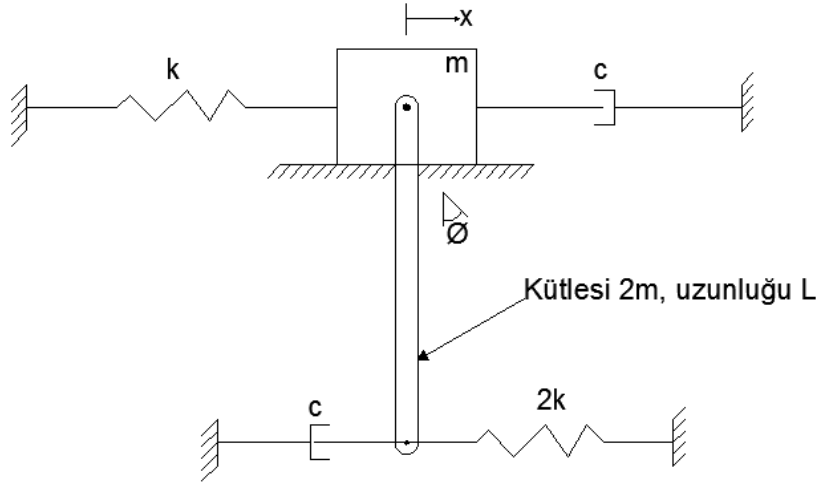


SORULAR ve CEVAPLAR

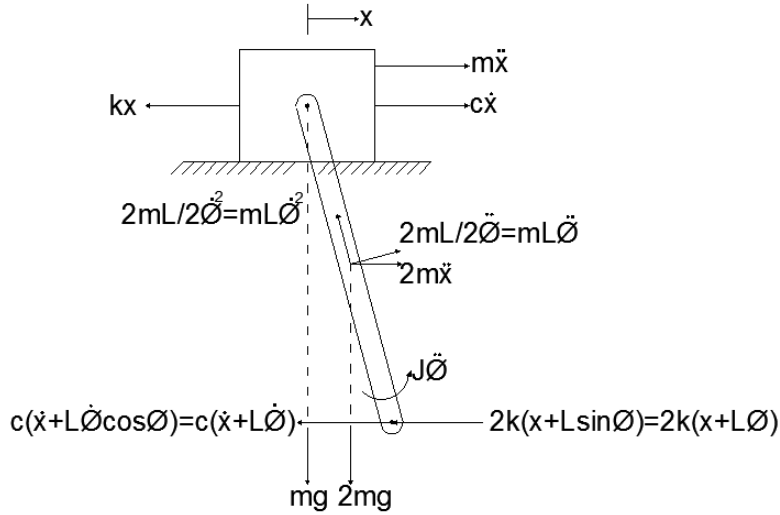
1 Aşağıdaki sistemin hareket denklemini elde ediniz.



Cevap

Newton Yöntemi ile çözüm

Verilen sistemin SCD'sini çizelim.



Küçük θ değerleri için $\sin \theta = \theta$, $\cos \theta = 1$, $\tan \theta = \theta$

m kütlesi için Newton Kanunu

$$\sum F = m.a \text{ (+ } \rightarrow \text{)}$$

$$m\ddot{x} + 2m\ddot{x} = -kx - 2k(x + L\theta) - c\dot{x} - c(\dot{x} + L\dot{\theta}) - mL\ddot{\theta} \cos \theta + mL\dot{\theta}^2 \sin \theta$$

$$3m\ddot{x} + mL\ddot{\theta} + 2c\dot{x} + cL\dot{\theta} + 3kx + 2kL\theta - mL\dot{\theta}^2 \theta = 0$$

$mL\dot{\theta}^2 \theta$ ihmal edilirse

$$3m\ddot{x} + mL\ddot{\theta} + 2c\dot{x} + cL\dot{\theta} + 3kx + 2kL\theta = 0 \dots \dots \dots (1)$$

$2m$ kütlesi için Newton Kanunu

$$\sum M = J\ddot{\theta} \text{ (Saat tersi yönü +)}$$

$$J\ddot{\theta} + mL\ddot{\theta} \frac{L}{2} + 2m\ddot{x} \frac{L}{2} = -c(\dot{x} + L\dot{\theta})L \cos \theta - 2k(x + L\theta)L \cos \theta - 2mg \frac{L}{2} \sin \theta$$

$$J = \frac{1}{12} 2mL^2 \quad J = \frac{mL^2}{6}$$

$$mL\ddot{x} + \left(\frac{mL^2}{6} + \frac{mL^2}{2}\right)\ddot{\theta} + c\dot{x} + cL^2\dot{\theta} + 2kx + (2kL^2 + mgL)\theta = 0 \dots\dots\dots(2)$$

