Quantum Interference and Entanglement

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Introduction

Background: Entanglement

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$(a_1|0\rangle + b_1|1\rangle) \otimes (a_2|0\rangle + b_2|1\rangle)$$

$$= a_1a_2|00\rangle + a_1b_2|01\rangle + b_1a_2|10\rangle + b_1b_2|11\rangle$$

E. Rieffel and W. Polak, “Quantum Computing: A Gentle Introduction.”

Background: History

- Nonlocality
- EPR paradox (Einstein, Podolsky, Rosen)
- Local, realistic HVT

How can we differentiate between realistic, local HVTs and quantum mechanics?

Bell states (aka EPR pairs)

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

E. Rieffel and W. Polak, “Quantum Computing: A Gentle Introduction.”

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Experimental Setup

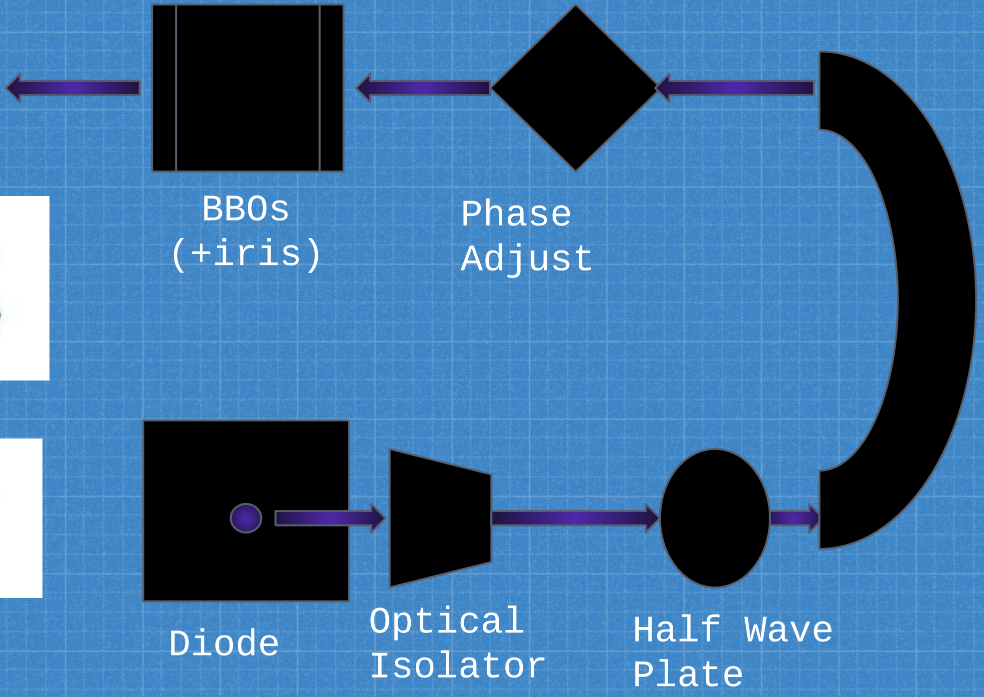
Experimental Setup: Generation

$$|\psi_{\mathrm{DC}}\rangle = \cos\ \theta_l|H\rangle\ |H\rangle\ + \exp[i\phi]\sin\ \theta_l|V\rangle\ |V\rangle$$

