

# UniScatter: Unifying Backscatter Communications, Reconfigurable Intelligent Surface, and Symbiotic Radio

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**Abstract**—Scatterers can harvest energy from, modulate information over, and influence propagation of surrounding radio waves. Backscatter Communications (BackCom) varies object impedance to manipulate the magnitude, phase, and/or frequency of scattered signal to encode information and deliver within coverage. Reconfigurable Intelligent Surface (RIS) adapts scattering antennas or programmable metamaterial to control propagation environment and boost signal strength in desired direction. Symbiotic Radio (SR) incorporates scatter nodes into active networks, which recycle ambient signal to transmit self information and enhance legacy multipath to the cooperative receiver. In this paper, we introduce UniScatter as a novel paradigm that bridges and generalizes those three concepts to support the high throughput, ultralow power, and ubiquitous connectivity of future wireless networks. Using shared spectrum, energy, and infrastructures, each UniScatter node adapts probability distribution of reflection states based on Channel State Information (CSI) to balance information encoding and passive beamforming in a mutualistic and flexible manner. The scatter source of BackCom/SR and reflecting element of RIS can be viewed as extreme cases of UniScatter node, where the input distribution boils down to equiprobable and degenerate, respectively. To accommodate signal characteristics, we propose a practical receiver that semi-coherently decodes all nodes from accumulated energy, re-encodes to determine the equivalent channel, then decodes the primary link. It reduces the complexity of cooperative decoding while preserves the benefit of backscatter modulation and passive beamforming. Moreover, we consider an application scenario where a multi-antenna Access Point (AP) serves a single-antenna user surrounded by multiple UniScatter nodes, and characterize the achievable primary-(total-)backscatter rate region by designing input distribution at the nodes, active beamforming at the AP, and backscatter decision design at the user. Simulation results demonstrate UniScatter can unify BackCom, RIS and SR to flexibly control the transmit-assist tradeoff via smart input distribution design.

## I. INTRODUCTION

**B**ACKSCATTER is recently re-innovated as a promising approach to support low-power communications and control wireless propagation environments. Ambient Backscatter Communications (AmBC) that enables battery-free communication between interactive nodes was first introduced and prototyped in [1], where devices harvest energy from and modulate information over ambient Radio-Frequency (RF) signals by switching between reflecting and absorbing states. To combat the strong direct-link interference of AmBC, [2] exploited the repeating structure of Orthogonal Frequency-Division Multiplexing (OFDM) symbol and proposed a multi-antenna

detector that only requires the backscatter channel strength. Cooperative AmBC that decodes primary and backscatter links using co-located receiver was proposed in [3], where the authors evaluated the error performance of Maximum-Likelihood (ML), linear, and Successive Interference Cancellation (SIC) detectors for flat fading channels and proposed a low-complexity ML detector for frequency-selective fading channels. The concept was then generalized to SR in [4] to “exploit the benefits and address the drawbacks” of Cognitive Radio (CR) and AmBC. The authors of [5] classified SR into commensal, parasitic, and competitive types based on link priority, and derived their instantaneous rates and optimal power allocations. The corresponding outage probabilities were also studied in [6]. Besides, [7] concluded that if the backscatter symbol period is sufficiently long, then the non-coherent primary rate would approach its coherent counterpart. The authors thus proposed to decode the primary link under backscatter uncertainty, then perform SIC and decode the backscatter link. [8] also explored the asymptotic impact of transmit/receive antenna and backscatter symbol period on the ergodic rate of primary and backscatter links. For a Multiple-Input Multiple-Output (MIMO) SR system with a multi-antenna backscatter node, [9] proposed a beamforming design to maximize the backscatter rate while guaranteeing the primary performance. However, those paper only considered one backscatter node and backscatter multiple access remains an open issue. In [10], a Non-Orthogonal Multiple Access (NOMA)-based SR was proposed and receive combining was investigated when SIC order follows equivalent channel strength. A Time-Division Multiple Access (TDMA)-based SR with energy harvesting constraints was also presented in [11], where transmit power, reflection efficiency, and time allocation were jointly optimized to maximize energy efficiency. To reduce coordination between passive nodes, [12] proposed a random code-assisted multiple access for SR and evaluated the asymptotic Signal-to-Interference-plus-Noise Ratio (SINR) using random matrix theory.

On the other hand, RIS alters propagation environment for legacy networks using numerous sub-wavelength passive elements with adaptive amplitudes and/or phases. Extensive research has been devoted to optimizing the phase shifts for the whole channel block to improve communication, sensing, and power performances [13]–[18]. Besides, [19] proposed a dynamic passive beamforming that further adjusts RIS over fine-grained time slots for OFDM systems to balance beamforming gain and multiuser diversity. The idea was further

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applied to Wireless Powered Communication Network (WPCN) to accommodate the downlink Wireless Power Transfer (WPT) phase and uplink Wireless Information Transfer (WIT) phase of the “harvest-then-transmit” protocol [20], [21]. For multiuser WPT, [22] also reported that dynamic beamforming can mimic multi-beam reflection in a time-division manner and reduce the phase extraction loss. Although dynamic passive beamforming artificially creates temporal diversity for flexible channel reconfiguration and resource allocation, it demands additional computational cost and control overhead, and the tradeoff deserves further attention especially for large RIS. In [23], [24], RIS was also introduced to single- and multi-node SR systems to reduce the total transmit power. When joint transmitter-RIS encoding is possible, [25] proved using RIS only for passive beamforming is generally suboptimal in terms of achievable rate for finite input constellations. Recently, RIS-empowered SR was introduced in [26]–[28] where independent passive beamforming and backscatter encoding were combined using advanced architectures. The authors of [26], [27] proposed the RIS to modulate binary message over the whole phase shift matrix and the receiver to decode from (non-coherent) primary to (coherent) backscatter link. In contrast, [28] divided the RIS into reflection and information elements, and evaluated the error performance of non-coherent backscatter detection.

To the best of our knowledge, all existing SR literatures assumed backscatter modulation employs either Gaussian codebook [5]–[11] or finite equiprobable inputs [3], [12], [26]–[29]. The former is impractical for passive nodes with constrained number of states, while the latter does not fully exploit CSI to boost achievable backscatter rate. Besides, most relevant designs [3], [5]–[12], [26], [27], [29] were built over ideal ML or SIC receiver. However, the advantage of SIC is questionable because 1) it requires non-coherent primary encoding at the transmitter and re-encoding, precoding, and subtraction at the receiver, 2) the primary and backscatter symbols are mixed by multiplication instead of superposition, and 3) the backscatter symbol period is typically much longer due to physical constraints. Motivated by those, we propose the concept of Metascatter, which adapts the input distribution of a passive backscatter node to generalize backscatter sources of SR and reflecting elements of RIS. The contributions of this paper is summarized as follows.

*First*, Metascatter adapts the input probability distribution of a finite-state passive backscatter device based on primary and (cascaded) backscatter CSI to unify and generalize parasitic source of SR and reflecting elements of RIS. The reflection pattern over time is no longer fully random or deterministic, but can be flexibly distributed to balance backscatter encoding and passive beamforming. For single-user scenario, when primary link is absolutely prioritized, the distribution falls on one state and Metascatter boils down to conventional RIS. When only considering backscatter performance, the distribution involves the highest entropy and Metascatter is essentially an AmBC node.

*Second*, we consider an application scenario where multiple Metascatters ride over a point-to-point transmission, exploiting additional propagation paths to simultaneously transmit and assist. To fully accommodate backscatter characteristics,

we also propose a novel receiving strategy that first jointly decodes all Metascatter from accumulated energy, then models their reflection patterns and backscatter paths within equivalent channel for primary decoding. Since backscatter message is modulated over primary signal, backscatter decoding is indeed part of primary channel training, and there is no need for operation-intensive SIC at the receiver.

*Third*, we evaluate the achievable primary-(total-)backscatter rate region by optimizing the input distribution at Metascatters, the active beamforming at the AP, and the decision regions at the user. Since the original problem is highly non-convex, we consider a suboptimal Block Coordinate Descent (BCD) algorithm where the Karush-Kuhn-Tucker (KKT) input distribution is numerically evaluated by limit of sequences, the active beamforming is iteratively updated by Projected Gradient Descent (PGD) accelerated by backtracking line search, and the decision regions are first restricted to convex, then refined by existing thresholding designs.

