Principles of Programming
(4190.210)

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Syllabus

- Lecture
  - Tue & Thu: 17:00 ~ 18:15 (@302-107)
  - https://github.com/snu-sf-class/pp202402
  - Bring your laptop to lectures
- Instructor
  - Chung-Kil Hur
  - http://sf.snu.ac.kr/gil.hur/
- Teaching Assistant
  - Yeji Han & Yeonwoo Nam
  - Contact TAs at pp@sf.snu.ac.kr
- Grading (tentative)
  - Attendance: 5%
  - Assignments: 25%
  - Midterm exam: 30%
  - Final exam: 40%

Who am I?

- Prof. Chung-Kil (Gil) Hur [허충길]
  - Education: KAIST (B.S.), Univ of Cambridge (Ph.D.)
  - (High school) Bronze medal in IMO 1994 @Hongkong.
  - Software Foundations Lab
    http://sf.snu.ac.kr
  - Research Topics
    - Software Verification
    - Low-level Language Semantics (C/C++/LLVM/Rust)
    - Relaxed-Memory Concurrency
  - Our collaborators
    - Many in USA, UK, Germany, France, Israel.
  - Publications in Programming Languages (PL) area
    - POPL and PLDI are the two most prestigious conferences in PL area (see https://csrankings.org)
    - Published 12 PLDI and 8 POPL papers
    - Received distinguished paper awards from PLDI 2017, 2021 & POPL 2020
    - General Chair for PLDI 2025 in Seoul

Introduction

Overview

- Part 1
  Functional Programming (with Scala)

- Part 2
  Object-Oriented Programming (with Scala)

- Part 3
  Type Class (Type-Oriented) Programming (with Scala)

- Part 4
  Imperative Programming (with Rust)

Imperative vs. Functional Programming

- Imperative Programming
  - Computation by memory reads/writes
  - Sequence of read/write operations
  - Repetition by loop
  - More procedural (ie, describe how to do)
  - Easier to write efficient code

```
sum = 0;
i = n;
while (i > 0) {
  sum = sum + i;
  i = i - 1;
}
```

- Functional Programming
  - Computation by function application
  - Composition of function applications
  - Repetition by recursion
  - More declarative (ie, describe what to do)
  - Easier to write safe code

```
def sum(n: Int): Int
  if (n <= 0)
    0
  else
    n + sum(n-1)
```

Both Imperative & Functional Style Supported

- Many languages support both imperative & functional style
  - More imperative: Java, Javascript, C++, Python, Rust, …
  - More functional: OCaml, SML, Lisp, Scheme, …
  - Middle: Scala
  - Purely functional: Haskell (monadic programming)

Object-Oriented vs. Type-Oriented Programming

- Goal
  - To support module systems for writing large software
  - Specifically, to support code abstraction & reuse

- Object-Oriented Programming
  - Class Instance (ie, Object): data + methods (ie, functions)
  - Code abstraction & reuse together via inheritance

- Type Class (Type-Oriented) Programming
  - Type Class Instance: type + methods (ie, functions)
  - Abstraction and Reuse are separated
    - Code abstraction via type class instantiation
    - Code reuse via composition

Why Scala & Rust?

- Why Scala?
  - Equally well support both imperative & functional style
  - Many advanced features (both OOP & Type class supported)
  - Compatible with Java
  - Memory management with Garbage Collection (automatic, dynamic)

- Why Rust?
  - Can write (low-level) efficient code like C and C++ but safely
  - Using ownership types
  - Memory management with ownership types (automatic, static) + lightweight Garbage Collection

PART 1

Functional Programming with Function Applications

Values, Names, Functions and Evaluations

Values, Expressions, Names

- Types and Values
  - A type is a set of values
  - `Int`: {-2147483648,…,-1,0,1, …,2147483647} //32-bit integers
  - `Double`: 64-bit floating point numbers // real numbers in practice
  - `Boolean`: {true, false}
  - …
- Expressions
  - Composition of
    values, names, primitive operations, …
- Name Binding
  - Binding expressions to names
- Examples

```
def a = 1 + (2 + 3)
def b = 3 + a * 4
```

Evaluation

- Evaluation
  - Reducing an expression into a value
  - Strategy
  1. Take a name or an operator (outer to inner)
  2. (name) Replace the name with its associated expression
  3. (name) Evaluate the expression
  4. (operator) Evaluate its operands (left to right)
  5. (operator) Apply the operator to its operands

- Examples

```
5+b ~ 5+(3+(a*4)) ~ 5+(3+(1+(2+3))*4) ~ … ~ 32
```

Functions and Substitution

- Functions
  - Expressions with Parameters
  - Binding functions to names

```
def f(x: Int): Int = x + a
```

