核物质集体态的相对论 RPA研究

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Outline

- Introduction to EOS
- Interaction matrix in relativistic RPA
- Thermodynamic properties of zero sound
- Sigma meson mass vs density of matter stability
- Summary

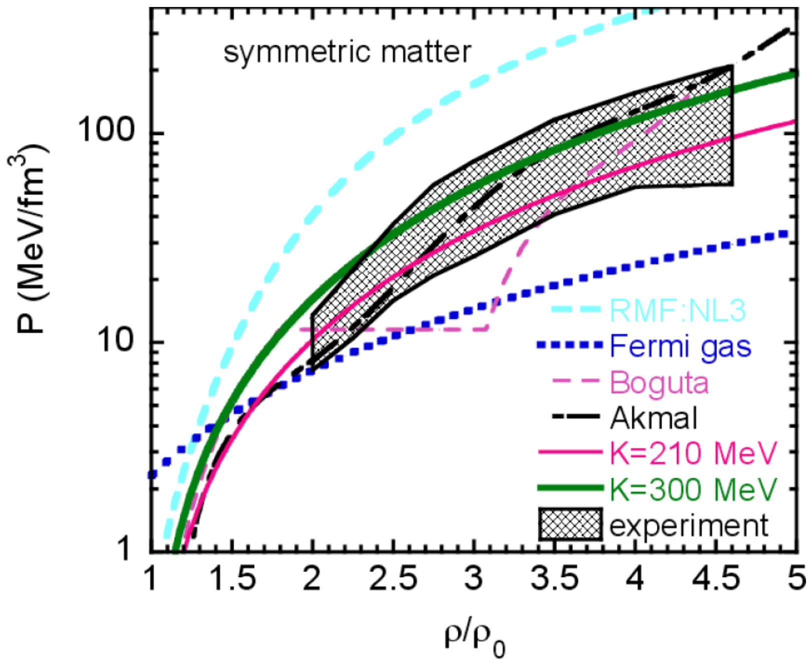
Nuclear EOS

$$E(\rho) = E_0(\rho) + E_{sym}(\rho)\delta^2 + \cdots$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \cdots,$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \cdots$$

$$\boldsymbol{E_{sym}\left(\approx \frac{\rho_0}{2}\right) = E_{sym}(\rho_0) - \frac{L}{6} + \cdots}$$



$\boldsymbol{\rho_o \cong 0.16 \pm 0.03\, fm^{-3}}$,
$\boldsymbol{\kappa \cong 230 \pm 5}$ **MeV, Yo99,PRL82,69;**

$\boldsymbol{240 \pm 20}$ **MeV**, Stone et al 2014, PRC89, 044316; Roca-Maza et al 2018, PPNP101, 96.

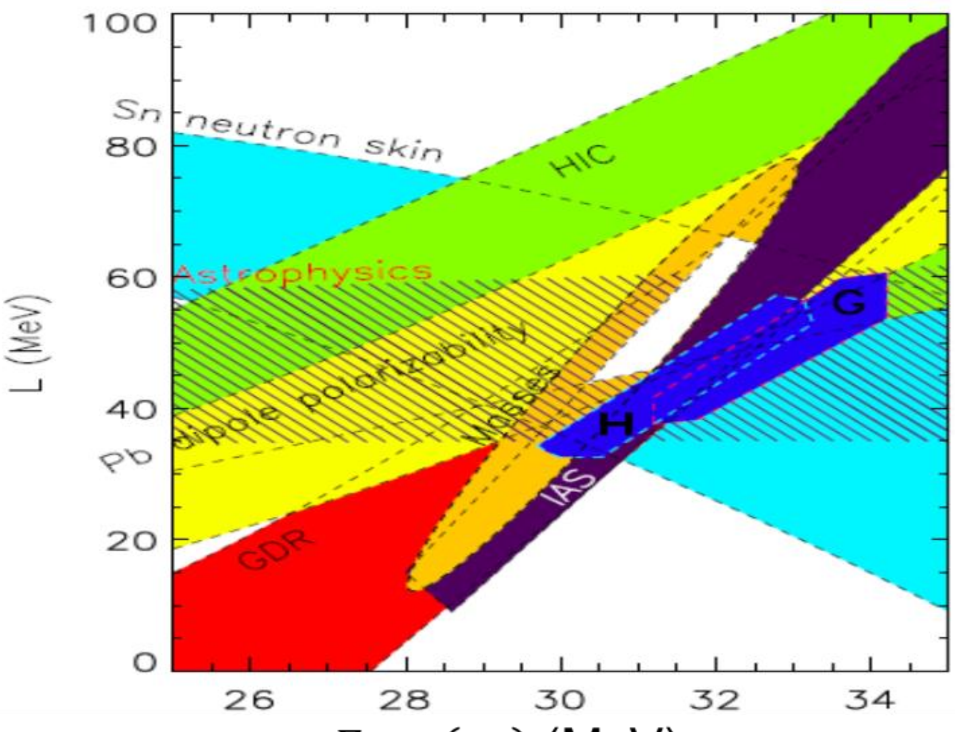
Danielewicz, et al. Science 298(2002)1592;
Collective flow data from high energy…

To Recognize the SYMMETRY ENERGY

Liquid-drop model for nuclei

E= $a_{\rm v}A - a_{\rm s}A^{2/3} - a_4\frac{(N-Z)^2}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}$

$\frac{Z^2}{A^{1/3}} + \frac{a_A^V}{1+A^{-1/3}\,a_A^V/a_A^S}\frac{(N-Z)^2}{A}$



...tecki, P. Danielewicz,……

$+E_{\rm sym}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$

MeV

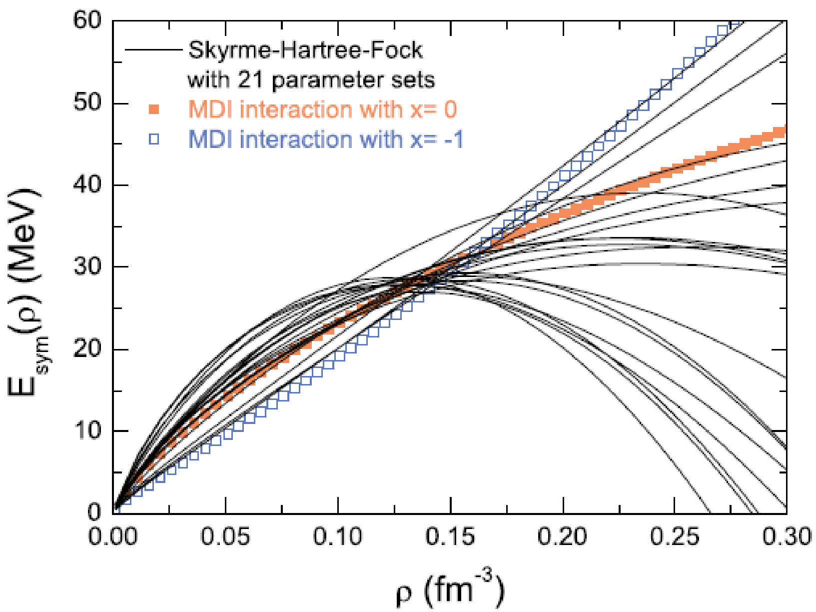


FIG. 1. (Color online) Density dependence of the nuclear symmetry energy $E_{\mathrm{sym}}(\rho)$ for 21 sets of Skyrme interaction parameters. The results from the MDI interaction with $x = -1$ (open squares) and 0 (solid squares) are also shown.

L.W.Chen,et.al., PRC72, 064309 (05)

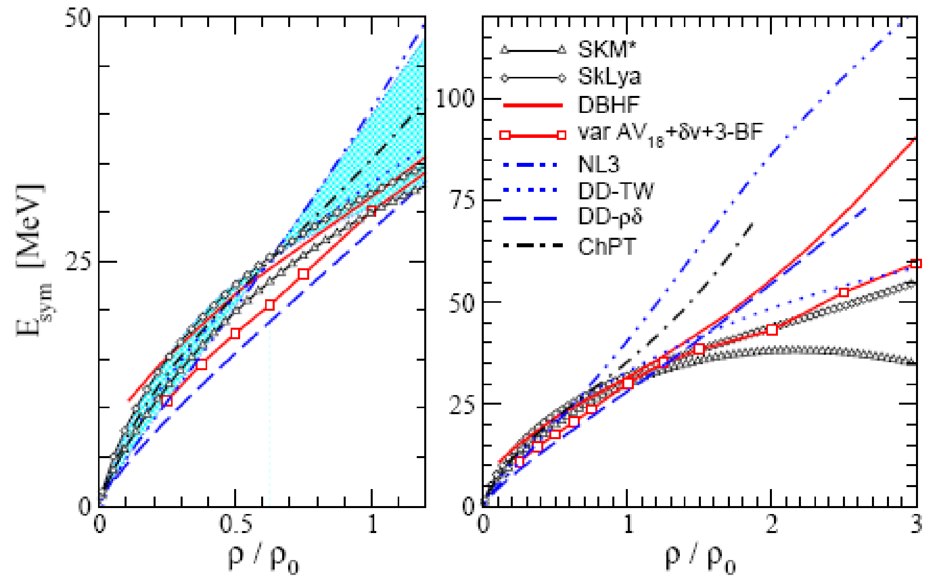


Fig. 5. Symmetry energy as a function of density as predicted by different models. The left panel shows the low density region while the right panel displays the high density range.

Fuchs, et.al., arXiv:nucl-th/0511070

- Uncertainties in EOS, either of symmetric or asymmetric matter, call for new signals.
- Check the collective modes in dense matter, as they arise from the residue interactions of neutrons and protons.

