

Principles of Programming

(4190.306)

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Syllabus

➤ Lecture & Practice Session

- Tue: 15:30 ~ 16:20 & 18:30 ~ 20:20
- Thu: 15:30 ~ 16:20 (302-209)
- <https://github.com/snu-sf-class/pp201902>
- Bring your laptop to lectures & practice sessions

➤ Instructor

- Chung-Kil Hur
- <http://sf.snu.ac.kr/gil.hur/>

➤ Teaching Assistant

- Yonghyun Kim, Dongjoo Kim.
Contact TAs at pp2019fall@sf.snu.ac.kr

➤ Grading (tentative)

- Attendance: 5%, Assignments: 25%
- Midterm exam: 30%, Final exam: 40%

Free Laptop Rental

You can rent a laptop from the department for free.

Who am I?

➤ Prof. Chung-Kil (Gil) Hur [허충길]

- Education: KAIST (B.S.), Univ of Cambridge (Ph.D.)
- Software Foundations Lab
<http://sf.snu.ac.kr>
- Research Topics
 - Software Verification
 - Low-level Language Semantics (C/C++/LLVM/Rust)
 - Relaxed-Memory Concurrency
- Our collaborators
 - [UK] U of Cambridge, Microsoft Research Cambridge
 - [Germany] Max Planck Institute for Software Systems
 - [France] INRIA
 - [USA] Princeton, Yale, Upenn, Utah, Google, IBM, Mozilla.
- Publications
 - 11 top conference papers (last 6 years at SNU).
PLDI(6), POPL(2), OOPSLA(1), ICFP(1), AAAI(1)
- (High school) Bronze medal in IMO 1994.

Introduction

Overview

➤ Part 1

Functional Programming with Function Applications

➤ Part 2

Object-Oriented Programming

➤ Part 3

Type Classes for Specifications

➤ Part 4

Imperative Programming with Memory Updates

Imperative vs. Functional Programming

➤ Imperative Programming

- Computation by memory reads/writes
- Sequence of read/write operations
- Repetition by loop
- More procedural
- Easier to write efficient code

```
sum = 0;
i = n;
while (i > 0) {
    sum = sum + i;
    i = i - 1;
}
```

➤ Functional Programming

- Computation by function application
- Composition of function applications
- Repetition by recursion
- More declarative
- Easier to write safe code

```
def sum(n) =
    if (n <= 0)
        0
    else
        n + sum(n-1)
```

Both Imperative & Functional Style Supported

- Many languages support both imperative & functional style
 - More imperative: Java, Javascript, C++, Python, Rust, ...
 - More functional: OCaml, SML, Lisp, Scheme, ...
 - Middle: Scala
 - Purely functional: Haskell (monadic programming)

Object-Oriented Programming

➤ Object-Oriented Programming

- Classes/Objects: data + methods
- Can separate Specification & Implementation

```
class Point(x: Int, y: Int) {  
  // data  
  val px : Int = x  
  val py : Int = y  
  // methods  
  def plus(q : Point) : Point =  
    new Point(px + q.px, py + q.py)  
}  
  
val p1 : Point = new Point(3,4)  
val p2 : Point = new Point(1,10)  
val p3 : Point = p1.plus(p2)
```

Why Scala?

➤ Why Scala?

- Equally well support both imperative & functional style
- Many advanced features (both OOP & Type class supported)
- Compatible with Java

PART 1

Functional Programming with Function Applications

Names, Functions and Evaluations

Values, Expressions, Names

➤ Types and Values

- A type is a set of values
- `Int`: $\{-2147483648, \dots, -1, 0, 1, \dots, 2147483647\}$ //32-bit integers
- `Double`: 64-bit floating point numbers // real numbers in practice
- `Boolean`: $\{\text{true}, \text{false}\}$
- ...

➤ Expressions

- Composition of
values, names, primitive operations, ...

➤ Name Binding

- Binding expressions to names

➤ Examples

```
def a = 1 + (2 + 3)
def b = 3 + a * 4
```

Evaluation

➤ Evaluation

- Reducing an expression into a value
- Strategy
 1. Take a name or an operator (outer to inner)
 2. (name) Replace the name with its associated expression
 3. (name) Evaluate the expression
 4. (operator) Evaluate its operands (left to right)
 5. (operator) Apply the operator to its operands

➤ Examples

$5+b \sim 5+(3+a*4) \sim 5+(3+(1+(2+3))*4) \sim \dots \sim 32$

Functions and Substitution

➤ Functions

- Expressions with Parameters
- Binding functions to names

```
def f(x: Int): Int = x + a
```

➤ Evaluation by substitution

- ...
- (function) Evaluate its operands (left to right)
- (function)
Replace the function application by the expression of the function
Replace its parameters with the operands

$$5 + f(f(3) + 1) \sim 5 + f((3 + a) + 1) \sim \dots \sim 5 + f(10) \sim 5 + (10 + a) \\ \sim \dots \sim 21$$

Simple Recursion

➤ Recursion

- Use X in the definition of X
- Powerful mechanism for repetition
- Nothing special but just rewriting

```
def sum(n: Int) : Int =  
  if (n <= 0)  
    0  
  else  
    n + sum(n-1)
```

```
sum(2) ~ if (2<=0) 0 else (2+sum(2-1)) ~  
2+sum(1) ~ 2+(if (1<=0) 0 else (1+sum(1-1))) ~  
2+(1+sum(0)) ~ 2+(1+(if (0<=0) 0 else (0+sum(0-1))))  
~ 2+(1+0) ~ 3
```


Termination/Divergence

Evaluation may not terminate

➤ Termination

- An expression may reduce to a value

➤ Divergence

- An expression may reduce forever

```
def loop: Int = loop
```

```
loop ~ loop ~ loop ~ ...
```

Evaluation strategy: Call-by-value, Call-by-name

$f(e1, e2)$

➤ Call-by-value

- Evaluate the arguments first, then apply the function to them

➤ Call-by-name

- Just apply the function to its arguments, without evaluating them.

```
def square (x: Int) = x * x
```

```
[cbv]square(1+1) ~ square(2) ~ 2*2 ~ 4
```

```
[cbn]square(1+1) ~ (1+1)*(1+1) ~ 2*(1+1) ~ 2*2 ~ 4
```

CBV, CBN: Differences

➤ Call-by-value

- Evaluates arguments once

➤ Call-by-name

- Do not evaluate unused arguments

➤ Question

- Do both always result in the same value?

Scala's evaluation strategy

➤ Call-by-value

- By default

➤ Call-by-name

- Use “=>”

```
def one(x: Int, y: =>Int) = 1
```

```
one(1+2, loop)
```

```
one(loop, 1+2)
```

Scala's name binding strategy

➤ Call-by-value

- Use “val” (also called “field”) e.g. `val x = e`
- Evaluate the expression first, then bind the name to it

➤ Call-by-name

- Use “def” (also called “method”) e.g. `def x = e`
- Just bind the name to the expression, without evaluating it
- Mostly used to define functions

```
def a = 1 + 2 + 3
```

```
val a = 1 + 2 + 3 // 6
```

```
def b = loop
```

```
val b = loop
```

```
def f(a: Int, b: Int): Int = a*b - 2
```

Conditional Expressions

➤ If-else

- `if (b) e1 else e2`
- `b` : Boolean expression
- `e1, e2`: expressions of the same type

➤ Rewrite rules:

- `if (true) e1 else e2 → e1`
- `if (false) e1 else e2 → e2`

```
def abs(x: Int) = if (x >= 0) x else -x
```

Boolean Expressions

➤ Boolean expression

- true, false
- !b
- b && b
- b || b
- e <= e, e >= e, e < e, e > e, e == e, e != e

➤ Rewrite rules:

- !true → false
- !false → true
- true && b → b
- false && b → false
- true || b → true
- false || b → b

true && (loop == 1) ~ loop == 1 ~ loop == 1

Exercise: and, or

➤ Write two functions

- `and(x,y) == x && y`
- `or(x,y) == x || y`
- Do not use `&&`, `||`

```
and(false,loop==1)
```

```
~ if (false) loop==1 else false
```

```
~ false
```

```
and(true,loop==1)
```

```
~ if (true) loop==1 else false
```

```
~ loop==1 ~ loop==1 ...
```


Solution

```
def and(x: Boolean, y: =>Boolean) =  
  if (x) y else false
```

```
def or(x: Boolean, y: =>Boolean) =  
  if (x) true else y
```

Exercise: square root calculation

➤ Calculate square roots with Newton's method

```
def isGoodEnough(guess: Double, x: Double) =  
    ??? // guess*guess is 99.9% close to x
```

```
def improve(guess: Double, x: Double) =  
    (guess + x/guess) / 2
```

```
def sqrtIter(guess: Double, x: Double): Double =  
    ??? // repeat improving guess until it is good enough
```

```
def sqrt(x: Double) =  
    sqrtIter(1, x)
```

```
sqrt(2)
```

Solution

➤ Calculate square roots with Newton's method

```
def isGoodEnough(guess: Double, x: Double) =  
    guess*guess/x > 0.999 && guess*guess/x < 1.001
```

```
def improve(guess: Double, x: Double) =  
    (guess + x/guess) / 2
```

```
def sqrtIter(guess: Double, x: Double): Double =  
    if (isGoodEnough(guess, x)) guess  
    else sqrtIter(improve(guess, x), x)
```

```
def sqrt(x: Double) =  
    sqrtIter(1, x)
```

```
sqrt(2)
```

Blocks and Name Scoping

Blocks in Scala

➤ Block

- { **val** x1 = e1
 def x2 = e2
 e
}
- Is an expression
- Allow nested name binding
- Allow arbitrary order of “**def**”s, but not “**val**”s (think about why)

Scope of names

➤ Block

```
val t = 0
def f(x: Int) = t + g(x)
def g(x: Int) = x * x
val x = f(5)
val r = {
  val t = 10
  val s = f(5)
  s - t
}
t + r
```

- A definition inside a block is only accessible within the block
- A definition inside a block shadows definitions of the same name outside the block
- A definition is accessible inside a nested block unless it is shadowed
- A function is evaluated under the environment where it is defined, not the environment where it is invoked.

Rewriting for Blocks

1: val t = 0

2: def f(x: Int) = t + g(x)

3: def g(x: Int) = x*x

4: val x = f(5)

5: val r = {

6: val t = 10

7: val s = f(5)

8: s - t }

9: t + r

➤ Evaluation with Environment

[], 1 ~ [t=0], 2 ~ [..., f=(x)t+g(x)], 3 ~ [..., g=(x)x*x], 4
~ [..., x=25], 5 ~ [...] : [], 6 ~ [...] : [t=10], 7
~ [...] : [..., s=25], 8 ~ [..., r=15], 9 ~ [...], 15

4: [t=0, f=..., g=...] : [x=5], t+g(x) ~ 0+g(5) ~ 25

g(5): [t=0, f=..., g=...] : [x=5], x*x ~ 5*5 ~ 25

7: [t=0, f=..., g=..., x=25] : [x=5], t+g(x) ~ 0+g(5) ~ 25

g(5): [t=0, f=..., g=..., x=25] : [x=5], x*x ~ 5*5 ~ 25

Note: The same name cannot be defined twice in the same block

Example: def with no arguments

```
1: val t = 0
2: def x = t+t      // is treated as def x() = t
3: val r = {
4:   val t = 10
5:   x }            // is treated as x()
```

➤ Evaluation with Environment

$[]$, 1 \sim $[t=0]$, 2 \sim $[..., x=()t+t]$, 3 \sim $[...]:[]$, 4
 \sim $[...]:[t=10]$, 5 \sim $[..., r=0]$, 6
5: $[t=0, x=()t+t]:[], t+t \sim 0$

Semi-colons and Parenthesis

➤Block

- Can write two definitions/expressions in a single line using ;
- Can write one definition/expression in two lines using (), but can omit () when clear

// ok

```
val r = {  
    val t = 10; val s = square(5); t +  
    s }
```

// Not ok

```
val r = {  
    val t = 10; val s = square(5); t  
    + s }
```

// ok

```
val r = {  
    val t = 10; val s = square(5); (t  
    + s) }
```

Exercise: Writing Better Code using Blocks

➤ Make the following code better

```
def isGoodEnough(guess: Double, x: Double) =  
    guess*guess/x > 0.999 && guess*guess/x < 1.001
```

```
def improve(guess: Double, x: Double) =  
    (guess + x/guess) / 2
```

```
def sqrtIter(guess: Double, x: Double): Double = {  
    if (isGoodEnough(guess,x)) guess  
    else sqrtIter(improve(guess,x),x)  
}
```

```
def sqrt(x: Double) =  
    sqrtIter(1, x)
```

```
sqrt(2)
```

Solution

```
def sqrt(x: Double) = {  
  def sqrtIter(guess: Double, x: Double): Double = {  
    if (isGoodEnough(guess, x)) guess  
    else sqrtIter(improve(guess, x), x)  
  }  
  def isGoodEnough(guess: Double, x: Double) = {  
    val ratio = guess * guess / x  
    ratio > 0.999 && ratio < 1.001  
  }  
  def improve(guess: Double, x: Double) =  
    (guess + x/guess) / 2  
  
  sqrtIter(1, x)  
}
```

```
sqrt(2)
```

Lazy Call-By-Value

Lazy call-by-value

➤ Lazy call-by-value

- Use “lazy val” e.g. `lazy val x = e`
- Evaluate the expression **first time it is used**, then bind the name to it

```
def f(c: Boolean, i: =>Int): Int = {  
  lazy val iv = i  
  if (c) 0  
  else iv * iv * iv  
}
```

```
f(true, {println("ok"); 100+100+100+100})  
f(false, {println("ok"); 100+100+100+100})
```

Tail Recursion

Recursion needs care

➤ Summation function

- Write a summation function `sum` such that
 $\text{sum}(n) = 1+2+\dots+n$
- Test
`sum(10), sum(100), sum(1000), sum(10000),`
`sum(100000), sum(1000000)`
- What's wrong? (Think about evaluation)

Recursion: Try

```
def sum(n: Int): Int =  
  if (n <= 0) 0 else (n+sum(n-1))
```


Recursion: Tail Recursion

```
import scala.annotation.tailrec
```

```
def sum(n: Int): Int = {  
    @tailrec def sumItr(res: Int, m: Int): Int =  
        if (m <= 0) res else sumItr(m+res,m-1)  
    sumItr(0,n)  
}
```

Higher-Order Functions

Functions as Values

➤ Functions

- Functions are normal values of function types $(A_1, \dots, A_n \Rightarrow B)$.
- They can be copied, passed and returned.
- Functions that take functions as arguments or return functions are called higher-order functions.
- Higher-order functions increase code reusability.