- Evaluation by substitution
  - …
  - (function) Evaluate its operands (left to right)
  - (function)
    Replace the function application by the expression of the function
    Replace its parameters with the operands

```
5+f(f(3)+1) ~ 5+f((3+a)+1) ~ … ~ 5+f(10) ~ 5+(10+a)
~ … ~ 21
```

Simple Recursion

- Recursion
  - Use X in the definition of X
  - Powerful mechanism for repetition
  - Nothing special but just rewriting

```
def sum(n: Int) : Int =
  if (n <= 0)
    0
  else
    n + sum(n-1)
```

```
sum(2) ~ if (2<=0) 0 else (2+sum(2-1)) ~
2+sum(1) ~ 2+(if (1<=0) 0 else (1+sum(1-1))) ~
2+(1+sum(0)) ~ 2+(1+(if (0<=0) 0 else (0+sum(0-1))))
~ 2+(1+0) ~ 3
```

Termination/Divergence

Evaluation may not terminate

- Termination
  - An expression may reduce to a value

- Divergence
  - An expression may reduce forever

```
def loop: Int = loop
```

```
loop ~ loop ~ loop ~ …
```

Evaluation strategy: Call-by-value, Call-by-name

`f(e1,e2)`

- Call-by-value
  - Evaluate the arguments first, then apply the function to them
- Call-by-name
  - Just apply the function to its arguments, without evaluating them.

```
def square (x: Int) = x * x

[cbv]square(1+1) ~ square(2) ~ 2*2 ~ 4

[cbn]square(1+1) ~ (1+1)*(1+1) ~ 2*(1+1) ~ 2*2 ~ 4
```

CBV, CBN: Differences

- Call-by-value
  - Evaluates arguments once
- Call-by-name
  - Do not evaluate unused arguments

- Question
  - Do both always result in the same value?

Scala’s evaluation strategy

- Call-by-value
  - By default
- Call-by-name
  - Use “=>”

```scala
def one(x: Int, y: =>Int) = 1

one(1+2, loop)

one(loop, 1+2)
```

Scala's name binding strategy

- Call-by-value
  - Use "val" (also called "field") e.g. `val x = e`
  - Evaluate the expression first, then bind the name to it
- Call-by-name
  - Use "def" (also called "method") e.g. `def x = e`
  - Just bind the name to the expression, without evaluating it
  - Mostly used to define functions

```
def a = 1 + 2 + 3
val a = 1 + 2 + 3 // 6
def b = loop
val b = loop

def f(a: Int, b: Int): Int = a*b - 2
```

Conditional Expressions

- If-else
  - `if` (b) e_1 `else` e_2
  - b : Boolean expression
  - e_1, e_2: expressions of the same type
- Reduction rules:
  - `if (true)` e_1 `else` e_2 $\rightarrow$ e_1
  - `if (false)` e_1 `else` e_2 $\rightarrow$ e_2

```
def abs(x: Int) = if (x >= 0) x else -x
```

Boolean Expressions

- Boolean expression
  - `true, false`
  - `!b`
  - `b && b`
  - `b || b`
  - `e <= e, e >= e, e < e, e > e, e == e, e != e`
- Reduction rules:
  - `!true → false`
  - `!false → true`
  - `true && b → b`
  - `false && b → false`
  - `true || b → true`
  - `false || b → b`

```
true && (loop == 1) ~ loop == 1 ~ loop == 1
```

Exercise: and, or

- Write two functions
  - `and(x,y) == x && y`
  - `or(x,y) == x || y`
  - Do not use `&&`, `||`

```
and(false,loop==1)
~ if (false) loop==1 else false
~ false

and(true,loop==1)
~ if (true) loop==1 else false
~ loop==1 ~ loop==1 …
```

Solution

```
def and(x: Boolean, y: =>Boolean) =
  if (x) y else false

def or(x: Boolean, y: =>Boolean) =
  if (x) true else y
```

Exercise: square root calculation

- Calculate square roots with Newton's method

```
def isGoodEnough(guess: Double, x: Double) =
  ??? // guess*guess is 99.9% close to x

def improve(guess: Double, x: Double) =
  (guess + x/guess) / 2

def sqrtIter(guess: Double, x: Double): Double =
  ??? // repeat improving guess until it is good enough

def sqrt(x: Double) =
  sqrtIter(1, x)

sqrt(2)
```