BBO-1: $|V\rangle \rightarrow |vv\rangle$

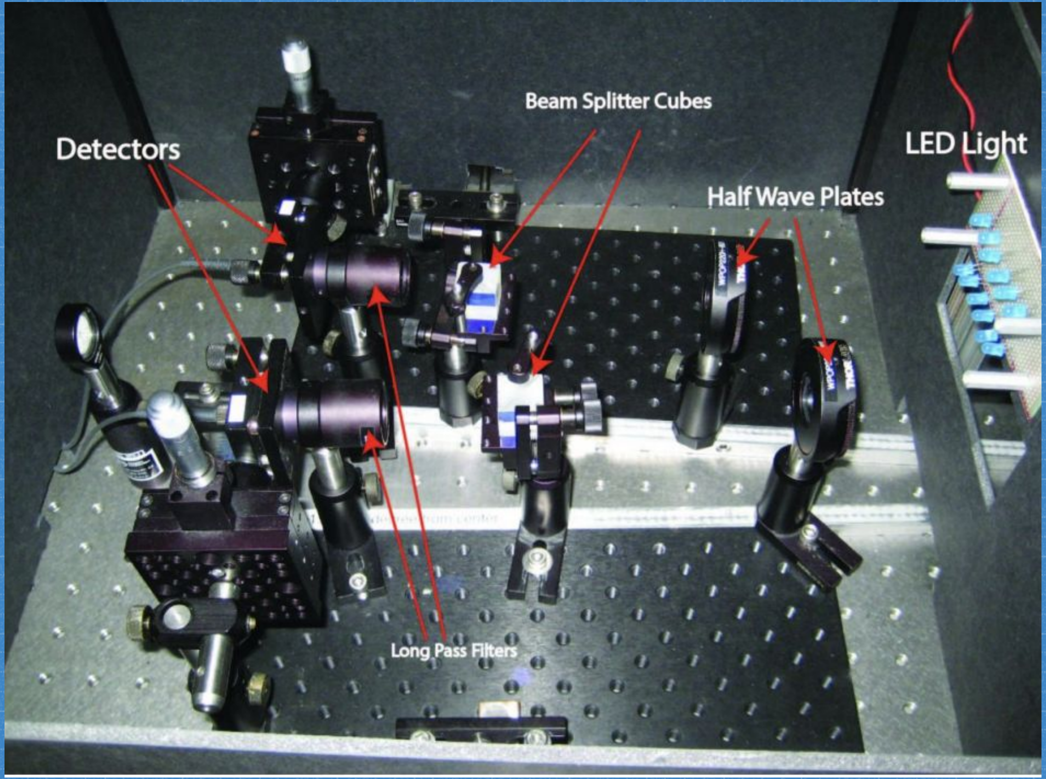
BBO-2: $|H\rangle \rightarrow |hh\rangle$

$$|\psi_{\mathrm{pump}}\rangle = \cos\ \theta_l|V\rangle\ + \exp[i\phi\ ]\sin\ \theta_l|H\rangle$$



D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”

Experimental Setup: Detection



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Predictions

Calculation (short example)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$

meas. bases: $|V_\alpha\rangle = \cos\alpha\,|V\rangle - \sin\alpha\,|H\rangle$

$|H_\alpha\rangle = \sin\alpha\,|V\rangle + \cos\alpha\,|H\rangle$

If $\alpha = 0^\circ$, $|V\rangle \to |V\rangle$ If $\alpha = 90^\circ$, $|V\rangle \to -|H\rangle$

$|H\rangle \to |H\rangle$ $|H\rangle \to |V\rangle$

$$\Rightarrow |\psi\rangle \to \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

Generalize:

$$\begin{aligned} P_{VV}(\alpha,\beta) = {} & \sin^2\alpha\,\sin^2\beta\,\cos^2\theta_l \\ & + \cos^2\alpha\,\cos^2\beta\,\sin^2\theta_l \\ & + \tfrac{1}{4}\sin 2\alpha\,\sin 2\beta\,\sin 2\theta_l\,\cos\phi_m\,. \end{aligned}$$

D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”

Local, Realistic HVT vs Quantum Mechanics

	P_{HH}	P_{HV}	P_{VH}	P_{VV}
(0°, 0°)	½	0	0	½
(0°, 90°)	0	½	½	0
(45°, 45°)	½	0	0	½
(90°, 0°)	0	½	½	0

	P_{HH}	P_{HV}	P_{VH}	P_{VV}
(0°, 0°)	½	0	0	½
(0°, 90°)	0	½	½	0
(45°, 45°)	¼	¼	¼	¼
(90°, 0°)	0	½	½	0

QM vs HVT

$$N(\alpha,\beta)=A(\sin^2\alpha\sin^2\beta\cos^2\theta_l+\cos^2\alpha\cos^2\beta\sin^2\theta_l+\tfrac{1}{4}\sin 2\alpha\sin 2\beta\sin 2\theta_l\cos\phi_m)+C,$$

$$C=N(0°,90°),$$

$$A=N(0°,0°)+N(90°,90°)-2C,$$

$$\tan^2\theta_l=\frac{N(90°,90°)-C}{N(0°,0°)-C},$$

$$\cos\phi_m=\frac{1}{\sin 2\theta_l}\left(4\frac{N(45°,45°)-C}{A}-1\right)$$

D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”

