QoS-adaptive input distribution and decision region design. On the other hand, it also relaxes the feasible domain of discrete RIS phase shift selection problem from vertices of M-dimensional probability simplex to the simplex itself. The original problem is highly non-convex and we propose a BCD algorithm that iteratively updates $\{p_k\}_{k\in\mathcal{K}}$, w and t.

A. Input Distribution

For any given w and t, we can formulate a DMMAC by (10) and simplify (17) to

$$\max_{\{\boldsymbol{p}_k\}_{k\in\mathcal{K}}} I(x_{\mathcal{K}}) \tag{18a}$$

$$\{p_k\}_{k\in\mathcal{K}}$$

s.t.
$$(17b),(17c),$$
 $(18b)$

which involves the product term (6) and is generally non-convex (unless K=1). Following [63], we first recast the KKT conditions to their equivalent forms, then propose a numerical method that guarantees those conditions on convergence of sequences.

Proposition 1. The KKT optimality conditions for problem (18) are equivalent to, $\forall k, m_k$,

$$I_k^{\star}(x_{m_k}) = I^{\star}(x_K), \quad \text{if } p^{\star}(x_{m_k}) > 0,$$
 (19a)

$$I_{k}^{\star}(x_{m_{k}}) \leq I^{\star}(x_{K}), \quad \text{if } p^{\star}(x_{m_{k}}) = 0,$$
 (19b)

where $I_k(x_{m_k})$ is the weighted sum marginal information

$$I_k(x_{m_k}) = \sum_{m_{\mathcal{K}\setminus\{k\}}} p(x_{m_{\mathcal{K}\setminus\{k\}}}) I(x_{m_{\mathcal{K}}}). \tag{20}$$

Proof. Please refer to Appendix A.

For each RIScatter node, (19a) suggests each probable state should produce the same marginal information (averaged over Algorithm 1: KKT Input Distribution Evaluation by Limits of Sequence

Input:
$$K, N, h_{D}^{H}, H_{C}, \alpha, \mathcal{X}, \sigma_{v}^{2}, \rho, w, t, \epsilon$$

Output: $\{p_{k}^{\star}\}_{k \in \mathcal{K}}$

1: Set $h^{H}(x_{m_{K}}), \forall m_{K}$ by (4)

2: $\sigma_{m_{K}}^{2}, \forall m_{K}$ by (7)

3: $f(z|x_{m_{K}}), \forall m_{K}$ by (8)

4: $q(\hat{x}_{m_{K}'}|x_{m_{K}}), \forall m_{K}, m_{K}'$ by (10)

5: Initialize $r \leftarrow 0$

6: $p_{k}^{(0)} > 0, \forall k$

7: Get $p^{(r)}(x_{m_{K}}), \forall m_{K}$ by (6)

8: $I^{(r)}(x_{m_{K}}), \forall m_{K}$ by (12), (14), (16)

9: $I_{k}^{(r)}(x_{m_{K}}), \forall k, m_{k}$ by (20)

10: $I^{(r)}(x_{K})$ by (11), (13), (15)

11: Repeat

12: Update $r \leftarrow r+1$

13: $p_{k}^{(r)}, \forall k$ by (21)

14: Redo step 7–10

15: Until $I^{(r)}(x_{K}) - I^{(r-1)}(x_{K}) < \epsilon$

all states of other nodes), while (19b) suggests any state with potentially less marginal information should not be used.

Proposition 2. For any strictly positive initializer $\{p_k^{(0)}\}_{k\in\mathcal{K}}$, the KKT input probability of node k at state m_k is given by the converging point of the sequence

$$p^{(r+1)}(x_{m_k}) = \frac{p^{(r)}(x_{m_k}) \exp\left(\frac{\rho}{1-\rho} I_k^{(r)}(x_{m_k})\right)}{\sum_{m'_k} p^{(r)}(x_{m'_k}) \exp\left(\frac{\rho}{1-\rho} I_k^{(r)}(x_{m'_k})\right)}, \quad (21)$$

where r is the iteration index.

At iteration r+1, the input distribution of node k is updated over $\{\{\boldsymbol{p}_q^{(r+1)}\}_{q=1}^{k-1}, \{\boldsymbol{p}_q^{(r)}\}_{q=k}^K\}$. The KKT input distribution design is summarized in Algorithm 1.

Remark 5. The insufficiency of the KKT conditions for problem (18) implies that the proposed method may not converge to the global-optimal solution. However, simulation results show that the average performance gap is indistinguishable at a moderate K.

B. Active Beamforming

For any given $\{p_k\}_{k\in\mathcal{K}}$ and t, problem (17) reduces to

$$\max I(x_K) \tag{22a}$$

$$\begin{array}{ll}
\max_{\boldsymbol{w}} & I(x_{\mathcal{K}}) \\
\text{s.t.} & (17d),
\end{array} (22a)$$

which is still non-convex due to the integration and entropy terms. Note the DMMAC $q(x_l|x_{m_{\mathcal{K}}})$ depends on the variance of accumulated receive energy $\sigma_{m_K}^2$, which is a function of \boldsymbol{w} . Plugging (8) into (10), we have

$$q(x_l|x_{m_K}) = \frac{\int_{t_{l-1}/\sigma_{m_K}^2}^{t_l/\sigma_{m_K}^2} z^{N-1} \exp(-z) dz}{(N-1)!}$$
(23)

$$=Q\left(N,\frac{t_{l-1}}{\sigma_{m_{\kappa}}^{2}},\frac{t_{l}}{\sigma_{m_{\kappa}}^{2}}\right),\tag{24}$$

¹²Joint encoding over multiple nodes can be viewed as its special case with an augmented source.

