Isospin splitting of nucleon effective mass in light of the decoded symmetry energy

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Outline

I. Motivation

II. ImQMD model (-23 version)

III. Effective mass splitting from HICs.

IV. Summary and Outlook

➢ Definition

The nonrelativistic nucleon effective mass is defined as:

$$\frac{m_q^*}{m} = [1 + \frac{m}{p}\frac{\partial V_q}{\partial p}]^{-1}, \qquad q = n, p.$$

The effective mass splitting:

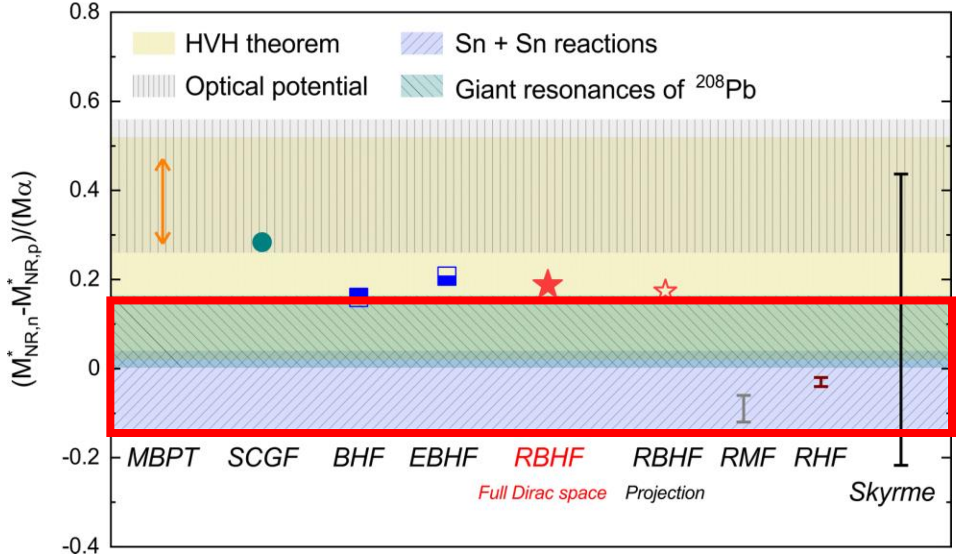
$$\Delta m_{np}^* = \frac{m_n^* - m_p^*}{m}.$$

➢ The study of effective mass splitting has important physical significance for us to understand various physical phenomena in nuclear physics and astrophysics.

- symmetry energy constraints
- properties of neutron stars
- properties of rare isotopes
- ...

Current status of constraints on effective mass splitting

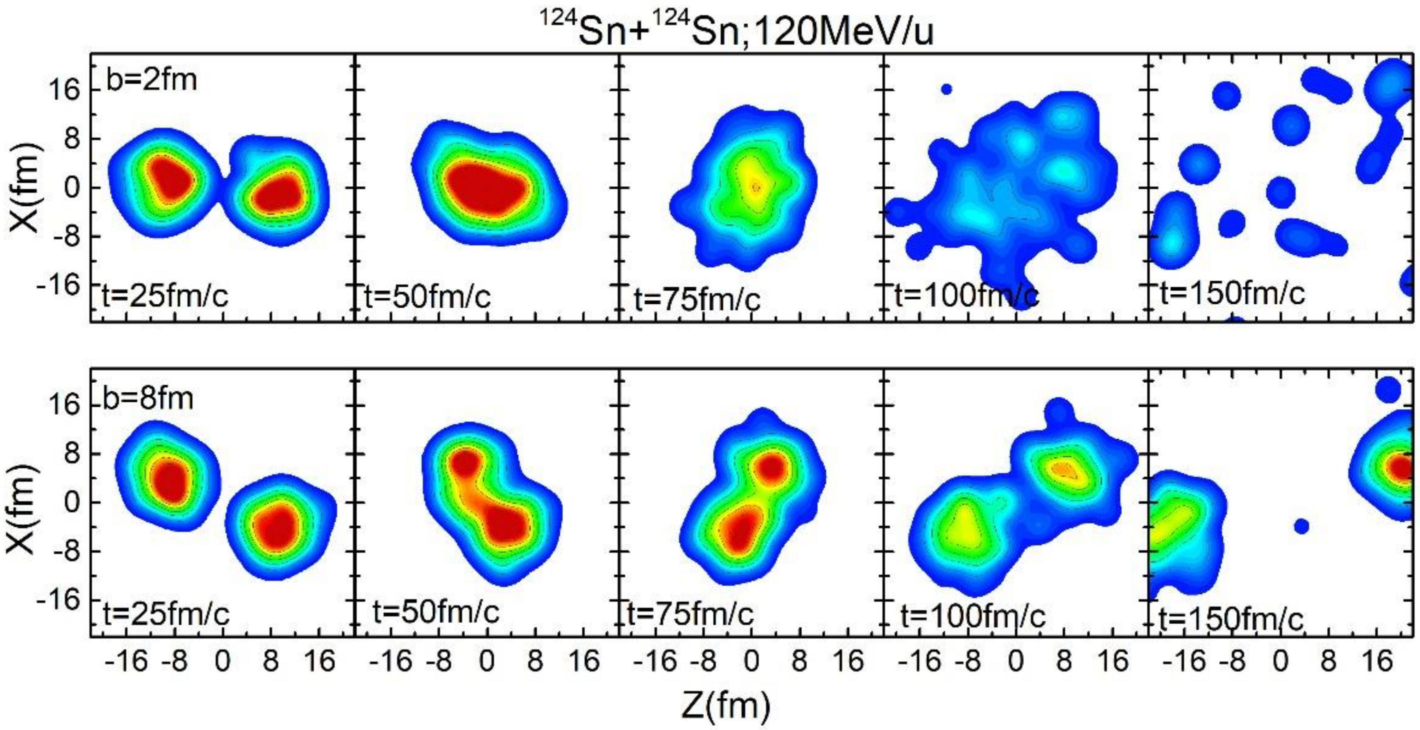
1) Microscopic model: MBPT, SCGF, BHF, EBHF, HVH, RBHF,($m_n^* > m_p^*$)
   RMF, RHF($m_n^* < m_p^*$) $\cdots$
2) Optical potential ($m_n^* > m_p^*$)
3) HICs ($m_n^* < m_p^*$, **with larger uncertianty**)



- The sign and magnitude of the effective mass splitting are still unclear.
- What is behind the door of the constraints of Δm_{np}^* from HICs?

Sibo Wang , Hui Tong, Qiang Zhao, Chencan Wang , Peter Ring , and Jie Meng. Phys. Rev. C. 108, L031303 (2023).

I. Motivation



The effective mass splitting constrained by HICs is not at a certain density or momentum.

