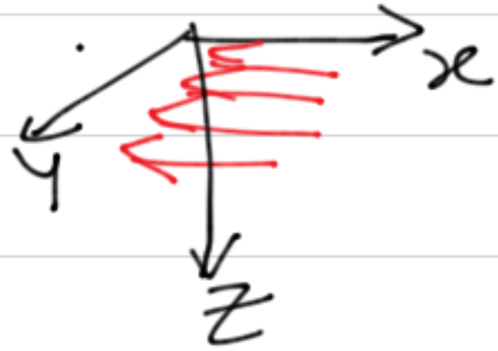
Consider an infinite plane sheet of current

$$\bar{J}_s = -J_s(t)\,\hat{x} \quad \text{at } z=0$$



$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}, \quad \nabla\times\bar{H} = \bar{J}+\frac{\partial\bar{D}}{\partial t}$$

Since only $\hat{x}$ of $\bar{J}$ exists, we can set all derivatives with respect to x and y to zero, i.e., wave propagates along z.

$$-\frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}, \quad -\frac{\partial H_y}{\partial z} = J_x + \frac{\partial D_x}{\partial t} \quad \|$$

$$\| \quad \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}, \quad \frac{\partial H_x}{\partial z} = \frac{\partial D_y}{\partial t}$$

$$0 = -\frac{\partial B_z}{\partial t}, \quad 0 = \frac{\partial D_z}{\partial t}$$

Only 2 equations involve J_x, but here J_x is a volume current density. So, we first solve with $J_x = 0$

$$\frac{\partial E_x}{\partial z} = -\mu_0\frac{\partial H_y}{\partial t}, \quad \frac{\partial H_y}{\partial z} = -\epsilon_0\frac{\partial E_x}{\partial t} \Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0\frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Define $\tau = z\sqrt{\mu_0 \epsilon_0} = \frac{z}{v_p}$

Also define intrinsic impedance $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$

Such that we can write the solutions

$$E_x(z,t) = A f\left(t - \frac{z}{v_p}\right) + B g\left(t + \frac{z}{v_p}\right)$$

$$H_y(z,t) = \frac{1}{\eta_0}\left[A f\left(t - \frac{z}{v_p}\right) - B g\left(t + \frac{z}{v_p}\right)\right]$$

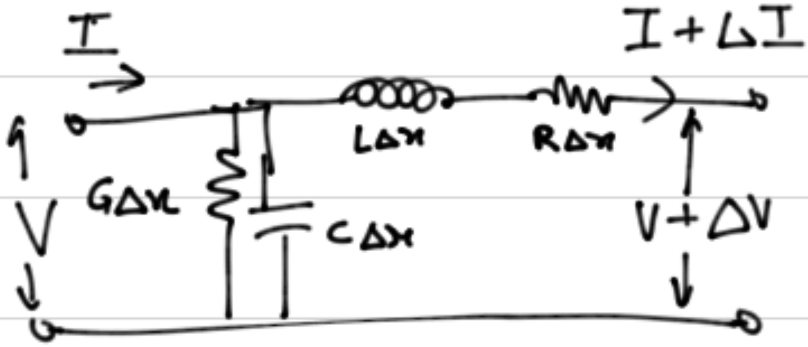
We will return to these equations later (when we do plane waves. But, we can see the analogy with TLs now

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t} \longleftrightarrow \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\epsilon_0 \frac{\partial E_x}{\partial t} \longleftrightarrow \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}$$

$E_x \to V$ $\quad \mu_0 \to L$ $\quad v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \to \frac{1}{\sqrt{LC}}$

$H_y \to I$ $\quad \epsilon_0 \to C$ $\quad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \to \sqrt{\frac{L}{C}}$



$$\left.\begin{aligned}\Delta V &= -(R\Delta x + j\omega L\Delta x)I \\ \Delta I &= -(G\Delta x + j\omega C\Delta x)V\end{aligned}\right\} \quad \frac{\Delta V}{\Delta x} = -(R+j\omega L)I \qquad \frac{\Delta I}{\Delta x} = -(G+j\omega C)V$$

$\lim \Delta x \to 0$

$$\frac{d^2V}{dx^2} = +(R+j\omega L)(G+j\omega C)\,V$$

$$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C)\,I$$

Define $\gamma^2 = (R+j\omega L)(G+j\omega C) = \alpha + j\beta$

$$V = V^+ e^{-\gamma x} + V^- e^{\gamma x}$$

$$I = I^+ e^{-\gamma x} + I^- e^{\gamma x}$$

To get instantaneous values insert time harmonics

$$V(t) = V^+ e^{j\omega t - \gamma x} + V^- e^{j\omega t + \gamma x}, \quad I(t) = I^+ e^{j\omega t - \gamma x} + I^- e^{j\omega t + \gamma x}$$

Arbitrary Constants $V^{\pm}, I^{\pm}$

$$\frac{d}{dx}\left[V^+e^{-\gamma x}+V^-e^{\gamma x}\right]=-(R+j\omega L)\left[I^+e^{-\gamma x}+I^-e^{+\gamma x}\right]$$

$$-\gamma\left[V^+e^{-\gamma x}-V^-e^{\gamma x}\right]=-(R+j\omega L)\left(I^+e^{-\gamma x}+I^-e^{\gamma x}\right)$$

Must be true for all x !! Can only happen if we equate coeffs of $e^{\pm\gamma x}$

$e^{-\gamma x}$: $-\gamma V^+=-(R+j\omega L)I^+$

$e^{+\gamma x}$: $\gamma V^-=-(R+j\omega L)I^-$

$$\left.\begin{aligned}\frac{V^+}{I^+}&=\frac{R+j\omega L}{\gamma}=\sqrt{\frac{R+j\omega L}{G+j\omega C}}\\ \frac{V^-}{I^-}&=-\left(\frac{R+j\omega L}{\gamma}\right)=-\sqrt{\frac{R+j\omega L}{G+j\omega C}}\end{aligned}\right\}\ Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$V=V^+e^{-\gamma x}+V^-e^{\gamma x}$$

$$I=\frac{V^+}{Z_0}e^{-\gamma x}-\frac{V^-}{Z_0}e^{\gamma x}$$

$$\boxed{\begin{aligned}f\lambda&=v_p\\ \frac{\omega}{\beta}&=v_p\end{aligned}}$$

