Universal relations in neutron stars

文德华

【1】Shen Yang, Dehua Wen∗, Jue Wang and Jing Zhang, Exploring the universal relations with the correlation analysis of neutron star properties, Phys. Rev. D 105, 063023 (2022)

【2】Yuxi Li, Jue Wang, Zehan Wu and Dehua Wen*, Inferring the gravitational binding energy and moment of inertia of PSR J0030 + 0451 and PSR J0740 + 6620 from new universal relations, Class. Quantum Grav. 39, 035014(2022)

【3】Y. X. Li, H. Y. Chen, D. H. Wen* and J. Zhang, Constraining the nuclear symmetry energy and properties of the neutron star from GW170817 by Bayesian analysis, Eur. Phys. J. A, 57, 31(2021)

【4】W. J. Sun, D. H. Wen* and J. Wang, New quasi-universal relations for static and rapid rotating neutron stars, Phys. Rev. D 102, 023039 (2020)

【5】R. R. Jiang, D. H. Wen* and H. Y. Chen, Universal behavior of compact star based upon gravitational binding energy, Phys. Rev. D, 100, 123010 (2019).

【6】N. Zhang, D. H. Wen* and H. Y. Chen，Imprint of the speed of sound in nuclear matter on global properties of neutron stars，Phys. Rev. C 99, 035803 (2019)

【7】D. H. Wen, B. A. Li, * H. Y. Chen and N. B. Zhang, GW170817 implications on the frequency and damping time of f -mode oscillations of neutron stars, Phys. Rev. C 99, 045806 (2019)

【8】H. Y. Chen, D. H. Wen* and N. Zhang, Suitable resolution of EOS tables for neutron star investigations, Chin. Phys. C 43, 054108(2019)

【9】B. A. Li, P. G. Krastev, D. H. Wen and N. B. Zhang, Towards understanding astrophysical effects of nuclear symmetry energy, Eur. Phys. J. A 55, 117 (2019)

【10】Jue Wang, Shen Yang, and Dehua Wen*, Constraining Properties of Massive Neutron Star through R-Mode , Eur. Phys. J. A, accepted (2022)

CONTENT

CONTENT

- ➢ **Brief introduction to neutron star research**

- ➢ **Universal relations (UR) and correlations of NS**

- • **UR of f-mode**

- • **UR of gravitational binding energy**

- • **UR of rotational neutron star**

- • **UR and correlation analysis**

- ➢ **Conclusion**

中子星基本结构

- 大气层：厚度仅有数厘米。

- 外壳层：$\rho = 1 \times 10^4 g/cm^3 \sim 4 \times 10^{11} g/cm^3$
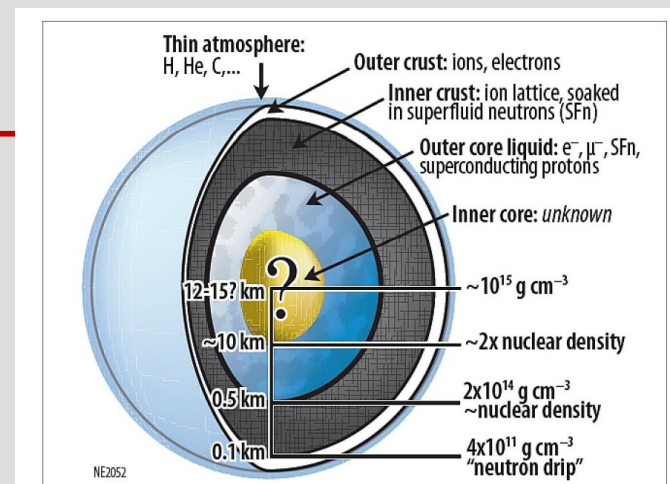
在此区域内，电子作为自由费米气可以在壳层内自由移动。随着密度不断增大，电子和质子将通过中子化过程产生中子并发射中微子，以降低能量。晶格内的原子核变得更加丰中子。

- 内壳层：$\rho = 4 \times 10^{11} g/cm^3 \sim 2 \times 10^{14} g/cm^3$

密度高于中子滴出密度，晶格将被中子气笼罩，晶格内的原子核将在库仑斥力下变形为"pasta-like"结构。

- 内核：$\rho > 2 \times 10^{14} g/cm^3$

由轻子，质子和中子组成。随密度增大，能量升高，超子或将产生。甚至在中心处夸克可能解禁闭，从而产生夸克胶子等离子体。

内壳层及内核高密部分的物态方程还具有很大的不确定性

Accurate observations of massive neutron stars

PSR J1614—2230 (original mass measurement in Demorest et al. 2010 and current mass measurement in Arzoumanian et al. 2018)

$$M = 1.908 \pm 0.016 \; M_\odot$$

PSR J0348+0432 (Antoniadis et al. 2013)

$$M = 2.01 \pm 0.04 \; M_\odot$$

PSR J0740+6620 (Cromartie et al. 2019)

$$M = 2.14^{+0.10}_{-0.09} \; M_\odot$$

修正后：$M = 2.08^{+0.07}_{-0.07} M_\odot$

Possible observations of massive neutron stars：

GW190814

$2.50 \; M_\odot$ —$2.67 \; M_\odot$

B. P. Abbott *et al:*, Astrophys. J. Lett. **896**, L44 3 (2020)

[1] Demorest, P. B., Pennucci, T., Ransom, S. M., et al., 2010, Nature, 467, 1081
[2] Arzoumanian, Z., Baker, P. T., Brazier, A., et al. 2018, ApJ, 859, 47
[3] Antoniadis, J., Freire, P. C. C., Wex, N., et al. 2013, Science, 340, 448
[4] Cromartie, H. T., Fonseca, E., Ransom, S. M., et al. 2020, NatAs, 4，27

Mass radius observation constraints from NICER
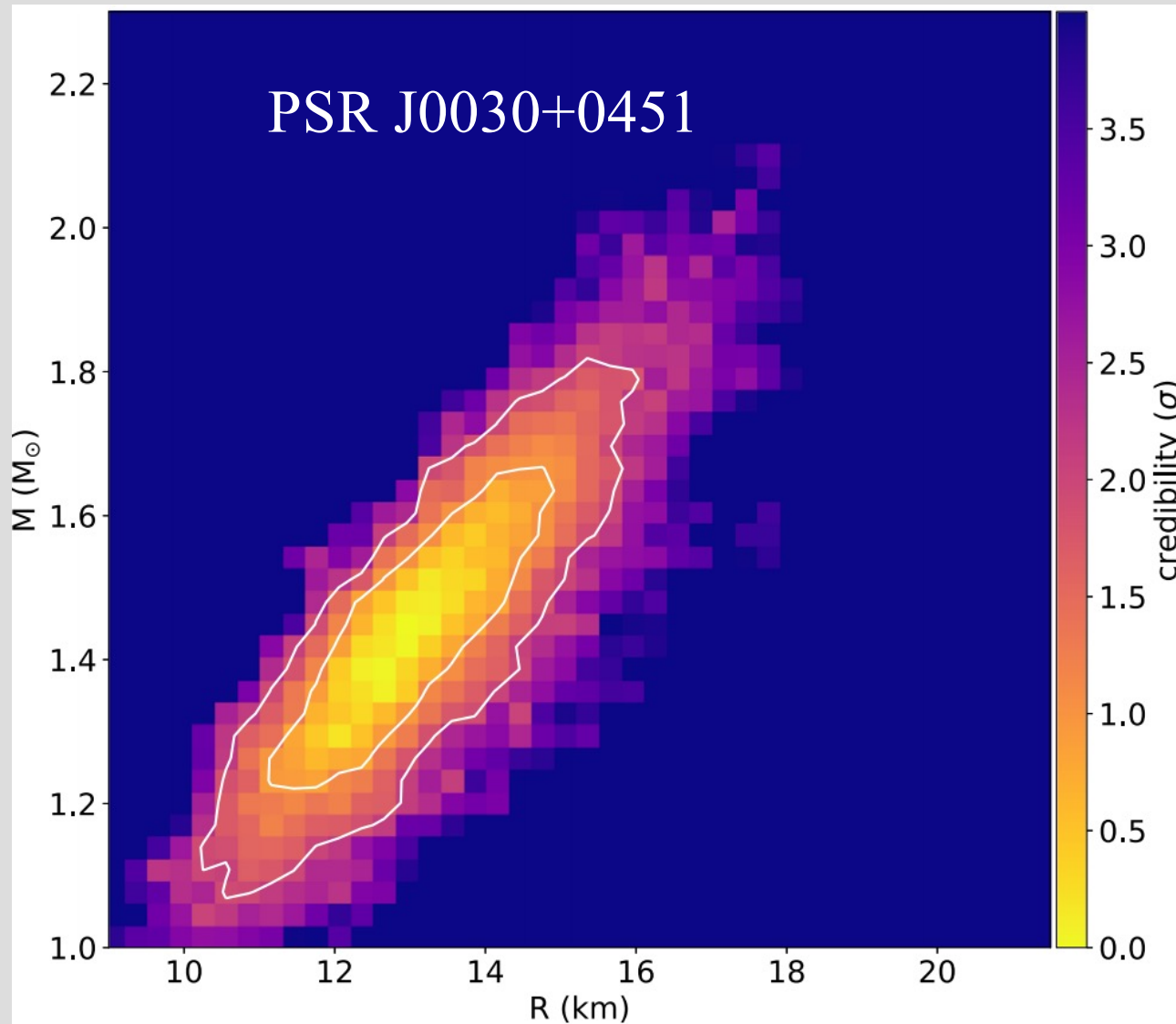
PSR J0030+0451

PSR J0030+0451 at 68% :

$$M = 1.44^{+0.15}_{-0.14} M_\odot \,,$$

$$R = 13.02^{+1.24}_{-1.06} \ km_\circ$$

M. C. Miller *et al*., Astrophys. J. Lett. **887**, L24 (2019); Riley T E, Watts A L, Bogdanov S, et al, Astrophys. J. Lett. 2019, 887(1): L21.

PSR J0740+6620 at 68% :

$$M = 2.08^{+0.07}_{-0.07} M_\odot \,,$$

$$R = 13.7^{+2.6}_{-1.5} \ km_\circ$$

M. C. Miller et al 2021 ApJL 918 L28

Neutron Star Interior Composition Explorer (NICER)