*Notations*: Scalars, vectors and matrices are respectively denoted by italic, bold lower-case, and bold upper-case letters.  $j$  represents the imaginary unit.  $\mathbb{R}_+^{x \times y}$  and  $\mathbb{C}^{x \times y}$  respectively denote the space of real nonnegative and complex  $x \times y$  matrices.  $\Delta^n = \{(p_0, \dots, p_n) \in \mathbb{R}_+^{n+1} | \sum_{i=0}^n p_i = 1\}$  denotes the standard  $n$ -simplex.  $(\cdot)^*$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $(\cdot)^+$ ,  $|\cdot|$ ,  $\|\cdot\|$  respectively represent the conjugate, transpose, conjugate transpose, ramp function, absolute value, and Euclidean norm. The distribution of a CSCG random vector with mean  $\mathbf{0}$  and covariance  $\Sigma$  is denoted by  $\mathcal{CN}(\mathbf{0}, \Sigma)$ .  $\sim$  means “distributed as”.  $(\cdot)^{(i)}$  represents the  $i$ -th iterated value and  $(\cdot)^*$  represents the end solution.

## II. BACKSCATTER PRINCIPLES

Passive backscatter nodes harvest energy from and modulate information over surrounding RF signals. As shown in Fig. 1(a), a typical passive node consists of a scattering antenna, an energy harvester, an integrated receiver, a load-switching modulator, and on-chip components (e.g., micro-controller and sensors) [31]. Its equivalent circuit is presented in Fig. 1(b). When illuminated, the node absorbs a portion of the impinging wave for information decoding and/or energy harvesting [32], and backscatters the remaining as *structural* and *antenna* components. The former consistently contributes to environment multipath and can be modelled by channel estimation [33], while the latter depends on antenna-load impedance mismatch and can be used for backscatter modulation [34] and/or channel reconfiguration [35]. Fig. 1(c) illustrates the scatter model of a node with  $M$  states, where the reflection coefficient of state  $m \in \mathcal{M} \triangleq \{1, \dots, M\}$  is defined as<sup>1</sup>

$$\Gamma_m = \frac{Z_m - Z_A^*}{Z_m + Z_A}, \quad (1)$$

where  $Z_m$  is the load impedance at state  $m$  and  $Z_A$  is the antenna input impedance.

<sup>1</sup>It corresponds to a linear backscatter model where the reflection coefficient is irrelevant to incident electromagnetic field strength.

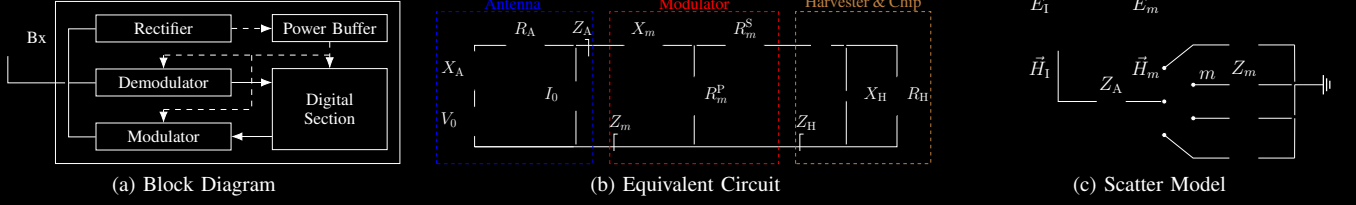


Fig. 1. Block diagram, equivalent circuit, and scatter model of a passive backscatter node. The solid and dashed vectors represent signal and energy flows. The backscatter antenna behaves as a constant power source, where the voltage  $V_0$  and current  $I_0$  are introduced by incident electric field  $\vec{E}_i$  and magnetic field  $\vec{H}_i$  [30].

### A. SR: Backscatter Modulation

Backscatter sources encode self message by *random reflection states variation*. For  $M$ -ary Quadrature Amplitude Modulation (QAM), reflection coefficient  $\Gamma_m$  maps to the corresponding *complex constellation point*  $c_m$  by [36]

$$\Gamma_m = \alpha \frac{c_m}{\max_{m'} |c_{m'}|}, \quad (2)$$

where  $0 \leq \alpha \leq 1$  is the amplitude reflect ratio that controls the harvest-scatter tradeoff at the direction of interest.

### B. IRS: Channel Reconfiguration

RIS elements assist legacy transmission by *deterministic phase shifts selection* based on relevant CSI. For a reflecting element with  $M$  candidate states, reflection coefficient  $\Gamma_m$  relates to the corresponding *phase shift*  $\theta_m$  by [13]

$$\Gamma_m = \beta_m \exp(j\theta_m), \quad (3)$$

where  $0 \leq \beta_m \leq 1$  is overall amplitude reflect ratio of state  $m$ .

### C. Metascatter: Bridge and Generalization

Metascatters simultaneously transmit and assist by *adaptive input distribution design* based on primary and (cascaded) backscatter CSI. Instead of using fully random or deterministic reflection pattern over time, as shown in Fig. 2, Metascatter semi-randomly selects reflection state for each backscatter block, with *guidance of input probability*  $P(\Gamma_m)$  for state  $m$ . In other words, it flexibly controls input distribution of candidate reflection states to balance backscatter encoding and passive beamforming. SR and RIS can be regarded as extreme cases of Metascatter, where node input distribution boils down to uniform and deterministic, respectively.

**Remark 1.** Compared to conventional RIS literatures that optimize phase shifts under unit-module constraint, Metascatter starts from predefined reflection coefficients and designs their input distribution under sum-probability constraint to achieve flexible primary-backscatter tradeoff.

## III. METASCATTER-ENABLED NETWORK

### A. System Model

As shown in Fig. 3, we propose a Metascatter-enabled single-user multi-node network where two coexisting systems share spectrum, energy and infrastructures. The primary point-to-point transmission from a  $Q$ -antenna AP to a single-antenna

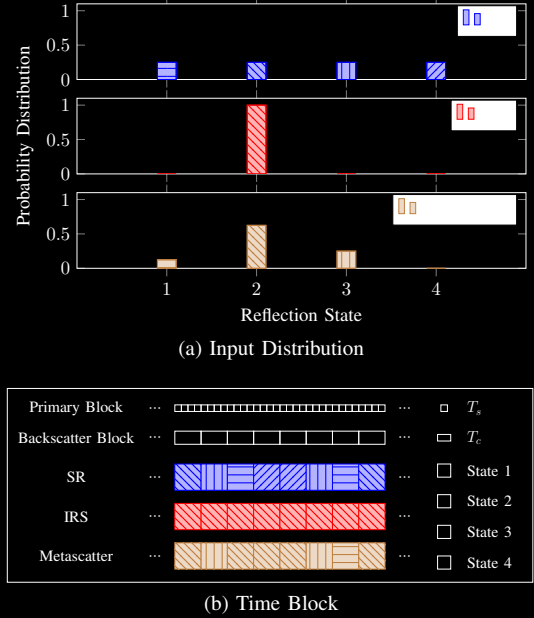


Fig. 2. Input probability distribution and time block structure of SR, RIS, and Metascatter.  $T_s$  and  $T_c$  respectively denote the primary and backscatter symbol period. Within channel coherence time, Metascatter semi-randomly selects reflection state for each backscatter block, with guidance from input probability distribution.