1 ve 2 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} 3m & mL \\ mL & \frac{2mL^2}{3} \end{bmatrix}}_{\text{Kütle Matrisi}} \underbrace{\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix}} + \underbrace{\begin{bmatrix} 2c & cL \\ cL & cL^2 \end{bmatrix}}_{\text{Sönüm Matrisi}} \underbrace{\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix}} + \underbrace{\begin{bmatrix} 3k & 2kL \\ 2kL & (2kL^2 + mgL) \end{bmatrix}}_{\text{Rijitlik Matrisi}} \underbrace{\begin{Bmatrix} x \\ \theta \end{Bmatrix}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Lagrange Yöntemi ile çözüm

Sistemin Kinetik Enerjisi:

m kütleli cisim öteleme hareketi yapmaktadır. 2m kütleli çubuk ise hem dönme hemde öteleme hareketi yapmaktadır

$$T = \underbrace{\frac{1}{2}m\dot{x}^2}_{m \text{ kütleli öteleme}} + \underbrace{\frac{1}{2}2m\left(\dot{x} + \frac{1}{2}L\dot{\theta}\right)^2}_{2m \text{ kütleli öteleme}} + \underbrace{\frac{1}{2}\frac{1}{12}2mL^2\dot{\theta}^2}_{2m \text{ kütleli dönme}} \dots\dots\dots(3)$$

Sistemin potansiyel enerjisi:

$$U = \frac{1}{2}kx^2 + \frac{1}{2}2k(x + L\theta)^2 - 2mg\frac{L}{2}\cos\theta \dots\dots\dots(4)$$

Vizkoz sönüm elemanının yaptığı iş

$$\begin{aligned} \delta W &= -c\dot{x}\delta x - c(\dot{x} + L\dot{\theta})\delta(x + L\theta) \\ &= -c(2\dot{x} + L\dot{\theta})\delta x - cL(\dot{x} + L\dot{\theta})\delta\theta \end{aligned}$$

Lagrange denkleminin uygulanması

$$L = T - U$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} = Q_1$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{x}}\right) - \frac{\delta T}{\delta x} + \frac{\delta U}{\delta x} = Q_1$$

$$\frac{d}{dt}\left[m\dot{x} + 2m\left(\dot{x} + \frac{L}{2}\dot{\theta}\right)(1)\right] + [kx + 2k(x + L\theta)(1)] = -2c\dot{x} - cL\dot{\theta}$$

$$3m\ddot{x} + mL\ddot{\theta} + 2c\dot{x} + cL\dot{\theta} + 3kx + 2kL\theta = 0 \dots\dots\dots(5)$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{\theta}}\right) - \frac{\delta T}{\delta \theta} + \frac{\delta U}{\delta \theta} = Q_2$$

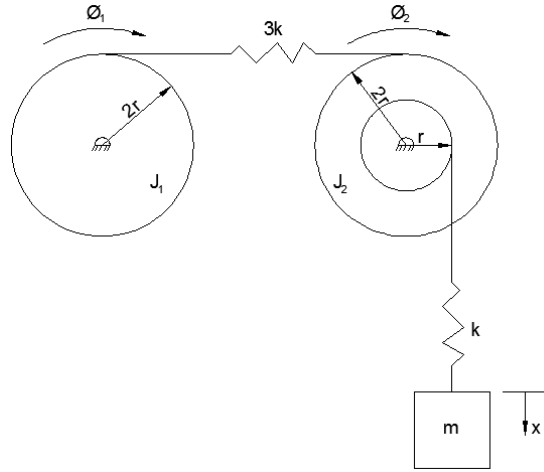
$$\frac{d}{dt}\left[2m\left(\dot{x} + \frac{1}{2}L\dot{\theta}\right)\left(\frac{L}{2}\right) + \frac{1}{12}2mL^2\dot{\theta}\right] + 2k(x + L\theta)(L) + mgL\sin\theta = -cL\dot{x} - cL^2\dot{\theta}$$

$$mL\ddot{x} + \frac{2mL^2}{3}\ddot{\theta} + cL\dot{x} + cL^2\dot{\theta} + 2kLx + (2kL^2 + mgL)\theta = 0 \dots\dots\dots(6)$$

