基于新普适性关系和天文观测精确约束中子星性质

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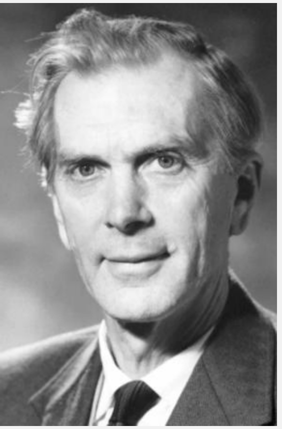
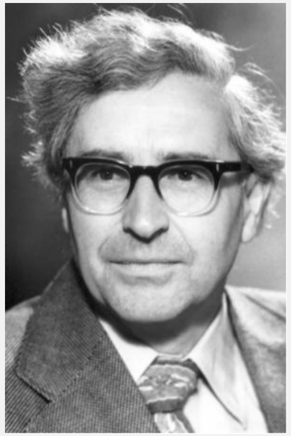
目录

1974 年诺贝尔物理学奖



Martin Ryle



Antony Hewish

Dame Jocelyn
Bell Burnell

中子星是大质量主序星在演化末期形成的致密天体，是20世纪60年代天文学的四大发现之一。中子星拥有地面实验难以实现的天然极端环境，因此借助NICER、LIGO、Virgo等大科学装置的天文观测，成为研究高密核物质性质的重要手段。

REPORTS

CORRECTED 9 MAY 2014

I-Love-Q: Unexpected Universal Relations for Neutron Stars and Quark Stars

Kent Yagi* and Nicolás Yunes

ries (*6*). Similarly, GWs from NS binary inspirals cannot be easily used to test general relativity (GR) because of EoS degeneracies (*7*, *8*). We here find a way to uniquely break these degeneracies through universal I-Love-Q relations between the reduced moment of inertia, $\bar{I}$; tidal Love number, $\bar{\lambda}^{(\mathrm{tid})}$; and quadrupole moment, $\bar{Q}$, that are essentially insensitive to the star's EoS (*9*).

Consider an isolated, slowly rotating NS or QS described by its mass, M_*; the magnitude of its spin angular momentum, J, and angular ve-

EOS-independent普适性关系

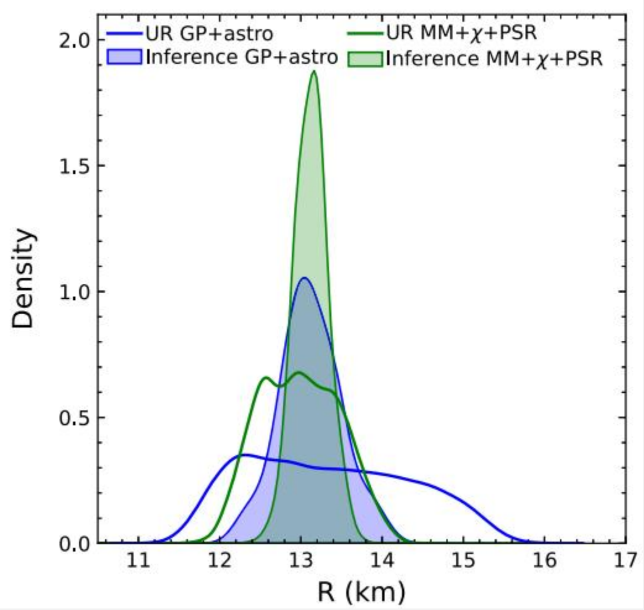
EOS： $P(\rho)$ $\xrightarrow{\text{determine}}$

- M (Mass)
- R (Radius)
- Λ (Tidal deformation)
- I (moment of inertia)
- oscillation mode (f-mode, g-mode,r-mode ...)

