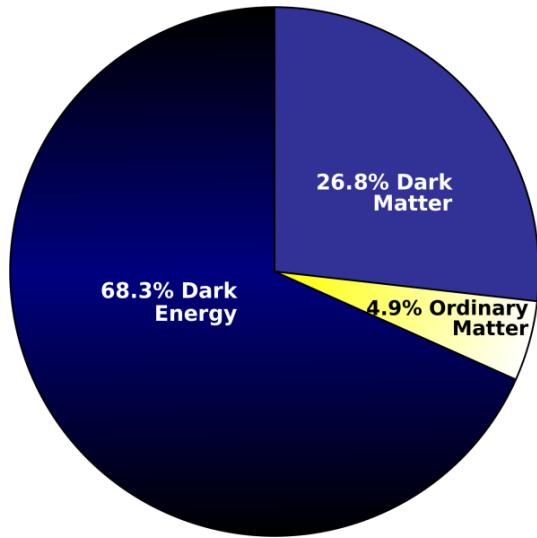


Oscillating Features in the Neutron Electromagnetic Structure

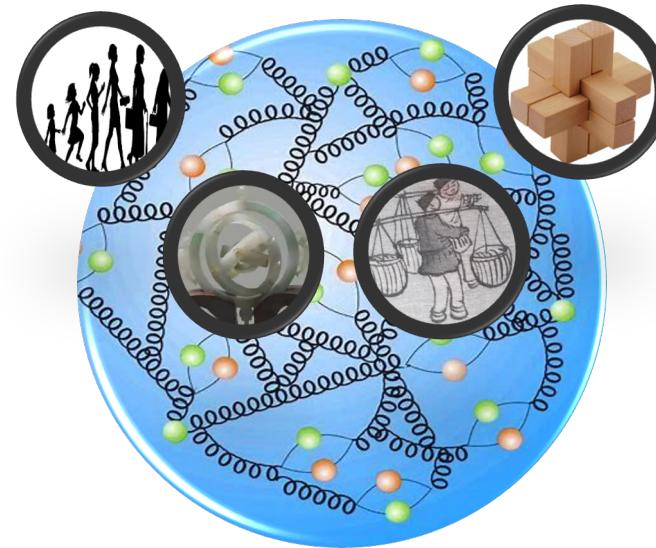
胡继峰(on behalf of BESIII Collaboration)

华南师范大学量子物质研究院

Motivation



**Major components of
visible matter**

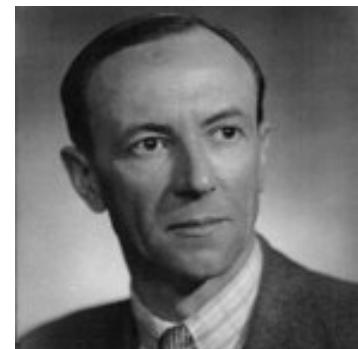


**Key of understanding
the strong interaction**

Before Discovery of the Neutron

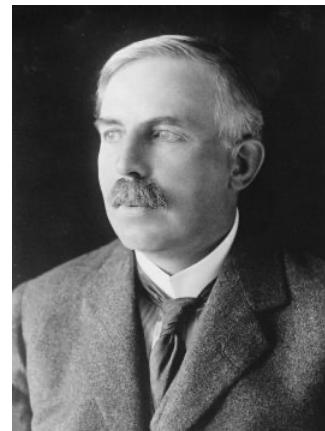


1896 β radiation, Henri
Becquerel, Marie/Pierre Curie



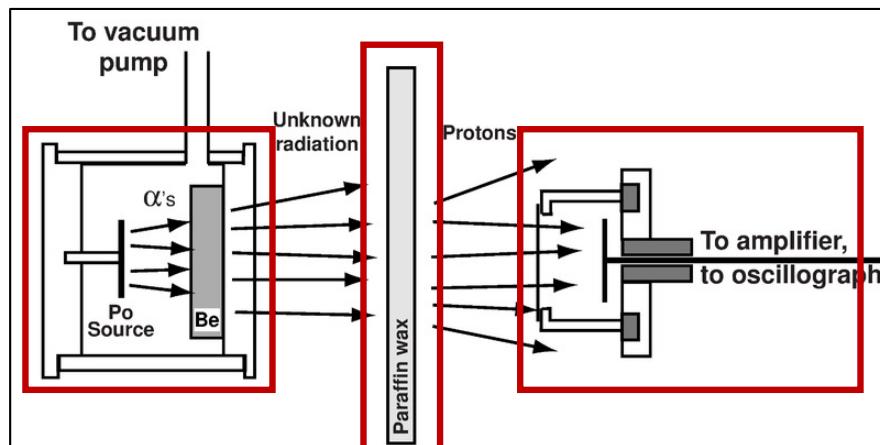
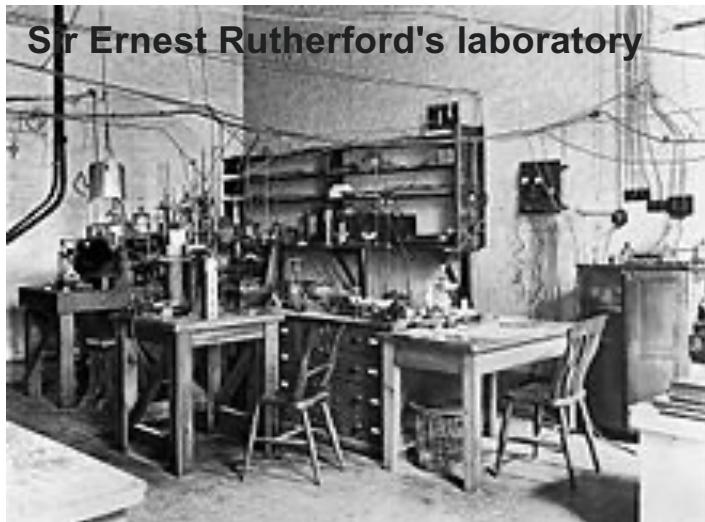
1932 discovery of the neutron,
James Chadwick

1919 discovery of the proton,
Ernest Rutherford



Discovery of the Neutron (1932)

[James Chadwick - Wikipedia](#)



early technique

<https://www.nature.com/articles/129312a0.pdf>

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NATURE

[FEBRUARY 27, 1932]

Letters to the Editor

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of, rejected manuscripts intended for this or any other part of NATURE. No notice is taken of anonymous communications.]

Possible Existence of a Neutron

It has been shown by Bothe and others that beryllium when bombarded by α -particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0.3 cm.^{-1} . Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when a layer containing hydrogen gas was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly $3 \times 10^8 \text{ cm. per sec.}$ They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of $50 \times 10^6 \text{ electron volts.}$

I have made some experiments using the valve counter to examine the properties of this radiation excited in beryllium. The valve counter consists of a small ionisation chamber connected to an amplifier, and the sudden production of ions by the entry of a particle, such as a proton or α -particle, is recorded by the deflexion of an oscillograph. These experiments have shown that the ionisation effects produced by hydrogen, helium, lithium, beryllium, carbon, air, and argon, The particles ejected from hydrogen, behave, as regards range and ionising power, like protons with speeds up to about $3.2 \times 10^8 \text{ cm. per sec.}$ The particles from the other elements have a large ionising power, and appear to be in each case recoil atoms of the elements.

If we ascribe the ejection of the proton to a Compton recoil from a quantum of 52×10^6 electron volts, then the nitrogen recoil atom arising by a similar process should have an energy not greater than about 400,000 volts, should produce not more than about 100,000 ions, and have a range in air at N.T.P. of about 1.3 mm. Actually, some of the recoil atoms in nitrogen produce at least 30,000 ions. In collaboration with Dr. Feather, I have observed the recoil atoms in an expansion chamber, and their range, estimated visually, was sometimes as much as 3 mm. at N.T.P.

These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions. The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. The capture of the α -particle by the Be^9 nucleus may be supposed to result in the formation of a Be^{10} nucleus and the emission of the neutron. From the energy relations of this process, the speed of the recoil neutron in the forward direction may well be about $3 \times 10^8 \text{ cm. per sec.}$ The collisions of this neutron with the atoms through which it passes give rise to the recoil atoms, and the observed energies of the recoil atoms are in fair agreement with this view. Moreover, I have observed that the protons ejected from hydrogen by the radiation emitted in the opposite direction to that of the exciting α -particle appear to have a much smaller range than those ejected by the forward radiation.

No. 3252, VOL. 129]

This again receives a simple explanation on the neutron hypothesis.

Let it be supposed that the radiation consists of quanta, then the capture of the α -particle by the Be^9 nucleus forms C^{10} . The defect of C^{10} is known with sufficient accuracy to show that the energy of the quantum emitted in this process cannot be greater than about 14×10^6 volts. It is difficult to make such a quantum responsible for the effects observed.

It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favour of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.

J. CHADWICK.

Cavendish Laboratory,
Cambridge, Feb. 17.

The Oldoway Human Skeleton

A LETTER appeared in NATURE of Oct. 24, 1931, signed by Messrs. Leakey, Hopwood, and Beck, in which, among other conclusions, it is stated that "there is no possible doubt that the human skeleton came from Bed No. 2 and not from Bed No. 4". This must be taken to mean that the skeleton is to be considered as a natural deposit in Bed No. 2, which is overlaid by the later beds Nos. 3 and 4, and that all consideration of human interment is ruled out.

If this be true, it is a most unusual occurrence. The skeleton, which is of modern type, with well-filled teeth, was found completely articulated, down even to the phalanges, and in a position of extraordinary contraction. Complete mammalian skeletons of any age are, as field palaeontologists know, of great rarity. When they occur, their perfection can usually be explained as the result of sudden death and immediate covering by volcanic dust. Many of the more or less perfect skeletons which may be seen in museums have been rearticulated from bones found somewhat scattered as the result of death from floods, or in the neighbourhood of drying water-holes. We know of one case of a perfect articulated skeleton being found in company with such broken and scattered remains as appear to be abundant at Oldoway. Either the skeleton was all deposited as in the *Sivapithecus* quarry at Sioux City, Nebraska, or all were scattered and broken in various degrees, as in ordinary bone beds. The probability, therefore, that the Oldoway skeleton represents an artificial burial is thus one that will occur to paleontologists.

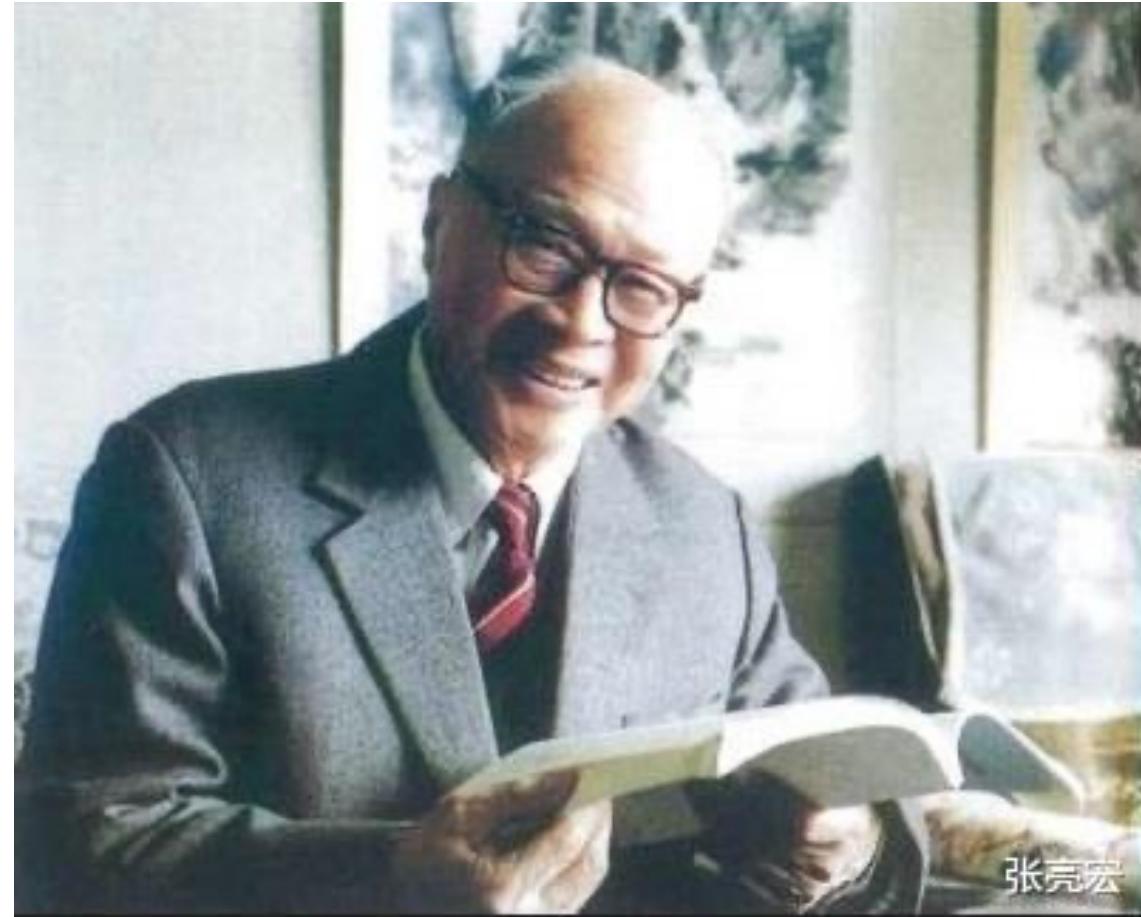
The skeleton was exhumed in 1913, and published photographs show that the excavation made for its disinterment was extensive. It is, therefore, very difficult to believe that in 1931 there can be reliable evidence left at the site as to the conditions under which it was deposited. If naturally deposited in Bed No. 2, the skeleton is of the highest possible importance, because it would be of pre-Mousterian age, and would fit in the company of *Pithecanthropus* and the Pithom skull described by J. P. Viret, all of whose remains are fragmentary to the last degree. Of the few other human remains for which such antiquity is claimed, the Galley Hill skeleton and the Ipswich skeleton are, or apparently were, complete. The first of these was never seen *in situ* by any trained observer, and the latter has, we believe, been withdrawn by its discoverer. The other fragments, found long ago, are entirely without satisfactory evidence as to their mode of occurrence.