Examples

```
def sumLinear(n: Int): Int =  
  if (n <= 0) 0 else n + sumLinear(n-1)
```

```
def sumSquare(n: Int): Int =  
  if (n <= 0) 0 else n*n + sumSquare(n-1)
```

```
def sumCubes(n: Int): Int =  
  if (n <= 0) 0 else n*n*n + sumCubes(n-1)
```

Q: How to write reusable code?

Examples

```
def sum(f: Int=>Int, n: Int): Int =  
  if (n <= 0) 0 else f(n) + sum(f, n-1)
```

```
def linear(n: Int) = n  
def square(n: Int) = n * n  
def cube(n: Int) = n * n * n
```

```
def sumLinear(n: Int) = sum(linear, n)  
def sumSquare(n: Int) = sum(square, n)  
def sumCubes(n: Int) = sum(cube, n)
```

Anonymous Functions

➤ Anonymous Functions

- Syntax

$(x_1: T_1, \dots, x_n: T_n) \Rightarrow e$

or

$(x_1, \dots, x_n) \Rightarrow e$

```
def sumLinear(n: Int) = sum((x: Int) => x, n)
```

```
def sumSquare(n: Int) = sum((x: Int) => x*x, n)
```

```
def sumCubes(n: Int) = sum((x: Int) => x*x*x, n)
```

Or simply

```
def sumLinear(n: Int) = sum((x) => x, n)
```

```
def sumSquare(n: Int) = sum((x) => x*x, n)
```

```
def sumCubes(n: Int) = sum((x) => x*x*x, n)
```

Exercise

```
def sum(f: Int=>Int, a: Int, b: Int): Int =  
  if (a <= b) f(a) + sum(f, a+1, b) else 0
```

```
def product(f: Int=>Int, a: Int, b: Int): Int =  
  if (a <= b) f(a) * product(f, a+1, b) else 1
```

DRY (Do not Repeat Yourself) using a higher-order function, called “mapReduce”.

Exercise

```
def mapReduce(reduce: (Int,Int)=>Int, inival: Int,  
              f: Int=>Int, a: Int, b: Int): Int = {  
  if (a <= b) reduce(f(a),mapReduce(reduce,inival,f,a+1,b))  
  else inival  
}
```

```
def sum(f: Int=>Int, a: Int, b: Int): Int =  
  mapReduce((x,y)=>x+y,0,f,a,b)
```

```
def product(f: Int=>Int, a: Int, b: Int): Int =  
  mapReduce((x,y)=>x*y,1,f,a,b)
```


Closures for functional values

```
1: val t = 0
2: val f : Int=>Int = {           // Note: What if using “def f” ?
3:     val t = 10
4:     def g(x: Int) : Int = x + t
5:     g _ }
6: f(20)
```

* Try: Evaluation without Closures

$[], 1 \sim [t=0], 2 \sim [\dots]:[], 3 \sim [\dots]:[t=10], 4$
 $\sim [\dots]:[\dots, g=(x)x+t], 5 \sim [t=0, f=(x)x+t], 6 \sim [\dots], 20$
 $6: [t=0, f=(x)x+t]:[x=20], x+t \sim 20+0 \sim 20$

* Evaluation with Closures

$E1[], 1 \sim E1[t=0], 2 \sim E1[\dots]:E2[], 3 \sim E1[\dots]:E2[t=10], 4$
 $\sim E1[\dots]:E2[t=10, g=(x)x+t], 5$
 $\sim E1[t=0, f=((x)x+t, E2)], 6 \sim E1[\dots], 30$
 $6: E1:E2:E3[x=20], x+t \sim 20+10 \sim 30$

Problem

// should be allowed

{

def f(x: Int) = g(x)

def g(x: Int) = 10

val x = f(10)

x

}

// should not be allowed

{

def f(x: Int) = g(x)

val x = f(10)

def g(x: Int) = 10

x

}

Safety Checking

Safety checking for ruling out accessing undefined values is simple.

- For “val x = e”,
all names in “e” should be defined before this definition.
- For “def x = e”,
all names in “e” should be defined before the next “val” definition.

/* The following code passes the safety checking */

```
{ def f(x: Int) = g(x)
  def g(x: Int) = 10
  val a = 10
  f(10) }
```

/* The following code fails at the safety checking */

```
{ def f(x: Int) = g(x)
  val a = 10
  def g(x: Int) = 10
  f(10) }
```

(Parameterized) code vs. values

- Functions defined using “def” are not values but parameterized code/expressions.
- Anonymous functions are values.
- Code f is converted to a value $(f \ _)$.
- The compiler often infers and inserts missing conversions automatically.

- Anonymous functions can be seen as syntactic sugar:

$(x:T) \Rightarrow e$

is equivalent to

$\{ \text{def } \text{noname}(x:T) = e; (\text{noname } _) \}$

where `noname` is not used in e .

- One can even write a recursive anonymous function in this way.
- Q: what's the difference between param. exps and function values?
A: function values are “closures” (ie, param. exp. + env.)
- Q: how to implement call-by-name?
A: The argument expression is converted to a closure.

Example: call by name with closures

```
1: val t = 0
2: def f(x: =>Int) = t + x
3: val r = {
4:   val t = 10
5:   f(t*t) }           // t*t is treated as ()=>t*t
```

➤ Evaluation with Closures

$E1[], 1 \sim E1[t=0], 2 \sim E1[t=0, f=(x:=>Int)t+x], 3$
 $\sim E1[\dots]:E2[], 4 \sim E1[\dots]:E2[t=10], 5 \sim E1[\dots, r=100], 6$

$5: E1:E3[x=(()t*t, E2)], t+x$
 $\sim 0 + x \sim 0 + 100 \sim 100$

$x: E1:E2, t*t \sim 10*10 \sim 100$

Currying

Motivation

```
def sum(f: Int=>Int, a: Int, b: Int): Int =  
  if (a <= b) f(a) + sum(f, a+1, b) else 0  
def linear(n: Int) = n  
def square(n: Int) = n * n  
def cube(n: Int) = n * n * n  
def sumLinear(a: Int, b: Int) = sum(linear, a, b)  
def sumSquare(a: Int, b: Int) = sum(square, a, b)  
def sumCubes(a: Int, b: Int) = sum(cube, a, b)
```

We want the following. How?

```
def sumLinear = sum(linear)  
def sumSquare = sum(square)  
def sumCubes = sum(cube)
```

Solution

```
def sum(f: Int=>Int): (Int,Int)=>Int = {  
  def sumF(a: Int, b: Int): Int =  
    if (a <= b) f(a) + sumF(a+1, b) else 0  
  sumF  
}
```

```
def sumLinear = sum(linear)  
def sumSquare = sum(square)  
def sumCubes = sum(cube)
```


Benefits

```
def sumLinear = sum(linear)
def sumSquare = sum(square)
def sumCubes = sum(cube)
```

`sumSquare(3, 10) + sumCubes(5, 20)`

We don't need to define the wrapper functions.

`sum(square)(3, 10) + sum(cube)(5, 20)`

Multiple Parameter List

```
def sum(f: Int=>Int): (Int,Int)=>Int = {  
  def sumF(a: Int, b: Int): Int =  
    if (a <= b) f(a) + sumF(a+1, b) else 0  
  sumF _  
}
```

Or simply:

```
def sum(f: Int=>Int)(a: Int, b: Int): Int =  
  if (a <= b) f(a) + sum(f)(a+1, b) else 0
```

Note that `sum(f)` is just code.

`(sum(f) _)` creates a closure like `(sumF _)`

The following code is inefficient. Think about why.

```
def sum(f: Int=>Int): (Int,Int)=>Int =  
  (a,b) => if (a <= b) f(a) + sum(f)(a+1, b) else 0
```

Currying and Uncurrying

- A function of type

$$(T_1, T_2, \dots, T_n) \Rightarrow T$$

can be turned into that of type

$$T_1 \Rightarrow (T_2 \Rightarrow (\dots \Rightarrow (T_n \Rightarrow T) \dots))$$

- This is called “currying” named after Haskell Brooks Curry.
- The opposite direction is called “uncurrying”.

Currying using Anonymous Functions

```
def foo(x: Int, y: Int, z: Int)(a: Int, b: Int) =  
  x + y + z + a + b
```

```
val f1 = (x: Int, z: Int, b: Int) => foo(x, 1, z)(2, b)
```

```
val f2 = foo(_: Int, 1, _: Int)(2, _: Int)
```

```
val f3 = (x: Int, z: Int) => ((b: Int) => foo(x, 1, z)(2, b))
```

```
f1(1, 2, 3)
```

```
f2(1, 2, 3)
```

```
f3(1, 2)(3)
```

Exercise

Curry the mapReduce function.

```
def mapReduce(reduce: (Int, Int) => Int, inival: Int,  
              f: Int => Int, a: Int, b: Int): Int = {  
  if (a <= b) reduce(f(a), mapReduce(reduce, inival, f, a+1, b))  
  else inival  
}
```

```
def sum(f: Int => Int, a: Int, b: Int): Int =  
  mapReduce((x, y) => x+y, 0, f, a, b)
```

```
def product(f: Int => Int, a: Int, b: Int): Int =  
  mapReduce((x, y) => x*y, 1, f, a, b)
```

Solution

```
def mapReduce(reduce:(Int,Int)=>Int,inival: Int)
    (f: Int=>Int) (a: Int, b: Int): Int = {
    if (a <= b) reduce(f(a),mapReduce(reduce,inival)(f)(a+1,b))
    else inival
}
```

// need to make a closure since mapReduce is param. code.

```
def sum = mapReduce((x,y)=>x+y,0) _
```

// val is better than def. Think about why.

```
val product = mapReduce((x,y)=>x*y,1) _
```

Exceptions

Exception & Handling

```
class factRangeException(val arg: Int) extends Exception
```

```
def fact(n : Int): Int =  
  if (n < 0) throw new factRangeException(n)  
  else if (n == 0) 1  
  else n * fact(n-1)
```

```
def foo(n: Int) = fact(n + 10)
```

```
try {  
  println (fact(3))  
  println (foo(-100))  
} catch {  
  case e : factRangeException => {  
    println("fact range error: " + e.arg)  
  }  
}
```


Datatypes

Types so far

Types have introduction operations and elimination ones.

- Introduction: how to construct elements of the type
- Elimination: how to use elements of the type

➤ Primitive types

- Int, Boolean, Double, String, ...
- Intro for Int: ..., -2, -1, 0, 1, 2,
- Elim for Int: +, -, *, /, <, <=, ...