ImQMD-23(Improved quantum molecular dynamics model-23**):**

1. The standard Skyrme potential energy density can be used: $u_{sky} = u_{loc} + u_{md}$, connect the nuclear reaction and structure based on the same footing.
2. Extended Skyrme MDI ($g(\mathbf{p}-\mathbf{p}') = \delta(\boldsymbol{r}-\boldsymbol{r}')\sum_{I=0}^{N} b_I(\boldsymbol{p}-\boldsymbol{p}')^{2I}$).
3. Accurate calculations on the nonlinear density-dependent interaction
4. New Pauli blocking with antisymmetry wave function effects.

ImQMD-23 team group:

Yingxun Zhang, Junping Yang, Xiang Chen, Cui Ying, Zhuxia Li, Kai Zhao

The Skyrme potential energy density is used: $u_{sky} = u_{loc} + u_{md}$

$$u_{loc} = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^{\eta}} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{sym}\frac{\rho^2}{\rho_0}\delta^2 + B_{sym}\frac{\rho^{\eta+1}}{\rho_0^{\eta}}\delta^2$$

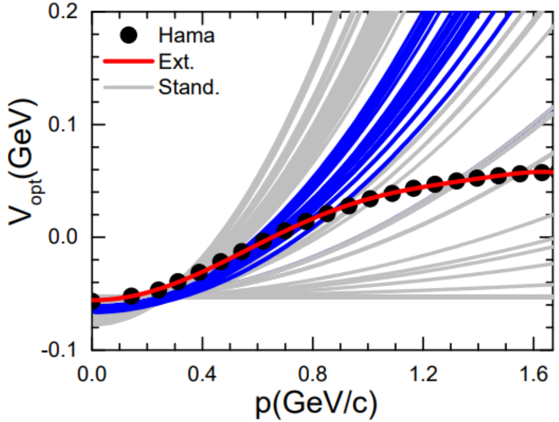
$$u_{md} = C_0 \sum_{ij} \int d^3p d^3p' f_i(\boldsymbol{r},\boldsymbol{p}) f_j(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}') + D_0 \sum_{ij\in n} \int d^3p d^3p' f_i(\boldsymbol{r},\boldsymbol{p}) f_j(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}') + D_0 \sum_{ij\in p} \int d^3p d^3p' f_i(\boldsymbol{r},\boldsymbol{p}) f_j(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}')$$

Momentum-dependent interaction:

$$\boxed{g(\boldsymbol{p}-\boldsymbol{p}') = (\boldsymbol{p}-\boldsymbol{p}')^2}$$

$V_q \rightarrow \infty$, **at large relative momentum**

$$\boxed{\mathrm{g}(\mathbf{p}-\mathbf{p}') = \delta(\boldsymbol{r}-\boldsymbol{r}') \sum_{I=0}^{N} b_I (\boldsymbol{p}-\boldsymbol{p}')^{2I}}$$



Equation of state and the density dependence of the symmetry energy:

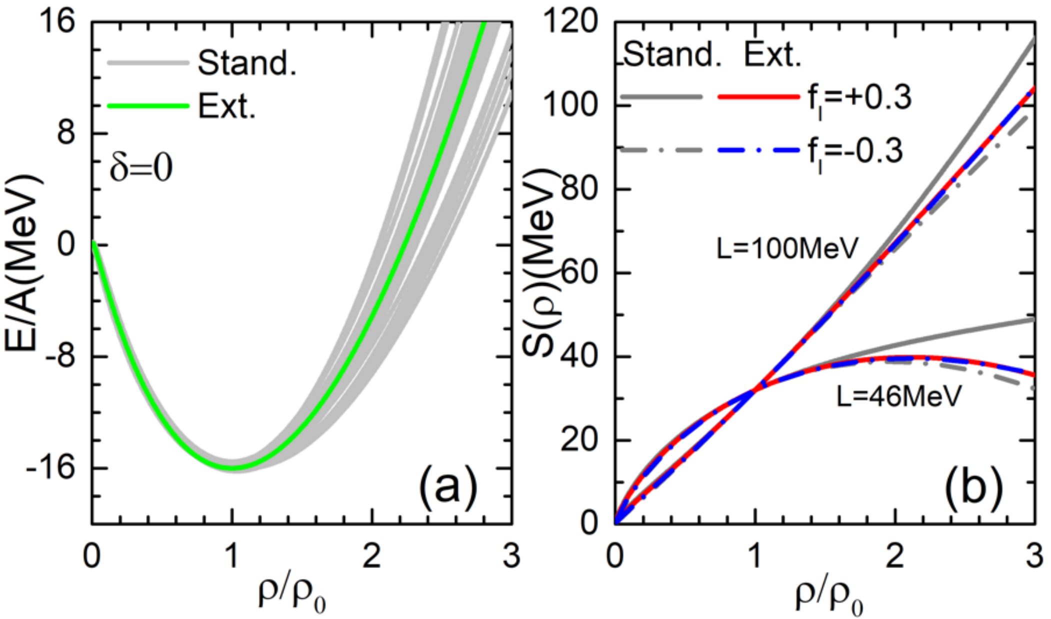
TABLE I: Four sets of nuclear matter parameters used in this work.

$\rho_0(\mathrm{fm}^{-3})$	$E_0(MeV)$	$K_0(\mathrm{MeV})$	$S_0(\mathrm{MeV})$	$L(MeV)$	m_s^*/m	f_I
0.16	-16	230	32	46,100	0.77	0.3
0.16	-16	230	32	46,100	0.77	-0.3