5 ve 6 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} 3m & mL \\ mL & \frac{2mL^2}{3} \end{bmatrix}}_{\text{Kütle Matrisi}} \underbrace{\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix}} + \underbrace{\begin{bmatrix} 2c & cL \\ cL & cL^2 \end{bmatrix}}_{\text{Sönüm Matrisi}} \underbrace{\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix}} + \underbrace{\begin{bmatrix} 3k & 2kL \\ 2kL & (2kL^2 + mgL) \end{bmatrix}}_{\text{Rijitlik Matrisi}} \underbrace{\begin{Bmatrix} x \\ \theta \end{Bmatrix}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

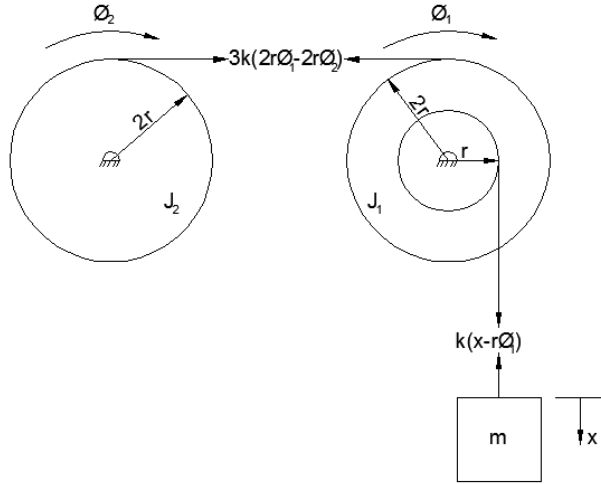
2 Aşağıdaki sistemin hareket denklemini elde ediniz.



Cevap

Newton Yöntemi ile çözüm

Verilen sistemin SCD'sini çizelim.



Küçük θ değerleri için $\sin \theta = \theta$, $\cos \theta = 1$, $\tan \theta = \theta$

m kütlesi için Newton Kanunu

$$\sum F = m \cdot a \quad (\downarrow +)$$

$$m\ddot{x} = -k(x - r\theta_1)$$

$$m\ddot{x} + kx - kr\theta_1 = 0 \dots \dots \dots (1)$$

$$\sum M = J_1 \ddot{\theta}_1 \quad (\text{Saat yönü} +)$$

$$J_1 \ddot{\theta}_1 = -6kr(2r\theta_1 - 2r\theta_2) + kr(x - r\theta_1)$$

$$J_1 \ddot{\theta}_1 = -12kr^2\theta_1 + 12kr^2\theta_2 + krx - kr^2\theta_1$$

$$J_1 \ddot{\theta}_1 + 13kr^2\theta_1 - 12kr^2\theta_2 - krx = 0 \dots \dots \dots (2)$$

$$\sum M = J_2 \ddot{\theta}_2 \quad (\text{Saat yönü} +)$$

$$J_2 \ddot{\theta}_2 = 6kr(2r\theta_1 - 2r\theta_2)$$

$$J_2 \ddot{\theta}_2 + 12kr^2\theta_2 - 12kr^2\theta_1 = 0 \dots \dots \dots (3)$$

1,2 ve 3 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} m & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_2 \end{bmatrix}}_{\text{Kütle Matrisi}} \underbrace{\begin{Bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix}} + \underbrace{\begin{bmatrix} k & -kr & 0 \\ -kr & 13kr^2 & -12kr^2 \\ 0 & -12kr^2 & 12kr^2 \end{bmatrix}}_{\text{Rijitlik Matrisi}} \underbrace{\begin{Bmatrix} x \\ \theta_1 \\ \theta_2 \end{Bmatrix}} = \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}}_{\text{Sıfır Vektörü}}$$

Lagrange Yöntemi ile çözüm

Sistemin Kinetik Enerjisi:

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} m \dot{x}^2 \dots\dots\dots (4)$$

Sistemin potansiyel enerjisi:

$$U = \frac{1}{2} k (x - r\theta_1)^2 + \frac{1}{2} 3k (2r\theta_1 - 2r\theta_2)^2 \dots\dots\dots (5)$$

