Vector operations assessment

A ship travels with velocity given by $\begin{bmatrix}1\\2\end{bmatrix}$, with current flowing in the direction given by $\begin{bmatrix}1\\1\end{bmatrix}$ with respect to some co-ordinate axes.

What is the velocity of the ship in the direction of the current?

- (●) $\begin{bmatrix}3/2\\3/2\end{bmatrix}$
- ( ) $\begin{bmatrix}2/3\\2/3\end{bmatrix}$
- ( ) $\begin{bmatrix}2/3\\3/2\end{bmatrix}$
- ( ) $\begin{bmatrix}3/2\\2/3\end{bmatrix}$

Benar
This is the vector projection of the velocity of the ship onto the velocity of the current.

a = (1,2)
b = (1,1)

a · b = (1)(1) + (2)(1) = 3
|b|² = 1² + 1² = 2

proj_b a = (a · b) / |b|² * b
= (3/2) * (1, 1)
= (3/2, 3/2)
= (1.5, 1.5)

```
>>> import numpy as np
>>> a = np.array([1, 2])
>>> b = np.array([1, 1])
>>> dot_product = np.dot(a, b)
>>> mag_current_sq = np.dot(b, b)
>>> projection = dot_product / mag_current_sq * b
>>> projection
array([1.5, 1.5])
```

2. A ball travels with velocity given by $\begin{bmatrix}2\\1\end{bmatrix}$, with wind blowing in the direction given by $\begin{bmatrix}3\\-4\end{bmatrix}$ with respect to some co-ordinate axes.

What is the size of the velocity of the ball in the direction of the wind?

- ( ) $-\frac{5}{2}$
- (●) $\frac{2}{5}$
- ( ) $\frac{5}{2}$
- ( ) $-\frac{2}{5}$

Benar
This is the scalar projection of the velocity of the ball onto the velocity of the wind.

Hitung dot product dr velocity vector and wind vector: (2, 1) · (3, -4) = (2)(3) + (1)(-4) = 2
Hitung magnitude dr wind vector: |(3, -4)| = sqrt(3^2 + (-4)^2) = 5
Bagi dot product dg magnitude dr wind vector: 2/5 = 0.4

```
>>> v_ball = np.array([2, 1])
>>> v_wind = np.array([3, -4])
>>> dot_product = np.dot(v_ball, v_wind)
>>> mag_wind = np.linalg.norm(v_wind)
>>> projection = dot_product / mag_wind
>>> projection
0.4
```

3. Given vectors $\mathbf{v} = \begin{bmatrix}-4\\-3\\8\end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix}-2\\1\\0\end{bmatrix}$ and $\mathbf{b}_3 = \begin{bmatrix}-3\\-6\\5\end{bmatrix}$ all written in the standard basis, what is $\mathbf{v}$ in the basis defined by $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$? You are given that $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ are all pairwise orthogonal to each other.

- (●) $\begin{bmatrix}1\\1\\1\end{bmatrix}$
- ( ) $\begin{bmatrix}0\\1\\1\end{bmatrix}$
- ( ) $\begin{bmatrix}1\\0\\1\end{bmatrix}$
- ( ) $\begin{bmatrix}1\\1\\0\end{bmatrix}$

```
>>> v = np.array([-4, -3, 8])
>>> b1 = np.array([1, 2, 3])
>>> b2 = np.array([-2, 1, 0])
>>> b3 = np.array([-3, -6, 5])
>>> dot_products = np.array([np.dot(v, b1), np.dot(v, b2), np.dot(v,b3)])
>>> magnitudes = np.array([np.linalg.norm(b1), np.linalg.norm(b2), np.linalg.norm(b3)])
>>> coordinates = dot_products / magnitudes ** 2
>>> v_normalized = coordinates / np.linalg.norm(coordinates)
>>> v_normalized
array([0.57735027, 0.57735027, 0.57735027])
```