$$f_I(\rho, p) = \frac{m}{m_s^*} - \frac{m}{m_v^*}$$

$(m_n^* < m_p^*)$

$(m_n^* > m_p^*)$

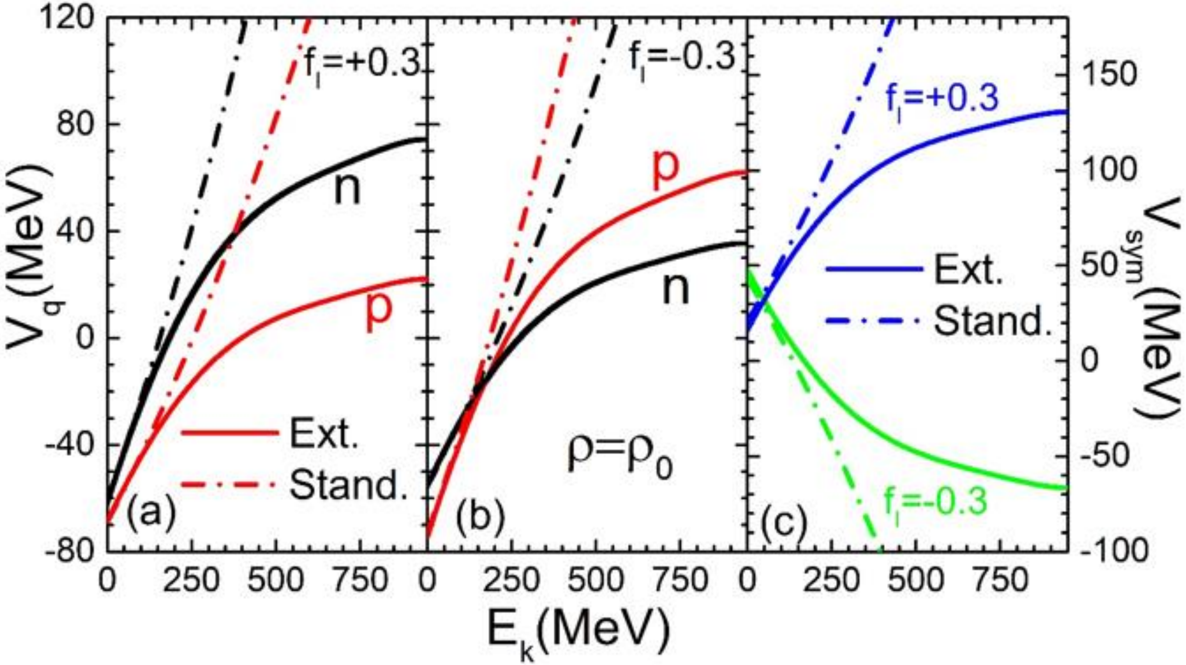


Junping Yang, Xiang Chen, Ying Cui, Yangyang Liu, Zhuxia Li, and Yingxun Zhang†, Phys. Rev. C **109**, 054624(2024).

Single-particle potential and symmetric potential:

$$\delta = 0.2$$

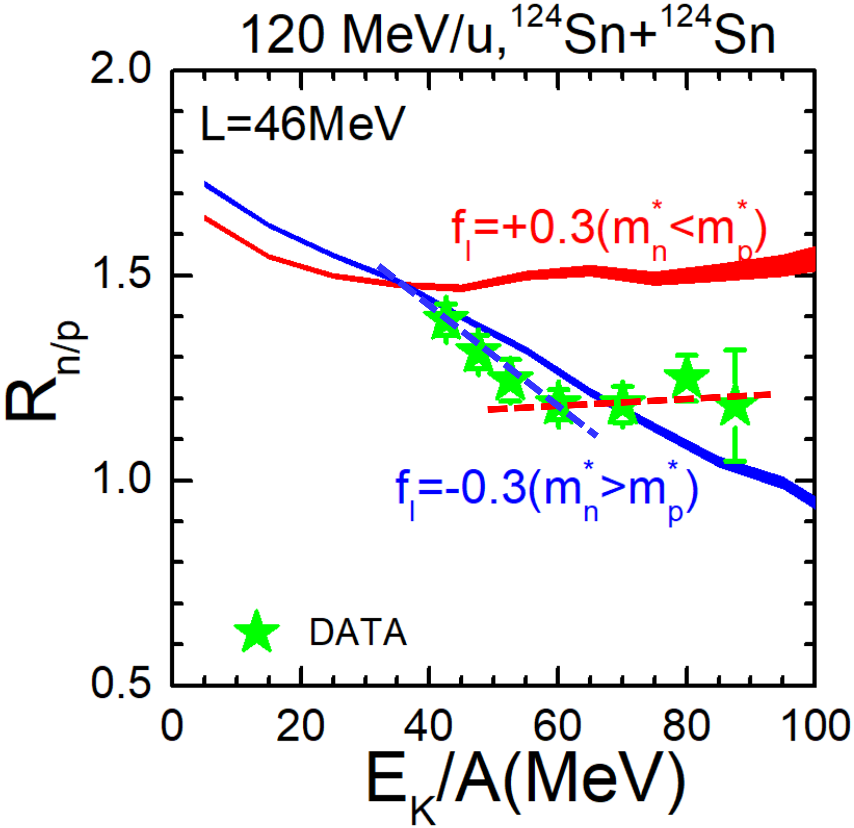
$$V_{sym} = \frac{V_n - V_p}{2\delta}$$



Junping Yang, Xiang Chen, Ying Cui, Yangyang Liu, Zhuxia Li, and Yingxun Zhang†, Phys. Rev. C **109**, 054624(2024).

- ✓ **Compared with the standard MDI, the symmetric potential obtained by the extended MDI flattens out with the increase of kinetic energy.**

III. Effective mass splitting from HICs.



$$R_{n/p} = \frac{\mathrm{Y(n)}}{\mathrm{Y(p)}} \propto \exp\left(\frac{\mu_n - \mu_p}{T}\right) = \exp\left(\frac{2V_{asy}\delta}{T}\right) \quad (1)$$

$$\Delta m^*_{np} \approx -(\frac{m^*}{m})^2 4m\delta \frac{\partial V_{asy}}{\partial p^2} \quad (2)$$

$$R_{n/p} \propto \exp\left[\frac{2(V^0_{asy} + \frac{\partial V_{asy}}{\partial p^2}p^2 + \cdots)\delta}{T}\right] \quad (3)$$

$$\approx \exp\left[\frac{2V^0_{asy}\delta}{T}\right] \exp\left[-\frac{(\frac{m}{m^*})^2 \Delta m^*_{np}}{T} E'_k\right].$$

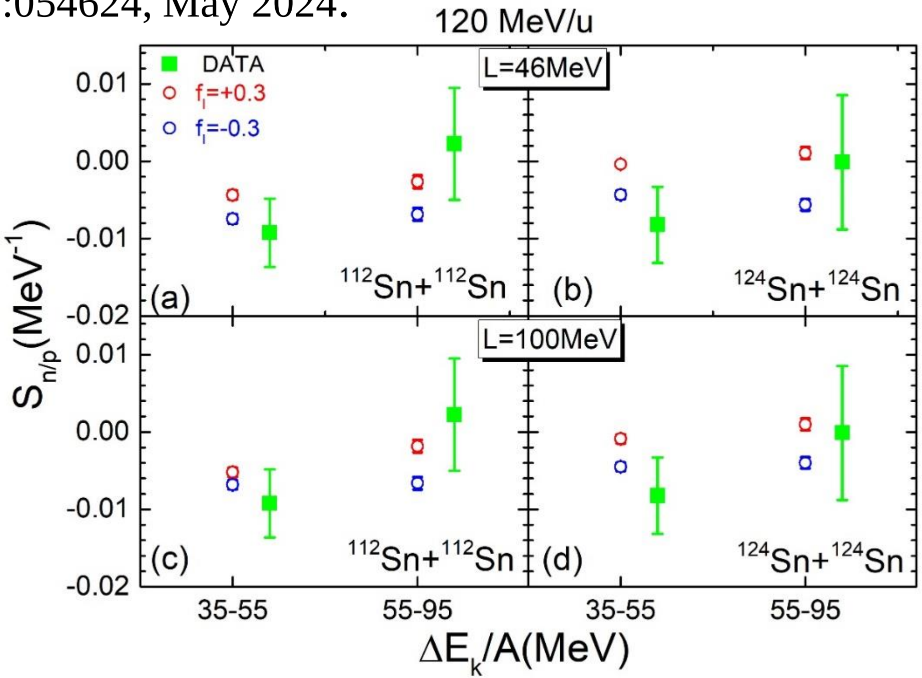
$E'_k = \lambda E_k/A$ (E'_k the relative kinetic energy between colliding nucleon pairs)

$$\boxed{S_{n/p} = \frac{\partial ln R_{n/p}}{\partial E_k/A} = -\frac{\lambda}{T}(\frac{m}{m^*})^2 \Delta m^*_{np}}$$