where $Q(N,b_1,b_2) \triangleq \int_{b_1}^{b_2} z^{N-1} \exp(-z)/(N-1)! dz$ is the regularized incomplete Gamma function. Its series expansion is given by [64, Theorem 3]

$$Q\left(N, \frac{t_{l-1}}{\sigma_{m\kappa}^2}, \frac{t_l}{\sigma_{m\kappa}^2}\right) = \exp\left(-\frac{t_{l-1}}{\sigma_{m\kappa}^2}\right) \sum_{n=0}^{N-1} \frac{\left(\frac{t_{l-1}}{\sigma_{m\kappa}^2}\right)^n}{n!} - \exp\left(-\frac{t_l}{\sigma_{m\kappa}^2}\right) \sum_{n=0}^{N-1} \frac{\left(\frac{t_l}{\sigma_{m\kappa}^2}\right)^n}{n!}, \quad (25)$$

whose gradient with respect to w^* is

$$\nabla_{\boldsymbol{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_l}{\sigma_{m_{\mathcal{K}}}^2}\right) = \frac{\boldsymbol{h}(x_{m_{\mathcal{K}}}) \boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}}) \boldsymbol{w}}{(\sigma_{m_{\mathcal{K}}}^2)^2} \times \left(g_{m_{\mathcal{K}}}(t_l) - g_{m_{\mathcal{K}}}(t_{l-1})\right), \quad (26)$$

where

$$g_{m_{\mathcal{K}}}(t_l) = t_l \exp\left(-\frac{t_l}{\sigma_{m_{\mathcal{K}}}^2}\right) \left(-1 + \sum_{n=1}^{N-1} \frac{\left(n - \frac{t_l}{\sigma_{m_{\mathcal{K}}}^2}\right) \left(\frac{t_l}{\sigma_{m_{\mathcal{K}}}^2}\right)^{n-1}}{n!}\right). \tag{27}$$

On top of (25) and (26), we rewrite $I(x_K)$ and $\nabla_{w^*}I(x_K)$ as (28) and (29) at the end of page 7, respectively. Problem (22) can thus be solved by the PGA method. At each iteration r, the unregulated active beamformer is updated by

$$\bar{\boldsymbol{w}}^{(r)} = \boldsymbol{w}^{(r-1)} + \gamma \nabla_{\boldsymbol{w}^*} I^{(r-1)}(x_{\mathcal{K}}), \tag{30}$$

where γ is the step size (refinable by backtracking line search [65, Section 9.2]). Then, \bar{w} is projected onto the feasible domain (17d) to retrieve the active beamformer

$$w = \frac{\sqrt{P}\bar{w}}{\max(\sqrt{P}, ||\bar{w}||)}.$$
 (31)

The PGA active beamforming design is summarized in Algorithm 2.

C. Decision Threshold

For any given $\{\boldsymbol{p}_k\}_{k\in\mathcal{K}}$ and $\boldsymbol{w},$ problem (17) reduces to

$$\max_{t} I(x_{\mathcal{K}}) \tag{32a}$$
s.t. (17e),(17f), (32b)

s.t.
$$(17e),(17f),$$
 $(32b)$

Algorithm 2: Active Beamforming Optimization by PGA

Input:
$$Q, N, h_{\rm D}^{\sf H}, H_{\sf C}, \alpha, \mathcal{X}, P, \sigma_v^2, \rho, \{p_k\}_{k \in \mathcal{K}}, t, \alpha, \beta, \gamma, \epsilon$$

Output: w^*

1: Set $h^{\sf H}(x_{m_{\mathcal{K}}})$, $\forall m_{\mathcal{K}}$ by (4)

2: $p(x_{m_{\mathcal{K}}})$, $\forall m_{\mathcal{K}}$ by (6)

3: Initialize $r \leftarrow 0$

4: $w^{(0)}$, $||w^{(0)}||^2 \leq P$

5: Get $(\sigma_{m_{\mathcal{K}}}^{(r)})^2$, $\forall m_{\mathcal{K}}$ by (7)

6: $Q^{(r)}(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_l}{\sigma_{m_{\mathcal{K}}}^2})$, $\forall m_{\mathcal{K}}, l$ by (25)

7: $I^{(r)}(x_{\mathcal{K}})$ by (28)

8: $\nabla_{w^*}Q^{(r)}(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_l}{\sigma_{m_{\mathcal{K}}}^2})$, $\forall m_{\mathcal{K}}, l$ by (26)

9: $\nabla_{w^*}I^{(r)}(x_{\mathcal{K}})$ by (29)

10: Repeat

11: Update $r \leftarrow r+1$

12: $\gamma^{(r)} \leftarrow \gamma$

13: $\bar{w}^{(r)}$ by (30)

14: $w^{(r)}$ by (31)

15: Redo step 5-7

16: While $I^{(r)}(x_{\mathcal{K}})$ $< I^{(r-1)}(x_{\mathcal{K}})$ $+ \alpha\gamma ||\nabla_{w^*}I^{(r-1)}(x_{\mathcal{K}})||^2$

17: Set $\gamma^{(r)} \leftarrow \beta\gamma^{(r)}$

which is still non-convex since t appears on the limits of integration (10). Instead of solving it directly, we constrain the feasible domain from continuous space \mathbb{R}^{L+1}_+ to discrete candidates (i.e., fine-grained energy levels) \mathcal{T}^{L+1} . As shown in Fig. 6, grouping adjacent energy bins formulates decision regions.