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《Possible existence of a Neutron》

与诺贝尔奖失之交臂

当时，正在柏林大学攻读博士学位的王淦昌就铍辐射现象向导师 Meitner 表达了自己的看法，还提出了进一步的实验方案。可惜的是，王先生的方案被导师屡次否决，而该方案与之后 Chadwick 所采用的实验方案正相同！后来，Meitner 为此特意向王先生表达了歉意。



张亮云

王淦昌先生，1907-1998，核物理学家、中国科学院院士。

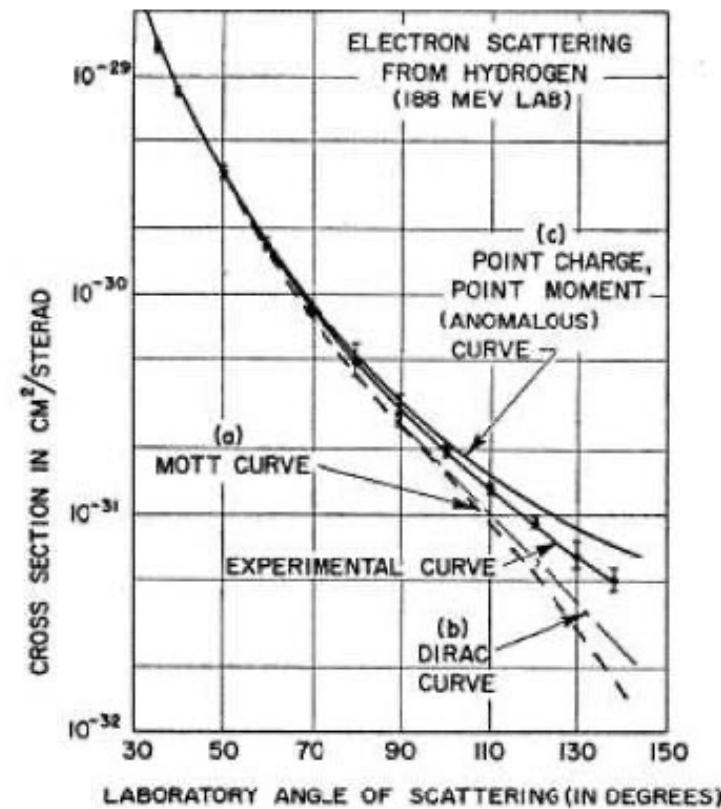
Electron Scattering (1950s)

Hofstadter R. etc., "electron scattering and nuclear structure" Rev. Mod. Phys. 28, 214–254 (1956)

[Robert Hofstadter - Biographical \(nobelprize.org\)](#)



Over a period of time lasting at least two thousand years, Man has puzzled over and sought an understanding of the composition of matter. ... Indeed this structure may be quite complex, so that the elegant idea of elementarity must be abandoned.



Electromagnetic Form Factor

Point-like scattered by point-like

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E \sin^4(\theta/2)}$$

Spin=0

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E \sin^4(\theta/2)}$$

Spin=½

$$|F(q)|^2 \equiv 1$$

The structured scattered by point-like

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E \sin^4(\theta/2)} \frac{E'}{E} \left[1 + \frac{Q^2}{2M^2} \tan^2(\theta/2) \right]$$

$d\sigma/d\Omega_{\text{Mott}}$

$$|F(q)|^2$$

Hofstadter R. etc., Rev. Mod. Phys. 28, 214–254 (1956)
Prog. Part. Nucl. Phys. 59:694-764, 2007

- A parameterized function depending on the momentum transfer q^2 .
- Imaging the nucleon structure using the electromagnetic probe q^2 .

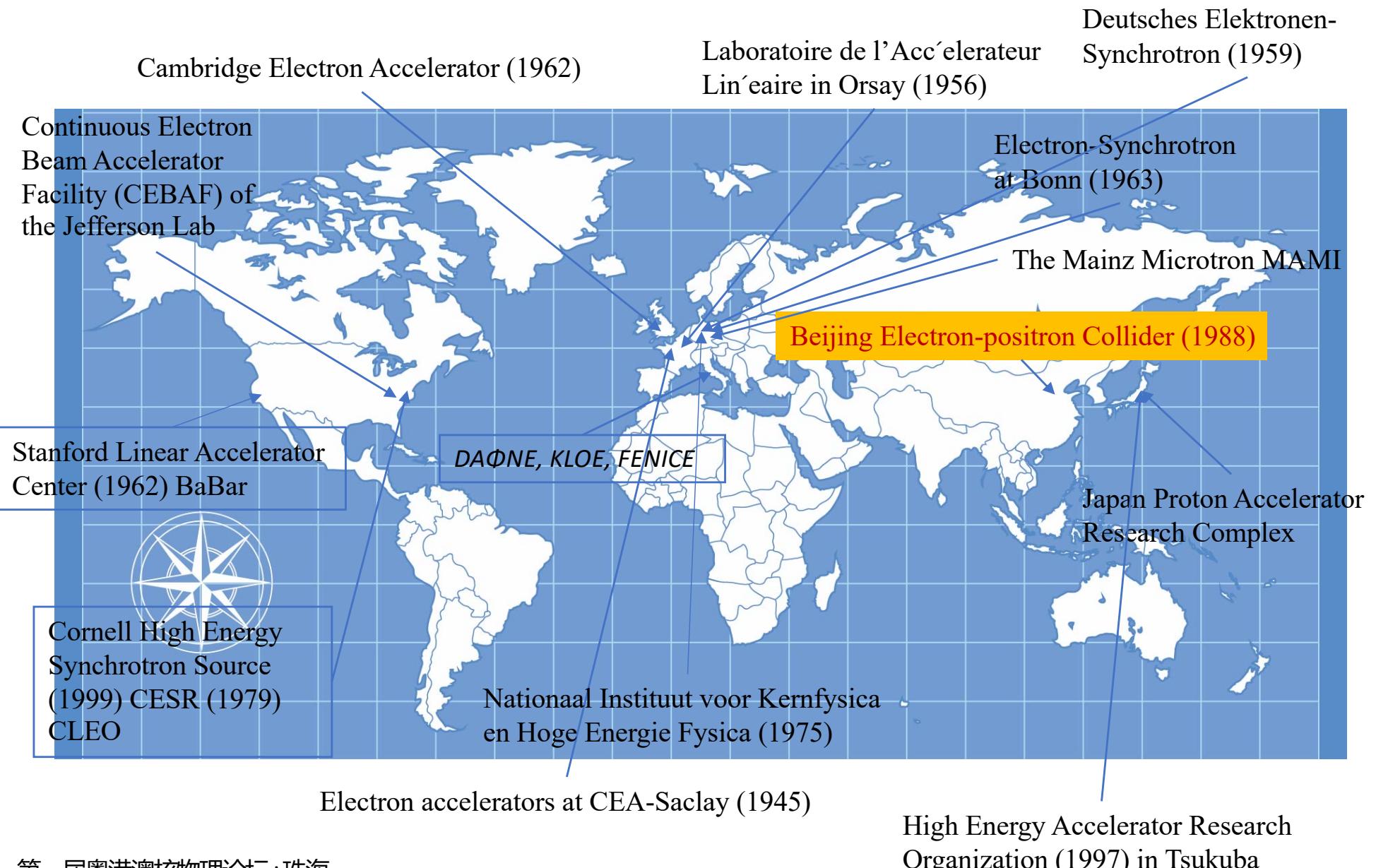
$$[F(q)]^2 = \frac{\sigma(q)}{\sigma_{\text{Mott}}(q)}$$

Simplest observable

$$\mathcal{J}_{\text{hadronic}}^\mu = e \overline{N}(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right] N(p)$$

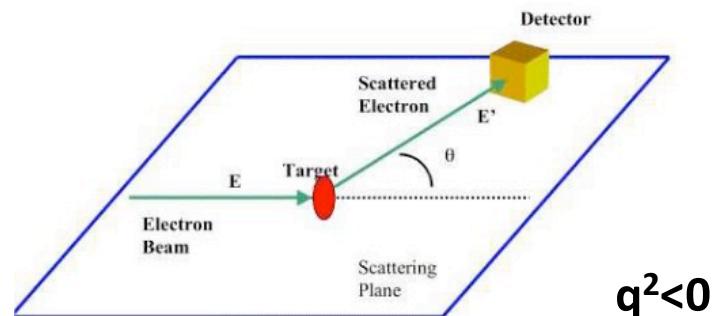
$$\begin{aligned} G_E &= F_1 + \tau F_2 \\ G_M &= F_1 + F_2 \end{aligned}$$

Developments



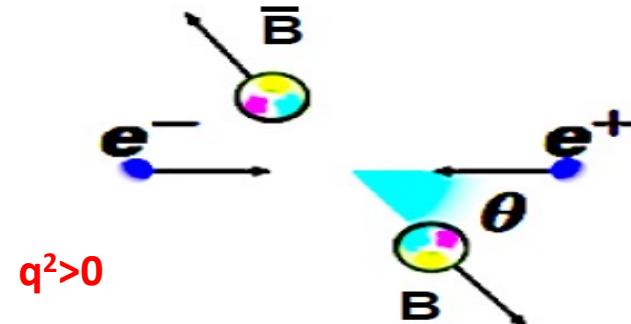
Experiments

Scattering experiment



$$q^2 < 0$$

Anihilation experiment



$$q^2 > 0$$

INSTRUMENTS AND TECHNIQUES

The Cambridge Electron Accelerator

This 6-billion-volt machine will be the world's highest energy electron synchrotron.

M. Stanley Livingston and William A. Shurcliff

Livingston, M. S., & Shurcliff, W. A. (1961). *The Cambridge Electron Accelerator*. *Science*, 134(3486), 1186–1193.
<http://www.jstor.org/stable/1707819>

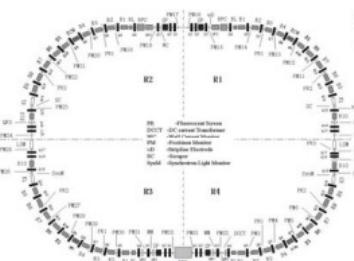
BEPCII : STATUS AND PROGRESS

BEPCII Team
Institute of High Energy Physics, CAS
P.O.Box 918, Beijing 100039, China

Abstract

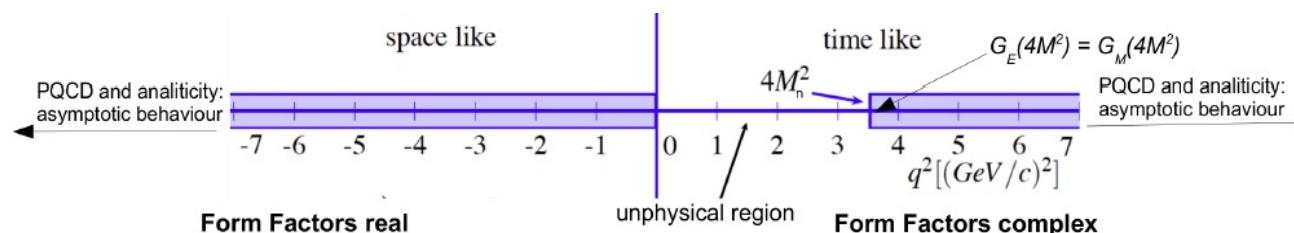
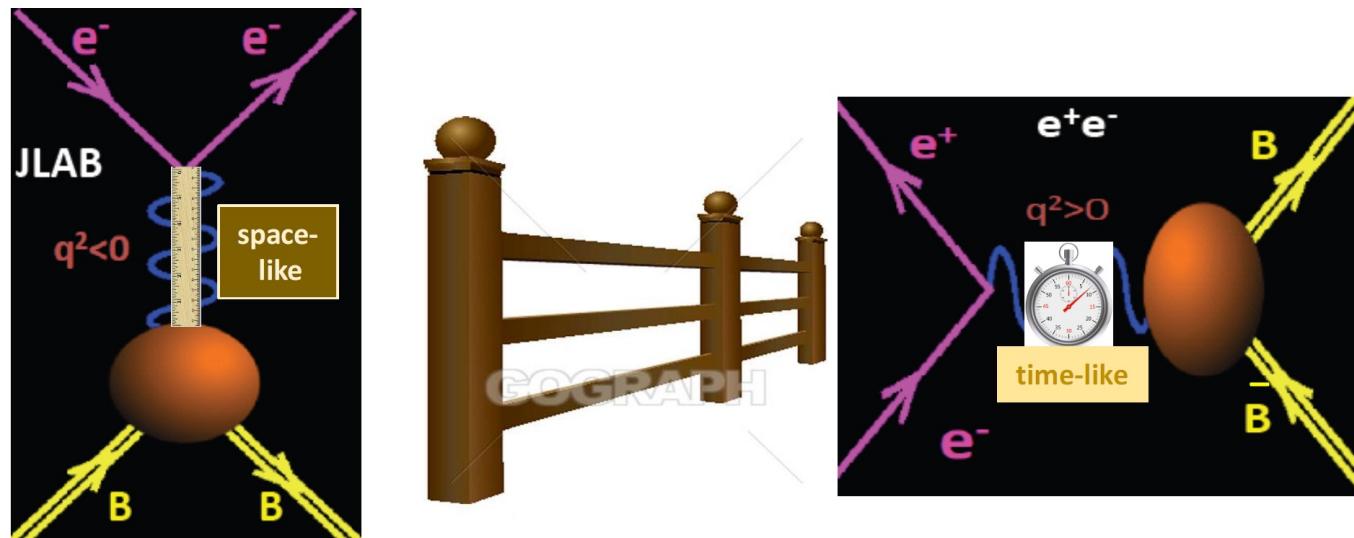
The status and progress of second phase construction of the Beijing Electron-Positron Collider (BEPC), i.e. the BEPCII, are reported. The design luminosity of the BEPCII is $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ at 1.89 GeV with a double-ring scheme. The performance of the BEPCII as a synchrotron radiation source will also be improved with the expected beam current of 250mA at 2.5 GeV. Some key technologies are being developed in order to achieve the goal of the project.

