

基于中子-质子平衡约束核物质状态方程

——第三届“粤港澳”核物理会议



广东·深圳

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指导老师：苏军

■ 研究背景

核物质状态方程

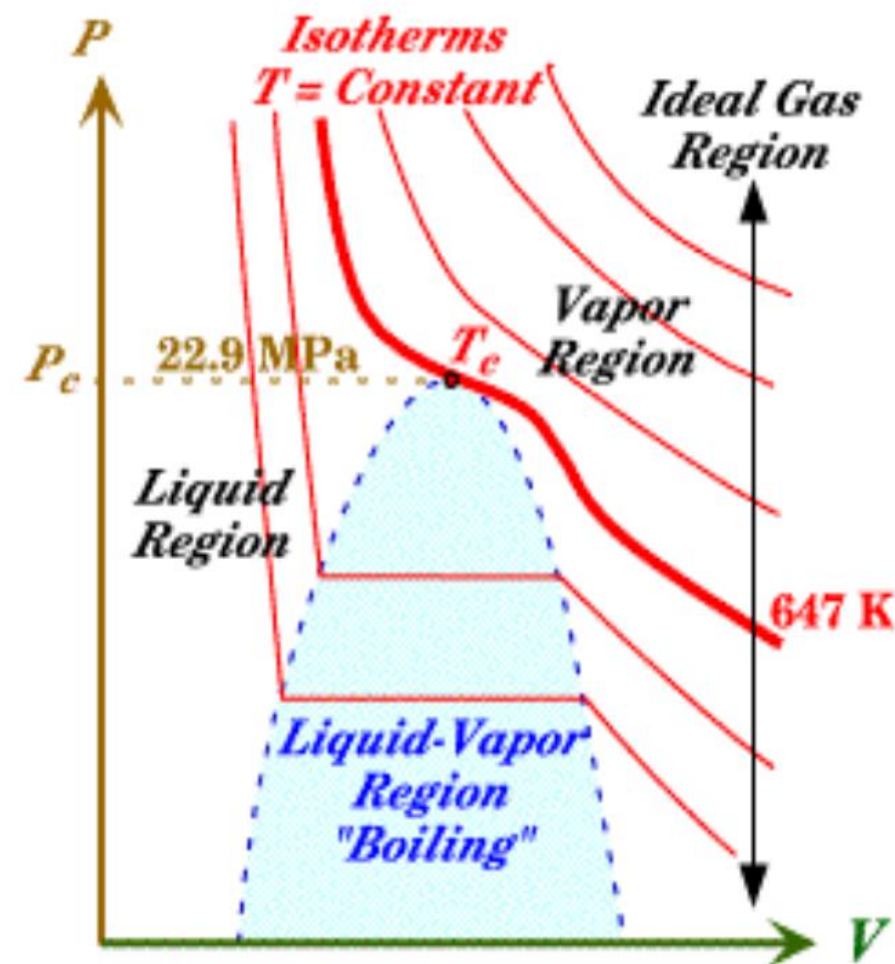
基于同位旋输运约束状态方程

■ 理论模型介绍

■ 结果与讨论

■ 总结

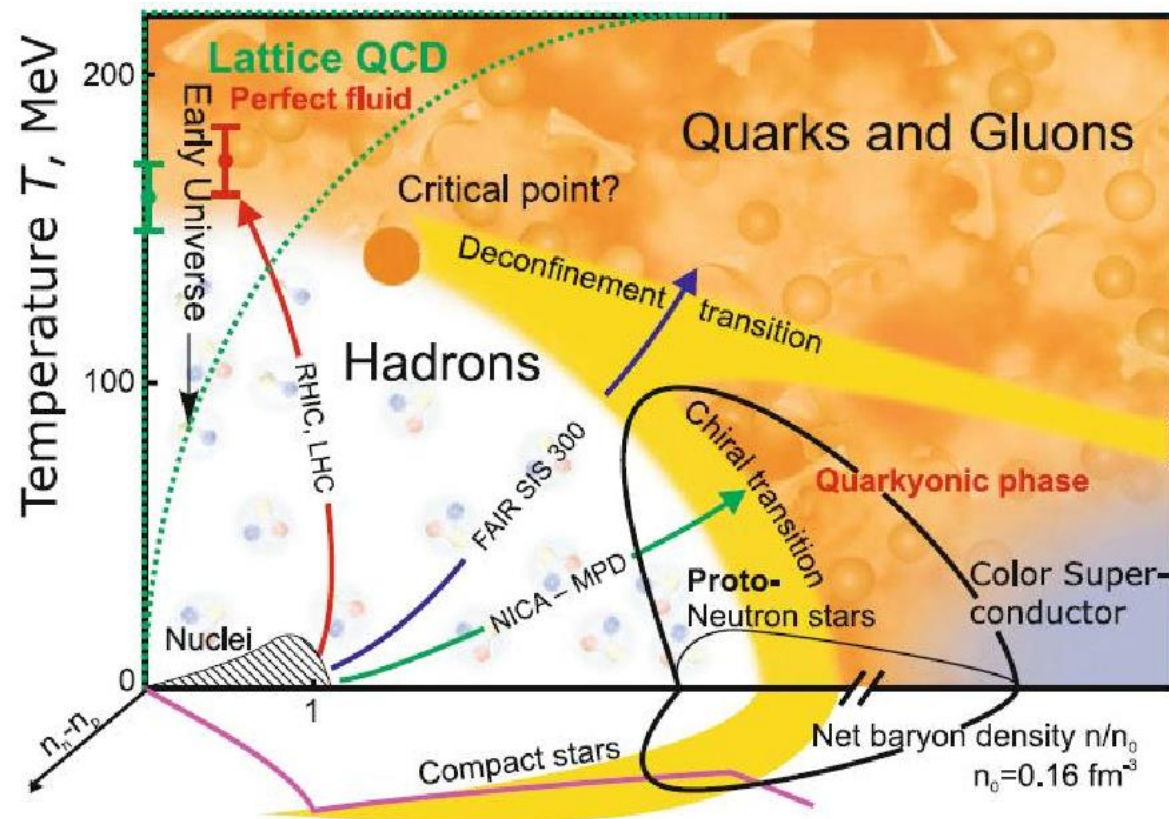
状态方程



van der Waals EOS:
$$\left[p + a \left(\frac{n}{V} \right)^2 \right] (V - nb) = nRT$$

↓
引力项

↓
斥力项



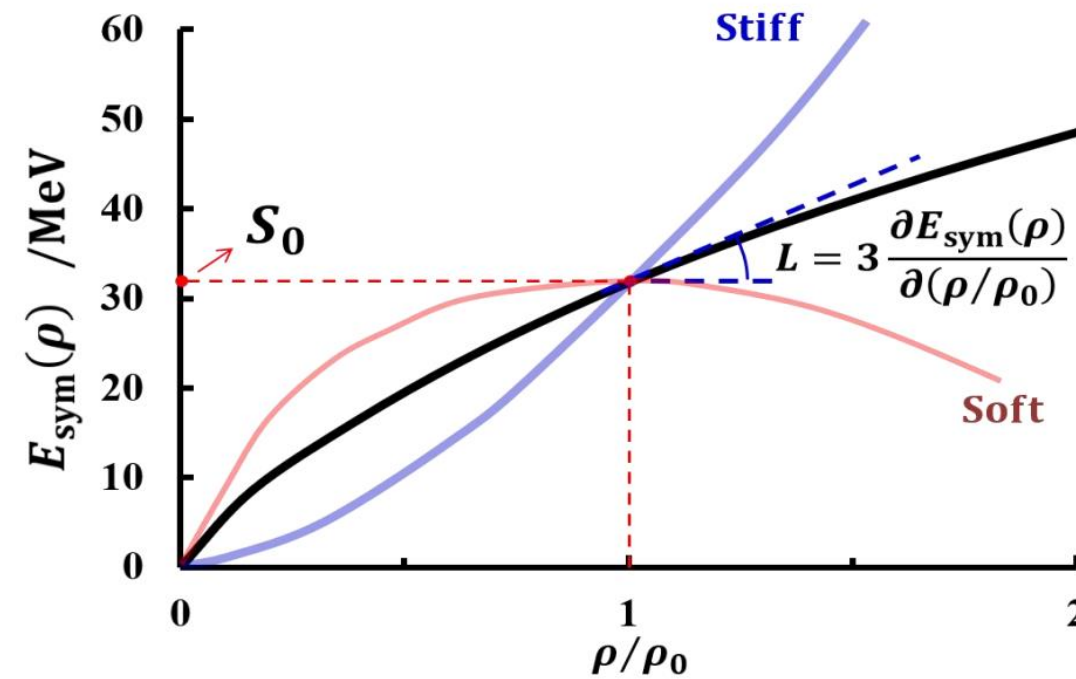
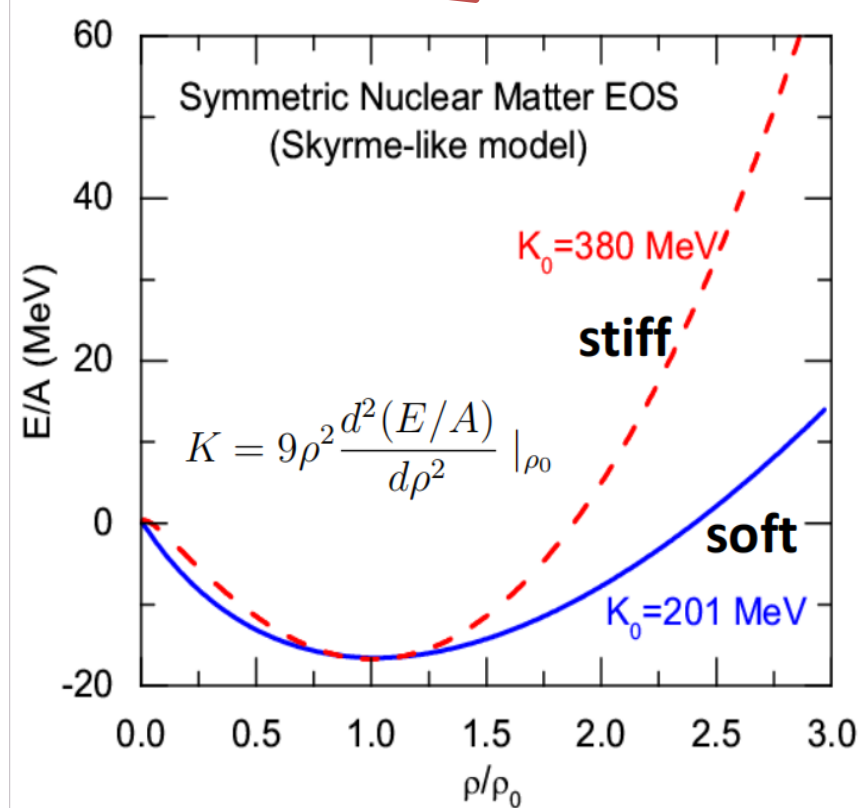
Fortov V E. Extreme states of matter : on Earth and in the cosmos.