Solution

➢Calculate square roots with Newton's method

```
def isGoodEnough(guess: Double, x: Double) =
  guess*guess/x > 0.999 && guess*guess/x < 1.001

def improve(guess: Double, x: Double) =
  (guess + x/guess) / 2

def sqrtIter(guess: Double, x: Double): Double =
  if (isGoodEnough(guess,x)) guess
  else sqrtIter(improve(guess,x),x)

def sqrt(x: Double) =
  sqrtIter(1, x)

sqrt(2)
```

Blocks and Name Scoping

Blocks in Scala

- Block
  - ```
    { val x1 = e1
      def x2 = e2
      e
    }
    ```
  - Is an expression
  - Allow nested name binding
  - Allow arbitrary order of "`def`"s, but not "`val`"s (think about why)
    ```

Scope of names

➢ Block

```
{ val t = 0
  def f(x: Int) = t + g(x+1)
  def g(y: Int) = y * y
  val x = f(5)
  val r = {
    val t = 10
    val s = f(5)
    s - t }
  t + r }
```

- Block-scoped definitions are only accessible within the block
- Inner definitions shadow outer ones of the same name
- Outer definitions are accessible in nested blocks unless shadowed
- No duplicate definitions allowed in the same block
- Functions are evaluated under the environment where they are defined, not where they are called

Problem

```
// should be allowed
{
  def f(x:Int) = g(x)
  def g(x: Int) = 10
  val x = f(10)
  x
}

// should not be allowed
{
  def f(x:Int) = g(x)
  val x = f(10)
  def g(x: Int) = 10
  x
}
```

Safety Checking

Safety checking rules out any use of undefined names.

- For “val x = e”,
  all names in “e” should be defined before this definition.
- For “def x = e”,
  all names in “e” should be defined before the next “val” definition.

/* The following code passes the safety checking */

```
{ def f(x:Int) = g(x)
  def g(x: Int) = 10
  val a = 10
  f(10) }
```

/* The following code fails at the safety checking */

```
{ def f(x:Int) = g(x)
  val a = 10
  def g(x: Int) = 10
  f(10) }
```

Evaluation for Blocks

```
1: { val t = 0
2:    def f(x: Int) = t + g(x+1)
3:    def g(y: Int) = y*y
4:    val x = f(5) + 7
5: val r = {
6:    val t = 10
7:    val s = f(5)
8:    s - t }
9: t + r }
```

- Evaluation with Environment

```
[E0|t=_,f=_,g=_,x=_ ,r=_ ],1 ~ 0
[E0|t=0,f=(x)t+g(x),g=(y)y*y,x=_ ,r=_ ],4 ~ f(5)+7 ~ 36+7~ 43
  f(5): E0::[E1|x=5],t+g(x+1) ~ 0+g(6) ~ 36
          g(6): E0::[E2|y=6],y*y ~ 6*6 ~ 36
[E0|t=0,f=(x)t+g(x),g=(y)y*y,x=43,r=_ ],5 ~ 26
  5: E0::[E3|t=_ ,s=_ ],6 ~ 10
     E0::[E3|t=10,s=_ ],7 ~ f(5) ~ 36
       f(5): E0::[E4|x=5],t+g(x+1) ~ 0+g(6) ~ 36
               g(6): E0::[E5|y=6],y*y ~ 6*6 ~ 36
     E0::[E3|t=10,s=36],8 ~ s-t ~ 26
[E0|t=0,f=(x)t+g(x),g=(y)y*y,x=43,r=26],9 ~ t+r ~ 26
```

Semi-colons and Parenthesis

- Block
  - Can write two definitions/expressions in a single line using ;
  - Can write one definition/expression in two lines using (), but can omit () when clear

```
// ok
val r = {
  val t = 10; val s = square(5); t +
  s }
// Not ok
val r = {
  val t = 10; val s = square(5); t
  + s }
// ok
val r = {
  val t = 10; val s = square(5); (t
  + s) }
```

Exercise: Writing Better Code using Blocks

- Make the following code better

```
def isGoodEnough(guess: Double, x: Double) =
  guess*guess/x > 0.999 && guess*guess/x < 1.001

def improve(guess: Double, x: Double) =
  (guess + x/guess) / 2

def sqrtIter(guess: Double, x: Double): Double = {
  if (isGoodEnough(guess,x)) guess
  else sqrtIter(improve(guess,x),x)
}

def sqrt(x: Double) =
  sqrtIter(1, x)

sqrt(2)
```

Solution

```
def sqrt(x: Double) = {
  def sqrtIter(guess: Double, x: Double): Double = {
    if (isGoodEnough(guess,x)) guess
    else sqrtIter(improve(guess,x),x)
  }
  def isGoodEnough(guess: Double, x: Double) = {
    val ratio = guess * guess / x
    ratio > 0.999 && ratio < 1.001
  }
  def improve(guess: Double, x: Double) =
    (guess + x/guess) / 2

  sqrtIter(1, x)
}

sqrt(2)
```

Lazy Call-By-Value

Lazy call-by-value

- Lazy call-by-value
  - Use "lazy val" e.g. `lazy val x = e`
  - Evaluate the expression first time it is used, then bind the name to it