M, R, Λ: observed(X)

I, oscillation mode: unobserved(Y)

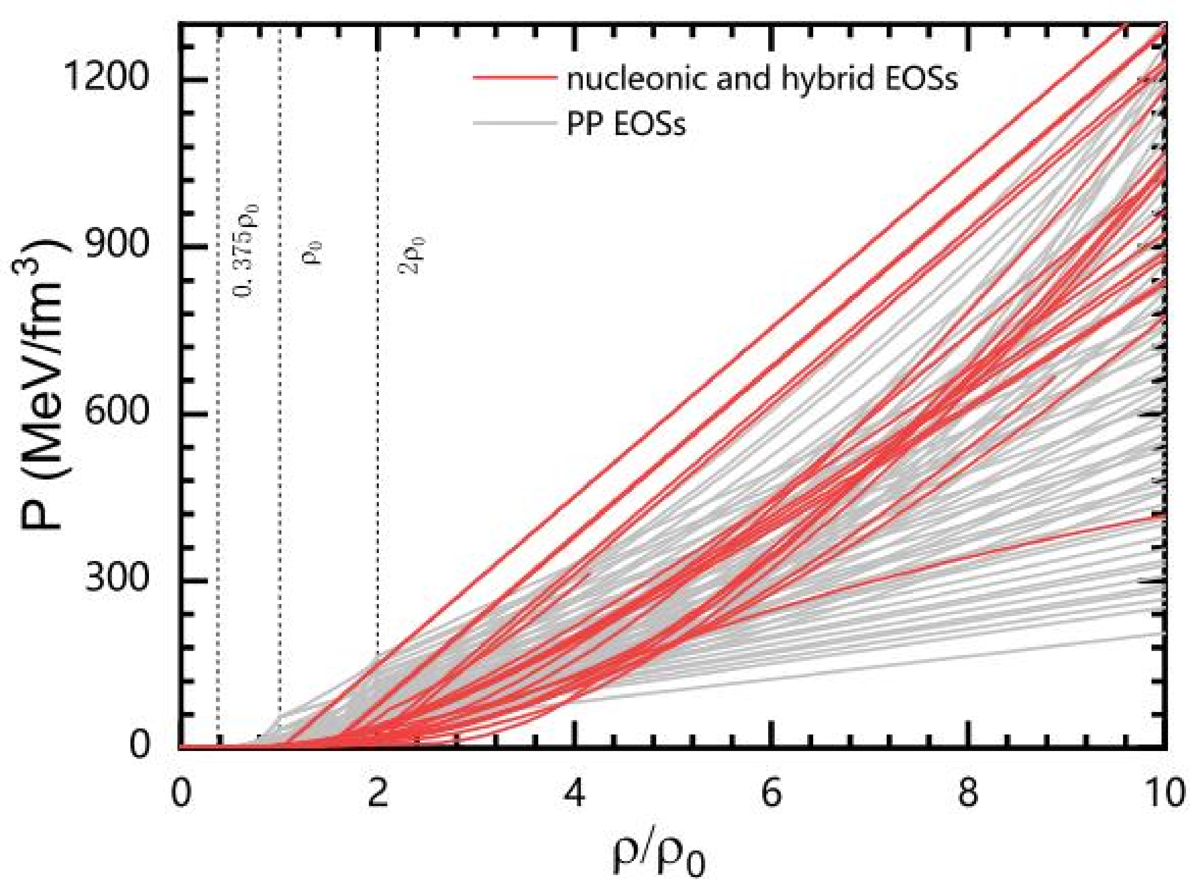
$$Y = F(X)$$



Suleiman PRD 2022

- 如何提高普适性关系的精度?
- 能否解释普适性关系建立的原因?

EOS模型



1.分段多方物态

(PP EOS)

