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CSE427

Answer

Polynomial kernel of degree 2,

$$K(x,y) = (x \cdot y + 1)^2$$

$$= (x \cdot y + 1)(x \cdot y + 1)$$

= ~~[illegible]~~

$$= 2xy + x^2y^2 + 1$$

$$= (\sqrt{2}x, x^2, 1) \cdot (\sqrt{2}y, y^2, 1)$$

$$\therefore \varphi(x) = \begin{pmatrix} \sqrt{2}x \\ x^2 \\ 1 \end{pmatrix}$$

Now transforming each data points,

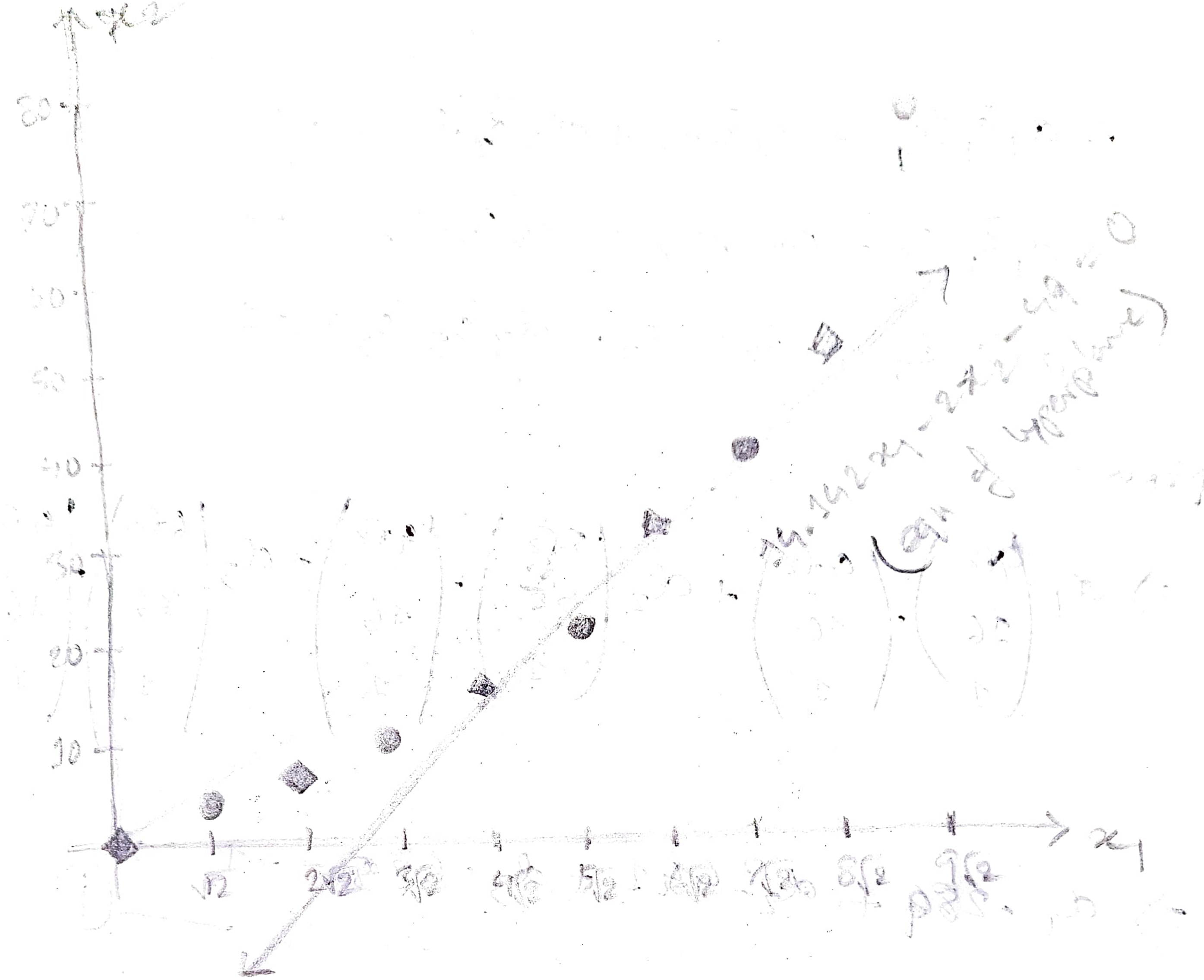
$$\varphi(x_1 = 0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \varphi(x_2 = 1) = \begin{pmatrix} \sqrt{2} \\ 1 \\ 1 \end{pmatrix}$$