Bell Inequality

$$E(\alpha,\beta) \equiv P_{VV}(\alpha,\beta) + P_{HH}(\alpha,\beta) - P_{VH}(\alpha,\beta) - P_{HV}(\alpha,\beta).$$

$$P_{VV}(\alpha,\beta) = N(\alpha,\beta)/N_{\mathrm{tot}}$$

$$P_{VH}(\alpha,\beta) = N(\alpha,\beta_\perp)/N_{\mathrm{tot}}$$

$$P_{HH}(\alpha,\beta) = N(\alpha_\perp,\beta_\perp)/N_{\mathrm{tot}}$$

$$P_{HV}(\alpha,\beta) = N(\alpha_\perp,\beta)/N_{\mathrm{tot}}$$

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)}$$

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”

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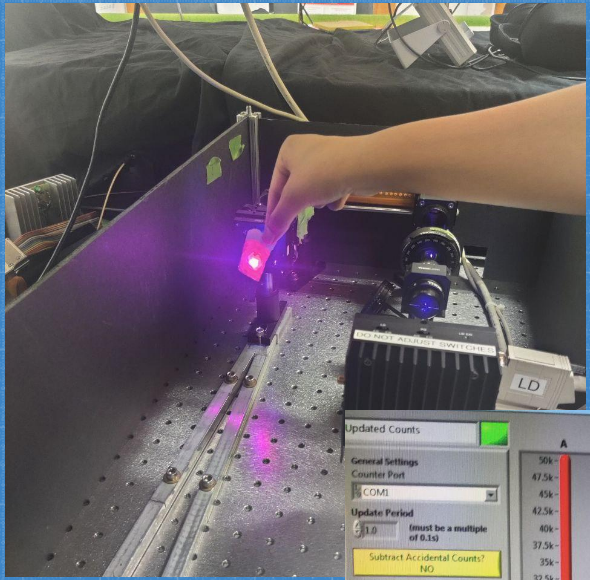
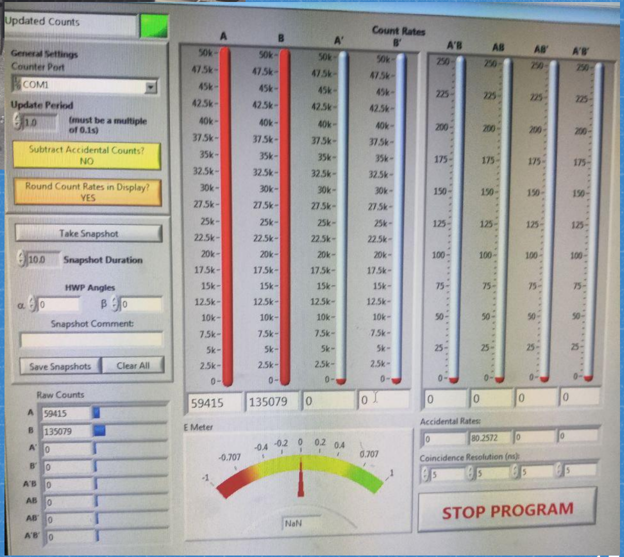
Procedure

Procedure

- Align components to maximize photon counts
  - Find offset on HWPs
- Tune Bell state by adjusting phase
- Measure values needed to get C, A, ϕ, and θ_L
- Measure at the appropriate HWP angles to calculate S