Redo step 13-15

End While

Redo step 8, 9

21: **Until** $\| \boldsymbol{w}^{(r)} - \boldsymbol{w}^{(r-1)} \| \le \epsilon$

Remark 6. The design of the energy detector does not affect the primary achievable rate, since the composite channel (4) can always be determined after backscatter decoding and re-encoding. This implies that any thresholding maximizing the total backscatter rate is optimal for problem (32).

Remark 7. In terms of total backscatter rate, the nodes can be viewed as an augmented source, and problem (32) becomes the rate-optimal quantizer design for DMTC.

$$I(x_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) \left(\rho \log \left(1 + \frac{|\boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}})\boldsymbol{w}|^{2}}{\sigma_{v}^{2}} \right) + (1 - \rho) \sum_{l} Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^{2}}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^{2}} \right) \log \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^{2}}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^{2}} \right)}{\sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^{2}}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^{2}} \right)} \right)$$
(28)

17:

18:

19:

20:

$$\nabla_{\boldsymbol{w}^*} I(x_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) \left(\rho \frac{\boldsymbol{h}(x_{m_{\mathcal{K}}}) \boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}}) \boldsymbol{w}}{\sigma_{m_{\mathcal{K}}}^2} + (1 - \rho) \sum_{l} \left(\log \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^2}\right)}{\sum_{m_{\mathcal{K}}'} p(x_{m_{\mathcal{K}}'}) Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^2}\right)} + 1 \right)$$

$$\times \nabla_{\boldsymbol{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^2}\right) - \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}}^2}\right) \sum_{m_{\mathcal{K}}'} p(x_{m_{\mathcal{K}}'}) \nabla_{\boldsymbol{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}'}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}'}}\right)}{\sum_{m_{\mathcal{K}}'} p(x_{m_{\mathcal{K}}'}) Q\left(N, \frac{t_{l-1}}{\sigma_{m_{\mathcal{K}}'}^2}, \frac{t_{l}}{\sigma_{m_{\mathcal{K}}'}}\right)} \right)$$

$$(29)$$

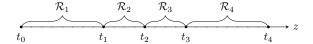


Fig. 6. The thresholds are chosen from fine-grained candidates instead of the continuous space. Each decision region consists of at least one bin.

Thanks to Remark 6 and 7, problem (32) can be recast as

$$\max_{\mathbf{t} \in \mathcal{T}^{L+1}} I_{\mathbf{B}}(x_{\mathcal{K}}; \hat{x}_{\mathcal{K}}) \tag{33a}$$

whose global optimal solution has been obtained in recent works. [66] started from the quadrangle inequality and proposed a Dynamic Programming (DP) method accelerated by the Shor-Moran-Aggarwal-Wilber-Klawe (SMAWK) algorithm with computational complexity $\mathcal{O}(L^2(\operatorname{card}(\mathcal{T})-L))$. On the other hand, [67] started from the optimality condition for three neighbor thresholds and presented a traverse-then-bisect algorithm with complexity $\mathcal{O}(\operatorname{card}(\mathcal{T})L\log(\operatorname{card}(\mathcal{T})L))$. In Section IV, both schemes will be compared with the ML scheme [68]

$$t_l^{\text{ML}} = N \frac{\sigma_{l-1}^2 \sigma_l^2}{\sigma_{l-1}^2 - \sigma_l^2} \log \frac{\sigma_{l-1}^2}{\sigma_l^2}, \quad l \in \mathcal{L} \setminus \{L\},$$
(34)

which is suboptimal for problem (32) unless all nodes are with equiprobable inputs.

IV. SIMULATION RESULTS

In this section, we provide numerical results to evaluate the proposed input distribution, active beamforming, and backscatter decision algorithms. We assume the AP-user distance is $10\,\mathrm{m}$ and at least one RIScatter nodes are randomly dropped in a disk centered at the user with radius r. The AP is with an average transmit power budget $P=36\,\mathrm{dBm}$ and all nodes employs M-QAM with $\alpha=0.5$. For all channels involved, we consider a distance-dependent path loss model

$$L(d) = L_0 \left(\frac{d_0}{d}\right)^{\gamma},\tag{35}$$

together with a Rician fading model

$$\boldsymbol{H} = \sqrt{\frac{\kappa}{1+\kappa}} \bar{\boldsymbol{H}} + \sqrt{\frac{1}{1+\kappa}} \tilde{\boldsymbol{H}}, \tag{36}$$

where d is the transmission distance, $L_0=-30\,\mathrm{dB}$ is the reference path loss at $d_0=1\mathrm{m}$, κ is the Rician K-factor, $\bar{\boldsymbol{H}}$ is the deterministic line-of-sight component with unit-magnitude entries, and $\tilde{\boldsymbol{H}}$ is the Rayleigh fading component with standard independent and identically distributed (i.i.d.) CSCG entries. We choose $\gamma_D=2.6$, $\gamma_F=2.4$, $\gamma_B=2$, and $\kappa_D=\kappa_F=\kappa_B=5$ for direct, forward and backward links. The finite decision threshold domain \mathcal{T} is obtained by b-bit uniform discretization over the critical interval defined by the confidence bounds of edge hypotheses (i.e., lower bound of x_1 and upper bound of x_L) with confidence $1-\varepsilon$. We set b=9 and $\varepsilon=10^{-3}$. All achievable rate regions are averaged over 10^3 channel realizations.

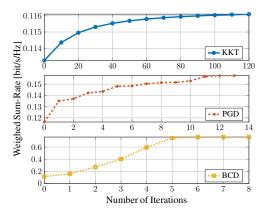


Fig. 7. Typical convergence curves at $\rho=0$ for $Q=4,~K=8,~M=2,~N=20,~\sigma_v^2=-40{\rm dBm}$ and $r\!=\!2{\rm m}.$

A. Evaluation of Proposed Algorithms

1) Initialization: To characterize the achievable rate region, we progressively obtain all boundary points by successively increasing ρ and solving problem (17). For $\rho=0$ where the backscatter link is prioritized, we initialize Algorithm 1 and 2 by uniform input distribution and Maximum Ratio Transmission (MRT) towards the sum cascaded channel $\sum_k h_{C,k}^H$, respectively. At the following points, both algorithms are initialized by the solutions at the previous point.