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Relativistic Interacting Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{int}} =& \overline{\psi}[\gamma_\mu(i\partial^\mu - g_\omega\omega^\mu - g_\rho\boldsymbol{\tau}\cdot\boldsymbol{b^\mu} + \frac{e}{2}(1+\tau_3)A^\mu) \\ & - (M - g_\sigma\phi)]\psi + 4g_\rho^2 b_\mu\cdot b^\mu\Lambda_v g_\omega^2\omega_\mu\omega^\mu \\ & - U(\phi,\omega),\end{aligned}$$

where $U(\phi,\omega)$ is the nonlinear σ and ω meson self-interactions,

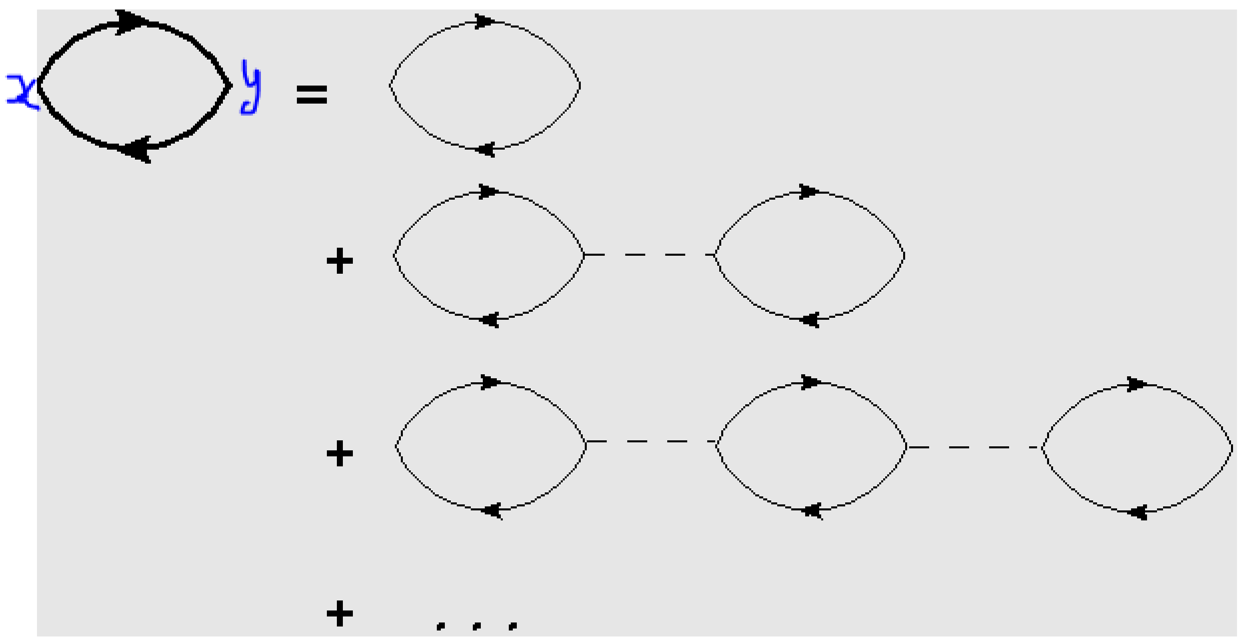
$$U(\phi,\omega) = -\frac{1}{2}g_{\sigma\omega}\omega^2\phi^2 + \frac{1}{3!}g_2\phi^3 + \frac{1}{4!}g_3\phi^4 + \frac{1}{4}c_3\omega^4.$$

Excitations in RRPA

- Relativistic RPA: Dyson equation

Lim &Horowitz, NPA 501, 729(89), Ma, et al, NPA 686, 173 (01)， Carriere, Horowitz, et al, ApJ 593, 463 (03)

相对论无规相位近似：给定初点与终点，包含各种可能的随机行走，幅度固定，相位随机



Dyson equation

- $\widetilde{\Pi}_L = \Pi_L + \widetilde{\Pi}_L D_L \Pi_L$
- $\widetilde{D}_L = D_L \ + D_L \ \Pi_L \widetilde{D}_L$

- $\widetilde{\Pi}_L = \left(1 - D_L \ \Pi_L\right)^{-1} \Pi_L; \ D_L = \left(1 - D_L \ \Pi_L\right)^{-1} D_L$
- Dielectric function: $\epsilon_L = \det(1 - D_L \ \Pi_L)$

Formulas of the RRPA method

- The longitudinal polarization tensor:

$$\Pi_L = \begin{pmatrix} \Pi_{00}^e & 0 & 0 & 0 \\ 0 & \Pi_s^n + \Pi_s^p & \Pi_m^p & \Pi_m^n \\ 0 & \Pi_m^p & \Pi_{00}^p & 0 \\ 0 & \Pi_m^n & 0 & \Pi_{00}^n \end{pmatrix}$$

Usually, the polarization just includes the density-dependent part.

$$D_L^0 = \begin{pmatrix} d_g & 0 & -d_g & 0 \\ 0 & -d_s^0 & 0 & 0 \\ -d_g & 0 & d_g + d_v^0 + d_\rho^0 & d_v^0 - d_\rho^0 \\ 0 & 0 & d_v^0 - d_\rho^0 & d_v^0 + d_\rho^0 \end{pmatrix}$$

Here D^0, d^0 just means no meson Self-interactions

Formulas of RRPA method

- Couplings between different mesons such as ω&ρ and σ&ω modify propagator matrix $D_L(q)$

$$\Gamma^0[\Phi] = S[\Phi] = \int d^4x\mathcal{L}$$

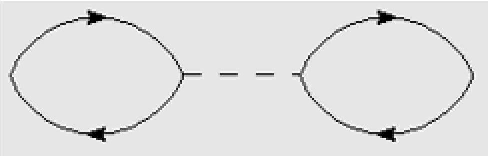
$$\int d^4z\Gamma_2(x,z)D(z-y) = \delta^4(x-y)$$

$$\Gamma_2^0(x,y) = D^{-1}(x,y) \quad \Gamma_2^0(x,y) = \frac{\delta^2\Gamma^0}{\delta\phi(x)\delta\phi(y)}$$

The two-point canonical vertex is first evaluated, then obtain the inverse of the matrix→D

The crossing term $d_{\sigma\omega}, d_{\omega\rho}$ included

$$D_L = \begin{pmatrix} d_g & 0 & -d_g & 0 \\ 0 & -d_\sigma & d_{\sigma\omega} & d_{\sigma\omega} \\ -d_g & d_{\sigma\omega} & d_g + d_V + 2d_{\omega\rho} & d_I \\ 0 & d_{\sigma\omega} & d_I & d_V - 2d_{\omega\rho} \end{pmatrix}$$



ω&ρ coupling is used to modifiy the density dependence of the symmetry energy; σ&ω coupling modifies in-medium nucleon mass.

Formulas of the RRPA method

$$d_g = -\frac{e^2}{q^2} = -\frac{4\pi\alpha}{q^2},$$

$$d_\sigma = \frac{g_\sigma^2((m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2)}{(m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2(m_\rho^{*2}-q^2)}$$

$$d_\omega = \frac{g_\omega^2(m_\rho^{*2}-q^2)(m_\sigma^{*2}-q^2)}{(m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2(m_\rho^{*2}-q^2)}$$

$$d_\rho = \frac{g_\rho^2((m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2)}{(m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2(m_\rho^{*2}-q^2)}$$

$$d_{\omega\rho} = \frac{-16g_\omega^2 g_\rho^2\Lambda_v W_0B_0(m_\sigma^{*2}-q^2)}{(m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2(m_\rho^{*2}-q^2)}$$

$$d_{\sigma\omega} = \frac{2g_\omega g_\sigma g_{\sigma\omega}\omega_0\phi_0(m_\rho^{*2}-q^2)}{(m_\rho^{*2}-q^2)(m_\omega^{*2}-q^2)(m_\sigma^{*2}-q^2)-(16g_\omega g_\rho\Lambda_v W_0B_0)^2(m_\sigma^{*2}-q^2)+(2g_{\sigma\omega}\omega_0\phi_0)^2(m_\rho^{*2}-q^2)}$$

$$m_\sigma^{*2} = -\frac{\partial^2\mathcal{L}}{\partial\phi^2}, \quad m_\omega^{*2} = \frac{\partial^2\mathcal{L}}{\partial\omega_0^2}, \quad m_\rho^{*2} = \frac{\partial^2\mathcal{L}}{\partial b_0^2}.$$

Li et al 2024, CPC48,034105.

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Zero sound and dispersion relation

- Zero sound meets the low energy limit: $\mathrm{q}_0 \to 0$, for $q \to 0$. It is a space-like excitation.
- While $q \to 0$, the limit $q_0 \to \omega_0$ defines an optical sound, originating from the time-like excitation.
- In nuclear matter, the matter fluctuation also gives rise to zero sound which meets the dispersion limit： $\mathrm{q}_0 \to 0$, for $q \to 0$.
- Question: how does zero sound behave at finite temperature?

A few examples

- Zero sound in cryogenic ^{3}He, by Landau in 1957， verified by Roach et al, PRL 736 (1976)
- Holographic quantum fluid： Karch, et al PRL102, 051602 (2009).
- Various zero-sound modes may increase the sub-threshold fusion rates in astrophysical nucleosynthesis: Tumino et al Nature 557, 7707 (2018), Zhang et al Nature 610, 7933 (2022)
- Zero-sound modes in nuclei: GMR and pigmy resonances
- Zero sound in neutron superfluidity of NS matter possibly affects neutron star cooling: Aguilera et al, PRL**102**, 091101 (2009)； Leinson, PRC83, 055803 (2011)
- Possible non-thermal irradiation from zero sound in pulsars: Svidzinsky, APJ590, 1 (2003)

