Gandaki University

Manju Subedi

Bachelor of Information Technology (BIT)

BSM 101

Exercise on Application of Derivatives

1. For the following problems use L'Hospital's Rule to evaluate the given limit.

   (a) $\lim_{x\to -4} \dfrac{x^3 + 6x^2 - 32}{x^3 + 5x^2 + 4x}$

   (b) $\lim_{x\to 1} \dfrac{x^2 + 8x - 9}{x^3 - 2x^2 - 5x + 6}$

   (c) $\lim_{t\to 2} \dfrac{t^3 - 7t^2 + 16t - 12}{t^4 - 4t^3 + 4t^2}$

   (d) $\lim_{x\to\infty} x^2 e^{-x}$

   (e) $\lim_{w\to -\infty} \dfrac{w^2 - 4w + 1}{3w^2 + 7w - 4}$

   (f) $\lim_{y\to\infty} \dfrac{y^2 - \mathbf{e}^{6y}}{4y^2 + \mathbf{e}^{7y}}$

2. For the followng problems determine all the number(s) c which satisfy the conclusion of Rolle's Theorem for the given function and interval.

   (a) $f(x) = x^2 - 2x - 8$ on $[-1, 3]$

   (b) $g(t) = 2t - t^2 - t^3$ on $[-2, 1]$

   (c) $f(x) = x^3 - 4x^2 + 3$ on $[0, 4]$

   (d) $Q(z) = 15 + 2z - z^2$ on $[-2, 4]$

   (e) $h(t) = 1 - \mathbf{e}^{t^2-9}$ on $[-3, 3]$

3. For the following problems determine all the number(s) c which satisfy the conclusion of the Mean Value Theorem for the given function and interval.

   (a) $f(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2, 5]$

   (b) $f(x) = x^3 - x^2 + x + 8$ on $[-3, 4]$

   (c) $g(t) = 2t^3 + t^2 + 7t - 1$ on $[1, 6]$

   (d) $P(t) = \mathbf{e}^{2t} - 6t - 3$ on $[-1, 0]$

   (e) $f(t) = 8t + \mathbf{e}^{-3t}$ on $[-2, 3]$

4. Show that the Taylor series about $x = 0$ for the function $f(x) = e^x$ is

   $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   Show also that this power series converges for all x.

5. Show that the Taylor series about $x = 0$ for the function $f(x) = \frac{1}{1-x}$ is

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$