```
def f(c: Boolean, i: =>Int): Int = {
  lazy val iv = i
  if (c) 0
  else   iv * iv * iv
}

f(true, {println("ok"); 100+100+100+100})
f(false, {println("ok"); 100+100+100+100})
```

Tail Recursion & Tail Call

Recursion needs care

- Summation function
  - Write a summation function `sum` such that
    `sum(n) = 1+2+…+n`
  - Test
    `sum(10),sum(100),sum(1000),sum(10000),`
    `sum(100000), sum(1000000)`
  - What’s wrong? (Think about evaluation)

Recursion: Try

```
def sum(n: Int): Int =
  if (n <= 0) 0 else (n+sum(n-1))
```

Recursion: Tail Recursion

```
import scala.annotation.tailrec

def sum(n: Int): Int = {
  @tailrec def sumItr(res: Int, m: Int): Int =
    if (m <= 0) res else sumItr(m+res,m-1)
  sumItr(0,n)
}
```

Mutual Recursion: Try

```
{
  def sum(acc: Int, n: Int): Int =
    if (n <= 0) acc else sum2(n + acc, n-1)

  def sum2(acc: Int, n: Int): Int =
    if (n <= 0) acc else sum(2*n + acc, n-1)

  sum(0, 20000)  // stack overflow
}
```

Mutual Recursion: Tail Call Optimization

```
import scala.util.control.TailCalls._

{
  def sum(acc: Int, n: Int): TailRec[Int] =
    if (n <= 0) done(acc) else tailcall(sum2(n + acc, n-1))

  def sum2(acc: Int, n: Int): TailRec[Int] =
    if (n <= 0) done(acc) else tailcall(sum(2*n + acc, n-1))

  sum(0, 20000).result
}
```

Higher-Order Functions

Functions as Values

- Functions
  - Functions are normal values of function types ($A_1,\ldots,A_n$=>B).
  - They can be copied, passed and returned.
  - Functions that take functions as arguments or return functions are called higher-order functions.
  - Higher-order functions increase code reusability.

Examples

```
def sumLinear(n: Int): Int =
  if (n <= 0) 0 else n + sumLinear(n-1)

def sumSquare(n: Int): Int =
  if (n <= 0) 0 else n*n + sumSquare(n-1)

def sumCubes(n: Int): Int =
  if (n <= 0) 0 else n*n*n + sumCubes(n-1)
```

Q: How to write reusable code?

Examples

```
def sum(f: Int=>Int, n: Int): Int =
  if (n <= 0) 0 else f(n) + sum(f, n-1)

def linear(n: Int) = n
def square(n: Int) = n * n
def cube(n: Int) = n * n * n

def sumLinear(n: Int) = sum(linear, n)
def sumSquare(n: Int) = sum(square, n)
def sumCubes(n: Int) = sum(cube, n)
```

Anonymous Functions

➢ Anonymous Functions

- Syntax

  $(x_1\text{: } T_1,\ldots,x_n\text{:}T_n) \Rightarrow e$

  or

  $(x_1,\ldots,x_n) \Rightarrow e$

```
def sumLinear(n: Int) = sum((x:Int)=>x, n)
def sumSquare(n: Int) = sum((x:Int)=>x*x, n)
def sumCubes(n: Int) = sum((x:Int)=>x*x*x, n)
```

Or simply

```
def sumLinear(n: Int) = sum((x)=>x, n)
def sumSquare(n: Int) = sum((x)=>x*x, n)
def sumCubes(n: Int) = sum((x)=>x*x*x, n)
```

Exercise

```
def sum(f: Int=>Int, a: Int, b: Int): Int =
  if (a <= b) f(a) + sum(f, a+1, b) else 0

def product(f: Int=>Int, a: Int, b: Int): Int =
  if (a <= b) f(a) * product(f, a+1, b) else 1
```

DRY (Do not Repeat Yourself) using a higher-order function, called “mapReduce”.

Exercise