$$P = K_i \rho^{\Gamma_i} \quad (\rho_i \leqslant \rho \leqslant \rho_{i+1})$$

$$K_i = \frac{P_i}{\rho_i^{\Gamma_i}}$$

$$\Gamma_i = \frac{\log_{10}(P_{i+1}/P_i)}{\log_{10}(\rho_{i+1}/\rho_i)}$$

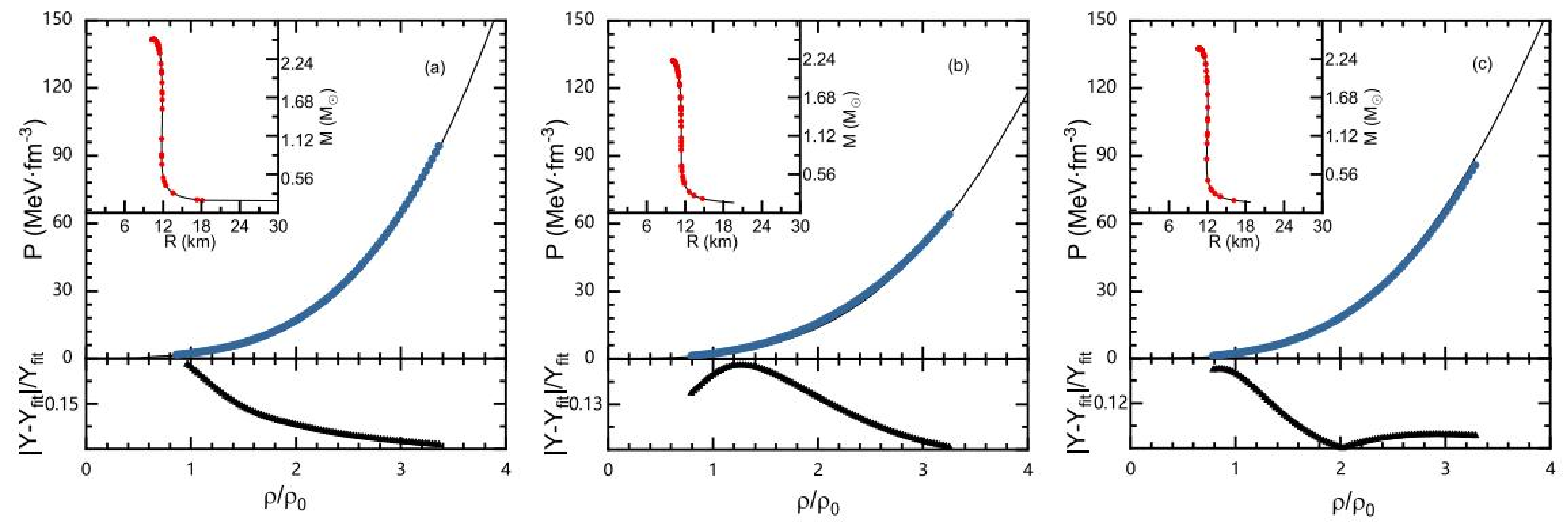
2.中子星及混合星物态

(nucleonic and hybrid EOS)

APR, APR3, APR4, ALF2, ENG, DDLZ1, DD-ME1, DD-ME2, MPA1, PKA1, PKO3, SLy, WFF1, WFF2, soft-EOS, stiff-EOS, APR3 + quark EOS, and DD-ME2 + quark EOS.

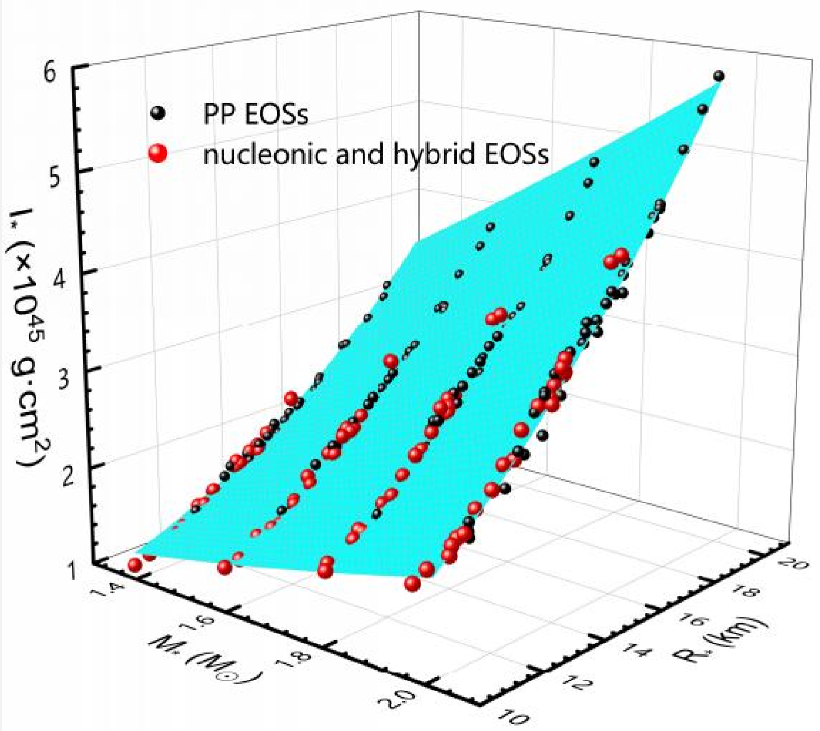
EOS-(in)dependent is equal to MR-(in)dependent