➤ Function types

- $\text{Int} \Rightarrow \text{Int}$, $(\text{Int} \Rightarrow \text{Int}) \Rightarrow (\text{Int} \Rightarrow \text{Int})$, ...
- Intro: $(x:T) \Rightarrow e$
- Elim: $f(v)$

Tuples

➤ Tuples

Intro:

- $(1, 2, 3) : (\text{Int}, \text{Int}, \text{Int})$
- $(1, \text{"a"}) : (\text{Int}, \text{String})$

Elim:

- $(1, \text{"a"}, 10)._1 = 1$
- $(1, \text{"a"}, 10)._2 = \text{"a"}$
- $(1, \text{"a"}, 10)._3 = 10$

Only up to length 22

Structural Types (a.k.a. Record Types): Examples

```
def bar (x: Int) = x+1
```

```
val foo = new //or, object foo
```

```
{ val a = 1+2  
  def b = a + 1  
  def f(x: Int) = b + x  
  def f(x: String) = "hello" + x  
  val g : Int => Int = bar _  
}
```

```
foo.b
```

```
foo.f(3)
```

```
foo.f("gil")
```

```
def g(x: {val a: Int; def b: Int;  
         def f(x: Int): Int; def f(x: String): String,  
         val g: Int => Int}) =
```

```
  x.f(3)
```

```
g(foo)
```

Structural Types: Evaluation

```
1: def bar (x: Int) = x+1
2: val foo = new //or, object foo
3: { val a = 2 + 1
4:   def f(x: Int) = a + x
5:   val g : Int => Int = bar _
6: }
7: val b = foo.f(1)
8: foo.g(b)
```

➤ Evaluation with Closures

```
E1[], 1 ~ E1[bar=(x)x+1], 2 ~ E1[...]:E2[], 3 ~
E1[...]:E2[a=3], 4 ~
E1[...]:E2[a=3, f=(x)a+x], 5 ~
E1[...]:E2[a=3, f=(x)a+x, g=((x)x+1, E1)], 6 ~
E1[bar=(x)x+1, foo=(E2)], 7 ~
E1[bar=(x)x+1, foo=(E2), b=4], 8 ~ 5
7: E1:E2:E3[x=1], a+x ~ 3+1 ~ 4
8: E1:E4[x=4], x+1 ~ 4+1 ~ 5
```

Type Alias

```
val gn = 0
object foo {
  val a = 3
  def b = a + 1
  def f(x: Int) = b + x + gn
}
```

foo.f(3)

```
type Foo = {val a: Int; def b: Int; def f(x: Int): Int}
```

```
def g(x: Foo) = {
  val gn = 10
  x.f(3)
}
```

g(foo)

Algebraic Datatypes

➤ Ideas

- $T = C \text{ of } T * \dots * T$
| $C \text{ of } T * \dots * T$
| \dots
| $C \text{ of } T * \dots * T$

- E.g.

Attr = Name of String
| Age of Int
| DOB of Int * Int * Int
| Height of Double

Intro:

Name(“Chulsoo Kim”), Name(“Younghee Lee”), Age(16),
DOB(2000,3,10), Height(171.5), ...

Algebraic Datatypes: Recursion

➤ Recursive ADT

- E.g.

$\text{IList} = \text{INil}$

| $\text{ICons of Int} * \text{IList}$

Intro:

$\text{INil}()$,

$\text{ICons}(3, \text{INil}())$,

$\text{ICons}(2, \text{ICons}(3, \text{INil}()))$,

$\text{ICons}(1, \text{ICons}(2, \text{ICons}(3, \text{INil}())))$,

...

Algebraic Datatypes In Scala

➤ Attr

```
sealed abstract class Attr
case class Name(name: String) extends Attr
case class Age(age: Int) extends Attr
case class DOB(year: Int, month: Int, day: Int) extends Attr
case class Height(height: Double) extends Attr
```

```
val a : Attr = Name("Chulsoo Kim")
val b : Attr = DOB(2000, 3, 10)
```

➤ IList

```
sealed abstract class IList
case class INil() extends IList
case class ICons(hd: Int, tl: IList) extends IList
```

```
val x : IList = ICons(2, ICons(1, INil()))
def gen(n: Int) : IList =
  if (n <= 0) INil()
  else ICons(n, gen(n-1))
```

Exercise

IOption = INone
 | ISome of Int

BTree = Leaf
 | Node of Int * BTree * BTree

```
sealed abstract class IList  
case class INil() extends IList  
case class ICons(hd: Int, tl: IList) extends IList
```

```
def x : IList = ICons(2, ICons(1, INil()))
```

Solution

```
sealed abstract class IOption  
case class INone() extends IOption  
case class ISome(some: Int) extends IOption
```

```
sealed abstract class BTree  
case class Leaf() extends BTree  
case class Node(value: Int, left: BTree, right: BTree)  
      extends BTree
```

Pattern Matching

➤ Pattern Matching

- A way to use algebraic datatypes

```
e match {  
  case C1(...) => e1  
  ...  
  case Cn(...) => en  
}
```

Pattern Matching: An Example

```
def length(xs: IList) : Int =  
  xs match {  
    case /Ni/() => 0  
    case /Cons(x, tl) => 1 + length(tl)  
  }
```

length(x)

Advanced Pattern Matching

➤ Advanced Pattern Matching

```
e match {  
  case P1 => e1  
  
  ...  
  
  case Pn => en  
}
```

- One can combine constructors and use `_` and `|` in a pattern.
(E.g) `case ICons(x, INil()) | ICons(x, ICons(_, INil())) => ...`
- The given value `e` is matched against the first pattern `P1`.
If succeeds, evaluate `e1`.
If fails, `e` is matched against `P2`.
If succeeds, evaluate `e2`.
If fails, ...
- The compiler checks exhaustiveness (ie, whether there is a missing case).

Advanced Pattern Matching: An Example

```
def secondElmt(xs: IList) : IOption =  
  xs match {  
    case /Nil() | /Cons(_, /Nil()) => /None()  
    case /Cons(_, /Cons(x, _)) => /Some(x)  
  }
```

Vs.

```
def secondElmt2(xs: IList) : IOption =  
  xs match {  
    case /Nil() | /Cons(_, /Nil()) => /None()  
    case /Cons(_, /Cons(x, /Nil())) => /Some(x)  
    case _ => /None()  
  }
```

Vs.

```
def secondElmt2(xs: IList) : IOption =  
  xs match {  
    case /Cons(_, /Cons(x, /Nil())) => /Some(x)  
    case _ => /None()  }
```

Pattern Matching on Int

```
def factorial(n: Int) : Int =  
  n match {  
    case 0 => 1  
    case _ => n * factorial(n-1)  
  }
```

```
def fib(n: Int) : Int =  
  n match {  
    case 0 | 1 => 1  
    case _ => fib(n-1) + fib(n-2)  
  }
```


Pattern Matching with If

```
def f(n: Int) : Int =  
  n match {  
    case 0 | 1 => 1  
    case _ if (n <= 5) => 2  
    case _ => 3  
  }
```

```
def f(t: BTree) : Int =  
  t match {  
    case Leaf() => 0  
    case Node(n,_,_) if (n <= 10) => 1  
    case Node(_,_,_) => 2  
  }
```

Exercise

Write a function “find(t: BTree, x: Int) : Boolean” that checks whether x is in t.

```
sealed abstract class BTree  
case class Leaf() extends BTree  
case class Node(value: Int, left: BTree, right: BTree)  
    extends BTree
```

Solution

```
def find(t: BTree, i: Int) : Boolean =  
  t match {  
    case Leaf() => false  
    case Node(n, lt, rt) =>  
      if (i == n) true  
      else find(lt, i) || find(rt, i)  
  }
```

```
def t: BTree = Node(5, Node(4, Node(2, Leaf(), Leaf()), Leaf()),  
  Node(7, Node(6, Leaf(), Leaf()), Leaf()))
```

```
find(t, 7)
```

```
find(t, 1)
```

Solution with Tail Recursion

```
def genTree(v: Int, n: Int) : BTree = {  
  def genTreeIter(t: BTree, m : Int) : BTree =  
    m match {  
      case 0 => t  
      case _ => genTreeIter(Node(v, t, Leaf()), m-1)  
    }  
  genTreeIter(Leaf(), n)  
}  
  
def find(t: BTree, x: Int) : Boolean = {  
  def findIter(ts: BList) : Boolean =  
    ts match {  
      case BNil() => false  
      case BCons(Leaf(), t1) => findIter(t1)  
      case BCons(Node(v, l, r), t1) =>  
        if (v == x) true  
        else findIter(BCons(l, BCons(r, t1)))  
    }  
  findIter(BCons(t, BNil()))  
}  
  
find(genTree(0,10000), 1)
```

Type Checking & Inference (Concept)

What Are Types For?

➤ Typed Programming

```
def id1(x: Int): Int = x
def id2(x: Double): Double = x
```

- At run time, type information is erased (ie, `id1 = id2`)

➤ Untyped Programming

```
def id(x) = x
```

- Do not care about types at compile time.
 - But, many such languages check types at run time paying cost.
 - Without run-time type check, errors can be badly propagated.
- ## ➤ What is compile-time type checking for?
- Can detect type errors at compile time.
 - Increase Readability (Give a good abstraction).
 - Soundness: Well-typed programs raise no type errors at run time.

Type Checking and Inference

➤ Type Checking

$$x_1:T_1, x_2:T_2, \dots, x_n:T_n \vdash e : T$$

- `def f(x: Boolean): Boolean = x > 3`
=> Type error
- `def f(x: Int): Boolean = x > 3`
=> OK. `f: (x: Int)Boolean`

➤ Type Inference

$$x_1:T_1, x_2:T_2, \dots, x_n:T_n \vdash e : ?$$

- `def f(x: Int) = x > 3`
=> OK by type inference. `f: (x: Int)Boolean`
- Too much type inference is not good. Why?

You can learn how type checking & inference work in
4190.310 Programming Languages

Parametric Polymorphism

Parametric Polymorphism: Functions

➤ Problem

```
def id1(x: Int): Int = x
def id2(x: Double): Double = x
```

- Can we avoid copy and paste?
- Polymorphism to the rescue!

➤ Parametric Polymorphism (a.k.a. For-all Types)

```
def id[A](x: A) : A = x
```

- The type of `id` is `[A](val x:A)=>A`
- `id` is a parametric expression.
- `id[T] _` is a value of type `T=>T` for any type `T`.

[We will learn other kinds of polymorphism later.]

Examples

```
def id[A](x:A) = x
id(3)
id("abc")
```

```
def applyn[A](f: A => A, n: Int, x: A): A =
  n match {
    case 0 => x
    case _ => f(applyn(f, n - 1, x))
  }
```

```
applyn((x:Int)=>x+1, 100, 3)
```

```
applyn((x:String)=>x+"!", 10, "gil")
```

```
applyn(id[String], 10, "hur")
```

```
def foo[A,B](f: A=>A, x: (A,B)) : (A,B) =
  (applyn[A](f, 10, x._1), x._2)
```

```
foo[String, Int]((x:String)=>x+"!", ("abc", 10))
```

Full Polymorphism using Scala's trick

```
type Applyn = {def apply[A](f: A=>A, n: Int, x: A): A}
object applyn { // val applyn = new {
  def apply[A](f: A=>A, n: Int, x: A): A =
    n match {
      case 0 => x
      case _ => f(apply(f, n-1, x))
    }
}
applyn((x:String)=>x+"!", 10, "gil")
// applyn.apply[String]((x:String)=>x+"!", 10, "gil")

def foo(f: Applyn): String = {
  val a:String = f[String]((x:String)=> x + "!", 10, "gil")
  val b:Int = f[Int]((x:Int)=> x + 2, 10, 5)
  a + b.toString()
}
foo(applyn)
```

Summary

➤ Main points

- Definitions allow type parameters at the beginning.
(E.g.) `def id[A](x: A) : A = x`
- Values does not allow type parameters.
(E.g.) `[A](A=>A)` is not a valid value type.
- Record types allow type parameters in their “def” fields.
(E.g.) `{def apply[A](x: A): A}` is a valid value type.

➤ Minor points

- The compiler automatically inserts “`.apply`” when a record is applied to arguments.
- The function types are encoded as “class types”, which we will learn in Part II.