Main observation conclusions of GW170817

GW170817 Source Properties

90% confidence intervals

$$D = 40^{+8}_{-14}\ \mathrm{Mpc}$$

Chirp mass

$$\mathcal{M} = 1.188^{+0.004}_{-0.002}M_\odot$$

$$m_1 = 1.42^{+0.18}_{-0.06}M_\odot$$

$$m_2 = 1.29^{+0.07}_{-0.13}M_\odot$$

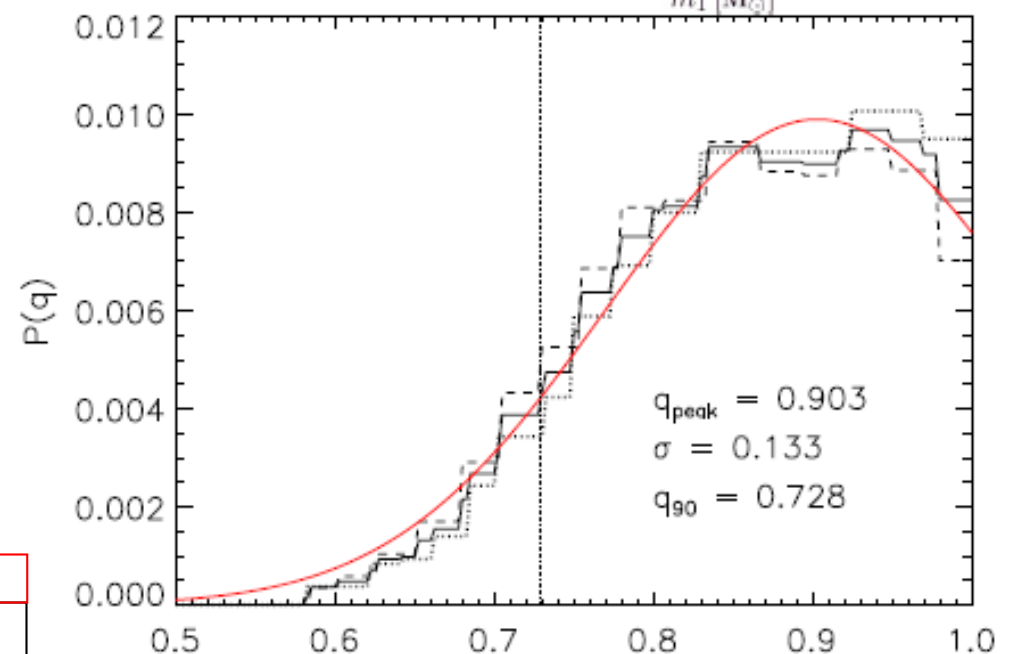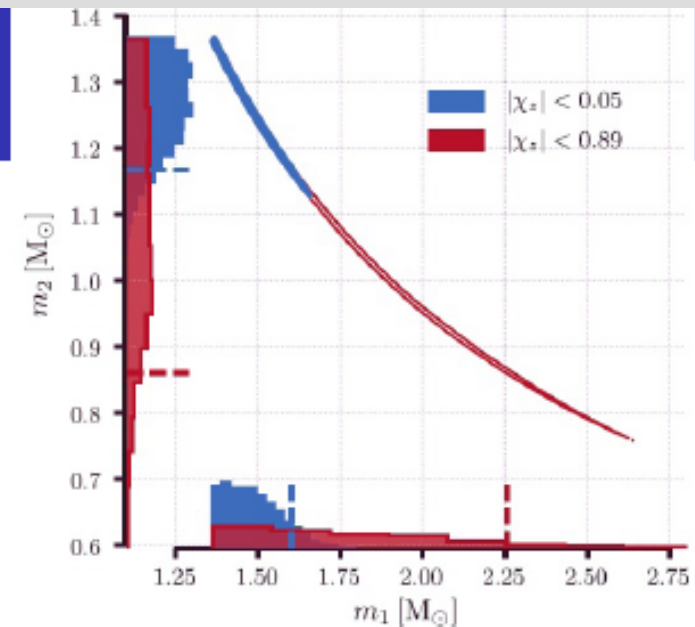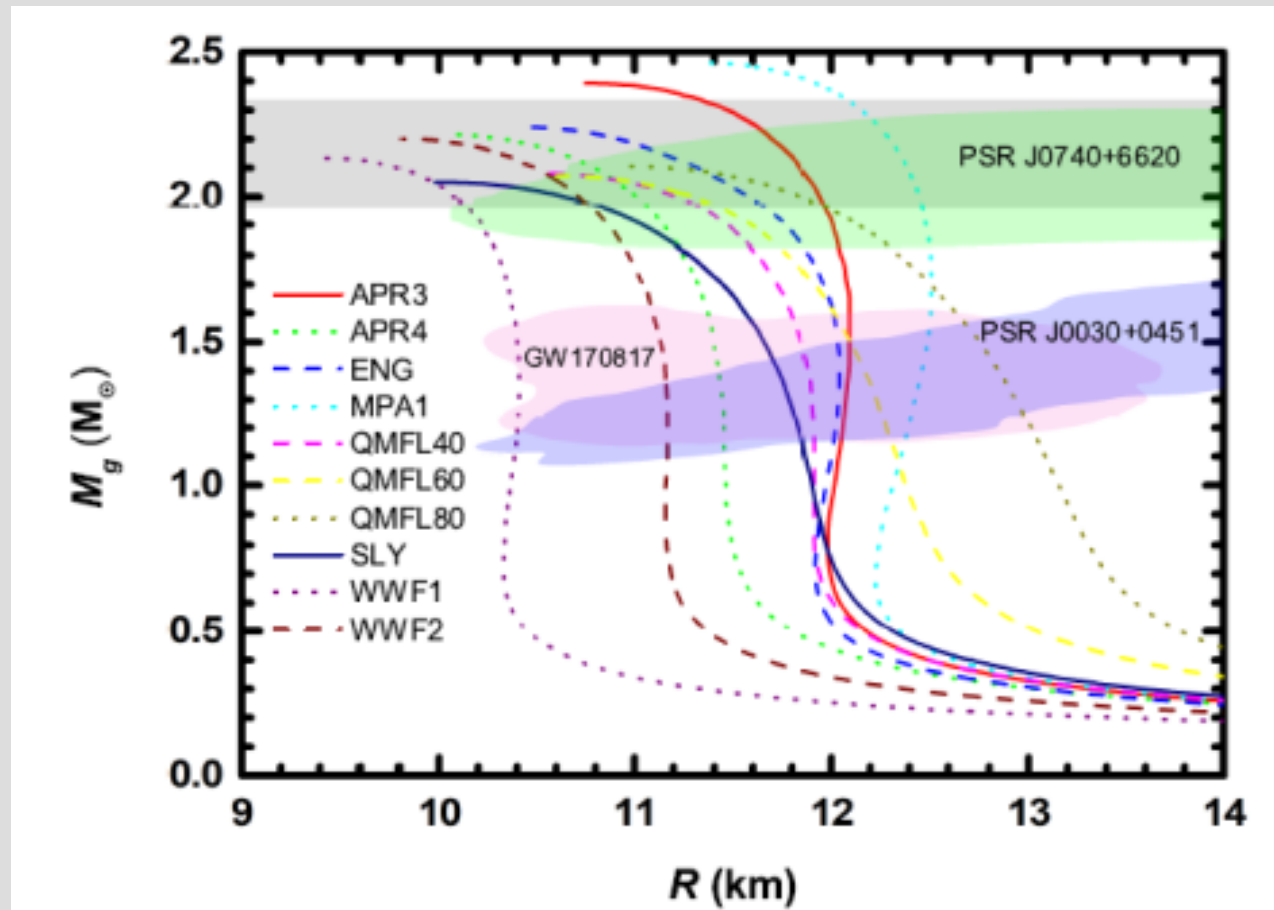$$q = \frac{m_2}{m_1} = 0.90^{+0.10}_{-0.17}$$

The binary tidal deformability

Phys. Rev. Lett. 121, 161101 (2018).

$$\bar{\Lambda} < 800 \qquad \text{New:}70<\Lambda_{1.4}<580$$

$|\chi_z| < 0.05$
$|\chi_z| < 0.89$

$q_{peak} = 0.903$
$\sigma = 0.133$
$q_{90} = 0.728$

# Observation constraints on the M-R relation

Mass–radius relations, where the gray area denotes the range of maximum mass accurately observed so far. The light red area denotes the marginalized posterior (Mg, R) of the merged binary neutron star released by LIGO and VIRGO collaboration in GW170817. The light blue and green areas denote the marginalized posterior (Mg, Re) released by NICER for PSR J0030 + 0451 and PSR J0740 + 6620, respectively.

Constraints of GW170817 observations on neutron star property

- If maximum mass above 2.2 M$_{sun}$, remnant lives too long and transfers too much rotational E to Kilonova. Thus it *appears* 2M$_{sun}$< M$_{max}$ < 2.2M$_{sun}$

- If R(1.6M$_{sun}$) < 10.5 km collapses too fast to BH with too little ejecta. Bauswein et al

- If R(1.4M$_{sun}$) > 13.7 km deformability too large.

- If R(0.5M$_{sun}$ NS) < 12.2 km, ^{208}Pb neutron skin too small for PREX

5
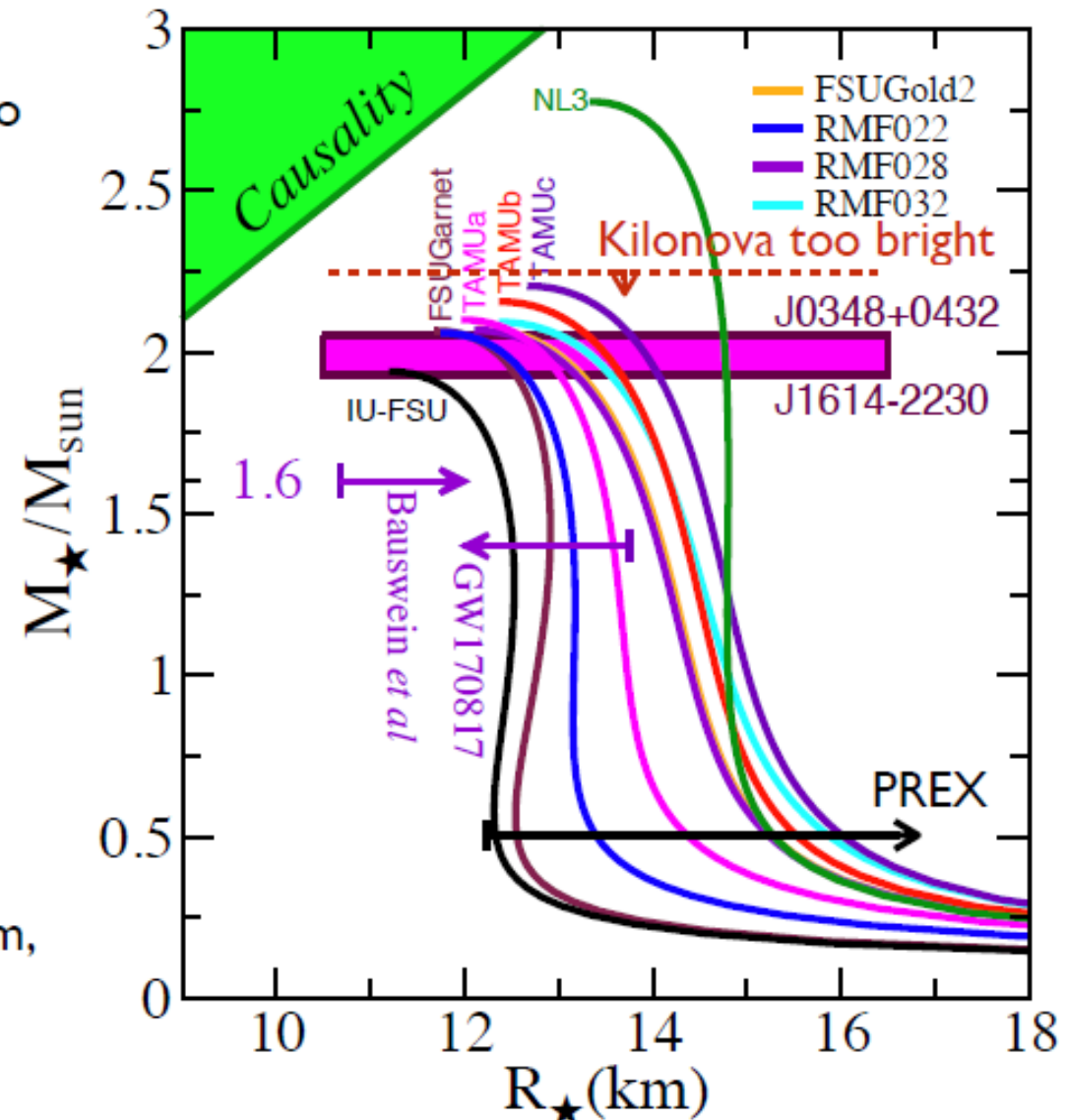
Universal relations among neutron star property (such as I-LOVE-Q)

Universal relation reflects the nature that some relations between the global properties of NS are not dependent on the EOS. This relation can be used to constrain the properties that cannot be directly observed (such as the gravitational binding energy) or cannot be accurately observed (such as the radius), and can be further reversely constrain the parameters of EOS (such as the symmetry energy).
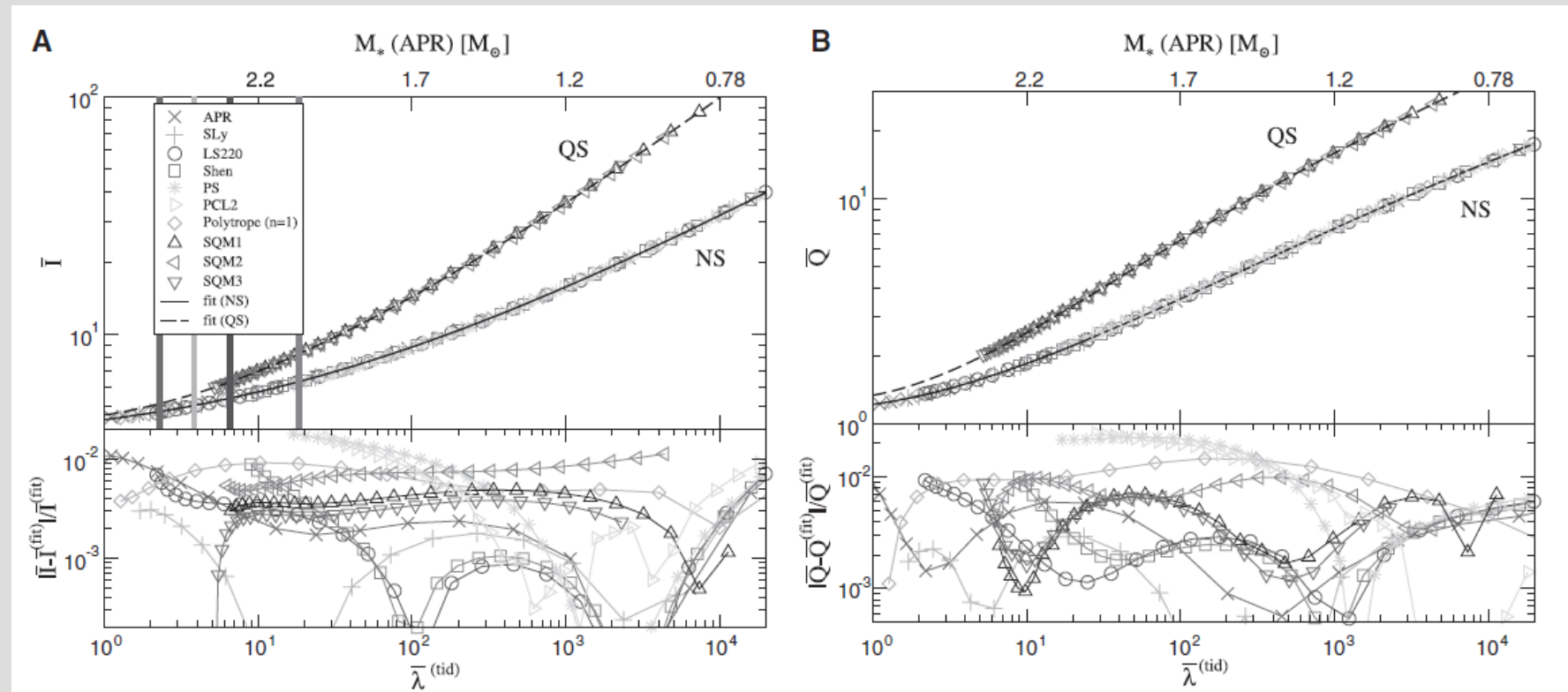

- These universal relations can be used to learn about neutron star deformability through observations of the moment of inertia, distinguish neutron stars from quark stars, and test general.