1 INTRODUCTION



Space-like versus Time-like

-- the view from the side of the fence -- *Borrowed from Prof. Olsen's talk.*

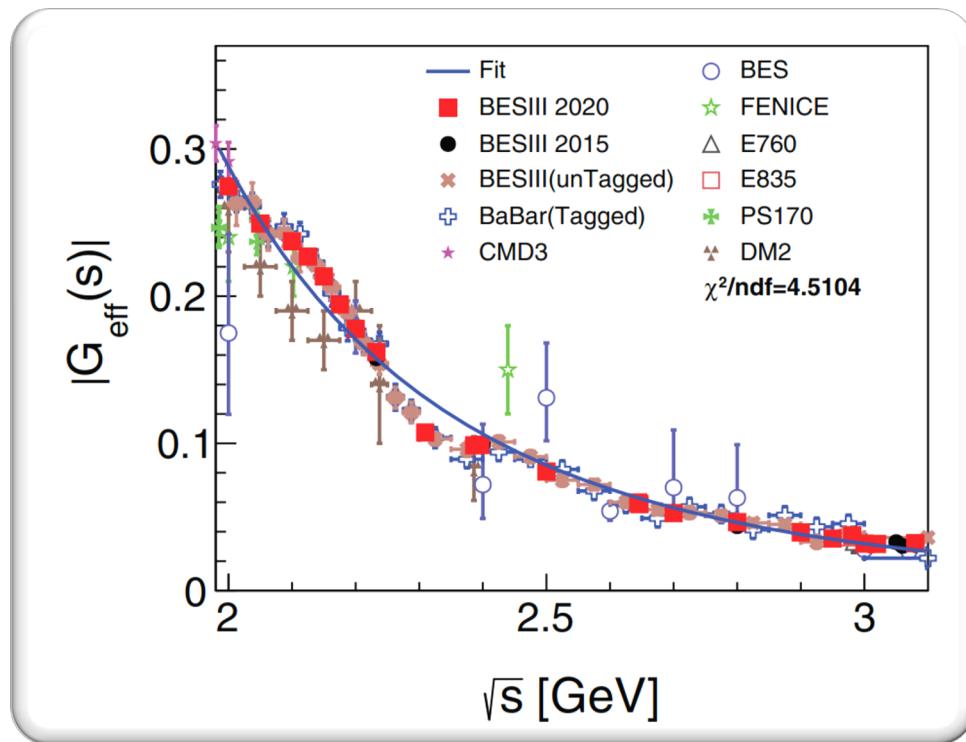


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \times \left(G_E^2 + \tau \left[1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2} \right] G_M^2 \right) / (1+\tau)$$

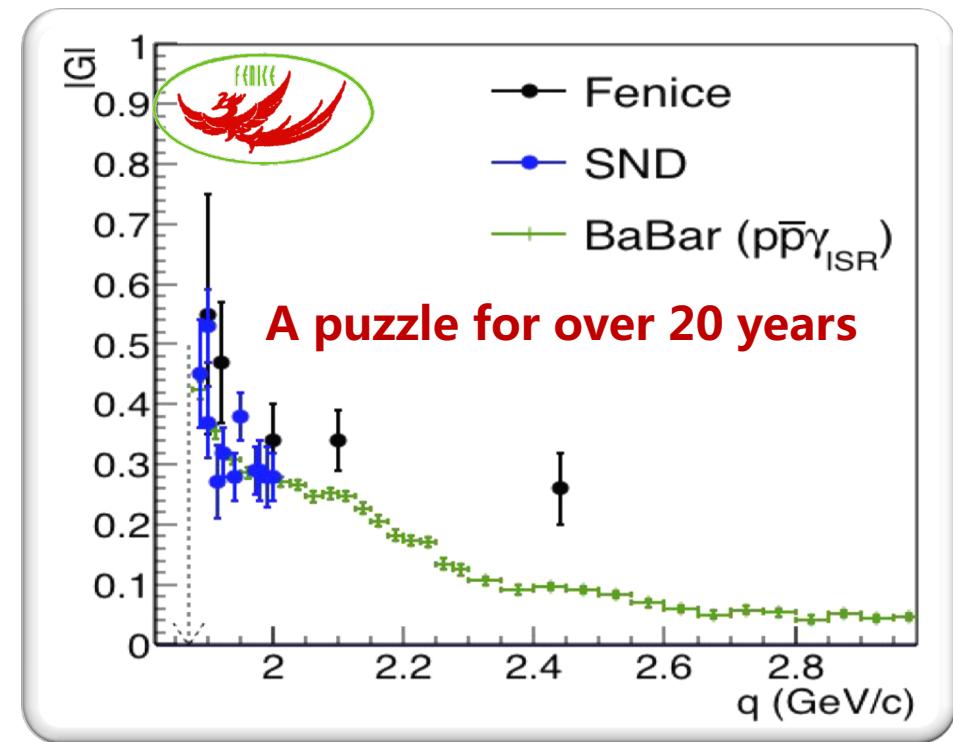
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{4s} C [|G_M(s)|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E(s)|^2 \sin^2 \theta]$$

Dispersion relation $F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im } F(t')}{t' - t - i\epsilon} dt'$

The γ^* -Nucleon Coupling Puzzle (1998)



well investigated



rarely investigated

Nucl. Phys. B 517, 3 (1998).
Phys. Rev. D 90, 112007 (2014).
EPJ Web Conf. 212, 07007 (2019).

Precision Era

20th century techniques was out

Borrowed from Prof. Olsen's talk.

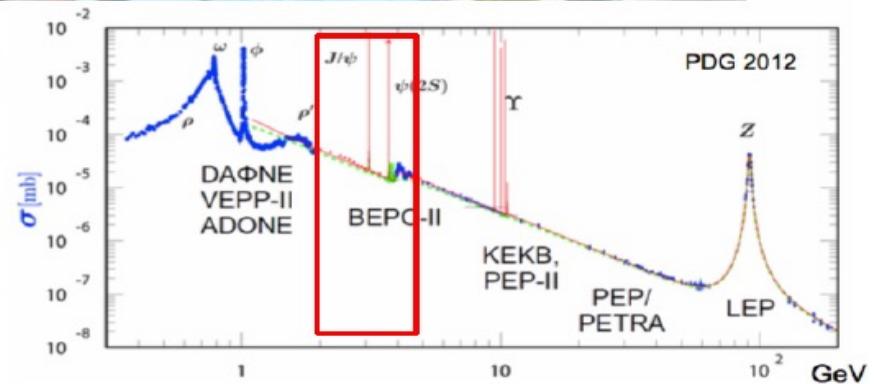
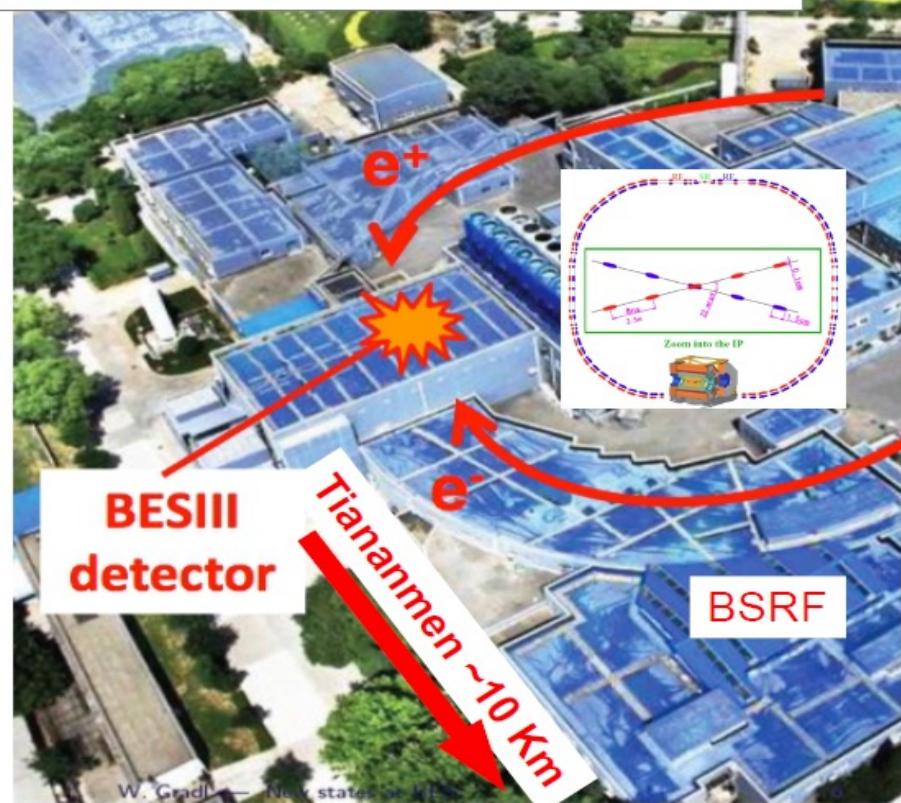


we need new, 21st century, techniques



The BEPCII Collider

Symmetric e^+e^- -collider (double rings)
Beam Energy: 1.0 – 2.3 GeV
Crossing angle: 11mrad
Design Luminosity $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
Energy spread: $5.16 \cdot 10^{-4}$

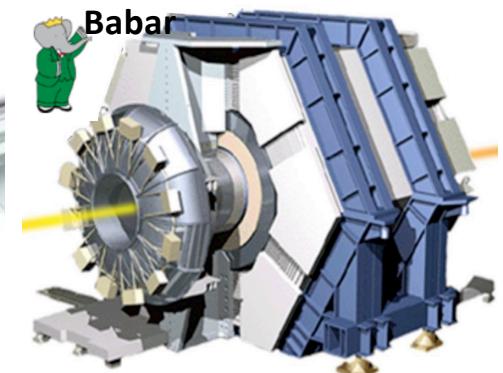
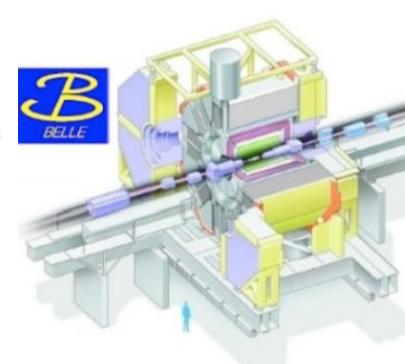
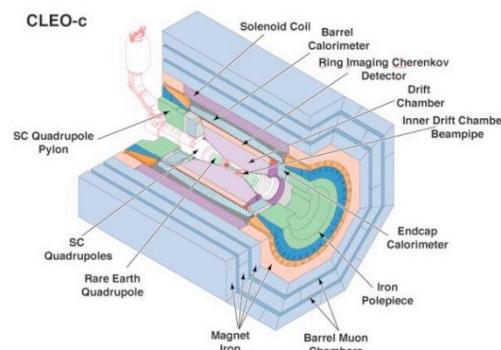
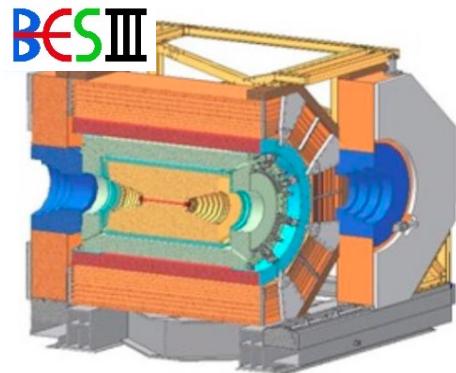


BEPCII is a high luminosity collider.

The BESIII Experiment

Nucl. Instrum. Meth. A 614, 345 (2010)

A precision frontier experiment under running in tau-charm energy region.



MDC	TOF	EMC	MUC	SC
Wire chamber	Scintillator	CsI crystal	RPC	
$\sigma_{xy} \sim 130 \mu m$ $\frac{\sigma_p}{p} < 0.5\% \text{ at } 1 \text{ GeV}/c$	Barrel $\sigma_t \sim 80 \text{ ps}$ Endcap $\sigma_t \sim 110 \text{ ps}$	$\frac{\sigma_E}{E} < 2.5\% \text{ at } 1 \text{ GeV}$ $\sigma_{\perp r} \sim 6 \text{ mm}$	$\sigma_{xy} \sim 1.5 \text{ cm}$	1.0 T 1.5 T

BESIII Collaboration



BES III

~500 members
From 76 institutions in 16 countries

Borrowed from Prof. Li Hai-bo's talk.