Lagrange denkleminin uygulanması

$$L = T - U$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{x}} \right) - \frac{\delta L}{\delta x} = 0$$

$$\frac{d}{dt} \left(\frac{\delta T}{\delta \dot{x}} \right) - \frac{\delta T}{\delta x} + \frac{\delta U}{\delta x} = 0$$

$$\frac{d}{dt} (m\dot{x}) + [k(x - r\theta_1)(1)] = 0$$

$$m\ddot{x} + kx - kr\theta_1 = 0 \dots\dots\dots (6)$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{\theta}_1} \right) - \frac{\delta L}{\delta \theta_1} = 0$$

$$\frac{d}{dt} \left(\frac{\delta T}{\delta \dot{\theta}_1} \right) - \frac{\delta T}{\delta \theta_1} + \frac{\delta U}{\delta \theta_1} = 0$$

$$\frac{d}{dt} (J_1 \dot{\theta}_1) + [k(x - r\theta_1)(-r) + 3k(2r\theta_1 - 2r\theta_2)(2r)]$$

$$J_1 \ddot{\theta}_1 - kr x + 13kr^2 \theta_1 - 12kr^2 \theta_2 = 0 \dots\dots\dots (7)$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{\theta}_2} \right) - \frac{\delta L}{\delta \theta_2} = 0$$

$$\frac{d}{dt} \left(\frac{\delta T}{\delta \dot{\theta}_2} \right) - \frac{\delta T}{\delta \theta_2} + \frac{\delta U}{\delta \theta_2} = 0$$

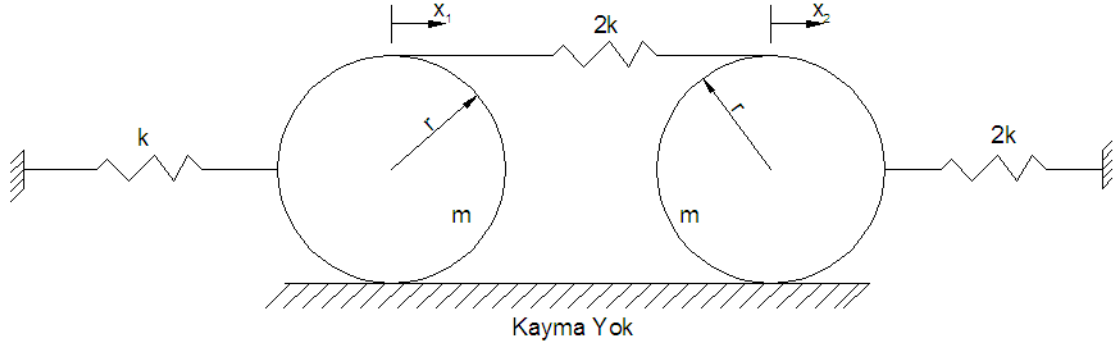
$$\frac{d}{dt} (J_2 \dot{\theta}_2) + [3k(2r\theta_1 - 2r\theta_2)(-2r)]$$

$$J_2 \ddot{\theta}_2 - 12kr^2 \theta_1 + 12kr^2 \theta_2 = 0 \dots\dots\dots (8)$$

6,7 ve 8 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} m & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_2 \end{bmatrix}}_{\text{Kütle Matrisi}} \underbrace{\begin{Bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix}} + \underbrace{\begin{bmatrix} k & -kr & 0 \\ -kr & 13kr^2 & -12kr^2 \\ 0 & -12kr^2 & 12kr^2 \end{bmatrix}}_{\text{Rijitlik Matrisi}} \underbrace{\begin{Bmatrix} x \\ \theta_1 \\ \theta_2 \end{Bmatrix}} = \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}}_{\text{Sıfır Vektörü}}$$