Kent Yagi* and Nicolás Yunes, Science 341, 365 (2013)

Observation of GW from oscillation modes in NS

All-sky search for short gravitational-wave bursts in the third Advanced LIGO and Advanced Virgo run

R. Abbott,[1] T. D. Abbott,[2] F. Acernese,[3,4] K. Ackley,[5] C. Adams,[6] N. Adhikari,[7] R. X. Adhikari,[1] V. B. Adya,[8] C. Affeldt,[9,10] D. Agarwal,[11] M. Agathos,[12,13] K. Agatsuma,[14] N. Aggarwal,[15] O. D. Aguiar,[16] L. Aiello,[17] A. Ain,[18] P. Ajith,[19] T. Akutsu,[20,21] S. Albanesi,[22] A. Allocca,[23,4] P. A. Altin,[8] A. Amato,[24] C. Anand,[5] S. Anand,[1] A. Ananyeva,[1] S. B. Anderson,[1] W. G. Anderson,[7] M. Ando,[25,26] T. Andrade,[27] N. Andres,[28] T. Andrić,[29] S. V. Angelova,[30] S. Ansoldi,[31,32] J. M. Antelis,[33] S. Antier,[34] S. Appert,[1] Koji Arai,[1] Koya Arai,[35]
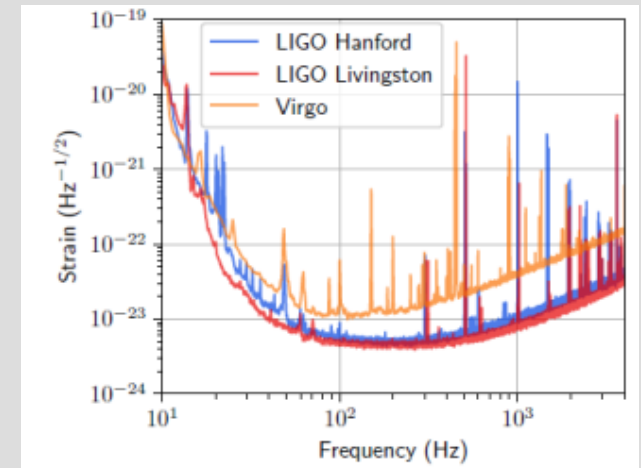
The neutron star signals considered are f-mode emissions…The sensitivities achieved by this search for generic bursts are still not suficient to be able to detect such high-frequency transients at the energy scale of pulsar glitches from e.g., the Vela Pulsar at high confidence. **Nevertheless the outlook is promising, since the expected improvements of the GW detectors in the high-frequency band for the next observation run are quite relevant** …

All-sky search for short gravitational-wave bursts in the third Advanced LIGO and Advanced Virgo run, arXiv:2107.03701v1 [gr-qc]

CONTENT

Basic equations of f-mode

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = 8\pi T^{\alpha\beta}$$

$$T^{\alpha\beta} = pg^{\alpha\beta} + (p+\rho)u^\alpha u^\beta,$$

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + d\sin^2\phi)$$

Static perturbation ⟹

$$ds^2 = -e^\nu(1+r^l H_0 Y_m^l e^{i\omega t})dt^2 - 2i\omega r^{l+1}H_1 Y_m^l e^{i\omega t}dtdr + e^\lambda(1-r^l H_0 Y_m^l e^{i\omega t})dr^2 + r^2(1-r^l K Y_m^l e^{i\omega t})(d\theta^2 + \sin^2\theta d\phi^2).$$

$$\xi^r = r^{l-1}e^{-\lambda/2}W Y_m^l e^{i\omega t},$$

$$\xi^\theta = -r^{l-2}V\partial_\theta Y_m^l e^{i\omega t},$$

$$\xi^\phi = -r^l(r\sin\theta)^{-2}V\partial_\phi Y_m^l e^{i\omega t}$$

⟹

$$H_1' = -r^{-1}[l+1+2Me^\lambda r^{-1} + 4\pi r^2 e^\lambda(p-\rho)]H_1 + r^{-1}e^\lambda[H_0 + K - 16\pi(\rho+p)V],$$

$$K' = r^{-1}H_0 + \tfrac{1}{2}l(l+1)r^{-1}H_1 - [(l+1)r^{-1} - \tfrac{1}{2}\nu']K - 8\pi(\rho+p)e^{\lambda/2}r^{-1}W,$$

$$W' = -(l+1)r^{-1}W + re^{\lambda/2}[\gamma^{-1}p^{-1}e^{-\nu/2}X - l(l+1)r^{-2}V + \tfrac{1}{2}H_0 + K],$$

$$X' = -lr^{-1}X + (\rho+p)e^{\nu/2}\{\tfrac{1}{2}(r^{-1} - \tfrac{1}{2}\nu')H_0 + \tfrac{1}{2}[r\omega^2 e^{-\nu} + \tfrac{1}{2}l(l+1)r^{-1}]H_1$$

$$+ \tfrac{1}{2}(\tfrac{3}{2}\nu' - r^{-1})K - \tfrac{1}{2}l(l+1)\nu'r^{-2}V - r^{-1}[4\pi(\rho+p)e^{\lambda/2} + \omega^2 e^{\lambda/2-\nu} - \tfrac{1}{2}r^2(r^{-2}e^{-\lambda/2}\nu')']W\}$$

$$V = \omega^{-2}(\rho+p)^{-1}e^\nu[e^{-\nu/2}X + r^{-1}p'e^{-\lambda/2}W - \tfrac{1}{2}(\rho+p)H_0].$$

$$\mathbf{Y}'(r,l,w) = \mathbf{Q}(r,l,w)\cdot\mathbf{Y}(r,l,w)$$

Definition and calculation of tidal deformability

$$\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}.$$

Cause: gravitational action from outside the star

$$Q_{ij} = \int d^3 x \delta\rho(\mathbf{x}) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right),$$

Effect: density disturbance inside star

$$Q_{ij} = -\lambda \mathcal{E}_{ij}.$$

Causal correlation: linear

$$k_2 = \frac{3}{2} G \lambda R^{-5}.$$

Definition of Tidal Love Number

Tidal deformability is a detectable quantity that both astrophysics and nuclear physics are concerned about

Definition and calculation of tidal deformability

$$k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y - 1) - y] \times$$

$$\left\{ 2C(6 - 3y + 3C(5y - 8)) + 4C^3 [13 - 11y + C(3y - 2) + 2C^2(1 + y)] \right.$$

$$\left. + 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \log(1 - 2C) \right\}^{-1},$$

where
$$C = m/R$$

$$y = \frac{R\beta(R)}{H(R)}$$

Parametric EOS model for neutron-rich nucleonic matter

The nucleon specific energy of neutron-rich matter with isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$ can be well approximated by the empirical parabolic law

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2 + O(\delta^4)$$

where

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^3$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\frac{\rho - \rho_0}{3\rho_0} + \frac{K_{sym}}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \frac{J_{sym}}{6}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^3$$

Parameters near the saturation density are constrained by existing knowledge as

$k_0:$ 230 ± 20 MeV

$E_{\text{sym}}(\rho_0): 31.7 \pm 3.2$ MeV

$L:$ 58.7 ± 28.1 MeV

$K_{\text{sym}}: -400 \sim 100$ MeV

$J_{\text{sym}}: -200 \sim 800$ MeV

$J_0: -300 \sim 400$ MeV

The frequency of f-mode for a neutron star with mass of $1.4M_\odot$ are constrained by the tidal polarizability $[70 < \Lambda_{1.4} < 580]$ of GW170817 in 1.67~2.18kHz;

D. H. Wen, B. A. Li, H. Y. Chen and N. B. Zhang, Phys. Rev. C 99, 045806 (2019)

Universal relation between f-mode DT and tidal deformability for 1.4M$_\odot$ NS
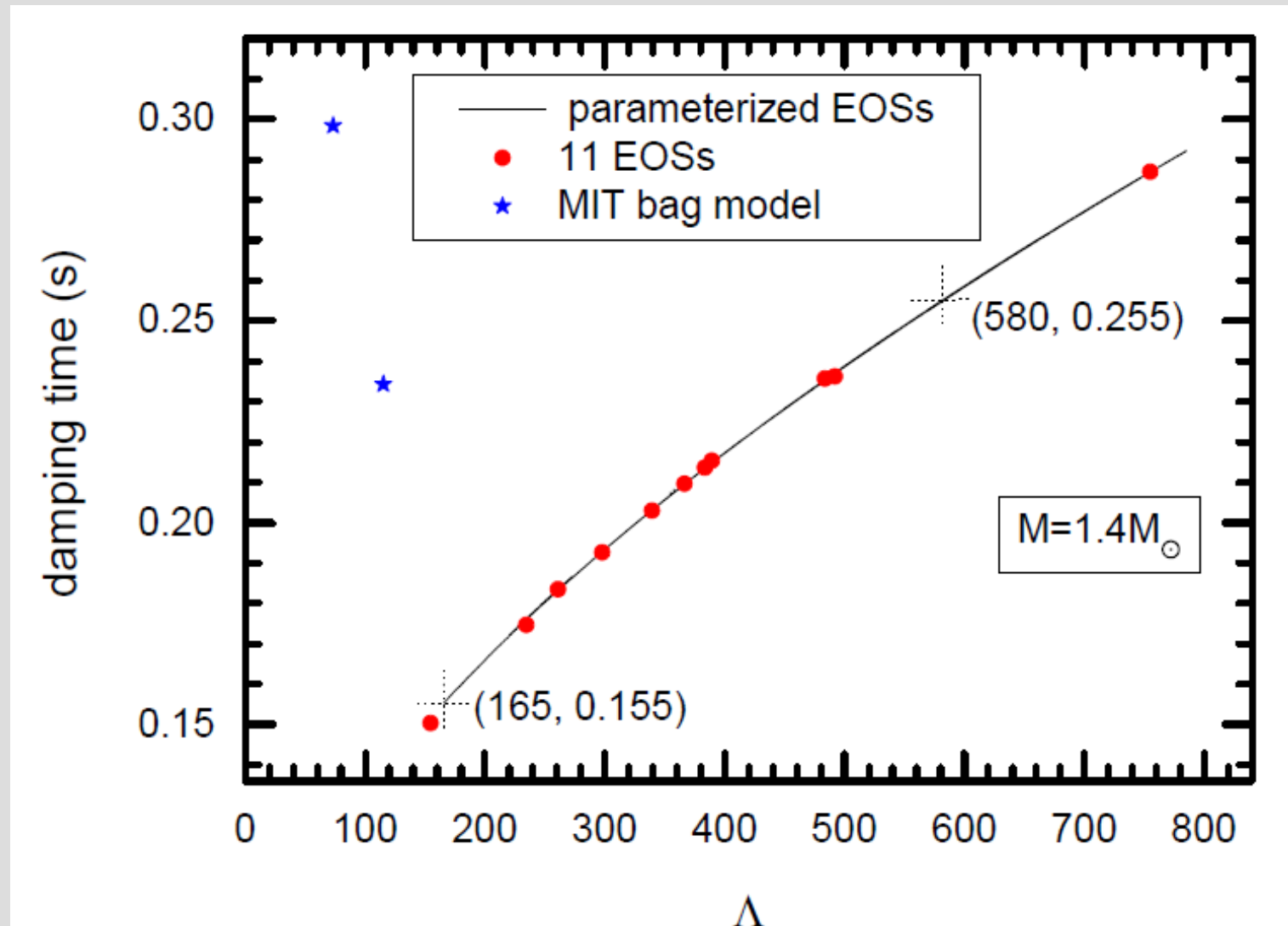
The damping time (DT) of f-mode for a neutron star with mass of 1.4M$_\odot$ are constrained by the tidal deformability [70<$\Lambda_{1.4}$<580] of GW170817 in 0.155~0.255s.