TOV方程: $$\frac{dP}{dr}=-\frac{G\rho m(r)}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}$$



Z.H. W & D.H.Wen CPC. 2024

转动惯量 $I_* - (M_*, R_*)$

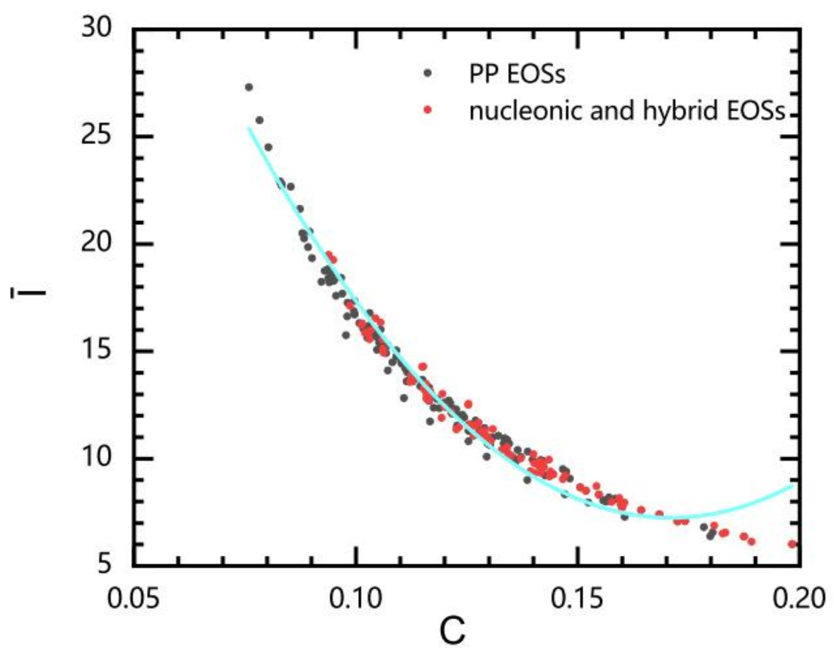


$$I_* = 4.417 - 2.696M_* - 0.513R_* + 0.427M_*^2 + 0.014R_*^2 + 0.249M_*R_*$$

其中$I_* = \frac{I}{10^{45}\ g \cdot cm^2}, M_* = \frac{M}{M_\odot}, R_* = \frac{R}{\mathrm{km}}$

$\bar{I} - C$

[Breu & Rezzolla, MNRAS. (2016)]



$$\bar{I} = 65.857 - 685.567C + 2004.727C^2$$

其中$\bar{I} = I/M^3$，$C = M/R$

误差比较：

	$I_* - (M_*, R_*)$	$\bar{I} - C$
Mean $\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$	0.019	0.774

$$\bar{I} - C$$

$$\bar{\mathbf{I}} = \mathbf{65.857} - \mathbf{685.567}\mathbf{C} + \mathbf{2004.727}\mathbf{C}^2$$

