Mathematics I (BSM 101)

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BSM 101

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January 4, 2023

Extreme Value Function:

The Max-Min Theorem for Continuous Functions:
If f is continuous at every point of a closed interval I, then f assumes both an absolute maximum value M and an absolute minimum value m somewhere in I. That is, there are numbers x_1 and x_2 in I with $f(x_1) = m, f(x_2) = M$, and $m \leq f(x) \leq M$ for every other x in I

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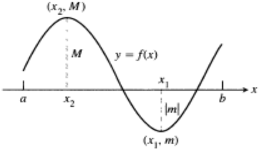
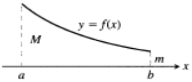
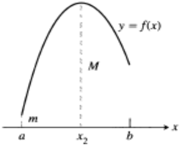


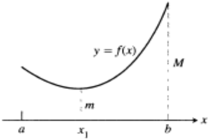
Maximum and minimum at interior points



Maximum and minimum at endpoints



Maximum at interior point, minimum at endpoint



Minimum at interior point, maximum at endpoint

Absolute Extreme Values

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Absolute maximum and minimum values are called absolute extrema. Absolute extrema are also called global extrema.

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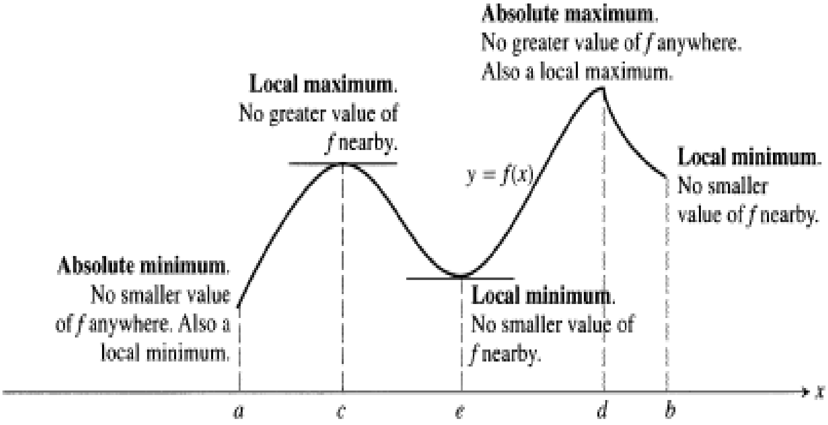

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The First Derivative Theorem for Local Extreme Values:
If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then

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Example 1: Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

Increasing and Decreasing Functions

Definition: Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. f increases on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$.
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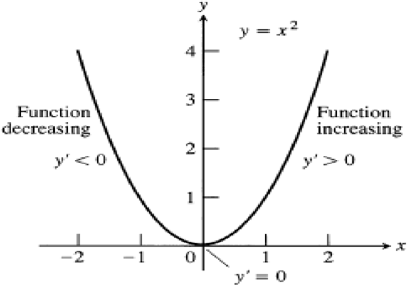
The First Derivative Test for Increasing and Decreasing: Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).
If $f'(x) > 0$ at each point of (a, b), then f increases on $[a, b]$.
If $f'(x) < 0$ at each point of (a, b), then f decreases on $[a, b]$.



Intervals of Increasing or Decreasing

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To find intervals on which f is increasing and decreasing:

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   a. If $f'(p) > 0$, then f is increasing on that subinterval.

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Example:2 Let $f(x) = x^3 + x^2 - x + 1$. Find intervals on which f is increasing or decreasing.

Finding Local Extrema(Maximum/Minimum)

First Derivative Test: Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

1. If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.

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2. If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima.

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3. If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called point of inflection.

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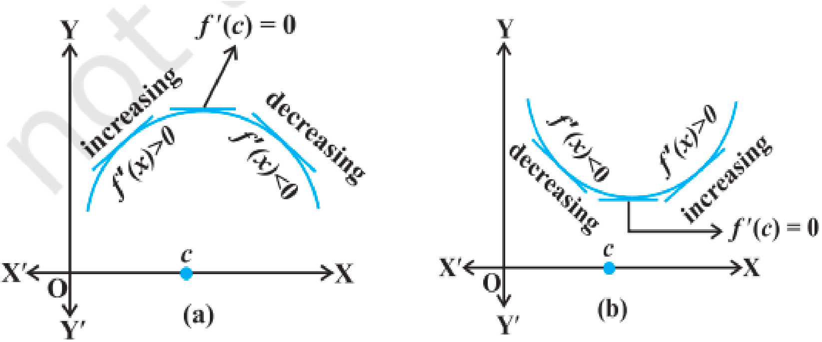
3. If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called point of inflection.

   Example 3: Find all points of local maxima and local minima of the function f given by

$$f(x) = x^3 - 3x + 3$$

Finding Local Extrema(Maximum/Minimum)



Finding Local Extrema(Maximum/Minimum)

Second Derivative Test: Let f be a function defined on an interval I and $c \in$ I. Let f be twice differentiable at c. Then

- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
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Example 4: Find local maximum and local minimum values of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

Example 5: Find all the points of local maxima and local minima of the function f given by

$$f(x) = 2x^3 - 6x^2 + 6x + 5$$

Thank you