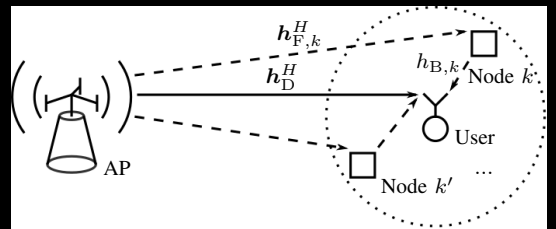


Fig. 3. A Metascatter-enabled single-user multi-node network.

user is assisted by  $K$  nearby single-antenna Metascatters. In the secondary backscatter Multiple Access Channel (MAC) system, the AP serves as the carrier emitter,  $K$  nearby single-antenna Metascatters modulate information over reradiated RF signals, and the user decodes their messages. For simplicity, we consider a quasi-static block fading model where channels remain constant within coherence interval while vary independently between consecutive blocks. Due to physical constraints on load switching, we assume the backscatter symbol period is  $N \gg 1$  times longer than primary and consider integer  $N$  without loss of generality. We also assume the direct channel and all

cascaded channels can be estimated and fed back to the AP.<sup>2</sup> Besides, we omit the signal reflected by two or more times [40] and ignore the time difference of arrival from different paths [5].

Denote the AP-user direct channel as  $\mathbf{h}_D^H \in \mathbb{C}^{1 \times Q}$ , the AP-node  $k \in \mathcal{K} \triangleq \{1, \dots, K\}$  forward channel as  $\mathbf{h}_{F,k}^H \in \mathbb{C}^{1 \times Q}$ , and the node  $k$ -user backward channel as  $h_{B,k}$ . Also, define the cascaded channel of tag  $k$  as  $\mathbf{h}_{C,k}^H \triangleq h_{B,k} \mathbf{h}_{F,k}^H \in \mathbb{C}^{1 \times Q}$ , and  $\mathbf{H}_C \triangleq [\mathbf{h}_{C,1}, \dots, \mathbf{h}_{C,K}]^H \in \mathbb{C}^{K \times Q}$ . Let  $\mathbf{x}_K \triangleq (x_1, \dots, x_K)$  be the backscatter symbol tuple of all Metascatters. Consider the signal model during one backscatter block (i.e.,  $N$  primary blocks). Under perfect synchronization, the equivalent primary channel is a function of *coded* backscatter symbols

$$\mathbf{h}_E^H(\mathbf{x}_K) \triangleq \mathbf{h}_D^H + \sum_{k \in \mathcal{K}} \alpha_k \mathbf{h}_{C,k}^H x_k \quad (4a)$$

$$= \mathbf{h}_D^H + \mathbf{x}^H \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_C, \quad (4b)$$

where  $\alpha_k$  is the amplitude reflect ratio of Metascatter  $k$ ,  $\boldsymbol{\alpha} \triangleq [\alpha_1, \dots, \alpha_K]^T \in \mathbb{R}_+^{K \times 1}$ ,  $x_k \in \mathcal{X} \triangleq \{c_1, \dots, c_M\}$  is the coded backscatter symbol of Metascatter  $k$ , and  $\mathbf{x} \triangleq [x_1, \dots, x_K]^H \in \mathbb{C}^{K \times 1}$ . The signal received by the user at primary block  $n \in \mathcal{N} \triangleq \{1, \dots, N\}$  is

$$y[n] = \mathbf{h}_E^H(\mathbf{x}_K) \mathbf{w} s[n] + v[n], \quad (5)$$

where  $s \sim \mathcal{CN}(0, 1)$  is the primary symbol,  $v \sim \mathcal{CN}(0, \sigma_v^2)$  is the Additive White Gaussian Noise (AWGN), and  $\mathbf{w} \in \mathbb{C}^{Q \times 1}$  is the active beamforming vector with average power constraint  $\|\mathbf{w}\|^2 \leq P$ .

**Remark 2.** Metascatter involves a symbiotic MAC where the primary and backscatter symbols of different duration are mixed by Multiplication Coding (MC) instead of Superposition Coding (SC). For each node, the reflection coefficient not only encodes the backscatter message, but also influences the equivalent primary channel (4). To accommodate such signal characteristics, novel receiving strategy apart from SIC is desired to better utilize the reflection pattern and boost the primary-backscatter tradeoff.

## B. Receiving Strategy

We propose a Metascatter receiver where the backscatter messages of all Metascatters are first jointly and semi-coherently detected using total received energy per backscatter block, then modeled within equivalent channel (4) as dynamic passive beamforming. Compared with ML and SIC, Metascatter receiver allows practical and low-complexity node multiple access with minor adjustment over legacy equipments.

At a specific backscatter block, denote  $m_k \in \mathcal{M}$  as the state index of Metascatter  $k$ , and let  $\mathbf{m}_K \triangleq (m_1, \dots, m_K)$  be the state index tuple of all Metascatters. Conditioned on  $\mathbf{m}_K$ , the received signal at primary block  $n$  is subject to the variation of  $s[n]$  and  $v[n]$ , distributed as  $y[n] \sim \mathcal{CN}(0, \sigma_{\mathbf{m}_K}^2)$  with

$$\sigma_{\mathbf{m}_K}^2 = |\mathbf{h}_E^H(\mathbf{x}_{\mathbf{m}_K}) \mathbf{w}|^2 + \sigma_v^2, \quad (6)$$

<sup>2</sup>Due to the lack of RF chains at the passive tag, accurate and efficient CSI acquisition at the AP can be challenging. One possibility is the AP sends training pilots, the tags respond in predefined manners, and the user performs least-square estimation with feedbacks [37]–[39].

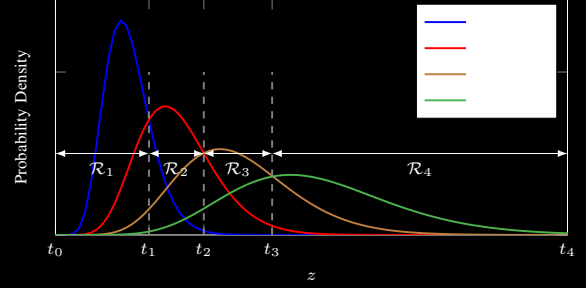


Fig. 4. PDF of total received energy per backscatter block, conditioned on different input hypothesis. Here, the convex ML decision regions are generally rate-suboptimal except for equiprobable inputs.

where  $x_{m_k}$  and  $\mathbf{x}_{\mathbf{m}_K}$  are the symbol and symbol tuple associated with state  $m_k$  and state tuple  $\mathbf{m}_K$ , respectively.<sup>3</sup> Also, denote the total received energy within backscatter block as  $z = \sum_{n=1}^N |y[n]|^2$ . As the sum of  $N$  independent and identically distributed (i.i.d.) exponential variables, the Probability Density Function (PDF) of  $z$  conditioned on  $\mathbf{m}_K$  follows Erlang distribution

$$f(z|\mathcal{H}_{\mathbf{m}_K}) = \frac{z^{N-1} \exp(-z/\sigma_{\mathbf{m}_K}^2)}{\sigma_{\mathbf{m}_K}^{2N} (N-1)!}, \quad (7)$$

where  $\mathcal{H}_{\mathbf{m}_K}$  denotes hypothesis  $\mathbf{m}_K$ . To accommodate backscatter characteristics and reduce decoding complexity, we consider a joint semi-coherent detection for all Metascatters over accumulated energy  $z$ . Once disjoint energy decision regions are determined, we can construct a Discrete Thresholding Multiple Access Channel (DTMAC) and formulate the transition probability from input  $\mathbf{x}_{\mathbf{m}_K}$  to output  $\hat{\mathbf{x}}_{\mathbf{m}'_K}$  as

$$P(\hat{\mathbf{x}}_{\mathbf{m}'_K} | \mathbf{x}_{\mathbf{m}_K}) = \int_{\mathcal{R}_{\mathbf{m}'_K}} f(z|\mathcal{H}_{\mathbf{m}_K}) dz, \quad (8)$$

where  $\mathcal{R}_{\mathbf{m}'_K}$  is the decision region of hypothesis  $\mathcal{H}_{\mathbf{m}'_K}$ . An example of ML energy decision is illustrated in Fig. 4.

**Remark 3.** The rate-optimal thresholding channel design remains under-investigated, and some attempts were made for single source with binary inputs in [41], [42]. For non-binary inputs with general distribution, the optimal decision region for each letter can be non-convex (i.e., with non-adjacent partitions) and the optimal number of thresholds is still unknown.