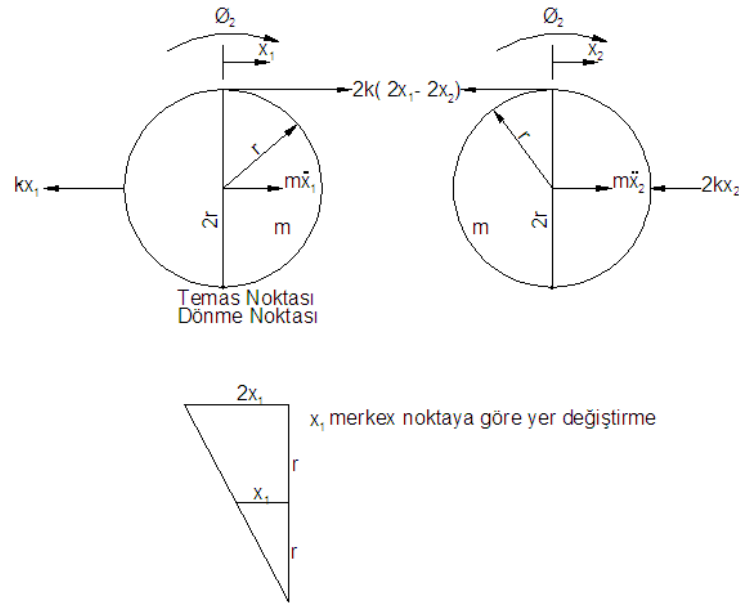
3 Aşağıdaki sistemin hareket denklemini elde ediniz.



Cevap

Newton Yöntemi ile çözüm

Verilen sistemin SCD'sini çizelim.



$$\theta_1 = \frac{x_1}{r} \quad \theta_2 = \frac{x_2}{r} \quad J = \frac{1}{2}mr^2$$

$$\sum M = J_1 \ddot{\theta}_1 \quad (\text{Saat yönü } +) \quad (\text{Dönme Noktasına Göre})$$

$$\frac{1}{2}mr^2 \frac{\ddot{x}_1}{r} + m\ddot{x}_1 r = 2k(2x_2 - x_1)2r - kx_1 r$$

$$\frac{3}{2}m\ddot{x}_1 = 8kx_2 - 8kx_1 - kx_1$$

$$\frac{3}{2}m\ddot{x}_1 + 9kx_1 - 8kx_2 = 0 \dots \dots \dots (1)$$

$$\sum M = J_2 \ddot{\theta}_2 \quad (\text{Saat yönü } +) \quad (\text{Dönme Noktasına Göre})$$

$$\frac{1}{2}mr^2 \frac{\ddot{x}_2}{r} + m\ddot{x}_2 r = -2k(2x_2 - x_1)2r - 2kx_2 r$$

$$\frac{3}{2}m\ddot{x}_2 = -10kx_2 + 8kx_1$$

$$\frac{3}{2}m\ddot{x}_2 - 8kx_1 + 10kx_2 = 0 \dots \dots \dots (2)$$

1 ve 2 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} \frac{3m}{2} & 0 \\ 0 & \frac{3m}{2} \end{bmatrix}}_{\text{Kütle Matrisi}} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} 9k & -8k \\ -8k & 10k \end{bmatrix}}_{\text{Rijitlik Matrisi}} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Lagrange Yöntemi ile çözüm

Sistemin Kinetik Enerjisi:

m kütleli cisimler hem öteleme hem dönme hareketi yapmaktadır.

$$T = \underbrace{\frac{1}{2} m \dot{x}_1^2}_{m \text{ kütleli } x_1 \text{ öteleme}} + \underbrace{\frac{1}{2} \frac{1}{r} m r^2 \left(\frac{\dot{x}_1}{r}\right)^2}_{m \text{ kütleli } \theta_1 \text{ dönme}} + \underbrace{\frac{1}{2} m \dot{x}_2^2}_{m \text{ kütleli } x_2 \text{ öteleme}} + \underbrace{\frac{1}{2} \frac{1}{r} m r^2 \left(\frac{\dot{x}_2}{r}\right)^2}_{m \text{ kütleli } \theta_2 \text{ dönme}} \dots (3)$$

Sistemin potansiyel enerjisi:

$$U = \frac{1}{2} k x_1^2 + \frac{1}{2} 2k x_2^2 + \frac{1}{2} 2k (2x_2 - 2x_1)^2 \dots (4)$$