Dyson equation

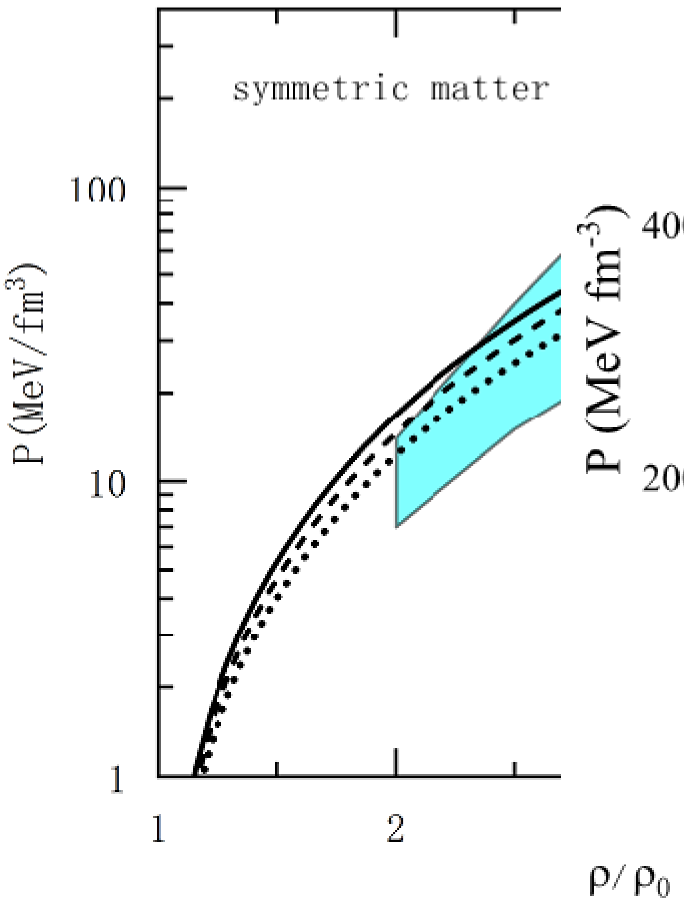
- $\widetilde{\Pi}_L = \Pi_L + \widetilde{\Pi}_L D_L \Pi_L \qquad vertex\text{:}\, \gamma_\mu, \gamma_\nu$

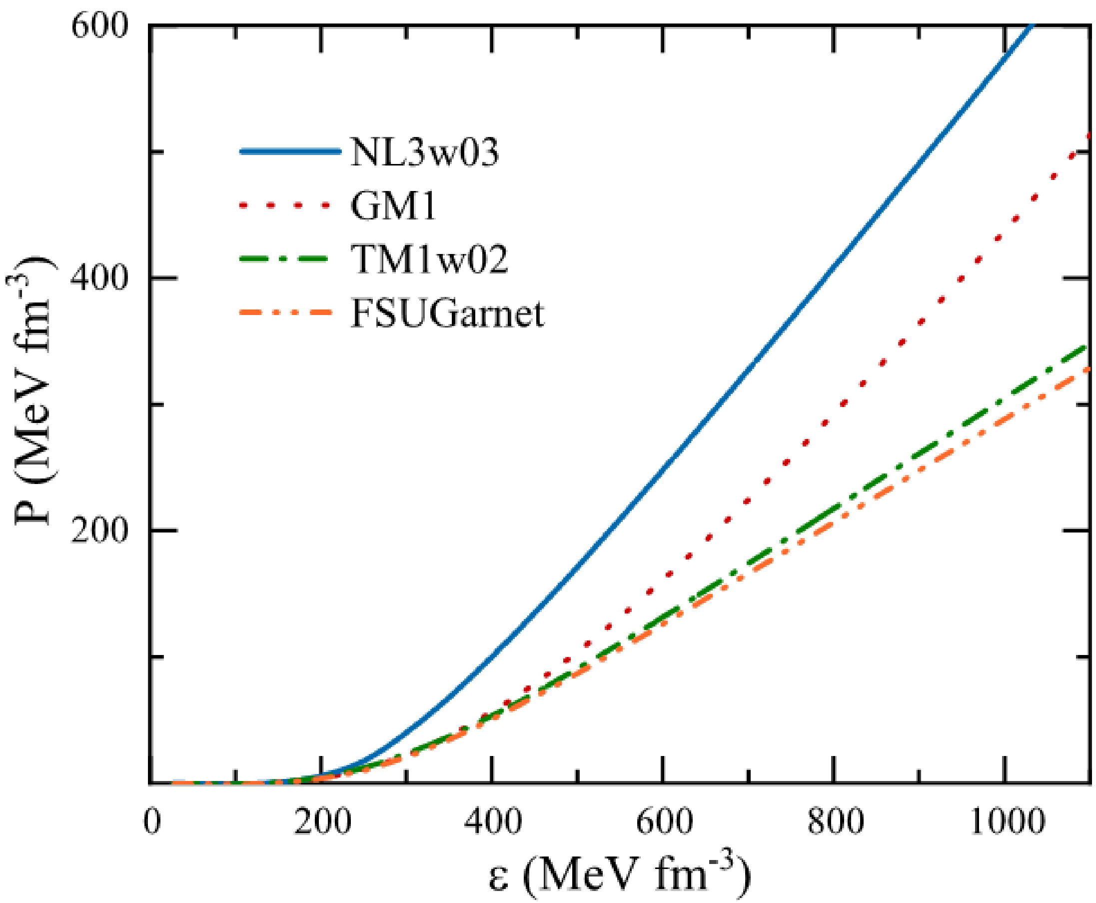
$$\widetilde{D}_L = D_L \ + D_L \ \Pi_L \widetilde{D}_L$$

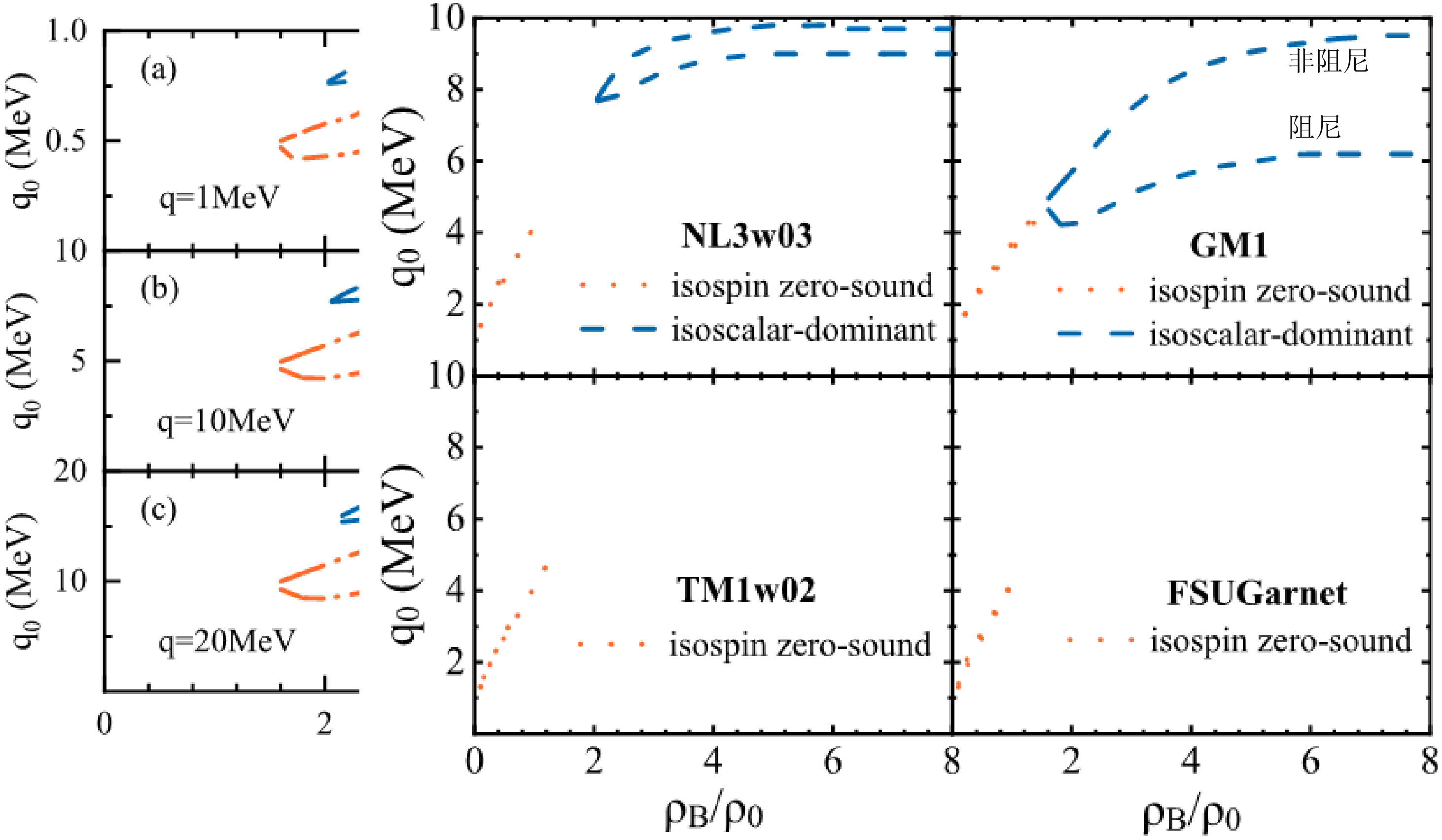
- $\widetilde{\Pi}_L = \left(1 - D_L \ \Pi_L\right)^{-1} \Pi_L; \ D_L = \left(1 - D_L \ \Pi_L\right)^{-1} D_L$
- Dielectric function: $\epsilon_L = \det(1 - D_L \ \Pi_L)$
- $\boldsymbol{\epsilon_L}$ **=0 determines collective modes. Non-vanishing imaginary gives the damping mode**
- $\widetilde{\boldsymbol{\Pi}}_{\mathbf{A}} = \boldsymbol{\Pi}_{\boldsymbol{A}} + \widetilde{\boldsymbol{\Pi}}_A D_A \Pi_A,$ Axial vertices: $\gamma_5 \gamma_\mu, \gamma_5 \gamma_\nu$