In the following context, we restrict all decision regions to convex and optimize decision thresholds accordingly. That is, for any bijective mapping  $f: \mathbf{m}_K \rightarrow \mathcal{L} \triangleq \{1, \dots, M^K\}$ , the decision region of letter  $l \in \mathcal{L}$  is defined as  $\mathcal{R}_l \triangleq [t_{l-1}, t_l]$ , where  $t_{l-1} \leq t_l$ . We also define the decision threshold vector as  $\mathbf{t} \triangleq [t_0, \dots, t_L]^T \in \mathbb{R}_+^{(L+1) \times 1}$ .

## C. Achievable Rates

Denote the input probability of state  $m_k$  of Metascatter node  $k$  as  $P_k(x_{m_k})$ , and define the input probability distribution vector of node  $k$  as  $\mathbf{p}_k \triangleq [P_k(c_1), \dots, P_k(c_M)]^T \in \mathbb{R}^{M \times 1}$ .

<sup>3</sup> $x_k$  and  $\mathbf{x}_K$  are random variables, while  $x_{m_k}$  and  $\mathbf{x}_{\mathbf{m}_K}$  are their instances indexed by  $m_k$  and  $\mathbf{m}_K$ .

With independent encoding at all nodes, the probability of backscatter symbol tuple  $x_{m_{\mathcal{K}}}$  is

$$P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) = \prod_{k \in \mathcal{K}} P_k(x_{m_k}). \quad (9)$$

Similar to [43], we define the backscatter information function between input symbol tuple instance  $x_{m_{\mathcal{K}}}$  and output symbol tuple  $\hat{x}_{\mathcal{K}}$  as

$$I^{\text{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}) \triangleq \sum_{m'_{\mathcal{K}}} P(\hat{x}_{m'_{\mathcal{K}}} | x_{m_{\mathcal{K}}}) \log \frac{P(\hat{x}_{m'_{\mathcal{K}}} | x_{m_{\mathcal{K}}})}{P(\hat{x}_{m'_{\mathcal{K}}})}, \quad (10)$$

where  $P(\hat{x}_{m'_{\mathcal{K}}}) = \sum_{m_{\mathcal{K}}} P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) P(\hat{x}_{m'_{\mathcal{K}}} | x_{m_{\mathcal{K}}})$ . We also define the backscatter marginal information of letter  $x_{m_k}$  of node  $k$  as

$$I_k^{\text{B}}(x_{m_k}; \hat{x}_{\mathcal{K}}) \triangleq \sum_{m_{\mathcal{K}} \setminus \{k\}} P_{\mathcal{K} \setminus \{k\}}(x_{m_{\mathcal{K}} \setminus \{k\}}) I^{\text{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}), \quad (11)$$

where  $P_{\mathcal{K} \setminus \{k\}}(x_{m_{\mathcal{K}} \setminus \{k\}}) = \prod_{q \in \mathcal{K} \setminus \{k\}} P_q(x_{m_q})$ . Moreover, we can write the backscatter mutual information as

$$I^{\text{B}}(x_{\mathcal{K}}; \hat{x}_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) I^{\text{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}). \quad (12)$$

Once backscatter messages are successfully decoded, we can re-encode to determine  $x_{\mathcal{K}}$  and retrieve equivalent primary channel by (4). We define the primary information function conditioned on backscatter symbol tuple  $x_{m_{\mathcal{K}}}$  as

$$I^{\text{P}}(s; y | x_{m_{\mathcal{K}}}) \triangleq \log \left( 1 + \frac{|\mathbf{h}_{\text{E}}^H(x_{m_{\mathcal{K}}}) \mathbf{w}|^2}{\sigma_v^2} \right), \quad (13)$$

the primary marginal information conditioned on letter  $x_{m_k}$  of node  $k$  as

$$I_k^{\text{P}}(s; y | x_{m_k}) \triangleq \sum_{m_{\mathcal{K}} \setminus \{k\}} P_{\mathcal{K} \setminus \{k\}}(x_{m_{\mathcal{K}} \setminus \{k\}}) I^{\text{P}}(s; y | x_{m_{\mathcal{K}}}), \quad (14)$$

and the primary ergodic mutual information as

$$I^{\text{P}}(s; y | x_{\mathcal{K}}) = \sum_{m_{\mathcal{K}}} P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) I^{\text{P}}(s; y | x_{m_{\mathcal{K}}}). \quad (15)$$

Finally, with a slight abuse of notation, we define the corresponding weighted sum information function, marginal information, and mutual information respectively as

$$I(x_{m_{\mathcal{K}}}) \triangleq \rho I^{\text{P}}(s; y | x_{m_{\mathcal{K}}}) + (1 - \rho) I^{\text{B}}(x_{m_{\mathcal{K}}}; \hat{x}_{\mathcal{K}}), \quad (16)$$

$$I_k(x_{m_k}) \triangleq \rho I_k^{\text{P}}(s; y | x_{m_k}) + (1 - \rho) I_k^{\text{B}}(x_{m_k}; \hat{x}_{\mathcal{K}}), \quad (17)$$

$$I(x_{\mathcal{K}}) \triangleq \rho I^{\text{P}}(s; y | x_{\mathcal{K}}) + (1 - \rho) I^{\text{B}}(x_{\mathcal{K}}; \hat{x}_{\mathcal{K}}), \quad (18)$$

where  $0 \leq \rho \leq 1$  is the relative priority of the primary link.

#### IV. INPUT DISTRIBUTION, ACTIVE BEAMFORMING, AND DECISION THRESHOLD DESIGN

To characterize the achievable primary-(total-)backscatter rate region of the proposed Metascatter-enabled network, we aim to maximize the weighted sum mutual information

with respect to node input distributions  $\{\mathbf{p}_k\}_{k \in \mathcal{K}}$ , active beamforming vector  $\mathbf{w}$ , and decision threshold vector  $\mathbf{t}$  as

$$\max_{\{\mathbf{p}_k\}_{k \in \mathcal{K}}, \mathbf{w}, \mathbf{t} \in \mathbb{R}_+^{L+1}} I(x_{\mathcal{K}}) \quad (19a)$$

$$\text{s.t.} \quad \sum_{m_k} P_k(x_{m_k}) = 1, \quad \forall k, \quad (19b)$$

$$P_k(x_{m_k}) \geq 0, \quad \forall k, m_k, \quad (19c)$$

$$\|\mathbf{w}\|^2 \leq P, \quad (19d)$$

$$t_{l-1} \leq t_l, \quad \forall l. \quad (19e)$$

Problem (19) is highly non-convex, and we propose a BCD algorithm that iteratively updates  $\{\mathbf{p}_k\}_{k \in \mathcal{K}}$ ,  $\mathbf{w}$  and  $\mathbf{t}$  until convergence.

##### A. Input Distribution

For any given  $\mathbf{w}$  and  $\mathbf{t}$ , we can construct the equivalent DTMAC by (8) and simplify (19) to

$$\max_{\{\mathbf{p}_k\}_{k \in \mathcal{K}}} I(x_{\mathcal{K}}) \quad (20a)$$

$$\text{s.t.} \quad (19b), (19c), \quad (20b)$$

which involves coupled term  $\prod_{k \in \mathcal{K}} P_k(x_{m_k})$  and is non-convex when  $K > 1$ . Next, we propose a numerical method that evaluate the KKT input distribution by limit of sequences.

**Remark 4.** As pointed out in [44], KKT conditions are generally necessary but insufficient for total rate maximization in discrete memoryless MAC. Therefore, KKT solutions may end up being saddle points of problem (20).

Following [43], we first recast KKT conditions to their equivalent form for problem (20), then propose an iterative method that guarantees input distribution satisfying above conditions on convergence.

**Proposition 1.** The KKT optimality conditions for problem (20) are equivalent to,  $\forall k, m_k$ ,

$$I_k^*(x_{m_k}) = I^*(x_{\mathcal{K}}), \quad P_k^*(x_{m_k}) > 0, \quad (21a)$$

$$I_k^*(x_{m_k}) \leq I^*(x_{\mathcal{K}}), \quad P_k^*(x_{m_k}) = 0. \quad (21b)$$

*Proof.* Please refer to Appendix A.  $\square$

For each node, (21a) suggests each probable state should produce the same marginal information (averaged over all states of other nodes), while (21b) implies any state with potentially less marginal information should not be used.

