Topological Quantum Computing: Basics and Implementations

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1 Introduction

Quantum computing is often heralded as the field that will change information science for good. It boasts a the potential to solve some of the most complex problems known to scientists and overtake the computation speeds of advanced supercomputers by harnessing properties of quantum states. Some of the biggest obstacles in this field so far are extremely small coherence times of quantum bits, or qubits, and their high susceptibility to noise. In fact, error rates are so high for qubits and quantum gates that error correction codes actively exacerbate and propagate errors. Topological quantum computing is a fairly recent sub-field of quantum computing that aims to create extremely fault-tolerant qubits and quantum gates using special quasiparticles and their topological properties.

2 Anyons

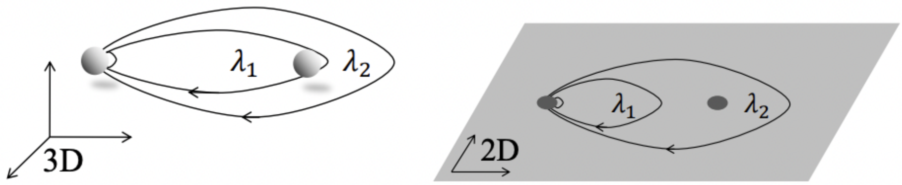
2.1 Introduction to Anyons

Anyons are quasiparticles that occur in two spatial dimensions and act as neither bosons nor fermions, but rather have unique statistics that are defined by fractional statistics [2]. Quasiparticles are groups of particles that have certain emergent phenomena when they interact with each other and their environment, effectively acting as a single particle with unique properties [2]. Quantum fractional statistics governs topological states where, upon interacting and exchanging along some arbitrary trajectory, the wavefunction of two particles will pick up a non-trivial phase $e^{i\phi}$ and retain a "memory" of the switch [2]. Such a full swap in three dimensions is not topologically distinct from the starting state of the unswapped particles; however, a full swap in two dimensions is a topologically nontrivial process where the ϕ in $e^{i\phi}$ can have any value – thus the name "anyon" [17].

2.2 Anyon "Memory" Retention

Consider two arbitrary particles on a sphere (so, two particles in three dimensions). Switching the particles and tracing their paths creates one arc per particle on the sphere. However, switching the particles again and continuing to trace their paths creates a closed loop per particle. This closed loop is topologically equivalent to simply a point on the sphere – it can be contorted into a point without its circumference being "torn" in any way. Thus, switching particles twice in a 3D space is topologically equivalent to not switching them at all [17]. This requirement implies that any exchange operator applied twice to the wavefunction of two particles must result in the identity operator (i.e., the original wavefunction must be returned). This gives us the restriction that $\phi = 0$ or $\phi = \pi$ are the only acceptable values of ϕ when phases to wavefunctions are defined to be of the form $e^{i\phi}$. Indeed, these are the exchange operators for fermions and bosons respectively [17].

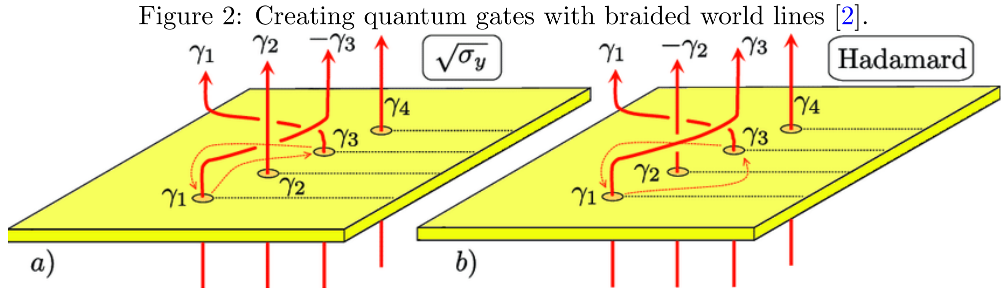
Figure 1: Examples of particle exchanges in 3D and 2D [17].



In two dimensions, however, such an exchange would be equivalent to one of the particles circling the other. In this case, the outer path cannot be contracted into a single point without passing through the other particle and being "torn." Thus, exchanging particles twice in two dimensions is not topologically equivalent to not switching them at all, and there is no restriction on the value of ϕ the way that there is in 3D. Thus, it is theoretically possible to have particles that exist in two dimensions that can have any arbitrary unitary exchange operator, though a stability consideration requires that ϕ be a rational multiple of 2π.