Numerical results

- Uncertainty of high-density EOS in RMF



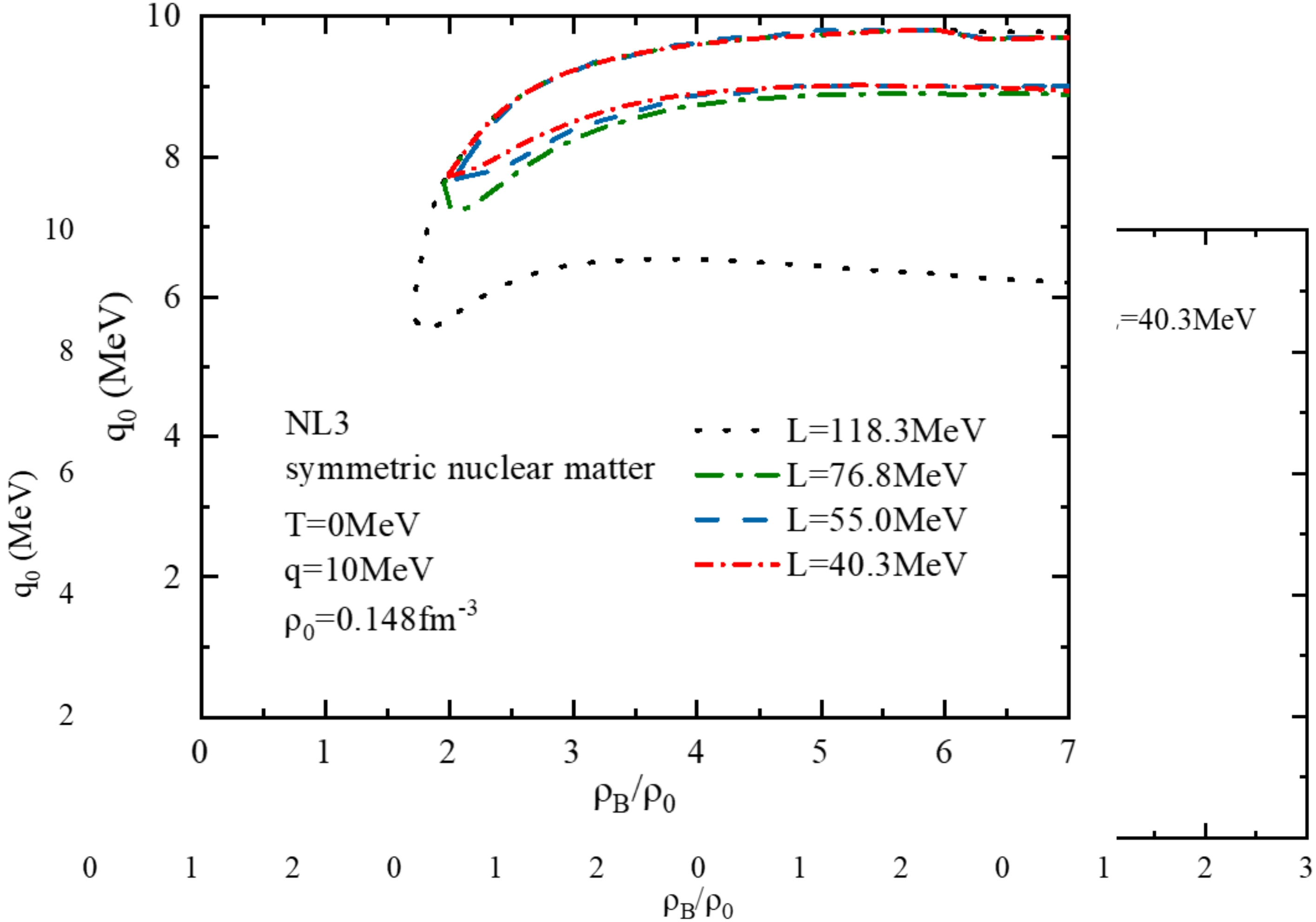


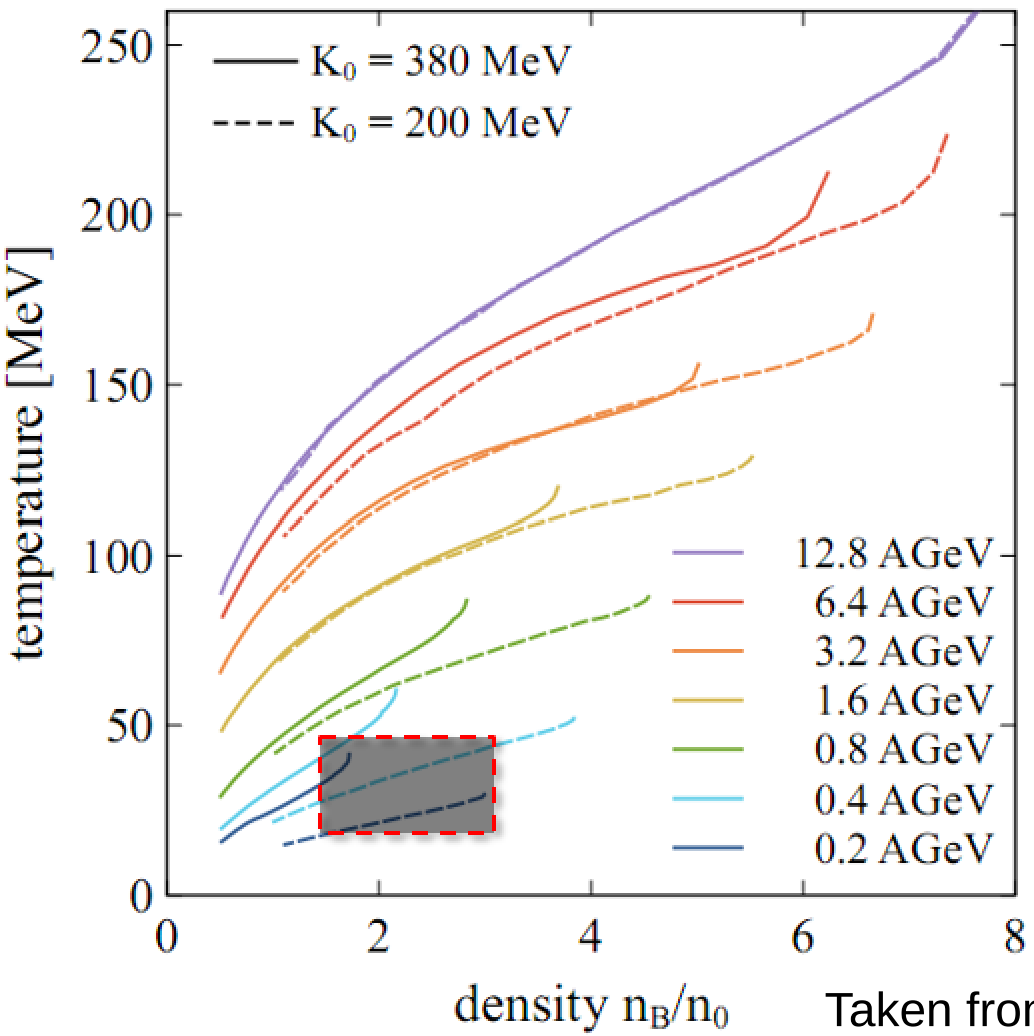


Damping：$Im\Pi \neq 0$

Zero sound vs. symmetry energy

At high density, the SE softening shifts the ZS energy upward; at low density, the SE softening suppresses the isovector ZS





Implication to heavy ion collision: whether there's effect?

Taken from Sorensen, et al, Dense Nuclear Matter Equation of State from Heavy-Ion Collisions , **arXiv: 2301.13253 (2023)**

Implication to heavy ion collision

- Soft EOS has no zero sound which arises by stiffening the EOS through reducing c_3.

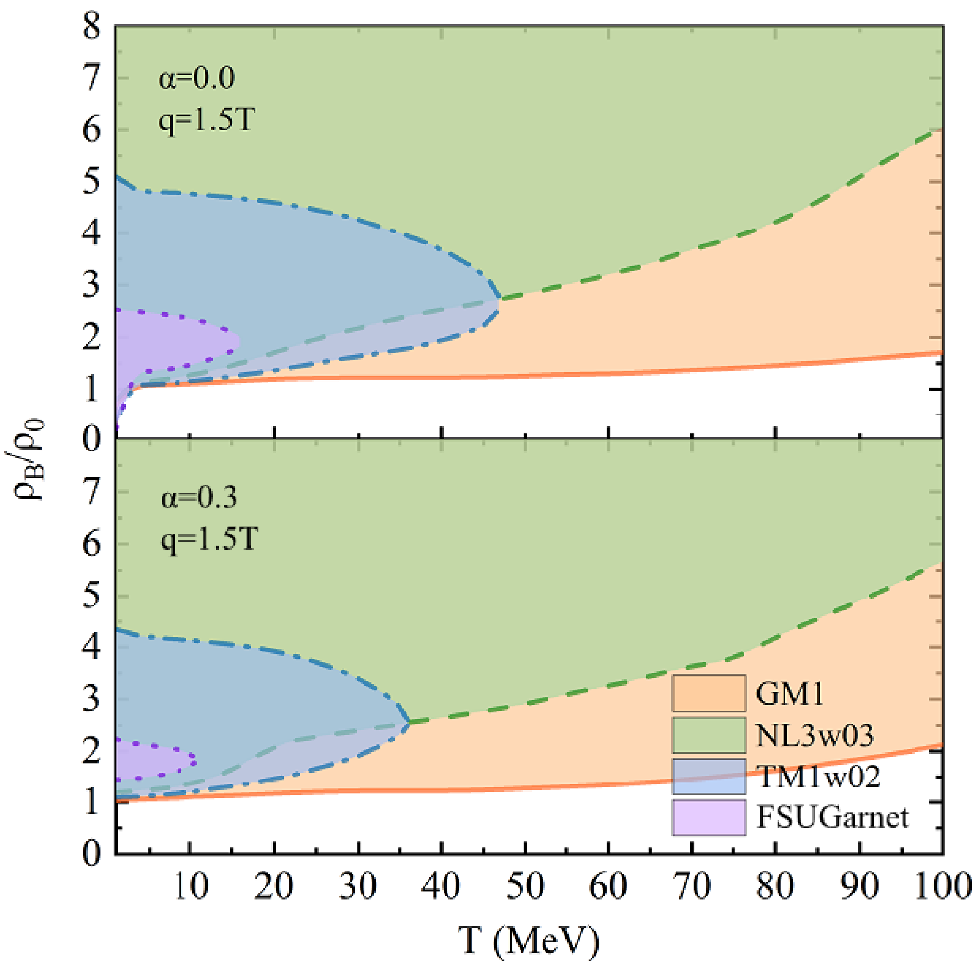
Isoscalar zero-sound			
	q (MeV)	The onset density(n0)	q 0(MeV)
TM1w02 C3=20	10	2.3	7.5
	20	2.4	15.1
	60	2.6	46.4
	80	2.8	63.5
	90	3.0	73.8

