

# First Class: Introduction

## Second Class: Cryptography Review

### Cryptographic Hash Function

Mathematical Function

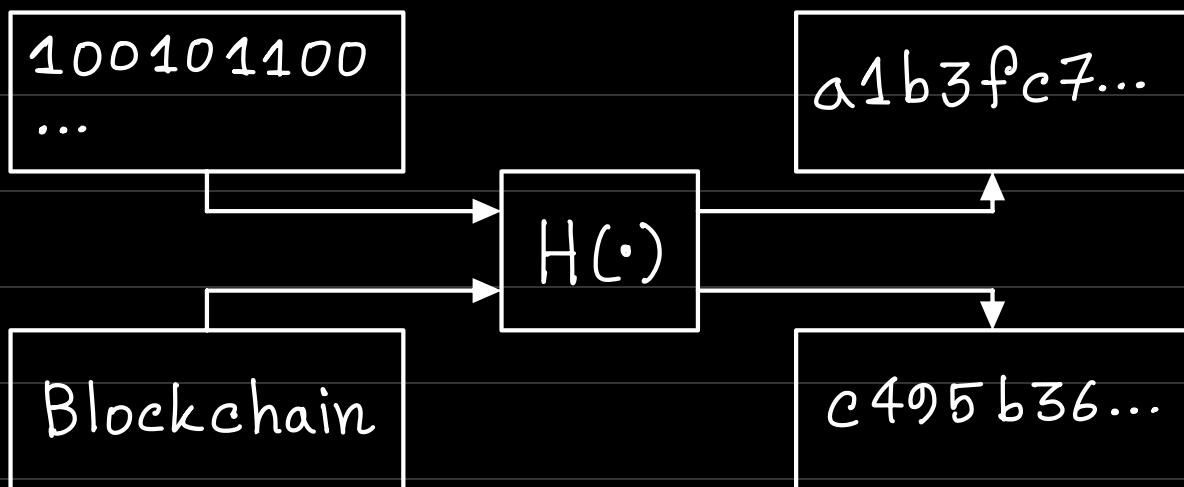
$$f: A \rightarrow B$$

$$f(a) = b; a \in A, b \in B$$

Mathematical function. Properties:

1. Input {any size}
2. Output {fixed size}
3. Time complexity:  $O(n)$

Compression:  $\{0,1\}^* \rightarrow \{0,1\}^n$



Collision-resistance } must-have  
 preimage resistance }  
 hiding } Desirable for  
 puzzle-friendliness } certain blockchain  
 system

Collision Resistance

$x \neq y ; H(x) = H(y)$  (must avoid this)

Very difficult to find collision,  
 So infeasible.

Infeasible: Hard to find a collision, but no,  
 no collisions exist.

input size > output size

{The pigeonhole principle}

To find collision,

Output,  $n = 3$  bits

Input,  $n_0 = 2^n + 1$  bits  
 $= 2^3 + 1$  bits  
 $= 9$  bits

Since  $\text{input\_size} > \text{output\_size}$ ,  
there must be a collision.

Worst Case:  $2^n + 1$  times

Best Case:  $2^{n/2} + 1$  times

The previous way was brute force.

Some Hash function:

$$H(x) = x \bmod 2^{256}$$

One collision:  $x \bmod 2^n \approx 2$  bits  
3 and  $3 + 2^{256}$   $x \bmod 4 \{0, 1, 2, 3\}$

Application:

Message Digest

Hash of any input, bits,  
random strings,  
characters, or even files.

Checksum: Hash of files

# Pre-Image Resistance:

H: Hash Functions

$$H(x) = y$$

One way function

For essentially all pre-specified outputs  $y$ , it is computationally infeasible to find an  $x$  such that

$$H(x) = y$$

$$a21b34c5 \rightarrow H^{-1}(\cdot) \rightarrow ???$$

Low min-entropy and High min-entropy

Sample Space:  $X = \{h, t\}$

$$H(x) = y$$

Can attacker find the value of  $x$  given  $y$ ?

Low min-entropy to High min-entropy

Hiding:  $H(r||x)$

$r$  is a secret value

Tackles  $x$  picking up  
from a low min-entropy  
distribution.

Hiding: Application

Commitment Scheme

Who will win?

Argentina

Hiding: Commitment Scheme

com := commit(msg, key)

↓  
hash function

Verification := verify (com, msg, key)

Security Properties:

1. Hiding
2. Binding

# Puzzle-Friendliness

$$H(K||x) = y$$

$K$  is chosen from a distribution with high min entropy.

Infeasible to find  $x$  in time significantly less than  $2^n$ .

## Puzzle-Friendliness: Application

A target set  $Y$ , for a valid solution

$z, z \in Y$

computation:  $z = H(\text{puzzle-ID}||x)$

$x$  changes until  $z \in Y$

## Family of hash functions:

\* MD5 (Message Digest 5) → Currently considered broken!

\* Secure Hashing Algorithm 1 (SHA1) → Currently considered broken!

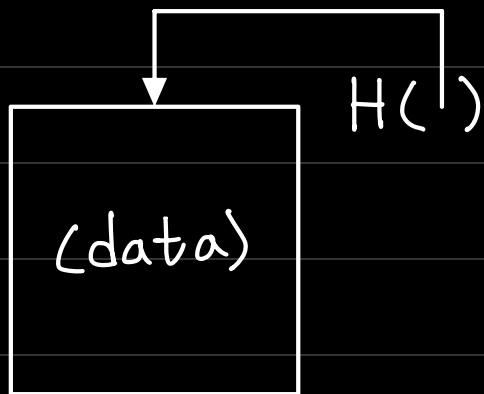
\* Secure Hashing Algorithm 2/3 (SHA2/3) → Safe to use, SHA3 preferable.

# Third Class: Cryptography Review

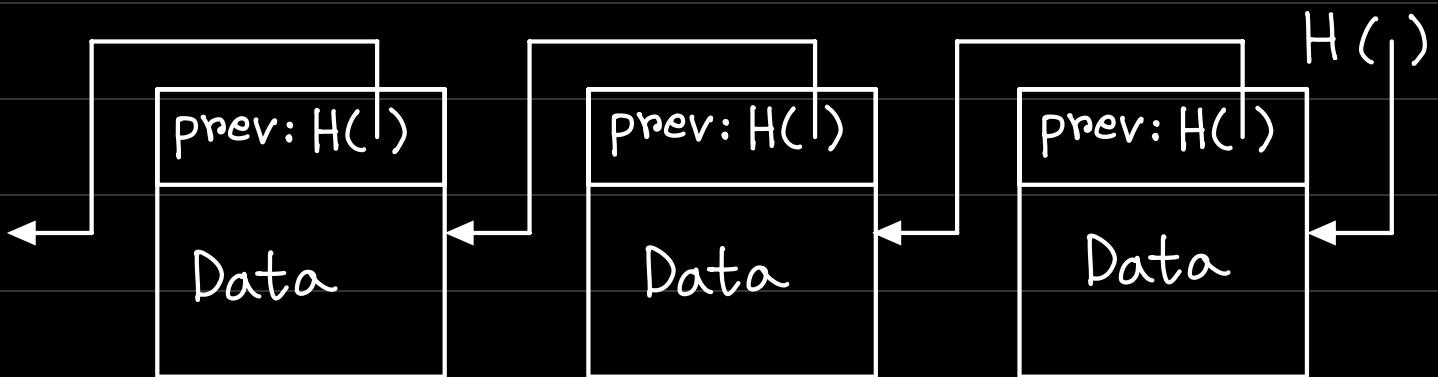
Hash pointer: A pointer to where some information is stored together with a cryptographic hash of the information.