D. H. Wen, B. A. Li, H. Y. Chen and N. B. Zhang, Phys. Rev. C 99, 045806 (2019)

Constraints of f-mode on symmetric energy of nuclear matter
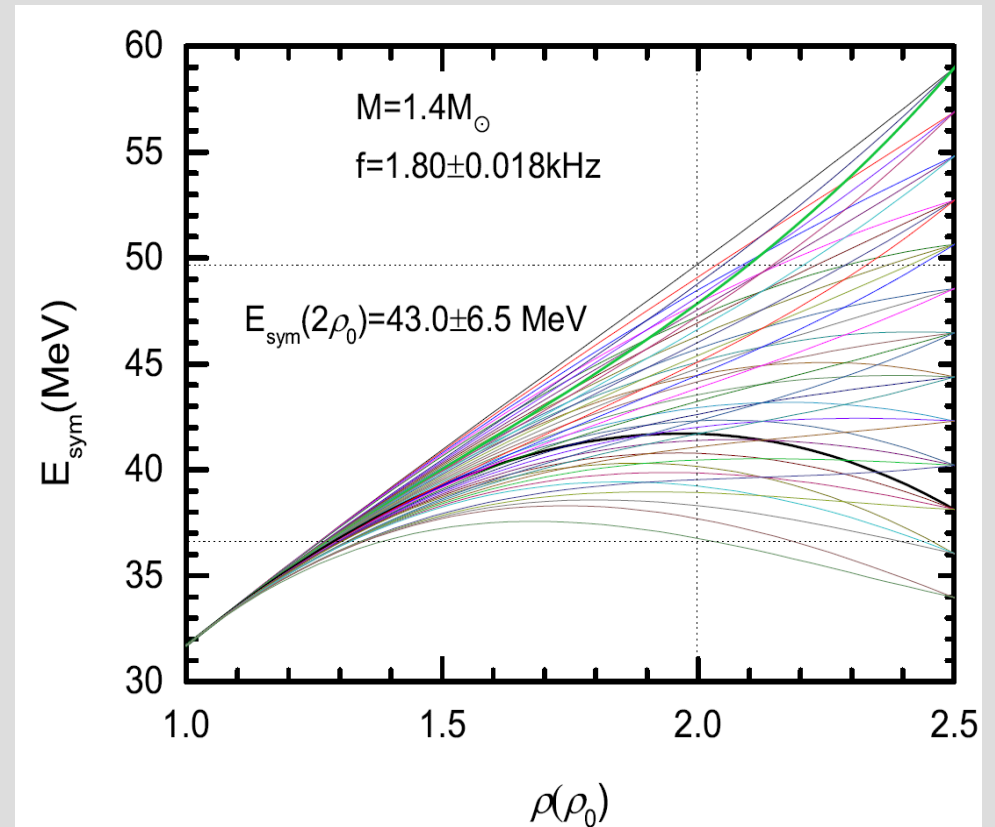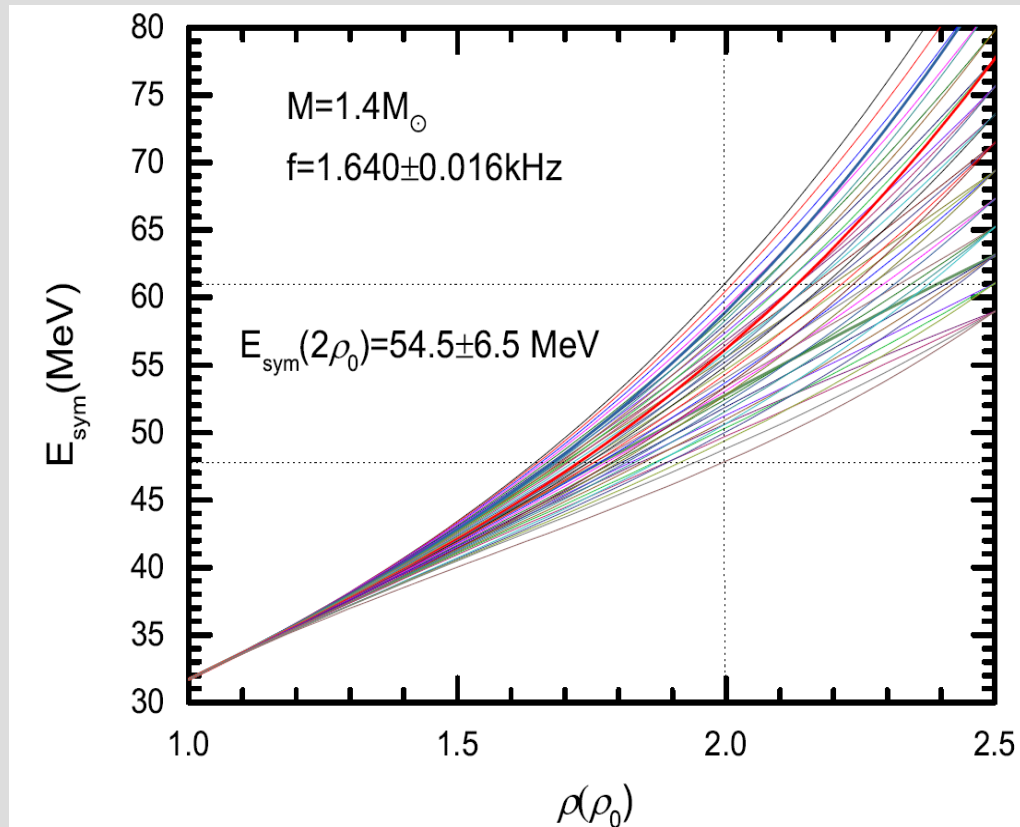
It is shown that if a lower frequency (1.640±0.016 kHz) is observed, there is a higher symmetry energy (54.5±6.5 MeV) at $2\rho_0$; if a higher frequency (1.800±0.018 kHz) is observed, there is a lower symmetry energy (43.0 ± 6.5 MeV).

D. H. Wen, B. A. Li, H. Y. Chen and N. B. Zhang, Phys. Rev. C 99, 045806 (2019)

CONTENT

- ➤ **Brief introduction to neutron star research**

- ➤ **Universal relations (UR) and correlations of NS**

- • **UR of f-mode**

- • **UR of gravitational binding energy**

- • **UR of rotational neutron star**

- • **UR and correlation analysis**

- ➤ **Conclusion**

Definition of gravitational binding energy and related quantities

Definition of the mass:

$$M = \int_0^R \rho \cdot 4\pi r^2 \, dr$$

Gravitational mass

$$M_p = \int_0^R \rho \cdot (1 - \frac{2M(r)}{r})^{-1/2} \cdot 4\pi r^2 \, dr$$

Proper mass

$$M_b = A m_b \qquad \boxed{m_b = 939 \text{MeV}}$$

Baryon mass

Definition of the binding energy (BE):

$$E_t = M - M_b$$

Total BE

$$E_g = M - M_p$$

Gravitational BE

$$E_n = E_t - E_g$$

Nuclear BE

Normally, the total BE and gravitational BE are negative, and the nuclear BE are positive (quark stars are excluded).

# Binding energy per baryon of neutron star sequence
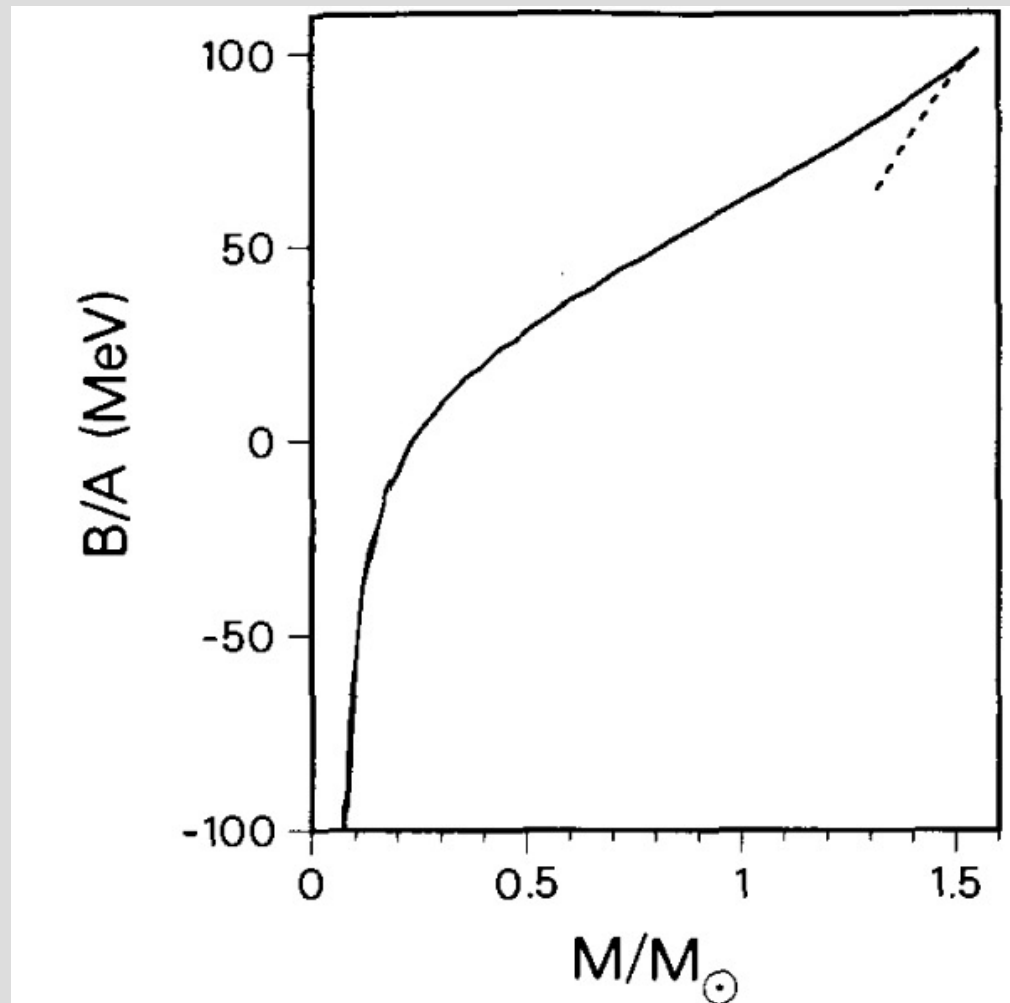
The dashed line represents the unstable configurations with central densities higher than that of the limiting mass star. The negative binding energy for stars with $M < 0.2 M_{sun}$.

Binding energies for 1.4M$_\odot$ NS under different EOSs

TABLE I. The binding energies of neutron stars and quark stars with a canonical gravitational mass 1.4 $M_\odot$.

EOS	M ($M_\odot$)	M_b ($M_\odot$)	M_p ($M_\odot$)	$-E_t$ ($M_\odot$)	$-E_g$ ($M_\odot$)	E_n ($M_\odot$)	$-E_t/A$ (MeV)	$-E_g/A$ (MeV)	E_n/A (MeV)
ALF2	1.40	1.59	1.59	0.194	0.191	−0.003	114.373	112.423	−1.95
APR3	1.40	1.56	1.62	0.160	0.218	0.057	96.34	130.90	34.56
APR4	1.40	1.57	1.64	0.170	0.236	0.065	101.74	140.78	39.04
ENG	1.40	1.55	1.62	0.151	0.218	0.067	91.56	131.93	40.37
MPA1	1.40	1.55	1.61	0.145	0.208	0.064	88.00	126.62	38.62
SLY	1.40	1.55	1.63	0.145	0.230	0.085	88.16	139.91	51.75
WWF1	1.40	1.58	1.66	0.178	0.262	0.084	105.97	155.63	49.67
WWF2	1.40	1.56	1.64	0.161	0.241	0.080	96.79	145.07	48.28
QMFL40	1.40	1.55	1.62	0.149	0.224	0.075	90.36	135.81	45.45
QMFL60	1.40	1.55	1.62	0.149	0.220	0.072	90.21	133.60	43.39
QMFL80	1.40	1.55	1.61	0.147	0.208	0.061	89.33	126.21	36.88
CIDDM	1.40	1.70	1.58	0.301	0.176	−0.125	165.94	96.83	−69.12
CDDM1	1.40	1.60	1.55	0.201	0.151	−0.051	117.91	88.27	−29.64
CDDM2	1.40	1.61	1.53	0.209	0.133	−0.076	122.24	77.71	−44.53