Zero Sound & Heavy-ion collision

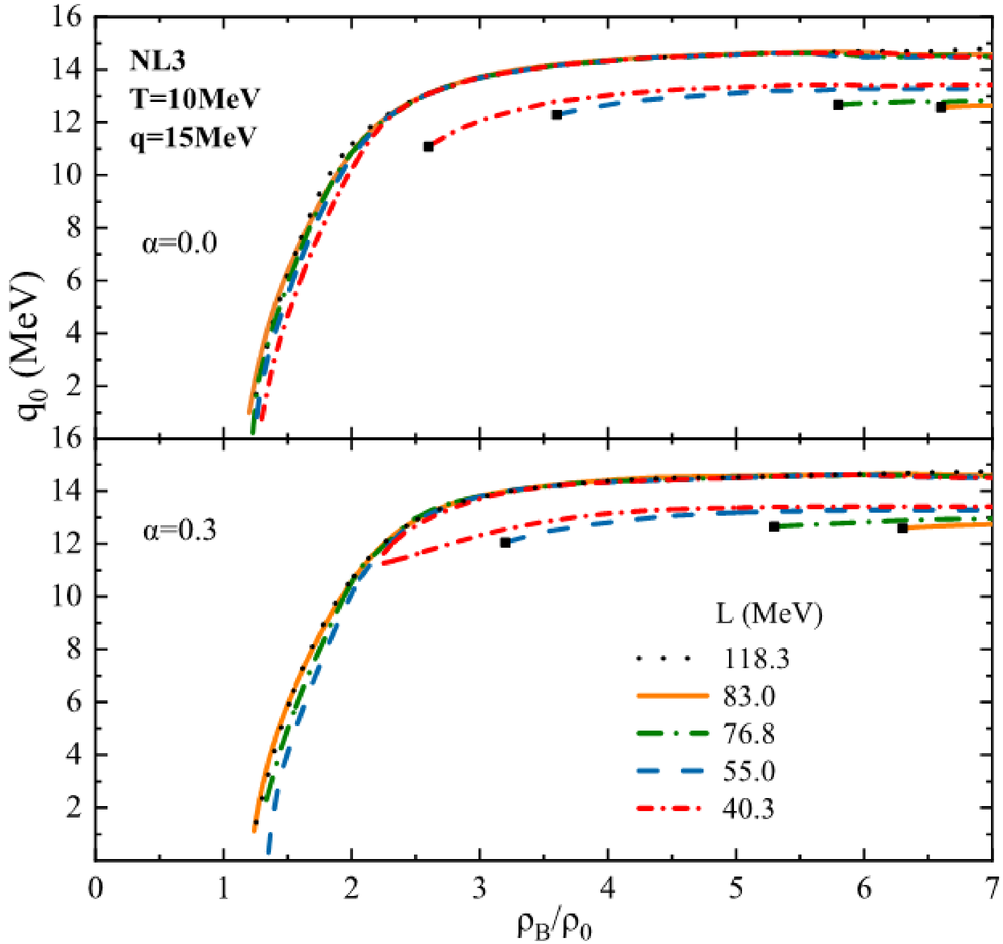
- Dense matter created by HIC， e.g. with 0.5AGeV: T=30-50 MeV， density > 2ρ_0
- Central energy of zero sound can exceed the thermalized energy (T) for large momenta, according to the dispersion relation $q \sim q_0$，
- As collective modes, energetic zero sound can affect the time scales of the formation & evolution of dense matter that are expected to be measurable.
- Precision measurement for constraining EOS at high density.

J. Ye et al. Phys. Rev. C108, 044312 (2023)

Zero sound at finite temperature

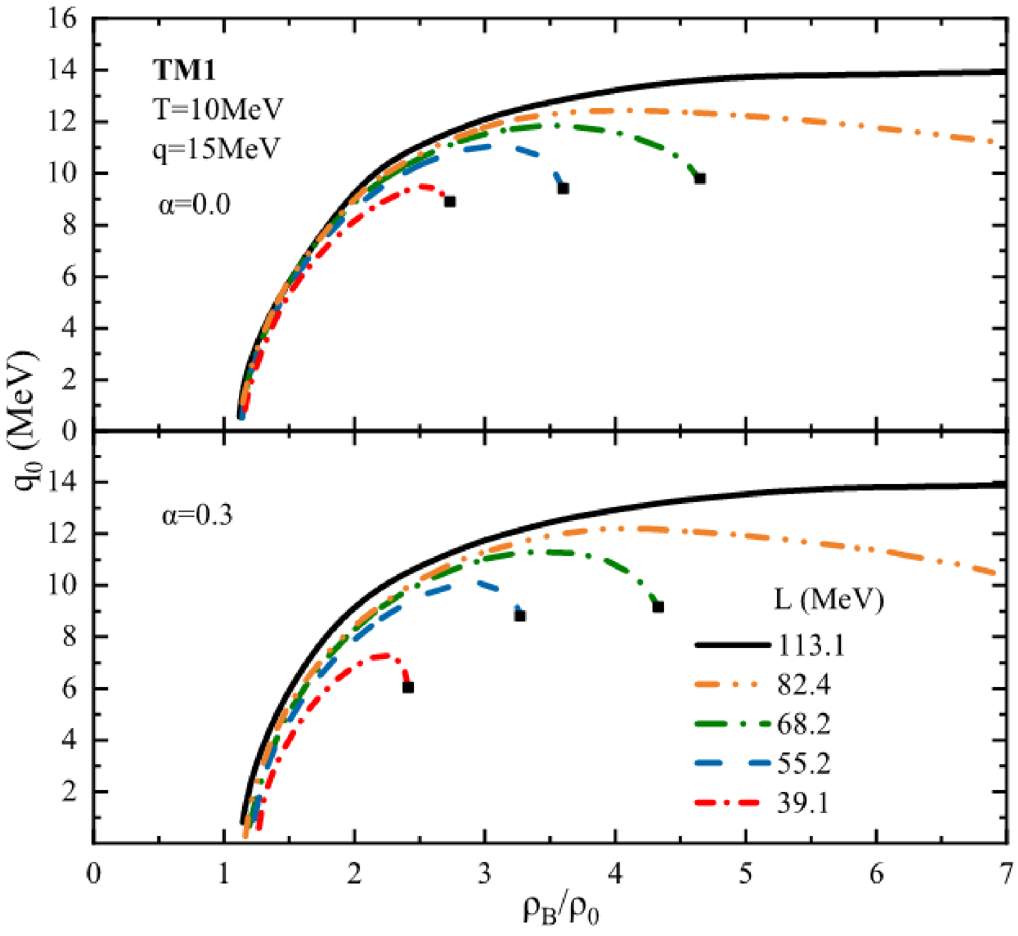


Sensitivity to symmetry energy in stiff RMF model NL3

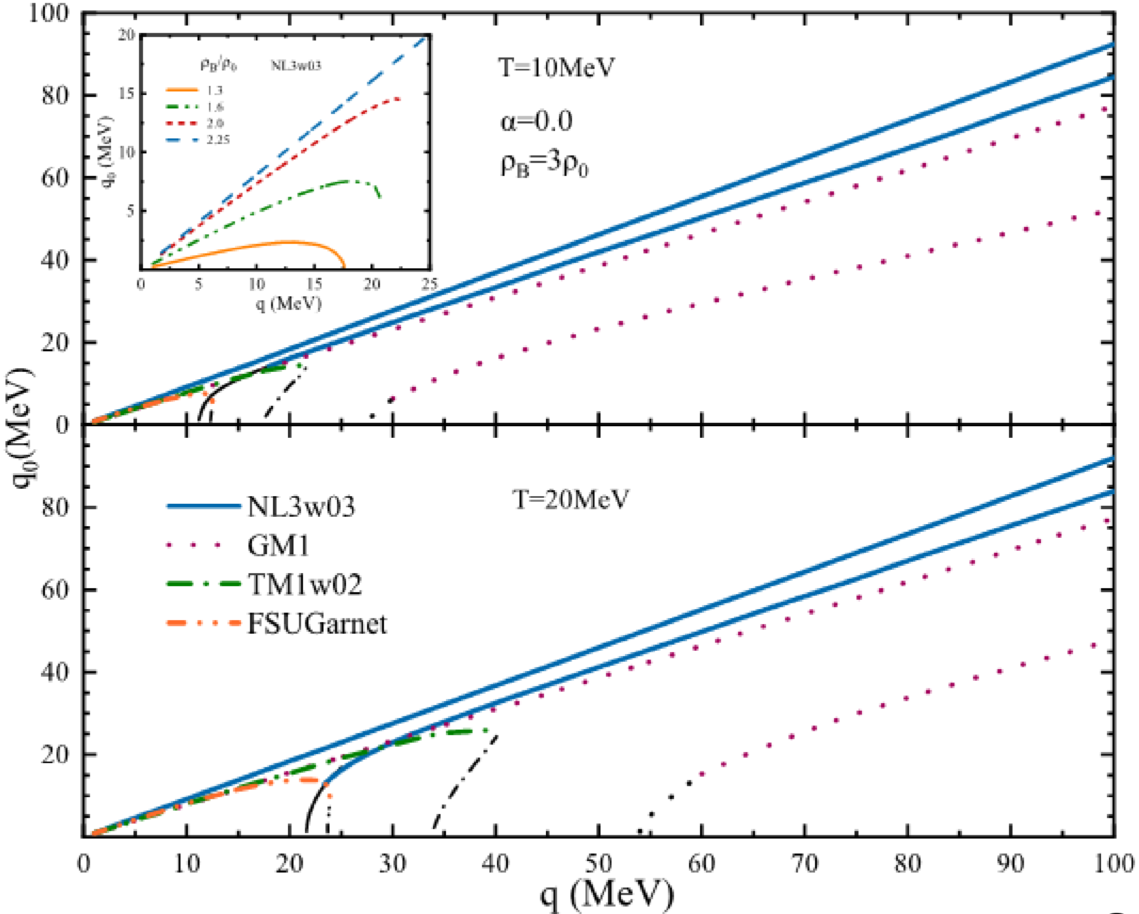


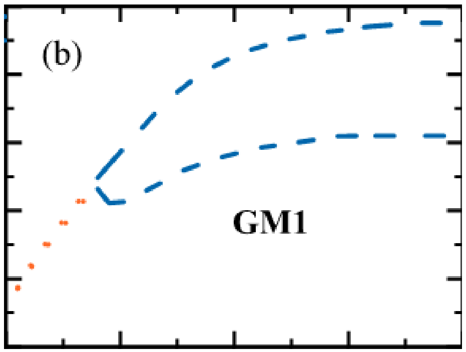
$$L = 3\rho_0 \left. \frac{\partial E_{sym}}{\partial \rho_B} \right|_{\rho_0}$$