Pros: To get the info back

To verify that it hasn't changed



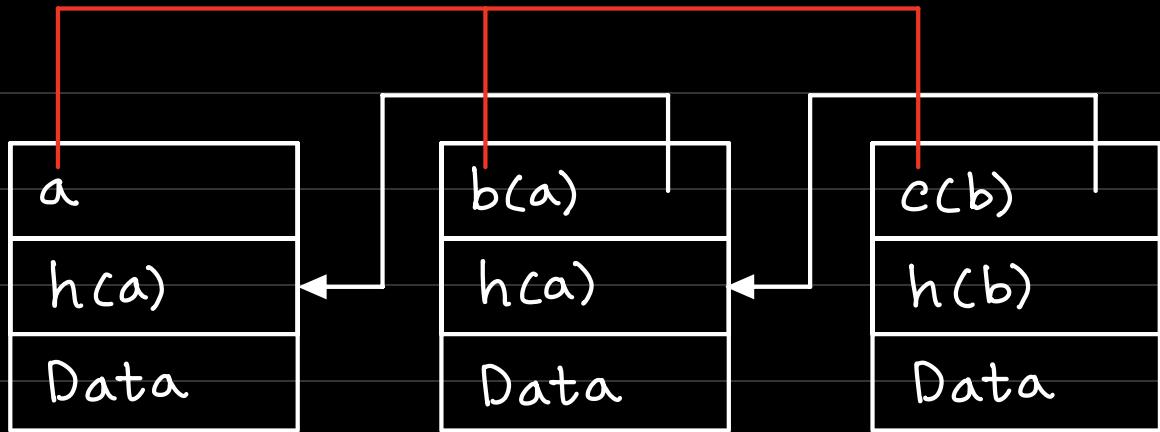
Linked List with Hash Pointer:



Temper-Evident Log

$$H(x) = y$$

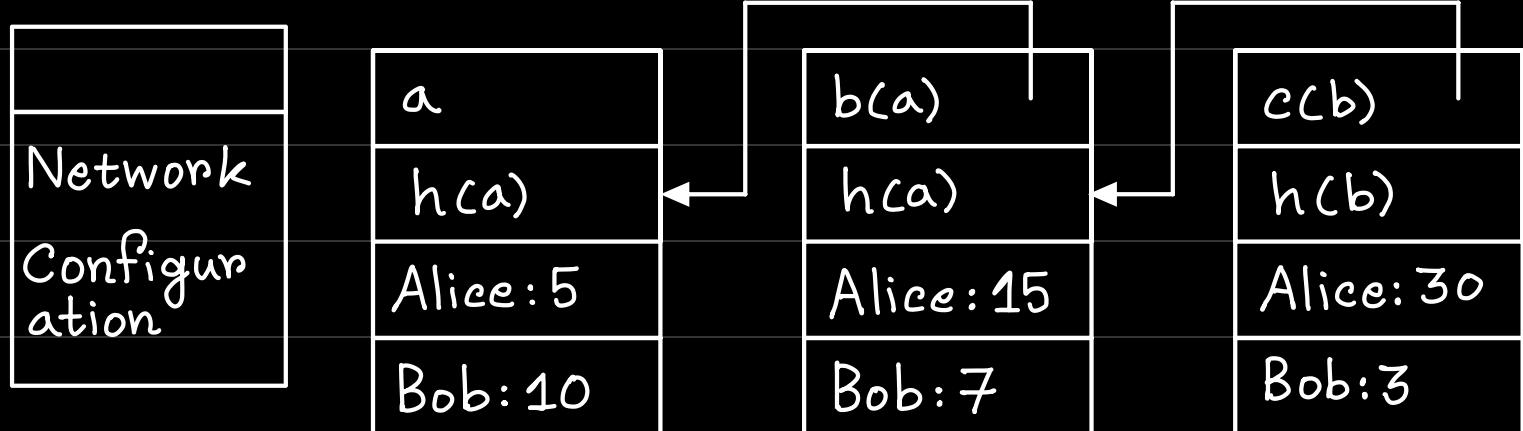
Address



$$H \left( \begin{array}{|c|c|c|} \hline \text{---} & \text{---} & \text{---} \\ \hline \text{---} & \text{---} & \text{---} \\ \hline \text{---} & \text{---} & \text{---} \\ \hline \end{array} \right) = y$$

block hash

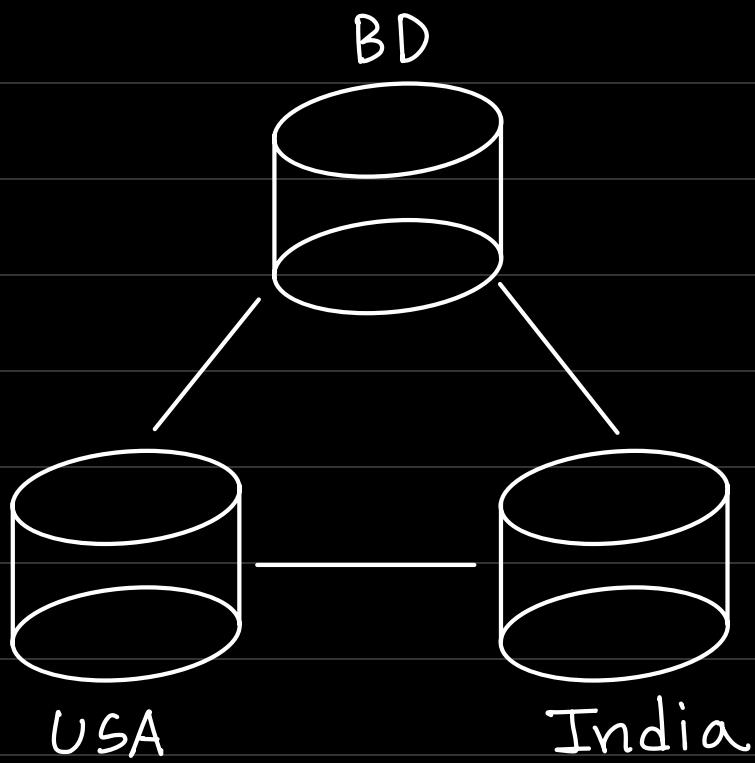
a	b	c
d	e	f
g	h	i



Blockchain!