核物质状态方程 → 核相互作用

核物质状态方程



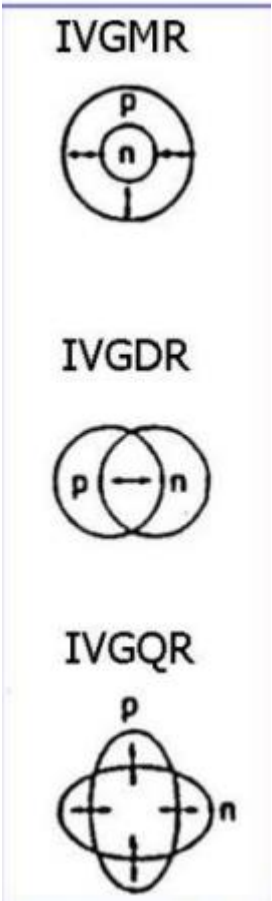
$$E/A(\rho, \delta) = E/A(\rho, 0) + \delta^2 \cdot S(\rho) ; \quad \delta = (\rho_n - \rho_p) / (\rho_n + \rho_p) = (N - Z) / A$$



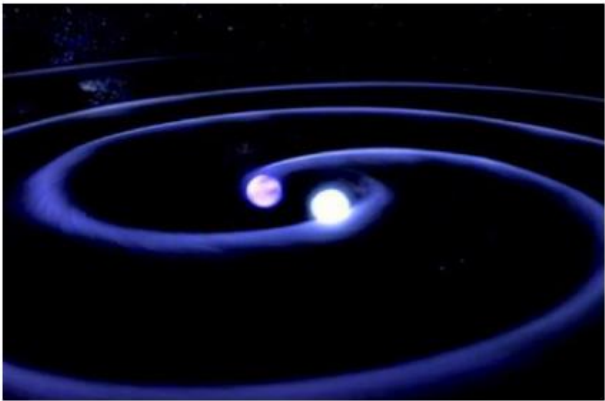
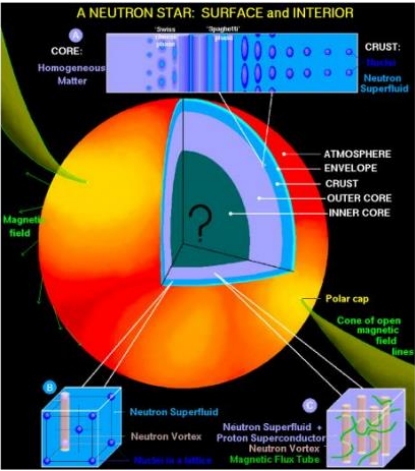
$$E/A(\rho) = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \rho \right)^{2/3} + \frac{\alpha \rho}{2\rho_0} + \frac{\beta \rho^\gamma}{(\gamma + 1)\rho_0^\gamma} + g_\tau \frac{\rho^\tau}{\rho_0^\tau}$$

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left(\frac{2}{3} \pi^2 \rho \right)^{2/3} + \frac{C_s}{2} \left(\frac{\rho}{\rho_0} \right)^{\gamma_i}$$

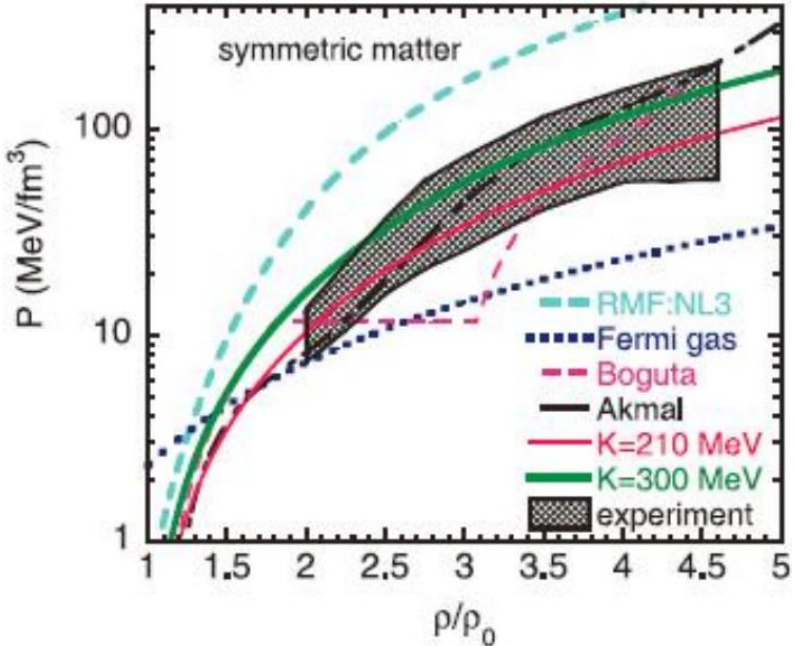
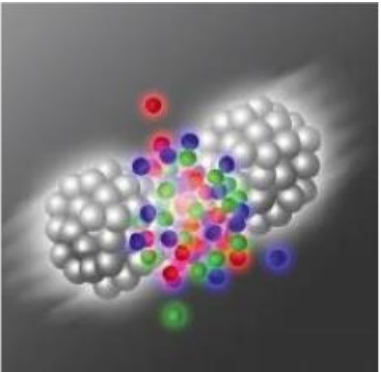
核结构