2) Convergence: The BCD algorithm is convergent for problem (17) since the input distribution and active beamforming subproblems converge and the thresholding subproblem attains global optimality. In company with BCD, we also plotted the convergence results of KKT and PGA algorithms in Fig. 7 to show how much performance is gained by solving each subproblem. It is observed that Algorithm 1 and 2 take around 100 and 10 iterations to converge, respectively. Overall, the BCD algorithm requires at most 5 iterations to converge. As ρ increases (not presented here), the convergence of all three algorithms are much faster thanks to the progressive initialization.

B. Comparison of Scattering Applications

On top of the setup in Fig. 4, we consider RIScatter and the following benchmark applications:

- Legacy: Active transmission without scatterers.
- BBC: The primary symbol becomes deterministic s[n] = 1 and the receive signal becomes

$$y^{\text{BBC}}[n] = \left(\boldsymbol{h}_{\text{D}}^{\text{H}} + \sum_{k} \alpha_{k} \boldsymbol{h}_{\text{C},k}^{\text{H}} x_{k}\right) \boldsymbol{w} + v[n]. \quad (37)$$

The total backscatter rate of coherent decoding approaches $K \log M$ when N is sufficiently large.

 AmBC: The user decodes both links independently and semi-coherently by treating the other as interference. The primary achievable rate is approximately¹³

$$I_{\mathrm{P}}^{\mathrm{AmBC}}(s;y) \approx \log\left(1 + \frac{|\boldsymbol{h}_{\mathrm{D}}^{\mathsf{H}}\boldsymbol{w}|^2}{\sum_{k}|\alpha_{k}\boldsymbol{h}_{\mathrm{C},k}^{\mathsf{H}}\boldsymbol{w}|^2 + \sigma_{v}^2}\right),$$
 (38)

 $^{13} \text{The scattered component is treated as interference with average power} \mathbb{E} \left\{ \sum_k \alpha_k \boldsymbol{h}_{\mathrm{C},k}^{\mathsf{H}} x_k \boldsymbol{w} s[n] \right\} = \sum_k |\alpha_k \boldsymbol{h}_{\mathrm{C},k}^{\mathsf{H}} \boldsymbol{w}|^2 \ [15].$

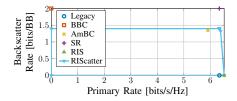


Fig. 8. Typical achievable rate region/points of scattering applications for $Q=1, K=1, M=4, N=10^3, \sigma_v^2=-40 \text{dBm}$ and r=2 m.

while the total backscatter rate follows (11) with uniform input distribution.

SR: For a sufficiently large N, the average primary rate under semi-coherent detection asymptotically approaches (13) with uniform input distribution [15]. When s[n] is successfully decoded and the direct interference h_D^Hws[n] is perfectly cancelled, the intermediate signal is

$$\hat{y}^{SR}[n] = \sum_{k} \alpha_k \boldsymbol{h}_{C,k}^{H} \boldsymbol{x}_k \boldsymbol{w} s[n] + v[n]. \tag{39}$$

The total backscatter rate of coherent decoding approaches $K\log M$ when N is sufficiently large.

• *RIS:* The reflection pattern is deterministic and the total backscatter rate is zero. The primary achievable rate is a special case of (13)

$$\begin{split} I_{\mathrm{P}}^{\mathrm{RIS}}(s; & y | x_{\mathcal{K}}) = I_{\mathrm{P}}(s; y | x_{m_{\mathcal{K}}^{\star}}) = \log \left(1 + \frac{|\boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}^{\star}}) \boldsymbol{w}|^{2}}{\sigma_{v}^{2}} \right), \\ & \text{where } m_{\mathcal{K}}^{\star} = & \operatorname{argmax}_{m_{\mathcal{K}}} I_{\mathrm{P}}^{\mathrm{RIS}}(s; y | x_{m_{\mathcal{K}}}). \end{split}$$

Fig. 8 compares the typical achievable rate region/points of RIScatter and those strategies. First, we observe BBC and SR achieve the best backscatter performance thanks to coherent decoding. For SR, this comes with the cost of N re-encoding, precoding, subtraction together with a time-domain MRC per backscatter symbol. Second, the average primary rate slightly decreases/increases in the presence of a AmBC/RIS node and the benefit of SR is not obvious. This is because the cascaded channel is usually orders of magnitude weaker than the direct channel. Here, RIS ensures constructive superposition of direct and scattered components while SR only creates a quasistatic rich-scattering environment that marginally enhances the average primary rate. When N is moderate, the randomly scattered signals should be modelled as interference rather than multipath components and the SR point should move towards the AmBC point. Third, RIScatter enables a flexible primarybackscatter tradeoff with adaptive input distribution design. In terms of maximum primary achievable rate, RIScatter coincides with RIS and outperforms the others using deterministic reflection pattern. On the other hand, for a large N, the maximum backscatter achievable rate of RIScatter is lower than BBC and SR but higher than AmBC. This is because both RIScatter and AmBC employ semi-coherent energy detection,

and the adaptive channel coding of RIScatter outperforms the

equiprobable inputs of AmBC, especially at the low backscatter SNR in the double fading scenario. When multiple antenna is

available at the AP, active beamforming can be optimized for

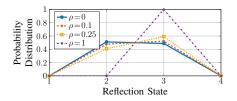


Fig. 9. Typical RIScatter reflection state distribution at different ρ for $Q\!=\!1$, $K\!=\!1$, $M\!=\!4$, $N\!=\!20$, $\sigma_v^2\!=\!-40 \mathrm{dBm}$ and $r\!=\!2\mathrm{m}$.