Parametric Polymorphism: Datatypes

```
sealed abstract class MyOption[A]  
case class MyNone[A]() extends MyOption[A]  
case class MySome[A](some: A) extends MyOption[A]
```

```
sealed abstract class MyList[A]  
case class MyNil[A]() extends MyList[A]  
case class MyCons[A](hd: A, tl: MyList[A]) extends MyList[A]
```

```
sealed abstract class BTree[A]  
case class Leaf[A]() extends BTree[A]  
case class Node[A](value: A, left: BTree[A], right: BTree[A])  
extends BTree[A]
```

```
def x: MyList[Int] = MyCons(3, MyNil())  
def y: MyList[String] = MyCons("abc", MyNil())
```

Exercise

BSTree[A] = Leaf

| Node of Int * A * BSTree[A] * BSTree[A]

```
def lookup[A](t: BSTree[A], k: Int) : MyOption[A] =  
    ???
```

```
def t : BSTree[String] =  
    Node(5, "My5",  
        Node(4, "My4", Node(2, "My2", Leaf(), Leaf()), Leaf()),  
        Node(7, "My7", Node(6, "My6", Leaf(), Leaf()), Leaf()))
```

lookup(t, 7)

lookup(t, 3)

Solution

```
sealed abstract class BSTree[A]
case class Leaf[A]() extends BSTree[A]
case class Node[A](key: Int, value: A, left: BSTree[A], right:
BSTree[A]) extends BSTree[A]
def lookup[A](t: BSTree[A], key: Int) : MyOption[A] =
  t match {
    case Leaf() => MyNone()
    case Node(k,v,l,t,r) =>
      k match {
        case _ if key == k => MySome(v)
        case _ if key < k => lookup(l,t,key)
        case _ => lookup(r,t, key)
      }
  }
def t : BSTree[String] =
  Node(5, "My5",
    Node(4, "My4", Node(2, "My2", Leaf(), Leaf()), Leaf()),
    Node(7, "My7", Node(6, "My6", Leaf(), Leaf()), Leaf()))
lookup(t, 7)
lookup(t, 3)
```

A Better Way

```
sealed abstract class BTree[A]  
case class Leaf[A]() extends BTree[A]  
case class Node[A](value: A, left: BTree[A], right: BTree[A])  
  extends BTree[A]
```

```
type BSTree[A] = BTree[(Int, A)]
```

```
def lookup[A](t: BSTree[A], k: Int) : MyOption[A] =  
  ???
```

```
def t : BSTree[String] =  
  Node((5, "My5"),  
    Node((4, "My4"), Node((2, "My2"), Leaf(), Leaf()), Leaf()),  
    Node((7, "My7"), Node((6, "My6"), Leaf(), Leaf()), Leaf()))
```

```
lookup(t, 7)
```


Solution

```
type BSTree[A] = BTree[(Int,A)]
```

```
def lookup[A](t: BTree[A], key: Int) : MyOption[A] =  
  t match {  
    case Leaf() => MyNone()  
    case Node((k,v), lt, rt) =>  
      k match {  
        case _ if key == k => MySome(v)  
        case _ if key < k => lookup(lt, key)  
        case _ => lookup(rt, key)  
      }  
  }
```

```
def t : BTree[String] =  
  Node((5, "My5"),  
    Node((4, "My4"), Node((2, "My2"), Leaf(), Leaf()), Leaf()),  
    Node((7, "My7"), Node((6, "My6"), Leaf(), Leaf()), Leaf()))
```

```
lookup(t, 7)  
lookup(t, 3)
```

Polymorphic Option (Library)

➤ Option[T]

Intro:

- None
- Some(x)
- Library functions

Elim:

- Pattern matching
- Library functions

Some(3) : Option[Int]

Some("abc"): Option[String]

None: Option[Int]

None: Option[String]

Polymorphic List (Library)

➤ List[T]

Intro:

- Nil
- $x :: L$
- Library functions

Elim:

- Pattern matching
- Library functions

`“abc”::Nil : List[String]`

`List(1,3,4,2,5) = 1::3::4::2::5::Nil : List[Int]`

PART 2

Object-Oriented Programming

Sub Type Polymorphism (Concept)

Motivation

We want:

```
object tom {  
  val name = "Tom"  
  val home = "02-880-1234"  
}
```

```
object bob {  
  val name = "Bob"  
  val mobile = "010-1111-2222"  
}
```

```
def greeting(r: ???) = "Hi " + r.name + ", How are you?"  
greeting(tom)  
greeting(bob)
```

Note that we have

```
tom: {val name: String; val home: String}  
bob: {val name: String; val mobile: String}
```

Sub Types to the Rescue!

```
type NameHome = { val name: String; val home: String }  
type NameMobile = { val name: String; val mobile: String }  
type Name = { val name: String }
```

NameHome <: Name (NameHome is a sub type of Name)

NameMobile <: Name (NameMobile is a sub type of Name)

```
def greeting(r: Name) = "Hi " + r.name + ", How are you?"  
greeting(tom)  
greeting(bob)
```

Sub Types

- The sub type relation is kind of the subset relation.
- But they are **NOT** the same.
- $T <: S$
Every element of T **can be used as** that of S.
- *Cf.* T is a subset of S.
Every element of T **is** that of S.
- Why polymorphism?
A function of type $S \Rightarrow R$ can be used as $T \Rightarrow R$ for many sub types T of S.
Note that $S \Rightarrow R <: T \Rightarrow R$ when $T <: S$.

Two Kinds of Sub Types

➤ Structural Sub Types

- The system implicitly determines the sub type relation by the structures of data types.
- Structurally equivalent types are treated the same.

➤ Nominal Sub Types

- The user explicitly specify the sub type relation using the names of data types.
- Structurally equivalent types with different names may be treated differently.

Structural Sub Types

General Sub Type Rules

- Reflexivity:

For any type T, we have:

$$T <: T$$

- Transitivity:

For any types T, S, R, we have:

$$T <: R \quad R <: S$$

=====

$$T <: S$$

Sub Types for Special Types

- Nothing: The empty set
- Any: The set of all values

- For any type T, we have:

$\text{Nothing} <: T <: \text{Any}$

- Example

```
val a : Int = 3
val b : Any = a
def f(a: Nothing) : Int = a
```

Sub Types for Records

- Permutation

$$\begin{array}{c} \text{=====} \\ \{...\; x: T1; y: T2; ...\} <: \{...\; y: T2, x: T1; ...\} \end{array}$$

- Width

$$\begin{array}{c} \text{=====} \\ \{...\; x: T; ...\} <: \{...\; ...\} \end{array}$$

- Depth

$$\begin{array}{c} T <: S \\ \text{=====} \\ \{...\; x: T ; ...\} <: \{...\; x: S; ...\} \end{array}$$

Sub Types for Records

- Example

`{val x: { val y: Int; val z: String}, val w: Int}`

`<: (by permutation)`

`{val w: Int; val x: { val y: Int; val z: String} }`

`<: (by depth & width)`

`{val w: Int; val x: {val z: String} }`

Sub Types for Tuples

- Depth

$$T <: S$$

=====

$$(\dots, T, \dots) <: (\dots, S, \dots)$$

Sub Types for Functions

- Function Sub Type

$$T <: T' \quad S <: S'$$

=====

$$(T' \Rightarrow S) <: (T \Rightarrow S')$$

- Example

```
def foo(s: {val a: Int; val b: Int}) :  
  {val x: Int; val y: Int} = {  
    object tmp {  
      val x = s.b  
      val y = s.a  
    }  
    tmp  
  }  
val gee:  
  {val a: Int; val b: Int; val c: Int} =>  
  {val x: Int} =  
  foo _
```


Classes

Class: Parameterized Record

```
type gee_type = {val name:String; val age: Int; def getPP(): String}
def gee_fun(_name: String, _age: Int) = {
  if (!(_age >= 0 && _age < 200)) throw new Exception("Out of range")
  new {
    val name : String = _name
    val age : Int = _age
    def getPP() : String = name + " of age " + age.toString() } }
```

```
class foo_type(_name: String, _age: Int) {
  if (!(_age >= 0 && _age < 200)) throw new Exception("Out of range")
  val name : String = _name
  val age : Int = _age
  def getPP() : String = name + " of age " + age.toString() }
val foo : foo_type = new foo_type("David Jones",25)
```

- use: foo.a foo.b foo.f
- foo is a value of foo_type
- gee is a value of gee_type

Class: No Structural Sub Typing

➤ Records: Structural sub-typing

`foo_type <: gee_type`

➤ Classes: Nominal sub-typing

`gee_type <: foo_type`

`val v1 : gee_type = foo`

`val v2 : foo_type = gee // type error`

```
def greet ing(r:{val name:String }) =  
  "Hi " + r.name + ", How are you?"  
greet ing(foo)
```

Structural Types vs. Nominal Types

➤ Structural Types

- Includes arbitrary values with the required structures as elements
- Allows arbitrary types with the required structures as sub types
- Cannot assume any properties on their elements

➤ Nominal Types

- Includes only specific values as elements
- Allows only specific types as sub types
- Can assume specific properties on their elements

Class: Can be Recursive!

```
class MyList[A](v: A, nxt: Option[MyList[A]]) {  
  val value : A = v  
  val next : Option[MyList[A]] = nxt  
}  
type YourList[A] = Option[MyList[A]]  
  
val t : YourList[Int] =  
  Some(new MyList(3, Some (new MyList(4, None))))
```

Note on Null value

- `null`: The special element of every class & structural type
- This value is needed to construct disjoint union types using classes in Java, which, however, is not as elegant and type safe as algebraic data types (ADTs):
 - Such disjoint union types can contain junk values (not elegant).
 - Null-point exception can be raised at run time (not type safe).
- For this reason, it is discouraged to use `null` in Scala although Scala supports `null` for compatibility with Java.
- Instead, it is encouraged to use ADTs, which themselves are classes and thus take advantages of both ADT and class.

Simplification using Argument Members

```
class MyList[A](v: A, nxt: Option[MyList[A]]) {  
  val value : A = v  
  val next : MyList[A] = nxt  
}
```

```
class MyList[A](val value:A, val next:Option[MyList[A]]) {  
}
```

```
class MyList[A](val value:A, val next:Option[MyList[A]])
```

Simplification using Companion Object

```
class MyList[A](v: A, nxt: Option[MyList[A]]) {  
  val value = v  
  val next = nxt  
}
```

```
object MyList // val MyList = new  
{ def apply[A](v: A, nxt: Option[MyList[A]]) =  
  new MyList(v,nxt)  
}
```

```
type YourList[A] = Option[MyList[A]]
```

```
val t0 = None
```

```
val t1 = Some(new MyList(3, Some (new MyList(4, None))))
```

```
val t2 = Some(MyList(3, Some (MyList(4, None))))
```


Exercise

Define a class “MyTree[A]” for binary trees:

```
MyTree[A] =  
  (value: A) *  
  (left: Option[MyTree[A]]) *  
  (right: Option[MyTree[A]])
```

Solution

```
class MyTree[A](v: A,  
                lt: Option[MyTree[A]],  
                rt: Option[MyTree[A]]) {  
    val value = v  
    val left = lt  
    val right = rt  
}
```

```
type YourTree[A] = Option[MyTree[A]]
```

```
val t0 : YourTree[Int] = None
```

```
val t1 : YourTree[Int] = Some(new MyTree(3, None, None))
```

```
val t2 : YourTree[Int] =  
    Some(new MyTree(3, Some (new MyTree(4, None, None)), None))
```

Nominal Sub Typing for Classes

Nominal Sub Typing, a.k.a. Inheritance

```
class foo_type(x: Int, y: Int) {  
  val a : Int = x  
  def b : Int = a + y  
  def f(z: Int) : Int = b + y + z  
}
```

```
class gee_type(x: Int) extends foo_type(x+1,x+2) {  
  val c : Int = f(x) + b  
}
```

```
gee_type <: foo_type
```

```
(new gee_type(30)).c  
def test(f: foo_type) = f.a + f.b  
test(new foo_type(10,20))  
test(new gee_type(30))
```

Overriding 1

```
class foo_type(x: Int, y: Int) {  
  val a : Int = x  
  def b : Int = a + y  
  def f(z: Int) : Int = b + y + z  
}  
class gee_type(x: Int) extends foo_type(x+1,x+2) {  
  override def f(z: Int) = b + z  
  // or, override def f(z: Int) = super.f(z) * 2  
  val c : Int = f(x) + b  
}  
(new gee_type(30)).c
```

Q: Can we override with a different type?

```
override def f(z: String): Int = 77    //No, arg: diff type  
def f(z: String): Int = 77             //Yes, arg: diff type  
override def f(z: Int): Nothing = ???  //Yes, ret: sub type
```

Overriding 2

```
class foo_type(x: Int, y: Int) {  
  val a : Int = x  
  def b : Int = a + y  
  def f(z: Int) : Int = b + y + z  
}
```

```
class gee_type(x: Int) extends foo_type(x+1,x+2) {  
  override def b = 10  
}
```

```
(new gee_type(30)).f(0)
```

Example: My List

```
class MyList[A]()  
class MyNil[A]() extends MyList[A]  
class MyCons[A](val hd: A, val tl: MyList[A])  
    extends MyList[A]  
val t: MyList[Int] =  
    new MyCons(3, (new MyCons(4, new MyNil()))))
```

Example: MyList

```
class MyList[A]
```

```
class MyNil[A]() extends MyList[A]
```

```
object MyNil { def apply[A]() = new MyNil[A]() }
```

```
class MyCons[A](val hd: A, val tl: MyList[A])  
  extends MyList[A]
```

```
object MyCons {  
  def apply[A](hd:A, tl:MyList[A]) = new MyCons[A](hd, tl)}
```

```
val t: MyList[Int] = MyCons(3, MyNil())
```

```
def length(x: MyList[Int]) = ???
```


Case Class

```
class MyList[A]() { ... }
```

```
case class MyNil[A]() extends MyList[A] { ... }
```

```
object MyNil { def apply[A]() = new MyNil[A]() }
```

```
case class MyCons[A](val hd: A, val tl: MyList[A])  
  extends MyList[A] { ... }
```

```
object MyCons {
```

```
  def apply[A](hd:A, tl:MyList[A]) = new MyCons[A](hd, tl)}
```

```
val t: MyList[Int] = MyCons(3, MyNil())
```

+ Pattern Matching

Cf. sealed abstract class MyList[A]

Exercise

Define “MyTree[A]” using sub class.

```
class MyTree[A](v: A,  
                lt: Option[MyTree[A]],  
                rt: Option[MyTree[A]]) {  
  val value = v  
  val left = lt  
  val right = rt  
}  
  
type YourTree[A] = Option[MyTree[A]]
```

Solution

```
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A,
                  left: MyTree[A],
                  right: MyTree[A])
    extends MyTree[A]

val t : MyTree[Int] =
    Node(3, Node(4, Empty(), Empty()), Empty())

t match {
  case Empty() => 0
  case Node(v, l, r) => v
}
```

Abstract Classes for Specification

Abstract Class: Specification

➤ Abstract Classes

- Can be used to abstract away the implementation details.

