Gandaki University

Manju Subedi

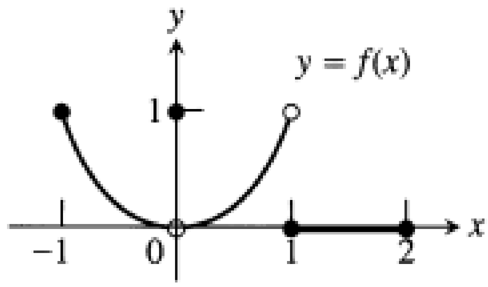
Bachelor of Information Technology

BSM 101

Exercise

Limits and Continuity

1. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



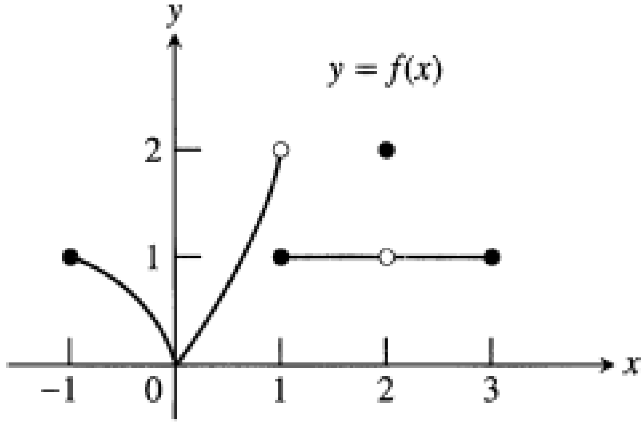
a) $\lim_{x\to -1^+} f(x) = 1$ b) $\lim_{x\to 0^-} f(x) = 0$ c) $\lim_{x\to 0^-} f(x) = 1$

d) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$ e) $\lim_{x\to 0} f(x)$ exists f) $\lim_{x\to 0} f(x) = 0$

g) $\lim_{x\to 0} f(x) = 1$ h) $\lim_{x\to 1} f(x) = 1$ i) $\lim_{x\to 1} f(x) = 0$

j) $\lim_{x\to 2^-} f(x) = 2$ k) $\lim_{x\to -1^-} f(x)$ does not exist. l) $\lim_{x\to 2^+} f(x) = 0$

2. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



a) $\lim_{x\to -1^+} f(x) = 1$ b) $\lim_{x\to 2} f(x)$ does not exist. c) $\lim_{x\to 2} f(x) = 2$

d) $\lim_{x\to 1^-} f(x) = 2$ e) $\lim_{x\to 1^+} f(x) = 1$ f) $\lim_{x\to 1} f(x)$ does not exist.

g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$ j) $\lim_{x\to -1^-} f(x) = 0$ k) $\lim_{x\to 3^+} f(x)$ does not exist.

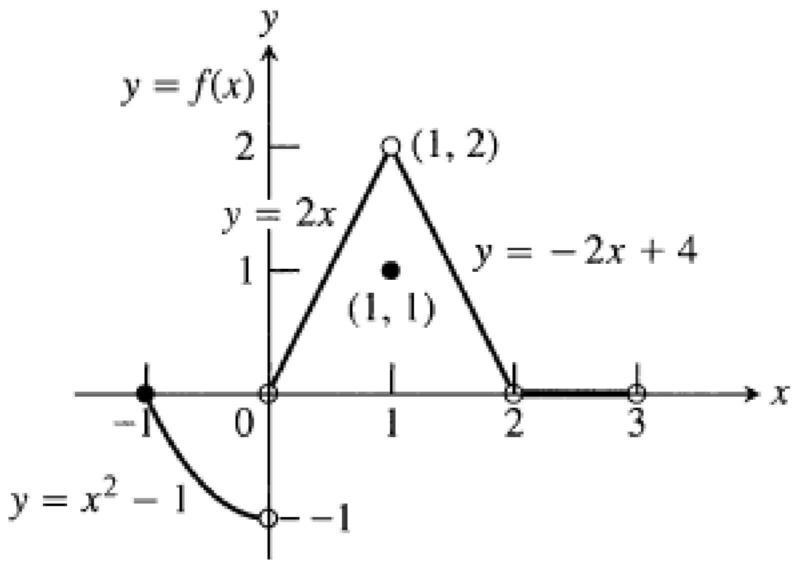
h) $\lim_{x\to c} f(x)$ exists at every c in the open interval $(-1, 1)$.