In this case, it is possible to use different types of exchanges as gates. Paths taken by particles are often referred to as "world lines" in topological quantum computing, with the movement of particles in specific ways to create unique exchange operators being called "braiding their world lines" [2]. For example, Figure 2 shows examples of quantum gates that can be created using this method.

Figure 2: Creating quantum gates with braided world lines [2].



This approach to creating qubits and quantum gates is astonishingly fault-tolerant. Disturbances to braids that keep gates and qubits remain topologically consistent are negligible - this accounts for most noise [14]! Furthermore, if noise causes world lines to "tear," as long as it does not somehow "glue" world lines back together in an order that corresponds to a different gate, errors are still detectable [14]. Even for the minuscule probability that this does happen, errors can still be detected at higher levels (e.g. algorithmically) and corrected for such stable qubits.

2.3 Abelian vs. Non-Abelian Anyons

Anyons are generally classified as either "abelian" or "non-abelian." Abelian anyons are anyons which have commuting exchange operators, while non-abelian anyons are anyons with exchange operators that do not commute [17]. This classification is important for topological implementations of quantum computing because if quantum gates (which generally do not commute) are to be defined in terms of exchange operators, non-abelian anyons must be used [17].

2.4 Experimental Evidence of Anyons

Though anyons were first predicted to exist in 1977, it wasn't until the 1982 discovery of the fractional quantum Hall effect that the search for them was exacerbated [3].

The fractional quantum Hall effect, discovered by Daniel Tsui and Horst Störmer in 1982, is a product of strong magnetic fields being applied to an electron gas [14]. This effect creates a new phase of matter that is topologically unique and has excitations in the form of quasiparticles that, upon being confined to two dimensions, obey fractional exchange statistics [14]. After further calculations, these quasiparticles were predicted to be anyons in 1985 [14].

Since then, the hunt for anyons has been active. As of 2020, there have been at least four separate experiments internationally that have verified the existence of abelian anyons [14].

Although quasiparticles that obey fractional statistics have been shown to exist, the little evidence for non-abelian anyons remains contested. Of the candidates for non-abelian anyons, Majorana quasiparticles are the most promising [13].

3 The Search for Majorana Particles

3.1 The Majorana Equation

If anyons are a class of particles and quasiparticles, then Majorana fermions are an example that fit in this class that. When confined to two dimensions, Majorana fermions theoretically exhibit the properties of non-abelian anyons [1].

Majorana fermions were first predicted by Ettore Majorana in the 1930s when he derived the Majorana equation (Figure 3) [1].

Figure 3: The Majorana equation [1].

$$\widetilde{\gamma^0} = \sigma_2 \otimes \sigma_1 \quad \widetilde{\gamma^1} = i\sigma_1 \otimes 1$$

$$\widetilde{\gamma^2} = i\sigma_3 \otimes 1 \quad \widetilde{\gamma^3} = i\sigma_2 \otimes \sigma_2$$

$$\left(i\widetilde{\gamma^\mu}\,\partial_\mu - m\right)\widetilde{\psi} = 0$$

This equation was a product of trying to derive a relativistic equation to describe photons [1]. Though it is not critical to understand this equation in order to understand Majorana fermions, it is important to note that this equation predicted the existence of a fermion that was its own antiparticle [1]. None of the fermions in the standard model are known to fit this description, but some quasiparticles do indeed match the predictions of this equation [1].

3.2 Neutrinos

There are a few candidates for Majorana quasiparticles, one of which is the neutrino. Though these are not the most popular candidates, they deserve a mention when talking about the search for Majorana quasiparticles. The effort by particle physicists to learn more about neutrinos in general has been coupled recently with the search for Majoranas, with many seeking to prove whether or not neutrinos may exhibit the properties of Majorana particles [5]. Particularly, it is hypothesized that the neutrino may be its own antiparticle, which is a defining property of Majorana fermions [13]. Much of this research involves underground observatories and includes questions about neutrinos and Majorana fermions as candidates for dark matter [5]. This report, however, focuses mostly on the much more thoroughly defined search for Majorana quasiparticles in superconductors.