$$\Downarrow$$

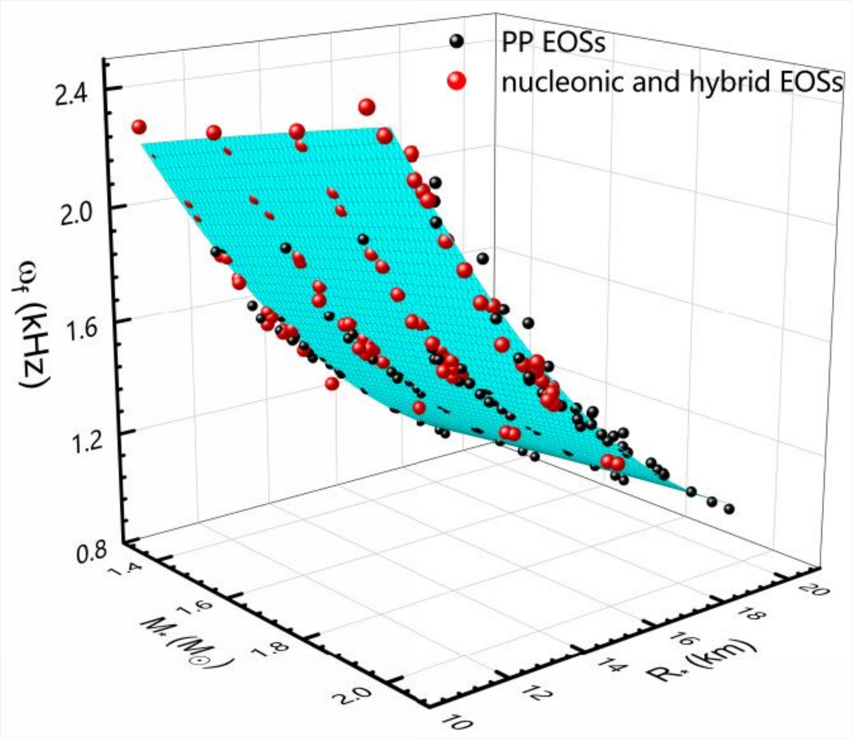
$$\bar{I} = \frac{I(G/c^2)}{(M(G/c^2))} = 65.857 - 685.567\frac{M}{R} + 2004.727\left(\frac{M}{R}\right)^2$$

$$I = [65.857 - 685.567MR^{-1} + 2004.727M^2R^{-2}]\frac{M^3}{23}$$

$$I = 2.863M^3 - 29.807M^4R^{-1} + 87.162M^5R^{-2}$$

C=M/R的形式导致难以存在R单独项

$f-mode$频率 $\omega_f-(M_*,\ R_*)$

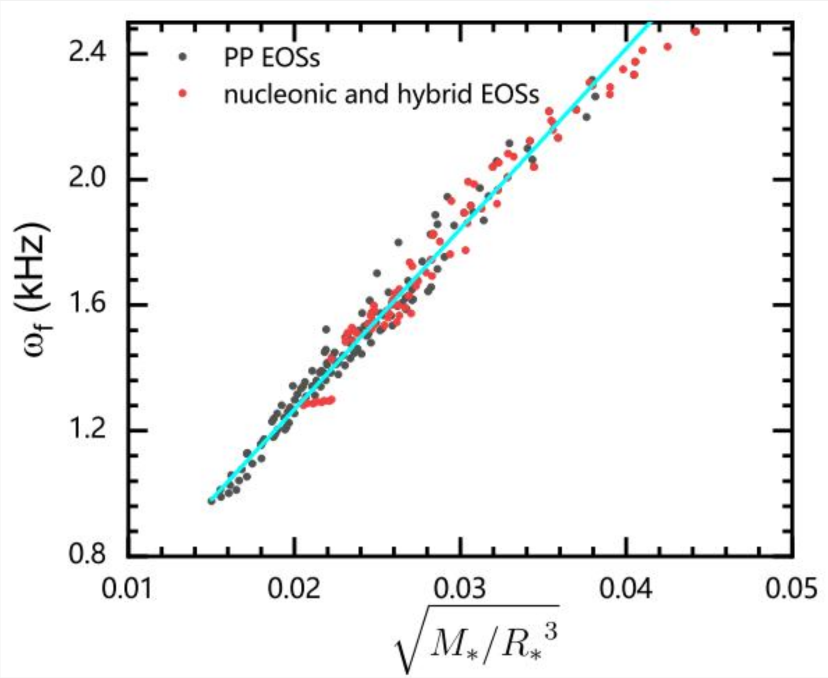


$$\omega_f(\mathrm{kHz})=4.66+0.838M_*-0.416R_*-0.0471M_*^{\,2}+0.0106R_*^{\,2}-0.0267M_*R_*$$