**Proposition 2.** The KKT input probability of node  $k$  of state  $m_k$  is given by the converging point of the sequence

$$P_k^{(r+1)}(x_{m_k}) = \frac{P_k^{(r)}(x_{m_k}) \exp\left(\frac{\rho}{1-\rho} I_k^{(r)}(x_{m_k})\right)}{\sum_{m'_k} P_k^{(r)}(x_{m'_k}) \exp\left(\frac{\rho}{1-\rho} I_k^{(r)}(x_{m'_k})\right)}, \quad (22)$$

where  $r$  is the iteration index and  $\mathbf{p}_k^{(0)} > \mathbf{0}$ ,  $\forall k$ .

*Proof.* Please refer to Appendix B.  $\square$

At each iteration, the input distribution of node  $k$  is evaluated over updated input distribution of node 1 to  $k-1$ , together

**Algorithm 1:** Numerical Evaluation of KKT Input Distribution**Input:**  $K, N, h_D^H, H_C, \alpha, \mathcal{X}, \sigma_v^2, \rho, w, t, \epsilon$ **Output:**  $\{p_k^*\}_{k \in \mathcal{K}}$ 

- 1: Set  $h_E^H(x_{m_K}), \forall m_K$  by (4)
- 2:  $\sigma_{m_K}^2, \forall m_K$  by (6)
- 3:  $f(z|\mathcal{H}_{m_K}), \forall m_K$  by (7)
- 4:  $P(\hat{x}_{m'_K}|x_{m_K}), \forall m_K, m'_K$  by (8)
- 5: Initialize  $r \leftarrow 0$
- 6:  $p_k^{(0)} > 0, \forall k$
- 7: Get  $P_K^{(r)}(x_{m_K}), \forall m_K$  by (9)
- 8:  $I_K^{(r)}(x_{m_K}), \forall m_K$  by (10), (13), (16)
- 9:  $I_k^{(r)}(x_{m_K}), \forall k, m_K$  by (11), (14), (17)
- 10:  $I^{(r)}(x_K)$  by (12), (15), (18)
- 11: **Repeat**
- 12:   Update  $r \leftarrow r+1$
- 13:    $p_k^{(r)}, \forall k$  by (22)
- 14:   Redo step 7–10
- 15: **Until**  $I^{(r)}(x_K) - I^{(r-1)}(x_K) \leq \epsilon$

with previous input distribution of node  $k+1$  to  $K$ . The KKT input distribution design is summarized in Algorithm 1.

**B. Active Beamforming**

For any given  $\{p_k\}_{k \in \mathcal{K}}$  and  $t$ , problem (19) reduces to

$$\max_w I(x_K) \quad (23a)$$

$$\text{s.t.} \quad (19d), \quad (23b)$$

which is still non-convex due to integration and entropy terms. To see this, we explicitly write (23a) as (24) at the bottom of page 6, where

$$Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) = \frac{\int_{t_{l-1}/\sigma_{m_K}^2}^{t_l/\sigma_{m_K}^2} z^{N-1} \exp(-z) dz}{(N-1)!} \quad (25)$$

is the regularized incomplete Gamma function that substitutes the DTMAC transition probability (8). Its series representation is given by [45, Theorem 3]

$$Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) = \exp\left(-\frac{t_{l-1}}{\sigma_{m_K}^2}\right) \sum_{n=0}^{N-1} \frac{\left(\frac{t_{l-1}}{\sigma_{m_K}^2}\right)^n}{n!} - \exp\left(-\frac{t_l}{\sigma_{m_K}^2}\right) \sum_{n=0}^{N-1} \frac{\left(\frac{t_l}{\sigma_{m_K}^2}\right)^n}{n!}. \quad (26)$$

Next, we derive the gradients of (26) and (24) w.r.t.  $w^*$  as (27) and (28) at the end of page 7 and 7, respectively. It allows problem (23) to be solved by the PGD method, where any

**Algorithm 2:** Iterative Active Beamforming Design by PGD with Backtracking Line Search**Input:**  $Q, N, h_D^H, H_C, \alpha, \mathcal{X}, P, \sigma_v^2, \rho, \{p_k\}_{k \in \mathcal{K}}, t, \alpha, \beta, \gamma, \epsilon$ **Output:**  $w^*$ 

- 1: Set  $h_E^H(x_{m_K}), \forall m_K$  by (4)
- 2:  $P_K(x_{m_K}), \forall m_K$  by (9)
- 3: Initialize  $r \leftarrow 0$
- 4:  $w^{(0)}, \|w^{(0)}\|^2 \leq P$
- 5: Get  $(\sigma_{m_K}^{(r)})^2, \forall m_K$  by (6)
- 6:  $Q^{(r)}\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right), \forall m_K, l$  by (25) or (26)
- 7:  $I^{(r)}(x_K)$  by (24)
- 8:  $\nabla_{w^*} Q^{(r)}\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right), \forall m_K, l$  by (27)
- 9:  $\nabla_{w^*} I^{(r)}(x_K)$  by (28)
- 10: **Repeat**
- 11:   Update  $r \leftarrow r+1$
- 12:    $\gamma^{(r)} \leftarrow \gamma$
- 13:    $\bar{w}^{(r)} \leftarrow w^{(r-1)} + \gamma \nabla_{w^*} I^{(r-1)}(x_K)$
- 14:    $w^{(r)}$  by (29)
- 15:   Redo step 5–7
- 16:   **While**  $I^{(r)}(x_K) < I^{(r-1)}(x_K) + \alpha \gamma \|\nabla_{w^*} I^{(r-1)}(x_K)\|^2$
- 17:     Set  $\gamma^{(r)} \leftarrow \beta \gamma^{(r)}$
- 18:     Redo step 13–15
- 19:   **End While**
- 20:   Redo step 8, 9
- 21: **Until**  $\|w^{(r)} - w^{(r-1)}\| \leq \epsilon$

unregulated beamforming vector  $\bar{w}$  can be projected onto the feasible domain of average transmit power constraint (19d) by

$$w = \sqrt{P} \frac{\bar{w}}{\max(\sqrt{P}, \|\bar{w}\|)}. \quad (29)$$

We present the iterative active beamforming design accelerated by backtracking line search in Algorithm 2.

**C. Decision Threshold**

For any given  $\{p_k\}_{k \in \mathcal{K}}$  and  $w$ , problem (19) reduces to

$$\max_{t \in \mathbb{R}_+^{L+1}} I(x_K) \quad (30a)$$

$$\text{s.t.} \quad (19e), \quad (30b)$$

where it is trivial to conclude  $t_0^* = 0$  and  $t_L^* = \infty$  for energy-based backscatter detection.

**Remark 5.** Backscatter detection (and decision design) has no impact on primary achievable rate. When nodes transmit at non-zero total rate, the user can re-encode backscatter messages to recover coded backscatter tuple  $x_K$  at each block. Otherwise,  $x_K$  can be fully deterministic and known to the user.

$$I(x_K) = \sum_{m_K} P_K(x_{m_K}) \left( \rho \log\left(1 + \frac{|h_E^H(x_{m_K})w|^2}{\sigma_v^2}\right) + (1-\rho) \sum_l Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) \log \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right)}{\sum_{m'_K} P_K(x_{m'_K}) Q\left(N, \frac{t_{l-1}}{\sigma_{m'_K}^2}, \frac{t_l}{\sigma_{m'_K}^2}\right)} \right) \quad (24)$$

Remark 5 suggests any  $t$  maximizes total backscatter mutual information  $I^B(x_K; \hat{x}_K)$  is also optimal for problem (30).

**Remark 6.** In terms of total backscatter rate, the nodes can be regarded as an equivalent source with augmented alphabet of symbol tuple  $x_{m_K}$ , and the DTMAC (8) essentially reduces to a Discrete Memoryless Thresholding Channel (DMTC).