3.3 Superconductors

Theoretically, it is fairly straightforward to find Majorana quasiparticles in superconductors. Here are the steps that would need to be taken:

3.3.1 Break spin-rotation symmetries

First, an experimenter interested in observing Majorana quasiparticles must break spin-rotation symmetry in a material [2]. Topological insulators are the best known candidate for such a material because they are the only materials that such symmetry breaking has previously been observed [11]. A 2015 paper from Cornell University detailed observed spontaneous symmetry breaking in the hexagonal plane of an electron-doped topological insulator when lowered to below the superconducting transition temperature of 3.4 K [11]. There have not yet been any successful attempts to break spin-rotation symmetry dependably, and as such this remains a bottleneck in the search for Majorana quasiparticles in topological materials [11].

3.3.2 Break time-reversal symmetries

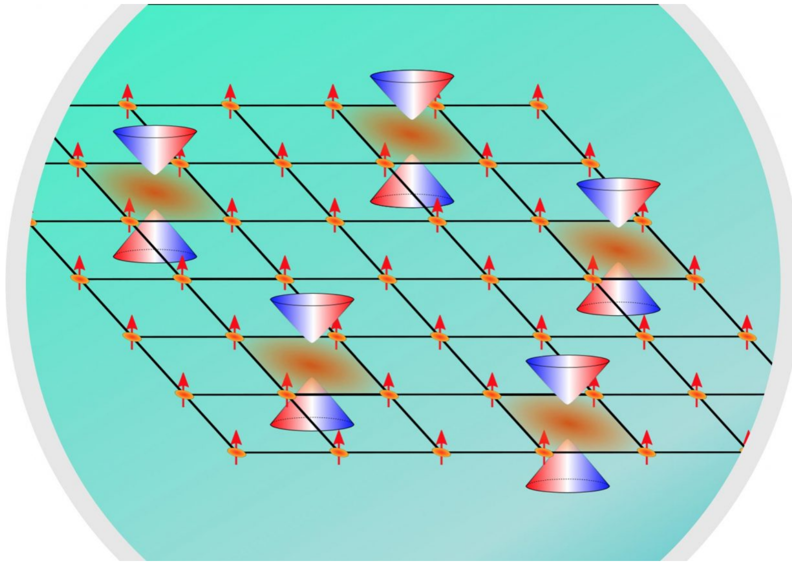
Next, an experimenter hunting for Majorana quasiparticles needs break time-reversal symmetry in a material [2]. Time-reversal symmetry is the inherent symmetry of materials wherein the laws of physics are the same whether they are considered going forward or backward in time [12]. One of the ways to break time-reversal symmetry is by developing ferromagnetism in a material, wherein all electron spins align parallel to each other [12]. Time-reversal inverts this effect, so ferromagnetism breaks time-reversal symmetry.This has previously been done in superconductors with reliable, reproducible results [12].

3.3.3 Close and reopen excitation gap

Superconductors have an energy gap in the excited states of their electrons that is caused by the extreme cooling they undergo [12]. This energy gap, sometimes known as the band gap, is an energy range where electrons are forbidden from being [12]. This gap generally exists between cones of allowed energies for electrons, as shown in Figure 4 [12].

Nanowires that are hybrids of superconductors and semiconductors can be created to experimentally test this [10]. During the transition between an open excitation gap to a closed

Figure 4: Topological surface state with an energy band gap between top and bottom cones [12].

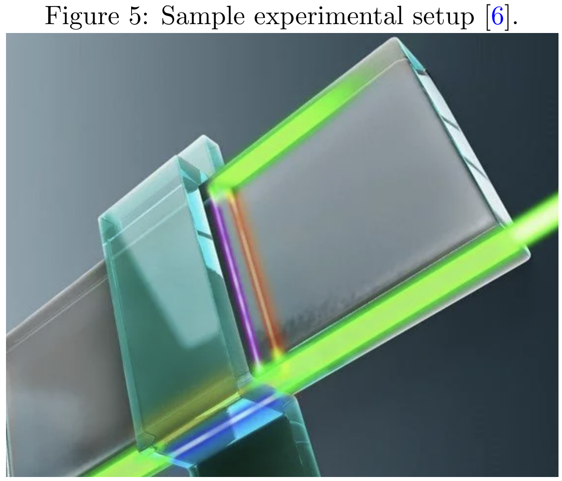


excitation gap to an open one again, such nanowires are expected to become topological superconductors [10]. This transition from trivial superconductors to topological superconductors is supposed to create Majorana bound states at the nanowire ends [15]. As the gap goes through zero, Majorana quasiparticles theoretically emerge due to magnetic or electrostatic defects [15].