Sensitivity to symmetry energy in soft RMF model TM1



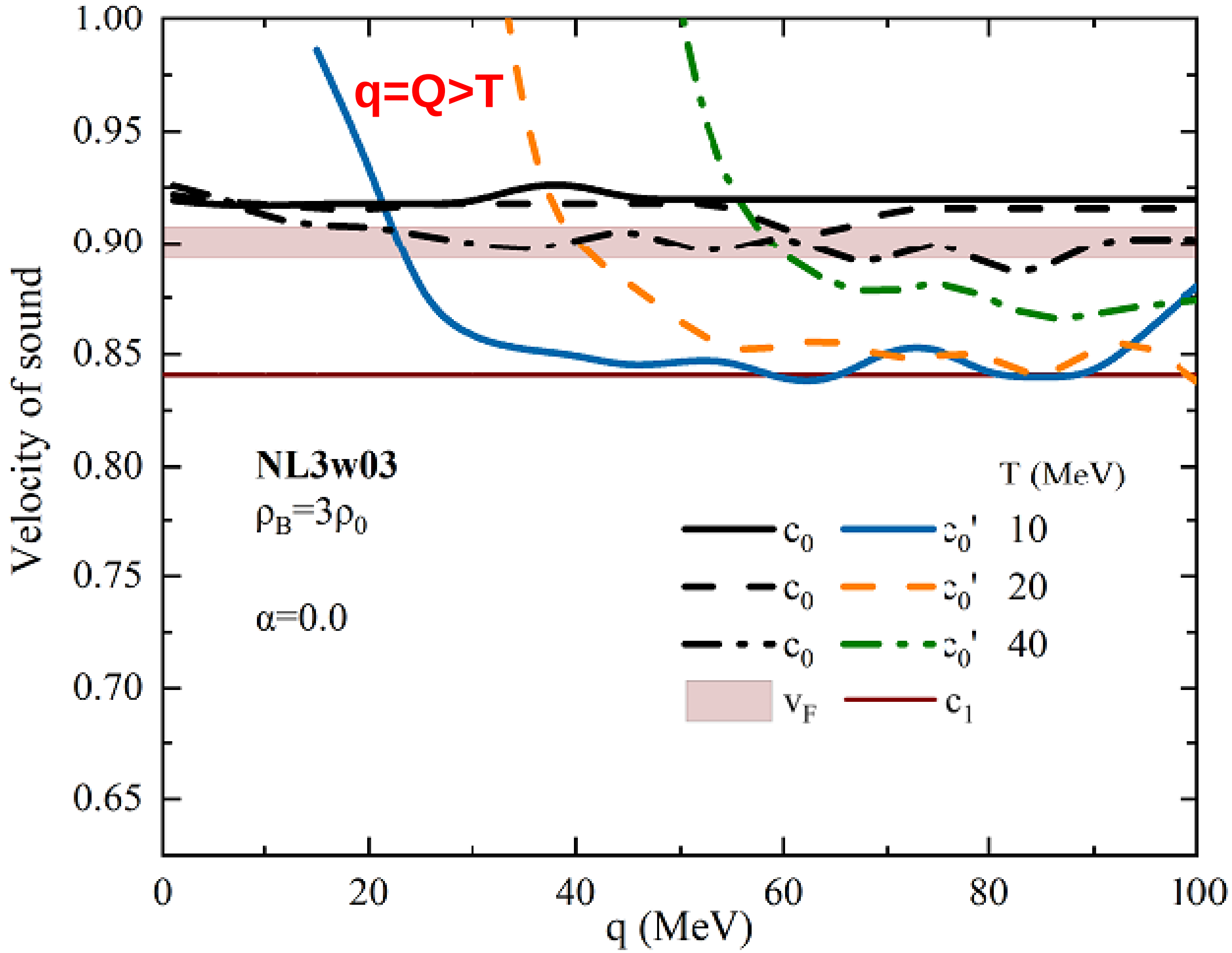
Thermal bifurcation of zero sound





Space-like condition: $q^2 \geq q_0^2$

Zero sound transforms into first sound



Jing Ye et al, 2024

Sound velocity $\frac{\partial q_0}{\partial q}, c_1^2 = \frac{\partial P}{\partial \epsilon}$

e^{iQr}: plane-wave harmonic oscillation & space-like condition

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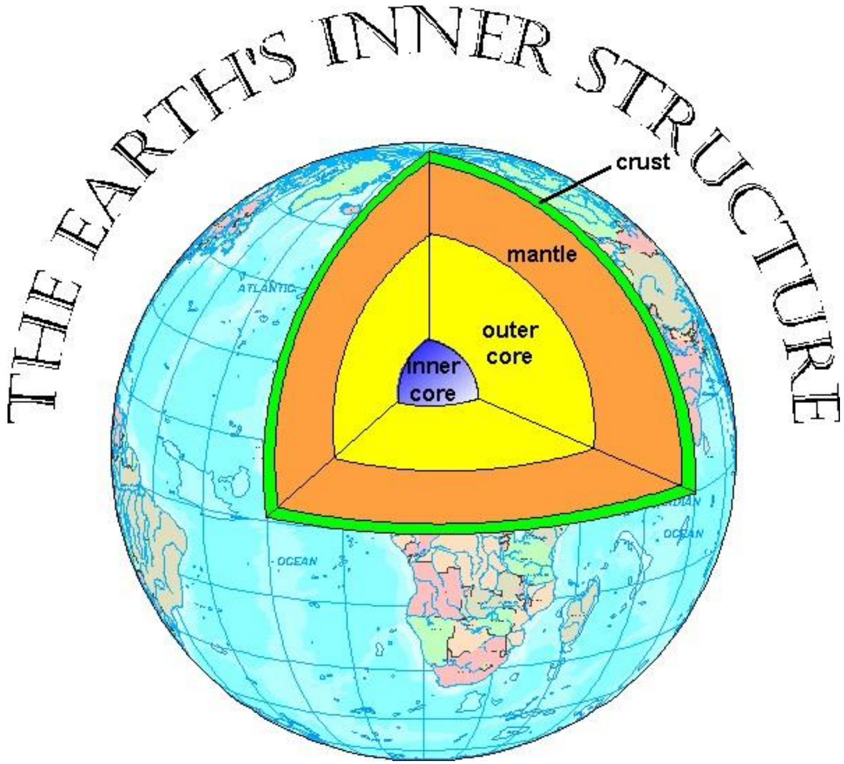
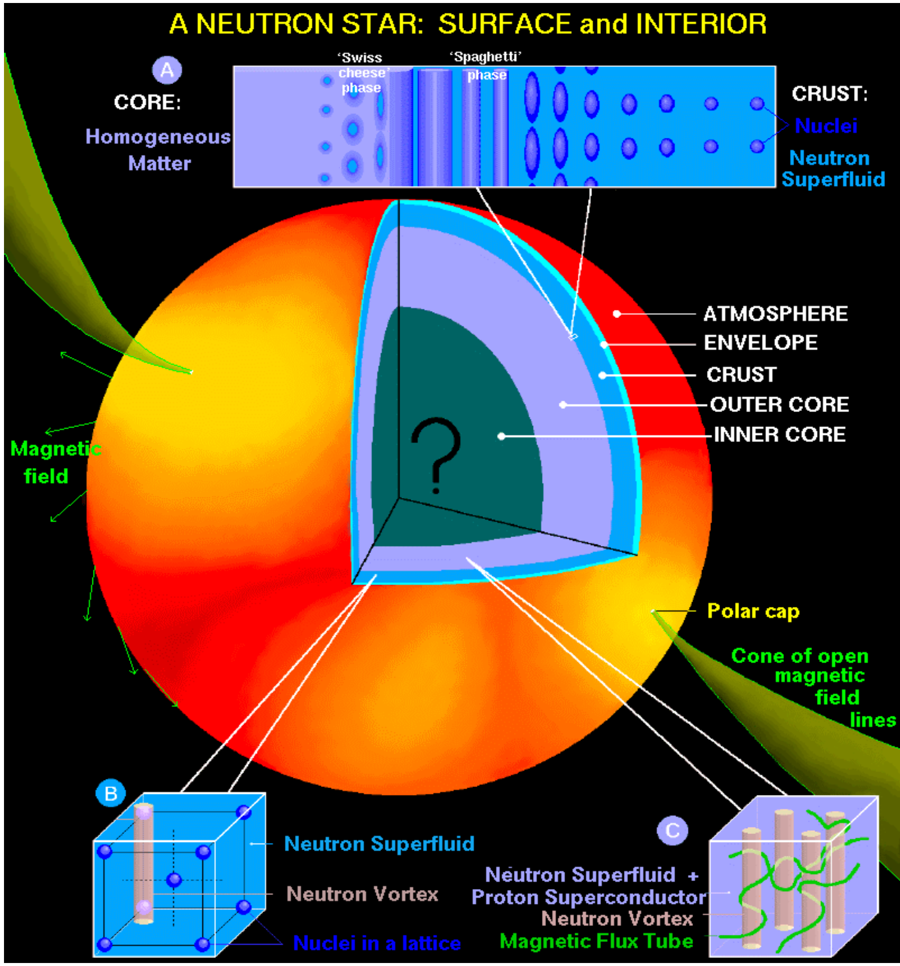


$M_{earth} = 5.9737 \times 10^{24}$ kg



Earth: mass $\approx 3 \bullet 10^{-6} M_{sun}$, radius=6378km, compactness $M/R \approx 5 \bullet 10^{-10}$

Neutron stars: $0.2\ M_{sun} \sim$ mass $\leq 2\ M_{sun}$, radius $\approx$ 12km, compactness $M/R \simeq 0.2$

Outer core of neutron star becomes unstable under the density fluctuation.