General solutions for the line voltage and current are

$$V(z,t) = A\cos\left[\omega\left(t - \frac{z}{v_p}\right) + \theta\right] + B\cos\left[\omega\left(t + \frac{z}{v_p}\right) + \phi\right]$$

$$I(z,t) = \frac{A}{Z_0}\cos\left[\omega\left(t - \frac{z}{v_p}\right) + \theta\right] - \frac{B}{Z_0}\cos\left[\omega\left(t + \frac{z}{v_p}\right) + \phi\right]$$

In phasor notation,

$$\left.\begin{aligned} \bar{V}(z) &= \bar{V}^+ e^{-j\beta z} + \bar{V}^- e^{+j\beta z} \\ \bar{I}(z) &= \frac{\bar{V}^+}{Z_0} e^{-j\beta z} - \frac{\bar{V}^-}{Z_0} e^{+j\beta z} \end{aligned}\right\} \quad \begin{aligned} \bar{V}^+ &= A e^{j\theta} \\ \bar{V}^- &= B e^{j\phi} \end{aligned}$$

Wave number $\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$ [rad/m]

In T.L.s we often define distances from the load end

$d = -z$; $V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$ (dropped $\bar{V}$)

Plane waves: $e^{-j\bar{\beta}\cdot\bar{r}}$ $\quad \bar{\beta}\cdot\bar{r} = \beta_x x + \beta_y y + \beta_z z$

Suppose we had $V_g(t)$ instead of the impulse $\delta(t)$

e.g. Freq response to $V_g = \cos \omega t$

$$\bar{V}_0(\omega) = \frac{4}{9} \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n e^{-j\omega(2nT_2 + T_0)}$$

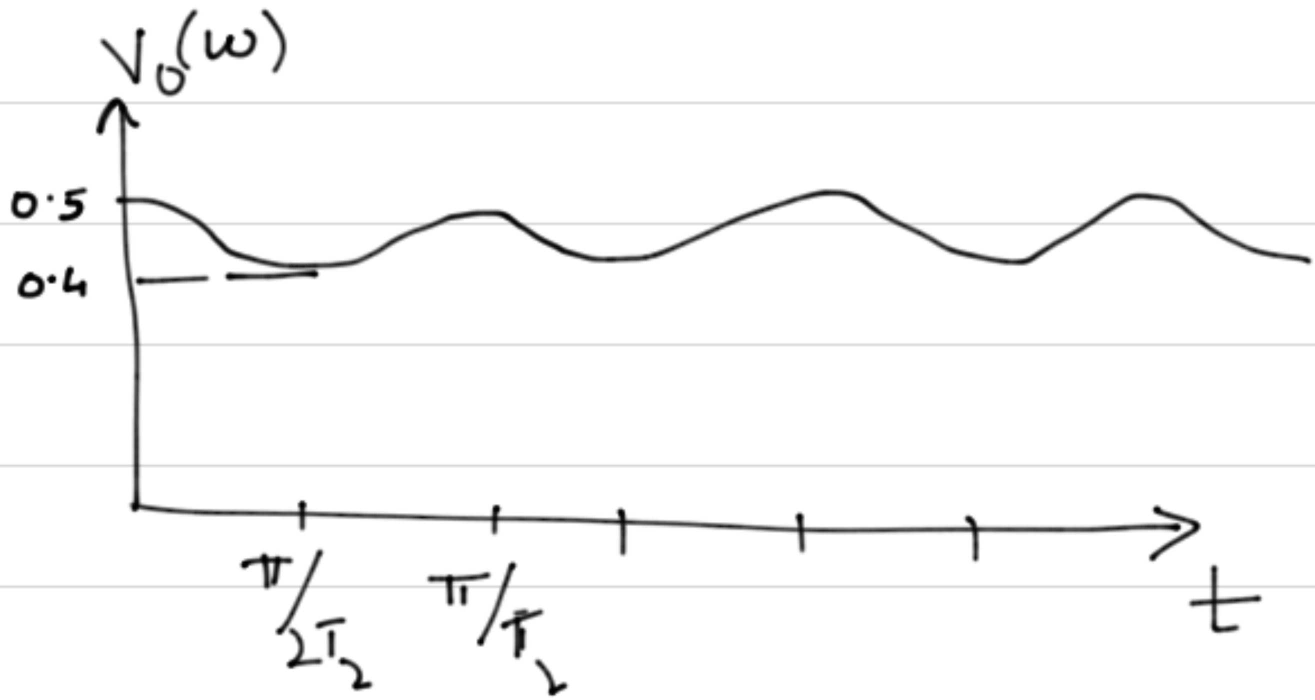
$$= \frac{4}{9} e^{-j\omega T_0} \sum_n \left(\frac{1}{9} e^{-j\omega T_2}\right)^n$$

$$= \frac{(4/9)\, e^{-j\omega T_0}}{1 - (1/9) e^{-2j\omega T_2}}$$

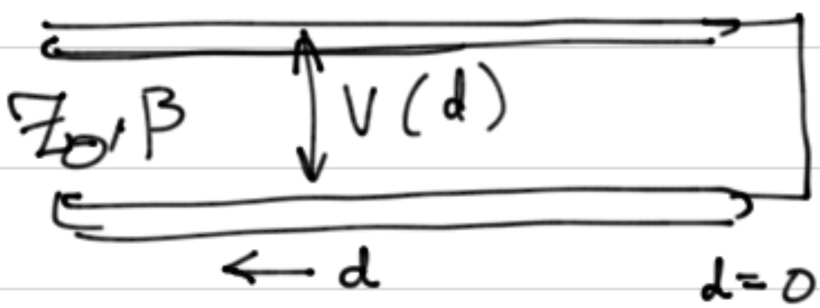
$$\boxed{\sum_n x^n = \frac{1}{1-x}}$$

Maxima at $2\omega T_2 = 2m\pi$, $m = 0, 1, 2 \ldots$

Minima at $2\omega T_2 = (2m+1)\pi$



T. L. shorted at load end.



$V(0)=0 \Rightarrow V^+ + V^- = 0$ Boundary Condition

$\Rightarrow V^- = -V^+$

$$V(d) = V^+ e^{j\beta d} - V^+ e^{-j\beta d} = 2jV^+ \sin \beta d$$

$$I(d) = \frac{1}{Z_0}\left(V^+ e^{j\beta d} + V^+ e^{-j\beta d}\right) = \frac{2V^+}{Z_0} \cos \beta d$$