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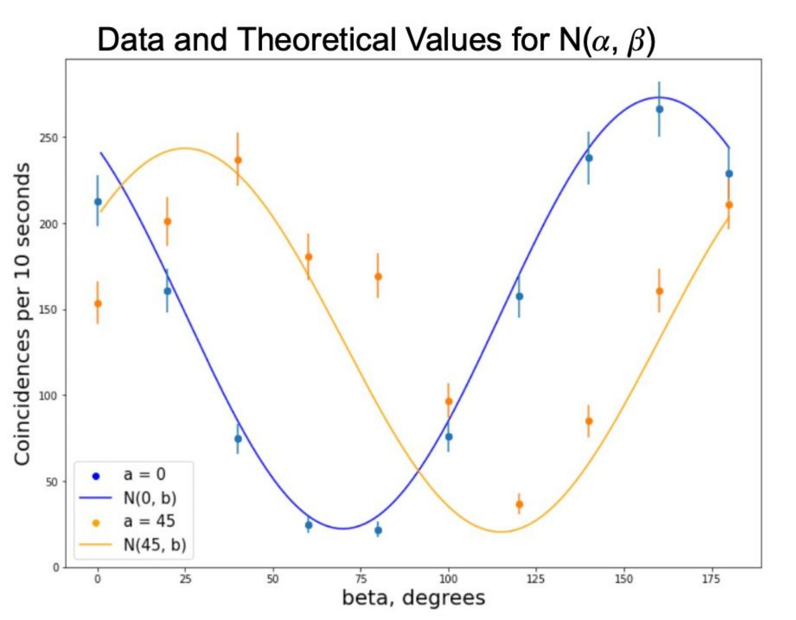
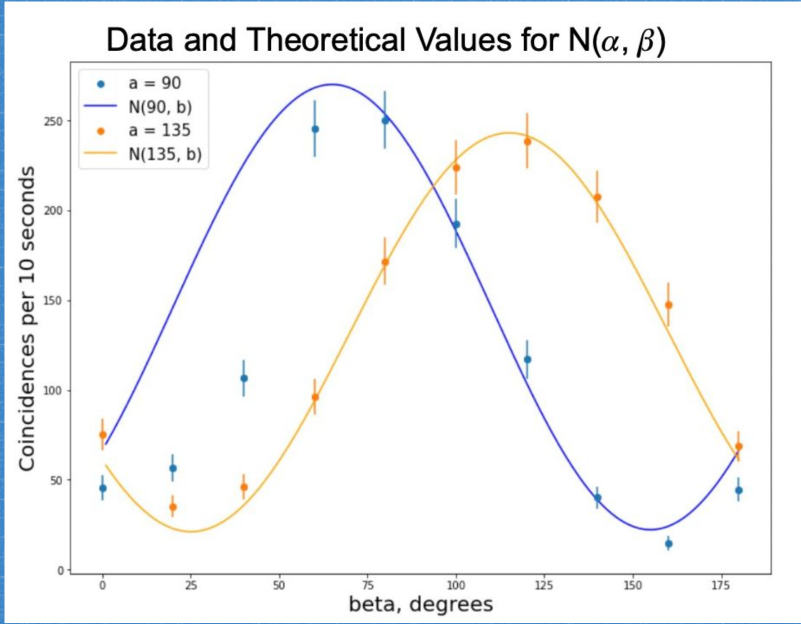
Results

Results

D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”

$$N(\alpha,\beta)=A(\sin^2\alpha\,\sin^2\beta\,\cos^2\theta_l+\cos^2\alpha\,\cos^2\beta\,\sin^2\theta_l+\tfrac{1}{4}\sin 2\alpha\,\sin 2\beta\,\sin 2\theta_l\,\cos\phi_m)+C$$





α = 0: $\chi 2$ = 1.32; α = 45: $\chi 2$ = 2.78; α = 90: $\chi 2$ = 3.81; α = 135: $\chi 2$ = 1.07

Results

Measurements

$E_{\alpha\beta}$ = 0.57 ± 0.06

$E_{\alpha'\beta}$ = -0.73 ± 0.05

$E_{\alpha\beta'}$ = 0.88 ± 0.03

$E_{\alpha'\beta'}$ = 0.46 ± 0.06

C = 22.30 ± 0.017

A = 468.8 ± 16.16

ϕ = 17.97 ± 1.39

θ_L = 43.45 ± 2.24

S = 2.64 ± 0.10

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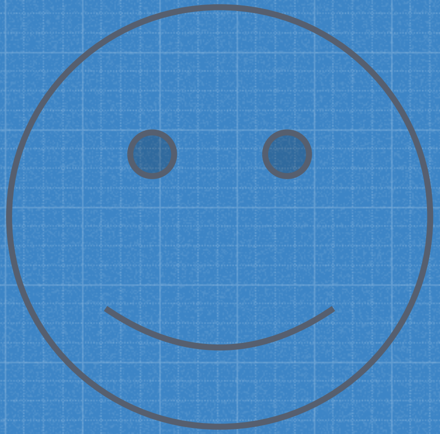
Conclusion

Conclusion

- We successfully created polarization-entangled photon pairs
- Demonstrated a clear Bell inequality violation
  - $S = 2.64 \pm 0.10 > 2$