Abstract classes for Specification
Concrete sub-classes for Implementation

Abstract Class: Specification

➤ Example Specification

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =  
  xs.getValue match {  
    case None => 0  
    case Some(n) => f(n) + sumElements(f)(xs.getNext)  
  }
```

```
def sumElementsId(xs: Iter[Int]) =  
  sumElements((x: Int)=>x)(xs)
```

Concrete Class: Implementation

```
sealed abstract class MyList[A] extends Iter[A]
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
}
case class MyCons[A](hd: A, tl: MyList[A])
  extends MyList[A]
{
  def getValue = Some(hd)
  def getNext = tl
}
```

```
val t1 = MyCons(3, MyCons(5, MyCons(7, MyNil())))
```

```
sumElementsId(t1)
```

Exercise

Define `IntCounter(n)` that implements the specification `Iter[A]`.

```
class IntCounter(n: Int) extends Iter[Int] {  
  def getValue = ???  
  def getNext = ???  
}
```

```
sumElementsId(new IntCounter(100))
```


Solution

Define `IntCounter(n)` that implements the specification `Iter[A]`.

```
class IntCounter(n: Int) extends Iter[Int] {  
  def getValue = if (n >= 0) Some(n) else None  
  def getNext = new IntCounter(n-1)  
}
```

```
sumElementsId(new IntCounter(100))
```

More on Abstract Classes

Problem: Iter for MyTree

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
sealed abstract class MyTree[A]  
case class Empty[A]() extends MyTree[A]  
case class Node[A](value: A,  
                   left: MyTree[A],  
                   right: MyTree[A]) extends MyTree[A]
```

Q: Can MyTree[A] implement Iter[A]?

Try it, but it is not easy.

Solution: Better Specification

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
abstract class Iterable[A] {  
  def iter : Iter[A]  
}
```

```
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =  
  xs.getValue match {  
    case None => 0  
    case Some(n) => f(n) + sumElements(f)(xs.getNext)  
  }
```

```
def sumElementsGen[A](f: A=>Int)(xs: Iterable[A]) : Int =  
  sumElements(f)(xs.iter)
```

Let's Use MyList

```
sealed abstract class MyList[A] extends Iter[A]
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
}
case class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A] {
  def getValue = Some(hd)
  def getNext = tl
}
```

MyTree <: Iterable (Try)

```
sealed abstract class MyTree[A] extends Iterable[A]
```

```
case class Empty[A]() extends MyTree[A] {  
  val iter = MyNil()  
}
```

```
case class Node[A](value: A,  
                  left: MyTree[A],  
                  right: MyTree[A]) extends MyTree[A] {  
  // "val iter" is more specific than "def iter",  
  // so it can be used in a sub type.  
  // In this example, "val iter" is also  
  // more efficient than "def iter".  
  val iter = MyCons(value, ???(left.iter, right.iter))  
}
```

Extend MyList with append

```
sealed abstract class MyList[A] extends Iter[A] {
  def append(lst: MyList[A]) : MyList[A]
}

case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
  def append(lst: MyList[A]) = lst
}

case class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A]
{
  def getValue = Some(hd)
  def getNext = tl
  def append(lst: MyList[A]) = MyCons(hd, tl.append(lst))
}
```

Note: tail-recursive “append”

```
sealed abstract class MyList[A] extends Iter[A] {
  def append(lst: MyList[A]) : MyList[A] =
    MyList.revAppend(MyList.revAppend(this, MyNil()), lst)
}

object MyList {
  // Tail-recursive functions should be written in “object”
  def revAppend[A](lst1: MyList[A], lst2: MyList[A]): MyList[A] =
    lst1 match {
      case MyNil() => lst2
      case MyCons(hd, tl) => revAppend(tl, MyCons(hd, lst2))
    }
}

case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
}

case class MyCons[A](val hd:A, val tl:MyList[A]) extends MyList[A] {
  def getValue = Some(hd)
  def getNext = tl
}
```


MyTree <: Iterable

```
sealed abstract class MyTree[A] extends Iterable[A] {  
  def iter : MyList[A]  
  // Note:  
  // def iter : Int // Type Error because not (Int <: Iter[A])  
}  
case class Empty[A]() extends MyTree[A] {  
  val iter = MyNil()  
}  
case class Node[A](value: A,  
                   left: MyTree[A],  
                   right: MyTree[A]) extends MyTree[A] {  
  def iter = MyCons(value, left.iter.append(right.iter))  
  // def iter = left.iter.append(MyCons(value, right.iter))  
  // def iter = left.iter.append(right.iter.append(  
  //   MyCons(value, MyNil())))  
}
```

Test

```
def generateTree(n: Int) : MyTree[Int] = {  
  def gen(lo: Int, hi: Int) : MyTree[Int] =  
    if (lo > hi) Empty()  
    else {  
      val mid = (lo+hi)/2  
      Node(mid, gen(lo,mid-1), gen(mid+1,hi))  
    }  
  gen(1,n)  
}
```

```
sumElementsGen((x: Int) => x)(generateTree(100))
```

Problem: Inefficiency

```
def time[R](block: => R): R = {  
  val t0 = System.nanoTime()  
  val result = block    // call-by-name  
  val t1 = System.nanoTime()  
  println("Elapsed time: " + ((t1 - t0)/1000000) + "ms"); result  
}  
  
def sumN[A](f: A=>Int)(n: Int, xs: Iterable[A]) : Int = {  
  def sumIter(res : Int, n: Int, xs: Iter[A]) : Int =  
    if (n <= 0) res  
    else xs.getValue match {  
      case None => res  
      case Some(v) => sumIter(f(v) + res, n-1, xs.getNext)  
    }  
  sumIter(0, n, xs.iter)  
}  
  
// Problem: takes a few seconds to get a single value  
{ val t: MyTree[Int] = generateTree(2000000)  
  time (sumN((x: Int) => x)(1, t)) }
```

Solution: Lazy List

```
abstract class LazyList[A] extends Iter[A] {
  def head : Option[A]
  def tail : LazyList[A]
  def append(lst: LazyList[A]) : LazyList[A]
  def getValue = head
  def getNext = tail
}

case class LNil[A]() extends LazyList[A] {
  val head = None
  val tail = this
  def append(lst: LazyList[A]) = lst
}

class LCons[A](ht: =>(A, LazyList[A])) extends LazyList[A] {
  lazy val (hd, tl) = ht
  def head = Some(hd)
  def tail = tl
  def append(lst: LazyList[A]) = LCons(hd, tl.append(lst))
}

object LCons { def apply[A](ht: =>(A, LazyList[A])) = new LCons(ht) }
```

Note: “append” is not recursive!!!

Lazy Iteration using LazyList

```
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : LazyList[A]
}
case class Empty[A]() extends MyTree[A] {
  val iter = LNil()
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A] {
  lazy val iter = LCons(value, left.iter.append(right.iter))
  // lazy val iter = left.iter.append(LCons(value, right.iter))
  // lazy val iter = left.iter.append(right.iter.append(
  //   LCons(value, LNil())))
}
{ val t: MyTree[Int] = generateTree(2000000)
  time (sumN((x: Int) => x)(100, t))
  time (sumN((x: Int) => x)(1000000, t))
}
```

Note: “i t e r” is not recursive!!!

Comparison: Using Lists of Trees

```
class MyTreeIter[A](val lst: MyList[MyTree[A]]) extends Iter[A] {
  val valAndNext : (Option[A], MyList[MyTree[A]]) = lst match {
    case MyNil() => (None, MyNil())
    case MyCons(hd, tl) => hd match {
      case Empty() => throw new Exception("...")
      case Node(v, Empty(), Empty()) => (Some(v), tl)
      case Node(v, lt, Empty()) => (Some(v), MyCons(lt, tl))
      case Node(v, Empty(), rt) => (Some(v), MyCons(rt, tl))
      case Node(v, lt, rt) => (Some(v), MyCons(lt, MyCons(rt, tl)))
    }
  }
  val getValue = valAndNext._1
  def getNext = new MyTreeIter(valAndNext._2)
}
```

Comparison: Using Lists of Trees

```
sealed abstract class MyTree[A] extends Iterable[A]
case class Empty[A]() extends MyTree[A] {
  val iter = new MyTreeIter(MyNil())
}
case class Node[A](value: A,
                  left: MyTree[A],
                  right: MyTree[A]) extends MyTree[A]
{
  val iter = new MyTreeIter(MyCons(this, MyNil()))
}

{ val t: MyTree[Int] = generateTree(2000000)
  time (sumN((x: Int) => x)(100, t))
  time (sumN((x: Int) => x)(1000000, t))
}
```

Iter <: Iterable

```
abstract class Iterable[A] {  
  def iter : Iter[A]  
}
```

```
abstract class Iter[A] extends Iterable[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
  def iter = this  
}
```

```
val lst : MyList[Int] =  
  MyCons(3, MyCons(4, MyCons(2, MyNil())))
```

```
sumElementsGen ((x:Int)=>x)(lst)
```


Wrapper for Inheritance

Using a Wrapper Class

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
class ListIter[A](val list: List[A]) extends Iter[A] {  
  def getValue = list.headOption  
  def getNext = new ListIter(list.tail)  
}
```

```
sumElements((x: Int) => x)(new ListIter(List(1, 2, 3, 4)))
```

MyTree Using ListIter

```
abstract class Iterable[A] {  
  def iter : Iter[A]  
}  
sealed abstract class MyTree[A] extends Iterable[A] {  
  override def iter : ListIter[A]  
}  
case class Empty[A]() extends MyTree[A] {  
  val iter : ListIter[A] = new ListIter(Ni)  
}  
case class Node[A](value: A,  
                   left: MyTree[A],  
                   right: MyTree[A])  
  extends MyTree[A] {  
  val iter : ListIter[A] = new ListIter(  
    value::(left.iter.list ++ right.iter.list))  
}
```

Test

```
val t : MyTree[Int] =  
  Node(3, Node(4, Node(2, Empty(), Empty()),  
    Node(3, Empty(), Empty()))),  
  Node(5, Empty(), Empty()))  
  
sumElementsGen((x: Int) => x)(t)
```

Abstract Class With Abstract Types

Using an Abstract Type

```
abstract class Iterable[A] {  
  type iter_t  
  def iter: iter_t  
  def getValue(i: iter_t) : Option[A]  
  def getNext(i: iter_t) : iter_t  
}  
  
def sumElements[A](f:A=>Int)(xs: Iterable[A]) : Int = {  
  def sumElementsIter(i: xs.iter_t) : Int =  
    xs.getValue(i) match {  
      case None => 0  
      case Some(n) => f(n) + sumElementsIter(xs.getNext(i))  
    }  
  sumElementsIter(xs.iter)  
}
```

MyTree Using ListIter

```
sealed abstract class MyTree[A] extends Iterable[A] {
  type iter_t = List[A]
  def getValue(i: List[A]): Option[A] = i.headOption
  def getNext(i: List[A]): List[A] = i.tail
}

case class Empty[A]() extends MyTree[A] {
  val iter: List[A] = Nil
}

case class Node[A](value: A,
                  left: MyTree[A], right: MyTree[A])
  extends MyTree[A] {
  val iter = value :: (left.iter ++ right.iter) //Pre-order
  //val iter = left.iter ++ (value :: right.iter) // In-order
  //val iter = left.iter ++ (right.iter ++ List(value))
  //Post-order
}
```

Test

```
val t : MyTree[Int] =  
  Node(3, Node(4, Node(2, Empty(), Empty()),  
    Node(3, Empty(), Empty()))),  
  Node(5, Empty(), Empty()))  
  
sumElements((x: Int) => x)(t)
```


Abstract Class with Arguments

Abstract Class with Arguments

```
abstract class IterableHE[A](eq: (A,A) => Boolean)
  extends Iterable[A]
{
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
}
```