Finally, we can employ existing thresholding design for DMTC to solve problem (30). For example, [46] first discretized the continuous energy  $z$  into numerous output bins, then grouped adjacent bins to maximize mutual information using Dynamic Programming (DP) accelerated by Shor-Moran-Aggarwal-Wilber-Klawe (SMAWK) algorithm. In [47], the authors first proved the optimality condition for any three neighbor thresholds, then fix  $t_0$ , traverse  $t_1$ , and sequentially optimizes the others by bisection. Both will be compared with ML decision [48]

$$t_l^{\text{ML}} = N \frac{\sigma_{l-1}^2 \sigma_l^2}{\sigma_{l-1}^2 - \sigma_l^2} \log \frac{\sigma_{l-1}^2}{\sigma_l^2}, \quad l \in \mathcal{L} \setminus \{0, L\}, \quad (31)$$

which is generally suboptimal for problem (30) except for equiprobable inputs at all nodes.

## V. SIMULATION RESULTS

In this section, we provide numerical results to evaluate the proposed input, beamforming and decision design over a single-user multi-node Metascatter-enabled network. We assume the distance between AP and user is 10 m, and  $K=2$  Metascatters are uniformly dropped within a disk centered at the user of radius 1 m. The carrier frequency is  $f=200\text{MHz}$ , and we consider i.i.d. Ricean fading between all terminals. For direct, forward and backward links, we set the path loss exponents to 2.6, 2.4 and 2, and the Ricean factor to 5, 5 and 10, respectively. The AP has  $Q=4$  antennas with maximum

average transmit power  $P=36\text{dBm}$ . All nodes have amplitude reflect ratio  $\alpha=0.5$ , symbol period ratio  $N=20$ , and perform QAM with  $M=2$  input states. The user is with average noise power  $\sigma_v^2=70\text{dBm}$  and receive antenna gain 3 dBi. All rate regions are averaged over at least 1000 instances, where ‘‘PSP’’ and ‘‘BSP’’ means primary and backscatter symbol periods, respectively. We choose decision design by SMAWK [46] as reference, and select discretize boundaries uniformly over the 95% confidence region of edge hypotheses. The parameters remain fixed unless otherwise specified.

### A. Input Distribution vs. Weight

In Fig. 5, we present typical input distributions of a single Metascatter with  $M=4$  inputs under different weight  $\rho$ . Fig. 5(a) and 5(b) are obtained from i.i.d. drop and channel realizations. We note that even for  $\rho=0$  (i.e., best backscatter performance), the optimal input distribution is not equiprobable like SR, and adaptive channel coding can further increase total backscatter rate based on CSI. On the other hand, the optimal input distribution becomes fully deterministic at  $\rho=1$  (i.e., best primary performance), where the state maximizes equivalent primary channel (4) strength is chosen with probability 1. In this case, Metascatter boils down to an RIS element with  $M$  discrete states. As  $\rho$  moves from 0 to 1, the optimal input distribution becomes gradually biased to one state and flexibly balances the primary-backscatter tradeoff.

### B. Rate Region vs. Input, Beamforming, and Decision Schemes

Fig. 6(a) compares the achievable rate regions by following input designs.

- **Exhaustion:**  $K$ -dimensional grid search over probability simplex;
- **KKT:** results of Algorithm 1;

$$\begin{aligned} \nabla_{\mathbf{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) &= \frac{\mathbf{h}_E(x_{m_K}) \mathbf{h}_E^H(x_{m_K}) \mathbf{w}}{(|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2)^2} \\ &\times \left( t_l \exp\left(-\frac{t_l}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right) \left(-1 + \sum_{n=1}^{N-1} \frac{n \left(\frac{t_l}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right)^{n-1} - \left(\frac{t_l}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right)^n}{n!}\right) \right. \\ &\quad \left. - t_{l-1} \exp\left(-\frac{t_{l-1}}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right) \left(-1 + \sum_{n=1}^{N-1} \frac{n \left(\frac{t_{l-1}}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right)^{n-1} - \left(\frac{t_{l-1}}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2}\right)^n}{n!}\right) \right) \end{aligned} \quad (27)$$

$$\begin{aligned} \nabla_{\mathbf{w}^*} I(x_K) &= \sum_{m_K} P_K(x_{m_K}) \left( \rho \frac{\mathbf{h}_E(x_{m_K}) \mathbf{h}_E^H(x_{m_K}) \mathbf{w}}{|\mathbf{h}_E^H(x_{m_K}) \mathbf{w}|^2 + \sigma_v^2} + (1-\rho) \sum_l \left( \log \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right)}{\sum_{m'_K} P_K(x_{m'_K}) Q\left(N, \frac{t_{l-1}}{\sigma_{m'_K}^2}, \frac{t_l}{\sigma_{m'_K}^2}\right)} + 1 \right) \right. \\ &\quad \left. \times \nabla_{\mathbf{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) - \frac{Q\left(N, \frac{t_{l-1}}{\sigma_{m_K}^2}, \frac{t_l}{\sigma_{m_K}^2}\right) \sum_{m'_K} P_K(x_{m'_K}) \nabla_{\mathbf{w}^*} Q\left(N, \frac{t_{l-1}}{\sigma_{m'_K}^2}, \frac{t_l}{\sigma_{m'_K}^2}\right)}{\sum_{m'_K} P_K(x_{m'_K}) Q\left(N, \frac{t_{l-1}}{\sigma_{m'_K}^2}, \frac{t_l}{\sigma_{m'_K}^2}\right)} \right) \end{aligned} \quad (28)$$



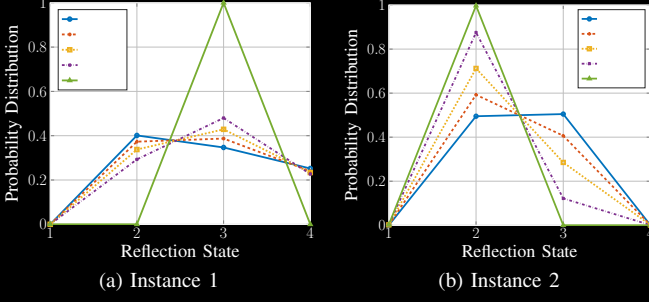


Fig. 5. Typical input distributions vs. weight  $\rho$  for a Metascatter with  $M=4$  inputs.

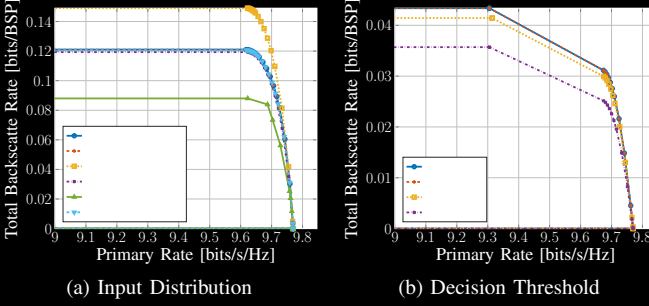


Fig. 6. Average primary-(total-)backscatter rate regions for different input distribution and decision threshold schemes.

- **Cooperation:** backscatter cooperation/joint encoding at all Metascatters, tuple input distribution/joint probability array optimization by Blahut-Arimoto algorithm [49], [50];
- **Marginalization:** marginal distributions of joint probability array;
- **Decomposition:** normalized tensors of rank-1 Canonical Polyadic (CP) decomposition of joint probability array;
- **Randomization:** Gaussian recovery from joint probability array [51].

We notice adaptive joint encoding at all Metascatters provides the outer bound of rate region. However, it involves arbitrarily dependent codewords, and backscatter cooperation between passive nodes is generally unaffordable. In contrast, the rate region of KKT input design in Algorithm 1 approaches that of exhaustive search, and the loss is negligible for  $K=2$ . Although the randomization method [51] returns similar rate region, it requires solving  $K+1$  linear programming problems before applying Gaussian recovery. The marginal distribution is slightly worse than KKT despite having the same computational complexity, while the approximation from CP decomposition is unsatisfying.

Fig. 6(b) compares the achievable rate region by following threshold schemes.

- **DP:** sequential quantizer grouping by dynamic programming [46];
- **SMAWK:** above accelerated by SMAWK algorithm;
- **Bisection:** sequential bisection threshold design [47];
- **ML:** maximum likelihood decision (31) [48].

