Mathematics I (BSM 101)

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MTH 101

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Outlines

- Introduction to Continuous Function
- Properties of Continuity
- Discontinuity and its types

Continuity

Definition: A function f is continuous at an interior point $x = c$ of its domain if

$$\lim_{x \to c} f(x) = f(c).$$

Continuity Test:

A function $f(x)$ is continuous at $x = c$ if and only if it meets the following three conditions.

1. $f(c)$ exists (c lies in the domain of f)
2. $\lim\limits_{x \to c} f(x)$ exists (f has a limit as $x \to c$)
3. $\lim\limits_{x \to c} f(x) = f(c)$ (the limit equals the functional value)

Properties of Continuity

Continuity of Algebraic Combinations:

If functions f and g are continuous at $x = c$, then the following functions are continuous at $x = c$:

1. $f + g$ and $f - g$
2. fg
3. kf, where k is any number
4. f/g (provided $g(c) \neq 0$)
5. $(f(x))^{m/n}$ (provided $f(x))^{m/n}$ is defined on an interval containing c, and m and n are integers)

Discontinuity & its Types

Discontinuity:

If a function $f(x)$ is not continuous at $x = c$ then it is called discontinuous at that point.

Discontinuity aries due to one of the following situation:

1. The right-hand limit or the left-hand limit or both of a function may not exist.

2. The right-hand limit and the left-hand limit of function may exist but are unequal.

3. The right-hand limit, as well as the left-hand limit of a function, may exist, but either of the two or both may not be equal to f(c).

Types of Discontinuity

- Jump Discontinuity
  - Discontinuity of the First Kind
  - Discontinuity of the Second Kind
- Infinite Discontinuity
- Removable Discontinuity

Types of Discontinuity

Jump Discontinuity: Left hand limit and right hand limit exist and finite but are not equal to each other.

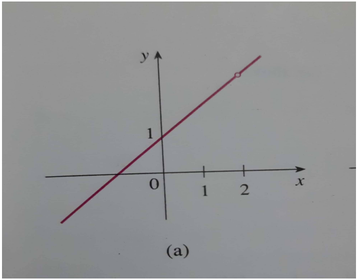
$$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$$

Removable Discontinuity: Left hand limit and right hand limit exist with equal finite limit value but their common value is not equal to $f(c)$

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \neq f(c)$$

Ex: $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

Infinite/Essential Discontinuity: Either one or both right hand and left hand limit do not exist or is Infinite.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0
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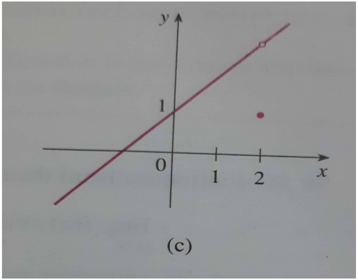
x
(a)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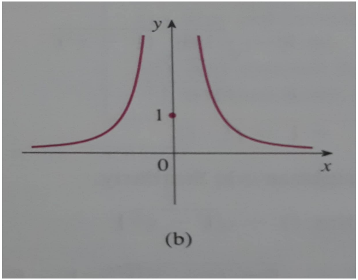
(c)


y
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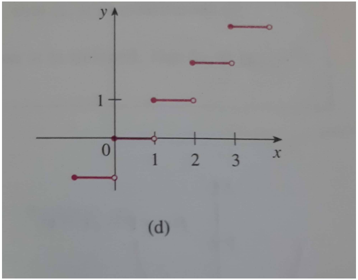
0
x
(b)


y
1
0
1
2
3
x
(d)

Discontinuity

- Graph (a) represent the function $f(x) = \dfrac{x^2 - x - 2}{x - 2}$ that shows removable discontinuity, because we could remove the discontinuity by redefining f(x) at 2 [i.e canceling (x-2) from numerator and denominator then redefine a function $g(x) = x + 1$]
- Discontinuity shown in graph (b) is an infinite/essential discontinuity.
- Graph (c) represent the function
  $$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{for} \quad x \neq 2 \\ 1 & \text{for} \quad x = 2 \end{cases}$$
  which shows the removable discontinuity, because we can redefing f(x) at 2 to make the function continuous.
- Graph (d) represent a jump discontinuity, because both limits exist but are not equal and function jumps from one value to another.

Note: If f is not continuous at c, we say f is discontinuous at c or f has discontinuity at c.

Thank You