$$H(A^5_{B10}) = abc65d\ldots \quad H(A^5_{B1000}) = cba79c\ldots$$

$$H(A^5 B^{10}) \neq H(A^5 B^{1000})$$



## Merkle Tree (Tree Algorithm)

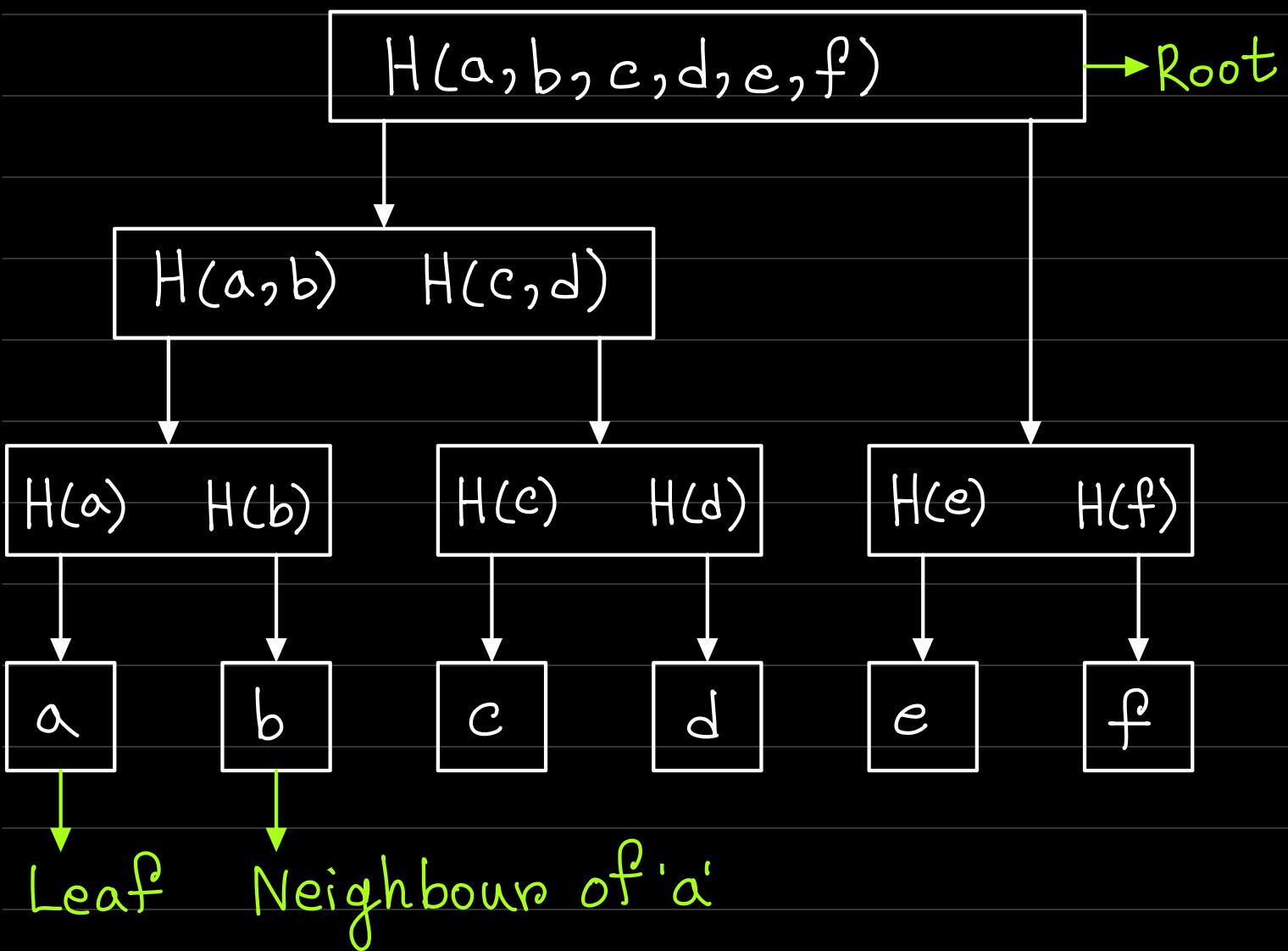
A data structure using cryptographic hashes, basically a binary tree (Almost two leaves) with hash pointer.

Time/Space Complexity:  $O(\log N)$

Used as an efficient and secure

way to verify large data structure.

Block  
 $S = \{a, b, c, d, e, f\}$  Target : (f)  
Block



Proof of membership:

1. Proof of membership for the leaf with knowledge of the root.
2. Using the neighbour nodes to verify if it leads to the root.

Bloom filter: a probabilistic data structure which allows to test if an element is a member of set.

Test if  $e \in S$

Returns either

True ("e possibly in set S")

or

False ("e definitely not in set S")

Set S: Apple

Element e: Apple

$H_1(\text{apple}) = \underline{1} \dots \text{MD5}$

$P_1(\text{add})$

$H_2(\text{apple}) = \underline{D} \dots \text{SHA1}$

$P_2(\text{evaluate})$

$H_3(\text{apple}) = \underline{3} \dots \text{SHA2}$

For 1D3,

	1	1											1		
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Binary Number: Either 1 or 0.

Set S: Apple, Lime

$$H_1(e) = \underline{6} \dots$$

Element e: Lime

$$H_2(e) = \underline{C} \dots$$

$$H_3(e) = \underline{E} \dots$$

For 6CE,

	1	1			1								1	1	1	
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	

Set S: Apple, Lime, Lemon  $H_1(e) = \underline{3} \dots$

Element e: Lemon  $H_2(e) = \underline{D} \dots$

$$H_3(e) = \underline{F} \dots$$

For 3DF,

~~X~~ ~~X~~ ✓

	1	1			1								1	1	1	1
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	

Finally,

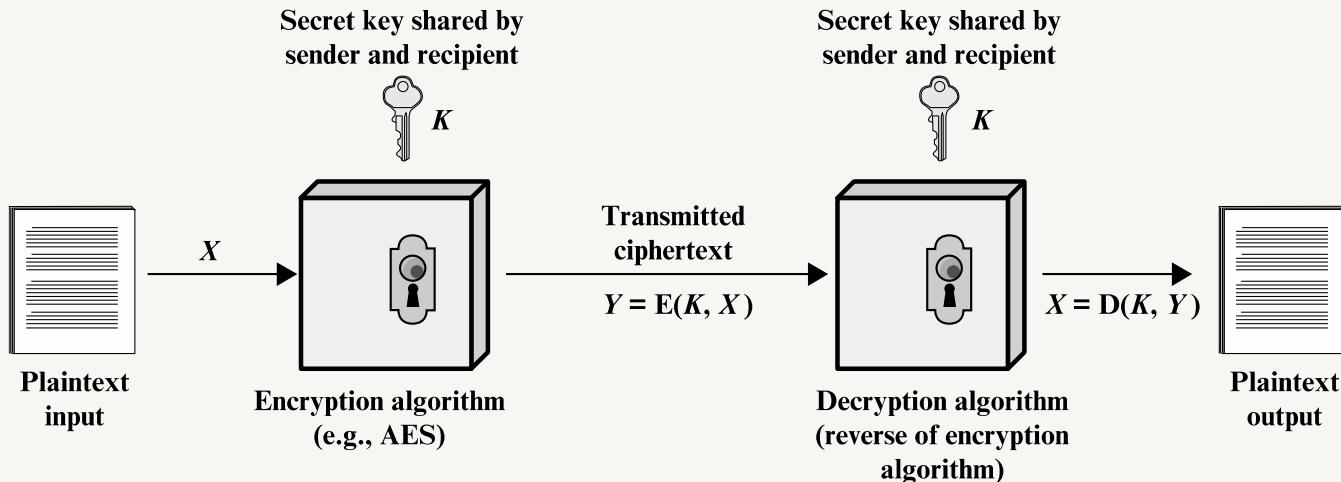
0	1	0	1	0	0	1	0	0	0	0	0	1	1	1	1
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

# Probabilistic Algorithm

Mango: A<sup>9</sup>6 {All should be True.}  
X X ✓

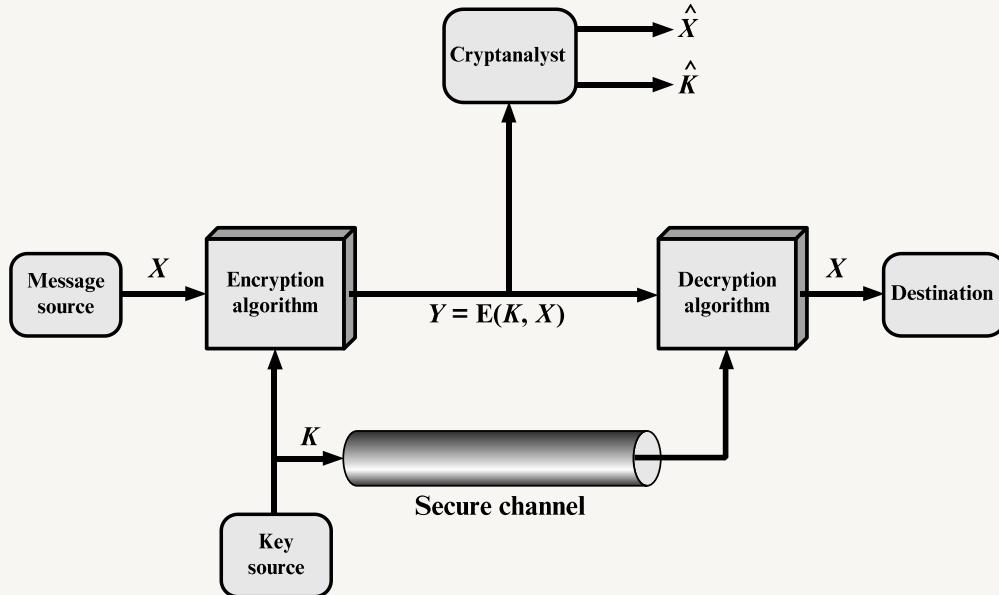
Grape: 6F3 {But there is no grape.  
✓✓✓ False Positive.}

