Impact of isovector pairing fluctuation on $0\nu\beta\beta$ decay in MR-CDFT

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CRD, X.Zhang, J.M.Yao, P.Ring, J.Meng. (2023), arXiv:2305.00742.

第二届“粤港澳”核物理论坛

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Double beta ($\beta\beta$) decay

* Two types of $\beta\beta$ decay:

✓ $2\nu\beta\beta$ decay

$(A, Z) \rightarrow (A, Z+2) + 2e^{-} + 2\bar{\nu}_e$

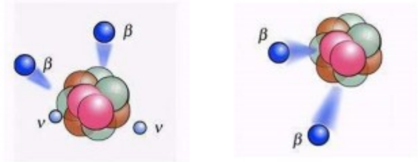
✓ $0\nu\beta\beta$ decay

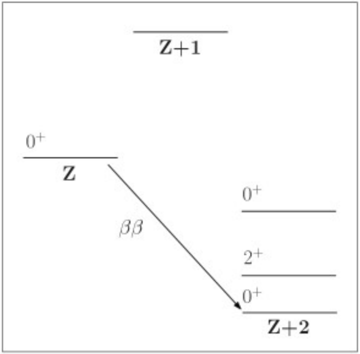
$(A, Z) \rightarrow (A, Z+2) + 2e^{-}$

* $\beta\beta$ decay occurs only when β decay is forbidden.

* $2\nu\beta\beta$ decay has been experimentally observed in a dozen of isotopes (^{40}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te, ^{130}Te...).

V.I.Tretyak, et al,At.Data Nucl.Data Tables 80(2002)83.





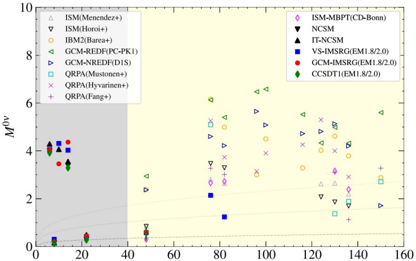
$0\nu\beta\beta$ decay and NMEs

Why search for $0\nu\beta\beta$ decay?

1. Lepton-number violation processes
2. Dirac or Majorana neutrinos
3. The absolute masses of neutrinos
4. The ordering of neutrinos masses

* Effective mass of neutrinos:

$$|\langle m_{\beta\beta}\rangle| = |\sum_{j=1}^{3} U_{ej}^2 m_j| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2}\right]^{1/2}$$

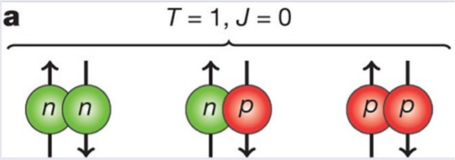


✓ Significant uncertainty in nuclear matrix elements (NMEs).

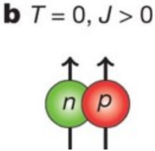
✓ How to reduce the discrepancy in NMEs among different models?

Pairing correlation

Nuclear pairing correlation

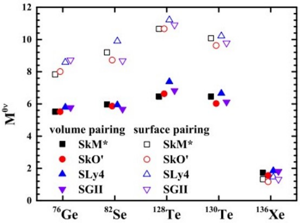


Isovector pairing

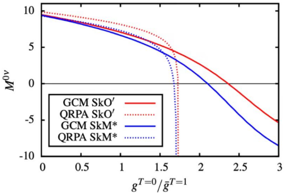


Isoscalar pairing

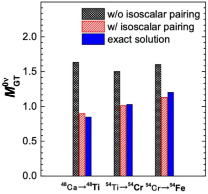
W.L.Lv et al.arXiv:2302.04423.



N. Hinohara & J. Engel, PRC90, 031301(R) (2014)



C. F. Jiao, J. Engel, and J. D. Holt, PRC96, 054310 (2017)



✓ Properly addressing pairing correlation is important in NMEs calculation.

MR-CDFT and pairing fluctuation

MR-CDFT:

J.M.Yao et al.PRC81:044311(2010).

CDFT → AMP+PNP → GCM

- ✓ Applies to nuclei of all mass regions, and transition to low-lying excitation states and the excitation energies can be calculated.

  J.M.Yao et al.PRC79:044312(2009). J.M.Yao et al.PRC81:044311(2010).

- ✓ Applies in the calculations of the NMEs of $0\nu\beta\beta$ decay for both the exchange of light neutrinos and heavy neutrinos mechanisms, with shape fluctuations.

  L.S.Song et al.PRC90:054309(2014). L.S.Song et al.PRC95:024305(2017).