MyTree

```
sealed abstract class MyTree[A](eq: (A, A) => Boolean)
  extends IterableHE[A](eq) {
    type iter_t = List[A]
    def getValue(i: List[A]) : Option[A] = i.headOption
    def getNext(i: List[A]) : List[A] = i.tail
  }

case class Empty[A](eq: (A, A) => Boolean)
  extends MyTree[A](eq) {
    val iter : List[A] = Nil
  }

case class Node[A](eq: (A, A) => Boolean,
                  value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A](eq) {
    val iter : List[A] = value :: (left.iter ++ right.iter)
  }
```

Test

```
val leq = (x: Int, y: Int) => x == y
```

```
val lEmpty = Empty(leq)
```

```
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =  
  Node(leq, n, t1, t2)
```

```
val t : MyTree[Int] =  
  lNode(3, lNode(4, lNode(2, lEmpty, lEmpty),  
                    lNode(3, lEmpty, lEmpty)),  
        lNode(5, lEmpty, lEmpty))
```

```
sumElements((x: Int) => x)(t)
```

```
t.hasElement(5)
```

```
t.hasElement(10)
```

Alternatively, Argument Elimination

```
abstract class IterableHE[A]
  extends Iterable[A]
{
  def eq(a:A, b:A) : Boolean
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
}
```

MyTree

```
sealed abstract class MyTree[A] extends Iterable[A] {  
  type iter_t = List[A]  
  def getValue(i : List[A]) : Option[A] = i.headOption  
  def getNext(i: List[A]) : List[A] = i.tail  
}  
  
case class Empty[A](_eq: (A,A)=>Boolean) extends MyTree[A] {  
  def eq(a:A, b:A) = _eq(a,b)  
  val iter : List[A] = Nil  
}  
  
case class Node[A](_eq: (A,A)=>Boolean,  
                  value: A, left: MyTree[A], right: MyTree[A])  
  extends MyTree[A] {  
  def eq(a:A, b:A) = _eq(a,b)  
  val iter : List[A] = value :: (left.iter ++ right.iter)  
}
```

Test

```
val leq = (x: Int, y: Int) => x == y
```

```
val lEmpty = Empty(leq)
```

```
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =  
  Node(leq, n, t1, t2)
```

```
val t : MyTree[Int] =  
  lNode(3, lNode(4, lNode(2, lEmpty, lEmpty),  
                    lNode(3, lEmpty, lEmpty)),  
        lNode(5, lEmpty, lEmpty))
```

```
sumElements((x: Int) => x)(t)
```

```
t.hasElement(5)
```

```
t.hasElement(10)
```

More on Classes

Motivating Example

```
class Primes(val prime: Int, val primes: List[Int]) {  
  def getNext: Primes = {  
    val p = computeNextPrime(prime + 2)  
    new Primes(p, primes ++ (p :: Nil))  
  }  
  def computeNextPrime(n: Int) : Int =  
    if (primes.forall((p: Int) => n%p != 0)) n  
    else computeNextPrime(n+2)  
}
```

```
def nthPrime(n: Int): Int = {  
  def go(primes: Primes, k: Int): Int =  
    if (k <= 1) primes.prime  
    else go(primes.getNext, k - 1)  
  if (n == 0) 2 else go(new Primes(3, List(3)), n)  
}  
nthPrime(10000)
```

Multiple Constructors

```
class Primes(val prime: Int, val primes: List[Int]) {  
  def this() = this(3, List(3))  
  def getNext: Primes = {  
    val p = computeNextPrime(prime + 2)  
    new Primes(p, primes ++ (p :: Nil))  
  }  
  def computeNextPrime(n: Int) : Int =  
    if (primes.forall((p: Int) => n%p != 0)) n  
    else computeNextPrime(n+2)  
}
```

```
def nthPrime(n: Int): Int = {  
  def go(primes: Primes, k: Int): Int =  
    if (k <= 1) primes.prime  
    else go(primes.getNext, k - 1)  
  if (n == 0) 2 else go(new Primes, n)  
}  
nthPrime(10000)
```

Access Modifiers

- Access Modifiers
 - Private: Only the class can access the member.
 - Protected: Only the class and its sub classes can access the member.

Using Access Modifiers

```
class Primes private (val prime: Int, protected val primes: List[Int])  
{  
  def this() = this(3, List(3))  
  def getNext: Primes = {  
    val p = computeNextPrime(prime + 2)  
    new Primes(p, primes ++ (p :: Nil))  
  }  
  private def computeNextPrime(n: Int) : Int =  
    if (primes.forall((p: Int) => n%p != 0)) n  
    else computeNextPrime(n+2)  
}
```

```
def nthPrime(n: Int): Int = {  
  def go(primes: Primes, k: Int): Int =  
    if (k <= 1) primes.prime  
    else go(primes.getNext, k - 1)  
  if (n == 0) 2 else go(new Primes, n)  
}  
nthPrime(10000)
```

Using Specification

➤ OOP-style Specification

```
abstract class PrimesSig {  
  // Problem: need an instance of PrimesSig to make a new instance  
  def makeNew : PrimesSig  
  def prime : Int  
  def getNext : PrimesSig  
}
```

➤ Type class-style Specification

```
abstract class PrimesSig[P] {  
  def makeNew : P  
  def prime(p: P) : Int  
  def getNext(p: P) : P  
}  
  
def nthPrime[P](tc: PrimesSig[P])(n: Int): Int = {  
  def go(p: P, k: Int): Int =  
    if (k <= 1) tc.prime(p) else go(tc.getNext(p), k - 1)  
  if (n == 0) 2 else go(tc.makeNew, n)  
}
```

Implementation

```
class Primes private (val prime: Int, protected val primes: List[Int])
{
  def this() = this(3, List(3))
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  private def computeNextPrime(n: Int) : Int =
    if (primes.forall((p: Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

class PrimesImpl extends PrimesSig[Primes] {
  def makeNew = new Primes
  def prime(p: Primes) = p.prime
  def getNext(p: Primes) = p.getNext
}

val primesTC = new PrimesImpl

nthPrime(primesTC)(10000)
```

Traits for Multiple Inheritance

Multiple Inheritance Problem

➤ Multiple Inheritance

- The famous “diamond problem”

```
class A(val a: Int)
class B extends A(10)
class C extends A(20)
class D extends B, C.
```

Problem 1: What is the value of (new D).a ?

Problem 2: The constructor of A must be executed once because A may contain side effects such as sending messages over the network.

Java's Solution: Interface

➤ Interface

- An interface cannot contain any implementation but only types of its methods.
- A class can inherit implementations from only one parent class but implement multiple interfaces.

Scala's Solution: Trait

➤ Traits

- A trait can implement any of its methods, but should have only one constructor with no arguments.
- A trait can
 - “extend” only one trait or (abstract) class with no arguments
 - “with” multiple traits.
- An (abstract) class can
 - “extend” only one trait or (abstract) class with any arguments
 - “with” multiple traits.

➤ Property

- Among the ancestors of a class, for the same class:
 - A constructor with arguments can appear at most once
 - A constructor with no argument can appear multiple times

Example

```
class A(val a : Int) {  
  def this () = this(0)  
}  
trait B {  
  def f(x: Int): Int = x  
}  
trait C extends A with B {  
  def g(x: Int): Int = x + a  
}  
trait D extends B {  
  def h(x: Int): Int = f(x + 50)  
}  
class E extends A(10) with C with D {  
  override def f(x: Int) = x * a  
}  
  
val e = new E
```

Algorithm for Multiple Inheritance

➤ Algorithm

- Give a linear order among all ancestors by “post-order” traversing without revisiting the same node.
- Invoke the constructors once in that order.

Note. Post-order traversal of a class C means

- Recursively post-order traverse C’s first parent; ...;
- Recursively post-order traverse C’s last parent; and
- Visit C.

By post-order traversing from “E” in the previous example, we have the order: A(10) → B → C → D → E

```
val e = new E
```

```
e.a // 10
```

```
e.f(100) // 100*10
```

```
e.g(100) // 100 + 10
```

```
e.h(100) // (100 + 50) * 10
```

- A constructor with arguments is always visited before the same constructor with no arguments.
- Compile error if the same field is implemented by multiple classes

A Simple Example With Traits

Motivation

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
class ListIter[A](val list: List[A]) extends Iter[A] {  
  def getValue = list.headOption  
  def getNext = new ListIter(list.tail)  
}
```

```
abstract class Dict[K,V] {  
  def add(k: K, v: V): Dict[K,V]  
  def find(k: K): Option[V]  
}
```

Q: How can we extend ListIter and implement Dict?

Specification using Traits

```
// abstract class Dict[K,V] {  
//   def add(k: K, v: V): Dict[K,V]  
//   def find(k: K): Option[V] }
```

```
trait Dict[K,V] {  
  def add(k: K, v: V): Dict[K,V]  
  def find(k: K): Option[V]  
}
```

Implementing Traits

```
class ListIterDict[K,V]  
  (eq: (K,K)=>Boolean, list: List[(K,V)])  
  extends ListIter[(K,V)](list)  
    with Dict[K,V]  
{  
  def add(k:K,v:V) = new ListIterDict(eq,(k,v)::list)  
  def find(k: K) : Option[V] = {  
    def go(l: List[(K, V)]): Option[V] = l match {  
      case Nil => None  
      case (k1, v1) :: tl =>  
        if (eq(k, k1)) Some(v1) else go(tl) }  
    go(list) }  
}
```


Test

```
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =  
  xs.getValue match {  
    case None => 0  
    case Some(n) => f(n) + sumElements(f)(xs.getNext)  
  }
```

```
def find3(d: Dict[Int,String]) = {  
  d.find(3)  
}
```

```
val d0 = new ListIterDict[Int,String]((x,y)=>x==y,Nil)  
val d = d0.add(4,"four").add(3,"three")
```

```
sumElements[(Int,String)](x=>x._1)(d)  
find3(d)
```

Mixin with Traits

Motivation: Mixin Functionality

```
abstract class Iter[A] {  
  def getValue: Option[A]  
  def getNext: Iter[A]  
}
```

```
class ListIter[A](list: List[A]) extends Iter[A]  
{  
  def getValue = list.headOption  
  def getNext: Iter[A] = new ListIter(list.tail)  
}
```

```
trait MRIter[A] extends Iter[A] {  
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = ???  
}
```

Mixin Composition

```
trait MRIter[A] extends Iter[A] {  
  override def getNext: MRIter[A]  
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C =  
    getValue match {  
      case None => ival  
      case Some(v) =>  
        combine(f(v), getNext.mapReduce(combine, ival, f))  
    }  
}
```

```
class MRListIter[A](list: List[A])  
  extends ListIter (list) with MRIter[A]  
{  
  override def getNext: MRIter[A] = new MRListIter(list.tail)  
}
```

```
val mr = new MRListIter[Int](List(3,4,5))  
mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Mixin Composition: A Better Way

```
trait MRIter[A] extends Iter[A] {  
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = {  
    def go(c: Iter[A]): C = c.getValue match {  
      case None => ival  
      case Some(v) => combine(f(v), go(c.getNext))  
    }  
    go(this)  
  }  
}
```

```
class MRListIter[A](list: List[A])  
  extends ListIter(list) with MRIter[A]
```

```
val mr = new MRListIter[Int](List(3,4,5))
```

```
// or, val mr = new ListIter(List(3,4,5)) with MRIter[Int]
```

```
mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Syntactic Sugar: new A with B with C { ... }

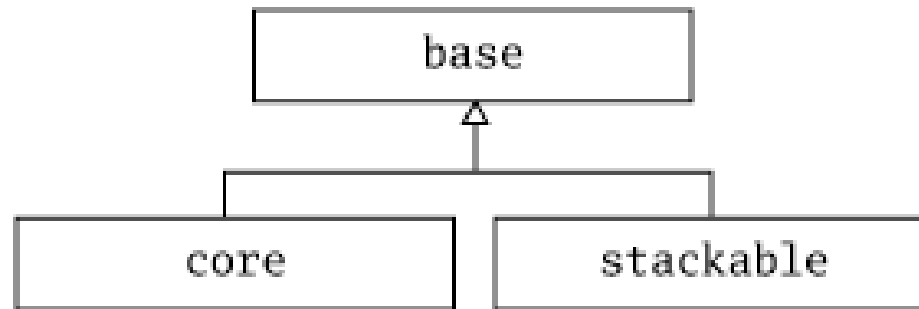
```
new A with B1 ... with Bm {  
  code  
}
```

is equivalent to

```
{  
  class _tmp_ extends A with B1 ... with Bm {  
    code  
  }  
  new _tmp_  
}
```

Stacking with Traits

Typical Hierarchy in Scala



- **BASE**
Interface (trait or abstract class)
- **CORE**
Functionality (trait or concrete class)
- **CUSTOM**
Modifications (each in a separate, composable trait)

IntStack: Base

➤BASE

```
trait IntStack {  
  def get(): (Int, IntStack)  
  def put(x: Int): IntStack  
}
```

IntStack: Core

➤CORE