# Symmetric encryption



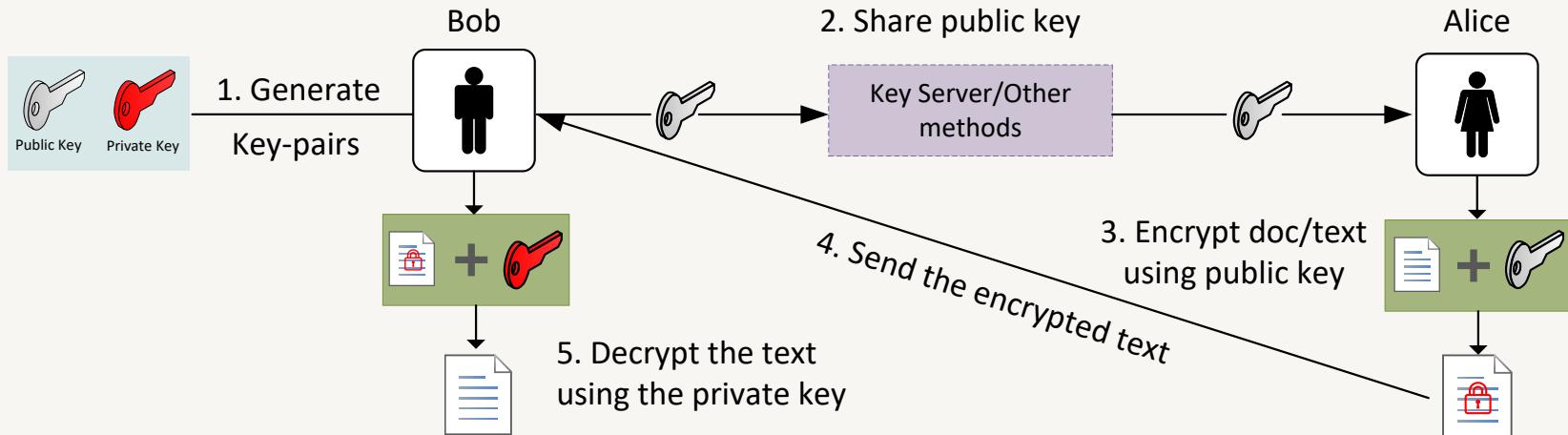
The main issue is key-management!

# Symmetric encryption

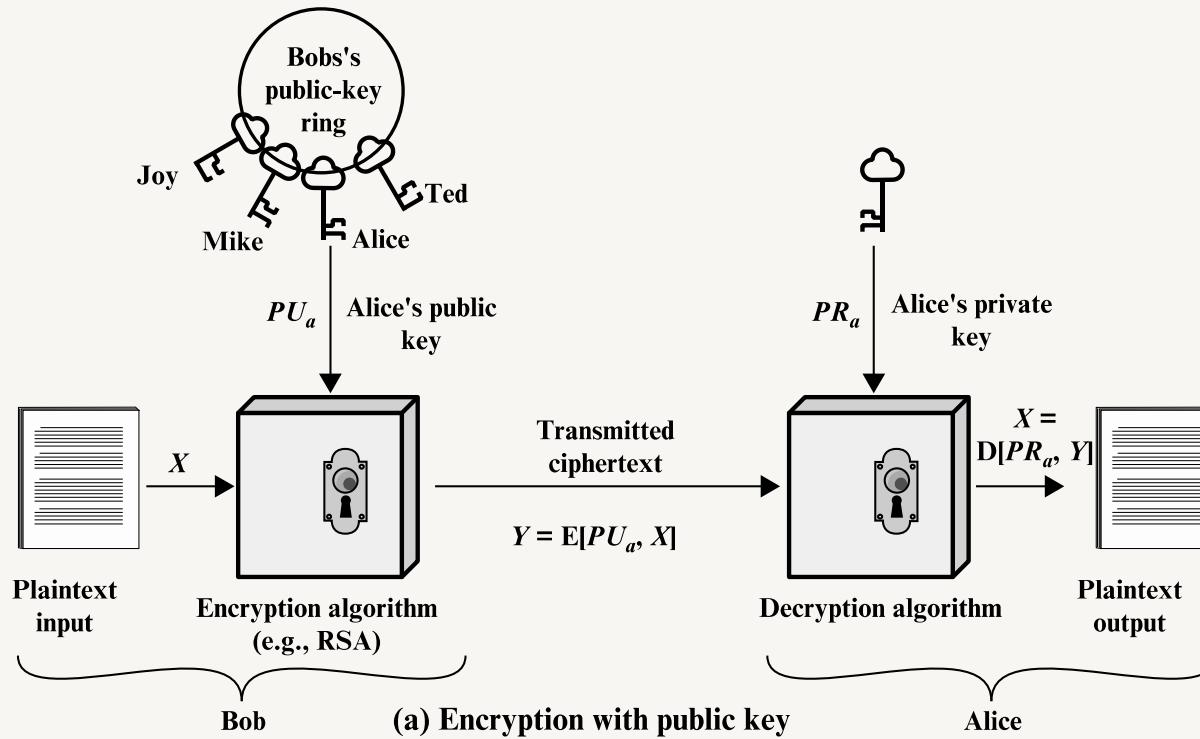


AES is currently the most widely used symmetric encryption

# Public key cryptography: encryption



# Public key cryptography: encryption



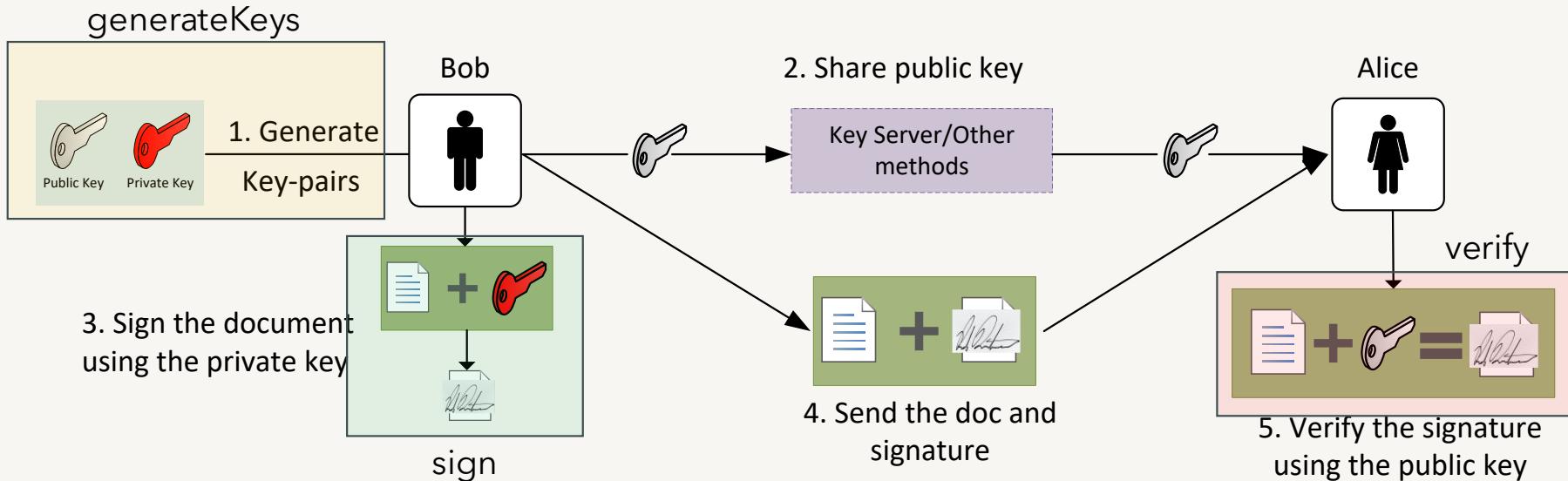
# Public key cryptography: digital signature