Fuente: <https://www.cia.gov/library/publications/cia-maps-publications>
Adaptación por: Colomer

Asia (6/10)

Pakistan (2): COMSATS Institute of Information Technology

University of the Punjab, University of Lahore

Mongolia (1): Institute of Physics and Technology

Korea (1): Chung-Ang University

India (1): Indian Institute of Technology madras

Thailand (1): Suranaree University of Technology

Beijing Institute of Petrochemical Technology,

China Center of Advanced Science and Technology, Fudan University,

Guangxi Normal University, Guangxi University,

Hangzhou Normal University, Henan Normal University,

Henan University of Science and Technology,

Huazhong Normal University, Huangshan College, Hunan University,

Hunan Normal University, Henan University of Technology

Institute of modern physics, Jilin University, Lanzhou University, Liaoning Normal University,

Liaoning University, Nanjing Normal University, Nanjing University, Nankai University, North China Electric Power University,

Peking University, Qufu normal university, Shanxi University,

Shanxi Normal University, Sichuan University, Shandong Normal University, Shandong University, Shanghai Jiaotong University, Soochow University,

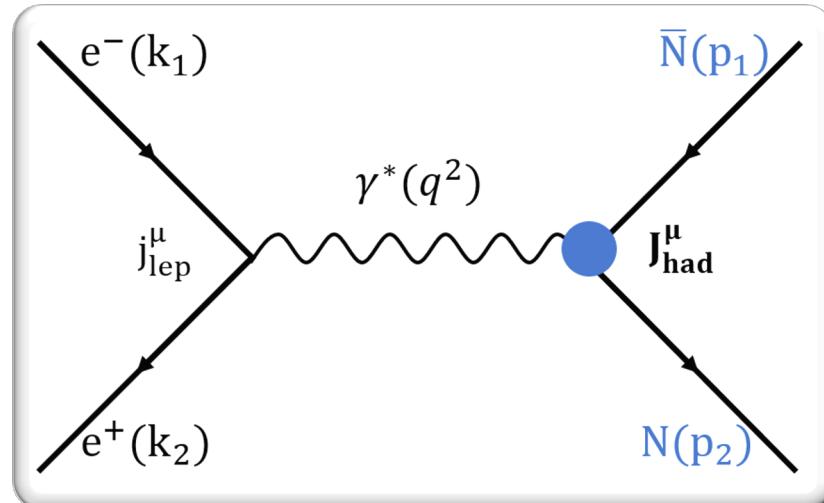
South China Normal University, Southeast University, Sun Yat-sen University, Tsinghua University, University of Chinese Academy of Sciences, Univ Jinan, University of Science and Technology of China,

University of Science and Technology Liaoning,

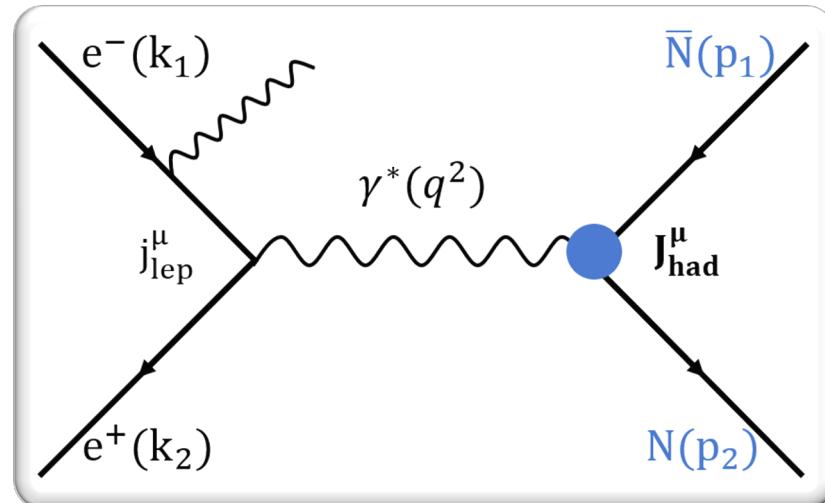
University of South China, Wuhan University, Xinyang Normal University, Zhejiang University, Zhengzhou University, YunNan University, China University of Geosciences

Access to Time-like Form Factor

Energy scan



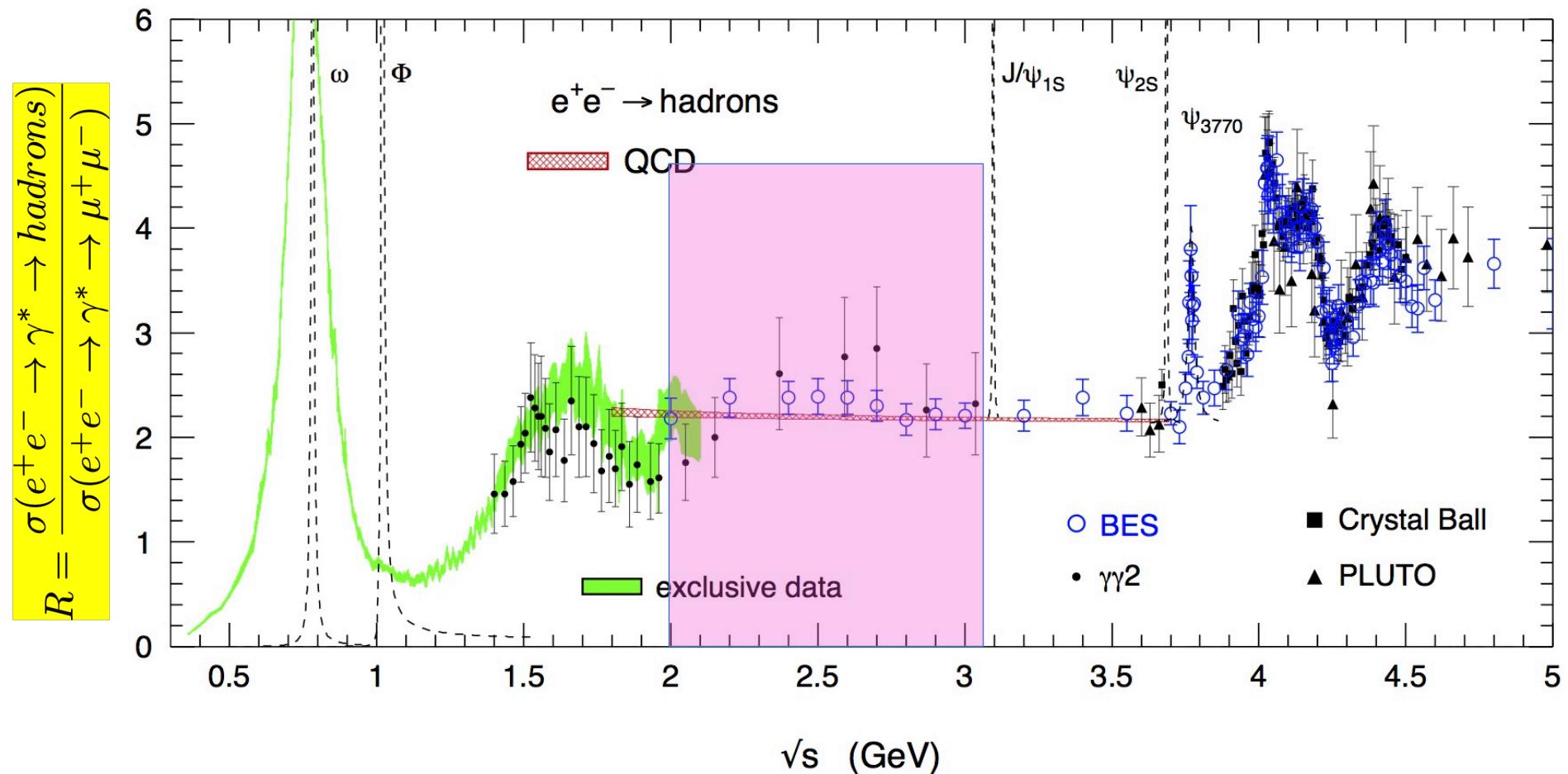
Initial-state-radiation



	Energy Scan	Initial State Radiation
E_{beam}	discrete	fixed
\mathcal{L}	low at each beam energy	high at one beam energy
σ	$\frac{d\sigma_{p\bar{p}}}{d(\cos \theta)} = \frac{\alpha^2 \beta C}{4q^2} [G_M ^2 (1 + \cos^2 \theta) + \frac{4m_p^2}{q^2} G_E ^2 \sin^2 \theta]$	$\frac{d^2\sigma_{p\bar{p}\gamma}}{dx d\theta_\gamma} = W(s, x, \theta_\gamma) \sigma_{p\bar{p}}(q^2)$ $W(s, x, \theta_\gamma) = \frac{\alpha}{\pi x} \left(\frac{2-2x+x^2}{\sin^2 \theta_\gamma} - \frac{x^2}{2} \right)$
q^2	single at each beam energy	from threshold to s

Taking Data

Borrowed from Prof. Li Hai-bo's talk.

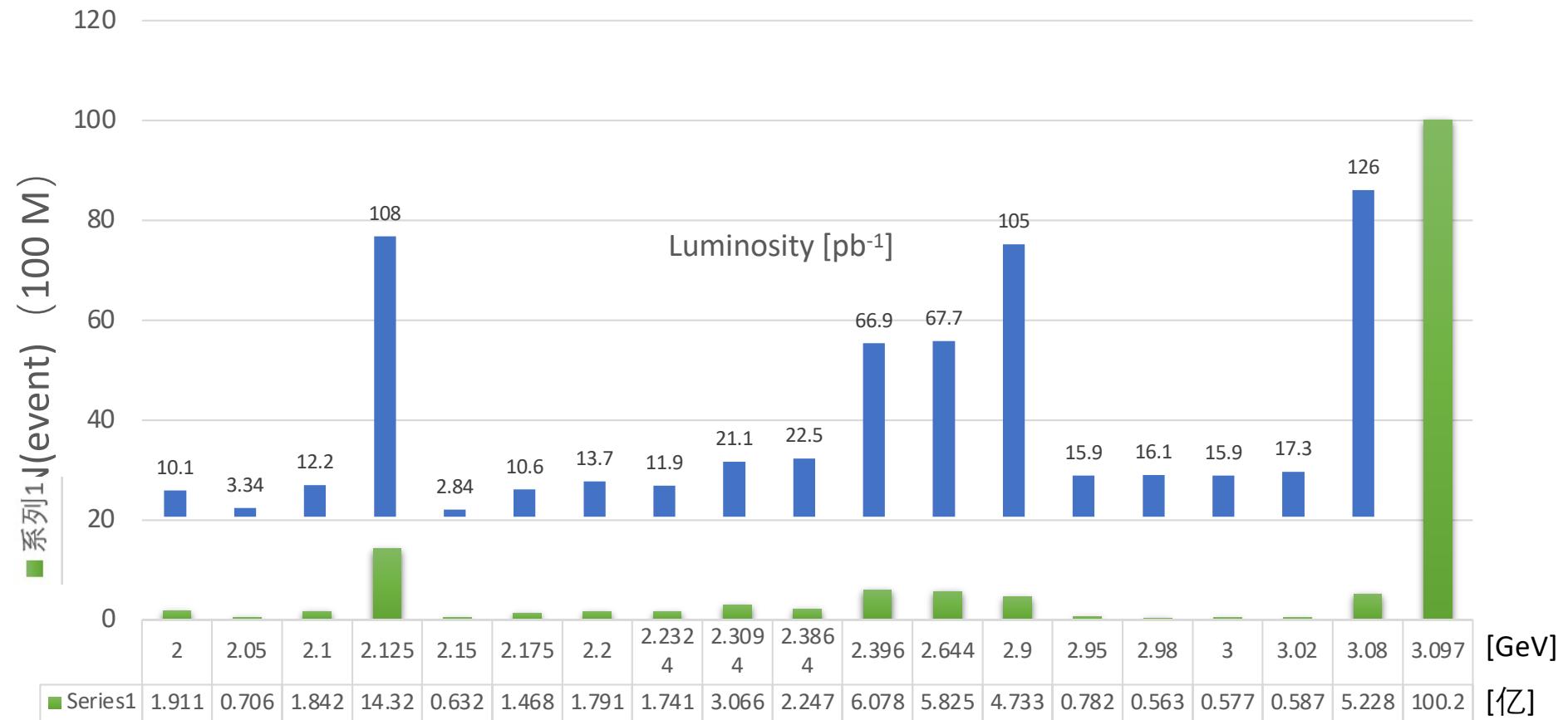


688 pb⁻¹ R-scan data and 10 billion J/Ψ data.

2.0 GeV 2.05 ... 3.02 3.08
Sampling 22 energies, 688 pb⁻¹

3.773 4.60
7 energies 7460 pb⁻¹

Taking Data



2011 tau mass scan, 2012 R scan, 2013-2014 R scan phase 2, 2014-2015 R scan phase 1.