```
class BasicIntStack protected (xs: List[Int]) extends IntStack
{
  override val toString = "Stack:" + xs.toString
  def this() = this(Nil)
  protected def mkStack(xs: List[Int]): IntStack =
    new BasicIntStack(xs)
  def get(): (Int, IntStack) = (xs.head, mkStack(xs.tail))
  def put(x: Int): IntStack = mkStack(x :: xs)
}
```

```
val s0 = new BasicIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

IntStack: Custom Modifications

➤CUSOM

```
trait Doubling extends IntStack {  
  abstract override def put(x: Int): IntStack = super.put(2 * x)  
}
```

```
trait Incrementing extends IntStack {  
  abstract override def put(x: Int): IntStack = super.put(x + 1)  
}
```

```
trait Filtering extends IntStack {  
  abstract override def put(x: Int): IntStack =  
    if (x >= 0) super.put(x) else this  
}
```

IntStack: Stacking

➤ Stacking

```
class DIFIntStack protected (xs: List[Int])  
  extends BasicIntStack(xs)  
  with Doubling with Incrementing with Filtering  
{  
  def this() = this(Nil)  
  override def mkStack(xs: List[Int]): IntStack =  
    new DIFIntStack(xs)  
}
```

```
val s0 = new DIFIntStack  
val s1 = s0.put(3)  
val s2 = s1.put(-2)  
val s3 = s2.put(4)  
val (v1,s4) = s3.get()  
val (v2,s5) = s4.get()
```

Intersection Types

Intersection Types

➤ Typing Rule

$$\frac{t : T1 \quad t : T2}{t : T1 \text{ with } T2}$$

➤ Example

```
trait A { val a: Int = 0 }  
trait B { val b: Int = 0 }  
class C extends A with B {  
  override val a = 10  
  override val b = 20  
  val c = 30  
}
```

```
val x = new C  
val y: A with B = x  
y.a // 10  
y.b // 20  
y.c // type error
```

Subtype Relation for “with”

The subtype relation for “with” is structural.

- Permutation

=====

$$\dots \text{ with } T1 \text{ with } T2 \dots <: \dots \text{ with } T2 \text{ with } T1 \dots$$

- Width

=====

$$\dots \text{ with } T \dots <: \dots \dots$$

- Depth

=====

$$T <: S$$

=====

$$\dots \text{ with } T \dots <: \dots \text{ with } S \dots$$

Additional Resources

➤ Traits

- <http://www.scala-lang.org/old/node/126>

➤ Mixin Composition

- <http://www.scala-lang.org/old/node/117>

➤ Stackable Trait Pattern

- http://www.artima.com/scalazine/articles/stackable_trait_pattern.html

➤ Multiple Inheritance via Traits

- <https://www.safaribooksonline.com/blog/2013/05/30/traits-how-scala-tames-multiple-inheritance/>

➤ UCSD CSE 130

- <http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/02-classes.html>

PART 3

Type Classes for Specifications

Specifications over Parameter Types

Subtype Polymorphism

```
trait Ord {  
  // this cmp that < 0 iff this < that  
  // this cmp that > 0 iff this > that  
  // this cmp that == 0 iff this == that  
  def cmp(that: Ord): Int  
  
  def ==(that: Ord): Boolean = (this.cmp(that)) == 0  
  def < (that: Ord): Boolean = (this.cmp(that) < 0  
  def > (that: Ord): Boolean = (this.cmp(that) > 0  
  def <= (that: Ord): Boolean = (this.cmp(that) <= 0  
  def >= (that: Ord): Boolean = (this.cmp(that) >= 0  
}  
  
def max3(a: Ord, b: Ord, c: Ord) : Ord =  
  if (a <= b) { if (b <= c) c else b }  
  else      { if (a <= c) c else a }
```

* Problem: hard (almost impossible) to define `OrdInt <: Ord`

Specification over Parameter Types

```
trait Ord[A] {  
  def cmp(that: Ord[A]): Int  
  def getValue : A  
  
  def ==(that: Ord[A]): Boolean = (this.cmp(that)) == 0  
  def < (that: Ord[A]): Boolean = (this.cmp that) < 0  
  def > (that: Ord[A]): Boolean = (this.cmp that) > 0  
  def <= (that: Ord[A]): Boolean = (this.cmp that) <= 0  
  def >= (that: Ord[A]): Boolean = (this.cmp that) >= 0  
}  
  
def max3[A](a: Ord[A], b: Ord[A], c: Ord[A]) : Ord[A] =  
  if (a <= b) { if (b <= c) c else b }  
  else      { if (a <= c) c else a }  
  
class OInt(val getValue : Int) extends Ord[Int] {  
  def cmp(that: Ord[Int]) = getValue - that.getValue  
}  
  
max3(new OInt(3), new OInt(2), new OInt(10)).getValue
```

Further example: Ordered Bag

```
class Bag[U, A <: Ord[U]] protected (val toList: List[A]) {  
  def this() = this(Nil)  
  def add(x: A) : Bag[U,A] = {  
    def go(elmts: List[A]): List[A] =  
      elmts match {  
        case Nil => x :: Nil  
        case e :: _ if (x < e) => x :: elmts  
        case e :: _ if (x == e) => elmts  
        case e :: rest => e :: go(rest)  
      }  
    new Bag(go(toList))  
  }  
}  
  
val emp = new Bag[Int,0Int]()  
val b = emp.add(new 0Int(3)).add(new 0Int(2)).add(new 0Int(10))  
b.toList.map((x)=>x.getValue)
```

Works, but Very Awkward!!!

Type Classes

Completely Separating Ord from Int

```
abstract class Ord[A] {  
  def cmp(me: A, you: A): Int  
  
  def ==(me: A, you: A): Boolean = cmp(me, you) == 0  
  def < (me: A, you: A): Boolean = cmp(me, you) < 0  
  def > (me: A, you: A): Boolean = cmp(me, you) > 0  
  def <= (me: A, you: A): Boolean = cmp(me, you) <= 0  
  def >= (me: A, you: A): Boolean = cmp(me, you) >= 0  
}  
  
def max3[A](a: A, b: A, c: A)(implicit ord: Ord[A]) : A =  
  if (ord.<=(a, b)) {if (ord.<=(b, c)) c else b }  
  else {if (ord.<=(a, c)) c else a }  
  
implicit val intOrd : Ord[Int] = new Ord[Int] {  
  def cmp(me: Int, you: Int) = me - you }
```

max3(3, 2, 10) // 10

Implicit

➤ Implicit

- An argument is given “implicitly”

```
def foo(s: String)(implicit t: String) = s + t
```

```
implicit val exclamation : String = "!!!!!!"
```

```
foo("Hi")
```

```
foo("Hi")("???) // can give it explicitly
```


Bag Example

```
class Bag[A] protected (val toList: List[A])(implicit ord: Ord[A])  
{ def this()(implicit ord: Ord[A]) = this(Nil)(ord)
```

```
  def add(x: A) : Bag[A] = {  
    def go(elmts: List[A]) : List[A] =  
      elmts match {  
        case Nil => x :: Nil  
        case e :: _ if (ord.<(x,e)) => x :: elmts  
        case e :: _ if (ord.==(x,e)) => elmts  
        case e :: rest => e :: go(rest)  
      }  
    new Bag(go(toList))  
  }  
}
```

```
implicit val intOrd : Ord[Int] = new Ord[Int] {  
  def cmp(me: Int, you: Int) = me - you }
```

```
(new Bag[Int]()).add(3).add(2).add(3).add(10).toList
```

Bootstrapping Implicits

// lexicographic order

```
implicit def tupOrd[A, B](implicit ordA: Ord[A], ordB: Ord[B])
  : Ord[(A, B)] =
  new Ord[(A, B)] {
    def cmp(me: (A, B), you: (A, B)) : Int = {
      val c1 = ordA.cmp(me._1, you._1)
      if (c1 != 0) c1
      else { ordB.cmp(me._2, you._2) }
    }
  }
```

```
val b = new Bag[(Int, (Int, Int))]
b.add((3, (3, 4))).add((3, (2, 7))).add((4, (0, 0))).toList
```

With Different Orders

```
val intOrdRev : Ord[Int] =  
  new Ord[Int] { def cmp(me: Int, you: Int) = you - me }  
  
(new Bag[Int])(intOrdRev).add(3).add(2).add(10).toList
```

Type Classes With Multiple Parameters

Iter

```
// trait Iter[A] {  
//   def getValue: Option[A]  
//   def getNext: Iter[A]  
// }
```

```
abstract class Iter[I, A] {  
  def getValue(i: I): Option[A]  
  def getNext(i: I): I  
}
```

```
def sumElements[I](xs: I)(implicit proxy: Iter[I, Int]) : Int =  
  proxy.getValue(xs) match {  
    case None => 0  
    case Some(n) => n + sumElements(proxy.getNext(xs)) }  
}
```

```
def printElements[I, A](xs: I)(implicit proxy: Iter[I, A]) : Unit =  
  proxy.getValue(xs) match {  
    case None =>  
    case Some(n) => {print/n(n); printElements(proxy.getNext(xs))}}
```

List

```
implicit def listIter[A] : Iter[List[A], A] =  
  new Iter[List[A],A] {  
    def getValue(a: List[A]) = a.headOption  
    def getNext(a: List[A]) = a.tail  
  }
```

```
val l = List(3,5,2,1)
```

```
sumElements(l) //sumElements(l)(listIter[Int])
```

```
printElements(l) //printElements(l)(listIter[Int])
```

Iterable

```
// trait Iterable[A] {  
//   def iter : Iter[A]  
// }
```

```
abstract class Iterable[R, I, A] {  
  def iter(a: R): I  
  def iterProxy: Iter[I, A]  
}
```

```
def sumElements2[R, I](xs: R)(implicit proxy: Iterable[R, I, Int]) =  
  sumElements(proxy.iter(xs))(proxy.iterProxy)  
//sumElements[I](proxy.iter(xs))(proxy.iterProxy)
```

```
def printElements2[R, I, A](xs: R)(implicit proxy: Iterable[R, I, A]) =  
  printElements(proxy.iter(xs))(proxy.iterProxy)  
//printElements[I, A](proxy.iter(xs))(proxy.iterProxy)
```

MyTree

```
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A]

implicit def treeIterable[A](implicit proxy: Iter[List[A], A]) :
  Iterable[MyTree[A], List[A], A] =
  new Iterable[MyTree[A], List[A], A] {
    def iter(a: MyTree[A]): List[A] = a match {
      case Empty() => Nil
      case Node(v, left, right) => v :: (iter(left) ++ iter(right))
    }
    val iterProxy = proxy }
val t : MyTree[Int] =
  Node(3, Node(4, Empty(), Empty()), Node(2, Empty(), Empty()))
sumElements2(t) //sumElements2(t)(treeIterable[Int](listIter[Int]))
printElements2(t)
//printElements2(t)(treeIterable[Int](listIter[Int]))
```


Iter being Iterable

```
implicit def iterIterable[I,A](implicit proxy: Iter[I,A]) :  
  Iterable[I,A] =  
  new Iterable[I,A] {  
    def iter(a: I) = a  
    val iterProxy = proxy  
  }
```

```
// val l = List(3,5,2,1)  
sumElements2(l) //sumElements2(iterIterable(listIter[Int]))  
printElements2(l) //printElements2(iterIterable(listIter[Int]))
```

Higher-kind Type Classes

Iter

```
import scala.language.higherKinds
//trait Iter[I,A] {
//  def getValue(a: I): Option[A]
//  def getNext(a: I): I }
abstract class Iter[I[_]] {
  def getValue[A](a: I[A]) : Option[A]
  def getNext[A](a: I[A]) : I[A]
}
def sumElements[I[_]](xs: I[Int])(implicit itr: Iter[I]): Int = {
  itr.getValue(xs) match {
    case None => 0
    case Some(n) => n + sumElements(itr.getNext(xs)) }
}
def printElements[I[_],A](xs: I[A])(implicit itr: Iter[I]): Unit = {
  itr.getValue(xs) match {
    case None =>
    case Some(n) => {print/n(n); printElements(itr.getNext(xs))}}
}
```

List

```
implicit val listIter : Iter[List] =  
  new Iter[List] {  
    def getValue[A](a: List[A]) = a.headOption  
    def getNext[A](a: List[A]) = a.tail  
  }
```

```
val l = List(3,5,2,1)
```

```
sumElements(l) //sumElements(l)(listIter)
```

```
printElements(l) //printElements(l)(listIter)
```

