Mathematics I (BSM 101)

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Common Integrals

- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \sin kx dx = -\dfrac{\cos kx}{k} + C$
- $\int \cos kx dx = \dfrac{\sin kx}{k} + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \csc^2 x dx = -\cot x + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \csc x \cot x dx = -\csc x + C$
- $\int \tan x dx = ln|\sec x| + C$
- $\int \sec x dx = ln|\sec x + \tan x| + C$

The Fundamental Theorem of Calculus:

If the function $f(x)$ is continuous on the interval $a \leq x \leq b$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$ on $a \leq x \leq b$.

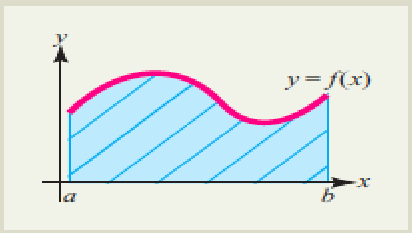

FIGURE 5.3 The region under the curve $y = f(x)$ over the interval $a \leq x \leq b$.

Rules for Definite Integrals

Let f and g be any functions continuous on $a \leq x \leq b$. Then,

- Constant multiple rule: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for constant k
- Sum rule: $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
- Difference rule: $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$
- $\int_a^a f(x)dx = 0$
- $\int_b^a f(x)dx = -\int_a^b f(x)dx$
- Subdivision rule: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Example

1. Use the fundamental theorem of calculus to find the area of the region under the line $y = 2x + 1$ over the interval $1 \leq x \leq 3$.

Solution

Since $f(x) = 2x + 1$ satisfies $f(x) \geq 0$ on the interval $1 \leq x \leq 3$, the area is given by the definite integral

$$A = \int_1^3 (2x + 1)dx$$

. Since an antiderivative of $f(x) = 2x + 1$ is $F(x) = x^2 + x$, the fundamental theorem of calculus tells us that

$$A = \int_1^3 (2x + 1)dx = x^2 + x\Big|_1^3$$
$$= \left[(3)^2 + (3)\right] - \left[(1)^2 + (1)\right] = 10$$

Example

2. Evaluate $\int_1^4 \left(\frac{1}{x} - x^2\right) dx$

Solution

An antiderivative of $f(x) = \frac{1}{x} - x^2$ is $F(x) = \ln|x| - \frac{1}{3}x^3$, so we have

$$\begin{aligned}
\int_1^4 \left(\frac{1}{x} - x^2\right) dx &= \left(\ln|x| - \frac{1}{3}x^3\right)\Bigg|_1^4 \\
&= \left[\ln 4 - \frac{1}{3}(4)^3\right] - \left[\ln 1 - \frac{1}{3}(1)^3\right] \\
&= \ln 4 - 21 \approx -19.6137
\end{aligned}$$

Evaluate $\int_0^1 8x\left(x^2+1\right)^3 dx$

Solution: The integrand is a product in which one factor $8x$ is a constant multiple of the derivative of an expression x^2+1 that appears in the other factor. This suggests that you let $u = x^2+1$. Then $du = 2xdx$, and so

$$\int 8x\left(x^2+1\right)^3 dx = \int 4u^3 du = u^4$$

The limits of integration, 0 and 1 , refer to the variable x and not to u. You can, therefore, proceed in one of two ways. Either you can rewrite the antiderivative in terms of x, or you can find the values of u that correspond to $x=0$ and $x=1$. If you choose the first alternative, you find that

$$\int 8x\left(x^2+1\right)^3 dx = u^4 = \left(x^2+1\right)^4$$

and so $\int_0^1 8x\left(x^2+1\right)^3 dx = \left.\left(x^2+1\right)^4\right|_0^1 = 16-1 = 15$

Example Continue

If you choose the second alternative, use the fact that $u = x^2 + 1$ to conclude that $u = 1$ when $x = 0$ and $u = 2$ when $x = 1$. Hence,

$$\int_0^1 8x\left(x^2+1\right)^3 dx = \int_1^2 4u^3 du = u^4\Big|^2 = 16 - 1 = 15$$

Classwork

1. Evaluate $\int_{1/4}^{2} \left(\frac{\ln x}{x}\right) dx$.


KEEP
CALM
AND
DON`T HESITATE TO
ASK QUESTIONS

The Improper Integral

If $f(x)$ is continuous for $x \geq a$, then

$$\int_a^{+\infty} f(x)dx = \lim_{N \to +\infty} \int_a^N f(x)dx$$

If the limit exists, the improper integral is said to converge to the value of the limit. If the limit does not exist, the improper integral diverges.
Example 1: Evaluate the improper integral

$$\int_1^{+\infty} \frac{1}{x^2} dx$$

Solution: First compute the integral from 1 to N and then let N approach infinity.

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{N \to +\infty} \int_1^N \frac{1}{x^2} dx = \lim_{N \to +\infty} \left(-\frac{1}{x} \Big|_1^N \right) = \lim_{N \to +\infty} \left(-\frac{1}{N} + 1 \right) =$$

Thank You