RIScatter nodes and the backscatter rate gain over AmBC is more prominent.

C. Input Distribution under Different QoS

The objective is to demonstrate RIScatter nodes can leverage CSI- and QoS-adaptive input distribution design to balance backscatter modulation and passive beamforming. For one RIScatter node with M=4, we evaluate the KKT input distribution at different QoS and present the result in Fig. 9. At $\rho = 0$ where the backscatter performance is prioritized, the optimal input distribution is 0 on two states and nearly uniform on the other two. This is inline with Shannon's observation that binary antipodal inputs is good enough for channel capacity at the low SNR [69]. Due to the weak scattered signal, the conditional energy PDF under different hypotheses can be closely spaced as in Fig. 5. The extreme states producing the lowest/highest energy are always assigned with non-zero probability, while the middles cannot provide enough energy difference and thus end up unused. At $\rho = 1$ where the primary link is prioritized, the optimal input distribution is 1 at state 3, since the corresponding reflection coefficient maximizes the primary SNR. That is, the reflection pattern becomes deterministic and the RIScatter node boils down to a discrete RIS element. Increasing ρ from 0 to 1 provides a smooth transition from backscatter modulation to passive beamforming, suggesting RIScatter unifies BackCom and RIS from a probabilistic perspective.

D. Rate Region by Different Schemes

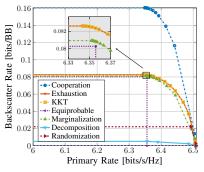
1) Input Distribution: We compare these input distribution designs for problem (18):

- Cooperation: Joint encoding using a joint probability array $p(x_{m_K})$ with M^K entries by Algorithm 1;
- Exhaustion: Exhaustive search over the M-dimensional probability simplex with resolution $\Delta p = 10^{-2}$;
- *KKT*: The KKT solution by Algorithm 1;
- Equiprobable: Uniform input distribution.

We also consider these independent distribution recovery methods from the joint probability array:

- Marginalization: Marginal probability distributions;
- *Decomposition:* Normalized rank-1 Canonical Polyadic (CP) decomposed tensors by Tensor Toolbox [70];
- *Randomization:* Gaussian randomization with the guidance of correlation matrix [71].

Fig. 10(a) shows their average achievable rate regions. It is observed that Cooperation achieves the outer bound of



(a) Input Distribution, Q = 1

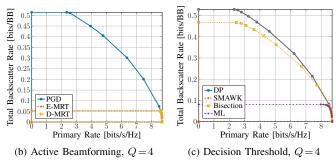


Fig. 10. Average primary-total-backscatter rate regions by different input distribution, active beamforming, and decision threshold schemes for K=2, M=4, N=20, $\sigma_v^2=-40 \mathrm{dBm}$ and $r=2 \mathrm{m}$.

all schemes, but its backscatter rate gain comes at the cost of implementation and computation complexity. Besides, the average rate performance of Exhaustion and KKT completely coincide with each other when $K\!=\!2$. This confirms Remark 5 that the KKT input distribution can be good enough when K is not very large. Equiprobable experiences minor backscatter and major primary rate losses without exploiting CSI and QoS. Those gaps should be larger when M and/or K increase. For the recovery methods, Marginalization provides a close performance to KKT, but Randomization and Decomposition fail our expectations for most channel realizations. Those observations emphasize the importance of adaptive RIScatter encoding and demonstrate the advantage of the proposed KKT input distribution design.

- 2) Active Beamforming: We consider three typical active beamforming schemes for problem (22):
 - PGA: Iterative PGA optimization by Algorithm 2;
 - *E-MRT*: MRT towards the ergodic composite channel $\sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) \boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}});$
 - *D-MRT*: MRT towards the direct channel $h_{\rm D}^{\rm H}$.

Fig. 10(b) presents the average achievable rate regions for those schemes. In the low- ρ regime, the proposed PGA beamformer significantly outperforms both MRT schemes in terms of total backscatter rate. This is because the semi-coherent backscatter decoding relies on the *relative difference* of receive energy under different backscatter symbol tuples. Such an energy diversity is enhanced by PGA, which effectively exploits backscatter constellation and input distribution knowledge rather than simply maximizing the direct/ergodic channel strength. As ρ increases, the primary SNR outweighs the backscatter energy difference in (28), and PGA beamformer

approaches both MRT schemes. At $\rho=1$, both PGA and E-MRT boil down to MRT towards the composite channel. The difference between E-MRT and D-MRT is insignificant when RIScatter nodes are dispersed. Those observations prove the proposed PGA active beamforming design can exploit the CSI, QoS, and backscatter constellation to enlarge the achievable rate region.

- 3) Decision Threshold: We evaluate the following decision threshold strategies for problem (33):
 - DP: Benchmark DP method for sequential quantizer [66];
 - SMAWK: DP accelerated by the SMAWK algorithm [66];
 - Bisection: The traverse-then-bisect algorithm [67];
 - ML: Maximum likelihood detector (34) [68].

Fig. 10(c) reveals the average achievable rate region for those strategies. The distribution-aware schemes DP, SMAWK and Bisection ensure higher total backscatter rate than ML. This is because the total backscatter rate (11) is a function of both input distribution and decision regions, and the rate-optimal threshold design heavily depends on input distribution. For example, the backscatter symbol tuples with zero input probability should be assigned with empty decision regions, in order to increase the success detection rates of other hypotheses. It highlights the importance of joint input distribution and decision threshold design in rate maximization problems.