Real voltage, current, power?

$$V(d,t) = \mathrm{Re}\left[\bar{V}(d) e^{j\omega t}\right]$$

$$= \mathrm{Re}\left(2 e^{j\pi/2} |\bar{V}^+| e^{j\theta} \sin \beta d \, e^{j\omega t}\right)$$

$$= -2|\bar{V}^+| \sin \beta d \sin(\omega t + \theta)$$

$$I(d,t) = \frac{2|\bar{V}^+|}{Z_0} \cos \beta d \cos(\omega t + \theta)$$

$$P(d,t) = V(d,t)\, I(d,t)$$ Instantaneous Power

$$= -\frac{|\bar{V}^+|^2}{Z_0} \sin 2\beta d \sin 2(\omega t + \theta)$$

Average Power

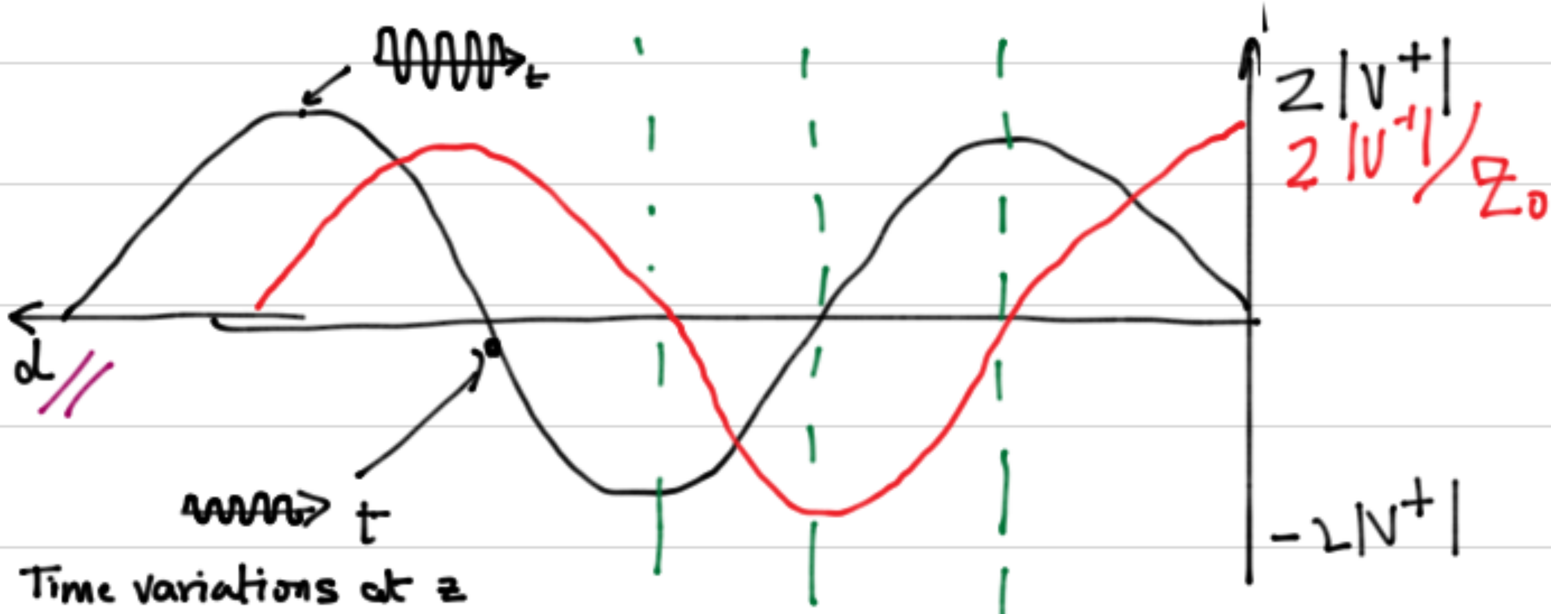
$$\langle P \rangle = \frac{1}{T}\int_0^T P(d,t)\,dt = \frac{\omega}{2\pi}\int_{t=0}^{2\pi/\omega} P(d,t)\,dt$$

$$= \frac{\omega}{2\pi}\frac{|\bar{V}^+|^2}{Z_0}\sin 2\beta d \int_0^{2\pi/\omega} \sin 2(\omega t+\theta)\,dt$$

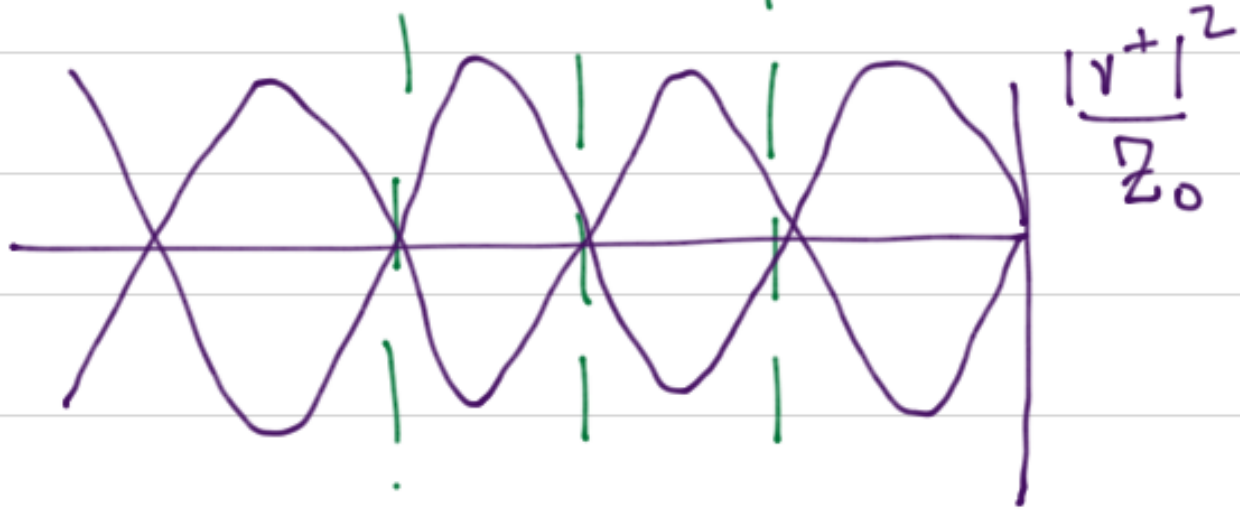
$$= 0 \;!!!$$

What does this mean?