- Digital signatures are based on asymmetric cryptography algorithms like RSA or ECC
- We need two properties of (analogue) signatures to hold in the digital world:
  - Only an entity is able to create a signature of its own, but everyone can verify it
  - This signature is tied to data that gets signed. A signature cannot be used for different data

# Digital signature: definition

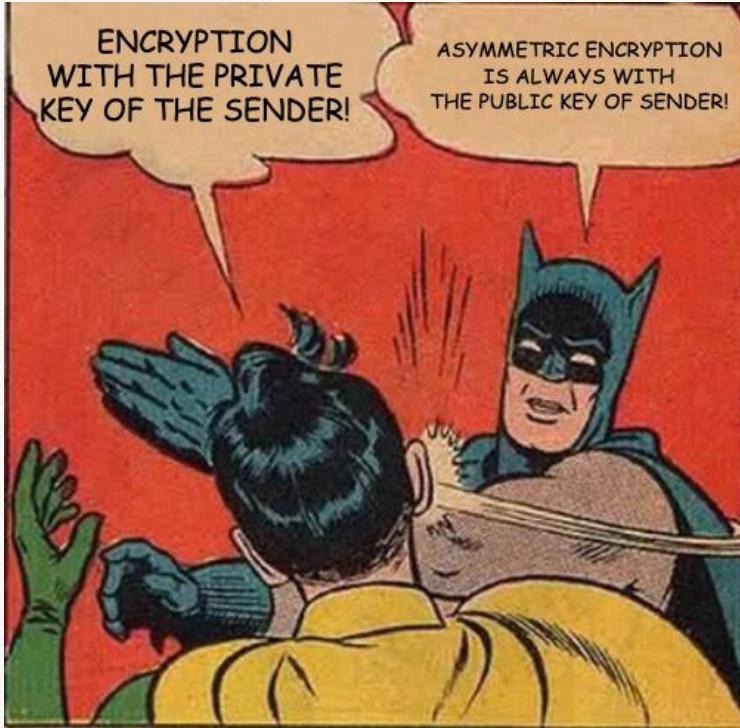
- Three algorithms:
- $(sk, pk) := \text{generateKeys}(\text{keysize})$ 
  - $sk$  is the secret key and is used to sign messages.  $pk$  is the public key and is given to everyone. With the  $pk$ , they can verify the signature
- $\text{sig} := \text{sign}(sk, \text{message})$ 
  - The `sign` method takes the message and the secret key,  $sk$ , as input and returns a signature for message under  $sk$
- $\text{isValid} := \text{verify}(pk, \text{message}, \text{sig})$ 
  - The `verify` method takes a message, a signature, and a public key as input
  - It will return true if the signature was generated out of the message and the secret key, otherwise false
- Such that  $\text{verify}(pk, \text{message}, \text{sign}(sk, \text{message})) == \text{true}$  and signatures are unforgeable

# Digital signature: definition



# Remember this....

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# Digital signature algorithms

- Two major digital signature schemes are available
- RSA-based signature schemes, such as RSA-PSS
  - RSA signature was invented 1977 by Rivest, Shamir and Adleman
  - Based on the assumption that the factorisation of large prime number multiplicated is very hard
  - ECC-based signature schemes, such as ECDSA
  - Suggested independently by Neal Koblitz and Victor S. Miller in 1985
  - Based on discrete logarithms
- The BSI recommends following key sizes for asymmetric cryptography
  - RSA: min. 2048 Bit
  - ECDSA: min. 256 Bit
- Due to smaller key sizes in ECC, many blockchain systems use ECC

# Cryptography resources

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- Introduction to Cryptography: With Coding Theory, by Lawrence C. Washington and Wade Trappe
- Cryptography and Network Security: Principles and Practice, by William Stallings

# Third Class: What is distributed system?

## Distributed system model

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- A distributed system consists of many computers (nodes)
- These nodes might be geographically located at different places, however, they are connected by a communication network
- All nodes are considered autonomous
  - They behave independently of each other
- They communicate with others using the network

Example : Torrent

# Distributed system model

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- Each node contains a processor, communication network, software and non-volatile storage
- Each processor within a node has volatile memory inaccessible by other nodes
- Each node has a network interface (NIC) through which it is connected to the network
- Software is mainly the OS
- The non-volatile is the storage used to store programs (other s/w) and data

# Distributed system model

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- Often a distributed system can be viewed as a logical construct, from an application viewpoint
- Such applications will be regarded as distributed applications
  - For this course, a distributed system = a distributed application
- A distributed application consists of concurrently executing processes
- A process is the execution of sequential program which is a list of statements or instructions
- Concurrent processes can be executed in a single processor, each sharing the processor, the *multiprogramming* approach
- However, we are more interested about processes running in parallel in different nodes

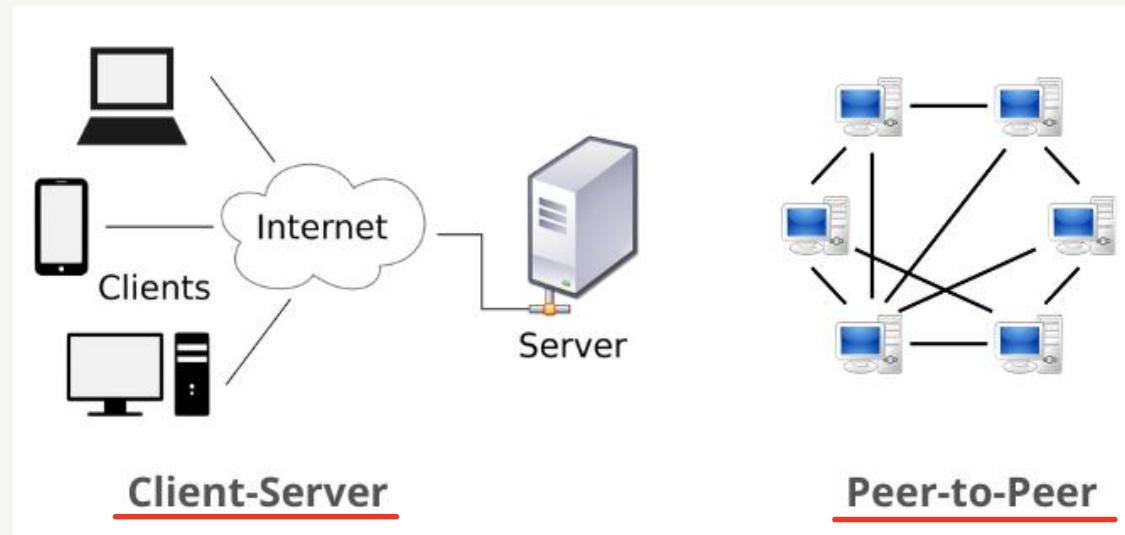
# Distributed system model

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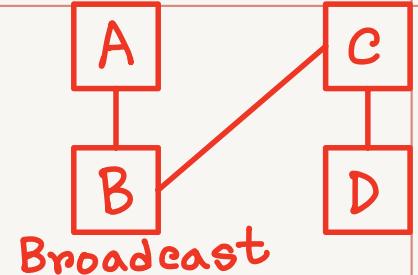
- There are three different types of concurrent processes:  
independent, competing and cooperating
- Concurrent processes are considered independent if the sets of objects accessed by them are disjoint
- Concurrent processes are considered competing if they share resources but they do not exchange information between them
- Cooperating processes exchange information between them either using shared data objects or via message passing