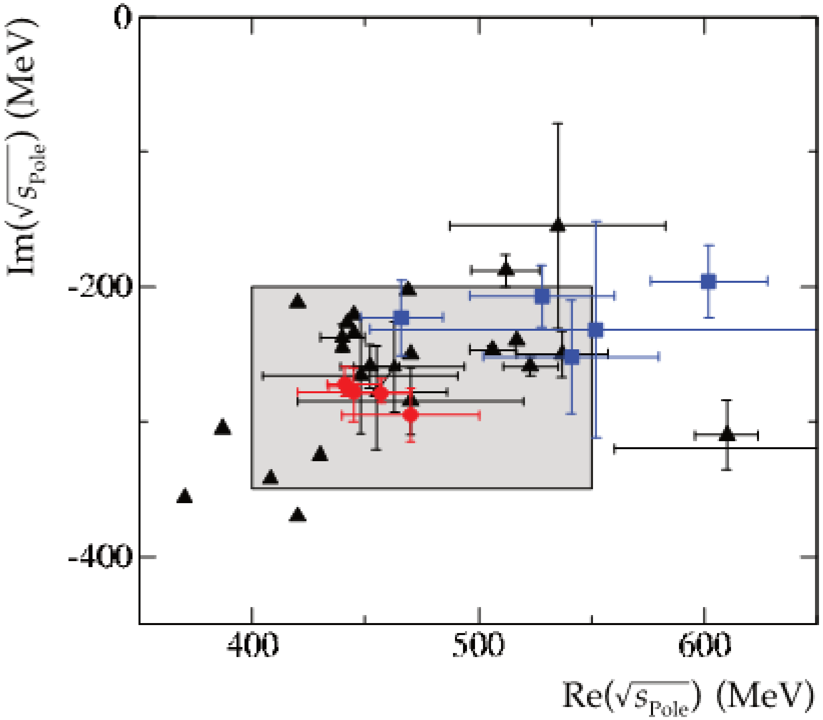
Courtesy of Prof. Bao-An Li

J.M. Lattimer and M. Prakash, *Science Vol. 304 (2004) 536-542.*

m_σ & vacuum

Sigma meson

- Higgs-like scalar in linear-σ model
- NJL model: $1 - \Re\Pi_\sigma(m_\sigma) = 0$; $< \bar{\psi}\psi > \chi$ order parameter
- Dilaton：SB of scale invariance, see Paeng, et al. PRC93,055203(16)
- Mixed with glueball, see Ochs, JPG40, 043001 (13)
- 2π resonance



PDG2022

Sigma meson in renormalized models

- The interacting lagrangian reads

$$\mathcal{L}_{\rm int} = \overline{\psi}[\gamma_\mu(i\partial^\mu - g_\omega V^\mu) - \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\boldsymbol{b}^{\boldsymbol{\mu}} - (M - g_\sigma\phi)]\psi - U(\phi), \quad (1)$$

where $U(\phi)$ is the nonlinear σ meson self-interactions $\frac{g_2}{3!}\phi^3 + \frac{g_3}{4!}\phi^4$. Without $U(\phi)$, the scalar self-energy is renormalized in a diagrammatic scheme with the counterterms [Chin, Ann. Phys. 1977]

$$\mathcal{L}_{CT} = \sum_{n=1}^{4} \frac{\alpha_n}{n!}\phi^n$$

With the inclusion of the nonlinear self-interactions, the divergent terms are different and can be worked out using the techniques of the path integral method**[See Serot&Walecka, Adv. Nucl. Phys. 16(86)].**

- With the counterterms as follows

$$L_{CT} = \Sigma_{n=1}^{4} \frac{\bar{\alpha}_n}{n!} \phi^n$$

In RHA, $U(\phi)$ is renormalized as

$$\begin{aligned} U^R(\phi) =& \frac{1}{2} m_\sigma^2 \phi^2 + \frac{1}{3!} g_2 \phi^3 + \frac{1}{4!} g_3 \phi^4 + \frac{1}{(8\pi)^2} \times \\ & \left[(m_\sigma^2 + g_2\phi + \frac{g_3\phi^2}{2})^2 \ln(1 + \frac{g_2\phi}{m_\sigma^2} + \frac{g_3\phi^2}{2m_\sigma^2}) - m_\sigma^2 (g_2\phi + \frac{g_3\phi^2}{2}) \right. \\ & \left. - \frac{3}{2}(g_2\phi + \frac{g_3\phi^2}{2})^2 - \frac{(g_2\phi)^2}{3m_\sigma^2}(g_2\phi + \frac{3g_3\phi^2}{2}) + \frac{(g_2\phi)^4}{12m_\sigma^4} \right]. \end{aligned}$$

The effective σ mass is defined $m_\sigma^{*2} = \partial^2 U^R(\phi)/\partial\phi^2$.

Longitudinal polarization with vacuum part

$$\Pi_L = \begin{pmatrix} \Pi^e_{00D} & 0 & 0 & 0 \\ 0 & \Pi^n_{sD} + \Pi^p_{sD} & \Pi^p_m & \Pi^n_m \\ 0 & \Pi^p_m & \Pi^p_{00D} & 0 \\ 0 & \Pi^n_m & 0 & \Pi^n_{00D} \end{pmatrix}$$

$$\Pi_L = \begin{pmatrix} \Pi^e_{00D} + \Pi^e_{00f} & 0 & 0 & 0 \\ 0 & \Pi^n_{sD} + \Pi^p_{sD} + \Pi^n_{sf} + \Pi^p_{sf} & \Pi^p_m & \Pi^n_m \\ 0 & \Pi^p_m & \Pi^p_{00D} + \Pi^p_{00f} & 0 \\ 0 & \Pi^n_m & 0 & \Pi^n_{00D} + \Pi^n_{00f} \end{pmatrix}$$

$$\begin{aligned} \Pi^0_{sf}(q^2) \quad = \quad & \frac{g^{*\,2}_\sigma}{\pi^2} \sum_{i=p,n} \left[\frac{m^2_\sigma - q^2}{8} - \frac{3}{4} \int_0^1 dx (M^{*\,2} + x(x-1)q^2) \ln \frac{x(x-1)q^2 + M^{*\,2}}{x(x-1)m^2_\sigma + M^2} \right. \\ & \left. - \frac{3}{4} \int_0^1 dx (M^{*\,2} - M^2) \ln(1 + \frac{x(x-1)m^2_\sigma}{M^2}) - \frac{1}{2}(3M\Sigma_s - \frac{9}{2}\Sigma^2_s) \right], \end{aligned}$$

where $\Sigma_s = M_i - M^*_i$.

Renormalization at $q^2 = 0, m^2_\sigma$?

Scalar polarization (0th order)

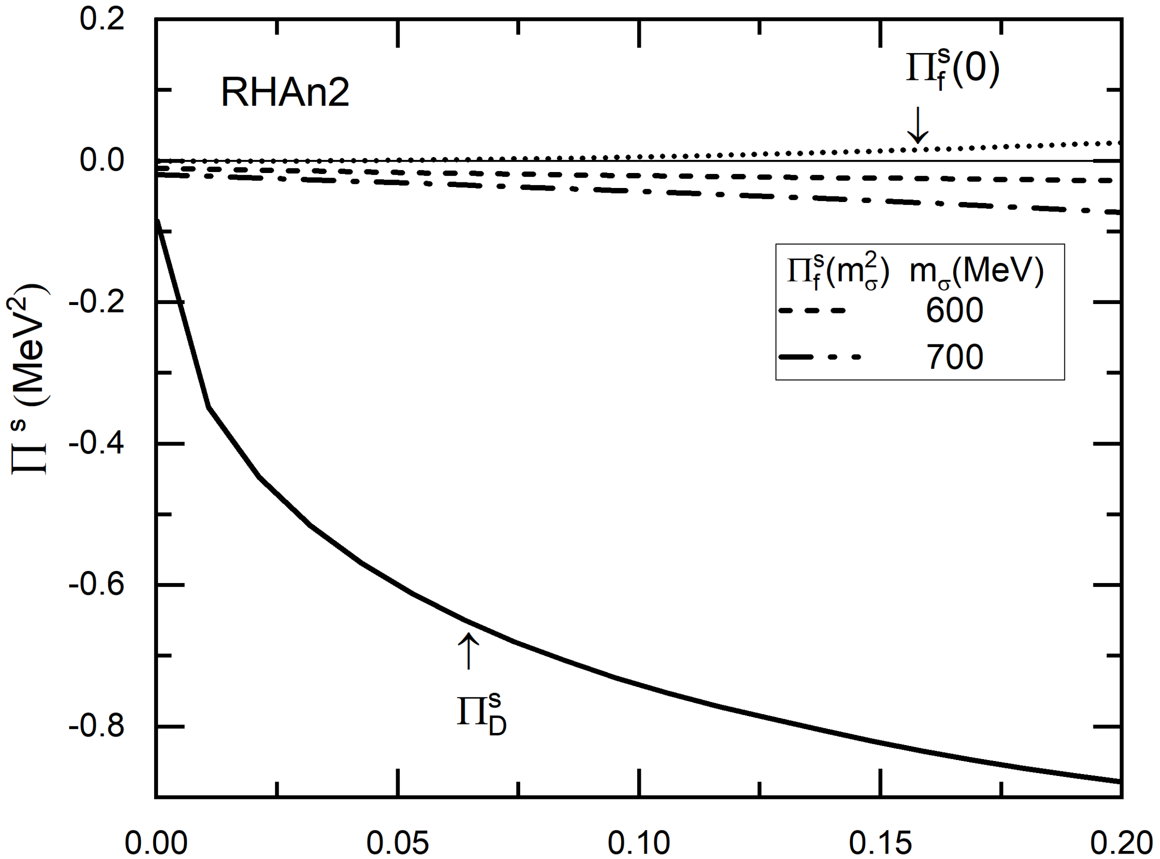


FIG. 1: The $\Pi_f^s(q^2)$ and Π_D^s as a function of density. Three curves of $\Pi_f^s(q^2)$ are evaluated at $q^2 = m_\sigma^2$ ($m_\sigma = 600, 700$ MeV) and 0, respectively.