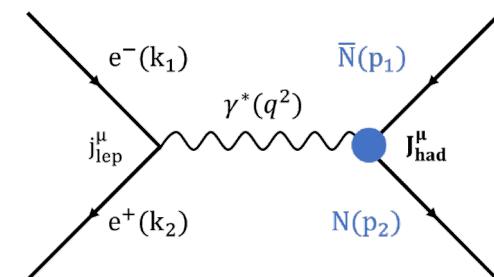
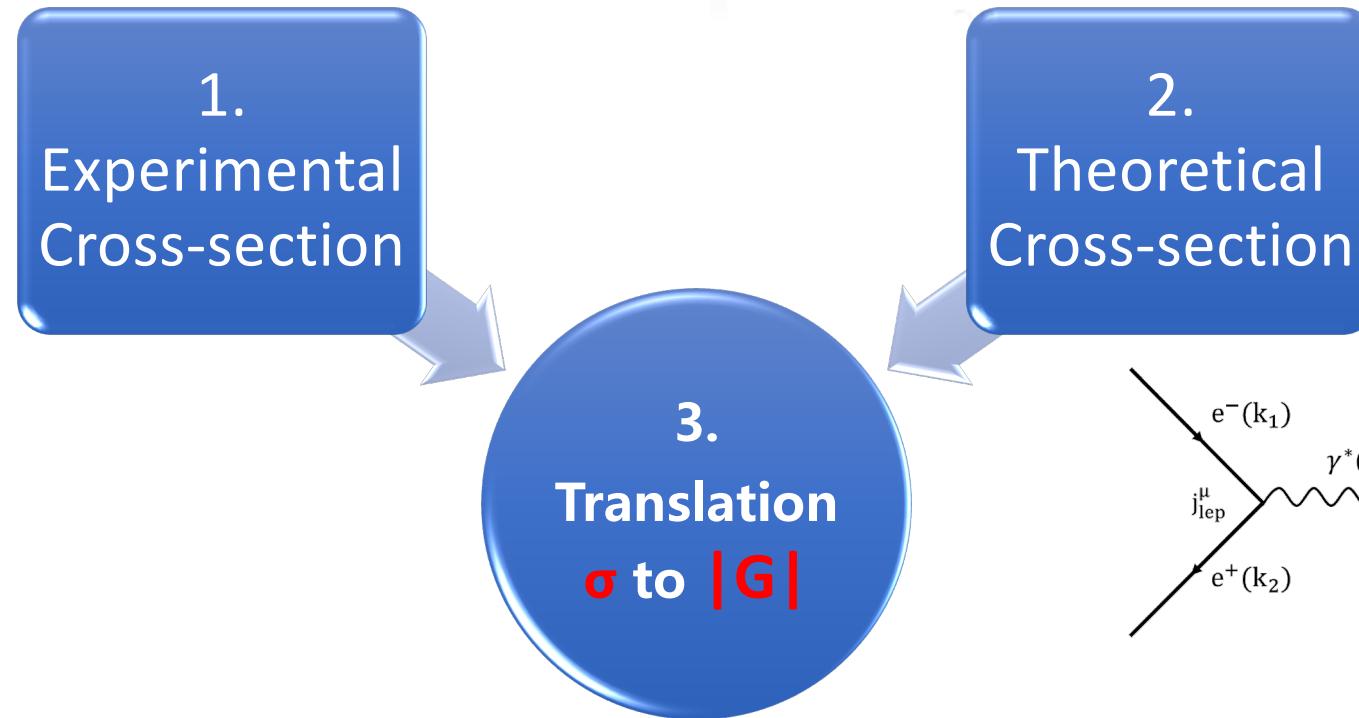
How to Measure ?

Hunt ...

$$\sigma(s) = \frac{N_{obs}}{\mathcal{L}_{int} \epsilon_{MC} (1 + \delta) C_{cor} C_{trg}}$$

Assumption

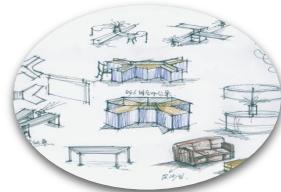
$$\sigma(s) = \frac{4\pi\alpha^2\beta C}{3s} (|G_M(s)|^2 + \frac{2M_n^2}{s} |G_E(s)|^2)$$



$$|G| = \sqrt{\frac{\sigma(s)}{(\frac{4\pi\alpha^2\beta}{3s})(1 + \frac{2M_n^2}{s})}}$$

This talk demonstrates effective form factor $|G|$.

What are Challenges ?



1. 重建



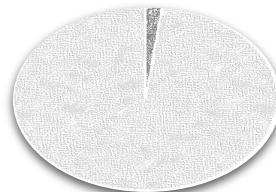
2. 粒子鉴别



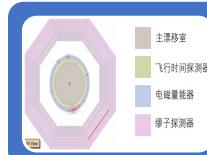
3. 事例分类



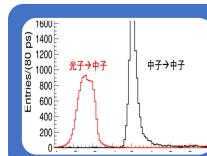
4. 校准



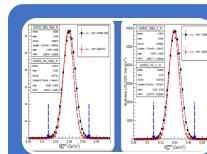
5. 模拟效率



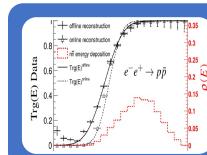
We develop a package for the neutral reconstruction, and verify it using $e^+e^- \rightarrow \gamma\gamma$.



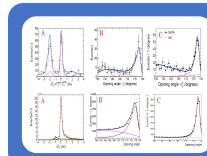
We use time-of-flight to identify photon/neutron with $e^+e^- \rightarrow \gamma\gamma$ and $J/\Psi \rightarrow \pi^+\pi^-\pi^0 \rightarrow \pi^+\pi^-\gamma\gamma$.



We calibrate detection efficiency using $J/\Psi \rightarrow p\pi n/\bar{n}$ taking advantage of 10 billion J/Ψ .



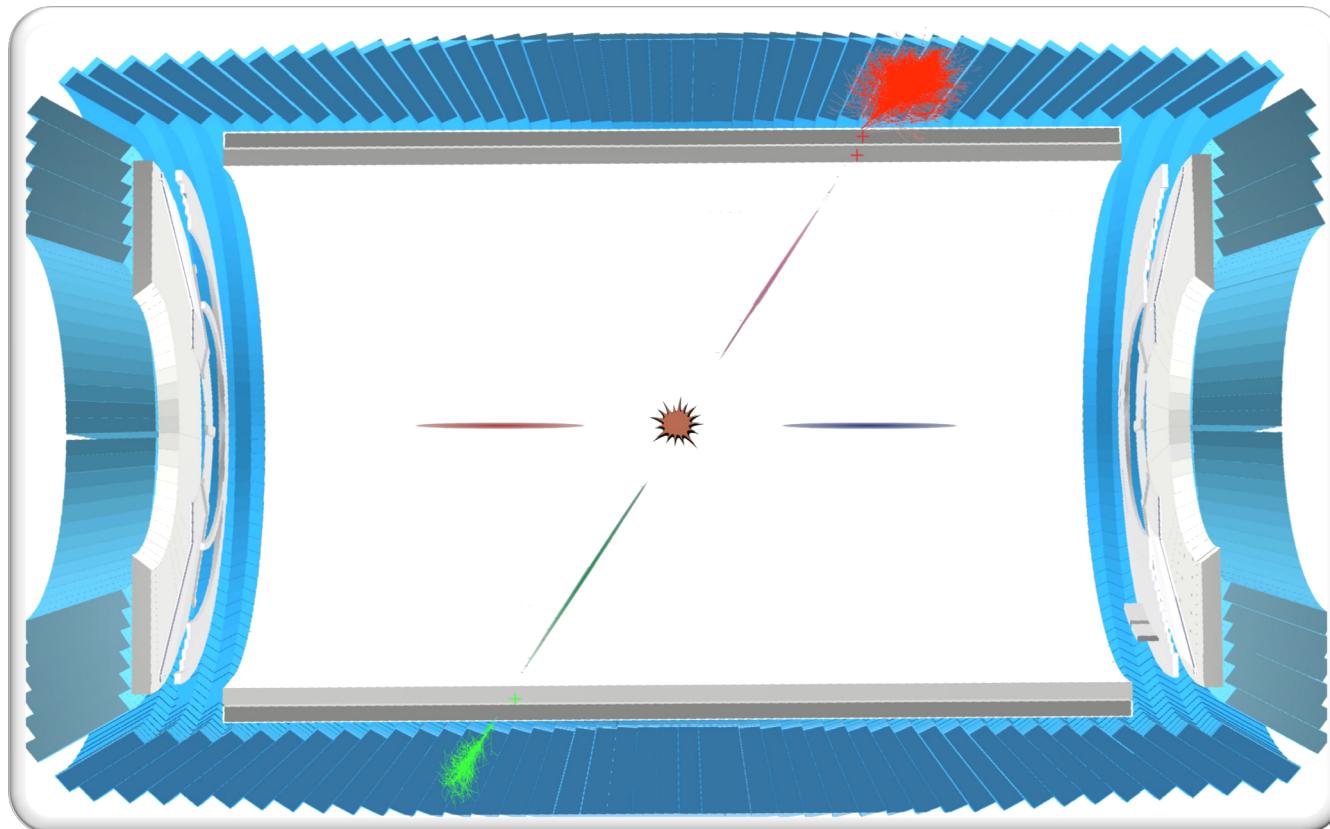
We calibrate trigger efficiency using $e^+e^- \rightarrow p\bar{p}$.



We try three solutions to minimize the statistical errors and verify each solution using $J/\Psi \rightarrow n\bar{n}$.