E. Rate Region under Different Configurations

In this study, we choose Q=4, K=8, M=2, N=20, $\sigma_v^2=-40 \mathrm{dBm}$ and $r=2 \mathrm{m}$ as a reference.

- 1) Number of Nodes: Fig. 11(a) reveals how the number RIScatter nodes K influence the primary-backscatter tradeoff. Interestingly, we observe that increasing K has a larger benefit on the total backscatter rate than primary. This is because each RIScatter node not only affects the primary SNR but also influences the relative energy difference that other nodes can make. To maximize the total backscatter rate, some nodes closer to the user may need to sacrifice their own rate and use the state that minimizes the composite channel strength, in order to increase the backscatter rate of other nodes. This accounts for the significant primary rate decrease in the low- ρ regime. On the other hand, when the primary link is prioritized, the RIScatter nodes boil down to RIS elements and enjoy a conventional passive array gain K^2 .
- 2) Number of States: Fig. 11(b) shows the relationship between available reflection states (i.e., QAM order) M and achievable rate regions when $K\!=\!1$. We notice that increasing the reflection states has a marginal effect on the primary rate but significantly improves the backscatter rate. This is because the maximum amplitude normalized-QAM (2) involves more weak reflection points as M increases. It greatly enhances the receive energy difference but cannot provide enough phase shift resolution with strong reflection.
- 3) Number of Transmit Antennas: Fig. 11(c) illustrates the impact of transmit antennas Q on the average performance. As Q increases, more scattered paths are available to PGA and the channel diversity can be further exploited to enhance the primary SNR and backscatter energy difference. It emphasizes the importance of multi-antenna RIScatter systems and demonstrate the effectiveness of the proposed PGA design.

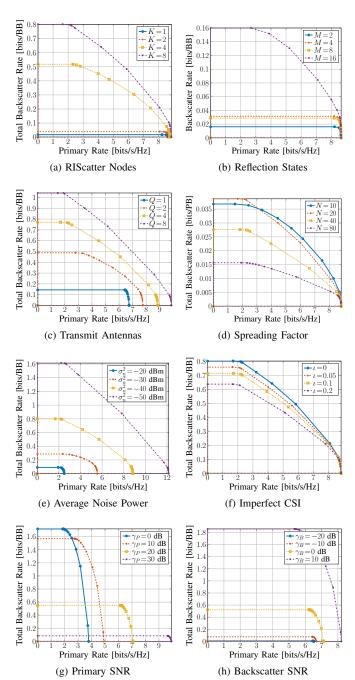


Fig. 11. Average primary-total-backscatter rate regions for different system configurations.

4) Spreading factor: Fig. 11(d) shows how the spreading factor N affects the achievable rate region. We using a very large N can severely constrain the backscatter throughput, since the gain in energy certainty (by law of large numbers) cannot withstand the loss in gross rate. As $N \to \infty$, RIScatter nodes boil down to static RIS elements and the total backscatter rate approaches 0. On the other hand, when N is too small, energy detection is error-prone as the DMMAC (10) is unreliable. It explains the observation that N=10 provides lower backscatter throughput than N=20. Therefore, we conclude the spreading

factor N should be carefully designed over multiple factors (e.g., data rate requirements, load switching speed at the nodes, and signal processing capability at the user).

- 5) Average Noise Power: Fig. 11(e) depicts the impact of average noise power σ_v^2 on average rate regions. Note that the noise influences both primary and backscatter SNR. When σ_v^2 relatively high, one can choose a larger N to improve the energy detection SNR.
- 6) Imperfect CSI: Due to the lack of RF chains at RIScatter nodes, fast and accurate acquisition of the cascaded CSI can be challenging, especially when the backscatter SNR is weak or the number of nodes is large. We consider an imperfect CSI model, where the cascaded channel of node k is estimated as

$$\hat{\boldsymbol{h}}_{\mathrm{C},k} = \boldsymbol{h}_{\mathrm{C},k} + \tilde{\boldsymbol{h}}_{\mathrm{C},k},\tag{41}$$

 $\tilde{h}_{C,k}$ is the estimation error with entries following i.i.d. CSCG distribution $\mathcal{CN}(0,\iota\Lambda_C)$, ι is the relative estimation error, and Λ_C is the cascaded path loss. In Fig. 11(f), it is observed that the channel estimation error mainly affects the backscatter rate. When ι increases from 0 to 0.2, the maximum total backscatter rate decreases by 20%. This is because the energy detector is sensitive to the DMMAC (10) and thus $\hat{h}_{C,k}$. On the other hand, a small estimation error may be insufficient to change the optimal passive beamforming state (especially for a small M) and the primary rate is almost unchanged in this case.

- 7) Primary SNR: The backscatter SNR is fixed to 0 dB in this study. Interestingly, Fig. 11(g) shows that increasing the primary SNR can improve the primary rate but degrade the backscatter rate. The reason is that the relative strength of the scattered signal compared to the direct signal is weakened, such that the nodes cannot make enough difference to the energy detector. This, together with Fig. 10(b) and 11(a), emphasizes the importance of accounting the primary-backscatter tradeoff in active-passive coexisting networks.
- 8) Backscatter SNR: The primary SNR is fixed to 20 dB in this study. Fig. 11(h) shows that the primary and backscatter rates are both improved when the backscatter SNR increases. This motivates one to use high-efficiency or semi-passive RIScatter nodes to improve the overall performance. In a multiuser RIScatter network, each node may be assigned to the nearest user to guarantee uniformly good performance for both links.