We observe that input-adaptive threshold designs achieve higher total backscatter rate than ML. This is because the

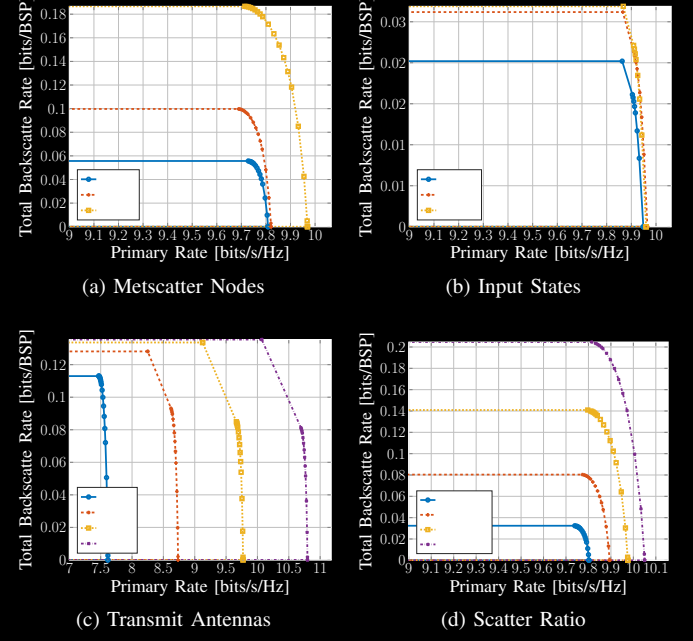


Fig. 7. Average primary-(total-)backscatter rate regions.

decision regions can be flexibly adjusted to enhance the capacity of DTMAC (8). For example, the tuples with negligible input probability should have narrower decision regions than those frequently employed, in order to improve detection performance. It emphasizes the importance of joint input distribution and decision region design.

### C. Rate Region vs. System Configuration

We present in 7 the impact of Metascatter nodes, input states, transmit antennas and scatter ratio on the achievable rate region. For the specified scenario, Fig. 7(a) shows the total backscatter rate almost scales proportionally with the number of Metascatter nodes, and the decrease of individual backscatter rate is unobvious. Besides, we conclude from 7(b) that increasing the reflection states has a marginal effect on both primary and backscatter rates. Those two facts motivates the use of numerous elementary/uncomplicated backscatter nodes, instead of high-order transponders or programmable high-resolution surfaces. Figs. 7(c) and 7(d) also prove increasing transmit antennas or scatter ratio can improve both primary and backscatter performance.

Fig. 8(a) shows the tradeoff between coverage disk radius  $r$  and achievable rate region. When nodes are far from the user, both primary and backscatter rates decrease due to the product pass loss of forward and backward channels. Under the assumption of ideal backscatter decoding and re-encoding, Fig. 8(b) suggests using lower symbol period ratio  $N$  can boost total backscatter rate per primary symbol. However, it requires more frequent detection and re-encoding at the user to maintain the primary rate. When  $N$  becomes sufficiently large, total backscatter rate approaches 0 and Metascatters boil down to conventional RIS elements with fixed reflection patterns during whole channel block. In Fig. 8(c), we observe that passive Metascatter achieves higher backscatter rate at lower carrier



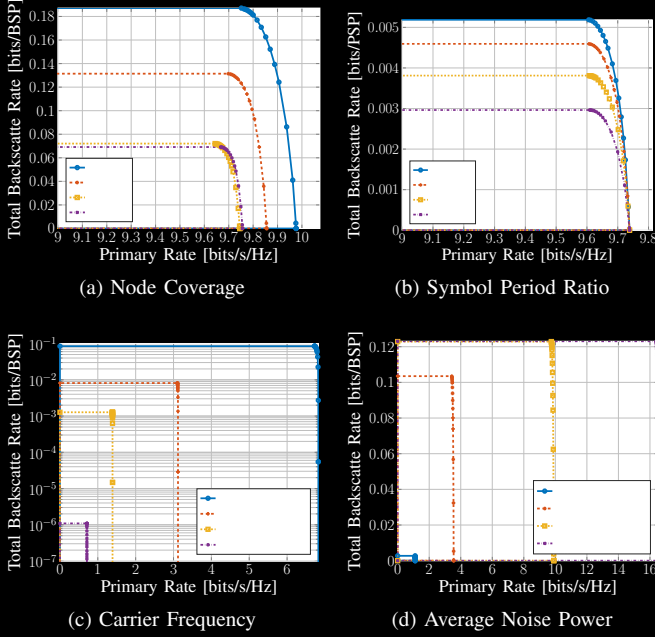


Fig. 8. Average primary-(total-)backscatter rate regions.

frequency because of preferable propagation loss. Finally, 8(d) prove the performance of energy detection is robust for a wide range of noise power.

## VI. CONCLUSION

This paper introduced Metascatter that adapts input distribution of passive backscatter nodes to simultaneously transmit and assist over existing wireless systems. Starting from backscatter principles, we showed how Metascatters bridges and generalizes parasitic source of SR and reflecting element of RIS via smart input design. An application scenario was considered where multiple Metascatters ride over a point-to-point transmission to simultaneously encode self message and perform passive beamforming. To characterize achievable primary-backscatter rate region, we proposed a BCD algorithm that evaluates KKT input distribution in closed form, optimizes active beamforming by PGD, and refines decision regions by existing methods. Numerical results demonstrated the advantage of adaptive node input distribution design for both primary and backscatter subsystems.

One particular interesting question is how to design Metascatters in a multi-user system. If one node can contribute to and be decoded by multiple users, its input distribution may be further adjusted to mimic multi-beam gain of dynamic beamforming [22].

## APPENDIX

### A. Proof of Proposition 1

Denote the Lagrange multipliers associated with (19b) and (19c) as  $\{\nu_k\}_{k \in \mathcal{K}}$  and  $\{\lambda_{k,m_k}\}_{k \in \mathcal{K}, m_k \in \mathcal{M}}$ , respectively. The

Lagrangian function of problem (20) is

$$L = -I(x_{\mathcal{K}}) + \sum_k \nu_k \left( \sum_{m_k \in \mathcal{M}} P_k(x_{m_k}) - 1 \right) - \sum_k \sum_{m_k} \lambda_{k,m_k} P_k(x_{m_k}), \quad (32)$$

and the KKT conditions are,  $\forall k, m_k$ ,

$$-\nabla_{P_k^*(x_{m_k})} I^*(x_{\mathcal{K}}) + \nu_k^* - \lambda_{k,m_k}^* = 0, \quad (33a)$$

$$\lambda_{k,m_k}^* = 0, \quad P_k^*(x_{m_k}) > 0, \quad (33b)$$

$$\lambda_{k,m_k}^* \geq 0, \quad P_k^*(x_{m_k}) = 0. \quad (33c)$$

The directional derivative can be explicitly expressed as

$$\nabla_{P_k^*(x_{m_k})} I^*(x_{\mathcal{K}}) = I_k^*(x_{m_k}) - (1 - \rho). \quad (34)$$

Combining (33) and (34), we have

$$I_k^*(x_{m_k}) = \nu_k^* + (1 - \rho), \quad P_k^*(x_{m_k}) > 0, \quad (35a)$$

$$I_k^*(x_{m_k}) \leq \nu_k^* + (1 - \rho), \quad P_k^*(x_{m_k}) = 0, \quad (35b)$$

which suggests

$$\sum_{m_k} P_k^*(x_{m_k}) I_k^*(x_{m_k}) = \nu_k^* + (1 - \rho). \quad (36)$$

On the other hand, by definition of weighted sum marginal information (17),

$$\sum_{m_k} P_k^*(x_{m_k}) I_k^*(x_{m_k}) = I^*(x_{\mathcal{K}}), \quad (37)$$

where the right-hand side is irrelevant to  $k$ . (35), (36), and (37) together complete the proof.