Extended MR-CDFT in our work:

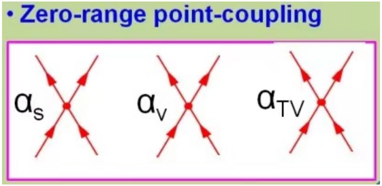
Including fluctuations in both quadrupole shapes and isovector pairing amplitudes.

The RMF+BCS theory

○ The mean-field wave functions $|\Phi(\mathbf{q})\rangle$ are generated from the (RMF+BCS) theory with constraints on both the mass quadrupole moment and pairing amplitude,

$$\langle\Phi|\hat{H}|\Phi\rangle = \langle\Phi|\hat{H}_0 - \sum_{\tau=n,p}\lambda_\tau \hat{N}|\Phi\rangle - \frac{1}{2}\lambda_Q\left(\langle\Phi|\hat{Q}_{20}|\Phi\rangle - q_{20}\right)^2 - \xi_p\left(\langle\Phi|\hat{P}_{T=1}|\Phi\rangle - P_1\right).$$

Here $\langle\Phi|\hat{H}_0|\Phi\rangle$ is given in the CDFT based on the *PC-PK*1 force.



$$\begin{aligned}\mathcal{L}_{point} =& \overline{\varphi}(i\gamma_\mu\partial^\mu - m)\varphi - e\overline{\varphi}\gamma^\mu\frac{1-\tau_3}{2}\varphi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\\ &- \frac{1}{2}\alpha_S(\overline{\varphi}\varphi)(\overline{\varphi}\varphi) - \frac{1}{2}\alpha_V(\overline{\varphi}\gamma_\mu\varphi)(\overline{\varphi}\gamma^\mu\varphi)\\ &- \frac{1}{2}\alpha_{TV}(\overline{\varphi}\vec{\tau}\gamma_\mu\varphi)(\overline{\varphi}\vec{\tau}\gamma^\mu\varphi) - \frac{1}{2}\delta_S\partial_\nu(\overline{\varphi}\varphi)\partial^\nu(\overline{\varphi}\varphi)\\ &- \frac{1}{2}\delta_V\partial_\nu(\overline{\varphi}\gamma_\mu\varphi)\partial^\nu(\overline{\varphi}\gamma^\mu\varphi) - \frac{1}{2}\delta_{TV}\partial_\nu(\overline{\varphi}\vec{\tau}\gamma_\mu\varphi)\partial^\nu(\overline{\varphi}\vec{\tau}\gamma^\mu\varphi)\\ &- \frac{1}{3}\beta_S(\overline{\varphi}\varphi)^3 - \frac{1}{4}\gamma_S(\overline{\varphi}\varphi)^4 - \frac{1}{4}\gamma_V[(\overline{\varphi}\gamma_\mu\varphi)(\overline{\varphi}\gamma^\mu\varphi)]^2.\end{aligned}$$

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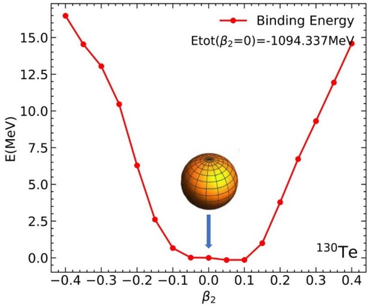
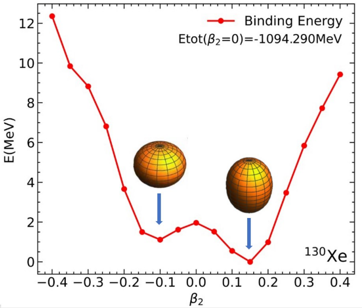
$$- \xi_p\left(\langle\Phi|\hat{P}_{T=1}|\Phi\rangle - P_1\right).$$

The second term generates mean-field states with different quadrupole deformations.





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The last constraint term generates mean-field states with different isovector pairing amplitudes by introducing the operator K.Sieja et al.Eur.Phys,J.A 20,413,2004.

$$\hat{P}_{T=1} = \frac{1}{2}\sum_{k>0}(c_k^\dagger c_{\bar{k}}^\dagger + c_{\bar{k}}c_k).$$

And with this constraints, the BCS equation

$$2(\epsilon_k - \lambda_F)v_k u_k + (f_k\Delta_k + \xi_p)(v_k^2 - u_k^2) = 0.$$

And the pairing gap $f_k\Delta_k$ is replaced by $f_k\Delta_k + \xi_p$.