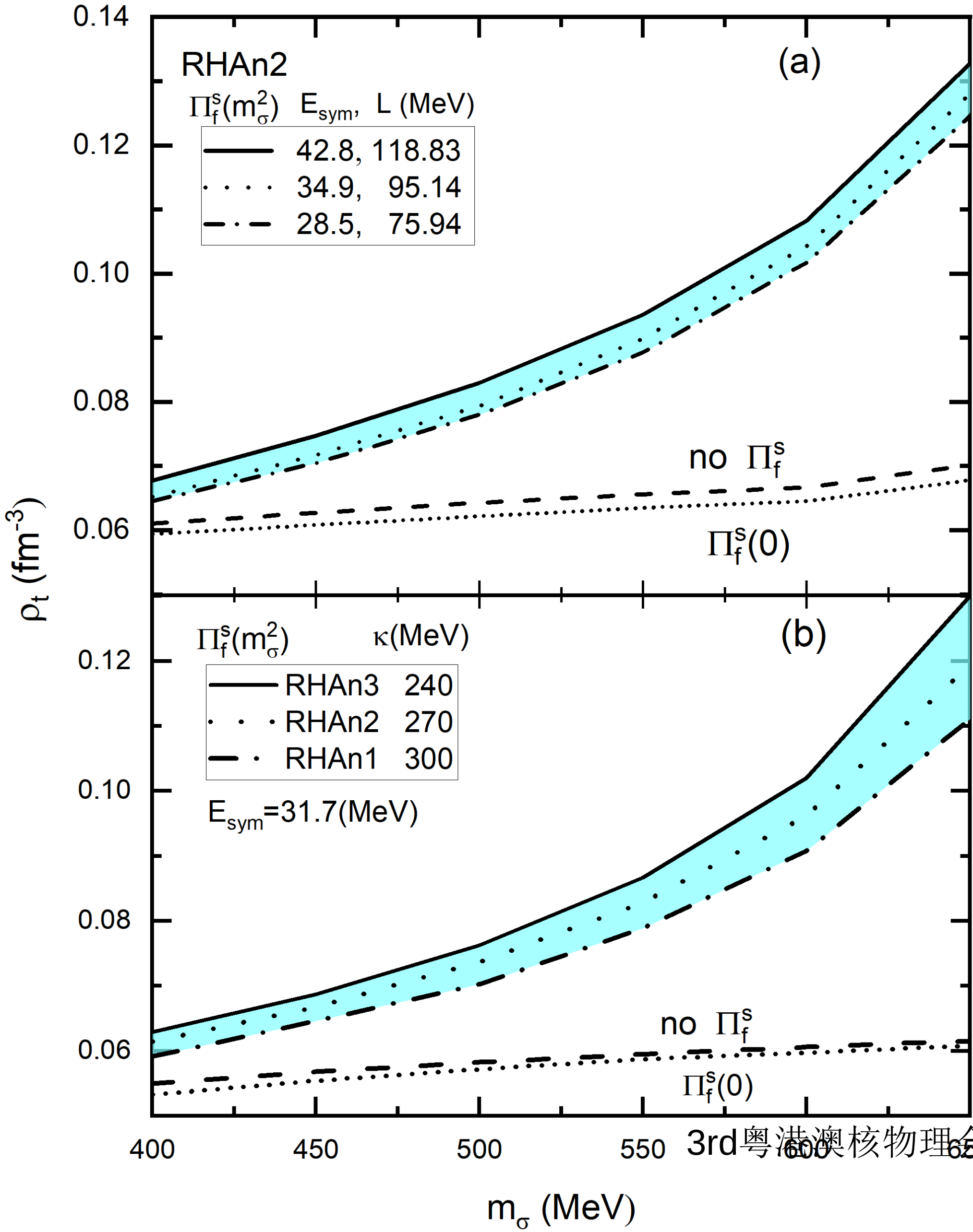
Parametrizations

Model	g_σ	g_ρ	g_ω	g_2	g_3	M^*/M	κ	ρ_t	P_t
RHA	7.99	3.91	9.79	0	0	0.726	461.1	0.078	0.265
RHAn1	7.70	4.03	8.39	25.9	24.5	0.784	300.0	0.073	0.321
RHAn2	7.51	4.09	7.71	40.8	-45.2	0.809	270.0	0.070	0.311
RHAn3	7.11	4.15	6.51	61.1	-75.7	0.847	240.0	0.067	0.276

TABLE I: Various RHA parametrizations and some properties of nuclear matter without (i.e., RHA) and with the inclusion of the σ-meson self-interacting terms. κ, ρ_t and P_t are the incompressibility (MeV), transition density (fm^{-3}) and pressure (MeV$/fm^3$), respectively. Here, the masses of σ, ω, and ρ mesons are 512, 783, and 770 MeV, respectively.

Where $\rho_0 = 0.16 \quad fm^{-3}, \quad E_b = -16.00 \quad MeV.$

Transition density vs m_σ



ρ_t versus m_σ moderately shaded by the EOS uncertainty: The influence of the symmetry energy (a) and of the incompressibility (b).

Sensitivity from cancellation in ϵ_L

- Though the difference between $\Pi_f^s(q^2)$ at q^2=0 and m_σ^2 is small, their effects on the ρ_t depart in sharp contrast.

$$\epsilon_L = [1 + d_s(\Pi_D^s + \Pi_f^s)] \cdot [1 + 4 d_\omega d_\rho \Pi_L^2 + 2(d_\omega + d_\rho)\Pi_L] - 4(1 + 2 d_\rho \Pi_L) d_s d_\omega \Pi_M^2 ,$$

- $\Pi_f^s(m_\sigma^2)$, which is of the same sign of Π_D^s, shifts the zero point of the dielectric function to a larger density, that is, the transition density ρ_t grows clearly with the rise of m_σ.

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NS crustal moment of inertia vs m_σ, L

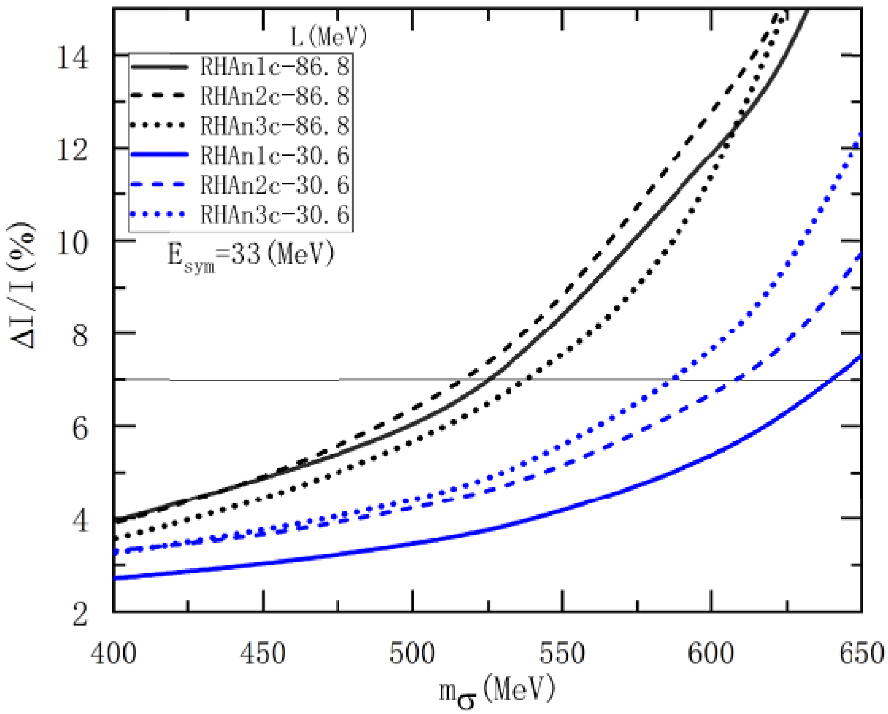
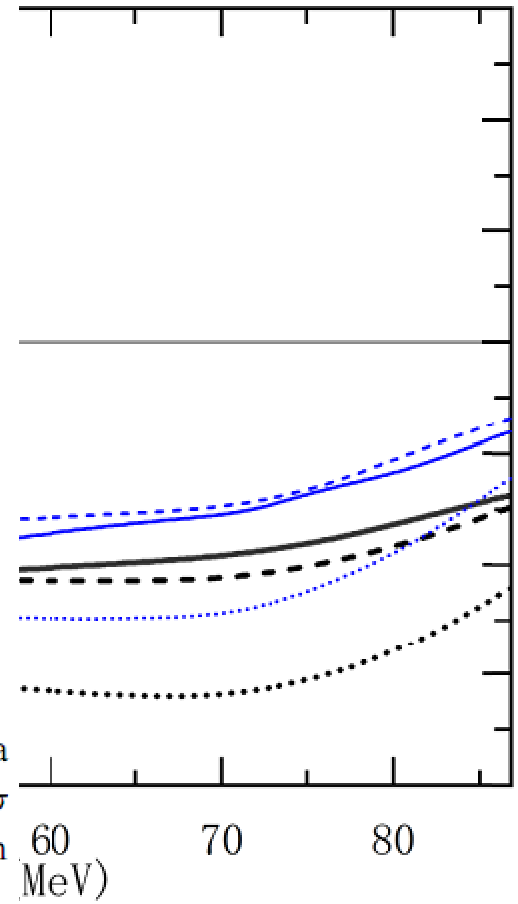


Fig. 7. (color online) Fraction of crustal moment of inertia $\Delta I/I$ in the $1.4M_\odot$ neutron star as a function of the mass of σ meson. The results are obtained using the complete formula in Eq. (21).



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Summary

- Relativistic RPA is used to determine collective in non-linear RMF models at finite temperature.
- The $\rho - T$ space for zero sounds are found to be sensitive to the EOS stiffness.
- Symmetry energy affects bifurcation or terminal density of zero sound.
- In renormalized relativistic models, the matter stability is found to be sensitively affected by the scalar polarization in the Dirac sea, leading to the dependence of the NS core-crust transition density on the scalar meson mass that is still of large uncertainty.

Thank you for your attention！