其中$M_*=\dfrac{M}{M_\odot},R_*=\dfrac{R}{\mathrm{km}}$

$\omega_f-\sqrt{M_*/R_*^{\,3}}$

[Andersson & Kokkotas, MNRAS (1998)]

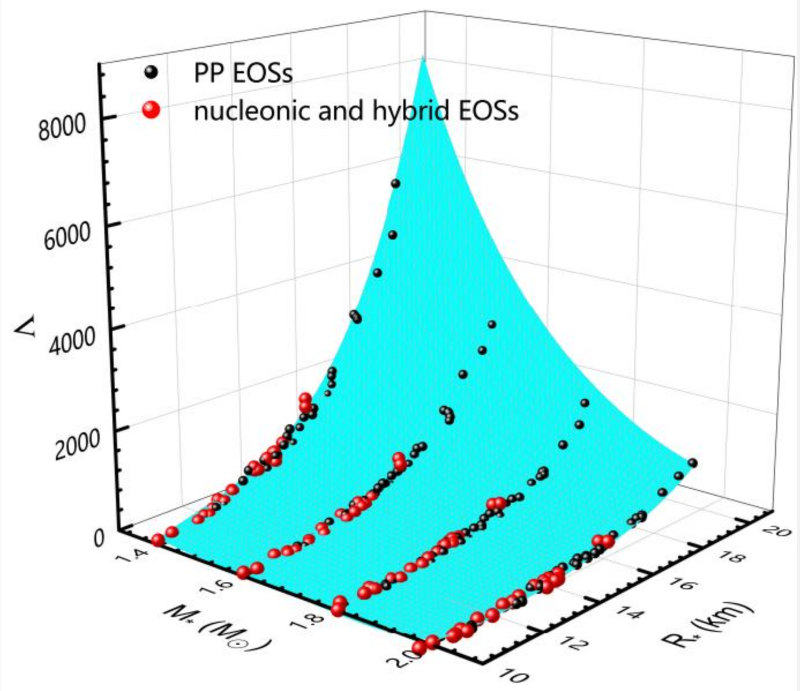


$$\omega_f(\mathrm{kHz})=0.118+57.478\sqrt{M_*/R_*^{\,3}}$$

误差比较：

	$\omega_f-(M_*,\ R_*)$	$\omega_f-\sqrt{M_*/R_*^{\,3}}$
Mean $\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)$	0.0698	0.106