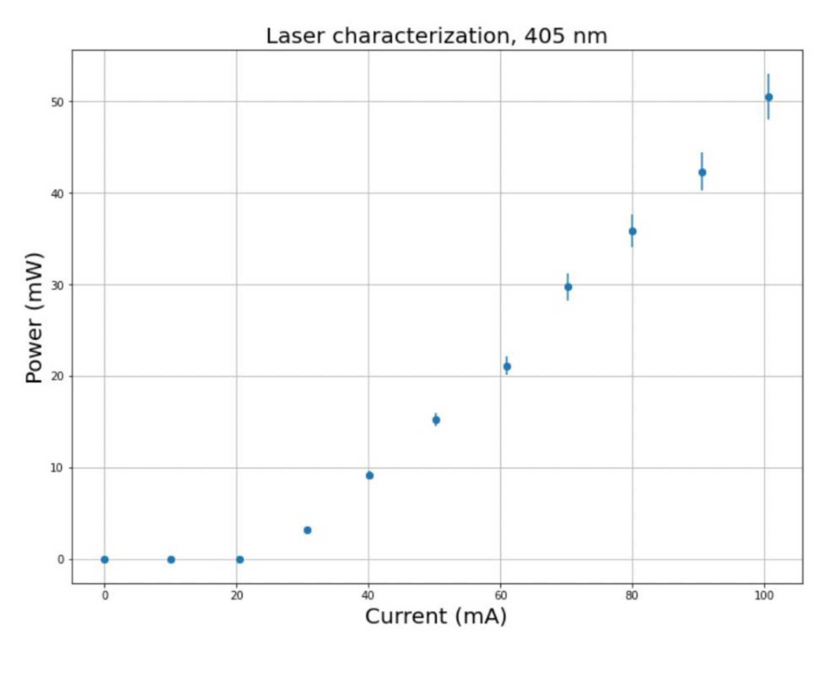
Thanks!

Questions?

Citations

- Physics 111B: Advanced Experimentation Laboratory “QIE - Quantum Interference & Entanglement.”
- D. Dehlinger and M.W. Mitchell, “Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory.”
- Svetlana G. Lukishova, “Quantum Optics and Quantum Information Teaching Laboratories at the Institute of Optics, University of Rochester.”
- E. Rieffel and W. Polak, “Quantum Computing: A Gentle Introduction.”
- J. Berkovitz, "Action at a Distance in Quantum Mechanics." In Edward N. Zalta (ed.). *The Stanford Encyclopedia of Philosophy* (Winter ed.).

Appendix



Error Propagation

$$\sigma_N = \sqrt{N}$$

$$\sigma_{S(\alpha,\beta)} = \sqrt{\sigma^2_{E_{\alpha,\beta}} + \sigma^2_{E_{\alpha',\beta}} + \sigma^2_{E_{\alpha,\beta'}} + \sigma^2_{E_{\alpha',\beta'}}}$$

$$\sigma_E = \frac{N(\alpha,\beta)}{N_{TOT}} \left(\sqrt{\sigma^2_{N(\alpha,\beta)} + \sigma^2_{N(\alpha',\beta')} + \sigma^2_{N(\alpha,\beta')} + \sigma^2_{N(\alpha',\beta)}} * \left(\frac{1}{N(\alpha,\beta)} + \frac{1}{N_{TOT}}\right)\right.$$

Appendix: more on purity graphs

```
beta = [0, 20, 40, 60, 80, 100, 120, 140, 160, 180]
coincidence_count_alpha_0 = [213.0, 160.79, 74.7, 24.8, 21.8, 75.8, 157.6, 238.1, 266.3, 229.3]
coincidence_count_alpha_45 = [153.7, 201.2, 237.1, 180.5, 169.3, 96.9, 37, 85, 160.9, 210.9]
coincidence_count_alpha_90 = [45.6, 56.8, 106.5, 245.5, 250.2, 192.5, 117, 40.1, 14.8, 44.7]
coincidence_count_alpha_135 = [75.1, 35.4, 46.3, 96.4, 171.5, 223.7, 238.5, 207.5, 147.7, 68.7]
```

$$N(\alpha,\beta)=A(\sin^2\alpha\sin^2\beta\cos^2\theta_l+\cos^2\alpha\cos^2\beta\sin^2\theta_l+\tfrac{1}{4}\sin 2\alpha\sin 2\beta\sin 2\theta_l\cos\phi_m)+C$$

α = 0: $\chi 2$ = 1.32
α = 45: $\chi 2$ = 2.78
α = 90: $\chi 2$ = 3.81
α = 135: $\chi 2$ = 1.07