Universal relation of gravitational binding energy
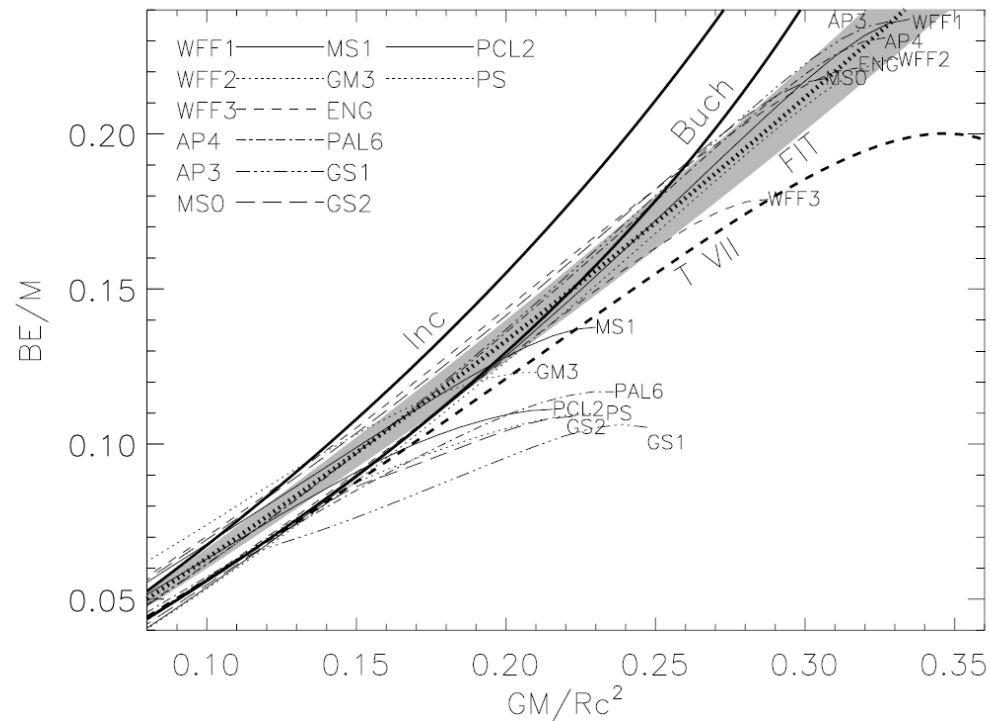
Eur. Phys. J. A (2016) **52**: 18
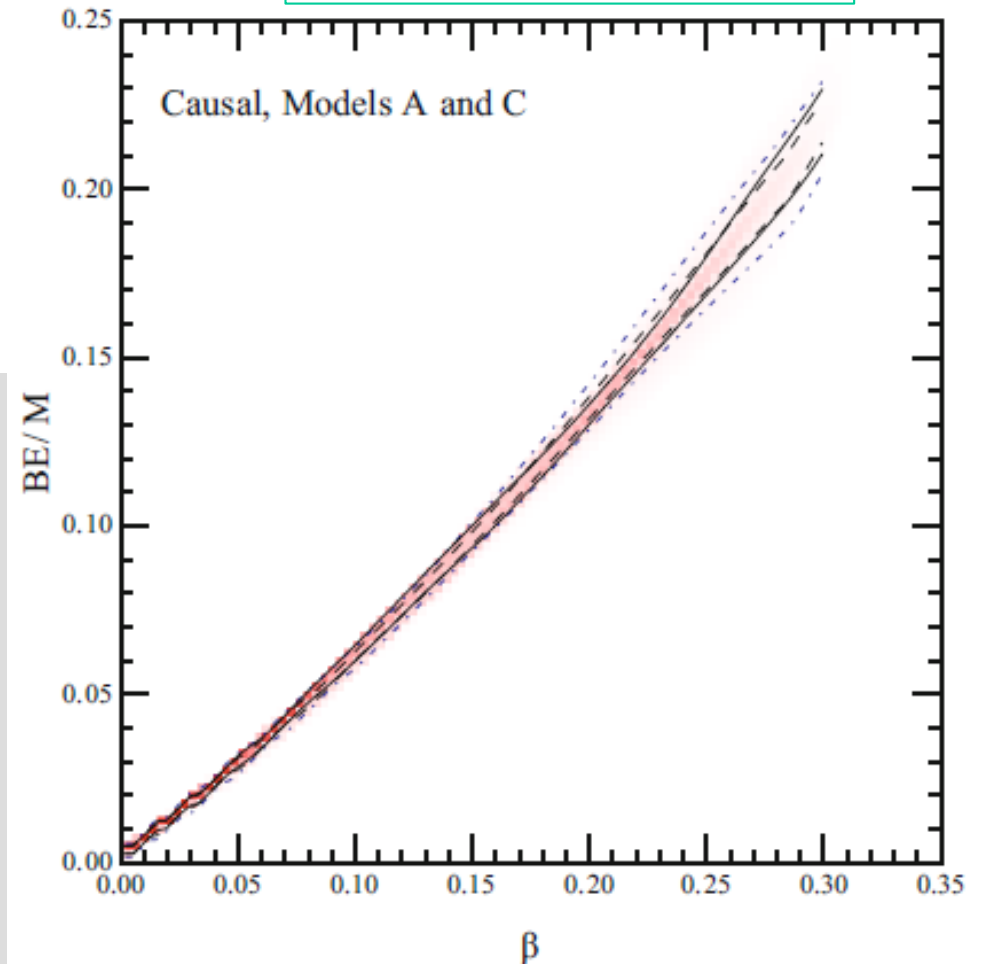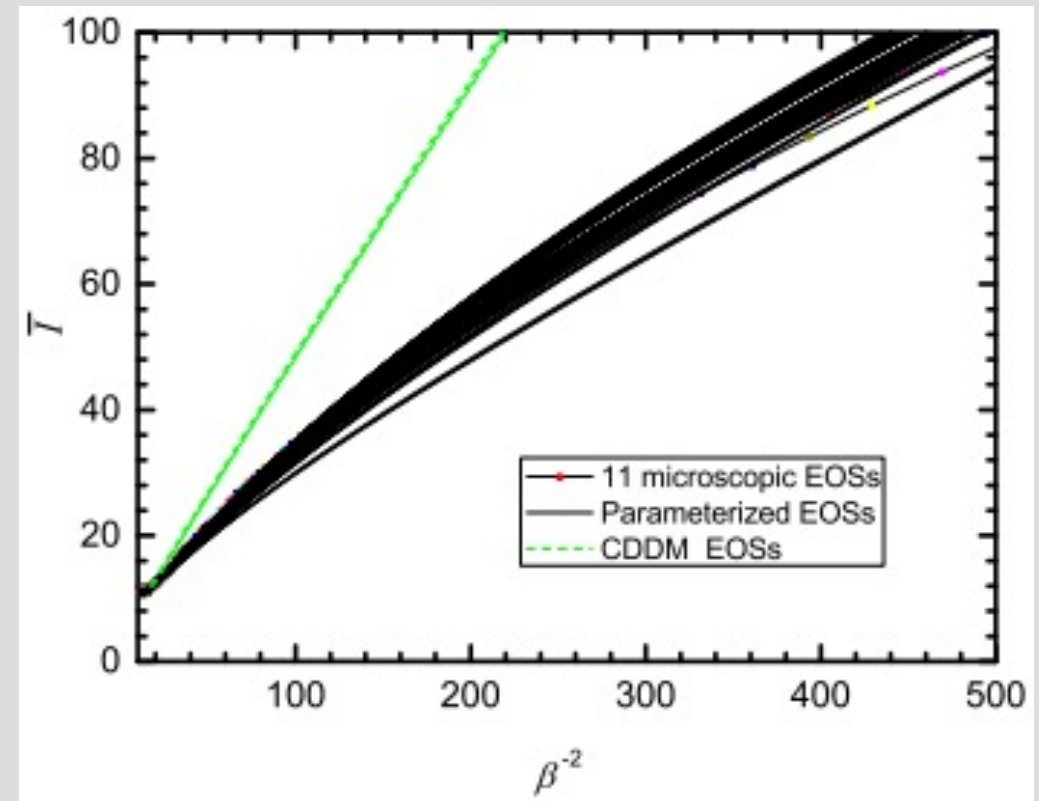
APJ, 550(2001)426

Universal relation between gravitational binding energy and moment of inertia
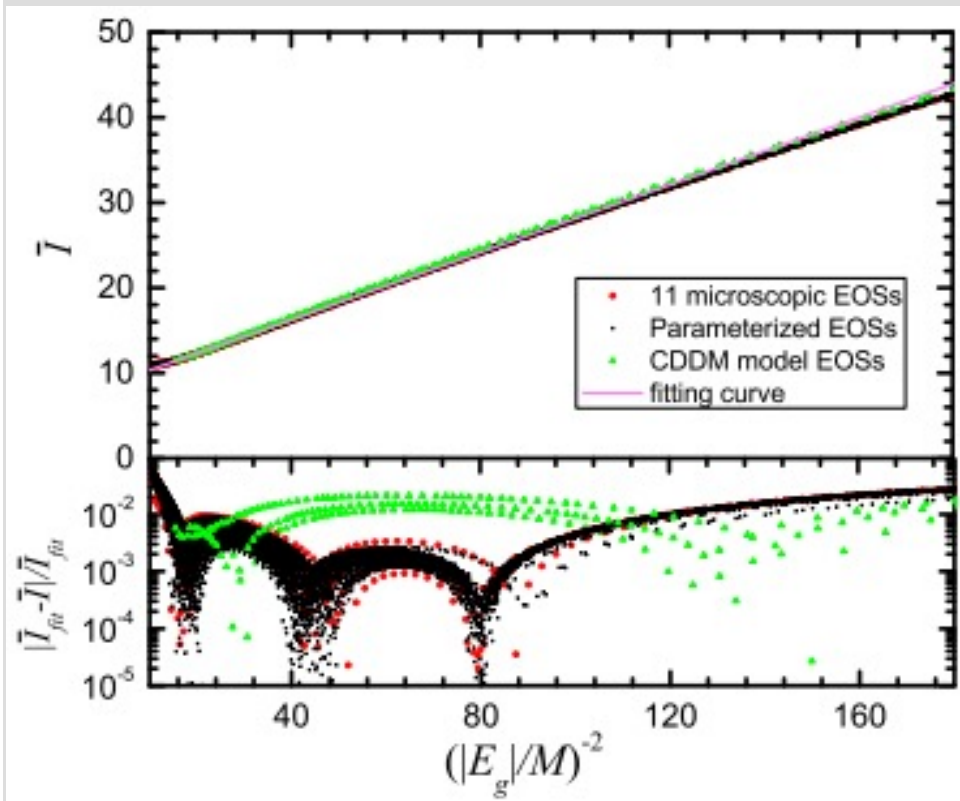
- There is a perfect linear universal relation between the dimensionless inertial and the dimensionless gravitational binding energy; the normal neutron stars and quark stars follow the same universal relation.

- It is sufficient to estimate the gravitational binding energy if the stellar mass M and the moment of inertia I are measured simultaneously, whether the compact star is a quark star or a neutron star.