潮汐形变 $\Lambda-(M_*, R_*)$

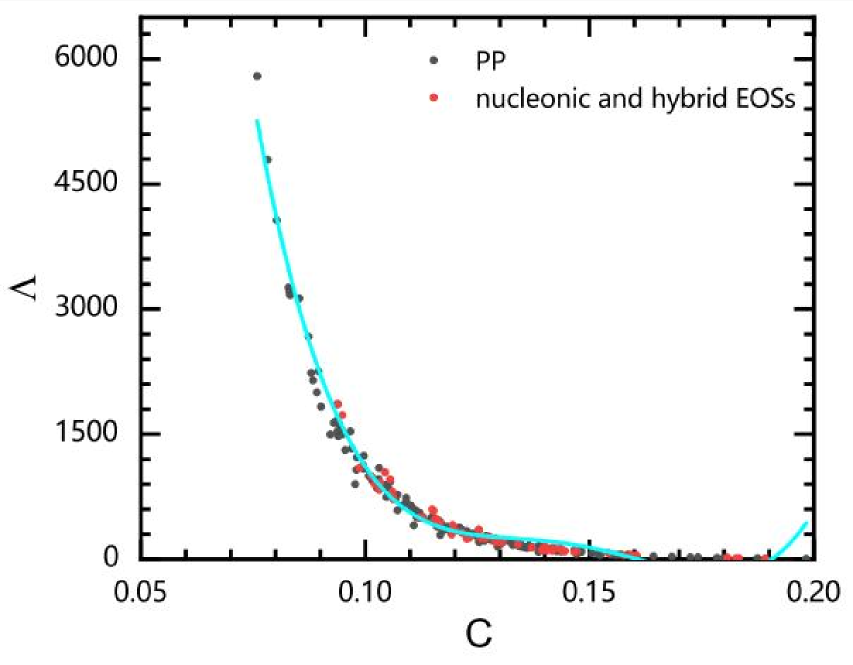


$$\Lambda = 108757.7751 - 299645.88054M_* + 4654.63671R_* - 5931.14445M_*R_* + 289044.79877M_*^2$$
$$-80.98134R_*^2 - 127741.20618M_*^3 + 18.16444R_*^3 + 5789.65511M_*^2R_* - 342.04328M_*R_*^2$$
$$+23162.31504M_*^4 + 0.35154R_*^4 + 295.79953M_*^2R_*^2 - 2493.99688M_*^3R_* - 18.85851M_*R_*^3$$

其中$M_* = \dfrac{M}{M_\odot}, R_* = \dfrac{R}{\text{km}}$

$\Lambda - C$

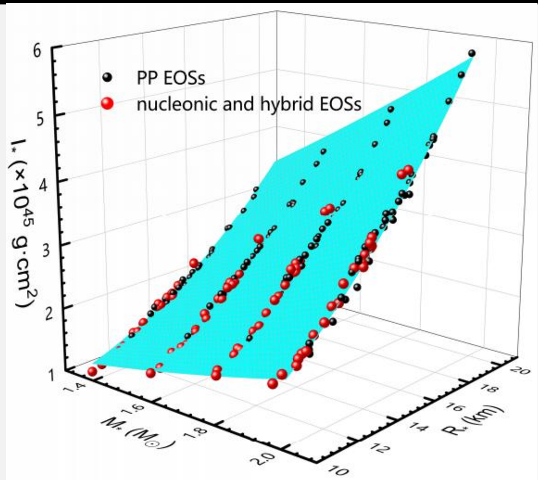
[Maselli et.al., PRD (2013).]

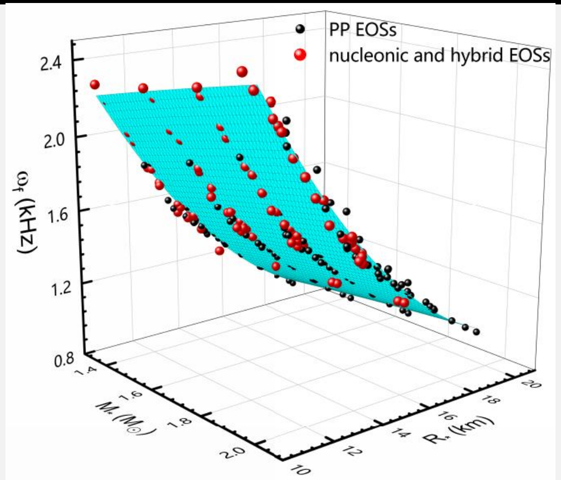


$$\Lambda = 1.009\times10^5 - 2.786\times10^6C + 2.878\times10^7C^2 - 1.312\times10^8C^3 + 2.223\times10^8C^4$$

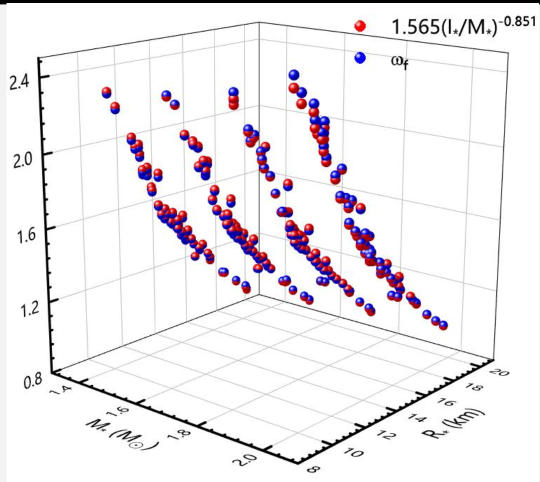
误差比较：

	$\Lambda-(M_*, R_*)$	$\Lambda-C$
Mean $\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$	0.0798	0.213