What do $V(d,t)$, $I(d,t)$ and $P(d,t)$ look like?



Time variations at z



$$-\frac{|\bar{V}^+|^2}{Z_0} \sin 2\beta d \sin 2(\omega t + \theta)$$

1. Voltage and current are out of phase by $\pi/2$

2. Since $P(z,t) = V(z,t)\, I(z,t)$, instantaneous power is zero when either voltage or current are zero

3. At any point along the line, there is also a sinusoidal time variation

Transmission line terminated in a short circuit.

Recall that $V(0)=0 \Rightarrow V^- = -V^+$

and $V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$

$= 2jV^+ \sin \beta d$

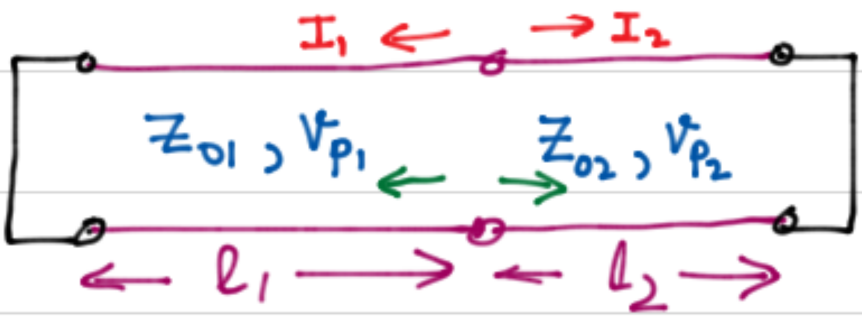
and $I(d) = \frac{2V^+}{Z_0} \cos \beta d$

$\left.\begin{matrix} V(d) = 2jV^+ \sin\beta d \\ I(d) = \frac{2V^+}{Z_0}\cos\beta d \end{matrix}\right\} Z(d) = jZ_0 \tan \beta d$

For a lossless line of length ℓ, the input impedance is purely reactive:

$$Z_{in} = jZ_0 \tan \beta \ell = jZ_0 \tan \frac{2\pi f}{v_p} \ell$$

What will happen if you connect two shorted lines?

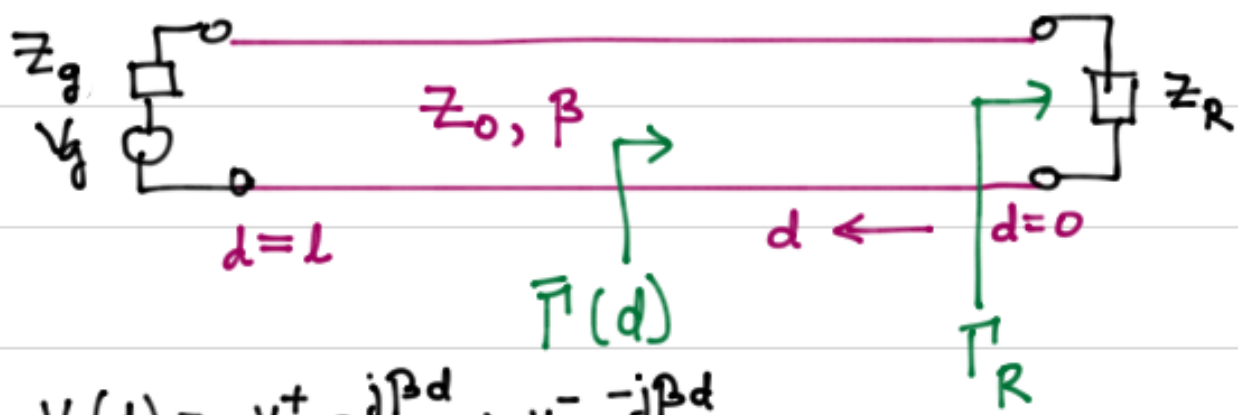


$I_1 + I_2 = 0$, $V_1 = V_2 \Rightarrow \frac{I_1}{V_1} + \frac{I_2}{V_2} = 0$ or $Y_1 + Y_2 = 0$

where Y is the admittance $\frac{1}{Z}$

$$\therefore Z_{01} \tan \beta_1 l_1 + Z_{02} \tan \beta_2 l_2 = 0$$

Line terminated with arbitrary load.



$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$I(d) = \frac{1}{Z_0}\left(V^+ e^{j\beta d} - V^- e^{-j\beta d}\right)$$

Boundary condition $V(0) = Z_R\, I(0)$

$$\therefore \quad V^+ + V^- = \frac{Z_R}{Z_0}\left(V^+ - V^-\right) \Rightarrow V^- = V^+ \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\Gamma_R = \frac{V^-}{V^+} = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\therefore \quad V(d) = V^+ e^{j\beta d} + \Gamma_R V^+ e^{-j\beta d}$$

$$I(d) = \frac{1}{Z_0}\left(V^+ e^{j\beta d} - \Gamma_R V^+ e^{-j\beta d}\right)$$

complex

We can also define $\bar{\Gamma}(d) = \frac{\bar{\Gamma}_R \bar{V}^+ e^{-j\beta d}}{\bar{V}^+ e^{j\beta d}} = \bar{\Gamma}_R e^{-2j\beta d}$