Universal relation between gravitational binding energy and tidal deformability
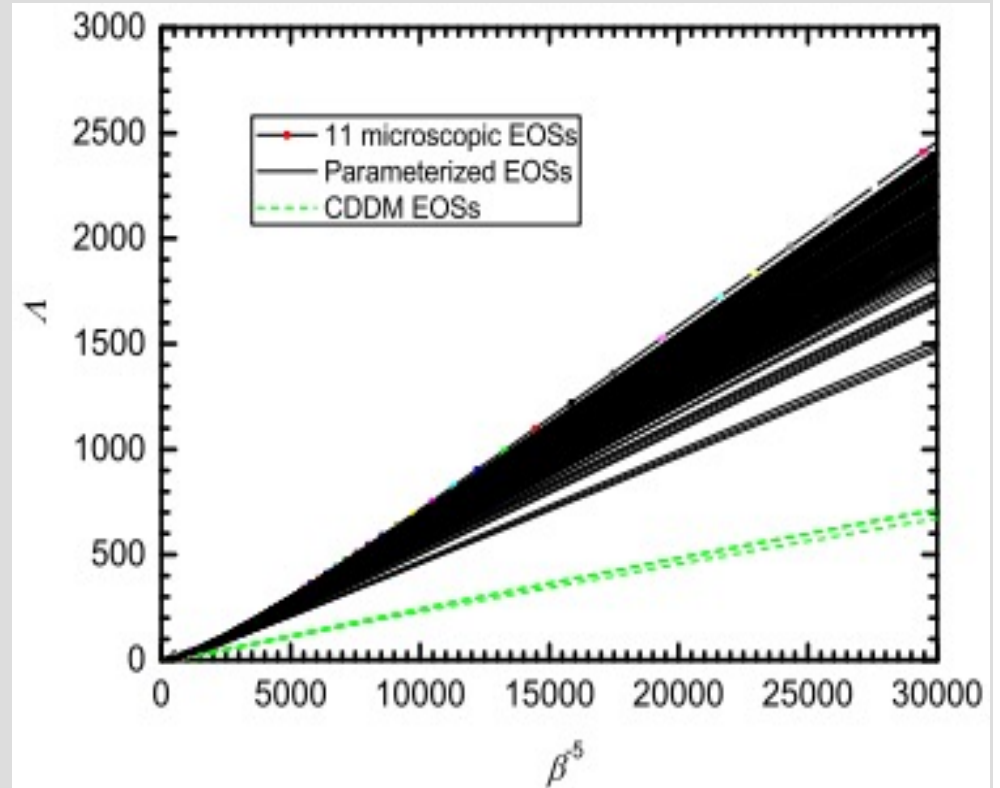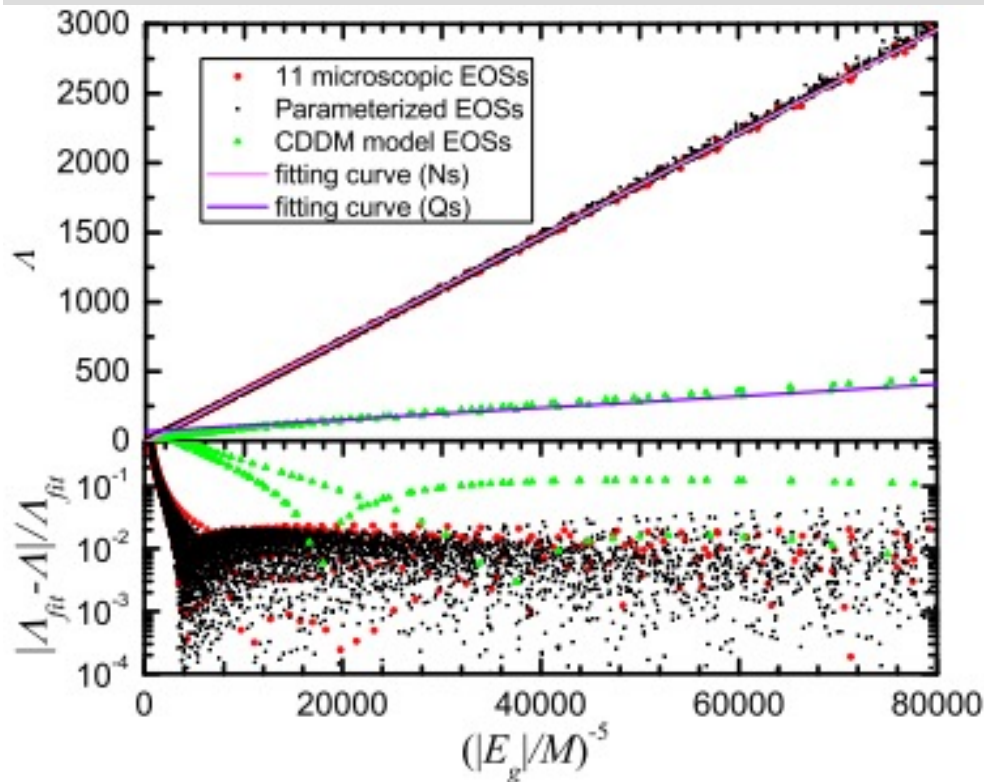
There exist ideal linear universal relations between dimensionless gravitational binding energy (negative fifth power) and the tidal deformability Λ. provide a potential way to distinguish normal neutron stars (with crust) and the quark stars (without crust) .
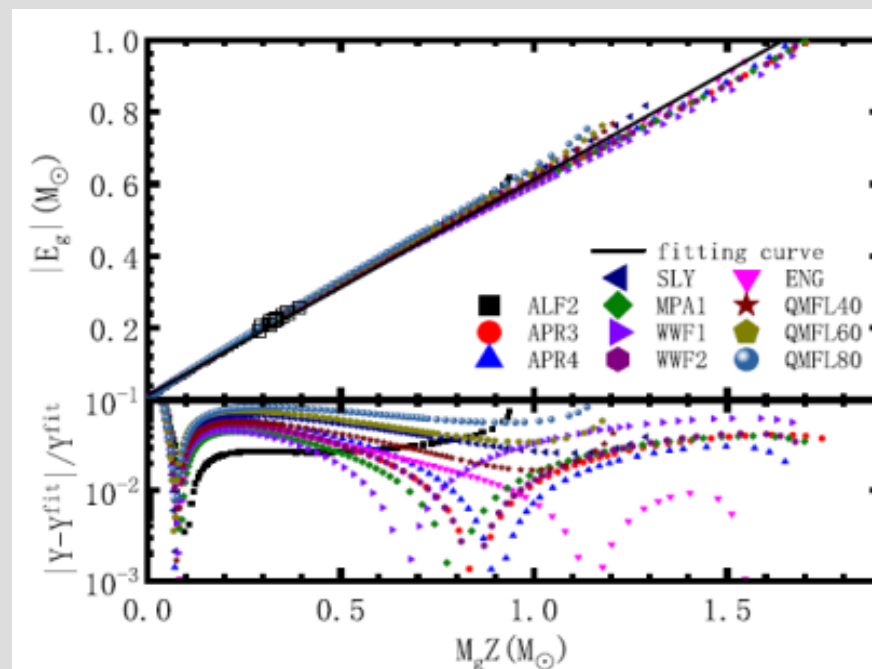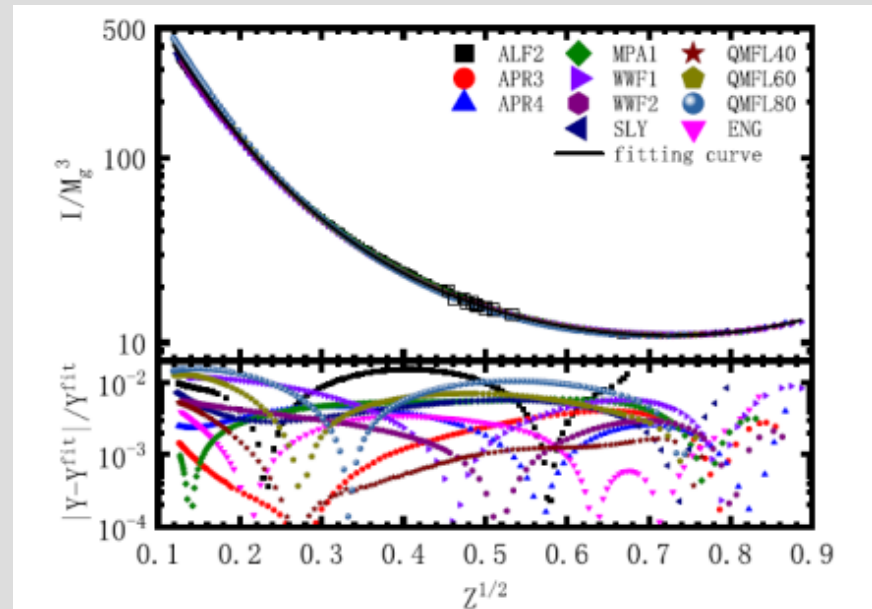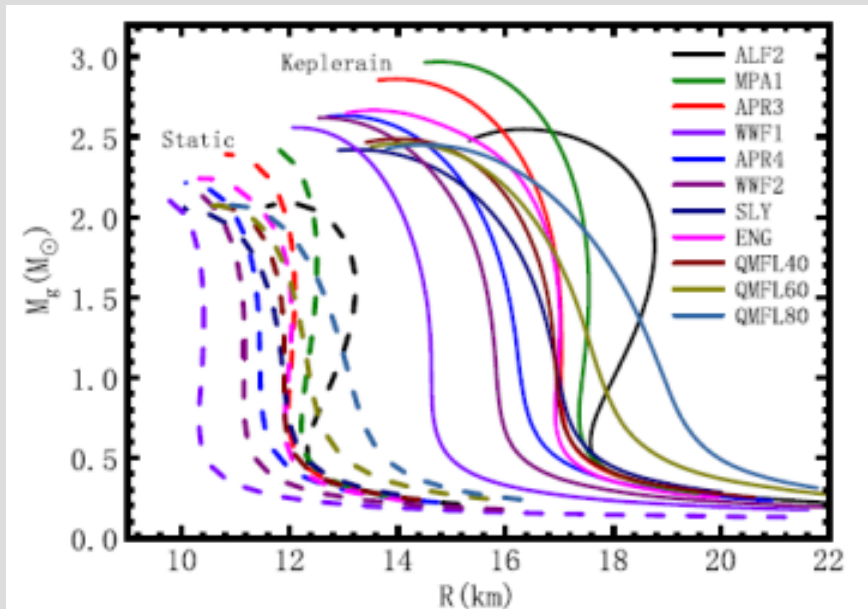
CONTENT

The ten fastest-spinning known radio pulsars

Pulsar	Spin Frequency (Hz)	P_b (days)	$M_{2,min}$ ($M_\odot$)	Eclipse Fraction	Location
J1748−2446ad	716.358	1.0944	0.14	0.4	Terzan 5
B1937+21	641.931		isolated		Galaxy
B1957+20	622.123	0.3819	0.021	0.1	Galaxy
J1748−2446O	596.435	0.2595	0.035	0.05	Terzan 5
J1748−2446P	578.496	0.3626	0.37	0.4	Terzan 5
J1843−1113	541.812		isolated		Galaxy
J0034−0534	532.714	1.5892	0.14	0	Galaxy
J1748−2446Y	488.243	1.17	0.14	0	Terzan 5
J1748−2446V	482.507	0.5036	0.12	0	Terzan 5
B0021−72J	476.048	0.1206	0.020	0.1*	47 Tucanae

Universal relation among *I, Z* and *E*$_g$— static case
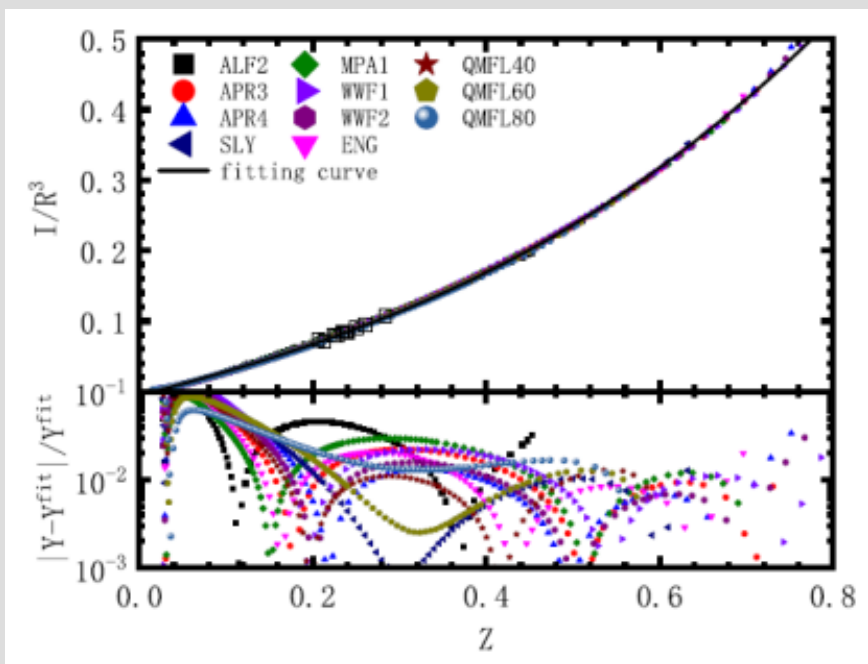
Universal relation for the rotating neutron stars
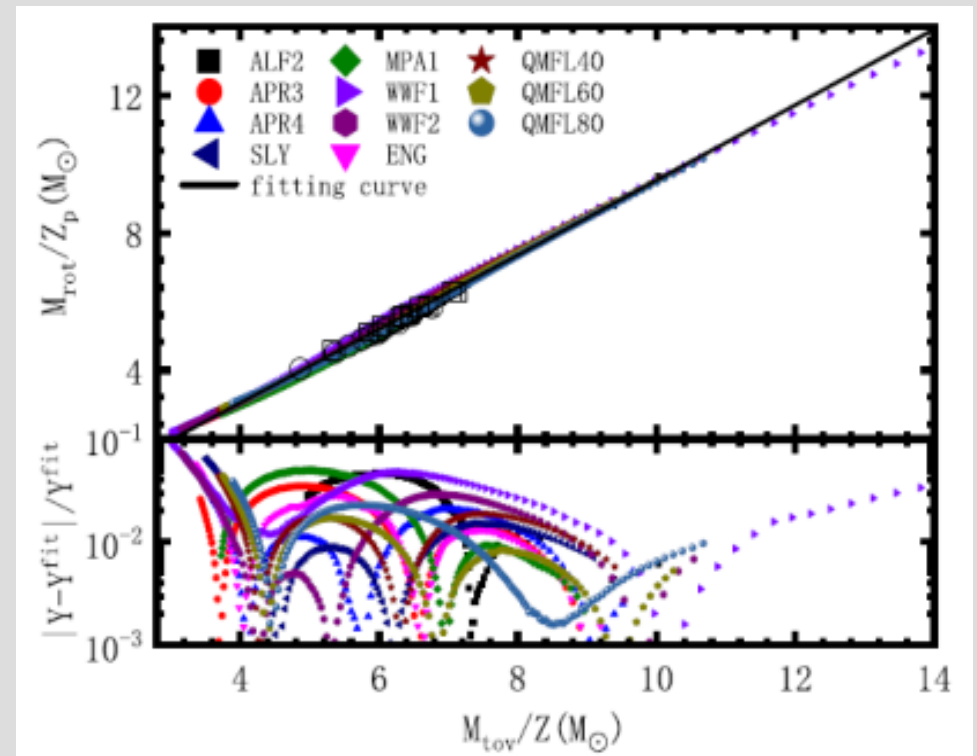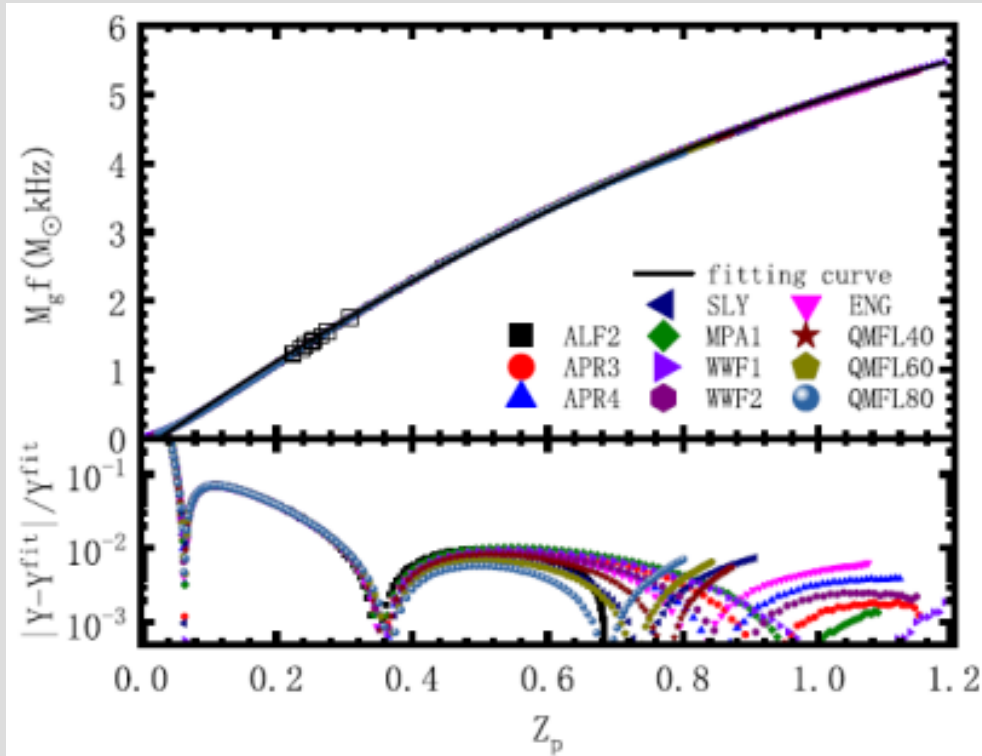
Rotating at Keplerian frequency