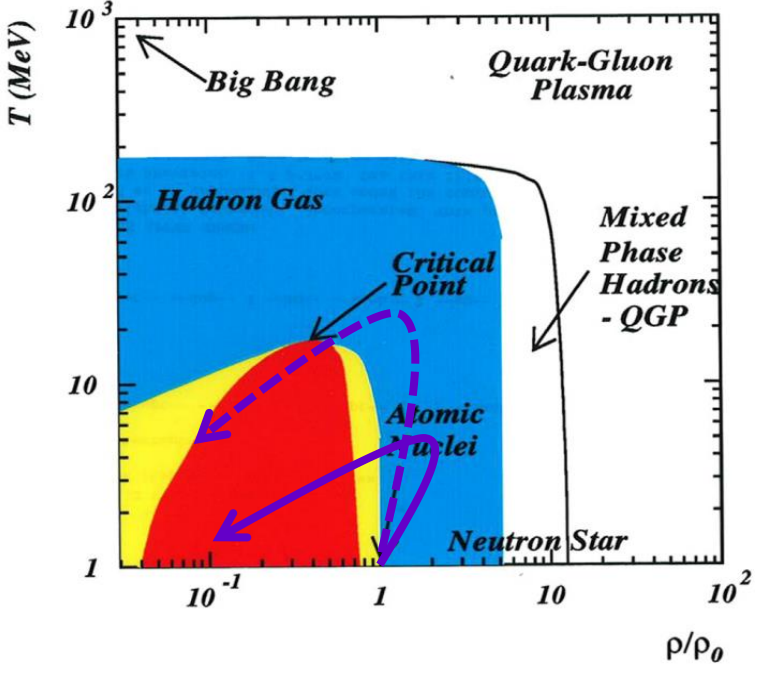
核天体



核反应



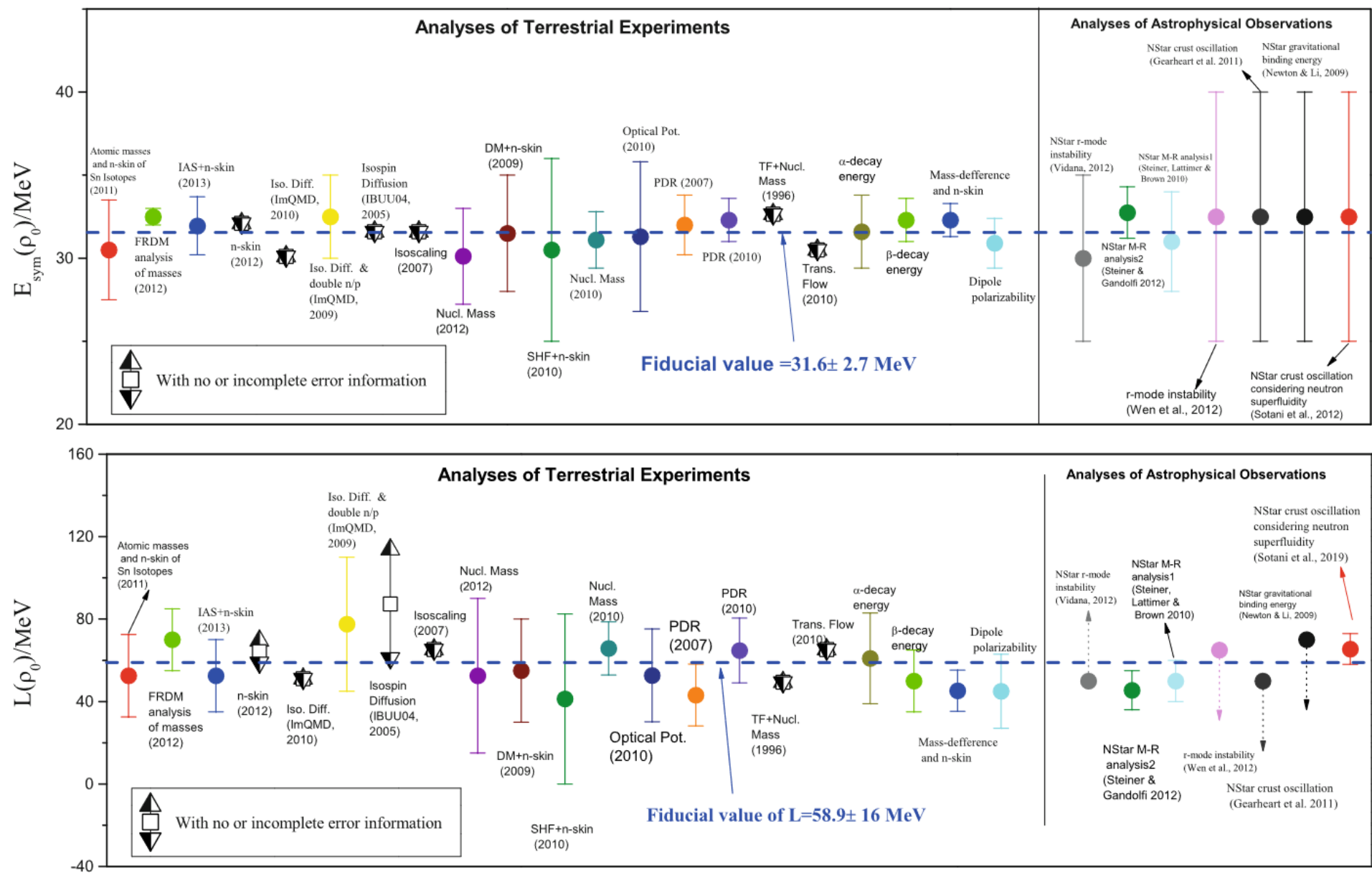
Science, 298,1592 (2002)



PPNP, 2019, 105: 82-138

核状态方程与核物理中的关键科学问题
紧密联系

对称能密度依赖——核结构、反应、天体

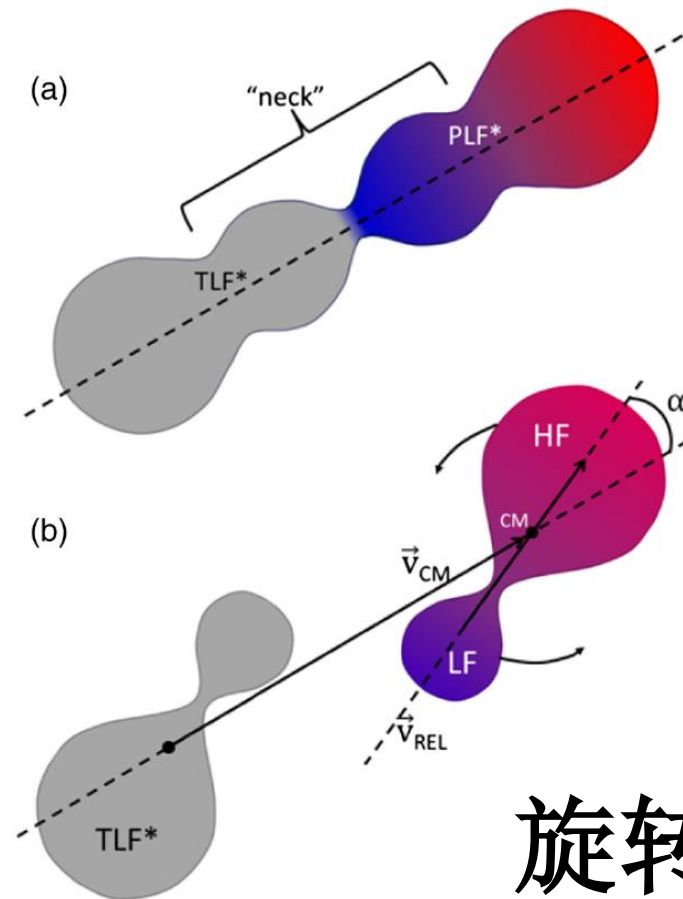


Eur. Phys. J. A (2019) 55: 117

对称能约束的不确定度很大

二元破裂反应中的中子-质子平衡

$^{70}\text{Zn} + ^{70}\text{Zn}$ @ 35A MeV by the K500 cyclotron at the Texas A&M University

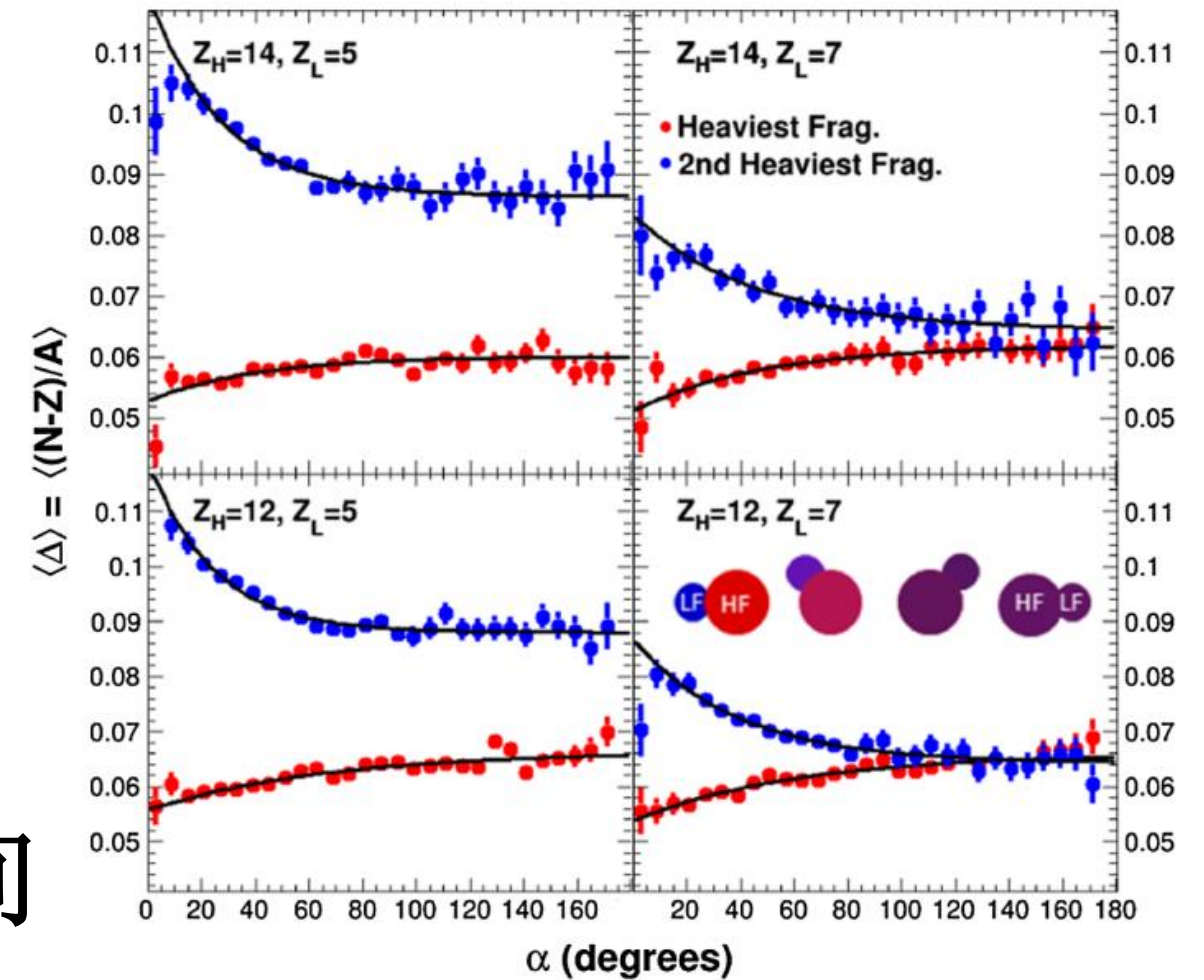


旋转角 \longrightarrow 时间

$$\alpha = \arccos \frac{\vec{v}_{cm} \cdot \vec{v}_{rel}}{|\vec{v}_{cm}| |\vec{v}_{rel}|}$$

