We solve the following ODE at $t = \tau$:

$$\frac{du}{dt} = a \exp\left(-\frac{u}{2}\right) - b \exp\left(\frac{u}{2}\right)$$

where $a, b > 0$ and $u(0) = u_0$ is given. The ODE is separable:

$$\frac{du}{a \exp(-u/2) - b \exp(u/2)} = dt,$$

$$\int_{u_0}^{u} \frac{du}{a \exp(-u/2) - b \exp(u/2)} = \int_0^{\tau} dt = \tau.$$

Let us evaluate the integral on the left hand side. Define a variable z by $u = 2\log z$ so that $du = 2\,dz/z$ and $z = \exp(u/2)$. Then,

$$\begin{aligned}\int_{u_0}^{u} \frac{du}{a \exp(-u/2) - b \exp(u/2)} &= 2\int_{z_0}^{z} \frac{dz/z}{a/z - bz} \\ &= \frac{2}{b}\int_{z_0}^{z} \frac{dz}{c^2 - z^2}\end{aligned}$$

where $z_0 = \exp(u_0/2)$ and $c = \sqrt{a/b}$. Proceed as follows:

$$\begin{aligned}\int_{z_0}^{z} \frac{dz}{c^2 - z^2} &= \int_{z_0}^{z} \frac{dz}{(c+z)(c-z)} \\ &= \frac{1}{2c}\int_{z_0}^{z} \left(\frac{1}{c+z} + \frac{1}{c-z}\right) dz \\ &= \frac{1}{2c}\int_{z_0}^{z} \left(\frac{1}{z+c} - \frac{1}{z-c}\right) dz \\ &= \frac{1}{2c}\left(\log\left|\frac{z+c}{z-c}\right| - \log\left|\frac{z_0+c}{z_0-c}\right|\right).\end{aligned}$$

Hence,

$$\frac{1}{bc}\left(\log\left|\frac{z+c}{z-c}\right| - \log\left|\frac{z_0+c}{z_0-c}\right|\right) = \tau,$$

$$\left|\frac{z+c}{z-c}\right| = \left|\frac{z_0+c}{z_0-c}\right| \exp(bc\tau).$$

The ODE represents a saturating dynamics towards the stationary soluation $u(\infty) = \log(a/b) = 2\log c$. Note here that $z(\infty) = c$. So, the sign of $z - c$ does not change in $t \in [0, \infty)$. Also, $z + c > 0$. Therefore, we may get rid of the absolute value:

$$\frac{z+c}{z-c} = \frac{z_0+c}{z_0-c} \exp(bc\tau).$$

Define

$$\alpha_0 = \frac{z_0+c}{z_0-c} = \frac{\exp(u_0/2) + c}{\exp(u_0/2) - c},$$

and rearrange the solution to get

$$z = c + \frac{2c}{\alpha_0 \exp(bc\tau) - 1}.$$

Or,

$$u = 2\log\left(c + \frac{2c}{\alpha_0 \exp(bc\tau) - 1}\right).$$