For a line of length l

$$\bar{Z}_{in} = \bar{Z}(l) = Z_0 \frac{1+\bar{\Gamma}(l)}{1-\bar{\Gamma}(l)}$$

$$= Z_0 \frac{1+\Gamma_R e^{-j2\beta d}}{1-\Gamma_R e^{-j2\beta d}} \quad , \quad \Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\frac{Z_{in}}{Z_0} = \frac{Z_R + Z_0 + (Z_R - Z_0) e^{-j2\beta d}}{Z_R + Z_0 - (Z_R - Z_0) e^{-j2\beta d}}$$

$$= \frac{(Z_R + Z_0) e^{j\beta d} + (Z_R - Z_0) e^{-j\beta d}}{(Z_R + Z_0) e^{j\beta d} - (Z_R - Z_0) e^{-j\beta d}}$$

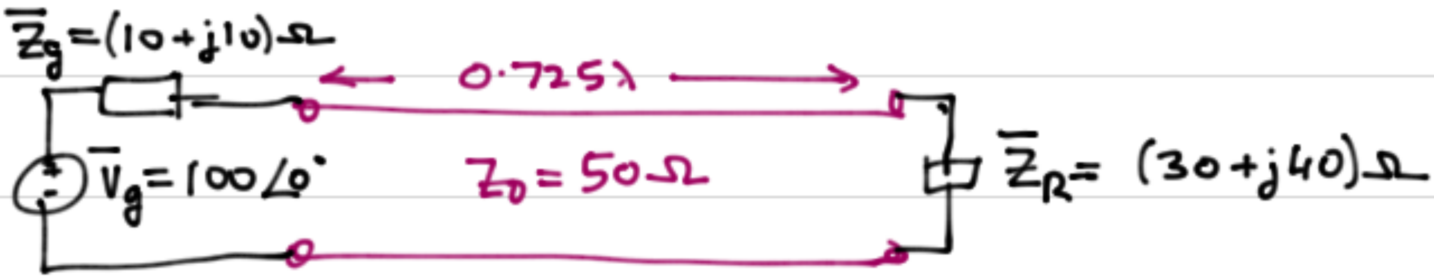
$$= \frac{Z_R \cos(\beta d) + j Z_0 \sin(\beta d)}{j Z_R \sin\beta d + Z_0 \cos\beta d}$$

$$\boxed{\frac{Z_{in}}{Z_0} = \frac{Z_R \cos(\beta d) + j Z_0 \sin(\beta d)}{Z_0 \cos(\beta d) + j Z_R \sin(\beta d)}}$$

<u>Standing Wave Ratio</u> : find $|\Gamma_R|$ through measurement

$$SWR = \frac{V_{max}}{V_{min}} = \frac{|\bar{V}^+|(1+|\bar{\Gamma}_R|)}{|\bar{V}^+|(1-|\bar{\Gamma}_R|)} = \frac{1+|\bar{\Gamma}_R|}{1-|\bar{\Gamma}_R|}$$

Power delivered to a load.



(a) $\bar{\Gamma}_R = \dfrac{\bar{Z}_R - \bar{Z}_0}{\bar{Z}_R + \bar{Z}_0} =$

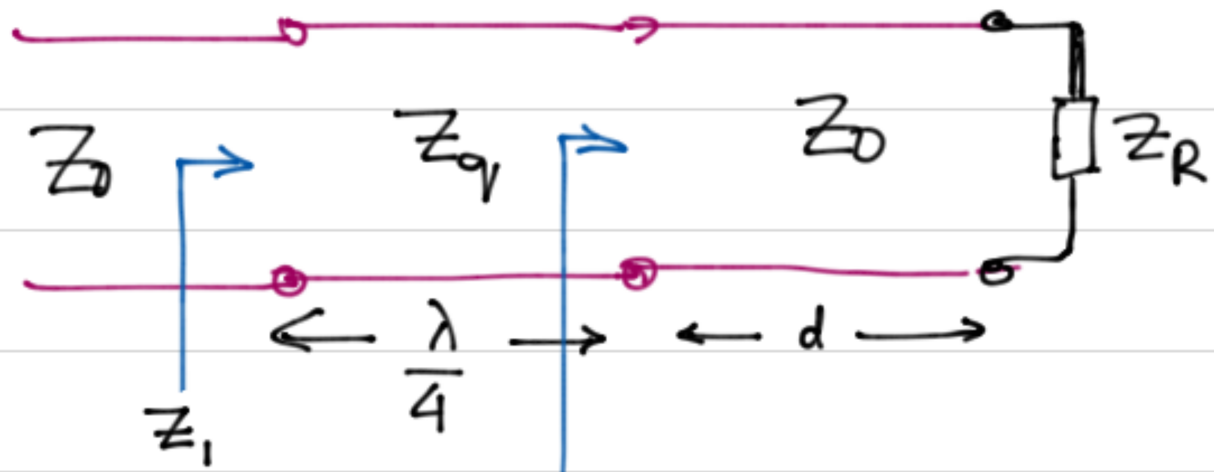
(b) $\bar{\Gamma}(l) =$

(c) $\bar{Z}_{in} = \bar{Z}(l) = Z_0 \dfrac{1 + \bar{\Gamma}(l)}{1 - \bar{\Gamma}(l)}$

(d) $\bar{I}(l) = \bar{I}_g = \dfrac{\bar{V}_g}{\bar{Z}_g + \bar{Z}_{in}}$

(e) $\bar{V}(l) = \bar{Z}_{in}\, \bar{I}(l)$

(f) $\langle P \rangle = \frac{1}{2} \mathrm{Re}\left[\bar{V}(l)\, \bar{I}^*(l)\right] = 48.26\ \mathrm{W}$



Z_1 : want to see Z_0 so that $\Gamma = 0$

Z_2

What do we see here? Impedance could be complex, unless we pick d correctly. So, we must choose d

Line impedance is real where voltage is at its max or min

$$\frac{Z_2}{Z_0} = \frac{Z_R \cos(\beta d) + jZ_0 \sin(\beta d)}{Z_0 \cos(\beta d) + jZ_R \sin(\beta d)}$$

