Implicitly Typed `miniJS`: SIF Version

1. Syntax

$$L \in \mathit{LevelVariable}$$
$$\ell \in \mathit{Level} ::= \textbf{public} \mid \textbf{secret} \mid \textbf{alice} \mid \textbf{bob} \mid L$$

$$n \in \mathbb{Z} \qquad b \in \mathit{Bool} \qquad s \in \mathit{String} \qquad x \in \mathit{Variable}$$

$$\begin{aligned}
t \in \mathit{Term} &::= c \mid e \\
c \in \mathit{Cmd} &::= \vec{t_i} \mid x := e \mid \textbf{while}\ e\ t \mid \textbf{output}\ \ell\ e \\
e \in \mathit{Exp} &::= n \mid b \mid s \mid \textbf{undef} \mid x \mid \ominus e \mid e_1 \oplus e_2 \\
&\mid \textbf{if}\ e\ t_1\ \textbf{else}\ t_2 \mid \textbf{input}\ \ell\ \mathit{typ} \mid \textbf{var}\ \overrightarrow{x_i = e_i}\ \textbf{in}\ t \\
\ominus \in \mathit{UnOp} &::= - \mid \neg \\
\oplus \in \mathit{BinOp} &::= + \mid - \mid \times \mid \div \mid \wedge \mid \vee \mid = \mid \neq \mid \leq \mid <
\end{aligned}$$

2. Domains

$$\begin{aligned}
\rho \in \mathit{TypeEnv} &= \mathit{Variable} \rightarrow \mathit{LevelVariable} \\
: \in \mathit{TypeEval} &= \mathit{TypeEnv} \times \mathit{Level} \times \mathit{Term} \rightarrow \mathit{Level} \\
\sqsubseteq \in \mathit{LevelComp} &= \mathit{Level} \times \mathit{Level} \rightarrow \mathit{Boolean}
\end{aligned}$$

We abuse notation in the following rules so that $\vec{x_i}$ means a vector of x's of length n with indices $1 \leq i \leq n$, and if x_i occurs free it means to iterate through all i. By convention, whenever one of the following rules uses an unbound level variable L it means to generate a fresh variable that's never been seen before. Initially $\rho = \emptyset$ and $\ell_w = \textbf{public}$.

3. Rules

$$\frac{\rho \cdot \ell_w \vdash t_i : \ell_i}{\rho \cdot \ell_w \vdash \vec{t_i} : \vec{\ell_i}.\text{last}}$$

$$\frac{\rho(x) = \ell_1 \qquad \rho \cdot \ell_w \vdash e : \ell_2 \qquad \ell_2 \sqsubseteq \ell_1 \qquad \ell_w \sqsubseteq \ell_1}{\rho \cdot \ell_w \vdash x := e : \ell_w}$$

$$\frac{\rho \cdot \ell_w \vdash e : \ell_1 \qquad \rho \cdot \ell_1 \vdash t : \ell_2}{\rho \cdot \ell_w \vdash \textbf{while}\ e\ t : \ell_w}$$

$$\frac{\rho \cdot \ell_w \vdash e : \ell_o \qquad \ell_w \sqsubseteq \ell \qquad \ell_o \sqsubseteq \ell}{\rho \cdot \ell_w \vdash \textbf{output}\ \ell\ e : \ell_w}$$

$$\rho \cdot \ell_w \vdash n : \ell_w$$

$$\rho \cdot \ell_w \vdash b : \ell_w$$

$$\rho \cdot \ell_w \vdash s : \ell_w$$

$$\rho \cdot \ell_w \vdash \textbf{undef} : \ell_w$$

$$\frac{\rho(x) = \ell \qquad \ell_w \sqsubseteq L \qquad \ell \sqsubseteq L}{\rho \cdot \ell_w \vdash x : L}$$

$$\frac{\rho \cdot \ell_w \vdash e : \ell}{\rho \cdot \ell_w \vdash \ominus e : \ell}$$

$$\frac{\rho \cdot \ell_w \vdash e_1 : \ell_1 \qquad \rho \cdot \ell_w \vdash e_2 : \ell_2 \qquad \ell_1 \sqsubseteq L \qquad \ell_2 \sqsubseteq L}{\rho \cdot \ell_w \vdash e_1 \oplus e_2 : L}$$

$$\frac{\begin{array}{c}\rho \cdot \ell_w \vdash e : \ell_1 \qquad \rho \cdot \ell_1 \vdash t_1 : \ell_2 \\ \rho \cdot \ell_1 \vdash t_2 : \ell_3 \qquad \ell_2 \sqsubseteq L \qquad \ell_3 \sqsubseteq L\end{array}}{\rho \cdot \ell_w \vdash \textbf{if}\ e\ t_1\ \textbf{else}\ t_2 : L}$$

$$\frac{\ell_w \sqsubseteq L \qquad \ell \sqsubseteq L}{\rho \cdot \ell_w \vdash \textbf{input}\ \ell\ \mathit{typ} : L}$$

$$\frac{\rho' = \rho[x_i \mapsto L_i] \qquad \ell_w \sqsubseteq L_i \qquad \rho' \cdot \ell_w \vdash t : \ell}{\rho \cdot \ell_w \vdash \textbf{var}\ \vec{x_i}\ \textbf{in}\ t : \ell}$$