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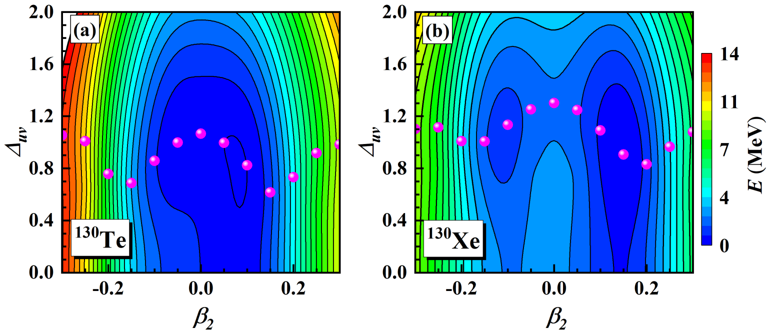


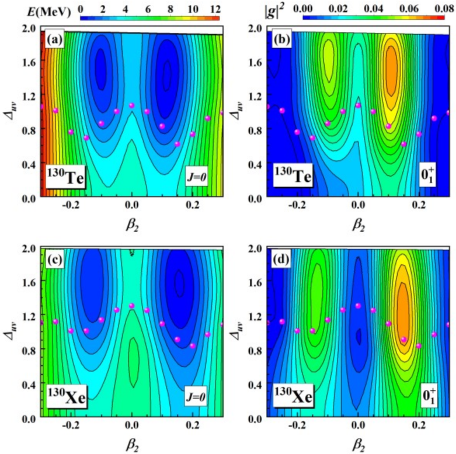
GCM and nuclear low-lying states

◯ The angular momentum projected and particle number projected basis function is constructed as

$$|JMNZ, \mathbf{q}\rangle = \hat{P}^J_{M0}\hat{P}^N\hat{P}^Z|\Phi(\mathbf{q})\rangle.$$

$J = 0$ potential energy surfaces

Collective ground state wave functions



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○ The collective wave functions of nuclear low-lying states within the GCM

$$|\Psi^{JMNZ}_\sigma\rangle = \sum_{\mathbf{q}} f^J_\sigma(\mathbf{q})|JMNZ, \mathbf{q}\rangle.$$

○ Through solving the HWG equation

P.Ring et al. *The nuclear many-body problem*, 1980.

$$\sum_{\mathbf{q}} \left[\mathcal{H}^J_{00}(\mathbf{q},\mathbf{q}') - E^J_\sigma \mathcal{N}^J_{00}(\mathbf{q},\mathbf{q}')\right] f^J_\sigma(\mathbf{q}') = 0, \quad \begin{cases} \mathcal{N}^J_{00}(\mathbf{q},\mathbf{q}') = \langle\Phi(\mathbf{q})|\hat{P}^J_{M0}\hat{P}^N\hat{P}^Z|\Phi(\mathbf{q}')\rangle \\ \mathcal{H}^J_{00}(\mathbf{q},\mathbf{q}') = \langle\Phi(\mathbf{q})|\hat{H}\hat{P}^J_{M0}\hat{P}^N\hat{P}^Z|\Phi(\mathbf{q}')\rangle \end{cases}$$

the weight functions $f^J_\sigma(\mathbf{q})$ and the energies of low-lying states E^J_σ are obtained.

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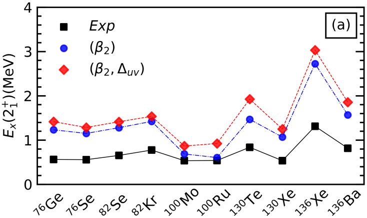
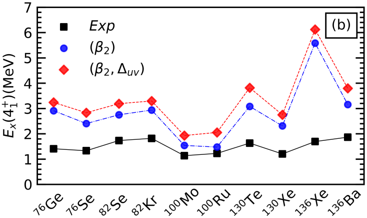




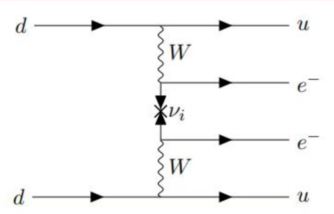
Transition operator and NMEs

* The half-life of $0\nu\beta\beta$ decay: $[T^{0\nu}_{1/2}]^{-1} = G_{0\nu} g_A^4 \eta_\alpha^2 |M^{0\nu}_\alpha|^2, \quad M^{0\nu}_\alpha = \langle \Psi_F | \hat{O}^{0\nu}_\alpha | \Psi_I \rangle.$