V. CONCLUSION

This paper introduced RIScatter as a novel scatter protocol that bridges backscatter modulation and passive beamforming. Starting from scattering principles, we showed how RIScatter nodes generalize information nodes of BackCom and reflect elements of RIS, how they can be built over existing passive scatter devices, and how they simultaneously encode self information and assist legacy transmission. We also proposed a practical SIC-free receiver that exploits the properties of active-passive coexisting networks to benefit both subsystems. The achievable primary-total-backscatter rate region was then studied for a single-user multi-node RIScatter network, where the input distribution, active beamforming, and decision thresholds are iteratively updated. Numerical results validated

¹⁴Please notice in Fig. 11(d) the unit of total backscatter rate is bits per primary block to indicate throughput.

the proposed algorithms and emphasized the importance of adaptive input distribution and cooperative receiver design.

To support massive RIScatter networks with large number of nodes and states, one possible future direction is to consider backscatter detection over the received signal domain rather than energy domain, where multi-antenna [72] and learning-based approaches can be promising. Another interesting question is how to design RIScatter node and receiver in a multi-user system to fully exploit the dynamic passive beamforming that naturally origins from backscatter modulation.

APPENDIX

A. Proof of Proposition 1

Denote the Lagrange multipliers associated with (17b) and (17c) as $\{\nu_k\}_{k\in\mathcal{K}}$ and $\{\lambda_{m_k}\}_{k\in\mathcal{K},m_k\in\mathcal{M}}$, respectively. The Lagrangian function of problem (18) is

$$I(x_{\mathcal{K}}) + \sum_{k} \nu_k \left(\sum_{m_k} p(x_{m_k}) - 1 \right) + \sum_{k} \sum_{m_k} \lambda_{m_k} p(x_{m_k}),$$
 (42)

and the KKT conditions are, $\forall k, m_k$,

$$I_k^{\star}(x_{m_k}) - (1-\rho) + \nu_k^{\star} + \lambda_{m_k}^{\star} = 0,$$
 (43a)

$$\lambda_{m_k}^{\star} = 0, \quad \text{if } p^{\star}(x_{m_k}) > 0,$$
 (43b)

$$\lambda_{m_k}^{\star} \ge 0$$
, if $p^{\star}(x_{m_k}) = 0$. (43c)

Plugging (43b) and (43c) into (43a) yields

$$I_k^{\star}(x_{m_k}) = 1 - \rho - \nu_k^{\star}, \quad \text{if } p^{\star}(x_{m_k}) > 0,$$
 (44a)

$$I_k^{\star}(x_{m_k}) \le 1 - \rho - \nu_k^{\star}, \quad \text{if } p^{\star}(x_{m_k}) = 0,$$
 (44b)

such that

$$\sum_{m_k} p^{\star}(x_{m_k}) I_k^{\star}(x_{m_k}) = 1 - \rho - \nu_k^{\star}. \tag{45}$$

On the other hand, by definition (20) we have

$$\sum_{m} p^{\star}(x_{m_k}) I_k^{\star}(x_{m_k}) = I^{\star}(x_{\mathcal{K}}), \tag{46}$$

where the right-hand side is irrelevant to k. (44), (45), and (46) together complete the proof.

B. Proof of Proposition 2

We first prove sequence (21) is non-decreasing in weighted sum mutual information. Let $p(x_{m_K}) = \prod_{q \in K} p(x_{m_q})$ and $p'(x_{m_K}) = p'(x_{m_k}) \prod_{q \in K \setminus \{k\}} p(x_{m_q})$ be two distributions potentially different at x_{m_k} , and $J(p(x_{m_K}), p'(x_{m_K}))$ be a joint function defined in (47) at the end of page 12. It is straightforward to verify $J(p(x_{m_K}), p(x_{m_K})) = I(x_K)$ and $J(p(x_{m_K}), p'(x_{m_K}))$ is a concave function for a given $p'(x_{m_K})$. Setting $\nabla_{p(x_{m_k})} J(p(x_{m_K}), p'(x_{m_K})) = 0$ yields

$$S'_k(x_{m_k}) - S'_k(x_{i_k}) + (1 - \rho) \log \frac{p(x_{i_k})}{p^*(x_{m_k})} = 0, \quad (48)$$

where $i_k \neq m_k$ is the reference state and

$$S'_{k}(x_{m_{k}}) \triangleq I'_{k}(x_{m_{k}}) + (1-\rho) \sum_{m_{\mathcal{K}} \setminus \{k\}} p(x_{m_{\mathcal{K}} \setminus \{k\}})$$

$$\times \sum_{m'_{\mathcal{K}}} q(\hat{x}_{m'_{\mathcal{K}}} | x_{m_{\mathcal{K}}}) \log p'(x_{m_{\mathcal{K}}}). \tag{49}$$

Evidently, $\forall m_k \neq i_k$, (48) boils down to

$$p^{\star}(x_{m_k}) = \frac{p'(x_{m_k}) \exp\left(\frac{\rho}{1-\rho} I'_k(x_{m_k})\right)}{\sum_{m'_k} p'(x_{m'_k}) \exp\left(\frac{\rho}{1-\rho} I'_k(x_{m'_k})\right)}.$$
 (50)

Since $p(x_{i_k}) = 1 - \sum_{m_k \neq i_k} p^*(x_{m_k})$ has exactly the same form as (50), the choice of reference does not matter and (50) is optimal $\forall m_k \in \mathcal{M}$. That is, for a fixed $p'(x_{m_k})$, (50) ensures

$$J(p(x_{m_{\mathcal{K}}}), p'(x_{m_{\mathcal{K}}})) \ge I'(x_{\mathcal{K}}). \tag{51}$$