Iterable

```
//trait Iterable[R, I, A] {  
  // def iter(a: R): I  
  // def iterProxy: Iter[I, A]  
  //}
```

```
abstract class Iterable[R[_], I[_]] {  
  def iter[A](a: R[A]): I[A]  
  def iterProxy: Iter[I]  
}
```

```
def sumElements2[R[_], I[_]](xs: R[Int])  
    (implicit proxy: Iterable[R, I]) =  
  sumElements(proxy.iter(xs))(proxy.iterProxy)  
//sumElements[I](proxy.iter(xs))(proxy.iterProxy)
```

```
def printElements2[R[_], I[_], A](xs: R[A])  
    (implicit proxy: Iterable[R, I]) =  
  printElements(proxy.iter(xs))(proxy.iterProxy)  
//printElements[I, A](proxy.iter(xs))(proxy.iterProxy)
```

MyTree

```
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A]

implicit val treeIterable : Iterable[MyTree, List] =
  new Iterable[MyTree, List] {
    def iter[A](a: MyTree[A]) : List[A] = a match {
      case Empty() => Nil
      case Node(v, left, right) => v :: (iter(left) ++ iter(right))
    }
    val iterProxy = implicitly[Iter[List]]
  }

val t : MyTree[Int] =
  Node(3, Node(4, Empty(), Empty()), Node(2, Empty(), Empty()))
sumElements2(t) //sumElements2(t)(treeIterable)
printElements2(t) //printElements2(t)(treeIterable)
```

Implicitly

➤ Definition

```
def implicitly[A](implicit proxy: A) : A = proxy
```

➤ Example

```
implicit val treeIterable :  
  Iterable[MyTree, List] = new Iterable[MyTree, List] {  
    def iter[A](a: MyTree[A]) : List[A] = a match {  
      case Empty() => Nil  
      case Node(v, left, right) => v :: (iter(left) ++ iter(right)) }  
    val iterProxy = implicitly[Iter[List]]  
  }
```

```
implicit def treeIterable(implicit proxy: Iter[List]) :  
  Iterable[MyTree, List] = new Iterable[MyTree, List] {  
    def iter[A](a: MyTree[A]) : List[A] = a match {  
      case Empty() => Nil  
      case Node(v, left, right) => v :: (iter(left) ++ iter(right)) }  
    val iterProxy = proxy  
  }
```

Iter being Iterable

```
implicit def iterIterable[I[_]](implicit proxy: Iter[I]) :  
  Iterable[I, I] = new Iterable[I, I]  
{  
  def iter[A](a: I[A]) = a  
  def iterProxy = proxy  
}
```

```
// val l = List(3, 5, 2, 1)  
sumElements2(l) //sumElements2(l)(iterIterable(listIter))  
printElements2(l) //printElements2(l)(iterIterable(listIter))
```


Example: Functor Specification

```
trait Functor[F[_]] {  
  def map[A,B](f: A=>B)(x: F[A]) : F[B]  
}
```

```
def compose[F[_],A,B,C](g: B=>C)(f: A=>B)(a: F[A])  
  (implicit proxy: Functor[F]): F[C] =  
  proxy.map(g)(proxy.map(f)(a))
```

Example: Functor Implementation

```
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A]

implicit val ListFunctor : Functor[List] = new Functor[List] {
  def map[A,B](f: A=>B)(x: List[A]) = x.map(f)
}

implicit val MyTreeFunctor : Functor[MyTree] = new Functor[MyTree] {
  def map[A,B](f: A=>B)(x: MyTree[A]) : MyTree[B] = x match {
    case Empty() => Empty()
    case Node(v,l,r) => Node(f(v),map(f)(l),map(f)(r)) }
}

compose((x: Int)=>x*x)((x: Int)=>x+x)(List(1,2,3))
val t : MyTree[Int] =
  Node(3, Node(4, Empty(), Empty()), Node(2, Empty(), Empty()))
compose((x: Int)=>x*x)((x: Int)=>x+x)(t)
```

Even Higher Kinds

```
// Iter: (* -> *) -> *
```

```
abstract class Iter[I[_]] {  
  def getValue[A](a: I[A]) : Option[A]  
  def getNext[A](a: I[A]) : I[A]  
}
```

```
// Foo: ((* -> *) -> *) -> *
```

```
abstract class Foo[I[_[_]]] {  
  def get : I[List]  
}
```

```
def f(x: Foo[Iter]) : Iter[List] = x.get
```

Turning Type Classes into OO Classes

Iter

```
// trait Iter[A] {  
//   def getValue: Option[A]  
//   def getNext: Iter[A]  
// }
```

```
abstract class Iter[I, A] {  
  def getValue(i: I): Option[A]  
  def getNext(i: I): I  
}
```

```
def sumElements[I](xs: I)(implicit proxy: Iter[I, Int]) : Int =  
  proxy.getValue(xs) match {  
    case None => 0  
    case Some(n) => n + sumElements(proxy.getNext(xs)) }  
}
```

```
def printElements[I, A](xs: I)(implicit proxy: Iter[I, A]) : Unit =  
  proxy.getValue(xs) match {  
    case None =>  
    case Some(n) => {print/n(n); printElements(proxy.getNext(xs))}}
```

List

```
implicit def listIter[A] : Iter[List[A], A] =  
  new Iter[List[A],A] {  
    def getValue(a: List[A]) = a.headOption  
    def getNext(a: List[A]) = a.tail  
  }
```

```
val l = List(3,5,2,1)
```

```
sumElements(l) //sumElements(l)(listIter[Int])
```

```
printElements(l) //printElements(l)(listIter[Int])
```

How to return data satisfying a specification?

```
def inclter(max: Int) : Iter[Int,Int] = new Iter[Int,Int] {  
  def getValue(i: Int) = if (i <= max) Some(i) else None  
  def getNext(i: Int) = i + 1  
}
```

```
def getMyIter(isInc: Boolean) : ? = {  
  if (isInc) ? // want to return 0 with inclter(10)  
  else      ? // want to return List(3,1,4) with listIter  
}
```

```
val i1 = getMyIter(true)  
printElements(i1)  
val i2 = getMyIter(false)  
printElements(i2)
```

What can we do?

Turning Type Classes into OO Classes

```
import scala.language.higherKinds
import scala.language.implicitConversions

abstract class Box2[S[_],A] {
  type Data
  val d: Data
  val i: S[Data,A]
}

object Box2 {
  implicit def apply[S[_],D,A](dd: D)(implicit ii: S[D,A]):
  Box2[S,A] = new Box2[S,A] {
    type Data = D
    val d = dd
    val i = ii
  }
  implicit def methods[S[_],A](b: Box2[S,A]): S[b.Data,A] = b.i
}
```


Test

```
def inclter(max: Int) : Iter[Int,Int] = new Iter[Int,Int] {  
  def getValue(i: Int) = if (i <= max) Some(i) else None  
  def getNext(i: Int) = i + 1  
}
```

```
def getMyIter(isInc: Boolean) : Box2[Iter,Int] = {  
  if (isInc) Box2(0)(inclter(10)) // Box2.apply(0)(inclter(10))  
  else List(3,1,4) // Box2(List(3,1,4))(listIter[Int])  
}
```

```
val i1 = getMyIter(true)  
printElements(i1.d)(i1.i)  
i1.getValue(i1.getNext(i1.d)) // i1.i.getValue(i1.i.getNext(i1.d))
```

```
val i2 = getMyIter(false)  
printElements(i2.d)(i2.i)  
i2.getValue(i2.getNext(i2.d))
```

Can define “Box” for different kinds of specs

```
abstract class Box[S[_]] {  
  type Data  
  val d: Data  
  val i: S[Data]  
}
```

```
object Box {  
  implicit def apply[D,S[_]](dd: D)(implicit ii: S[D]):  
    Box[S] = new Box[S] {  
    type Data = D  
    val d = dd  
    val i = ii  
  }  
  implicit def methods[S[_]](d: Box[S]): S[d.Data] = d.i  
}
```

Iterable

```
// abstract class Iterable[R,I,A] {  
//   def iter(a: R): I  
//   def iterProxy: Iter[I, A]  
// }
```

```
abstract class Iterable[R,A] {  
  def iter(a: R): Box2[Iter,A]  
}
```

```
def sumElements2[R](xs: R)(implicit proxy: Iterable[R,Int]) = {  
  val cs = proxy.iter(xs)  
  sumElements(cs.d)(cs.i)  
}
```

```
def printElements2[R,A](xs: R)(implicit proxy: Iterable[R,A]) = {  
  val cs = proxy.iter(xs)  
  printElements(cs.d)(cs.i)  
}
```

MyTree

```
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A]

implicit def treeIterable[A] :
  Iterable[MyTree[A], A] = new Iterable[MyTree[A], A] {
  def iter(a: MyTree[A]) : Box2[Iter, A] = {
    def go(l: MyTree[A]) : List[A] =
      l match {
        case Empty() => Nil
        case Node(v, left, right) => v :: (go(left) ++ go(right))
      }
    go(a) // Box2(go(a))(listIter[A])
  }
}

val t : MyTree[Int] =
  Node(3, Node(4, Empty(), Empty()), Node(2, Empty(), Empty()))
sumElements2(t) //sumElements2(t)(treeIterable[Int])
printElements2(t) //printElements2(t)(treeIterable[Int])
```

Iter being Iterable

```
implicit def iterIterable[I,A](implicit proxy: Iter[I,A]) :  
  Iterable[I,A] = new Iterable[I,A] {  
    def iter(a: I) = a // Box2(a)(proxy)  
  }
```

```
// val l = List(3,5,2,1)
```

```
sumElements2(l)
```

```
printElements2(l)
```

Stacking with Type Classes

IntStack Spec

```
trait IntStack[A] {  
  def empty : A  
  def get(s: A): (Int, A)  
  def put(s: A)(x: Int): A  
}
```

```
def testStack[A](stk: IntStack[A]) = {  
  val s0 = stk.empty  
  val s1 = stk.put(s0)(3)  
  val s2 = stk.put(s1)(-2)  
  val s3 = stk.put(s2)(4)  
  val (v1, s4) = stk.get(s3)  
  val (v2, s5) = stk.get(s4)  
  (v1, v2)  
}
```

Modifying Traits

```
trait Doubling[A] extends IntStack[A] {  
  abstract override def put(s: A)(x: Int): A = super.put(s)(2 * x)  
}
```

```
trait Incrementing[A] extends IntStack[A] {  
  abstract override def put(s: A)(x: Int): A = super.put(s)(x + 1)  
}
```

```
trait Filtering[A] extends IntStack[A] {  
  abstract override def put(s: A)(x: Int): A =  
    if (x >= 0) super.put(s)(x) else s  
}
```


Implementation using List

```
class ListStack extends IntStack[List[Int]] {  
  val empty = List()  
  def get(s: List[Int]) = (s.head, s.tail)  
  def put(s: List[Int])(x: Int) = x :: s  
}
```

```
val stkDIF : IntStack[List[Int]] =  
  new ListStack  
    with Doubling[List[Int]]  
    with Incrementing[List[Int]]  
    with Filtering[List[Int]]
```

```
testStack(stkDIF)
```

Implementation: Sorted Stack

```
class SortedStack extends IntStack[List[Int]] {  
  def empty = List()  
  def get(s: List[Int]) : (Int, List[Int]) = (s.head, s.tail)  
  def put(s: List[Int])(x: Int) : List[Int] = {  
    def go(l: List[Int]) : List[Int] = l match {  
      case Nil => x :: Nil  
      case hd :: tl => if (x <= hd) x :: l else hd :: go(tl)  
    }  
    go(s)  
  }  
}  
  
val sortedDIF : IntStack[List[Int]] =  
  new SortedStack  
    with Doubling[List[Int]]  
    with Incrementing[List[Int]]  
    with Filtering[List[Int]]  
  
testStack(sortedDIF)
```

PART 4

Imperative Programming with Memory Updates

Mutable Variables

➤ Mutable Variables

- Use “var” instead of “val” and “def”
- We can update the value stored in a variable.

```
class Main(i: Int) {  
  var a = i  
}
```

```
val m = new Main(10)  
m.a    // 10  
m.a = 20  
m.a    // 20  
m.a += 5    // m.a = m.a + 5  
m.a    // 25
```

While loop

➤ While loop

- Syntax: **while** (*cond*) *body*
Executes *body* while *cond* holds.
- It is equivalent to:

```
def mywhile(cond: =>Boolean)(body: =>Unit) : Unit =  
  if (cond) { body; mywhile(cond)(body) } else ()
```

➤ Example

```
var i = 0  
var sum = 0  
while (i <= 100) { // mywhile (i <= 100) {  
  sum += i  
  i += 2  
}  
sum // 2550
```

For loop

➤ For loop

- Syntax: `for (i <- collection) body`
Executes *body* for each *i* in *collection*.
- It is equivalent to:

```
def myfor[A](xs: Traversable[A])(f: A => Unit) : Unit =  
  xs.foreach(f)
```

➤ Example

```
var sum = 0  
for (i <- 0 to 100 by 2) { // myfor (0 to 100 by 2) { i =>  
  sum += i  
}  
sum // 2550
```

Additional Resources

➤ UCSD CSE 130

- <http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/00-crash.html>
- <http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/01-iterators.html>

Thanks for your hard work!