### B. Proof of Proposition 2

We first prove sequence (22) is non-decreasing in weighted sum mutual information. Let  $P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) = \prod_{q \in \mathcal{K}} P_q(x_{m_q})$  and  $P'_{\mathcal{K}}(x_{m_{\mathcal{K}}}) = P'_k(x_{m_k}) \prod_{q \in \mathcal{K} \setminus \{k\}} P_q(x_{m_q})$  be two probability distributions with potentially different marginal for tag  $k \in \mathcal{K}$  at state  $m_k \in \mathcal{M}$ , and define an intermediate function  $J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P'_{\mathcal{K}}(x_{m_{\mathcal{K}}}))$  as (38) at the end of page 10. It is straightforward to verify  $J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P_{\mathcal{K}}(x_{m_{\mathcal{K}}})) = I(x_{\mathcal{K}})$  and  $J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P'_{\mathcal{K}}(x_{m_{\mathcal{K}}}))$  is a concave function for a fixed  $P'_{\mathcal{K}}(x_{m_{\mathcal{K}}})$ . By choosing  $\nabla_{P_k^*(x_{m_k})} J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P'_{\mathcal{K}}(x_{m_{\mathcal{K}}})) = 0$ , we have

$$S'_k(x_{m_k}) - S'_k(x_{i_k}) + (1 - \rho) \log \frac{P_k(x_{i_k})}{P_k^*(x_{m_k})} = 0, \quad (39)$$

where  $i_k \neq m_k$  is the reference state and

$$S'_k(x_{m_k}) \triangleq I'_k(x_{m_k}) + (1 - \rho) \sum_{m_{\mathcal{K} \setminus \{k\}}} P_{\mathcal{K} \setminus \{k\}}(x_{m_{\mathcal{K} \setminus \{k\}}}) \times \sum_{m'_k} P(\hat{x}_{m'_k} | x_{m_{\mathcal{K}}}) \log P'_{\mathcal{K}}(x_{m_{\mathcal{K}}}). \quad (40)$$

Evidently,  $\forall m_k \neq i_k$ , (39) boils down to

$$P_k^*(x_{m_k}) = \frac{P'_k(x_{m_k}) \exp\left(\frac{\rho}{1-\rho} I'_k(x_{m_k})\right)}{\sum_{m'_k} P'_k(x_{m'_k}) \exp\left(\frac{\rho}{1-\rho} I'_k(x_{m'_k})\right)}. \quad (41)$$

We also notice  $P_k(x_{i_k}) = 1 - \sum_{m_k \neq i_k} P_k^*(x_{m_k})$  has exactly the same expression as (41). Therefore, the result is irrelevant to the choice of reference state, and (41) is indeed optimal  $\forall m_k \in \mathcal{M}$ . That is, for a fixed  $P_{\mathcal{K}}'(x_{m_{\mathcal{K}}})$ , choosing  $P_k(x_{m_k})$  by (41) ensures

$$J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P_{\mathcal{K}}'(x_{m_{\mathcal{K}}})) \geq I'(x_{\mathcal{K}}). \quad (42)$$

On the other hand, it also guarantees

$$\Delta \triangleq I(x_{\mathcal{K}}) - J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P_{\mathcal{K}}'(x_{m_{\mathcal{K}}})) \quad (43a)$$

$$= (1-\rho) \sum_{m_k} \frac{P_k'(x_{m_k}) f_k'(x_{m_k})}{\sum_{m'_k} P_k'(x_{m'_k}) f_k'(x_{m'_k})} \sum_{m'_k} P(\hat{x}_{m'_k} | x_{m_k}) \times \log \frac{\sum_{m'_k} P_k'(x_{m'_k}) P(\hat{x}_{m'_k} | x_{m'_k}) f_k'(x_{m'_k})}{\sum_{m'_k} P_k'(x_{m'_k}) P(\hat{x}_{m'_k} | x_{m'_k}) f_k'(x_{m'_k})} \quad (43b)$$

$$\geq (1-\rho) \sum_{m_k} \frac{P_k'(x_{m_k}) f_k'(x_{m_k})}{\sum_{m'_k} P_k'(x_{m'_k}) f_k'(x_{m'_k})} \sum_{m'_k} P(\hat{x}_{m'_k} | x_{m_k}) \times \left( 1 - \frac{\sum_{m'_k} P_k'(x_{m'_k}) P(\hat{x}_{m'_k} | x_{m'_k}) f_k'(x_{m'_k})}{\sum_{m'_k} P_k'(x_{m'_k}) P(\hat{x}_{m'_k} | x_{m'_k}) f_k'(x_{m'_k})} \right) \quad (43c)$$

$$= 0, \quad (43d)$$

where  $f_k'(x_{m_k}) \triangleq \exp\left(\frac{\rho}{1-\rho} I_k'(x_{m_k})\right)$  and the equality holds if and only if (41) converges. (42) and (43) together imply  $I(x_{\mathcal{K}}) \geq I'(x_{\mathcal{K}})$ . Since mutual information is bounded above, we conclude the sequence (22) is non-decreasing and convergent in mutual information.

Next, we prove any converging point of sequence (22), denoted as  $P_k^*(x_{m_k})$ , fulfills KKT conditions (21). To see this, define

$$D_k^{(r)}(x_{m_k}) \triangleq \frac{P_k^{(r+1)}(x_{m_k})}{P_k^{(r)}(x_{m_k})} = \frac{f_k^{(r)}(x_{m_k})}{\sum_{m'_k} P_k^{(r)}(x_{m'_k}) f_k^{(r)}(x_{m'_k})}. \quad (44)$$

As sequence (22) is convergent, any state with  $P_k^*(x_{m_k}) > 0$  need to satisfy  $D_k^*(x_{m_k}) \triangleq \lim_{r \rightarrow \infty} D_k^{(r)}(x_{m_k}) = 1$ , namely

$$I_k^*(x_{m_k}) = \frac{1-\rho}{\rho} \log \sum_{m'_k} P_k^*(x_{m'_k}) f_k^*(x_{m'_k}), \quad (45)$$

which is reminiscent of (35a) and (21a). That is to say, given  $P_k^{(0)}(x_{m_k}) > 0$ , any converging point with  $P_k^*(x_{m_k}) > 0$  must satisfy (21a). On the other hand, we assume  $P_k^*(x_{m_k})$  does not satisfy (21b), such that for any state with  $P_k^*(x_{m_k}) = 0$ ,

$$I_k^*(x_{m_k}) > I^*(x_{\mathcal{K}}) = \sum_{m'_k} P_k^*(x_{m'_k}) I_k^*(x_{m'_k}), \quad (46)$$

where the equality inherits from (18). Since exponential function is monotonically increasing, we have  $f_k^*(x_{m_k}) >$

$\sum_{m'_k} P_k^*(x_{m'_k}) f_k^*(x_{m'_k})$  and  $D_k^*(x_{m_k}) > 1$ . Considering  $P_k^{(0)}(x_{m_k}) > 0$  and  $P_k^*(x_{m_k}) = 0$ , it contradicts with

$$P_k^{(r)}(x_{m_k}) = P_k^{(0)}(x_{m_k}) \prod_{n=1}^r D_k^{(n)}(x_{m_k}). \quad (47)$$

Therefore, given  $P_k^{(0)}(x_{m_k}) > 0$ , any converging point with  $P_k^*(x_{m_k}) = 0$  must satisfy (21b). This completes the proof.

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$$J(P_{\mathcal{K}}(x_{m_{\mathcal{K}}}), P_{\mathcal{K}}'(x_{m_{\mathcal{K}}})) \triangleq \rho \sum_{m_{\mathcal{K}}} P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) \log \left( 1 + \frac{|\mathbf{h}_E^H(x_{m_{\mathcal{K}}}) \mathbf{w}|^2}{\sigma_v^2} \right) + (1-\rho) \sum_{m_{\mathcal{K}}} P_{\mathcal{K}}(x_{m_{\mathcal{K}}}) \sum_{m'_k} P(\hat{x}_{m'_k} | x_{m_{\mathcal{K}}}) \log \frac{P(\hat{x}_{m'_k} | x_{m_{\mathcal{K}}}) P_{\mathcal{K}}'(x_{m_{\mathcal{K}}})}{P'(\hat{x}_{m'_k}) P_{\mathcal{K}}(x_{m_{\mathcal{K}}})}. \quad (38)$$

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