✓ $S_{n/p}$ **is directly related to the** Δm^*_{np} **！！！**

III. Effective mass splitting from HICs.

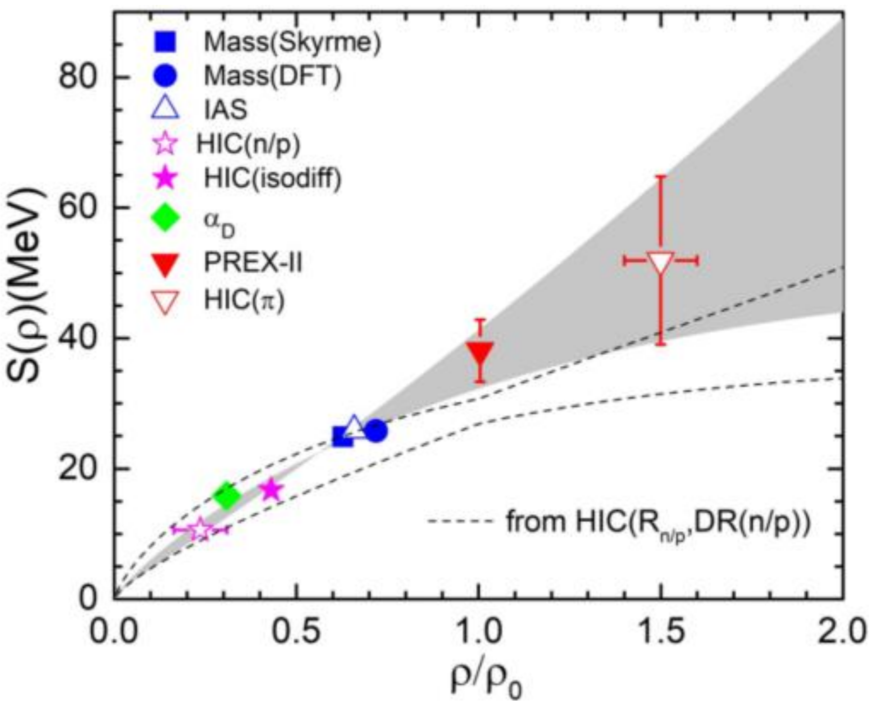
Junping Yang, Xiang Chen,Ying Cui,Yangyang Liu, Zhuxia Li, and Yingxun Zhang.
Phys. Rev. C, 109:054624, May 2024.



- ✓ **The calculations tend to favor different effective mass splitting at different kinetic energy regions.**
- ✓ $m_n^* > m_p^*$**(35~55MeV);** $m_n^* < m_p^*$**(55~95MeV);**

Determination of multi-dimensional parameter space:

$\rho_0(fm^{-3})$	$E_0(MeV)$	$K_0(MeV)$	m_s^*/m	f_I	$S_0(MeV)$	L(MeV)
0.16	-16	230	[0.6,1.0]	[-0.5,0.4]	[32,41]	[54.5,134]



HIC(n/p) :W. Lynch, M.B. Tsang, arXiv:1805.10757.

HIC(isodiff): P. Morfouace, C. Tsang, Y. Zhang, W. Lynch, et al., Phys. Lett. B 799, 135045 (2019)

α_D(electric dipole polarization): A. Tamii, et al., Phys. Rev. Lett. 107 (2011) 062502

MASS(Skyrme): B. A. Brown, Phys. Rev. Lett. 111, 232502 (2013).

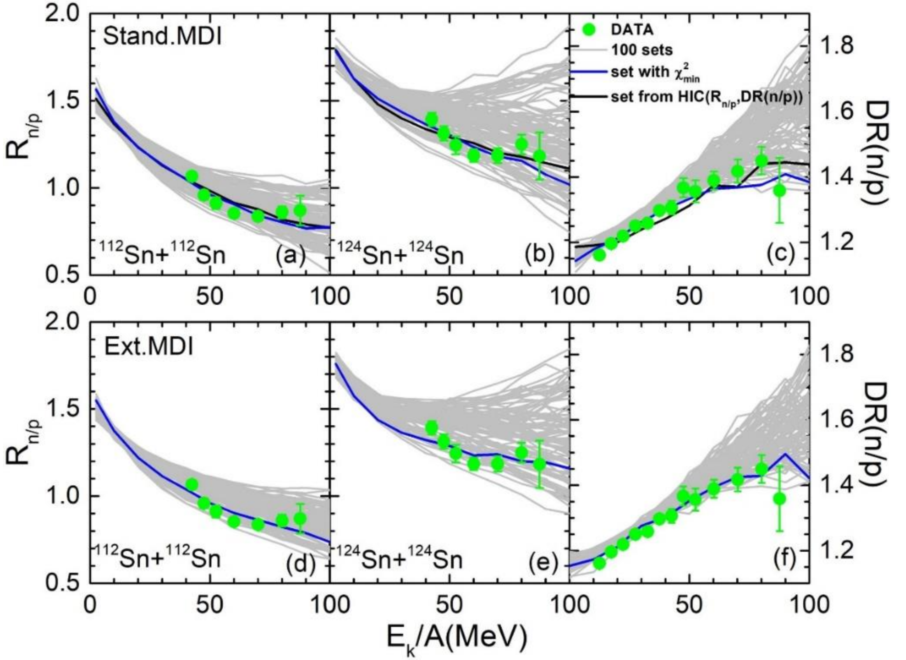
MASS(DFT): M. Kortelainen, J. McDonnell, W. Nazarewicz, et al., Phys. Rev. C. 85, 024304 (2012).