# Distributed system model: network

- P2P and client/server Network



# Network type



- Dwork et al.\* categorised three types of networks exhibiting different properties: synchronous, asynchronous, and partially/eventually synchronous
- The latency involved in delivering a message to all nodes in a synchronous network is bound by some time denoted as  $\Delta$ 
  - Any message sent at time  $T$  will be delivered by  $T + \Delta$
- On the other hand, the latency in an asynchronous network cannot be reliably bound by any  $\Delta$ , message will eventually be delivered

\*C. Dwork, N. Lynch, and L. Stockmeyer "Consensus in the presence of partial synchrony". Journal of the ACM (JACM), 35(2):288-323, 1988.

*Sent time: T*  
*Delivery time: T+Δ*

# Network type

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- In a partially/eventually synchronous network, it is assumed that
  - the network will eventually act as a synchronous network
  - even though it might be asynchronous over some arbitrary period of time

process node  
synchronous

## Fault model

- In a distributed system, a node (process) might behave differently for various reasons (e.g. intentional or unintentional corruption)
- When this happens, we call these nodes as faulty nodes
- Up to  $f$  nodes out of  $N$  (total number) may fail
  - $f$  is usually a function of  $N$ , like  $f < N/2$  or  $f < N/3$
- We mostly look at two types of faults:
  - Crash failure
  - Byzantine failure
- Nodes that do not fail are called “honest” or “correct” nodes

# Fault model: crash failure

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- The crash failure model deals with nodes that simply fail to respond due to some hardware or software failure
  - E.g. Hardware crash, hard disk bad sector, software crash, etc.
- It may happen any time without any prior warning
- The corresponding faulty node remains unresponsive until further actions are taken

# Fault model: BGP {Byzantine General Problem}

- The Byzantine Generals Problem, by Leslie Lamport, Robert Shostak, and Marshall Pease. ACM TOPLAS 1982
- Byzantine army divisions camped outside the walls of an enemy city
- Each division is led by a general
- Generals decide on a common plan of action



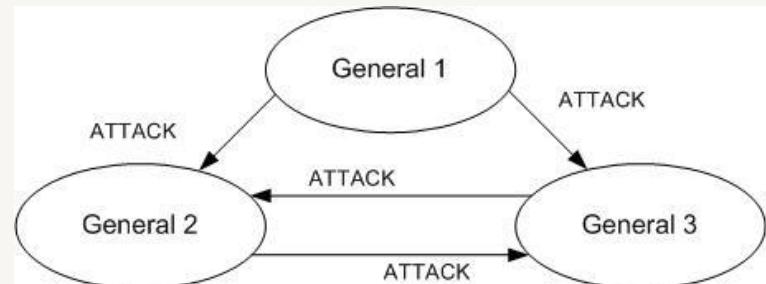
# Fault model: byzantine fault

- There are two types of generals: Loyal or Traitor
- Conditions needed to be met:
  - Loyal generals decide upon the same plan of action
  - Small number of traitors should not be able to lead the loyal generals make a bad decision



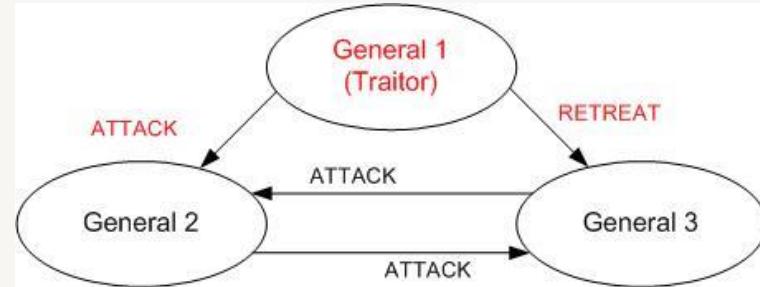
# Fault model: byzantine fault

- General 2 receives ATTACK, ATTACK
- General 3 receives ATTACK, ATTACK
- So ATTACK is Not a Bad Decision



# Fault model: byzantine fault

- General 2 receives ATTACK, ATTACK
- General 3 receives RETREAT, ATTACK
- Now, ATTACK or RETREAT?



The need for  
consensus algorithm

# Fault model: byzantine fault

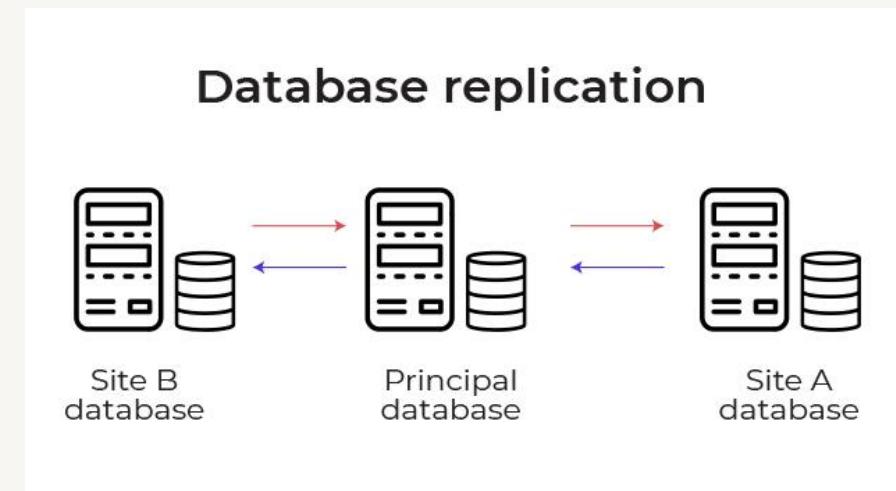
- Coping with failures in nodes not related to crash
- A (faulty) byzantine node sends conflicting information to different parts of system (the byzantine behavior)
  - Non-malicious: Software bugs
  - Malicious reasons: Machine compromised
- P2P Networks:
  - Faulty nodes generate corrupted and misleading messages
  - Good nodes have to “agree to do the same thing” (agreement)
- Agreement in the presence of faults is challenging

# Fault model: byzantine failure

- Byzantine failure deals with nodes that misbehave due to some software bugs or because of the nodes being compromised by an adversary
- A Byzantine node can behave maliciously by arbitrarily sending deceptive messages to others
  - This might affect the security of distributed systems
  - Hence, such nodes are mostly relevant in application with security implications