i) $\lim_{x\to c} f(x)$ exists at every c in the open interval $(1, 3)$.

j) $\lim_{x\to -1^-} f(x) = 0$ k) $\lim_{x\to 3^+} f(x)$ does not exist.

Exercises 3 and 4 are about the function and its graph.
For part 3(d) and 4(d) first answer by looking at the graph and then verify is algebraically.

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



3. a) Does $f(-1)$ exist?
   b) Does $\lim_{x\to -1^+} f(x)$ exist?
   c) Does $\lim_{x\to -1^+} f(x) = f(-1)$?
   d) Is f continuous at $x = -1$?

4. a) Does $f(1)$ exist?
   b) Does $\lim_{x\to 1} f(x)$ exist?
   c) Does $\lim_{x\to 1} f(x) = f(1)$?
   d) Is f continuous at $x = 1$?

5. Find the limits in following exercise:

a. $\lim_{x\to 4} \frac{x-4}{x^2-16}$ b. $\lim_{x\to -3} \frac{x+3}{x^2+4x+3}$ c. $\lim_{x\to -5} \frac{3x^2+9x-12}{3x+3}$

d. $\lim_{x\to 2} \frac{x^2-7x+10}{x-2}$ e. $\lim_{x\to 1}(x^3-x^2+2)$ f. $\lim_{t\to -1} \frac{t^2+3t+2}{t^2-t-2}$

g. $\lim_{x\to -2} \frac{-2x-4}{x^3+2x^2}$ h. $\lim_{y\to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$ i. $\lim_{u\to 1} \frac{u^4-1}{u^3-1}$

j. $\lim_{v\to 3} \frac{v^3-27}{v^4-81}$ k. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$ l. $\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}}$

m. $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$ n. $\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1}$ o. $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

p. $\lim_{x\to\infty} \frac{x^2+5x}{x^2-3}$ q. $\lim_{x\to\infty} \sqrt{x}(\sqrt{x}-\sqrt{x-a})$ r. $\lim_{x\to\infty} \sqrt{3x}-\sqrt{x-a})$

s. $\lim_{x\to\infty} \frac{2x^2+5x+2}{x^2-3}$

6. Determine whether the following function is continuous at $x = 1$.

$$f(x) = \begin{cases} x-1 & \text{for } 0 \le x < 1 \\ 1 & \text{for } x = 1 \\ 2x-2 & \text{for } x > 1 \end{cases}$$

7. A function is defined as

$$f(x) = \begin{cases} x+1 & \text{for} \quad -1 \le x < 0 \\ x & \text{for} \quad 0 \le x \le 1 \\ 2-x & \text{for} \quad 1 < x \le 2 \end{cases}$$

Discuss the contintity of the function at $x = 0$ and at $x = 1$.

8. Determine whether the function $f(x)$ defined by

$$f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \end{cases}$$ is continuous or discontinuous at $x = 0$

9. A function $f(x)$ is defined as follows

$$f(x) = \begin{cases} \frac{1}{2}-x, & \text{when } 0 < x < \frac{1}{2} \\ \frac{1}{2}, & \text{when } \quad x = \frac{1}{2} \\ \frac{3}{2}-x, & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

Is the function $f(x)$ continuous at $x = 1/2$?

10. Let $f(x) = \begin{cases} x^2-5 & \text{for } x > 4 \\ 8 & \text{for } x = 4 \\ 2x+3 & \text{for } x > 4 \end{cases}$

Show that the function is discontinuous at $x = 4$. Which type of discontinuity does f(x) has? Is it possible to make f(x) continuous at $x = 4$? if so how?