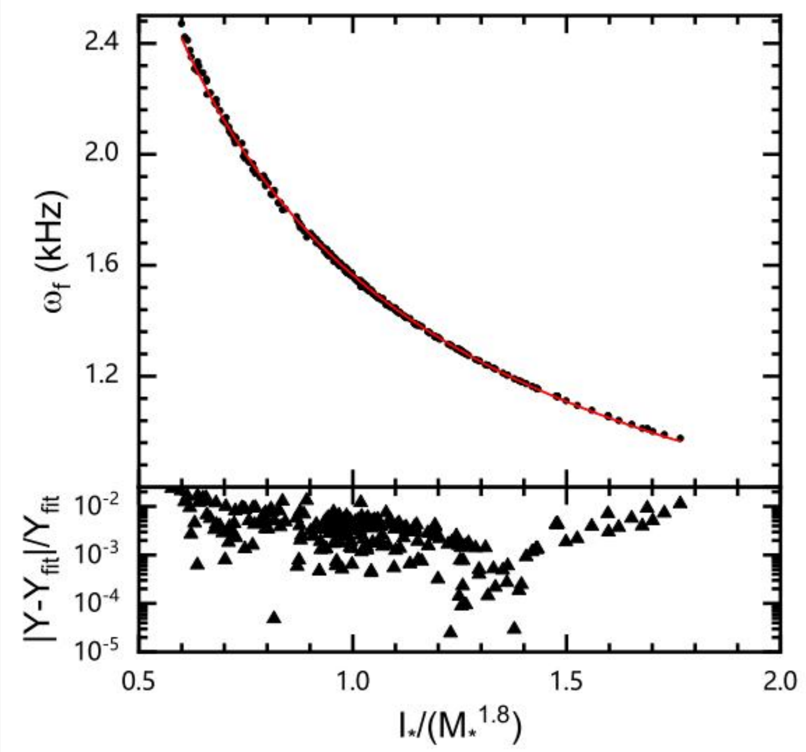




$\xrightarrow{3D\ affine\ transformation}$



基于MR投影建立新普适性关系



$$\omega_f\,(\mathrm{kHz}) = 1.565\,(I_*/M_*^{1.8})^{-0.851}$$

最大相对误差：2%

平均相对误差：0.37%

多元多项式除法

建立普适性关系就是建立函数式$Y=F(X)$，当我们把 $X(M_*, R_*)$ 和 $Y(M_*, R_*)$ 视作二变量的多项式，就可以利用多元多项式除法进行求解。

$$Y(M_*, R_*)=d(M_*, R_*)X(M_*, R_*)+r(M_*, R_*)$$

其中 $d(M_*, R_*)$ 为商，$r(M_*, R_*)$为余数。

对于I_*和ω_f有 $\omega_f(\mathrm{kHz})=d(M_*, R_*)I_*+r(M_*, R_*)$

其中

$$d(M_*, R_*)=-\frac{471}{4270}$$

$$r(M_*, R_*)=\frac{327}{247000}M_*R_*+\frac{577111}{1067500}M_*+\frac{463}{38125}R_*-\frac{2017943}{4270000}R_*+\frac{3139801}{610000}$$

在静态中子星条件下，分别建立了转动惯量，f-mode频率，潮汐形变与质量半径的普适性关系。我们发现：

- 质量半径投影建立的普适性关系具有更高的精度
- 以 $\omega_f - I_*$ 为例，展示了高精度普适性关系建立的过程，提供了新的视角解释普适性关系的建立原因
- 理论上可以推广至其他条件。例如转动条件下，将自旋频率对质量半径的改变视作额外添加的投影维度(即M_* ,R_*, Ω_*)，或视作MR轴的坐标变换。

结合NICER天文观测结果的预测

	质量半径	预测		
		转动惯量 ($\times 10^{45}$ g·cm^2)	f-mode频率 (kHz)	无量纲潮汐形变
PSR J0030+0451	1.44±0.15 $M_\odot$ 11.2-13.3 km [Miller et al. APJL 2021]	0.980 - 3.408	1.296 - 2.111	284.713 - 2833.076
PSR J0740+6620	2.08±0.07 $M_\odot$ 12.2-16.3 km [Riley et al. APJL 2021]	1.859 - 7.364	0.966 - 2.479	<1885.358

End