For $\beta d = \frac{\pi}{2} \Rightarrow d = \frac{\pi}{2} \cdot \frac{\lambda}{2\pi} = \frac{\lambda}{4}$

$$\frac{Z_2}{Z_0} = \frac{jZ_0}{jZ_R} \Rightarrow Z_2 = \frac{Z_0^2}{Z_R}$$

For $\beta d = \pi$, $d = \frac{\lambda}{2}$

$$\frac{Z_2}{Z_0} = \frac{Z_R}{Z_0} \Rightarrow Z_2 = Z_R$$

Similarly $Z_q^2 = Z_1 Z_2$; Freedom to choose we just want $Z_1 = Z_0$

$$Z_q = Z_0 \frac{Z_0^2}{Z_R}$$

$$Z_q = Z_0 Z_R$$

Bandwidth estimates:

How robust is the impedance match?

$$z = \frac{Z_{in}}{Z_0} = \frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)} \qquad l = \frac{\lambda}{4}$$

Suppose $\beta = \beta_0 + \delta\beta$, $\quad \beta_0 l = \frac{\pi}{2}$

$$\cos(\beta l) = \cos(\beta_0 l + \delta\beta l)$$

← $\cos(A+B) = \cos A \cos B - \sin A \sin B$; $\cos(\delta\beta l) \simeq 1$

$$= -\sin(\delta\beta l) \simeq -(\delta\beta) l$$

Could also do $\frac{\partial \cos(\beta l)}{\partial(\beta l)} = -\sin(\beta l)\Big|_{\pi/2} = -1$.

This is the first term in a Taylor series expansi

$$\cos(\beta l) \approx \cos(\beta_0 l) + \frac{\partial \cos(\beta l)}{\partial(\beta l)}\Big|_{\beta_0 l} \delta(\beta l) + O(\delta^2(\beta l))$$

order of

remember to evaluate the derivative term

$$\therefore \cos(\beta l) \simeq \cos(\pi/2) - \delta(\beta l) = -(\delta\beta) l$$

Impedance mismatch due to small variations

$$z = \frac{Z_{in}}{Z_0} = \frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)}$$

$\sin(\pi/2)\cos(\delta\beta l) + \cos(\pi/2)\sin(\delta\beta l)$

$$\approx \frac{-Z_L\,\delta(\beta l) + jZ_0}{-Z_0\,\delta(\beta l) + jZ_L} = \frac{jZ_0\left(1 + j\frac{Z_L}{Z_0}\delta(\beta l)\right)}{jZ_L\left(1 + j\frac{Z_0}{Z_L}\delta(\beta l)\right)}$$

$$= \frac{Z_0\left[1 + j\frac{Z_L}{Z_0}\delta(\beta l)\right]}{Z_L\left[1 + j\frac{Z_0}{Z_L}\delta(\beta l)\right]}$$

Assuming $Z_0 \sim Z_L$ such that we can use $\frac{1}{1+x} \simeq 1 - x$

$$z = \frac{Z_0}{Z_L}\left[1 + j\frac{Z_L}{Z_0}\delta(\beta l)\right]\left[1 - j\frac{Z_0}{Z_L}\delta(\beta l)\right]$$

$$= \frac{Z_0}{Z_L}\left[1 + j\left(\frac{Z_L}{Z_0} - \frac{Z_0}{Z_L}\right)\delta(\beta l)\right]$$

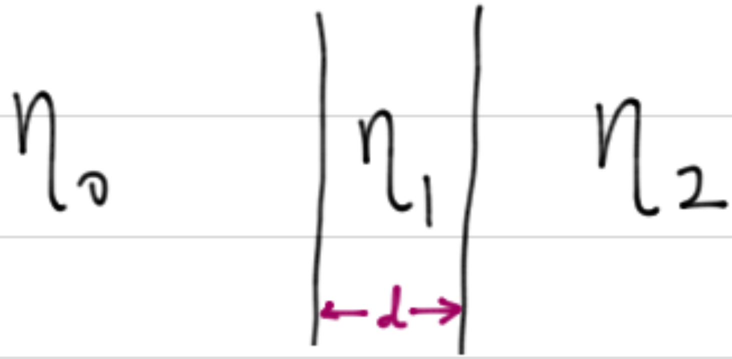
We have thrown away all $O(\delta^2(\beta l))$ terms

Since $z = \frac{Z_{in}}{Z_0}$, $Z_{in} \simeq \frac{Z_0^2}{Z_L} + jZ_0\underbrace{\left(\frac{Z_L}{Z_0} - \frac{Z_0}{Z_L}\right)}\delta(\beta l)$

The only real way to remove the first order variation is to actually have $Z_L = Z_0$!

We can vary $\delta(\beta l)$ by changing $\omega = v_p\beta$ or by assuming errors in setting l, or both.

Analogy with EM Waves.