Tough work 3 years+

Three Categories



Category A

$(EMC + TOF)^{\bar{n}} + TOF^n$

Category B

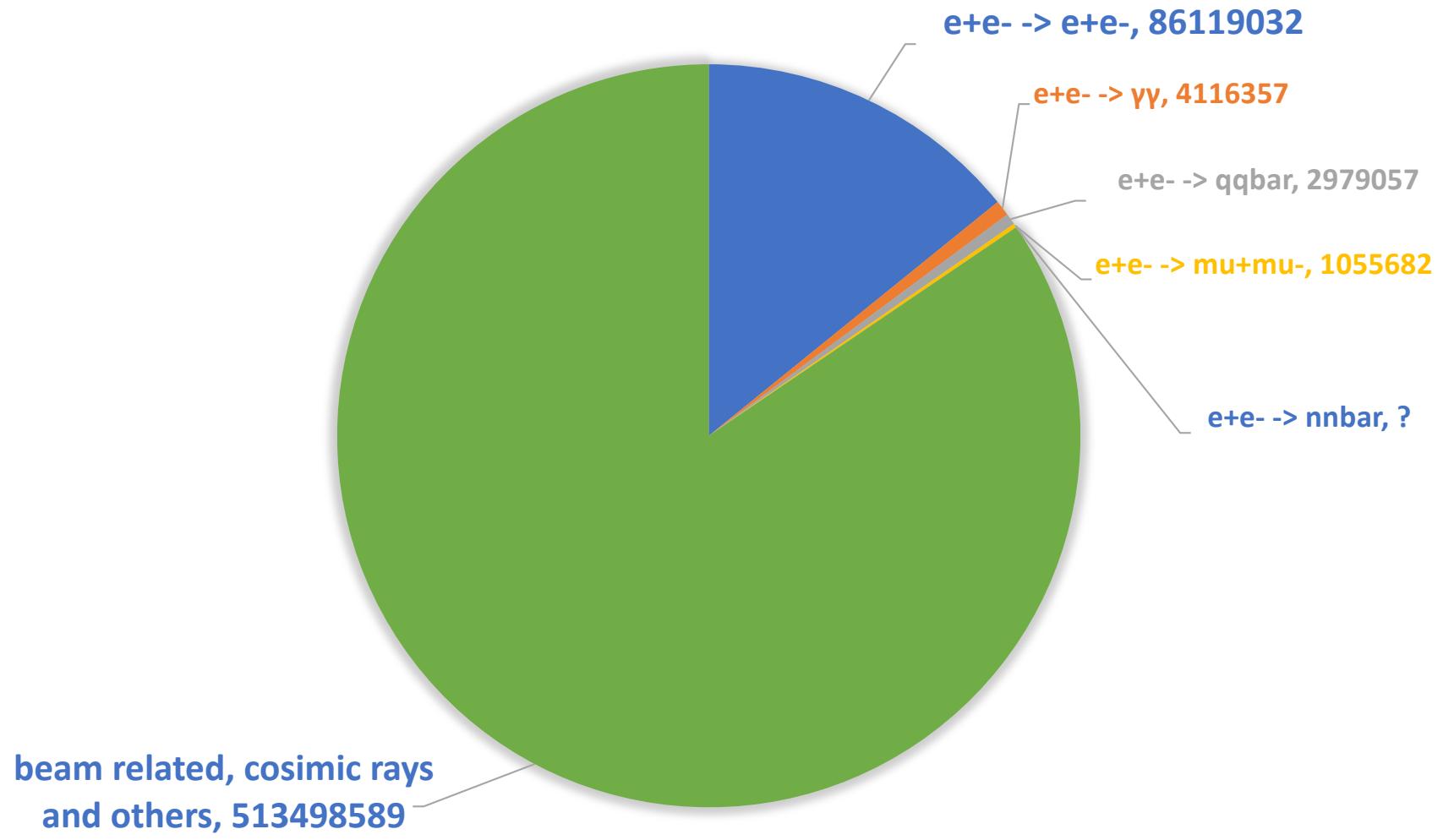
$(EMC + TOF)^{\bar{n}} + EMC^n + MUC$

Category C

$EMC^{\bar{n}} + EMC^n + MUC$

Feeling the Challenge

464 SIGNAL IN 607802167 EVENTS AT ECM=2.396 GEV

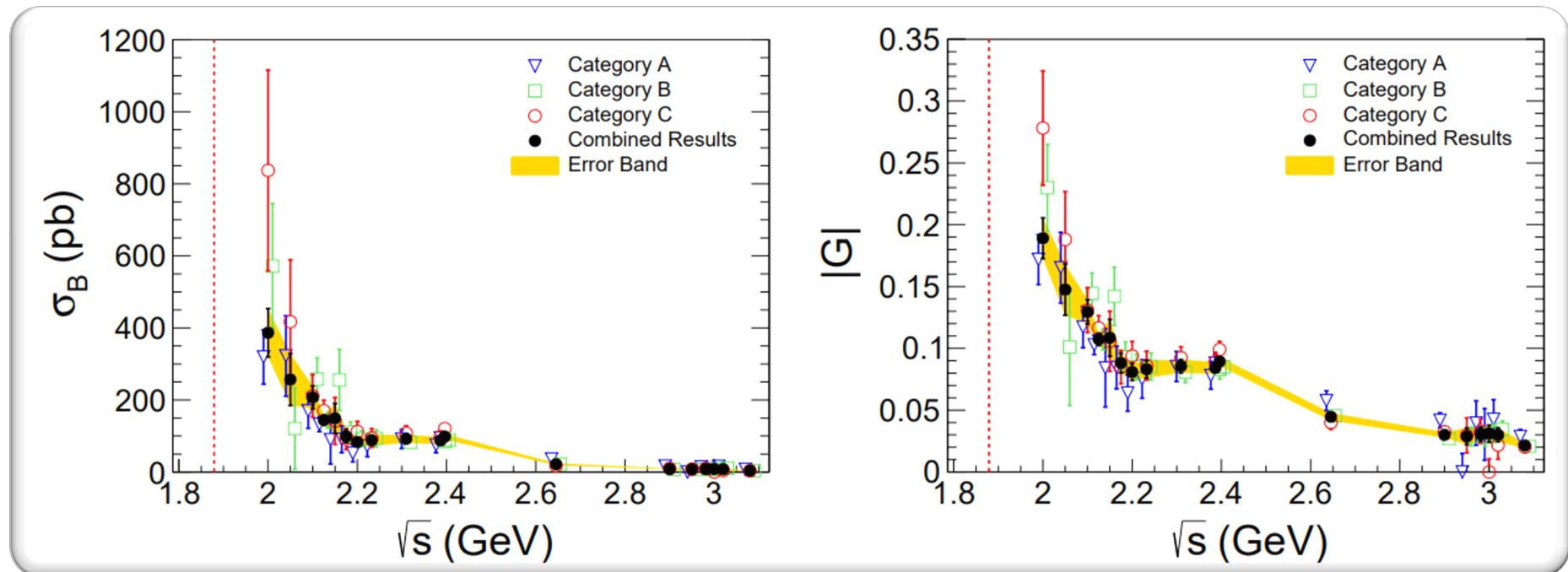


百万里挑一

Results of Three Categories

Nature Physics volume 17, pages 1200–1204 (2021)

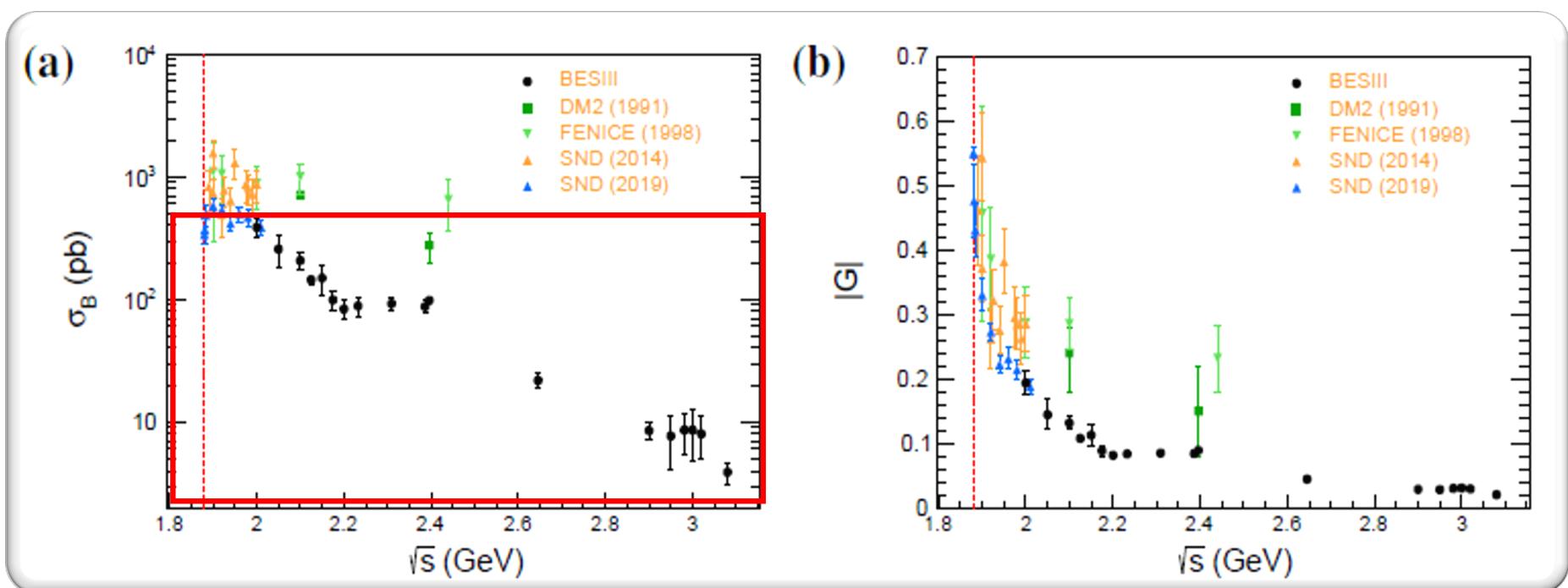
$$\sigma_B^i = \frac{\mathcal{N}_i^s}{\mathcal{L}_{\text{int}} \mathcal{E}_i (1 + \delta)_i}, \quad |G^i| = \sqrt{\frac{\sigma_B^i}{\frac{4\pi\alpha_{em}^2\beta}{3q^2} \left(1 + \frac{1}{2\tau}\right)}} \quad i = A, B, C$$



We combine 3 categories to get the best measurement.

Cross-sections and Form Factors

- ✓ 1st highlight of the results: most precise in the time-like region.
- ✓ The precision improved > a factor of ~30 over previous results.



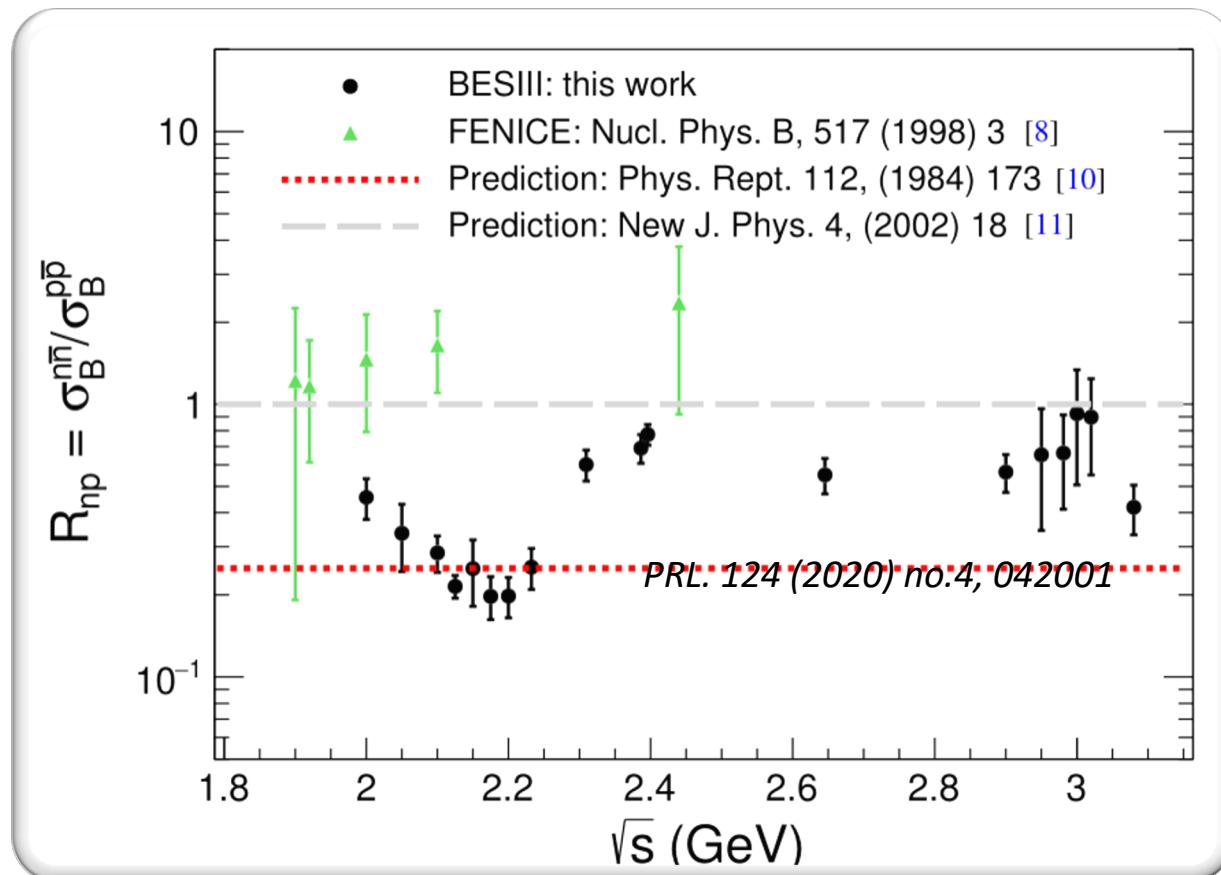
(a) Cross-section (b) Effective Form Factor

Nature Physics volume 17, pages 1200–1204 (2021)

Cross-section Ratios

- ✓ 2nd highlight of the results: precise ratios in 2.0~3.08 GeV.
- ✓ Clearly clarifying the photo-nucleon interaction puzzle.

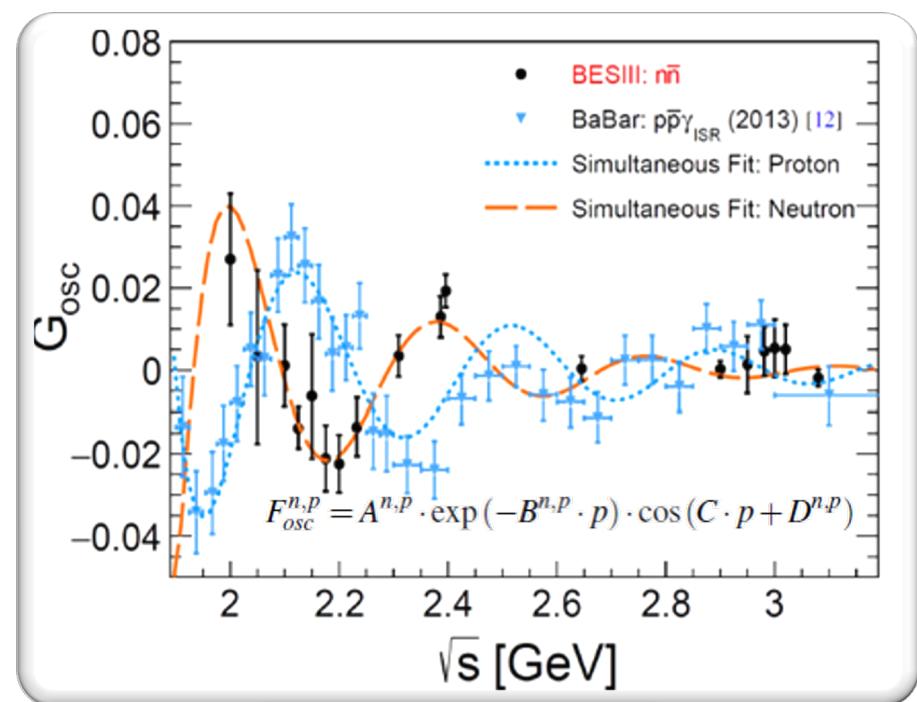
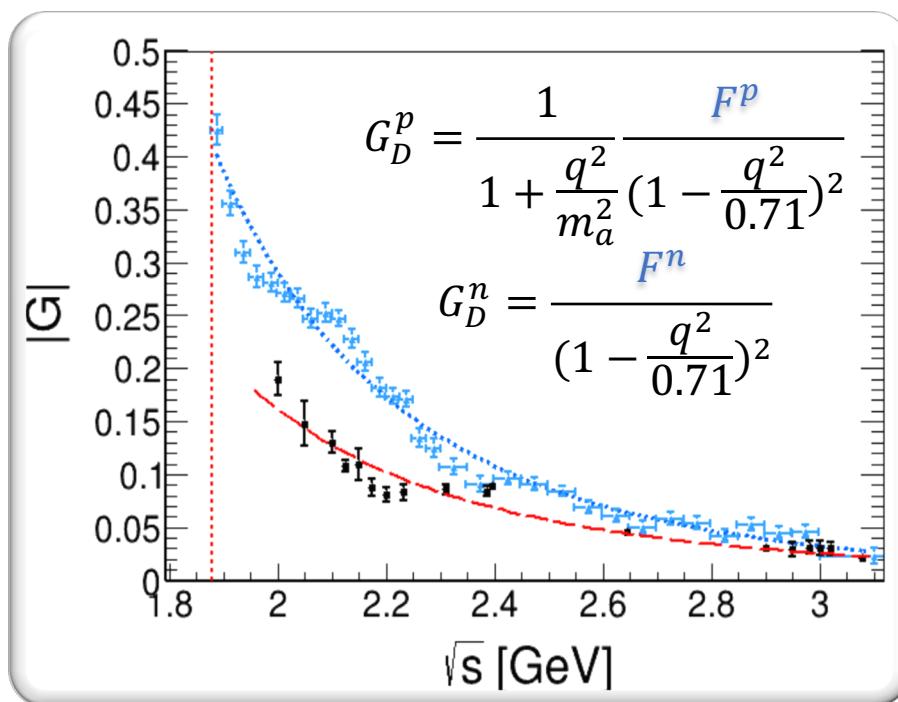
Nature Physics volume 17, pages 1200–1204 (2021)



[8]: *Phys. Rev. D* 99, no. 1, 014510 (2019); [10]: *Z. Phys. C* 52, 631 (1991).; [11]: *Phys. Rept.* 112, 173420 (1984)

Oscillation in the Form Factors

- ✓ 3rd highlight of the results: oscillation around the dipole law.
- ✓ A phase shift around $(125 \pm 12)^\circ$ is observed.