$$\phi(x_3=2) = \begin{pmatrix} 2\sqrt{2} \\ 4 \\ 1 \end{pmatrix}, \quad \phi(x_4=3) = \begin{pmatrix} 3\sqrt{2} \\ 9 \\ 1 \end{pmatrix}$$

$$\phi(x_5=4) = \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix}, \quad \phi(x_6=5) = \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix}$$

$$\phi(x_7=6) = \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix}, \quad \phi(x_8=7) = \begin{pmatrix} 7\sqrt{2} \\ 49 \\ 1 \end{pmatrix}$$

$$\phi(x_9=8) = \begin{pmatrix} 8\sqrt{2} \\ 64 \\ 1 \end{pmatrix}, \quad \phi(x_{10}=9) = \begin{pmatrix} 9\sqrt{2} \\ 81 \\ 1 \end{pmatrix}$$



$$s_1 = \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} \qquad s_2 = \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} \qquad s_3 = \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix}$$

(-1) $\qquad$ $(+1)$ $\qquad$ (-1)

$$\Rightarrow \alpha_1 \bar{s}_1 \cdot \bar{s}_1 + \alpha_2 \bar{s}_2 \cdot \bar{s}_1 + \alpha_3 \bar{s}_3 \cdot \bar{s}_1 = -1$$

$$\alpha_1 \bar{s}_1 \cdot \bar{s}_2 + \alpha_2 \bar{s}_2 \cdot \bar{s}_2 + \alpha_3 \bar{s}_3 \cdot \bar{s}_2 = +1$$

$$\alpha_1 \bar{s}_1 \cdot \bar{s}_3 + \alpha_2 \bar{s}_2 \cdot \bar{s}_3 + \alpha_3 \bar{s}_3 \cdot \bar{s}_3 = -1$$

Now,

$$\Rightarrow \alpha_1 \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} = -1$$

$$\Rightarrow \alpha_1 \cdot 289 + \alpha_2 \cdot 441 + \alpha_3 \cdot 625 = -1 \quad \text{—— (I)}$$

Again,

$$\alpha_1 \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} = +1$$

$$\Rightarrow \alpha_1 \cdot 441 + \alpha_2 \cdot 676 + \alpha_3 \cdot 961 = +1 \quad \text{—— (II)}$$

~~Again,~~

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Again,

$$\alpha_1 \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix} = -1$$

$$\Rightarrow \alpha_1 \cdot 625 + \alpha_2 \cdot 961 + \alpha_3 \cdot 1369 = -1 \quad \text{(iii)}$$

solving (i), (ii), (iii) we get,

$$\therefore \alpha_1 = -791$$

$$\alpha_2 = 1278$$

$$\alpha_3 = -536$$

Now $\bar{w} = \sum_i \alpha_i \bar{s}_i$

$$= -791 \begin{pmatrix} 4\sqrt{2} \\ 16 \\ 1 \end{pmatrix} + 1278 \begin{pmatrix} 5\sqrt{2} \\ 25 \\ 1 \end{pmatrix} - 536 \begin{pmatrix} 6\sqrt{2} \\ 36 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 14.142 \\ -2 \\ -49 \end{pmatrix}$$

$\therefore w = \begin{pmatrix} 14.142 \\ -2 \end{pmatrix}$

and bias, $b = -49$.

$\therefore$ The equation of the hyperplane is,

$$\begin{pmatrix} 14.142 \\ -2 \end{pmatrix}^T \cdot x - 49 = 0$$

or $14.142x_1 - 2x_2 - 49 = 0$

or $x_2 = \frac{14.142}{2}x_1 - \frac{49}{2}$

$\therefore x_2 = 7.071x_1 - 24.5$

(Ans)