Mathematics I (BSM 101)

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Outlines

- Recap (Average rate of change)
- Rules of Differentiation
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  - Power rule
  - Constant multiple rule
  - Sum or difference rule
  - Product rule
  - Quotient rule
  - General power rule
  - Chain rule
  - Implicit differentiation
  - Derivative of parametric function
  - Exponential/ logarithmic function
- Higher Order Derivatives

Recap from last Class:

The Slope of a curve at any point: The slope provides an idea of the measure of the rate of change in the value of y with respect to a change in the value of x.

Let (x_1, y_1) and $x_2, y_2)$ be any two points on a straight line, then the slope of the line is defined as $m = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Definition: The derivative of the function f with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

If the limits on the right hand side exist (i.e $f'(x)$ exist) then we say f has a derivative (is differentiable) at x.

Notation: The symbol $\frac{dy}{dx}$, $\frac{df(x)}{dx}$, $f'(x)$, y', f' are use to express the derivative.

The process of obtaining the derivative is called differentiation of the function $f(x)$

Rules of Differentiation

- Constant Function Rule: If $f(x) = c$, where c is any constant then, $f'(x) = 0$
- Power Rule: If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$
- Constant Multiple (Coefficient) Rule: If $f(x) = c \cdot g(x)$, where c is any constant, then $f'(x) = c \cdot g'(x)$
- Sum or Difference Rule: If $f(x) = u(x) \pm v(x)$, where u, and v are differentiable functions, then $f'(x) = u'(x) \pm v'(x)$
- Product Rule: If $f(x) = u(x) \cdot v(x)$, where u and v are differentiable functions, then $f'(x) = u(x)v'(x) + v(x)u'(x)$
- Quotient Rule: If $f(x) = \dfrac{u(x)}{v(x)}$, where u and v are differentiable functions and $v(x) \neq 0$, then $f'(x) = \dfrac{vu' - uv'}{v^2}$
- General Power Rule: If $f(x) = [u(x)]^n$, where u is a differentiable function and n is a real number, then $f'(x) = n[u(x)]^{n-1} \cdot u'(x)$

Derivatives Continue...

- Chain Rule: If $z = f(y)$ and $y = g(x)$, then $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$
- Implicit Differentiation: A Function which is expressed in the form $f(x, y) = 0$, say $x^2 + xy + xy^2 + y^2 = 0$ is called an implicit function. The process of finding value of $\frac{dy}{dx}$ without solving the equation for y is called implicit differentiation.
- Derivative of Parametric Function: Parametric funcitons are those in which both variables x and y are expressed in terms of a third variable called, the parameter (t) i.e. $x = f(t),\ y = g(t)$. To find
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \text{provided } \frac{dx}{dt} \neq 0$$
- Derivative of Logarithmic Function: The derivative of $ln\, x$ is
$$\frac{d}{dx}(ln\, x) = \frac{1}{x}, \quad x > 0$$
Also, $\frac{d}{dx}[ln\, h(x)] = \dfrac{1}{h(x)}[\dfrac{d}{dx}h(x)]$, Where h is a differentiable function of x

Derivatives Continue...

- Derivative of Exponential Function: The Derivative of e^x is
$\dfrac{d}{dx}(e^x) = e^x$
Also, $\dfrac{d}{dx}[e^{h(x)}] = e^{h(x)}\frac{d}{dx}\{h(x)\}$, Where h is a differentiable function of x.
- Derivative of One Function with respect to another Function: Suppose $y_1 = f_1(x)$ and $y_2 = f_2(x)$ are two functions. Derivative of y_1 with respect to y_2 is denoted by $\frac{dy_1}{dy_2}$ and defined by $\frac{dy_1}{dy_2} = \dfrac{\left(\frac{dy_1}{dx}\right)}{\left(\frac{dy_2}{dx}\right)}$

Increasing and Dercreasing Functions

A function $f(x)$ is increasing in an interval if the graph continuously rises as x goes from left to right through the interval. Similarly, a function $f(x)$ is decreasing in an interval if the graph continuously falls as x goes from left to right through the interval.

Higher Order Derivatives

$\dfrac{d}{dx}(f'(x)) = f''(x) = \dfrac{d^2 f}{dx^2}$ is the second order derivative of $f(x)$

$\dfrac{d}{dx}(f''(x)) = f'''(x) = \dfrac{d^3 f}{dx^3}$ is the third order derivative of $f(x)$ and so on.

Thank You