PRC 93, 035201 (2016).

PRL 114, 232301 (2015).

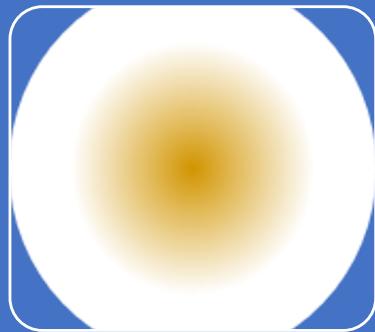
PRD 92, 198 034018 (2015).

$$G_{osc}(q^2) = |G| - G_D$$

Nobody predicted this

Understanding the Form Factor

Physics Reports 550–551 (2015) 1–103



Charge density distribution (Fourier transformation), or time-space revolution (generalized Fourier transformation).

Semi-classical: $F(q) = \int e^{iqr} \rho(r) d^3 r \xrightarrow{\text{proton}} F(q) \sim 1 - \frac{1}{6} q^2 \langle r_c^2 \rangle + O(q^4)$

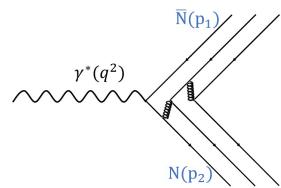
Generalized: $F(q^2) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} \int d^3 \vec{r} \rho(\vec{r}, t) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} \mathcal{Q}(t),$

$$G_D^p = \frac{1}{(1 + \frac{q^2}{m_a^2})} \times \frac{F^p}{(1 - \frac{q^2}{0.71})^2}$$

$$G_D^n = 1 \times \frac{F^n}{(1 - \frac{q^2}{0.71})^2}$$

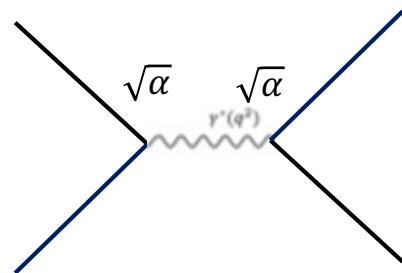
$$F[f(x) \otimes g(x)] = F[f(x)] * F[g(x)]$$

Understanding the Form Factor

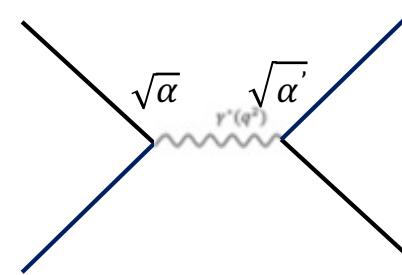


Quark-antiquark creation and transition to nucleon, and effective coupling

$$\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-} \propto \alpha^2$$



$$\sigma_{e^+ e^- \rightarrow n\bar{n}} \propto \alpha^2 |G|^2$$



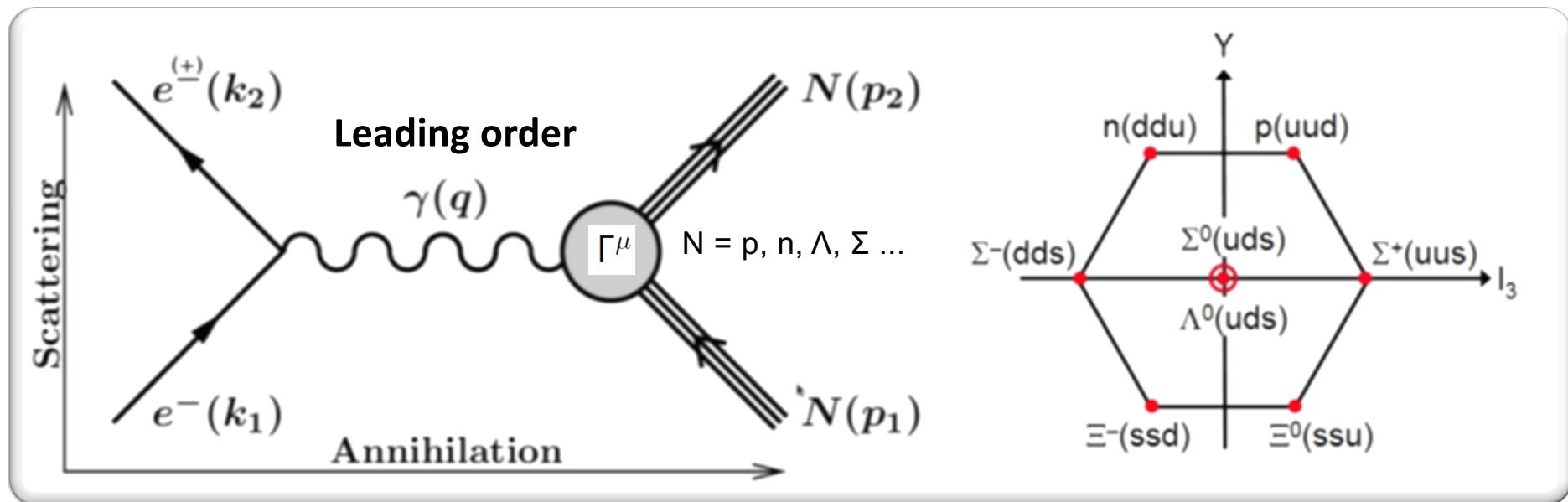
$$e' = \sqrt{|G|^2} e$$

$$e'(q^2 \rightarrow 4M_p^2) \sim 0.47 \sim \frac{1}{2} e \text{ for the proton}$$

$$e'(q^2 \rightarrow 4M_n^2) \sim 0.25 \sim \frac{1}{4} e \text{ for the neutron}$$

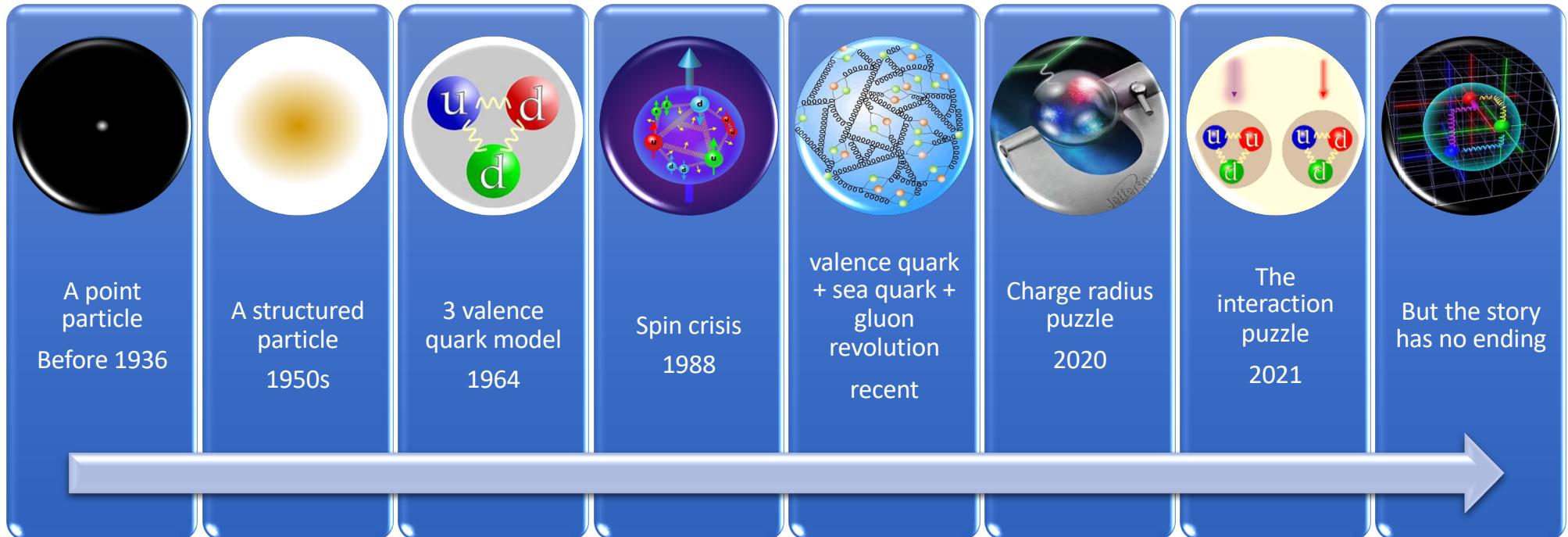
Understanding the Form Factor

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\kappa\sigma^{\mu\nu}q_\nu}{2M_B} F_2(q^2)$$



**Unique access to non-stable baryons
in the TL region at BESIII.**

A Story of the Nucleon Picture



BESIII achieved a mile stone in understanding the neutron.

From Idea to Fruit

Carrying out a typical research @BESIII

1. Proposal and
data-taking plan

Before 2012

2. Taking data
2012-2015

3. Data analysis,
publication
2016-2021

$$\langle i | H | f \rangle^{\textcolor{red}{2}} \rightarrow M_{\textit{easurment}}$$

Summary

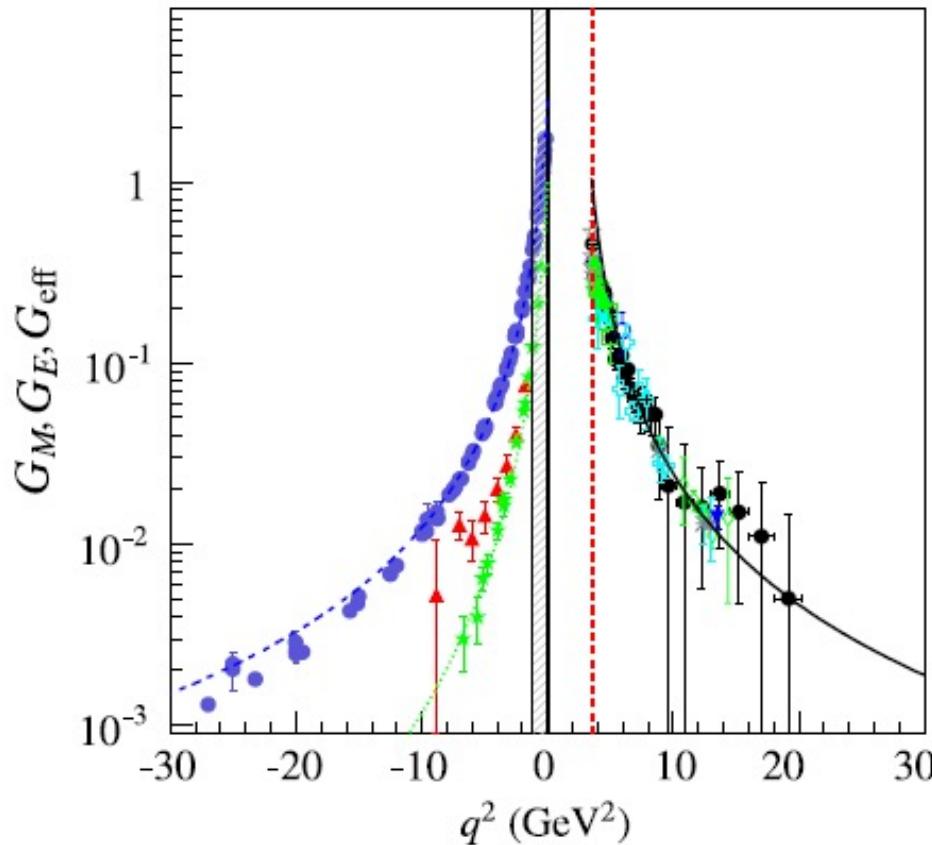
For the neutron time-like form factor, **BESIII collaboration**

1. released novel results with the best precision so far.
2. clarified the interaction puzzle persistent for over 20 years.
3. observed an oscillating behavior similar to the one in the proton.