CONTENT

- ➤ **Brief introduction to neutron star research**

- ➤ **Universal relations (UR) and correlations of NS**

- • **UR of f-mode**

- • **UR of gravitational binding energy**

- • **UR of rotational neutron star**

- • **UR and correlation analysis**

- ➤ **Conclusion**

Universal relations and correlation analysis

Shen Yang, Dehua Wen∗, Jue Wang and Jing Zhang, Phys. Rev. D 105, 063023 (2022)

- The linear correlation, which reflects the strength between two quantities, can be expressed as

$$r(X,Y) = \frac{N\sum\limits_{i=1}^{N} X_i Y_i - \sum\limits_{i=1}^{N} X_i \sum\limits_{i=1}^{N} Y_i}{\sqrt{N\sum\limits_{i=1}^{N} X_i^2 - \left(\sum\limits_{i=1}^{N} X_i\right)^2}\sqrt{N\sum\limits_{i=1}^{N} Y_i^2 - \left(\sum\limits_{i=1}^{N} Y_i\right)^2}}$$

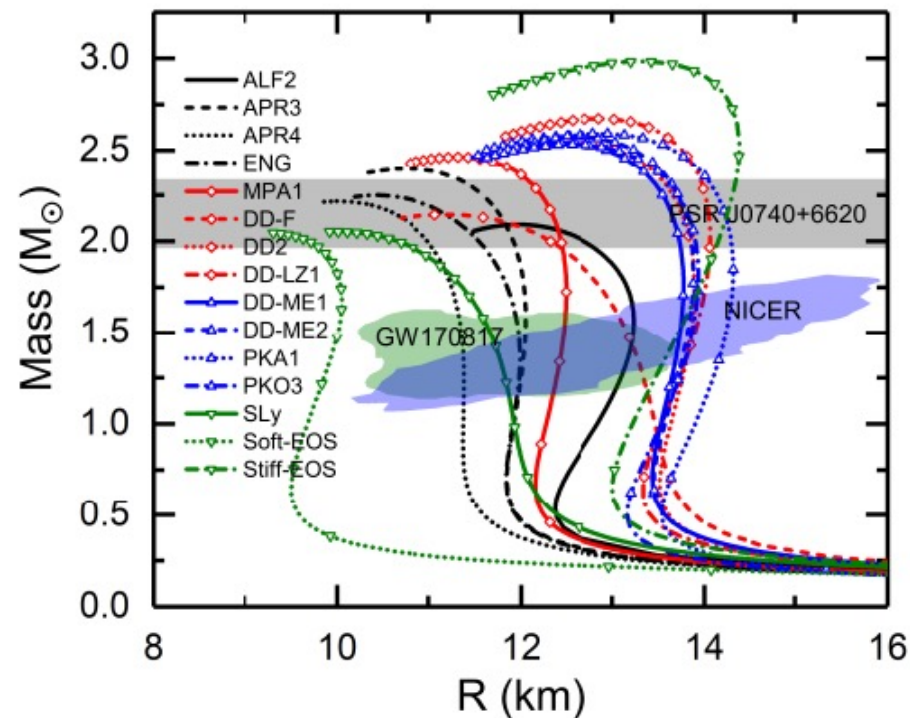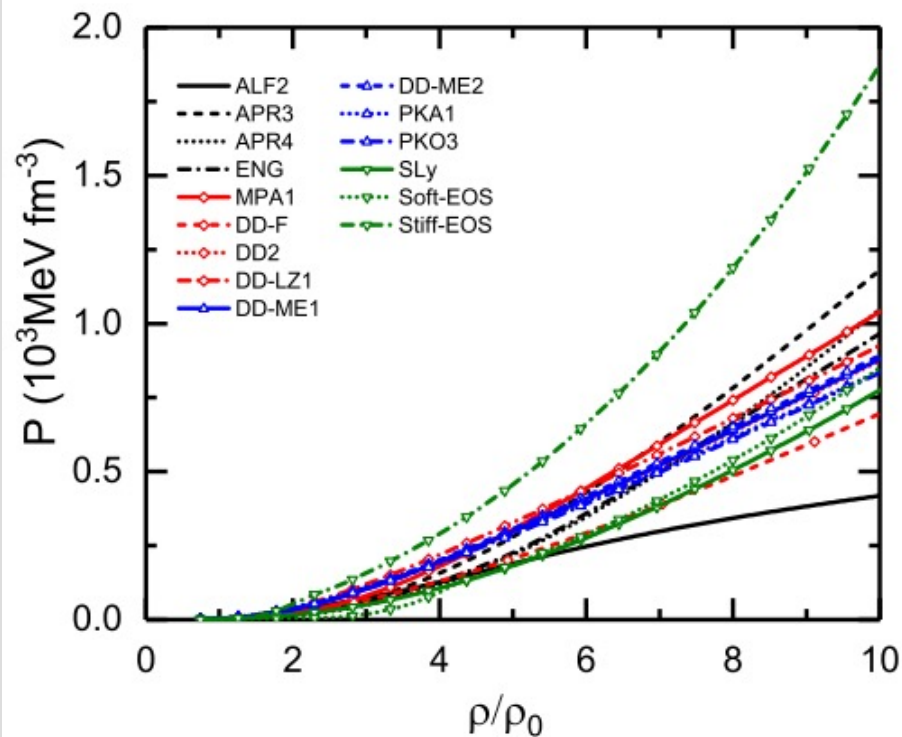简记为 $\qquad r(X,Y) = \dfrac{\mathrm{Cov}(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$

Pearson相关系数

The closer the absolute value of the correlation coefficient |r| is to 1, the greater the correlation strength between the two quantities will be.

M. Ferreira, M. Fortin, T. Malik, B. K. Agrawal, and C. Providencia, Phys. Rev. D 101, 043021 (2020).
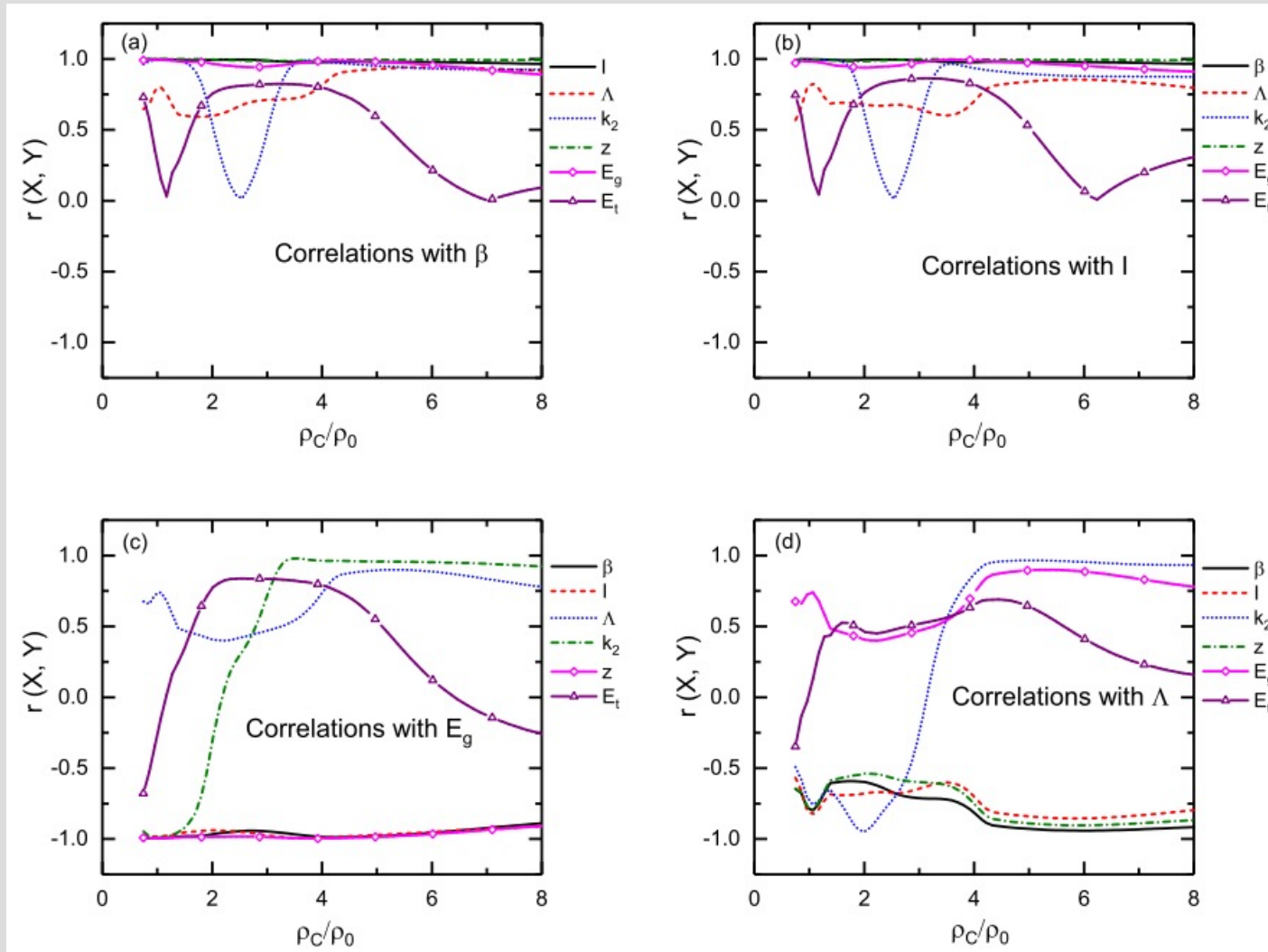
物态方程及对应的质量半径关系