3.4 Experimental Setup

An example of an experiment to search for Majorana fermions is as follows:

Figure 5: Sample experimental setup [6].



The horizontal bar would be a topological insulator cooled to break spin-rotation symmetries. The vertical bar would be a superconductor intended to break time-reversal symmetries [6].

Electrons would run along their edges and, if all steps are followed accurately, Majorana fermions should show up at the edges where they intersect as a result of electromagnetic defects [6].

3.5 Claimed Majorana Sightings

The experimental setup seems straightforward enough, but it has not yet yielded confirmed experimental observations of Majorana fermions.

3.5.1 2017 and 2018 Claims

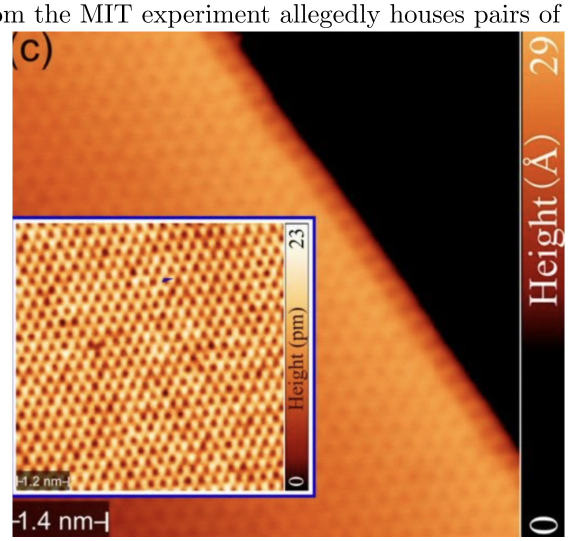
In 2017, UCLA published a paper claiming to have observed Majorana fermions using the exact experimental setup shown in Figure 5, with the topological insulator being a quantum anomalous Hall insulator [6]. Quantum anomalous Hall insulators are topological insulators in which current only flows at the edges [6]. However, this paper was debunked three years later, in January of 2020, by a collaboration between Pennsylvania State University and the University of Wurzburg in Germany when they attempted to reproduce the results [16][4].

In 2018, Microsoft published a paper in *Nature* claiming to have sighted Majorana fermions in a superconductor [9] . This claim, however, was retracted in April of 2021 amidst accusations of data fabrication and cherry picking [9].

3.5.2 MIT 2020 Paper

In April of 2020, an MIT group published a paper in *Proceedings* of the National Academy of Sciences claiming to have observed evidence of Majorana fermions in a system of materials that they fabricated [7]. This system consisted of nanowires of gold grown on vanadium, which is a superconducting material, and dotted with small ferromagnetic “islands” of europium sulfide [7].

Figure 6: Gold film from the MIT experiment allegedly houses pairs of Majorana fermions [7].



Upon scanning the surface of this material system, researchers saw signal spikes near the islands near zero energy which, according to theory, are signatures that can only be created by pairs of Majorana fermions [7]. This paper has not yet been disproven or retracted, and research to confirm these findings is ongoing.

4 Conclusion

Although topological quantum computing is a beautiful idea, it is yet to be backed up by experimental evidence. Theorists have fleshed it out an impressive amount, with many papers and articles written about experimental uses of Majorana fermions if they are ever discovered [8]. Experimental efforts to prove the existence of Majorana quasiparticles over the years have provided tantalizing glimpses of a world that we do not fully understand yet, and leaves us with more questions than ever. How can spin-rotation symmetry be broken? Can neutrinos be used to implement fault-tolerant qubits? Does the ever-evasive Majorana fermion exist? These are all questions that we can only continue to work on and hope that the answers are just around the corner.

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