```
def mapReduce(reduce: (Int,Int)=>Int,inival: Int,
              f: Int=>Int, a: Int, b: Int): Int = {
  if (a <= b) reduce(f(a),mapReduce(reduce,inival,f,a+1,b))
  else inival
}

def sum(f: Int=>Int, a: Int, b: Int): Int =
  mapReduce((x,y)=>x+y,0,f,a,b)

def product(f: Int=>Int, a: Int, b: Int): Int =
  mapReduce((x,y)=>x*y,1,f,a,b)
```

Evaluation for functional values

```
1:  { val t = 0
2:      val f : Int=>Int = {      // Note: What if using "def f" ?
3:          val t = 10
4:          def g(x: Int) : Int = x + t
5:          g _ }
6:      f(20) }
```

* Try: Evaluation without Closures

```
[E0|t=_,f=_],1 ~ 0
[E0|t=0,f=_],2 ~ (x)x+t
  E0::[E1|t=_,g=(x)x+t],3 ~ 10
  E0::[E1|t=10,g=(x)x+t],5 ~ (x)x+t
[E0|t=0,f=(x)x+t],6 ~ f(20) ~ 20
  f(20): E0::[E2|x=20],x+t ~ 20+0 ~ 20
```

Closures for functional values

```
1:  { val t = 0
2:      val f : Int=>Int = {      // Note: What if using "def f" ?
3:          val t = 10
4:          def g(x: Int) : Int = x + t
5:          g _ }
6:      f(20) }
```

* Evaluation with Closures

```
[E0|t=_,f=_],1 ~ 0
[E0|t=0,f=_],2 ~ (E1,(x)x+t)
  E0::[E1|t=_,g=(x)x+t],3 ~ 10
  E0::[E1|t=10,g=(x)x+t],5 ~ (E1,(x)x+t)
[E0|t=0,f=(E1,(x)x+t)],6 ~ f(20) ~ 30
  f(20): E1::[E2|x=20],x+t ~ 20+10 ~ 30
```

(Parameterized) expression vs. (closure) value

- Functions defined using “def” are not values but parameterized expressions.
- Parameterized expression `f` can be converted to a value `(f _)`.
- The compiler often infers and inserts missing conversions automatically.
- Anonymous functions are values.
- Anonymous functions can be seen as syntactic sugar:

  ```
  (x:T)=>e
  ```

  is equivalent to

  ```
  { def noname(x:T) = e; (noname _) }
  ```

  where `noname` is not used in `e`.
- One can even write a recursive anonymous function in this way.
- Q: what’s the difference between param. exps and function values?
  A: function values are “closures” (ie, param. exp. + env.)
- Q: how to implement call-by-name?
  A: The argument expression is converted to a closure.

Example: call by name with closures

```
1: { val t = 0
2:     def f(x: =>Int) = t + x
3:     val r = {
4:       val t = 10
5:       f(t*t) }                // t*t is treated as ()=>t*t
6:     r }
```

➢Evaluation with Closures

```
[E0|t=_,f=(x:=>Int)t+x,r=_],1 ~ 0
[E0|t=0,f=(x:=>Int)t+x,r=_],3 ~ 100
  E0::[E1|t=_],4 ~ 10
  E0::[E1|t=10],5 ~ f(t*t) ~ 100
  f(t*t): E0::[E2|x=(E1,t*t)],t+x ~ 0+x ~ 0+100 ~ 100
    x: E1,t*t ~ 10*10 ~ 100
[E0|t=0,f=(x)t+x,r=100],6 ~ r ~ 100
```

Currying

Motivation

```
def sum(f: Int=>Int, a: Int, b: Int): Int =
  if (a <= b) f(a) + sum(f, a+1, b) else 0
def sumLinear(a: Int, b: Int) = sum((n)=>n, a, b)
def sumSquare(a: Int, b: Int) = sum((n)=>n*n, a, b)
def sumCubes(a: Int, b: Int) = sum((n)=>n*n*n, a, b)
```

We want the following. How?

```
val sumLinear = sum(linear)
val sumSquare = sum(square)
val sumCubes = sum(cube)
```

Solution

```
def sum(f: Int=>Int): (Int,Int)=>Int = {
  def sumF(a: Int, b: Int): Int =
    if (a <= b) f(a) + sumF(a+1, b) else 0
  sumF  // sumF _
}

def sumLinear = sum((n)=>n)
def sumSquare = sum((n)=>n*n)
def sumCubes = sum((n)=>n*n*n)
```

Benefits

```
def sumLinear = sum((n)=>n)
def sumSquare = sum((n)=>n*n)
def sumCubes = sum((n)=>n*n*n)

sumSquare(3,10) + sumCubes(5,20)
```

We don't need to define the wrapper functions.