$$\vec{v}_{cm} = (m_H \vec{v}_H + m_L \vec{v}_L) / (m_H + m_L)$$

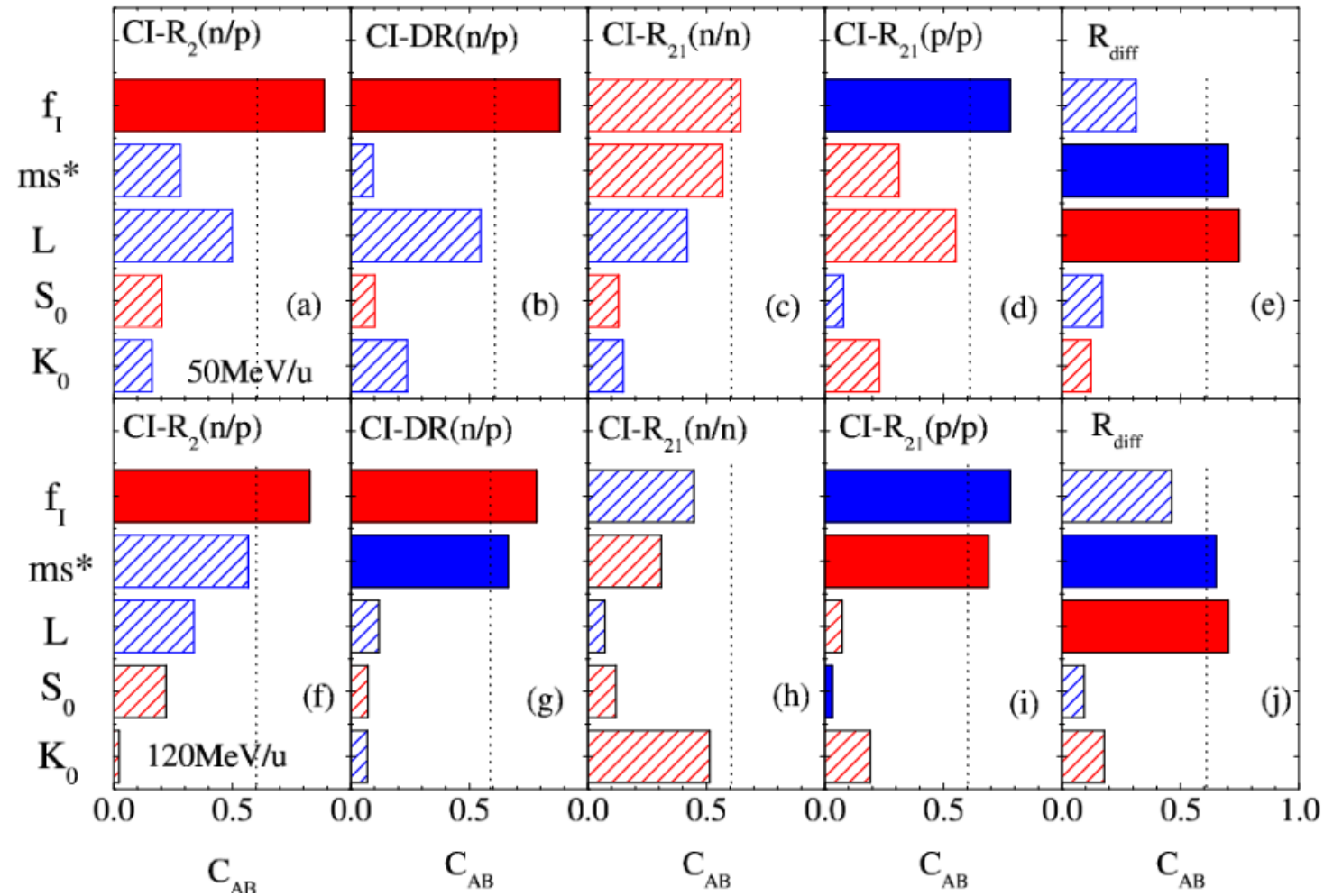
$$\vec{v}_{rel} = (\vec{v}_H - \vec{v}_L)$$



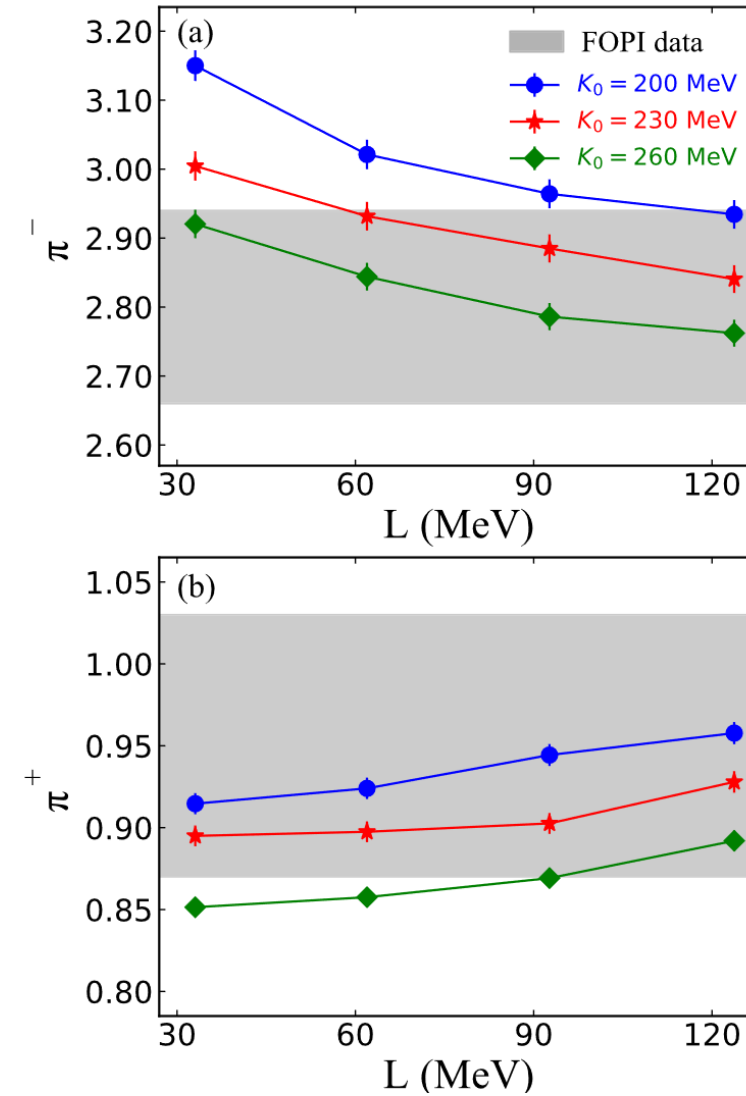
Phys. Rev. Lett. 118, 062501 (2017);
Phys. Rev. C 95, 044604 (2017)

拟合: $(N - Z)/A = a + b \exp(-c\alpha)$

EOS参数关联性



PLB 749 (2015) 262–266



PRC 109, 054619 (2024)

对称能密度依赖的约束需要考虑EOS参数关联性

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Isospin-dependent Quantum Molecular Dynamics model

N体薛定谔方程

$$i\hbar \frac{\partial}{\partial t} |\psi_{1\dots N}^{(k)}(t)\rangle = H_{1\dots N} |\psi_{1\dots N}^{(k)}(t)\rangle$$

高斯波包

$$\phi_i(\mathbf{r}, \mathbf{r}_i, \mathbf{p}_i, t) = \frac{1}{(2\pi L)^{3/4}} \exp\left[-\frac{(\mathbf{r} - \mathbf{r}_i(t))^2}{4L}\right] \exp\left[\frac{i\mathbf{p}_i(t) \cdot \mathbf{r}}{\hbar}\right]$$

直积

$$\Phi(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_N, \mathbf{p}_1, \dots, \mathbf{p}_N, t) = \prod_i \phi_i(\mathbf{r}, \mathbf{r}_i, \mathbf{p}_i, t)$$

