

Assignment 2: Asymptotic Analysis and Divide and Conquer

- Due by: Friday 23rd September, 2022 by 5pm IST.
- To be submitted to the following email address: office.of.gr@gmail.com
- The subject of the email should be: Assignment Number [2]: Algorithms, 2022
- Please clearly mention your name and roll number.
- Submit your work as a single pdf file. Additional material, code, etc can/should also be submitted, but there should be atleast 1 pdf, which has the entire assignment.
- Wherever there is code, in the assignments, the code should be well documented and easy to understand / follow.

Questions

1. Order the following functions by asymptotic growth rate.

$$\begin{array}{llll} 4n \log n + 2n & 2^{10} & 2^{\log n} & \\ 3n + 100 \log n & 4n & 2^n & \\ n^2 + 10n & n^3 & n \log n & \end{array}$$

2. In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Justify your answer.

| $f(n)$ | $g(n)$ |
|-----------------|-----------------|
| (i) $n - 100$ | $n - 200$ |
| (ii) $\log 2n$ | $\log 3n$ |
| (iii) $n^{0.1}$ | $(\log n)^{10}$ |
| (iv) $n2^n$ | 3^n |

3. Describe an efficient algorithm for finding the ten largest elements in a sequence of size n . What is the running time of your algorithm?
4. Use the divide and conquer integer multiplication algorithm to multiply the two binary integers 10011011 and 10111010.
5. You are given a unimodal array of n distinct elements, meaning that its entries are in increasing order up until its maximum elements, after which its elements are in decreasing order. Give an algorithm to compute the maximum element of a unimodal array that runs in $O(\log n)$ time.
6. **Towers of Hanoi.** Given a game board with three pegs and a set of disks of different diameter all stacked from smallest to largest on the leftmost peg, moves all of the disks to the rightmost peg following these two rules. First, only one disk may be moved at a time. Second, a larger diameter disk may never be placed on a smaller disk. Any number of disks can be used. Implement this in Python.