```
sum((n)=>n*n)(3,10) + sum((n)=>n*n*n)(5,20)
```

Multiple Parameter List

```
def sum(f: Int=>Int): (Int,Int)=>Int = {
  def sumF(a: Int, b: Int): Int =
    if (a <= b) f(a) + sumF(a+1, b) else 0
  sumF _
}
```

Or simply:

```
def sum(f: Int=>Int)(a: Int, b: Int): Int =
  if (a <= b) f(a) + sum(f)(a+1, b) else 0
```

Note that `sum(f)` is just a parameterized expression.
`(sum(f) _)` creates a closure like `(sumF _)`

The following code is slightly inefficient. Think about why.

```
def sum(f: Int=>Int): (Int,Int)=>Int =
  (a,b) => if (a <= b) f(a) + sum(f)(a+1, b) else 0
```

Comparison between Param. Exp vs. Closure

```
{
  def sum(f: Int=>Int)(a: Int, b: Int): Int =
    if (a <= b) f(a) + sum(f)(a+1, b) else 0
  def sumLinear = sum((n)=>n) _ // sum((n)=>n) incorrect

  sumLinear(0,100)
}
{
  def sum(f: Int => Int): (Int, Int) => Int = {
    def sumF(a: Int, b: Int): Int =
      if (a <= b) f(a) + sumF(a + 1, b) else 0
    sumF _
  }
  def sumLinear = sum((n)=>n) // sum((n)=>n) _ incorrect

  sumLinear(0,100)
}
```

Currying and Uncurrying

- A function of type

$$(T_1,T_2,\ldots,T_n)\text{=>}T$$

  can be turned into that of type

$$T_1\text{=>}(T_2\text{=>}(\ldots\text{=>}(T_n\text{=>}T)\ldots))$$

- This is called “currying” named after Haskell Brooks Curry.
- The opposite direction is called “uncurrying”.

Currying using Anonymous Functions

```
def foo(x: Int, y: Int, z: Int)(a: Int, b: Int) =
  x + y + z + a + b

val f1 = (x: Int, z: Int, b: Int) => foo(x,1,z)(2,b)

val f2 = foo(_:Int,1,_:Int)(2, _:Int)

val f3 = (x: Int, z: Int) => ((b: Int) => foo(x,1,z)(2,b))

f1(1,2,3)
f2(1,2,3)
f3(1,2)(3)
```

Exercise

Curry the mapReduce function.

```
def mapReduce(reduce: (Int,Int)=>Int,inival: Int,
              f: Int=>Int, a: Int, b: Int): Int = {
  if (a <= b) reduce(f(a),mapReduce(reduce,inival,f,a+1,b))
  else inival
}

def sum(f: Int=>Int, a: Int, b: Int): Int =
  mapReduce((x,y)=>x+y,0,f,a,b)

def product(f: Int=>Int, a: Int, b: Int): Int =
  mapReduce((x,y)=>x*y,1,f,a,b)
```

Solution

```
def mapReduce(reduce:(Int,Int)=>Int,inival: Int)
             (f: Int=>Int) (a: Int, b: Int): Int = {
  if (a <= b) reduce(f(a),mapReduce(reduce,inival)(f)(a+1,b))
  else inival
}

// need to make a closure since mapReduce is param. code.
def sum = mapReduce((x,y)=>x+y,0) _

// val is better than def. Think about why.
val product = mapReduce((x,y)=>x*y,1) _
```

Exceptions

Exception & Handling

```
class factRangeException(val arg: Int) extends Exception

def fact(n : Int): Int =
  if (n < 0) throw new factRangeException(n)
  else if (n == 0) 1
  else n * fact(n-1)

def foo(n: Int) = fact(n + 10)

try {
  println (fact(3))
  println (foo(-100))
} catch {
  case e : factRangeException => {
    println("fact range error: " + e.arg)
  }
}
```

Datatypes

Types so far

Types have introduction operations and elimination ones.

- Introduction: how to construct elements of the type
- Elimination: how to use elements of the type

➢ Primitive types
  - Int, Boolean, Double, String, …
  - Intro for Int: …,-2,-1,0,1,2,
  - Elim for Int: +,-,*,/,<,<=,…

➢ Function types
  - Int=>Int, (Int=>Int)=>(Int=>Int), …
  - Intro: (x:T)=>e
  - Elim: f(v)

Tuples

- Tuples

  Intro:
  - (1,2,3) : (Int, Int, Int)
  - (1,“a”) : (Int, String)

  Elim:
  - (1, “a”,10)._1 = 1
  - (1, “a”,10)._2 = “a”
  - (1, “a”,10)._3 = 10

  Only up to length 22

Structural Types (a.k.a. Record Types): Examples