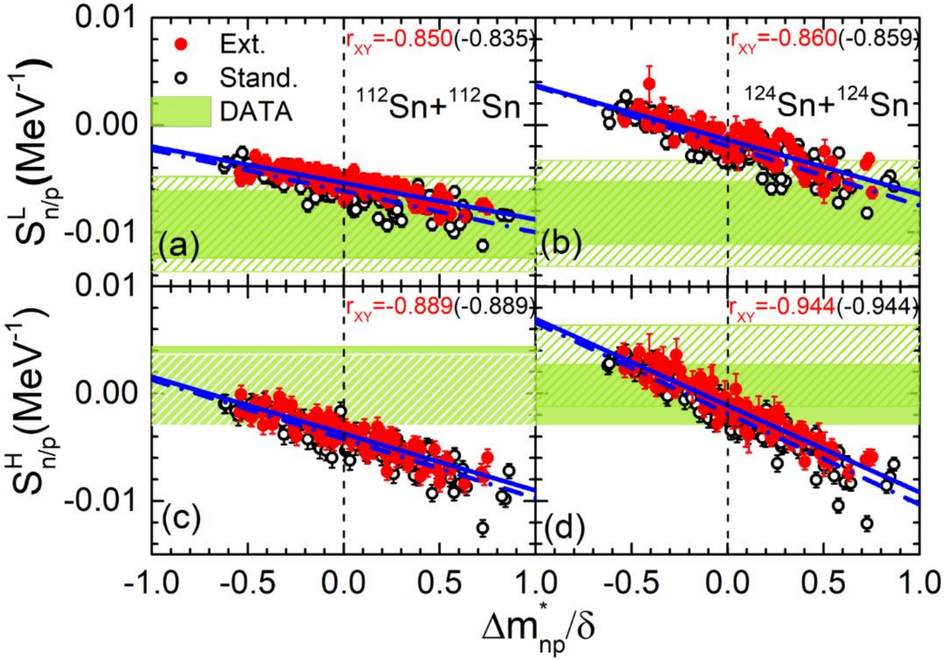
IAS(Isobaric analog state): P. Danielewicz, P. Singh, and J. Lee, Nucl. Phys. A 958, 147 (2017).

PREX-II: D. Adhikari, H. Albataineh, D. Androic, K. Aniol, et al., Phys. Rev. Lett. 126, 172502 (2021).

HIC(π): J. Estee, W. G. Lynch, C. Y. Tsang, et al., Phys. Rev. Lett. 126, 162701 (2021).

$R_{n/p}$, $DR_{n/p}$ and $S_{n/p}$ calculated by 100 parameter sets:





- ✓ $m_n^* > m_p^*$(low energy region); $m_n^* < m_p^*$(high energy region);
- ✓ $S_{n/p}$ is strongly correlated to the neutron-proton effective mass splitting!

IV. Summary and Outlook

1) **The slope of lnRn/p as a function of Ek/A, i.e., $S_{n/p}$ is directly related to the Δm^{*}_{np} ($r_{XY} = 0.944$ for neutron−rich system)**
2) **mn*>mp*(low energy region); mn*<mp*(high energy region).**

Future plan:

1) **The form of symmetry potential could first decrease and then increase with momentum. (the finite range Gogny force D1S and D250)**
2) **The effects of high-momentum tail in the initial nucleus could also influence the energy spectra of the emitted nucleons.**

Thank you for your attention!

$R_{n/p}$ and $DR_{n/p}$ calculated by 100 parameter sets:

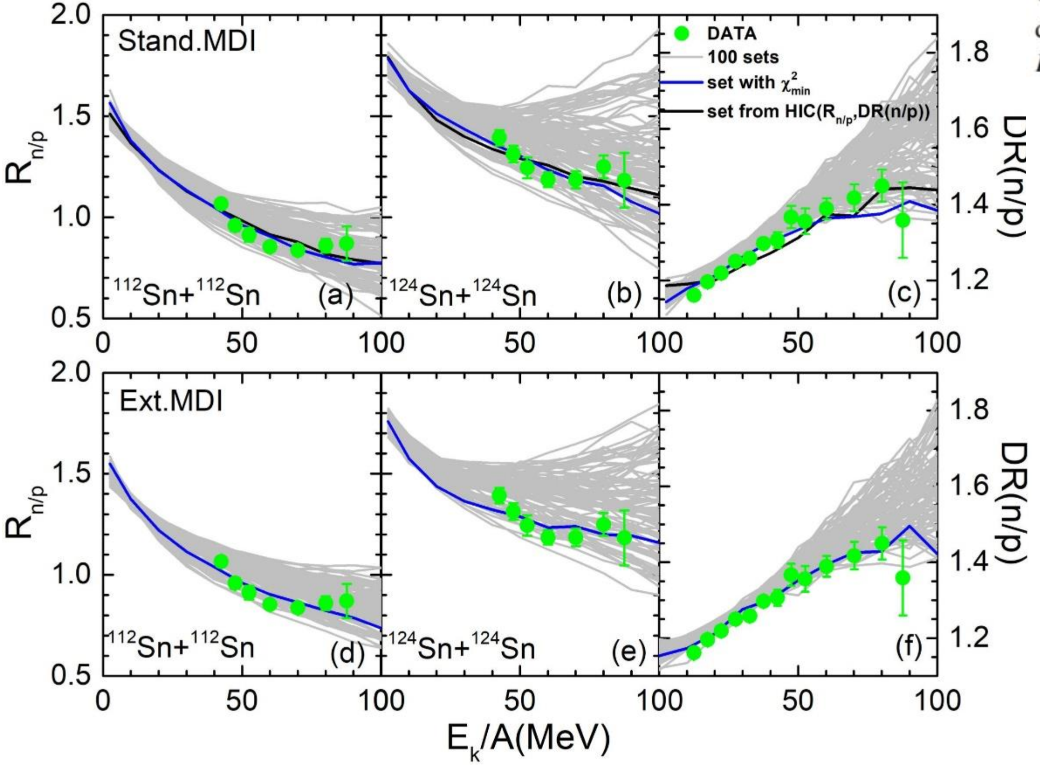


Table 2: The constraints on S_0, L, m_s^*/m and Δm_{np}^* obtained by simultaneously describing $R_{n/p}^{112}$, $R_{n/p}^{124}$, and $DR(n/p)$. m_s^*/m and Δm_{np}^* are in dimensionless, S_0, L are in MeV.