Lagrange denkleminin uygulanması

$$L = T - U$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{x}_1} \right) - \frac{\delta L}{\delta x_1} = 0$$

$$\frac{d}{dt} \left(\frac{\delta T}{\delta \dot{x}_1} \right) - \frac{\delta T}{\delta x_1} + \frac{\delta U}{\delta x_1} = 0$$

$$\frac{d}{dt} \left[m \dot{x}_1 + \frac{1}{2} m \dot{x}_1 (1) \right] + [k x_1 + 2k (2x_2 - 2x_1)(2)] = 0$$

$$\frac{3}{2} m \ddot{x}_1 + 9k x_1 - 8k x_2 = 0 \dots (5)$$

$$\frac{d}{dt} \left(\frac{\delta L}{\delta \dot{x}_2} \right) - \frac{\delta L}{\delta x_2} = 0$$

$$\frac{d}{dt} \left(\frac{\delta T}{\delta \dot{x}_2} \right) - \frac{\delta T}{\delta x_2} + \frac{\delta U}{\delta x_2} = 0$$

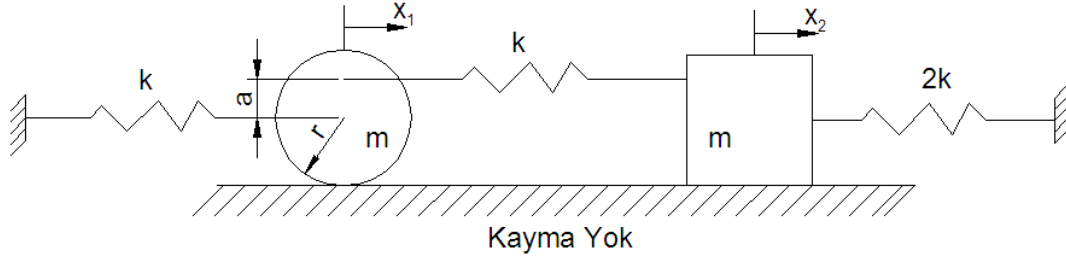
$$\frac{d}{dt} \left[m \dot{x}_2 + \frac{1}{2} m \dot{x}_2 (1) \right] + [2k x_2 + 2k (2x_2 - 2x_1)(2)] = 0$$

$$\frac{3}{2} m \ddot{x}_2 - 8k x_1 + 10k x_2 = 0 \dots (6)$$

5 ve 6 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} \frac{3m}{2} & 0 \\ 0 & \frac{3m}{2} \end{bmatrix}}_{\text{Kütle Matrisi}} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} 9k & -8k \\ -8k & 10k \end{bmatrix}}_{\text{Rijitlik Matrisi}} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

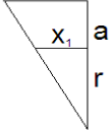
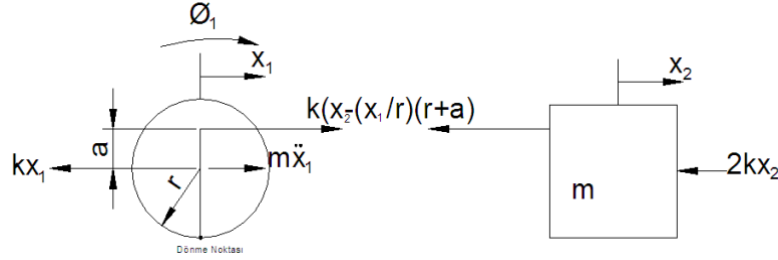
4 Aşağıdaki sistemin hareket denklemini elde ediniz.



Cevap

Newton Yöntemi ile çözüm

Verilen sistemin SCD'sini çizelim.



$x_1 = \theta r$ Benzerlikten

$$\frac{r}{(r+a)} = \frac{\theta r}{y} \quad y = \theta(r+a), \quad y = \frac{x_1}{r}(r+a), \quad \theta_1 = \frac{x_1}{r}, \quad J = \frac{1}{2}mr^2$$

$$\sum M = J\ddot{\theta} \quad (\text{Saat yönü } +) \quad (\text{Dönme Noktasına Göre})$$

$$\frac{1}{2}mr^2 \frac{\ddot{x}_1}{r} + m\ddot{x}_1 r = -kx_1 r + k \left(x_2 - \frac{x_1}{r}(r+a) \right) (r+a)$$

$$\frac{3}{2}m\ddot{x}_1 = -kx_1 + kx_2 \left(1 + \frac{a}{r} \right) - kx_1 \frac{(r+a)^2}{r^2}$$

$$\frac{3}{2}m\ddot{x}_1 = -kx_1 \left(1 + \frac{r^2 + 2ar + a^2}{r^2} \right) + kx_2 \left(1 + \frac{a}{r} \right)$$

$$\frac{3}{2}m\ddot{x}_1 + kx_1 \left(1 + \frac{r^2 + 2ar + a^2}{r^2} \right) - kx_2 \left(1 + \frac{a}{r} \right) = 0 \dots \dots \dots (1)$$

$$\sum F = m\ddot{x}_2 \quad (+ \rightarrow)$$

$$m\ddot{x}_2 = -2kx_2 - k \left(x_2 - \frac{x_1}{r}(r+a) \right)$$

$$m\ddot{x}_2 = -2kx_2 - kx_2 + kx_1 \left(1 + \frac{a}{r} \right)$$

$$m\ddot{x}_2 - kx_1 \left(1 + \frac{a}{r} \right) + 3kx_2 = 0 \dots \dots \dots (2)$$