# The need for consensus algorithm

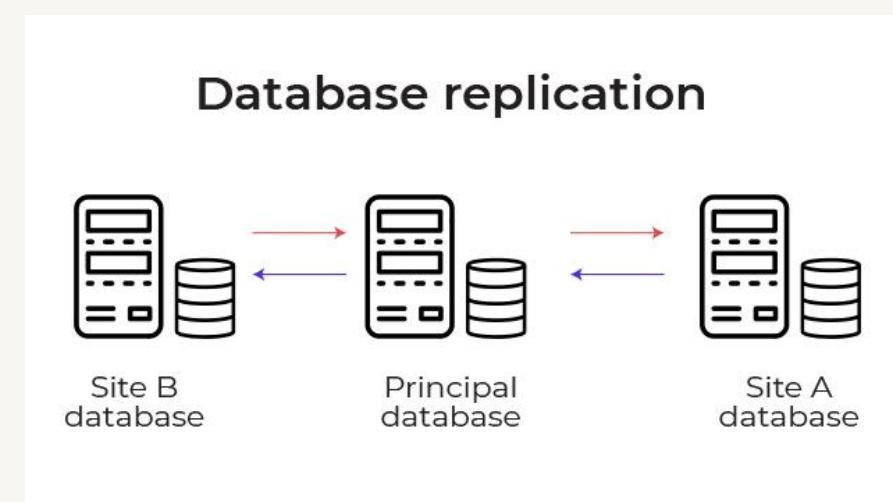
- Consensus is a fundamental problem in distributed applications
  - One use-case is database replication (aka Replicated Database)
- Database replication is the process of storing data in more than one site or node
- It is useful in ensuring resilience against node failures within a network
  - E.g. data are not lost when one or more nodes fail to function in an expected fashion
  - This improves the availability of data



<https://databand.ai/blog/data-replication-the-basics-risks-and-best-practices/>

# The need for consensus algorithm

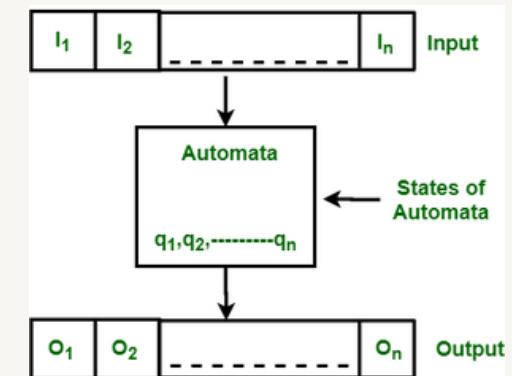
- It is simply copying data from one server to another server
  - So that all the users can share the same data without any inconsistency
- To ensure synchronisation across multiple nodes
  - The mechanism of consensus is used
- Consensus enables all nodes agree to a certain shared state/data among a set of distributed nodes



<https://databand.ai/blog/data-replication-the-basics-risks-and-best-practices/>

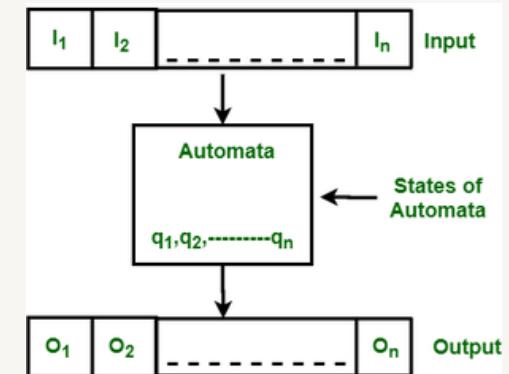
# State machine

- The finite automata or finite state machine is an abstract machine
  - Simply, it is an abstract model of a digital computer
- It has a set of states
- It contains rules for moving from one state to another but it depends upon the applied input symbol
- It can be deterministic or non-deterministic



# State machine

- A deterministic finite automata is a 5-tuple,  $(Q, \Sigma, \delta, q_0, F)$ , consisting of
  - a finite set of states  $Q$
  - a finite set of input symbols called the alphabet  $\Sigma$
  - a transition function  $\delta : Q \times \Sigma \rightarrow Q$
  - an initial or start state  $q_0 \in Q$
  - a set of accept states  $F \subseteq Q$



# Atomic broadcast

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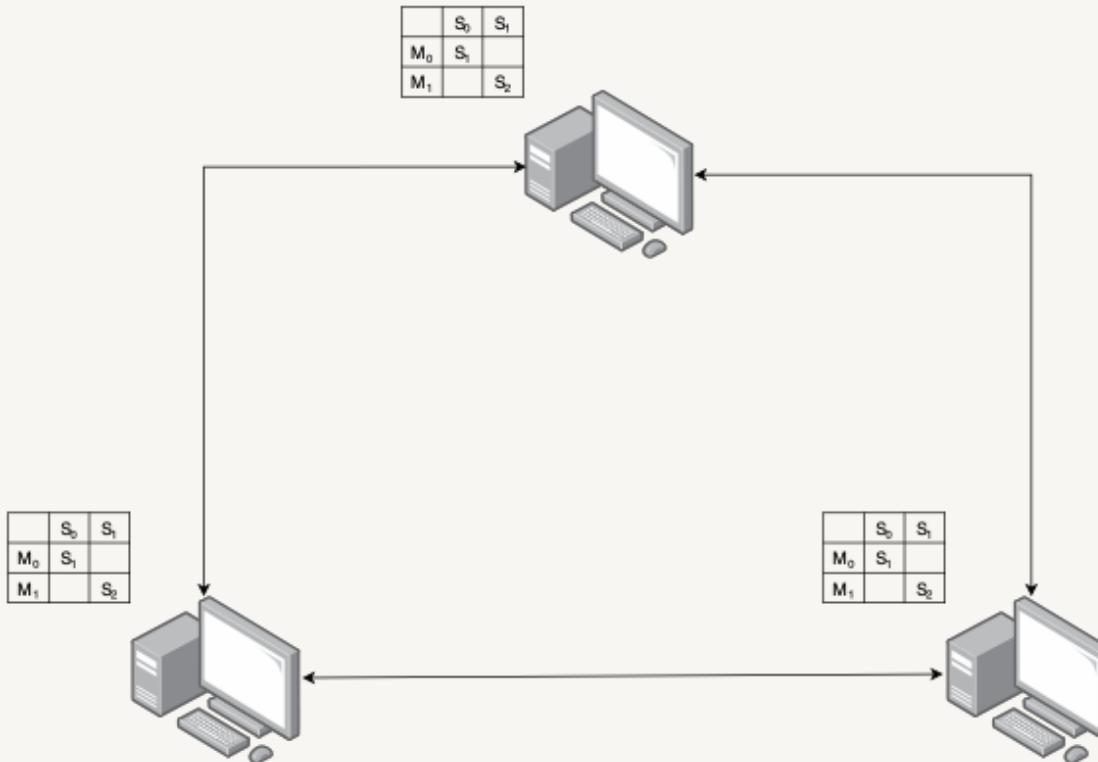
- In fault-tolerant distributed computing, an atomic broadcast or total order broadcast is a broadcast where
  - all nodes receive the same set of input messages in the same order (i.e. the same sequence of messages)