魏格纳变换

$$f(\mathbf{r}, \mathbf{p}, t) = \sum_i \frac{1}{(\pi\hbar)^3} e^{-\frac{[\mathbf{r}-\mathbf{r}_i(t)]^2}{2L}} e^{-\frac{[\mathbf{p}-\mathbf{p}_i(t)]^2 \cdot 2L}{\hbar^2}}$$

广义变分原理

$$S = \int_{t_1}^{t_2} \mathbf{L}[\Phi, \Phi^*] dt \quad \mathbf{L} = \langle \Phi | i\hbar \frac{d}{dt} - H | \Phi \rangle$$

IQMD 模型

$$\dot{\mathbf{r}}_i = \frac{\partial \langle H \rangle}{\partial \mathbf{p}_i}$$

$$\dot{\mathbf{p}}_i = \frac{\partial \langle H \rangle}{\partial \mathbf{r}_i}$$

IQMD模型

基本相互作用:

$$\langle H \rangle = \langle T \rangle + \langle U^{nucl} \rangle + \langle U^{Coul} \rangle$$

$$\langle T \rangle = \int f(\mathbf{r}, \mathbf{p}, t) \frac{\mathbf{p}^2}{2m} d^3r d^3p = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + C$$

$$\langle U^{Coul} \rangle = \frac{e^2}{4} \sum_i \sum_{j, j \neq i} \frac{(1 - t_{zi})(1 - t_{zj})[1 - \text{erfc}(|\mathbf{r}_i - \mathbf{r}_j|/\sqrt{4L})]}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$$\begin{aligned} \langle U^{nucl} \rangle = & \frac{\alpha}{2} \sum_i \sum_{j, j \neq i} \frac{\rho_{ij}}{\rho_0} + \frac{\beta}{1 + \gamma} \sum_i \left(\sum_{j, j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^\gamma + g_\tau \sum_i \left(\sum_{j, j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^\eta \\ & + \frac{C_{sym}}{2} \sum_i \sum_{j, j \neq i} t_{zi} t_{zj} \frac{\rho_{ij}}{\rho_0} \\ & + \frac{g_{sur}}{2} \sum_i \sum_{j, j \neq i} \frac{\rho_{ij}}{\rho_0} \left[\frac{3}{2L} - \left(\frac{|\mathbf{r}_i - \mathbf{r}_j|}{2L} \right)^2 \right] \end{aligned}$$

相互作用
势能密度

$$\rho_{ij} = \frac{1}{(4\pi L)^{3/2}} \exp\left[-\frac{(\mathbf{r}_i - \mathbf{r}_j)^2}{4L}\right]$$

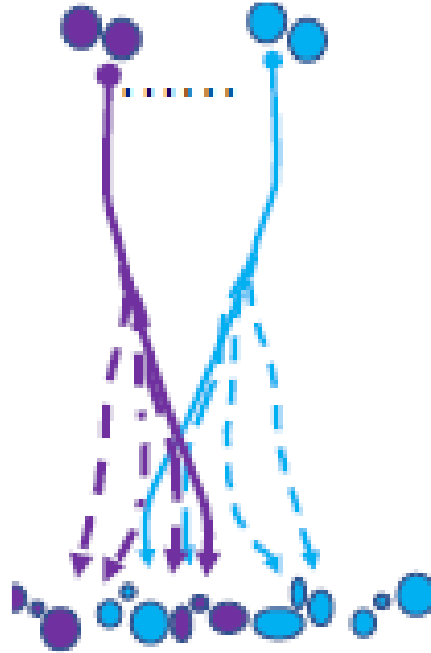
核子-核子碰撞

$$d \leq \sqrt{\sigma_{NN}/\pi}$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{el(inel)} = \sigma_{el(inel)}^{free} f_{el(inel)}^{med} f_{el(inel)}^{angl}$$

$$N + N \rightarrow N + N$$

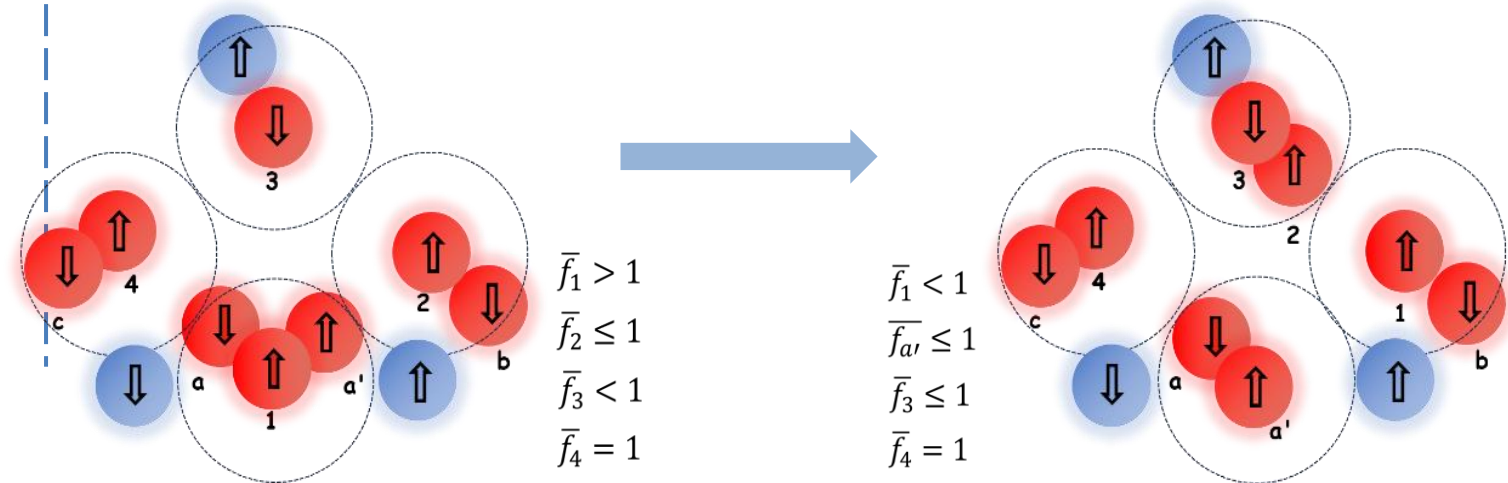
$$N + N \rightarrow N + \Delta, N + \Delta \rightarrow N + N$$



相空间约束方法(费米子属性改善)

Physical Review C 64(2): 024612

$$\bar{f}_i = 0.621 + \sum_{j \neq i}^N \frac{\delta_{\tau_j, \tau_i}}{2} \int_{h^3} \frac{1}{\pi^3 \hbar^3} e^{-\frac{(r_j - r_i)^2}{2L} - \frac{(p_j - p_i)^2}{\hbar^2/2L}} d^3r d^3p.$$



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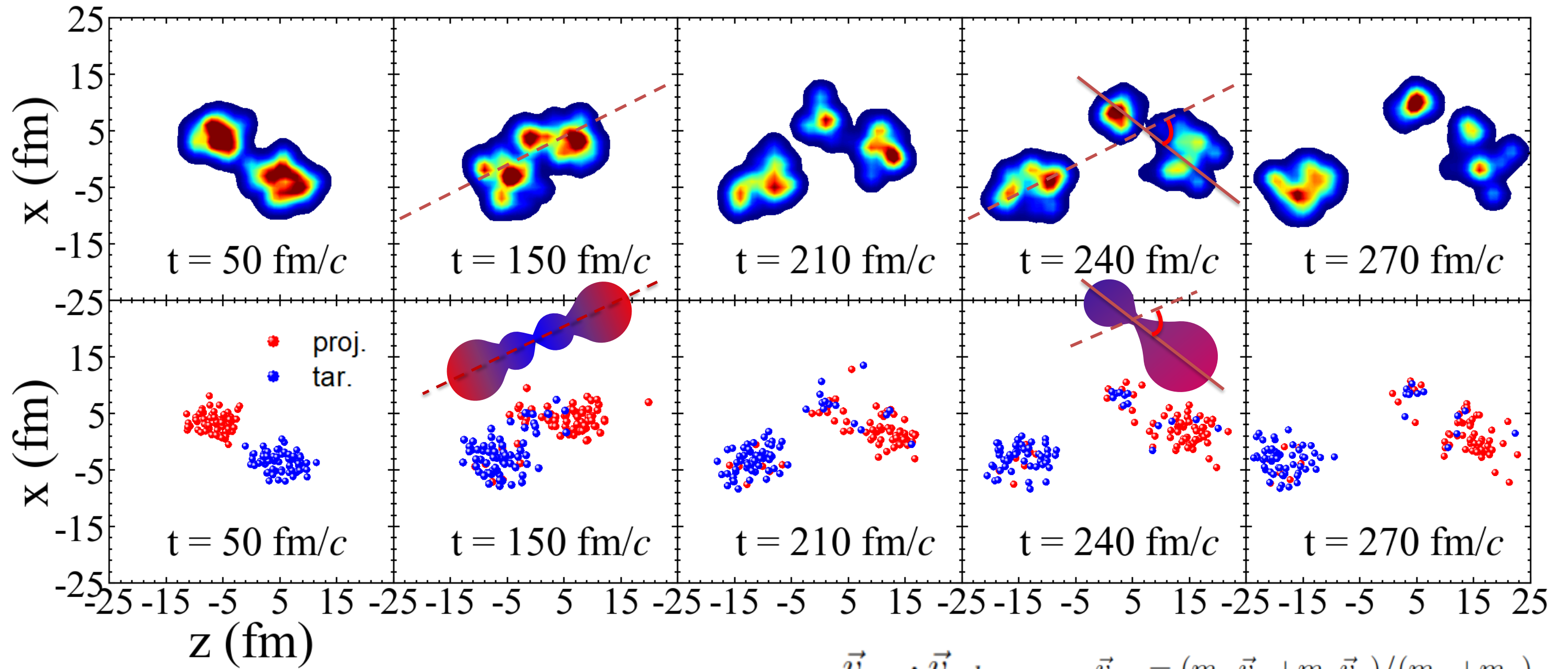
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弹核二元破裂反应