采用15组代表性EOS。除Soft EOS外，其它EOS的M-R关系均符合 GW170817, NICER以及 PSR J0740+6620（最大质量）的观测约束。
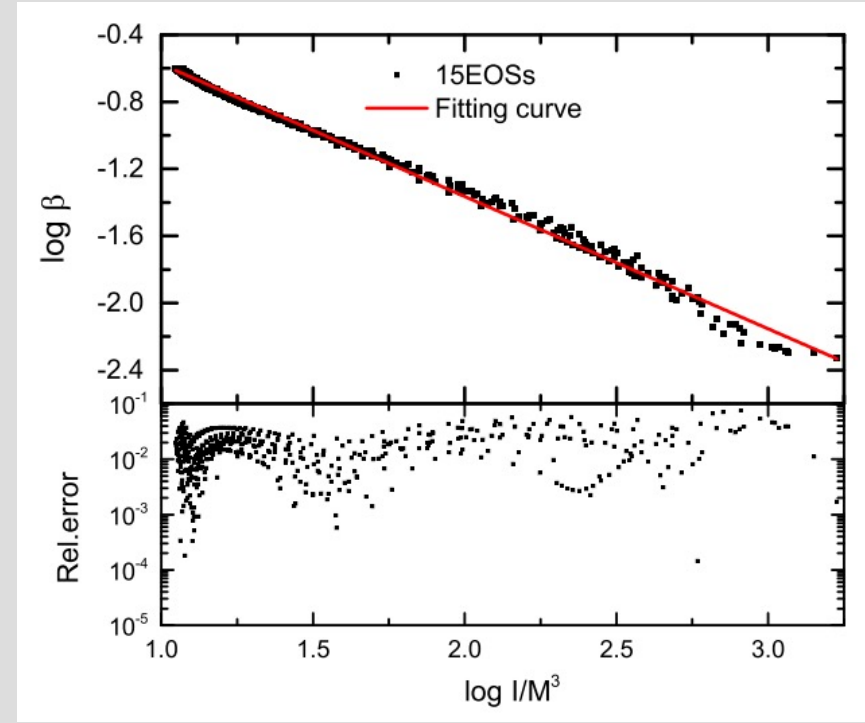
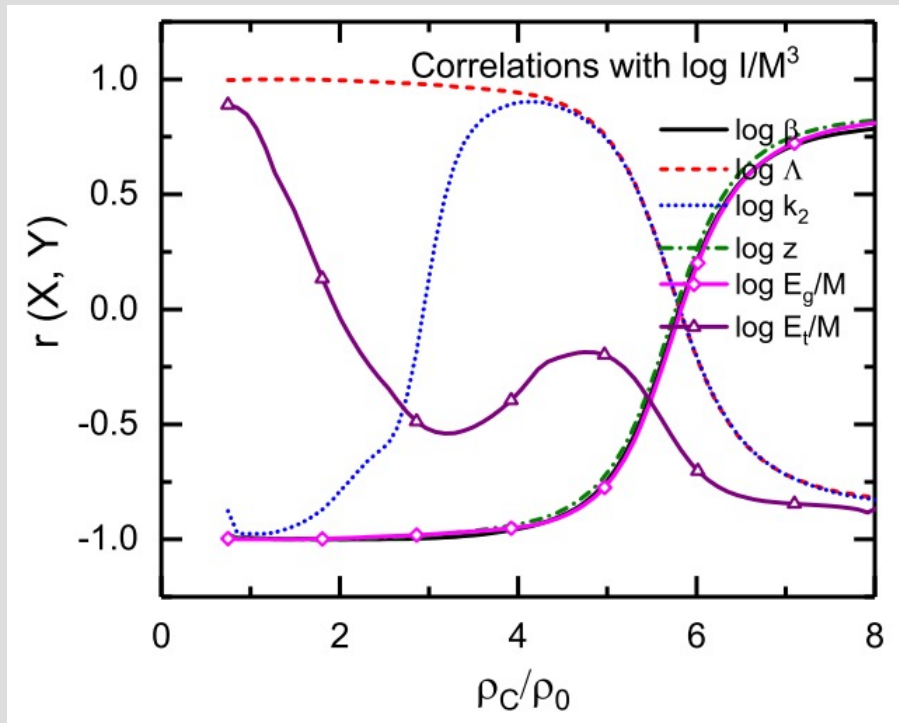# Universal relations and correlation analysis

Linear correlation properties of the neutron star properties. It is shown that the quantities possessing desired linear correlation properties are the compactness β, moment of inertia I, gravitational redshift z and gravitational binding energy Eg.

Universal relations and correlation analysis

Predicting the new universal (I/M^3– β) relations based on the linear correlation analysis

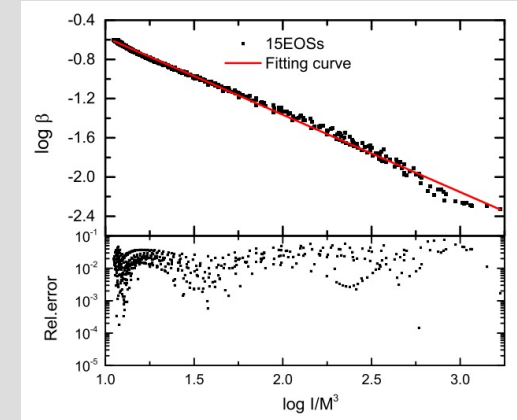The universal relation I/M^3- β can be expressed as

$$\log I/M^3 = -1.261 \log \beta + 0.277$$

Universal relations and correlation analysis

According to the universal relation I/M^3- β,

$$\log I/M^3 = -1.261 \log \beta + 0.277$$

The moment of inertia of PSR J0030+0451 can be constrained as :

$R_{1.4}$	$\beta_{1.4}$	$I_{1.4}$
$12.1^{+1.2}_{-0.8}$km	$0.171^{+0.012}_{-0.015}$	$2.09^{+0.27}_{-0.17}\times10^{45}$g cm^2
$13.00^{+2.09}_{-1.77}$km	$0.159^{+0.025}_{-0.022}$	$2.29^{+0.47}_{-0.39}\times10^{45}$g cm^2

CONTENT

- ➢ **Brief introduction to neutron star research**

- ➢ **Universal relations (UR) and correlations of NS**

- • **UR of f-mode**

- • **UR of gravitational binding energy**

- • **UR of rotational neutron star**

- • **UR and correlation analysis**

- ➢ **Conclusion**

Conclusion

➢ **The frequency and damping time of f-mode for NS with 1.4M$_\odot$ can be effectively constrained by the universal relation and observation of GW 170817.**

➢ **Gravitational binding energy is an ideal quantity to construct the universal relations. These general relations can be used to constrain global quantities of NS that are difficult to observe or cannot be observed directly.**

➢ **linear correlation analysis can be used to predict new universal relations and constrain the properties of NS.**

Thanks!

附录

Neutron Star Interior Composition Explorer (NICER)

- The main idea behind measuring stellar radii with NICER, or more appropriately the stellar compactness, is the identification and monitoring of "hot spots" on the stellar surface. Magnetic fields in pulsars are so strong and complex that charged particles that are ripped away from the star often crash back into the stellar surface creating hot spots, namely, regions within the star that glow brighter than the rest of the star.

- As the neutron star spins, the hot spots come in and out of view producing periodic variations in the brightness that are recorded by NICER.

- And just as the Shapiro delay takes advantage of general relativistic effects, so does NICER. Indeed, the gravitational field around the neutron star is so strong, that x-rays emitted from the back of the star get bent and are eventually detected by NICER's sophisticated instruments. For a highly compact neutron star, the hot spots never disappear: NICER actually sees the back of the star!

arXiv:2202.13191v1
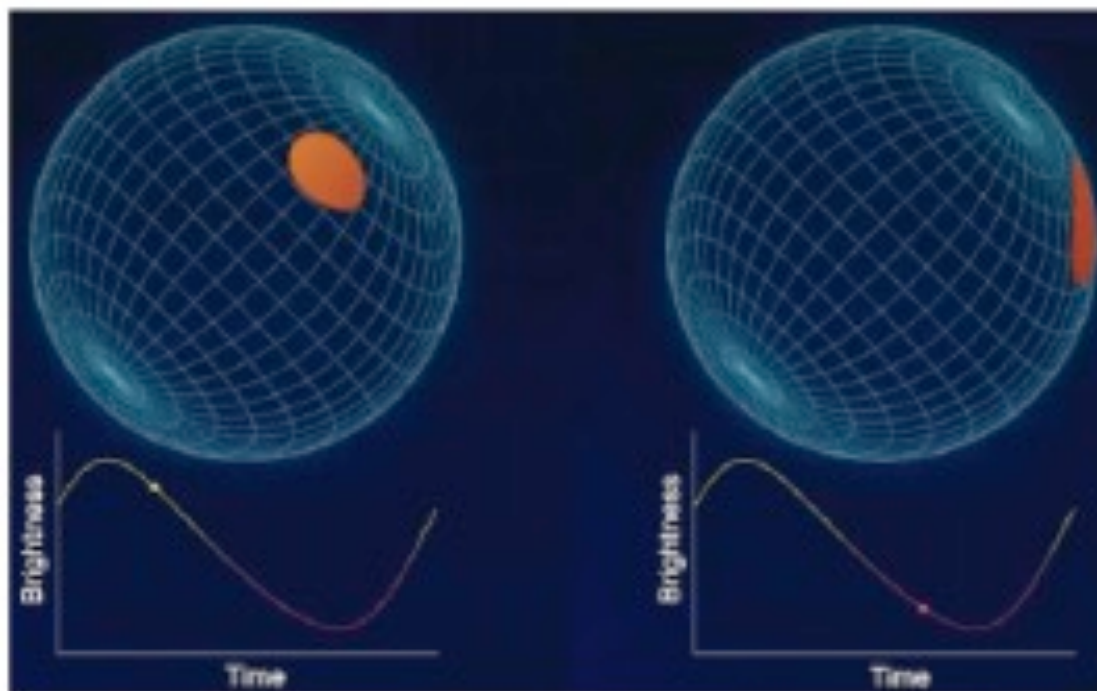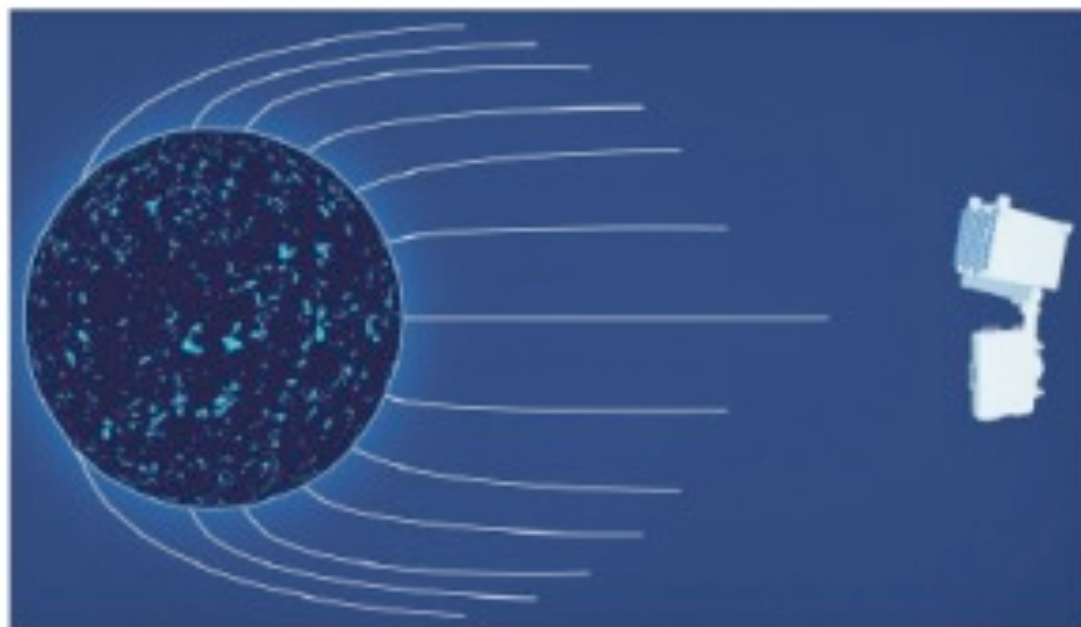
宏观可观测量通过Pearson相关系数的方法量化相关性

$$Corr[X,Y] = \frac{< (X - \mu_X)(Y - \mu_Y) >}{\sigma_X \sigma_Y}$$

- 其中，$\langle (X - \mu_x)(Y - \mu_y) \rangle$为变量之间的协方差，$\sigma_x$和$\sigma_y$分别为变量X和Y的标准差。当变量Y对X的增大而增大时，协方差为正值。反之，则为负值。为了衡量不同量纲的线性关系的好坏，需要将所有的协方差都统一进行归一化，以使相关系数在-1到+1之间的范围内取值。

- 线性程度越好的关系，其相关系数的绝对值越趋近于1。相关系数为正值，变量之间则为正相关; 反之，则为负相关。

$$\Lambda = \frac{2}{3} k_2 \cdot \left(\frac{R}{M} \right)^5$$

$$Q_{ij} = -\lambda \mathcal{E}_{ij}.$$

$$k_2 = \frac{3}{2} G \lambda R^{-5}.$$

$$k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y - 1) - y] \times$$
$$\left\{ 2C (6 - 3y + 3C(5y - 8)) + 4C^3 [13 - 11y + C(3y - 2) + 2C^2(1 + y)] \right.$$
$$\left. + 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \log (1 - 2C) \right\}^{-1},$$