```scala
import reflect.Selectable.reflectiveSelectable
def bar (x: Int) = x+1
val foo = new {
  val a = 1+2
  def b = a + 1
  def f(x: Int) = b + x
  val g : Int => Int = bar _
}
foo.b
foo.f(3)
val ff : Int=>Int = foo.f _
def g(x: {val a: Int; def b: Int;
          def f(x:Int): Int; val g: Int => Int}) =
  x.f(3)
g(foo)
```

Type Alias

```scala
import reflect.Selectable.reflectiveSelectable
type Foo = {val a: Int; def b: Int; def f(x:Int):Int}

val gn = 0
val foo : Foo = new {
  val a = 3
  def b = a + 1
  def f(x: Int) = b + x + gn
}

foo.f(3)

def g(x: Foo) = {
  val gn = 10
  x.f(3)
}
g(foo)
```

Structural Types: Evaluation

```
1: def bar (x: Int) = x+1
2: val foo = new //or, object foo
3: { val a = 2 + 1
4:     def f(x: Int) = a + x
5:     val g : Int => Int = bar _
6: }
7: val b = foo.f(1)
8: foo.g(b)
```

- Evaluation with Closures

```
E1[],1 ~ E1[bar=(x)x+1],2 ~ E1[…]:E2[],3 ~
E1[…]:E2[a=3],4 ~
E1[…]:E2[a=3,f=(x)a+x],5 ~
E1[…]:E2[a=3,f=(x)a+x,g=((x)x+1,E1)],6 ~
E1[bar=(x)x+1,foo=(E2)],7 ~
E1[bar=(x)x+1,foo=(E2),b=4],8 ~ 5
7: E1:E2:E3[x=1],a+x ~ 3+1 ~ 4
8: E1:E4[x=4],x+1 ~ 4+1 ~ 5
```

Algebraic Datatypes

- Ideas
  - T = C of T * … * T
    | C of T * … * T
    | …
    | C of T * … * T

  - E.g.
    Attr = Name of String
    | Age of Int
    | DOB of Int * Int * Int
    | Height of Double

    Intro:
    Name("Chulsoo Kim"), Name("Younghee Lee"), Age(16), DOB(2000,3,10), Height(171.5), …

Algebraic Datatypes: Recursion

- Recursive ADT
  - E.g.

    IList = INil

    | ICons of Int * IList

    Intro:

    INil(),

    ICons(3, INil()),

    ICons(2, ICons(3, INil())),

    ICons(1, ICons(2, ICons(3, INil()))),

    …

Algebraic Datatypes In Scala

- Attr

```scala
sealed abstract class Attr
case class Name(name: String) extends Attr
case class Age(age: Int) extends Attr
case class DOB(year: Int, month: Int, day: Int) extends Attr
case class Height(height: Double) extends Attr

val a : Attr = Name("Chulsoo Kim")
val b : Attr = DOB(2000,3,10)
```

- IList

```scala
sealed abstract class IList
case class INil() extends IList
case class ICons(hd: Int, tl: IList) extends IList

val x : IList = ICons(2, ICons(1, INil()))
def gen(n: Int) : IList =
  if (n <= 0) INil()
  else ICons(n, gen(n-1))
```

Exercise

IOption = INone
| ISome of Int

BTree = Leaf
| Node of Int * BTree * BTree

```
sealed abstract class IList
case class INil() extends IList
case class ICons(hd: Int, tl: IList) extends IList

def x : IList = ICons(2, ICons(1, INil()))
```

Solution

```
sealed abstract class IOption
case class INone() extends IOption
case class ISome(some: Int) extends IOption

sealed abstract class BTree
case class Leaf() extends BTree
case class Node(value: Int, left: BTree, right: BTree)
  extends BTree
```

Pattern Matching

- Pattern Matching
  - A way to use algebraic datatypes

```
e match {
  case C1(…) => e1 : T
   …
  case Cn(…) => en : T
} : T
```

Pattern Matching: An Example

```
def length(xs: IList) : Int =
  xs match {
    case INil() => 0
    case ICons(x, tl) => 1 + length(tl)
  }

length(x)
```

Advanced Pattern Matching

- Advanced Pattern Matching

```
e match {
  case P1 => e1
   …
  case Pn => en
}
```

  - One can combine constructors and use _ and | in a pattern.
    (E.g) case ICons(x, INil()) | ICons(x, ICons(_, INil())) => …
  - The given value `e` is matched against the first pattern `P1`.
    If succeeds, evaluate `e1`.
    If fails, `e` is matched against `P2`.
    If succeeds, evaluate `e2`.
    If fails, …
  - The compiler checks exhaustiveness (ie, whether there is a missing case).

Advanced Pattern Matching: An Example

```
def secondElmt(xs: IList) : IOption =
  xs match {
    case INil() | ICons(_,INil()) => INone()
    case ICons(_, ICons(x, _)) => ISome(x)
  }
```

Vs.