1 ve 2 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} \frac{3m}{2} & 0 \\ 0 & m \end{bmatrix}}_{\text{Kütle Matrisi}} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} k \left(2 + \frac{2a}{r} + \frac{a^2}{r^2} \right) & -k \left(1 + \frac{a}{r} \right) \\ -k \left(1 + \frac{a}{r} \right) & 3k \end{bmatrix}}_{\text{Rijitlik Matrisi}} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Lagrange Yöntemi ile çözüm

Sistemin Kinetik Enerjisi:

m kütleli disk hem öteleme hem dönme hareketi yapmaktadır.

$$T = \underbrace{\frac{1}{2}m\dot{x}_1^2}_{m \text{ kütleli } x_1 \text{ öteleme}} + \underbrace{\frac{1}{2} \frac{1}{2}mr^2\left(\frac{\dot{x}_1}{r}\right)^2}_{m \text{ kütleli } \theta_1 \text{ dönme}} + \underbrace{\frac{1}{2}m\dot{x}_2^2}_{m \text{ kütleli } x_2 \text{ öteleme}} \dots\dots\dots(3)$$

Sistemin potansiyel enerjisi:

$$U = \frac{1}{2}kx_1^2 + \frac{1}{2}k\left(x_2 - \frac{x_1}{r}(r+a)\right)^2 + \frac{1}{2}2kx_2^2 \dots\dots\dots(4)$$

Lagrange denkleminin uygulanması

$$L = T - U$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}_1}\right) - \frac{\delta L}{\delta x_1} = 0$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{x}_1}\right) - \frac{\delta T}{\delta x_1} + \frac{\delta U}{\delta x_1} = 0$$

$$\frac{d}{dt}\left[m\dot{x}_1 + \frac{1}{2}m\dot{x}_1(1)\right] + \left[kx_1 - k\left(x_2 - \frac{x_1}{r}(r+a)\right)\frac{(r+a)}{r}\right] = 0$$

$$\frac{3}{2}m\ddot{x}_1 + kx_1 - kx_2\left(1 + \frac{a}{r}\right) + kx_1\left(1 + \frac{2a}{r} + \frac{a^2}{r^2}\right) = 0$$

$$\frac{3}{2}m\ddot{x}_1 + kx_1\left(2 + \frac{2a}{r} + \frac{a^2}{r^2}\right) - kx_2\left(1 + \frac{a}{r}\right) = 0 \dots\dots\dots(5)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}_2}\right) - \frac{\delta L}{\delta x_2} = 0$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{x}_2}\right) - \frac{\delta T}{\delta x_2} + \frac{\delta U}{\delta x_2} = 0$$

$$\frac{d}{dt}[m\dot{x}_2] + k\left(x_2 - \frac{x_1}{r}(r+a)\right)(1) + 2kx_2 = 0$$

$$m\ddot{x}_2 + kx_2 - kx_1\left(1 + \frac{a}{r}\right) + 2kx_2 = 0$$

$$m\ddot{x}_2 - kx_1\left(1 + \frac{a}{r}\right) + 3kx_2 = 0 \dots\dots\dots(6)$$

5 ve 6 nolu denklemleri matris formatında yazarsak

$$\underbrace{\begin{bmatrix} \frac{3m}{2} & 0 \\ 0 & m \end{bmatrix}}_{\text{Kütle Matrisi}} \underbrace{\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}} + \underbrace{\begin{bmatrix} k\left(2 + \frac{2a}{r} + \frac{a^2}{r^2}\right) & -k\left(1 + \frac{a}{r}\right) \\ -k\left(1 + \frac{a}{r}\right) & 3k \end{bmatrix}}_{\text{Rijitlik Matrisi}} \underbrace{\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}} = \underbrace{\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$