On the other hand, we notice

$$I(x_{K}) - J(p(x_{m_{K}}), p'(x_{m_{K}}))$$

$$= (1 - \rho) \sum_{m_{k}} \frac{p'(x_{m_{k}}) f'_{k}(x_{m_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \sum_{m''_{K}} q(\hat{x}_{m''_{K}} | x_{m_{k}})$$

$$\times \log \frac{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{K}} | x_{m'_{k}}) f'_{k}(x_{m_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{K}} | x_{m'_{k}}) f'_{k}(x_{m_{k}})} \qquad (52a)$$

$$\geq (1 - \rho) \sum_{m_{k}} \frac{p'(x_{m_{k}}) f'_{k}(x_{m_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \sum_{m''_{K}} q(\hat{x}_{m''_{K}} | x_{m_{k}})$$

$$\times \left(1 - \frac{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{K}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{K}} | x_{m'_{k}}) f'_{k}(x_{m_{k}})} \right) \qquad (52b)$$

$$= (1 - \rho) \left(1 - \sum_{m_{k}} \frac{p'(x_{m_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \sum_{m''_{k}} q(\hat{x}_{m''_{K}} | x_{m_{k}})} \right) \qquad (52c)$$

$$= (1 - \rho) \left(1 - \sum_{m'_{k}} \frac{\sum_{m_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}} \right) \qquad (52c)$$

$$= (1 - \rho) \left(1 - \sum_{m'_{k}} \frac{\sum_{m_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}})} \right) \qquad (52d)$$

$$= (1 - \rho) \left(1 - \frac{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \sum_{m''_{k}} q(\hat{x}_{m''_{k}} | x_{m'_{k}})} \right) \qquad (52d)$$

$$= (1 - \rho) \left(1 - \frac{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \right) \qquad (52d)$$

$$= (1 - \rho) \left(1 - \frac{\sum_{m'_{k}} p'(x_{m'_{k}}) q(\hat{x}_{m''_{k}} | x_{m'_{k}}) f'_{k}(x_{m'_{k}})}{\sum_{m'_{k}} p'(x_{m'_{k}}) f'_{k}(x_{m'_{k}})} \right) \qquad (52e)$$

where $f_k'(x_{m_k}) \triangleq \exp\left(\frac{\rho}{1-\rho}I_k'(x_{m_k})\right)$ and the equality holds if and only if $p(x_{m_K})$ and $p'(x_{m_K})$ equals (i.e., (50) converges). (51) and (52) together imply $I(x_K) \geq I'(x_K)$. Since mutual information is bounded above, we conclude the sequence (21) is non-decreasing and convergent.

$$J\left(p(x_{m_{\mathcal{K}}}), p'(x_{m_{\mathcal{K}}})\right) \triangleq \sum_{m_{\mathcal{K}}} p(x_{m_{\mathcal{K}}}) \left(\rho \log\left(1 + \frac{|\boldsymbol{h}^{\mathsf{H}}(x_{m_{\mathcal{K}}})\boldsymbol{w}|^{2}}{\sigma_{v}^{2}}\right) + (1 - \rho) \sum_{m_{\mathcal{K}}'} q(\hat{x}_{m_{\mathcal{K}}'}|x_{m_{\mathcal{K}}}) \log\frac{q(\hat{x}_{m_{\mathcal{K}}'}|x_{m_{\mathcal{K}}})p'(x_{m_{\mathcal{K}}})}{p'(\hat{x}_{m_{\mathcal{K}}'})p(x_{m_{\mathcal{K}}})}\right). \tag{47}$$

Next, we prove any converging point of sequence (21), denoted as $p^*(x_{m_k})$, fulfills KKT conditions (19). Let

$$D^{(r)}(x_{m_k}) \triangleq \frac{p^{(r+1)}(x_{m_k})}{p^{(r)}(x_{m_k})} = \frac{f_k^{(r)}(x_{m_k})}{\sum_{m'_k} p^{(r)}(x_{m'_k}) f_k^{(r)}(x_{m'_k})}.$$
(53)

As sequence (21) is convergent, any state with $p^*(x_{m_k}) > 0$ need to satisfy $D^*(x_{m_k}) \triangleq \lim_{r \to \infty} D^{(r)}(x_{m_k}) = 1$, namely

$$I_{k}^{\star}(x_{m_{k}}) = \frac{1-\rho}{\rho} \log \sum_{m'_{k}} p^{\star}(x_{m'_{k}}) \exp\left(\frac{\rho}{1-\rho} I_{k}^{\star}(x_{m'_{k}})\right), \quad (54)$$

The right-hand side is a constant for node k and implies (44a). That is, any converging point with nonzero probability must satisfy (19a). On the other hand, we assume $p^*(x_{m_k})=0$ does not satisfy (19b), namely

$$I_k^{\star}(x_{m_k}) > I^{\star}(x_{\mathcal{K}}) = \sum_{m_k'} p^{\star}(x_{m_k'}) I_k^{\star}(x_{m_k'}), \tag{55}$$

Since the exponential function is monotonically increasing, (55) implies $f_k^{\star}(x_{m_k}) > \sum_{m_k'} p^{\star}(x_{m_k'}) f_k^{\star}(x_{m_k'})$ and $D^{\star}(x_{m_k}) > 1$. It contradicts with

$$p^{(r)}(x_{m_k}) = p^{(0)}(x_{m_k}) \prod_{n=1}^r D^{(n)}(x_{m_k}),$$
 (56)

since the left-hand side is zero while all terms on the right-hand side are strictly positive. The proof is completed.

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