# State machine replication

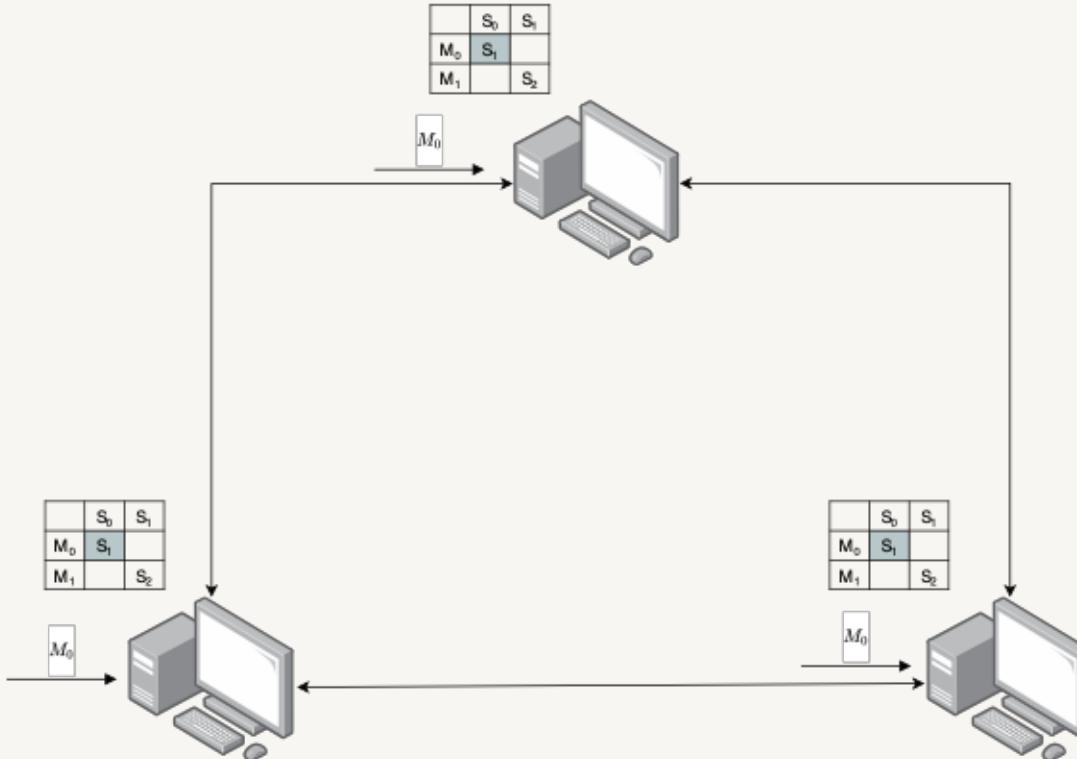
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- The notion of the replicated database can be generalised with the concept of State Machine Replication (SMR)
- The core idea behind SMR is that a computing machine can be expressed as a deterministic state machine
- The machine accepts an input message, performs its predefined computation, and might produce an output/response
  - These actions essentially change its state
- SMR conceptualises that such a state machine, with an initial state, can be replicated among different nodes

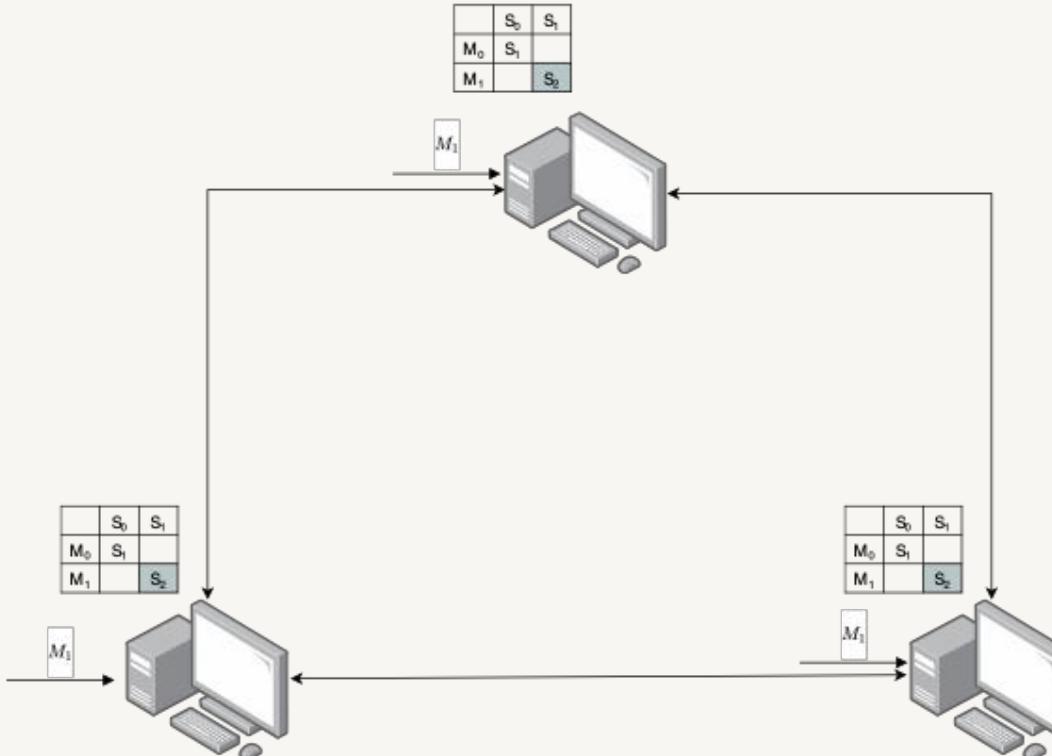
# State machine replication



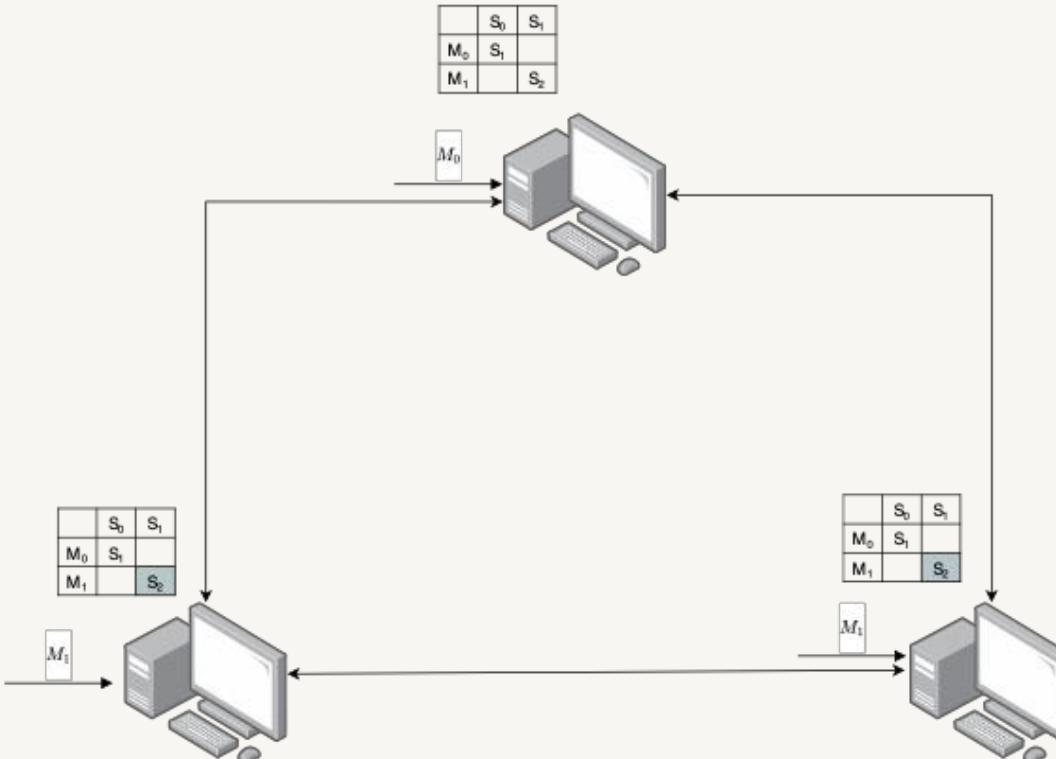
# State machine replication



# State machine replication



# State machine replication



# State machine replication

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- If we can ensure an atomic broadcast of messages then each node would be able to evolve the states of its state machine individually in exactly the same fashion
  - All nodes behave similarly even though they are different
- This can guarantee consistency and availability regarding the state of the machine (as well as data it holds)
  - For example, if a node fails, other nodes can be used to recover its states or the data
- Once this occurs, it can be said that a distributed consensus has emerged among the participating nodes

# State machine replication

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- To make this happen, a protocol is required
- The protocol needs to ensure the timely dissemination and atomic broadcast of input messages among the nodes
- Such a protocol is called a consensus protocol