Para.	Ext.	Stand.	Ref.[28]
S_0	34.9±2.2	35.5±2.2	28.8±1.9
L	105.1±22.5	102.0±22.3	49.6±13.7
m_s^*/m	0.80±0.11	0.76±0.12	0.67±0.03
Δm_{np}^*	(0.31±0.46)δ	(0.18±0.44)δ	(-0.05±0.09)δ

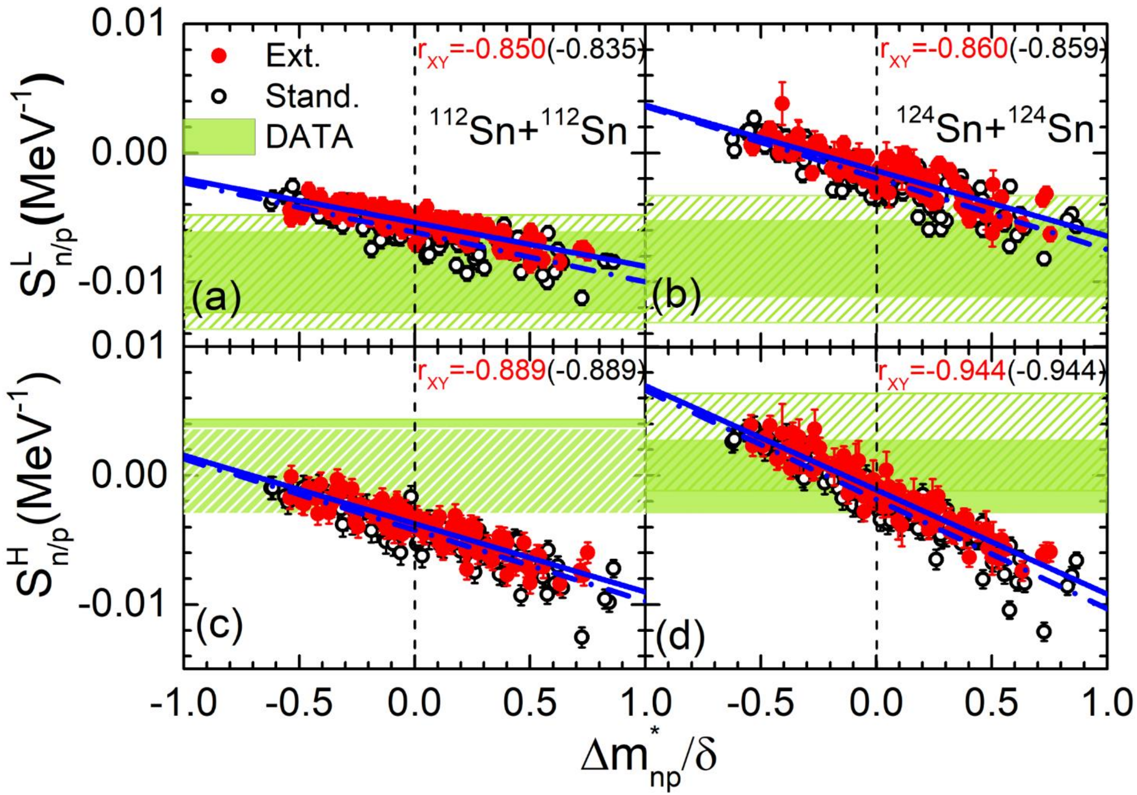
$S_0 \approx 35MeV$

$\mathrm{L} \approx 104MeV$

$m_s^*/m \approx 0.78$

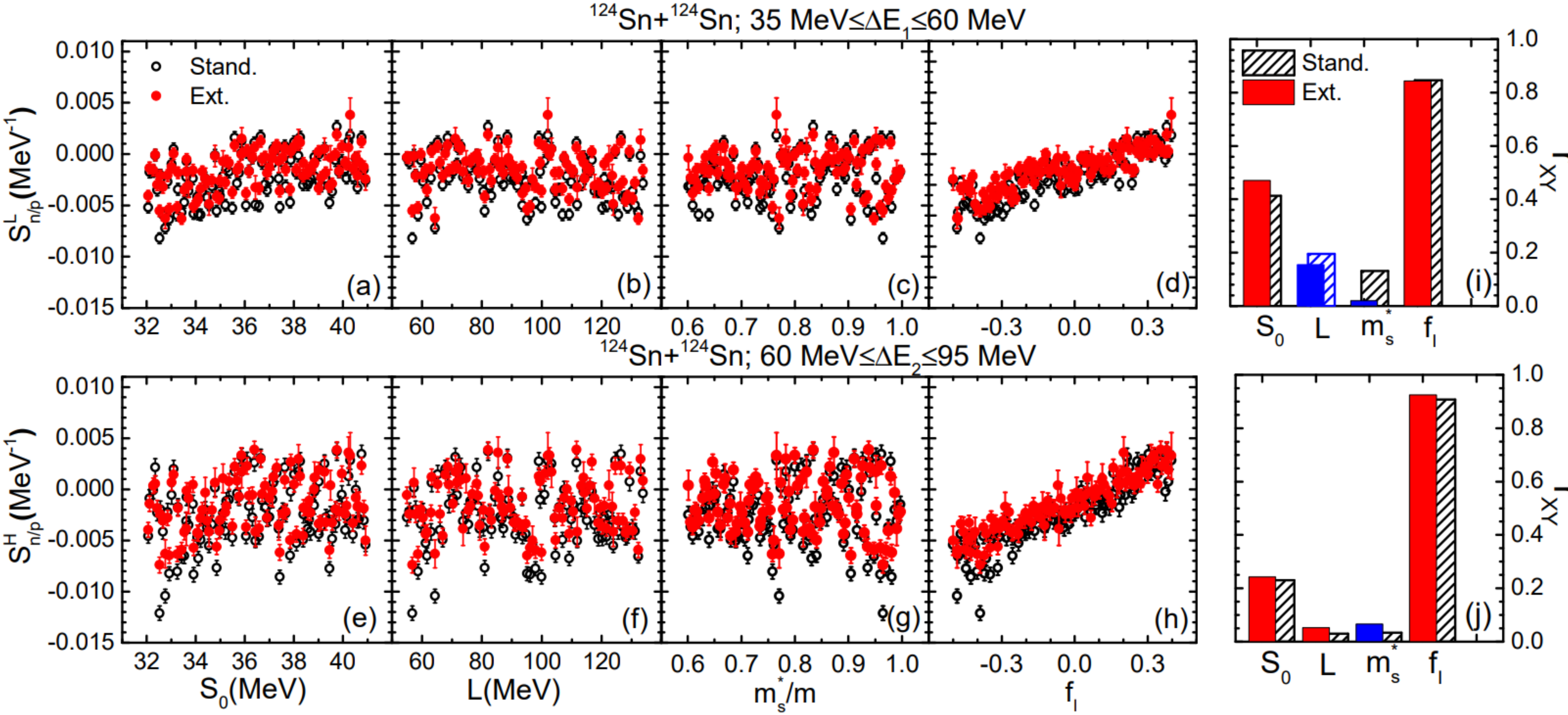
$\Delta m_{np}^* = 0.18 - 0.31$

III. Effective mass splitting from HICs.



- ✓ $m_n^* > m_p^*$(low energy region); $m_n^* < m_p^*$(high energy region);
- ✓ $S_{n/p}$ is strongly correlated to the neutron-proton effective mass splitting!