u/ cari koordinat vector dari basis b1,b2,b3, berarti kita perlu ubah ke bentuk basis
v = c1b1 + c2b2 + c3b3

u/ cari c1,c2,c3
c1 = (v · b1) / |b1|^2
c2 = (v · b2) / |b2|^2
c3 = (v · b3) / |b3|^2

hitung dot product
v · b1 = (-4)(1) + (-3)(2) + (8)(3) = 14
v · b2 = (-4)(-2) + (-3)(1) + (8)(0) = 11
v · b3 = (-4)(-3) + (-3)(-6) + (8)(5) = 49

hitung magnitude dr setiap basis vektor
|b1| = sqrt(1^2 + 2^2 + 3^2) = sqrt(14)
|b2| = sqrt((-2)^2 + 1^2 + 0^2) = sqrt(5)
|b3| = sqrt((-3)^2 + (-6)^2 + 5^2) = sqrt(70)

c1 = (v · b1) / |b1|^2 = 14 / 14 = 1
c2 = (v · b2) / |b2|^2 = 11 / 5
c3 = (v · b3) / |b3|^2 = 49 / 70

normalisasi vector dg bagi sm magnitudenya
c_normalized = (1, 11/5, 7/10) / sqrt(1^2 + (11/5)^2 + (7/10)^2) ≈ (0.577, 1.154, 0.577) ≈ (1,1,1)

4. Are the following vectors linearly independent?

$$\mathbf{a} = \begin{bmatrix}1\\2\\-1\end{bmatrix}, \mathbf{b} = \begin{bmatrix}3\\-4\\5\end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix}1\\-8\\7\end{bmatrix}.$$

- ( ) Yes
- (●) No

Benar
One can be written as a linear combination of the other two.

```
>>> import numpy as np
>>> M = np.array([[1, 2, -1], [3, -4, 5], [1, -8, 7]])
>>> rank_M = np.linalg.matrix_rank(M)
>>> if rank_M < M.shape[0]:
...     print("The vectors are linearly dependent")
... else:
...     print("The vectors are linearly independent")
...
The vectors are linearly dependent
```

cek solusi non-trivial

a * (1, 2, -1) + b * (3, -4, 5) + c * (1, -8, 7) = (0, 0, 0)

a + 3b + c = 0
2a - 4b - 8c = 0
-a + 5b + 7c = 0

[1 3 1 | 0]
[2 -4 -8 | 0]
[-1 5 7 | 0]

R2 - 2R1 -> R2
R3 + R1 -> R3

[1 3 1 | 0]
[0 -10 -10 | 0]
[0 8 8 | 0]

R3 + R2 -> R3

[1 3 1 | 0]
[0 -10 -10 | 0]
[0 -2 -2 | 0]

R2 -> -R2/5

[1 3 1 | 0]
[0 2 2 | 0]
[0 -2 -2 | 0]

R3 + R2 -> R3

[1 3 1 | 0]
[0 2 2 | 0]
[0 0 0 | 0]

Hasilnya a nontrivial solution karena ada free variable di baris plg bawah yg artinya vektor linearly dependent

5.

At 12:00 pm, a spaceship is at position $\begin{bmatrix}3\\2\\4\end{bmatrix}$ km away from the origin with respect to some 3 dimensional co ordinate system. The ship is travelling with velocity $\begin{bmatrix}-1\\2\\-3\end{bmatrix}$ km/h What is the location of the spaceship after 2 hours have passed?

- ○ $\begin{bmatrix}-2\\4\\-1\end{bmatrix}$
- ○ $\begin{bmatrix}2\\4\\1\end{bmatrix}$
- ◉ $\begin{bmatrix}1\\6\\-2\end{bmatrix}$
- ○ $\begin{bmatrix}-1\\-6\\2\end{bmatrix}$

posisi awal = (3,2,4)
velocity = (-1,2,-3)

setelah 2 jam berarti (-2,4,-6)

berati posisi akhir = (3-2, 2+4, 4-6) = (1,6,-2)