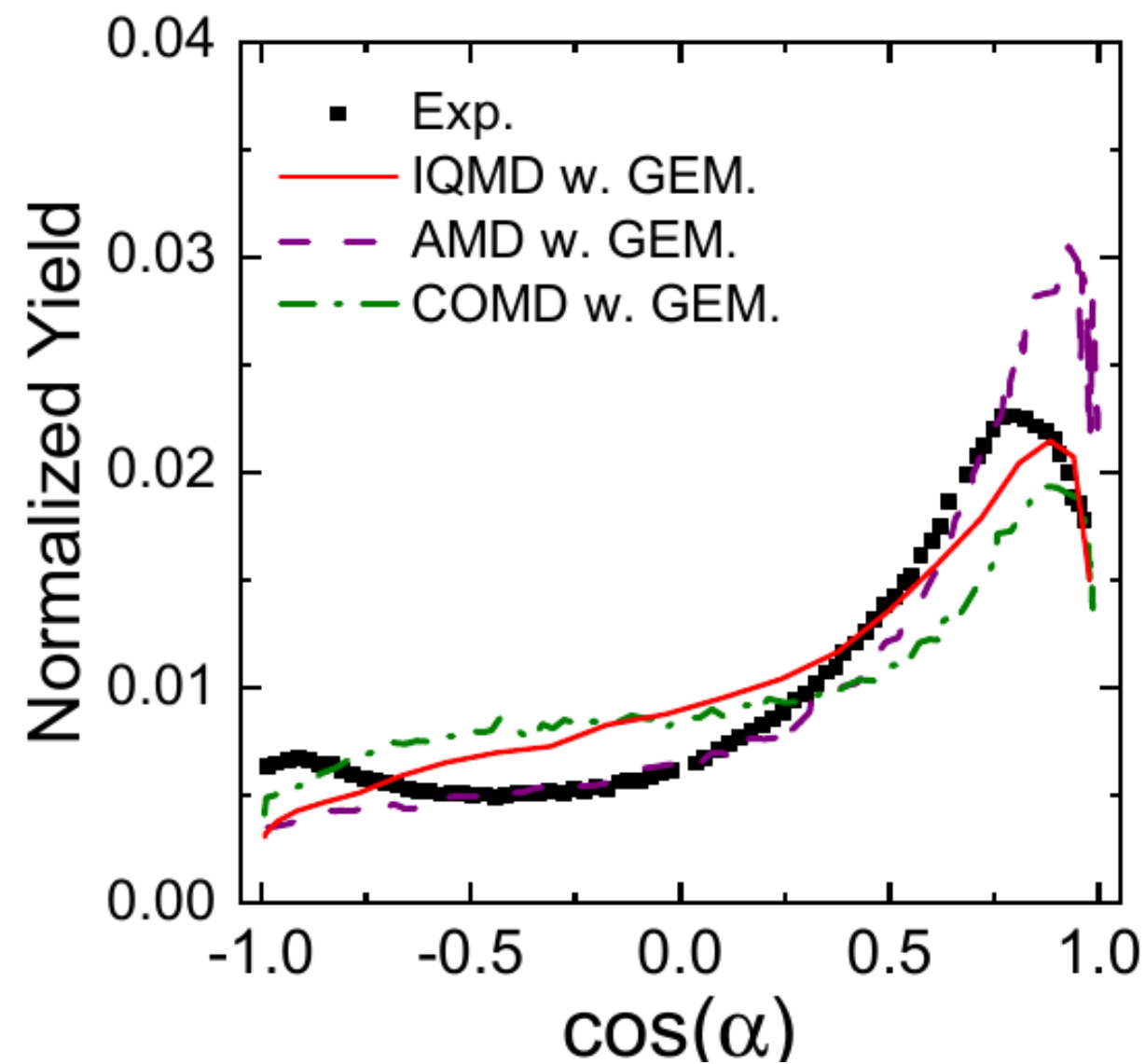
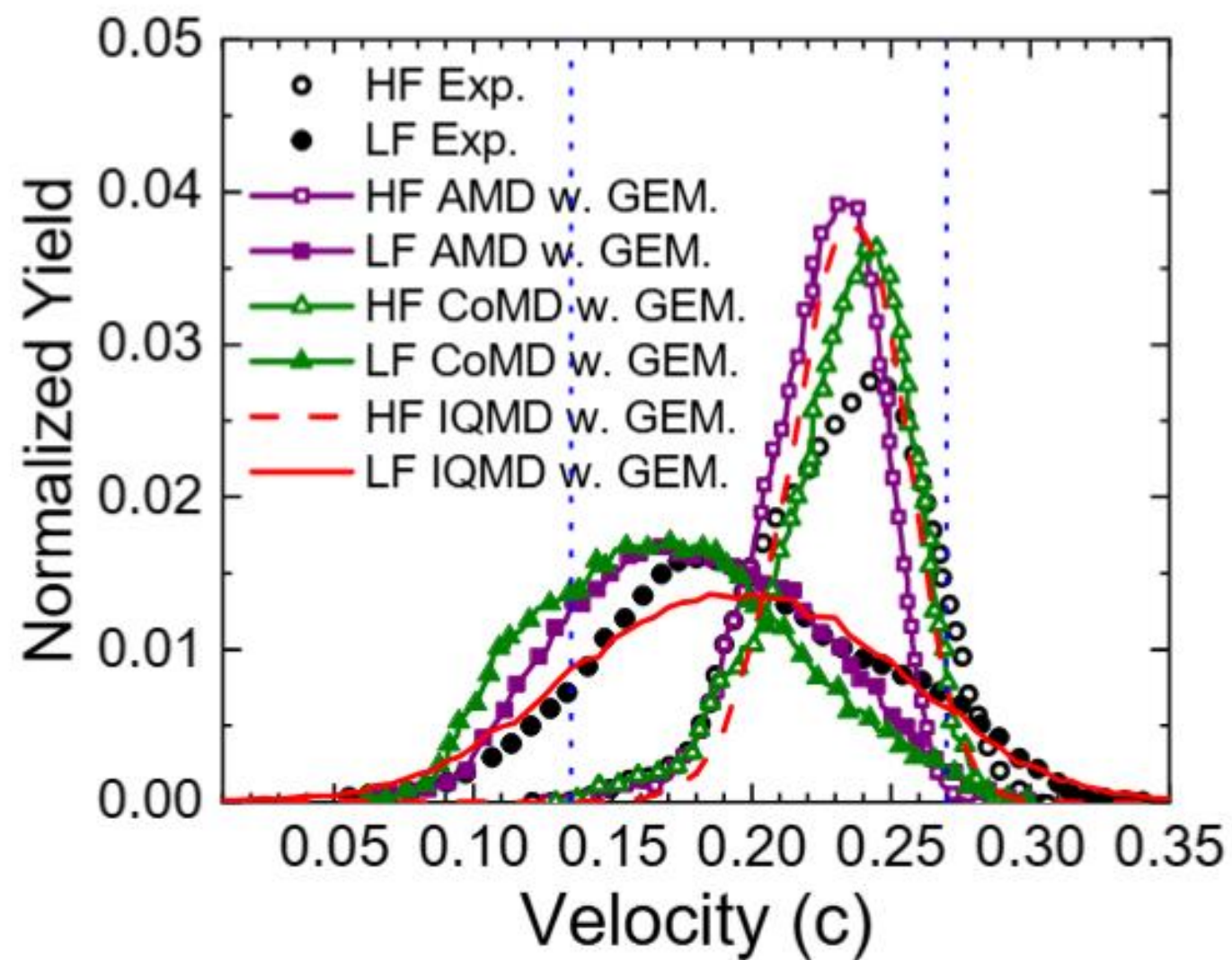


$^{70}\text{Zn} + ^{70}\text{Zn}$ @ 35A MeV

$$\alpha = \arccos \frac{\vec{v}_{cm} \cdot \vec{v}_{rel}}{|\vec{v}_{cm}| |\vec{v}_{rel}|}$$