The sensitivity of the nuclear matter parameters to $S_{n/p}$:



Extend Skyrme MDI(N^{3}LO pseudo-potential):

$$
\begin{aligned}
V_{Ext} = {} & t_0^{(0)}(1 + x_0^{(0)} P_\sigma)\delta(r) \\
& + \frac{1}{2} t_1^{(2)} \left(1 + x_1^{(2)} P_\sigma\right) \delta(r)[k'^2 + k^2] \\
& + t_2^{(2)} \left(1 + x_2^{(2)} P_\sigma\right) [k' \cdot \delta(r) k] \\
& + \frac{1}{4} t_1^{(4)} \left(1 + x_1^{(4)} P_\sigma\right) \delta(r)[(k'^2 + k^2)^2 + 4(k' \cdot k)^2] \\
& + t_2^{(4)} \left(1 + x_2^{(4)} P_\sigma\right) \delta(r)(k' \cdot k)\ (k'^2 + k^2) \\
& + \frac{1}{4} t_1^{(6)} \left(1 + x_1^{(6)} P_\sigma\right) \delta(r)(k'^2 + k^2)[(k'^2 + k^2)^2 + 12(k' \cdot k)^2] \\
& + t_2^{(6)} \left(1 + x_2^{(6)} P_\sigma\right) \delta(r)(k' \cdot k)\ [3(k'^2 + k^2)^2 + 4(k' \cdot k)^2] \\
& + \frac{1}{6} t_3 (1 + x_3 P_\sigma)[\rho(R)]^\sigma \delta(r) \\
& + i W_0 \sigma \cdot [k' \times \delta(r) k]
\end{aligned}
$$

In our work, we assume a phenomenological momentum-dependent interaction as:

$$
\boxed{\mathrm{g}(\mathbf{p} - \mathbf{p}') = \delta(r - r') \sum_{I=0}^{N} b_I (p - p')^{2I}}
$$

◻ **Appendix B.** Determination of the expansion number N and coefficient bI.

The MDI potential energy density is taken as,

$$u_{md} = \tilde{C}_0 \int d^3p d^3p' f(\boldsymbol{r},\boldsymbol{p}) f(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\widetilde{D}_0 \int d^3p d^3p' f_n(\boldsymbol{r},\boldsymbol{p}) f_n(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\widetilde{D}_0 \int d^3p d^3p' f_p(\boldsymbol{r},\boldsymbol{p}) f_p(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}').$$

The nonlocal part of single-particle potential of nucleon in cold uniform nuclear matter is,

$$V_q^{md}(\rho,\delta,p) = \frac{\delta u_{md}}{\delta f_q} = 2\tilde{C}_0 \sum_{\tau=n,p} \int_0^{p_{Fq}} d^3p' f_\tau(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}')$$

$$\boldsymbol{f_q = \frac{2}{(2\pi\hbar)^3}\Theta(p-p_{Fq}), q = n,p}$$

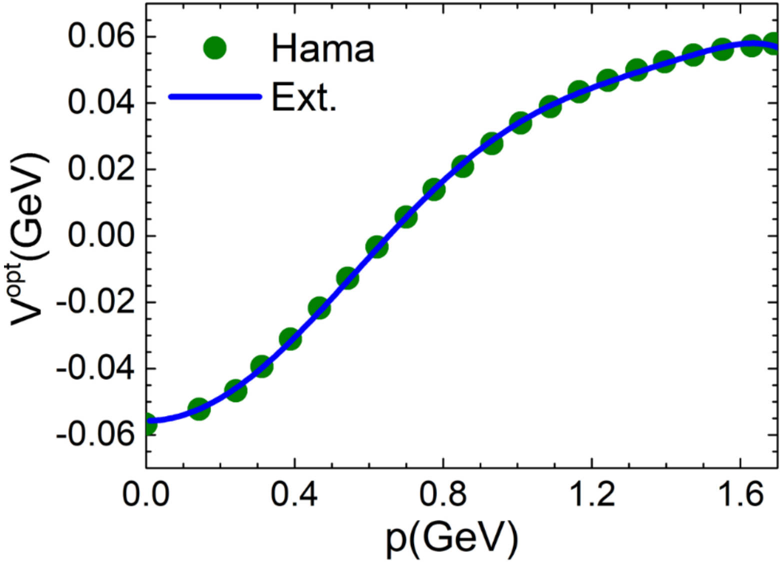
$$+2\widetilde{D}_0 \int_0^{p_{Fq}} d^3p' f_q(\boldsymbol{r},\boldsymbol{p}') g(\boldsymbol{p}-\boldsymbol{p}')$$

$$= V_{md}^0 \pm V_{md}^{asy}\delta + \cdots$$

Fitting Hama data

By fitting the Hama data we can get the values of the parameters b_0, b_1, b_2, b_3, b_4.

$b_0(GeV^2)$	$b_1(GeV^0)$	$b_2(GeV^{-2})$	$b_3(GeV^{-4})$	$b_4(GeV^{-4})$	$\widetilde{C}_0(fm^3/GeV)$	$\widetilde{D}_0(fm^3/GeV)$
-1.105	3.649	-2.608	0.826	-0.093	0.182	0.00



The experimental datas from:

S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif, and R. L. Mercer, Phys. Rev. C 41, 2737 (1990).

☐ **Appendix C.** Nuclear matter parameters and its relation to the interaction parameters

Given the values of nuclear matter parameters at normal density, i.e., S_0, L, K_0, E_0 and ρ_0, and the effective mass at normal density and fermi momentum,

(1) $P = \rho_0^2 \frac{dE/A(\rho_0,\delta=0)}{d\rho} = 0;$

(2) $E_0 = E/A\,(\rho_0);$

(3) $K_0 = 9\rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2}|_{\rho_0};$

(4) $\frac{m}{m_s^*}(\rho_0, p_F);$

(5) $f_I(\rho_0, p_F);$

(6) $S_0 = S(\rho_0);$

(7) $L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}|_{\rho_0}.$

$\Rightarrow$

$$\boxed{\alpha, \beta, \gamma, A_{sym}, B_{sym}, \tilde{C}_0 \text{ and } \tilde{D}_0}$$

Nuclear matter parameters and its relation to the interaction parameters

The saturation density ρ_0 is obtained by

$$P = \rho_0^2 \frac{dE/A(\rho_0,\delta=0)}{d\rho} = 0;\ \ \frac{2}{5}\epsilon_F^0 + \frac{\alpha}{2} + \frac{\beta}{\gamma+1}\gamma + \sum_{I=1}^{N} \tilde{g}_{md}^I \left(\frac{2I}{3} + 1\right) \rho_0^{2I/3+1} = 0. \quad (1)$$

The binding energy E_0 is,

$$E_0 = E/A\,(\rho_0) = \frac{3}{5}\epsilon_F^0 + \frac{\alpha}{2} + \frac{\beta}{\gamma+1} + \sum_{I=1}^{N} \tilde{g}_{md}^I \rho_0^{2I/3+1}. \quad (2)$$

The incompressibility K_0 is,

$$K_0 = 9\rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2}|_{\rho_0} = -\frac{6}{5}\epsilon_F^0 + 9\frac{\beta}{\gamma+1}\gamma(\gamma-1) + 6\sum_{I=1}^{N} \tilde{g}_{md}^I \left(\frac{2I}{3} + 1\right) I\rho_0^{2I/3+1}. \quad (3)$$