Appendix: code for calculating S

```
def p (n_a_b, n_ap_bp, n_a_bp, n_ap_b):
    p_vv = n_a_b/(n_a_b + n_ap_bp + n_a_bp + n_ap_b)
    p_hh = n_ap_bp/(n_a_b + n_ap_bp + n_a_bp + n_ap_b)
    p_vh = n_a_bp/(n_a_b + n_ap_bp + n_a_bp + n_ap_b)
    p_hv = n_ap_b/(n_a_b + n_ap_bp + n_a_bp + n_ap_b)
    return np.array([p_vv, p_hh, p_vh, p_hv])

def e_ab(n_a_b, n_ap_bp, n_a_bp, n_ap_b):
    e = (n_a_b + n_ap_bp - n_a_bp - n_ap_b)/(n_a_b + n_ap_bp + n_a_bp + n_ap_b)
    return e

def s(e_a0_b0, e_a0_b1, e_a1_b0, e_a1_b1):
    s = e_a0_b0 - e_a0_b1 + e_a1_b0 + e_a1_b1
    return s
```

Appendix: data and code for calculating S

```
# E(a,b) --> a = -45, b = -22.5, a perp = 45, b perp = 67.5
n_a_b_0 =  150.5  #-45, -22.5
n_ap_bp_0 = 134 #45, 67.5
n_a_bp_0 =  45.2 #-45, 67.5
n_ap_b_0 =  32.3 #45, -22.5

P_a_b = p (n_a_b_0, n_ap_bp_0, n_a_bp_0, n_ap_b_0)
print("P_vv, P_hh, P_vh, P_hv:", P_a_b)
E_a_b = e_ab(n_a_b_0, n_ap_bp_0, n_a_bp_0, n_ap_b_0)
print("E_ab:", E_a_b)
```

```
P_vv, P_hh, P_vh, P_hv: [0.41574586 0.37016575 0.12486188 0.08922652]
E_ab: 0.5718232044198895
```

```
# E(a, b') --> a = -45, b = 22.5, a_perp = 45, b perp = 112.5
n_a_b_1 = 40.60 #-45, 22.5
n_ap_bp_1 = 35.90 #45, 112.5
n_a_bp_1 = 252.30 #-45, 112.5
n_ap_b_1 = 231.20 #45, 22.5

P_a_b_prime = p (n_a_b_1, n_ap_bp_1, n_a_bp_1, n_ap_b_1)
print("P_vv, P_hh, P_vh, P_hv:",P_a_b_prime)
E_a_b_prime = e_ab(n_a_b_1, n_ap_bp_1, n_a_bp_1, n_ap_b_1)
print("E_a_b_prime:", E_a_b_prime)
```

```
P_vv, P_hh, P_vh, P_hv: [0.0725     0.06410714 0.45053571 0.41285714]
E_a_b_prime: -0.7267857142857143
```

```
# E(a', b) --> a = 0, b = -22.5, a perp = 90, b perp = 67.5
n_a_b_2 = 272.50  #0, -22.5
n_ap_bp_2 = 263.10 #90, 67.5
n_a_bp_2 = 16.0 #0, 67.5
n_ap_b_2 = 17.40 #90, -22.5

P_a_prime_b = p(n_a_b_2, n_ap_bp_2, n_a_bp_2, n_ap_b_2)
print("P_vv, P_hh, P_vh, P_hv:",P_a_prime_b)
E_a_prime_b = e_ab(n_a_b_2, n_ap_bp_2, n_a_bp_2, n_ap_b_2)
print("E_a_prime_b:", E_a_prime_b)
```

```
P_vv, P_hh, P_vh, P_hv: [0.47891037 0.46239016 0.02811951 0.03057996]
E_a_prime_b: 0.8826010544815467
```

```
# E(a', b') --> a = 0, b = 22.5
n_a_b_3 = 151.20 #0, 22.5
n_ap_bp_3 = 126.90 #90, 112.5
n_a_bp_3 = 58.60 #0, 112.5
n_ap_b_3 = 44.50 #90, 22.5

P_a_prime_b_prime = p(n_a_b_3, n_ap_bp_3, n_a_bp_3, n_ap_b_3)
print("P_vv, P_hh, P_vh, P_hv:",P_a_prime_b_prime)
E_a_prime_b_prime = e_ab(n_a_b_3, n_ap_bp_3, n_a_bp_3, n_ap_b_3)
print("E_a_prime_b_prime:",E_a_prime_b_prime)
```

```
P_vv, P_hh, P_vh, P_hv: [0.39664218 0.33289612 0.15372508 0.11673662]
E_a_prime_b_prime: 0.45907660020986363
```

```
print("S:", s(E_a_b, E_a_b_prime, E_a_prime_b, E_a_prime_b_prime))
```

```
S: 2.640286573397014
```

Appendix: Predicted polarization correlations