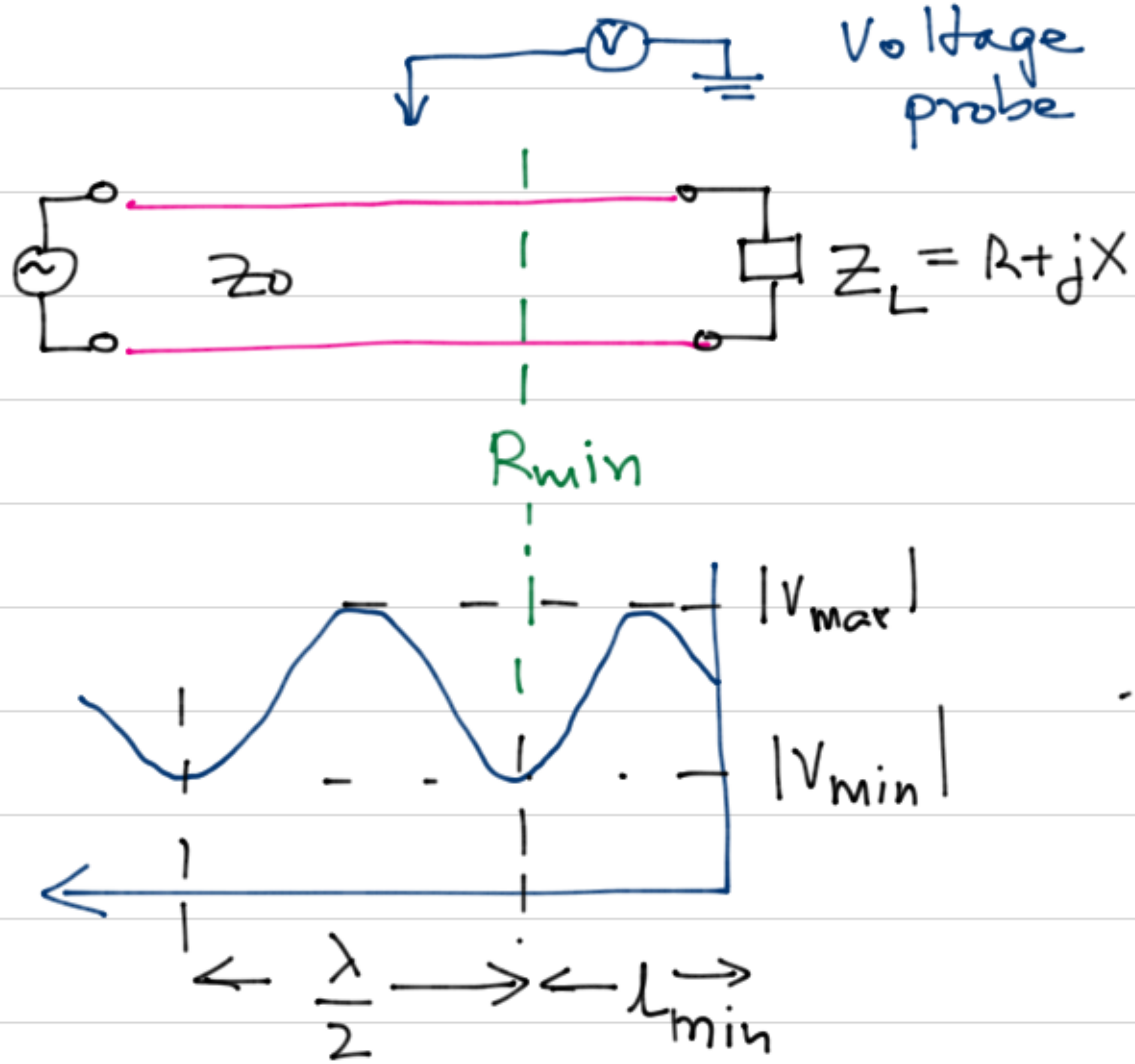
$\eta_0 = 377\Omega$ (air)

$\eta_1^2 = \eta_0 \eta_2$, $d = \frac{\lambda}{4}$ Anti reflection coating

What is its bandwidth?

Can also ask "what angle does it work at"? A wave could be incident at any angle → when we do WAVES.

Measurement of an unknown impedance



$$Z_L \equiv R + jX = Z_0 \left[\frac{R_{min} \cos\beta(-l_{min}) + jZ_0 \sin\beta(-l_{min})}{Z_0 \cos\beta(-l_{min}) + jR_{min} \sin\beta(-l_{min})} \right]$$

Made a measurement at l_{min} and transformed back to the load.

Voltage standing wave ratio

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}, \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

load end refl^n

Recall $V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l}$

$$= V^+ e^{j\beta l}\left[1 + \Gamma(l)\right]$$

$$= V^+ e^{j\beta l}\left[1 + \Gamma_L e^{-j2\beta l}\right]$$

$$= V^+ e^{j\beta l}\left[1 + \underbrace{|\Gamma_L| e^{j(\phi - 2\beta l)}}\right]$$

magnitude is same at every point on the line. Phase changes.

$$\therefore |V_{min}| = |V^+|\left[1 - |\Gamma_L|\right]$$

$$|V_{max}| = |V^+|\left[1 + |\Gamma_L|\right]$$

V repeats $2\beta(\Delta L) = 2\pi$

$\therefore \Delta L = \frac{\lambda}{2}$

$$|I_{max}| = \frac{|V_{max}|}{Z_0} = \frac{|V^+|}{Z_0}(1 + |\Gamma_L|)$$

$$|I_{min}| = \frac{|V_{min}|}{Z_0} = \frac{|V^+|}{Z_0}(1 - |\Gamma_L|)$$

More interesting stuff!

$$R_{min} = \frac{|V_{min}|}{|I_{max}|} = \frac{1 - |\Gamma_L|}{1 + |\Gamma_L|} Z_0 = \frac{Z_0}{\rho}$$

← measure!

$$\therefore \quad Z_L \equiv R + jX_L = Z_0 \left[\frac{\frac{Z_0}{\rho} \cos \beta l_{min} - jZ_0 \sin \beta l_{min}}{Z_0 \cos \beta l_{min} - j \frac{Z_0}{\rho} \sin \beta l_{min}} \right]$$

$$= Z_0 \left[\frac{1 - j\rho \tan \beta l_{min}}{\rho - j \tan \beta l_{min}} \right]$$

$$\therefore \quad R = \frac{\rho \left(1 + \tan^2 \beta l_{min}\right)}{\rho^2 + \tan^2 \beta l_{min}}$$

$$X = \frac{(1 - \rho^2) \tan \beta l_{min}}{\rho^2 + \tan^2 \beta l_{min}}$$

By measuring VSWR and finding l_{min}, we can determine Z_L (assuming we know β)

Lossy transmission line.

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \approx \sqrt{\frac{R}{G}} \quad \text{(real)} \quad \text{for } \begin{array}{l} R \gg \omega L \\ G \gg \omega C \end{array}$$

just being real does not guarantee being lossless

$$\approx \sqrt{\frac{L}{C}} \quad \text{(real)} \quad \text{for } \begin{array}{l} R \ll \omega L \\ G \ll \omega C \end{array}$$