The neutron/proton effective mass is,

Definition of m_s^* and m_v^* from:
E Chabanat,et,al, NPA 635(1998) 231-256

$$\frac{m}{m_q^*}(\rho,p) = 1 + \frac{m}{p}\frac{\partial V_{md}^q}{\partial p}, q = n, p.$$

$$= 1 + 2\tilde{C}_0 m\left[\sum_{I=1}^{N} b_I \sum_{k=0,k\in even}^{2I} \tilde{\mathcal{A}}_{Ik} \sum_q \rho_q^{(2I-k+3)/3} k \times p^{k-2}\right]$$

$$+2\widetilde{D}_0 m\left[\sum_{I=1}^{N} b_I \sum_{k=0,k\in even}^{2I} \tilde{\mathcal{A}}_{Ik} \rho_q^{(2I-k+3)/3} k \times p^{k-2}\right].$$

The isoscalar effective mass m_s^* can be obtained at $\rho_q = \frac{\rho}{2}$

$$\frac{m}{m_s^*}(\rho_0,p_F) = 1 + 4(\tilde{C}_0 + \frac{\widetilde{D}_0}{2})m\left[\sum_{I=1}^{N} b_I \sum_{k=0,k\in even}^{2I} \tilde{\mathcal{A}}_{Ik} (\frac{\rho_0}{2})^{(2I-k+3)/3} k \times p_F^{k-2}\right]. \quad (4)$$

The isovector effective mass m_v^* can be obtained at $\rho_q = 0$

$$\frac{m}{m_v^*}(\rho_0,p_F) = 1 + 4\tilde{C}_0 m\left[\sum_{I=1}^{N} b_I \sum_{k=0,k\in even}^{2I} \tilde{\mathcal{A}}_{Ik} (\frac{\rho_0}{2})^{(2I-k+3)/3} k \times p_F^{k-2}\right].$$

we define a quantity f_I,

$$f_I(\rho_0,p_F) = \frac{m}{m_s^*} - \frac{m}{m_v^*} = 2\widetilde{D}_0 m\left[\sum_{I=1}^{N} b_I \sum_{k=0,k\in even}^{2I} \tilde{\mathcal{A}}_{Ik} \left(\frac{\rho_0}{2}\right)^{(2I-k+3)/3} k \times p_F^{k-2}\right] \quad (5)$$

The symmetry energy coefficient S_0 is,

$$S_0 = S(\rho_0) = \frac{1}{3}\epsilon_F^0 + A_{sym} + B_{sym} + \sum_{I=1}^{N} \tilde{C}_{sym}^{I}\, \rho_0^{2I/3+1}. \quad (6)$$

The slope of the symmetry energy $\boldsymbol{L}$ is,

$$L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}\Big|_{\rho_0} = \frac{2}{9}\epsilon_F^0 + 3A_{sym} + 3B_{sym}\gamma + 3\sum_{I=1}^{N} \tilde{C}_{sym}^{I} \left(\frac{2I}{3} + 1\right) \rho_0^{2I/3+1} \quad (7)$$

Given the values of nuclear matter parameters at normal density, i.e., $\boldsymbol{S_0}$, $\boldsymbol{L}$, $\boldsymbol{K_0}$, $\boldsymbol{E_0}$ and $\boldsymbol{\rho_0}$, and the effective mass at normal density and fermi momentum, the coefficients $\boldsymbol{\alpha, \beta, \gamma, A_{sym}, B_{sym}, \tilde{C}_0}$ *and* $\boldsymbol{\tilde{D}_0}$ can be obtained as follows.

Parameters in Calculations:

TABLE I: Four sets of nuclear matter parameters used in this work.

$\rho_0(\mathrm{fm}^{-3})$	$E_0(MeV)$	$K_0(\mathrm{MeV})$	$S_0(\mathrm{MeV})$	$L(MeV)$	m_s^*/m	f_I
0.16	-16	230	32	46,100	0.77	0.3
0.16	-16	230	32	46,100	0.77	-0.3

$$\boxed{f_I(\rho,p) = \frac{m}{m_s^*} - \frac{m}{m_v^*}}$$

$(m_n^* < m_p^*)$

$(m_n^* > m_p^*)$

TABLE II. The parameters used in the calculations, i.e., $K_0 = 230$ MeV, $m_s^*/m = 0.77$, $S_0 = 32$ MeV, and varies both L and f_I. The parameters α, β, A_{sym}, B_{sym} are in MeV. $\tilde{C}_0$ and $\tilde{D}_0$ are $\mathrm{fm}^3\mathrm{GeV}^{-1}$.

Para.	(L=46, f_I=0.3)	(L=46, f_I=-0.3)	(L=100, f_I=0.3)	(L=100, f_I=-0.3)
α	-236.58 (-265.78)			
β	163.95 (194.93)			
γ	1.26 (1.22)			
A_{sym}	83.65 (108.44)	58.57 (62.73)	14.41 (25.32)	-10.67 (-20.40)
B_{sym}	-79.48 (-103.69)	-30.52 (-35.38)	-10.25 (-20.34)	38.72 (47.96)
$\tilde{C}_0$	-7.92×10^{-4} (-2.08×10^{-3})	0.37 (1.00)	-7.92×10^{-4} (-2.08×10^{-3})	0.37 (1.00)
$\tilde{D}_0$	0.37 (1.00)	-0.37 (-1.00)	0.37 (1.00)	-0.37 (-1.00)

Relation between the effective mass splitting and symmetry potential

$$\frac{m}{m_q^*} = 1 + \frac{m}{p}\frac{\partial V_q}{\partial p}, \quad q = n, p.$$

According to the definition of neutron/proton effective mass,

$$\frac{m}{m_n^*} - \frac{m}{m_p^*} = 2m\left(\frac{\partial V_n}{\partial p^2} - \frac{\partial V_p}{\partial p^2}\right) = 4m\delta\frac{\partial V_{asy}}{\partial p^2}.$$

In addition,

$$\frac{m}{m_n^*} - \frac{m}{m_p^*} = -\frac{m(m_n^* - m_p^*)}{m_n^* m_p^*} \approx -\left(\frac{m}{m^*}\right)^2 \Delta m_{np}^*.$$

The approximation comes from the $\frac{m_q^*}{m} = \frac{m^*}{m} \pm \frac{\delta m^*}{m}$, and the product of $\frac{m_n^* m_p^*}{m^2} = \left(\frac{m^*}{m}\right)^2 - \left(\frac{\delta m^*}{m}\right)^2 \approx \left(\frac{m^*}{m}\right)^2$.

$$\boxed{\Delta m_{np}^* \approx -\left(\frac{m^*}{m}\right)^2 4m\delta\frac{\partial V_{asy}}{\partial p^2},}$$