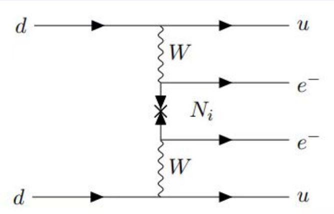
* In mechanism of exchanging either light $(\alpha = \nu)$ or heavy $(\alpha = N)$ Majorana neutrinos, the $0\nu\beta\beta$ decay operator

L.S.Song et al.PRC95:024305(2017).

$$\hat{O}^{0\nu}_\alpha = \frac{4\pi R}{g_A^2} \int\int d^3x_1 d^3x_2 \int \frac{d^3q}{(2\pi)^3} h_\alpha(q) \hat{\mathcal{J}}^\dagger_\mu(\mathbf{x}_1) \hat{\mathcal{J}}^{\mu\dagger}(\mathbf{x}_2) e^{i\mathbf{q}\cdot(\mathbf{x}_1-\mathbf{x}_2)}.$$



✓ $\eta_\nu = \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right| = \left|\frac{\sum_{\nu_j=1}^{3} U^2_{e\nu_j} m_{\nu_j}}{m_e}\right|$



✓ $\eta_N = \left|\sum_{N_j=1}^{3} \frac{U^2_{eN_j} m_p}{M_{N_j}}\right|$

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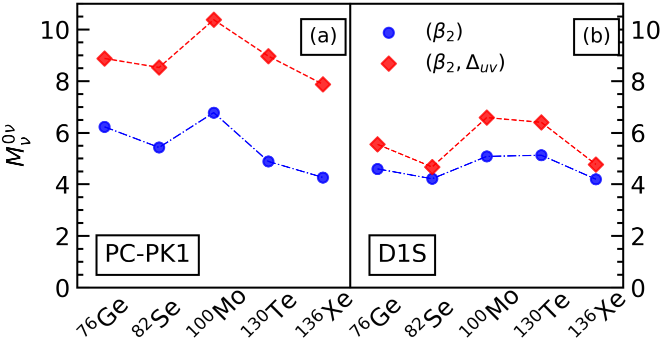


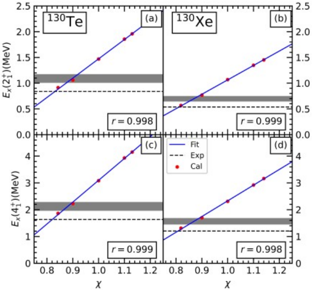
N.L.Vaquero et al.PRL.,2013,111:142501.

* According to the latest experimental measurement, the upper limit of effective mass of neutrinos: S.Abe et al.(KamLAND-Zen Collaboration) PRL.,2023,130:051801.

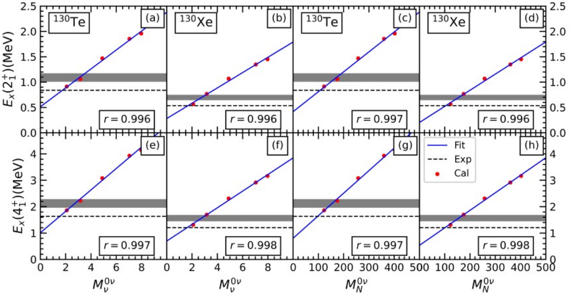
$$|\langle m_{\beta\beta} \rangle| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T^{0\nu}_{1/2} |M^{0\nu}_\nu|^2} \right]^{1/2} < 22.5 \text{ (meV)}$$

Correlation between NMEs and low-lying states

* The pairing field: $V_\tau^{pp}(\mathbf{r}_1, \mathbf{r}_2) = \chi V_\tau^{pp} \delta(\mathbf{r}_1 - \mathbf{r}_2)$.



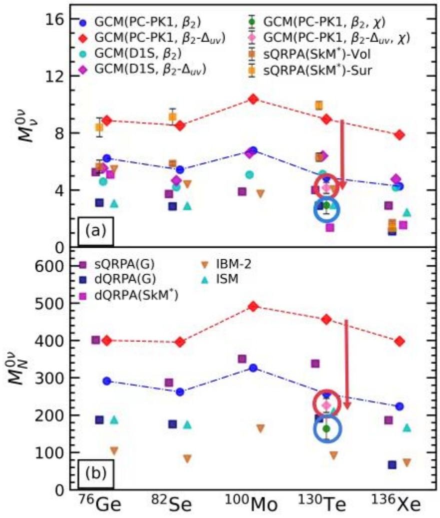
Correlation between pairing factor χ and the excitation energies



Correlation between nuclear matrix elements $M_{\nu/N}^{0\nu}$ of $0\nu\beta\beta$ decay and the excitation energies

Through the linear correlation, the pairing strength can be readjusted by the the excitation energies of 2_1^+ and 4_1^+ states, where the effect of the cranking states is estimated. M.Borrajo et al.PLB.,2015,746.341.