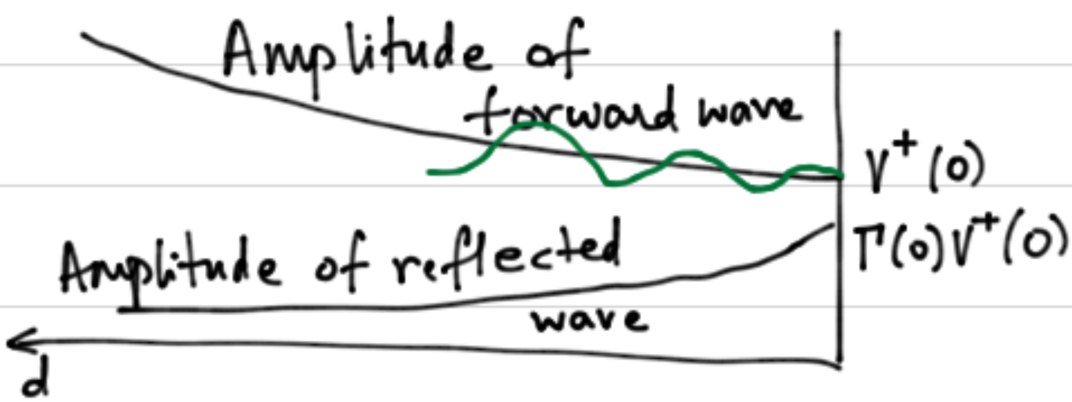
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \approx \sqrt{RG} \quad \text{(real)} \quad \text{for } \begin{array}{l} R \gg \omega L \\ G \gg \omega C \end{array}$$

$$\approx j\sqrt{LC} \quad \text{(imag)} \quad \text{for } \begin{array}{l} R \ll \omega L \\ G \ll \omega C \end{array}$$

In general, $Z_o = R_o + jX_o$, $\gamma = \alpha + j\beta$

$$\Gamma(l) = \Gamma(0) e^{-2\gamma l}$$

$$|\Gamma(l)| = |\Gamma(0)| e^{-2\alpha l}$$



Low loss transmission line

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j\omega L\left\{1-\frac{R}{j\omega L}\right\} j\omega C\left\{1-j\frac{G}{\omega C}\right\}}$$

$$= j\omega\sqrt{LC}\left\{1-j\frac{R}{\omega L}\right\}^{1/2}\left\{1-j\frac{G}{\omega C}\right\}^{1/2}$$

$$\approx j\omega\sqrt{LC}\left\{1-j\frac{R}{2\omega L}\right\}\left\{1-j\frac{G}{2\omega C}\right\} + O\left(\frac{1}{\omega^2}\right)$$

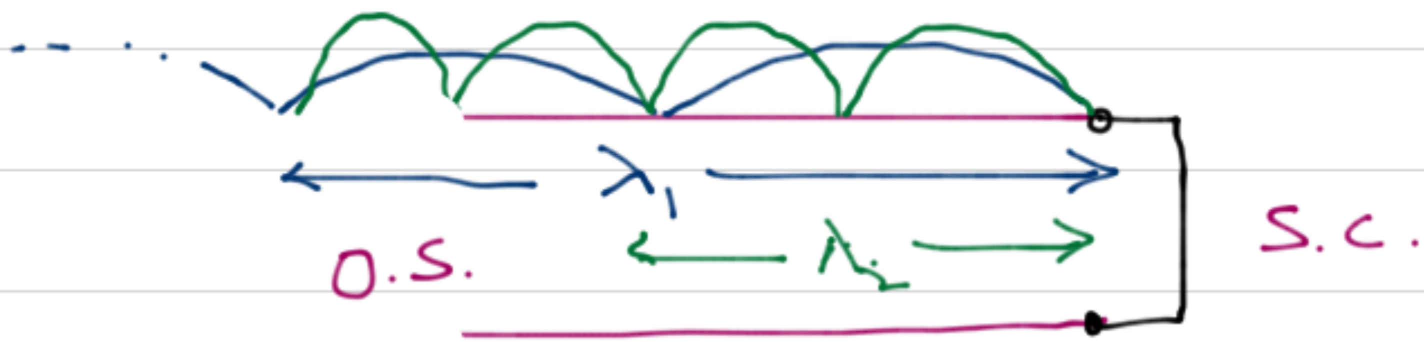
$$= j\omega\sqrt{LC}\left\{1-j\frac{R}{2\omega L}-j\frac{G}{2\omega C}\right\} + O\left(\frac{1}{\omega^2}\right)$$

$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$ $\leftarrow$ attenuation constant

$\beta = \omega\sqrt{LC}$ $\leftarrow$ phase constant, wave number

Transmission Lines

1. For an air dielectric line, the frequency is varied upwards from 50 MHz. The current reaches a minimum at 50·01 MHz and a maximum for 50·04 MHz. What is the distance of the short circuit from the "generator"?



Open circuit @ generator, short circuit @ Load

$$l = (2n-1)\frac{\lambda_n}{4}, \quad n = 1, 2, 3 \dots$$

$$\lambda_n = \frac{4l}{2n-1} \Rightarrow f_n = \frac{v_p}{\lambda_n} = \frac{(2n-1)}{4l} v_p, \quad n = 1, 2, 3 \dots$$

$$\frac{v_p}{4l} = (50\cdot 04 - 50\cdot 01)10^6 = 3\times 10^4$$

If $v_p = 3\times 10^8$ m/s, $l = 2\cdot 5$ km

2. Find the resonant frequencies for a transmission line res resonant system with $Z_{01} = 2Z_{02} = 60\Omega$, $l_1 = 5\text{cm}$. $l_2 = 2\text{cm}$ and $v_{p_1} = v_{p_2} = c/2$

Transcedental equation $\tan \frac{0.2\pi f}{c} + \frac{1}{2} \tan \frac{0.08\pi f}{c} = 0$

Plot $\tan \frac{0.2\pi f}{c}$ and $-\frac{1}{2} \tan \frac{0.08\pi f}{c}$ vs f

intersection points give the solutions.

$f = 1.1869, 2.0861, 3.1324, 4.3676$ GHz

3. A 50Ω line is terminated in a load impedance of $(75-j69)\Omega$. The line is 3·5 m long, excited by a source at 50 MHz. $v_p = 3\times10^8$ m/s. Find the input impedance, the complex input reflection coefficient, the VSWR and the position of the voltage minimum.

4. A 70Ω transmission line is terminated at $(50+j10)\Omega$. Find the position and length of a short circuited stub required for matching if the stub is to be added in

a. series   b. parallel

5. A transmission line has $L = 0{\cdot}25\ \mu H/m$, $C = 100\ pF/m$ and $G = 0$. What should the value of R be for the line to be treated as low loss? The freq of operation is 100 MHz