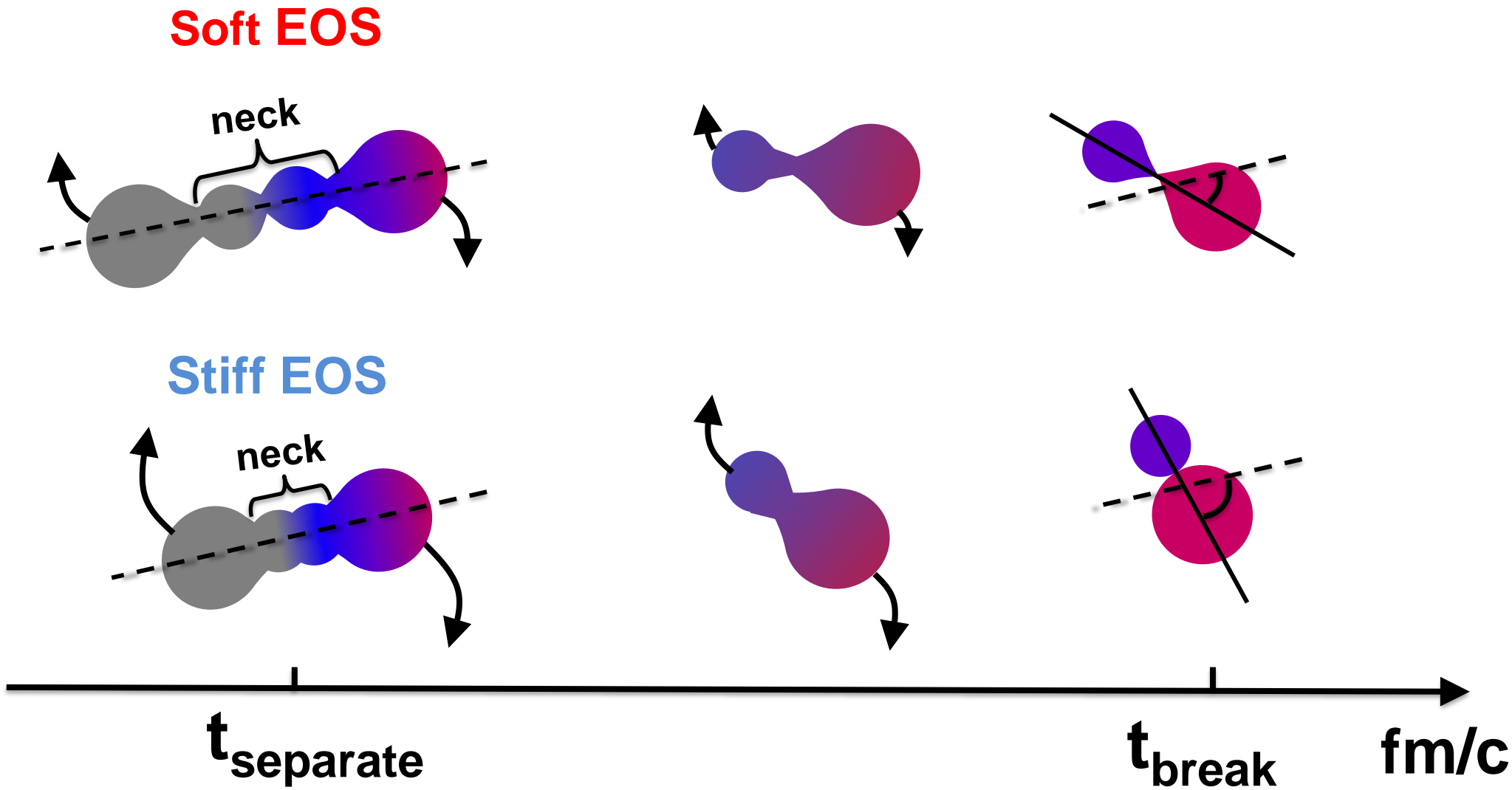
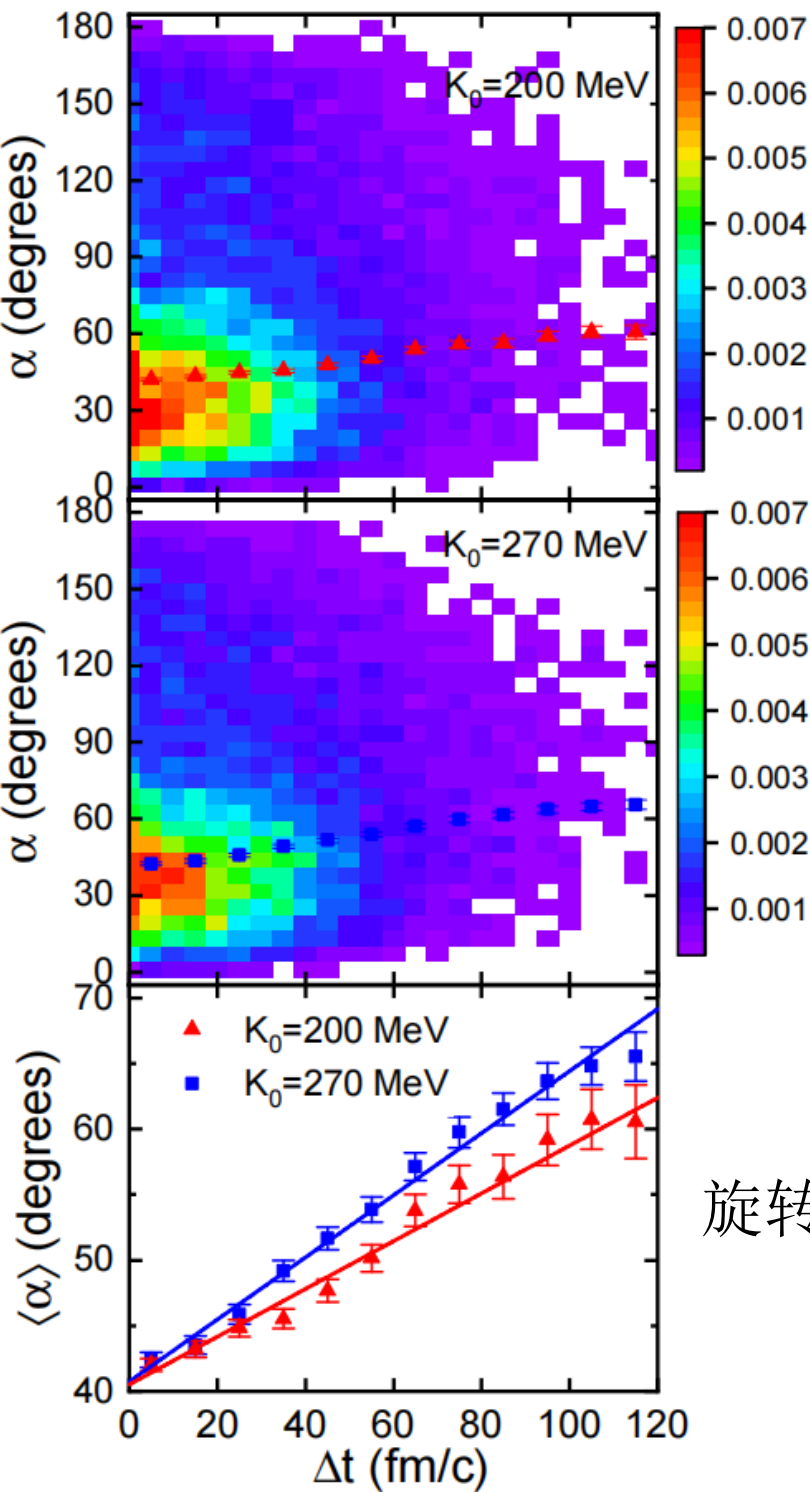
$$\vec{v}_{cm} = (m_H \vec{v}_H + m_L \vec{v}_L) / (m_H + m_L)$$