Thanks for your attention

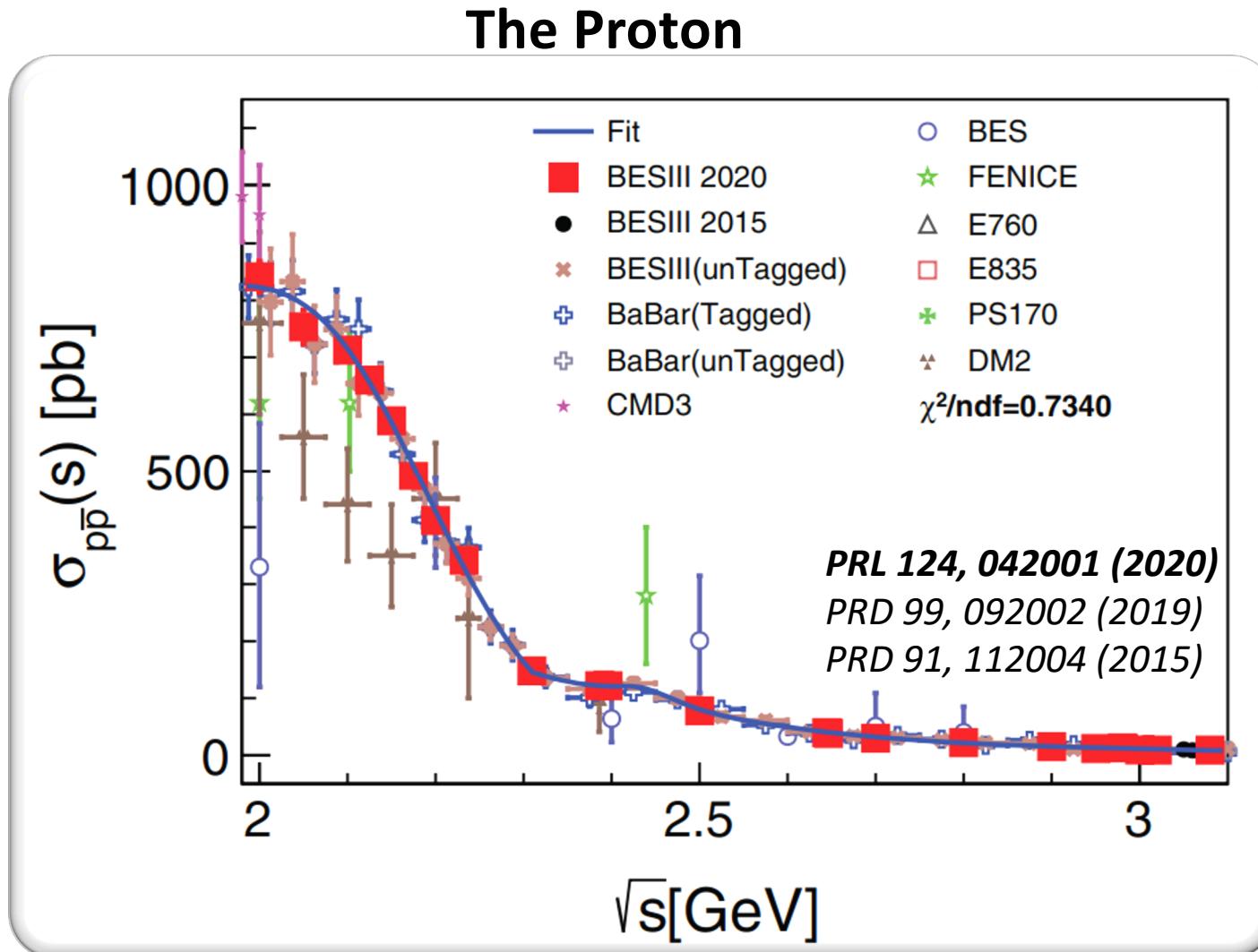
Acknowledgement to the owners because some pictures
are borrowed from the internet without prior information.

Appendix



point	$\rho(r) = \delta(r - r_o)$	$F(q^2) = 1$	unity
exponential	$\rho(r) = \frac{a^3}{8\pi} e^{-ar}$	$F(q^2) = \left[\frac{1}{1+q^2/a^2} \right]^2$	dipole
Yukawa	$\rho(r) = \frac{a^2}{4\pi r} e^{-ar}$	$F(q^2) = \frac{1}{1+q^2/a^2}$	pole
Gaussian	$\rho(r) = \left(\frac{a^2}{2\pi} \right)^{3/2} e^{-(a^2 r^2/2)}$	$F(q^2) = e^{-(q^2/2a^2)}$	Gaussian

The Proton Time-like Form Factor

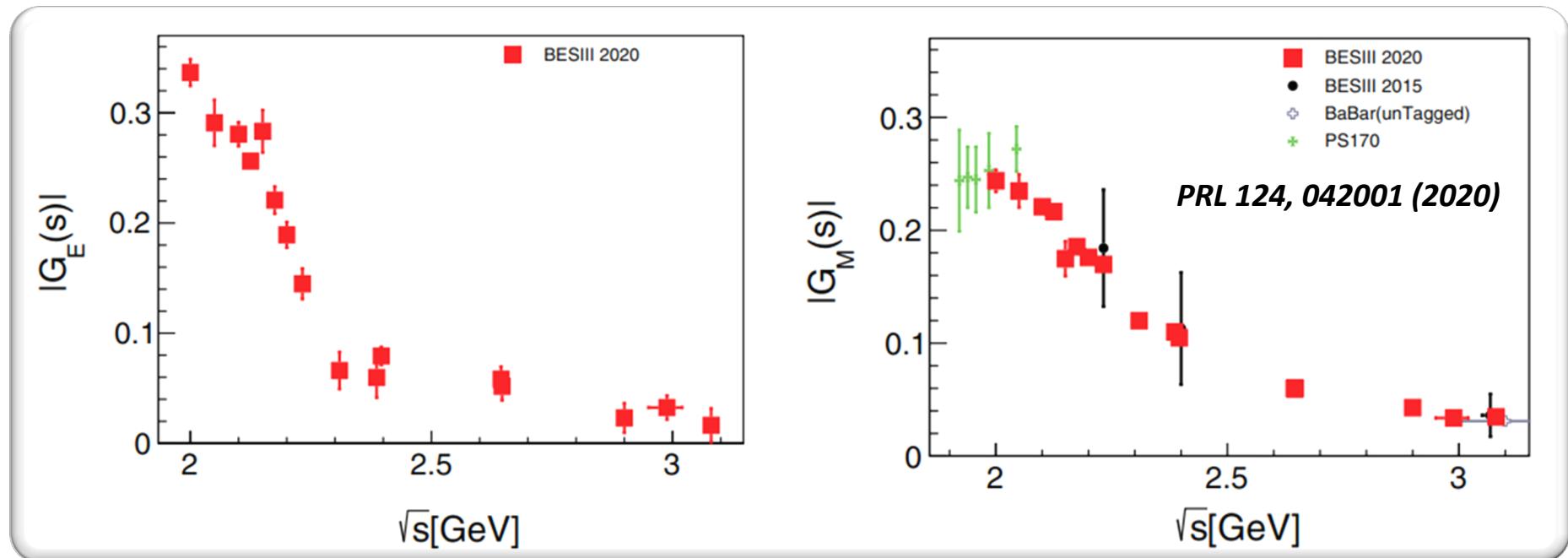


Most precise measurements in the time-like region

1st Precise Separation G_E from G_M

The Proton

$$\frac{d\sigma_{pp}}{d\Omega} = \frac{4\pi\alpha_{em}^2\beta C}{3q^2} [|G_M|^2(1 + \cos^2\theta) + \frac{4M_n^2}{q^2} |G_E|^2 \sin^2\theta]$$

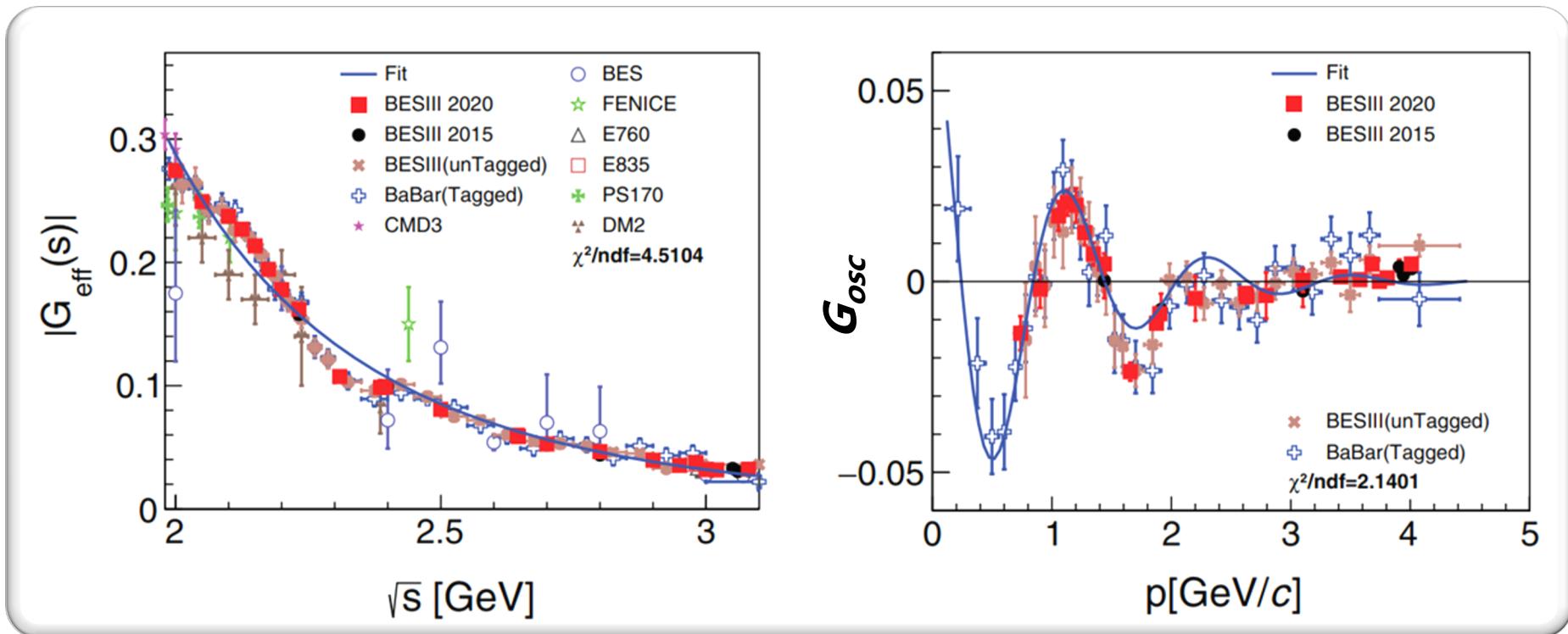


Precision: $|G_E|$ 2-94%, $|G_M|$ 1.5-9%

the precision comparable to the one in spacelike region

Confirmation

The Proton



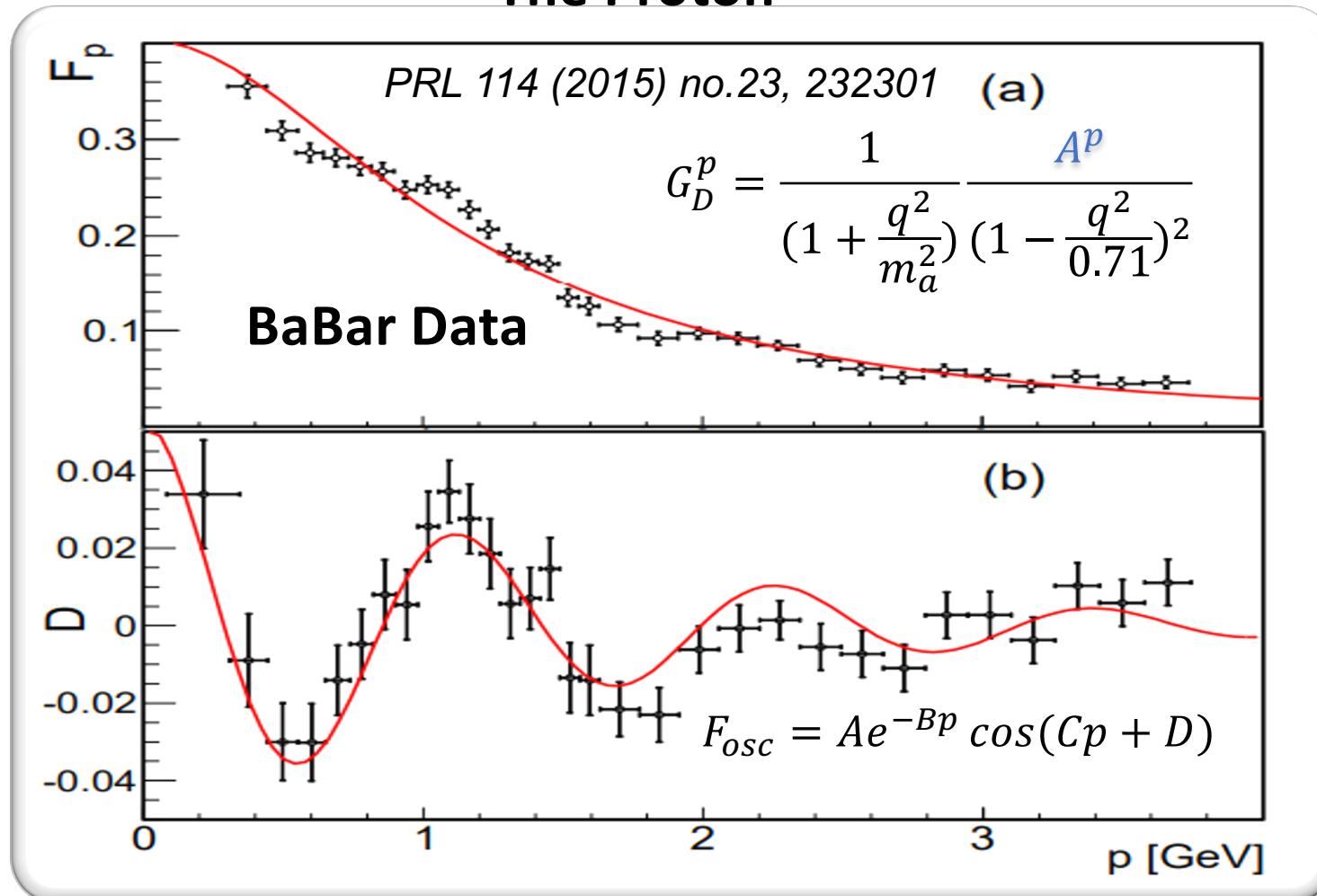
$$G_D^p = \frac{1}{(1 + \frac{q^2}{m_a^2})} \frac{F^p}{(1 - \frac{q^2}{0.71})^2}$$

$$G_{\text{osc}}(q^2) = |G| - G_D$$

Oscillation exists and needs more understanding.

Periodic Interference Structures

The Proton



$$p \equiv \sqrt{E^2 - M^2}, E \equiv \frac{q^2}{2M} - M$$

Nobody predicted this