Comparison of nuclear matrix elements



✓ Multiplied by the scaling factor χ, the pseudo data of NMEs $M_\nu^{0\nu} = 4.16(0.39)$ and $M_N^{0\nu} = 226(19)$ are obtained.

✓ The pseudo NMEs are about 54% and 50% smaller than that without the factor.

Summary and outlook

- ✓ We have extended the MR-CDFT for nuclear low-lying states and the NMEs of $0\nu\beta\beta$ decay by including the isovector pairing fluctuations.
- ✓ The inclusion of isovector pairing fluctuation stretches the low-lying energy spectra and enhances the NMEs by about 40%-80%.
- ✓ Using the linear correlation between the excitation energies of low-lying states and the NMEs, when evaluating the effect of cranking states, the NMEs can be significantly reduced.
- ◯ A strict calculation considering the cranking states?
- ◯ Including the effects of isoscalar pairing in MR-CDFT?
- ◯ The effect of fluctuations in higher-order deformations? (triaxial deformation...)

THANK YOU FOR YOUR ATTENTION!

NME distribution of $0\nu\beta\beta$ decay

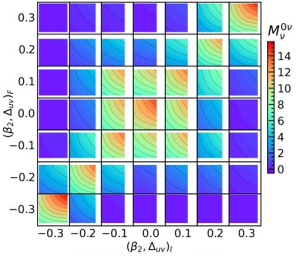
* With the nuclear wave functions constructed by GCM, the NME can be written as

$$M_{\alpha}^{0\nu} = \sum_{\mathbf{q}_I,\mathbf{q}_F} f_1^{0_F^+}(\mathbf{q}_F) f_1^{0_I^+}(\mathbf{q}_I)\sqrt{\mathcal{N}_{00}^{J=0}(\mathbf{q}_I,\mathbf{q}_I)\mathcal{N}_{00}^{J=0}(\mathbf{q}_F,\mathbf{q}_F)} \times \tilde{M}_{\alpha}^{0\nu}(\mathbf{q}_F,\mathbf{q}_I),$$

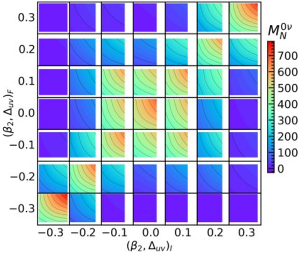
and the normalized NME defined as

$$\tilde{M}_{\alpha}^{0\nu}(\mathbf{q}_F,\mathbf{q}_I) = \frac{\langle \Phi_F(\mathbf{q}_F)|\hat{O}_{\alpha}^{0\nu}\hat{P}^{J=0}\hat{P}^N\hat{P}^Z|\Phi_I(\mathbf{q}_I)\rangle}{\sqrt{\mathcal{N}_{00}^{J=0}(\mathbf{q}_I,\mathbf{q}_I)\mathcal{N}_{00}^{J=0}(\mathbf{q}_F,\mathbf{q}_F)}},$$

which gives the distribution of NME for different collective parameters $\mathbf{q}$.



(a) Light neutrino exchange NMEs



(b) Heavy neutrino exchange NMEs

Numerical Details

a. The Dirac equation is solved by expanding the wave functions in the three-dimensional harmonic oscillator basis with 10 major shells. The point-coupling type of relativistic effective force PC-PK1 is adopted.

b. Pairing correlations between nucleons are treated with the BCS approximation using a density independent δ force. The pairing strength parameter V_τ^{pp} is -314.550 MeV fm^3 and -346.500 MeV fm^3 for neutrons and protons, respectively.

c. The generator coordinates are chosen in the interval of $\beta_2 \in [-0.3, 0.3]$ with a step size $\Delta\beta_2 = 0.1$ for ^{76}Ge, ^{82}Se and $\Delta\beta_2 = 0.05$ for ^{100}Mo, ^{130}Te, ^{136}Xe. The pairing parameters are chosen in the interval of $\delta \in [-0.2, 0.4]$ with a step size $\Delta\delta = 0.2$.

d. The CD Bonn is used as the parametrization for the Jastrow SRC function. The NMEs for both light neutrino exchange and heavy neutrino exchange are calculated.