$$\vec{v}_{rel} = (\vec{v}_H - \vec{v}_L)$$



arXiv: 2410.18569

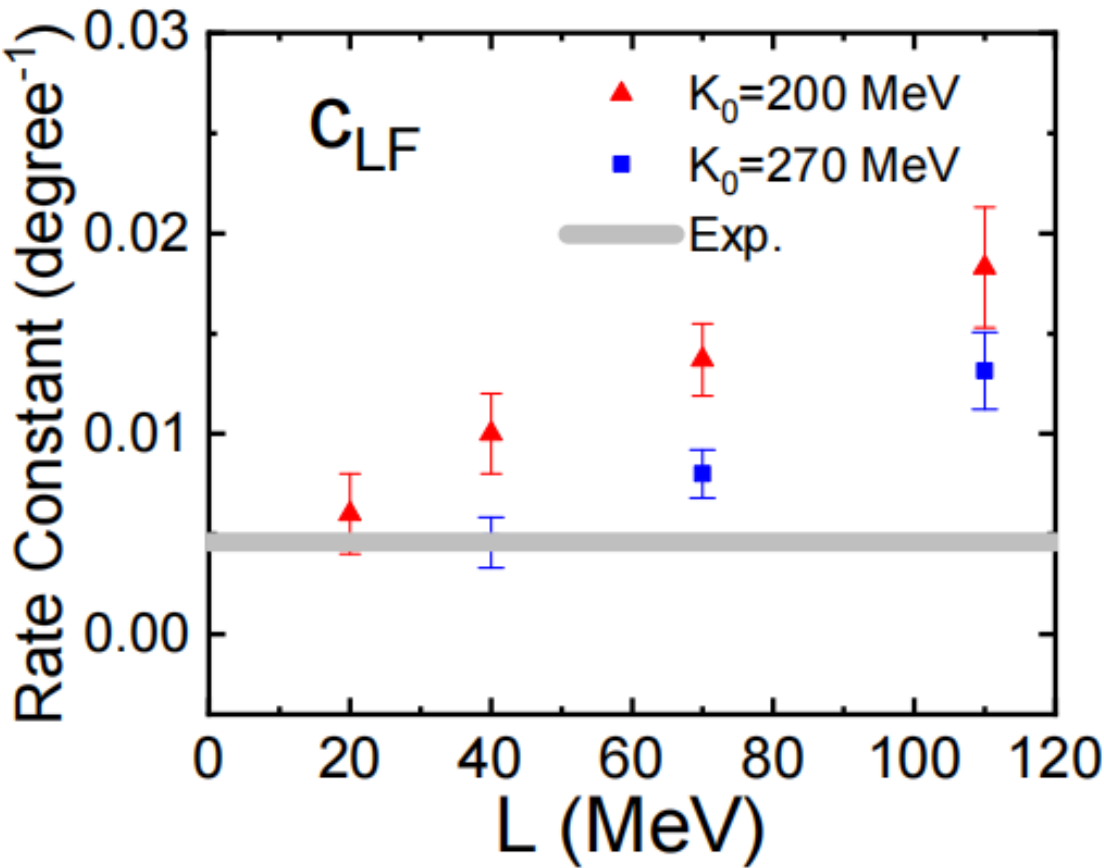
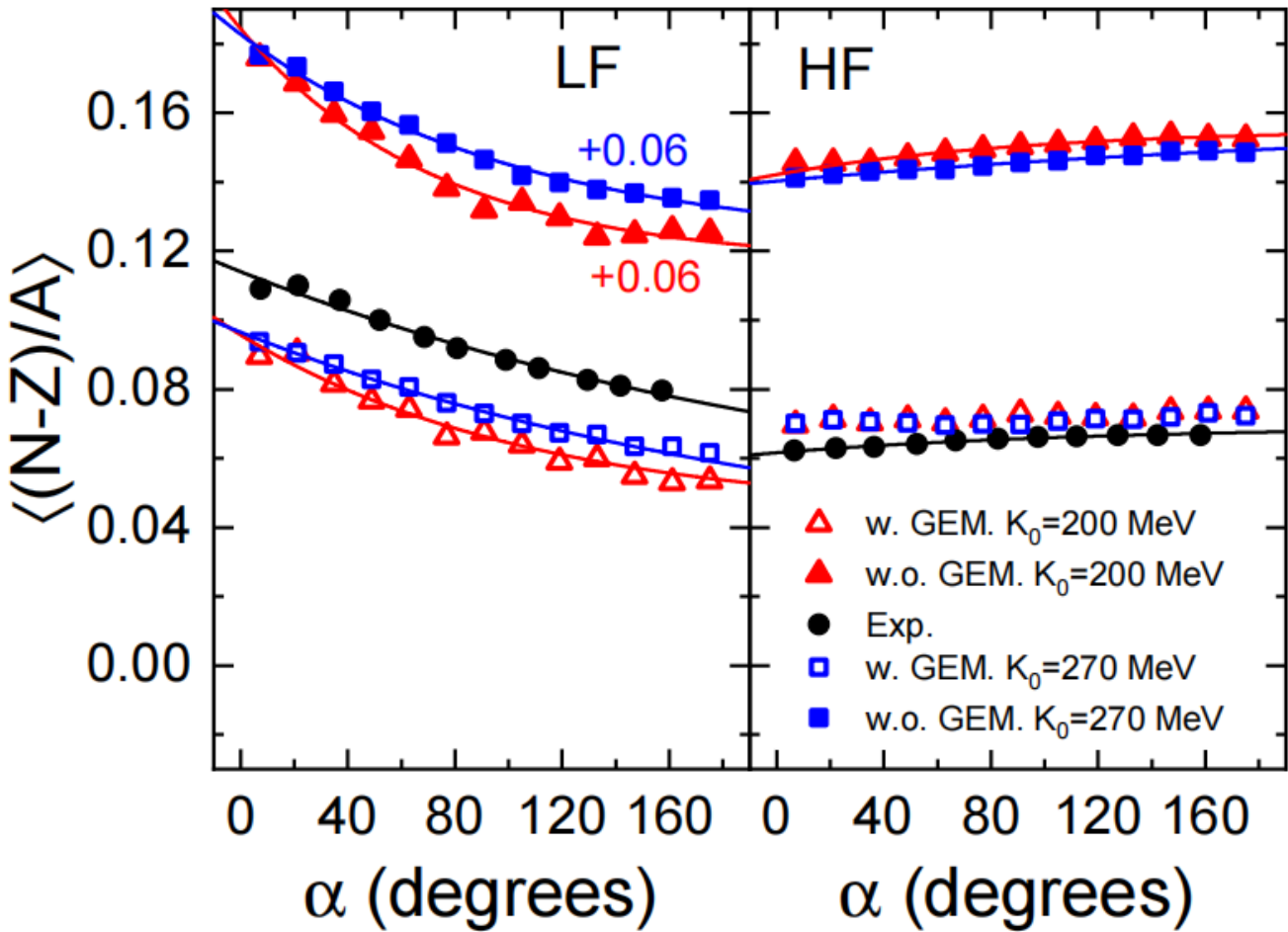
不可压缩性系数对二元破裂反应的影响



旋转时间 Δt = 弹核二元破裂时刻 - 弹靶分离时刻

不可压缩性 K_0 影响了系统角动量的耗散

基于中子-质子平衡约束对称能的密度依赖



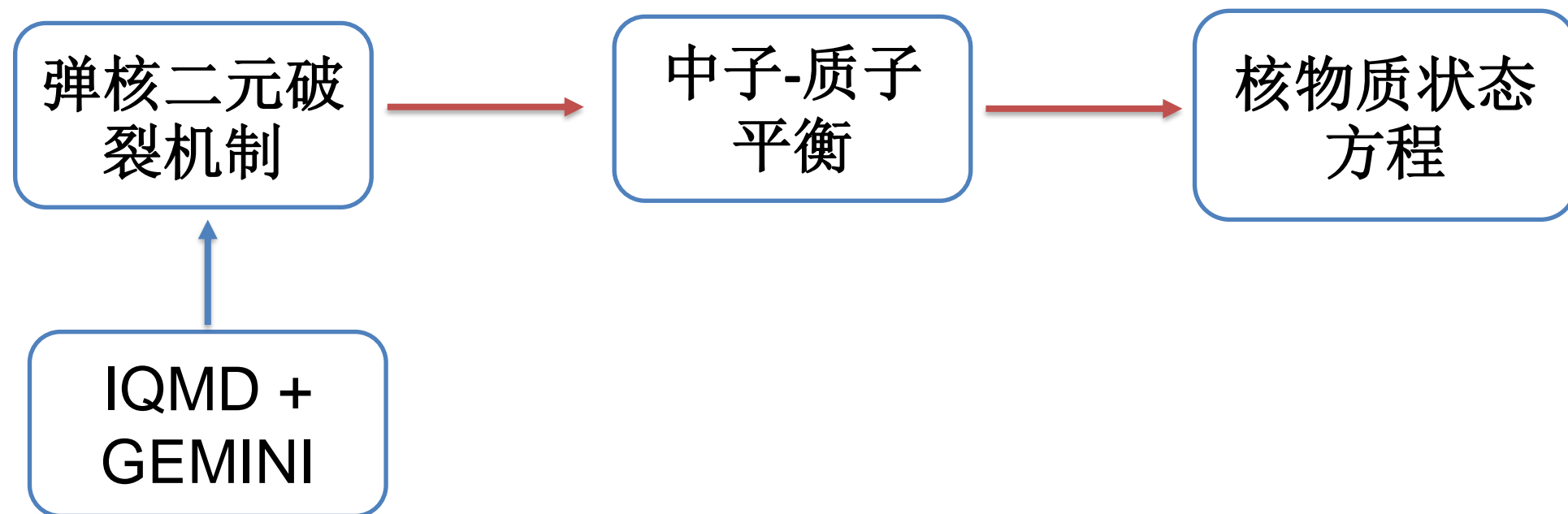
$$\begin{aligned} (N - Z)/A &= a + b \exp(-\alpha) \\ &= a + b \exp\left(-\frac{t}{\tau}\right) \\ &= a + b \exp\left(-\frac{1}{k\tau} \alpha\right) \end{aligned}$$

不可压缩性 $K_0 \longrightarrow$ 旋转角/时间 \longrightarrow 平衡速率

$$L = 20 \sim 40 \text{ MeV}$$

arXiv: 2410.18569

总结



- 不可压缩性系数 K_0 通过影响旋转角与时间线性关系进而改变中子-质子平衡性质
- 利用中子-质子平衡约束对称能需考虑 K_0 与 L 的关联
- 基于 K_0 与 L 关联, $L = 20 \sim 40 \text{ MeV}$

请各位专家批评指正！