```
def secondElmt2(xs: IList) : IOption =
  xs match {
    case INil() | ICons(_,INil()) => INone()
    case ICons(_, ICons(x, INil())) => ISome(x)
    case _ => INone()
  }
```

Vs.

```
def secondElmt2(xs: IList) : IOption =
  xs match {
    case ICons(_, ICons(x, INil())) => ISome(x)
    case _ => INone()  }
```

Pattern Matching on Int

```
def factorial(n: Int) : Int =
  n match {
    case 0 => 1
    case _ => n * factorial(n-1)
  }

def fib(n: Int) : Int =
  n match {
    case 0 | 1 => 1
    case _ => fib(n-1) + fib(n-2)
  }
```

Pattern Matching with If

```
def f(n: Int) : Int =
  n match {
    case 0 | 1 => 1
    case _ if (n <= 5) => 2
    case _ => 3
  }

def f(t: BTree) : Int =
  t match {
    case Leaf() => 0
    case Node(n,_,_) if (n <= 10) => 1
    case Node(_,_,_) => 2
  }
```

Exercise

Write a function “find(t: BTree, x: Int) : Boolean” that checks whether x is in t.

```
sealed abstract class BTree
case class Leaf() extends BTree
case class Node(value: Int, left: BTree, right: BTree)
  extends BTree
```

Solution

```
def find(t: BTree, i: Int) : Boolean =
  t match {
    case Leaf() => false
    case Node(n,lt,rt) =>
      i == n || find(lt, i) || find(rt, i)
  }

def t: BTree = Node(5,Node(4,Node(2,Leaf(),Leaf()),Leaf()),
  Node(7,Node(6,Leaf(),Leaf()),Leaf()))
find(t,7),find(t,1)
```

Type Checking & Inference (Concept)

What Are Types For?

- Typed Programming

```
def id1(x: Int): Int = x
def id2(x: Double): Double = x
```

  - At run time, type information is erased (ie, `id1` = `id2`)

- Untyped Programming

```
def id(x) = x
```

  - Do not care about types at compile time.
  - But, many such languages check types at run time paying cost.
  - Without run-time type check, errors can be badly propagated.

- What is compile-time type checking for?
  - Can detect type errors at compile time.
  - Increase Readability (Give a good abstraction).
  - Soundness: Well-typed programs raise no type errors at run time.

Type Checking and Inference

- Type Checking

$$x_1{:}T_1,\ x_2{:}T_2,\ \ldots,\ x_n{:}T_n\ \vdash\ e : T$$

  - `def f(x: Boolean): Boolean = x > 3`
    => Type error
  - `def f(x: Int): Boolean = x > 3`
    => OK. `f: (x: Int)Boolean`

- Type Inference

$$x_1{:}T_1,\ x_2{:}T_2,\ \ldots,\ x_n{:}T_n\ \vdash\ e : ?$$

  - `def f(x: Int) = x > 3`
    => OK by type inference. `f: (x: Int)Boolean`
  - Too much type inference is not good. Why?

You can learn how type checking & inference work in
4190.310 Programming Languages

Parametric Polymorphism

Parametric Polymorphism: Functions

- Problem

```
def id1(x: Int): Int = x
def id2(x: Double): Double = x
```

  - Can we avoid copy and paste?
  - Polymorphism to the rescue!

- Parametric Polymorphism (a.k.a. For-all Types)

```
def id[A](x: A) : A = x
```

  - The type of `id` is `[A](val x:A)=>A`
  - `id` is a parametric expression.
  - `id[T] _` is a value of type `T=>T` for any type `T`.
  - Function types do not support polymorphism.
    (E.g.) `[A](A=>A)` is not a valid function value type.

[We will learn other kinds of polymorphism later.]

Examples

```
def id[A](x:A) = x
id(3)
id("abc")

def applyn[A](f: A => A, n: Int, x: A): A =
  n match {
    case 0 => x
    case _ => f(applyn(f, n - 1, x))
  }
applyn((x:Int)=>x+1,100,3)
applyn((x:String)=>x+"!", 10, "gil")
applyn(id[String], 10, "hur")

def foo[A,B](f: A=>A, x: (A,B)) : (A,B) =
  (applyn[A](f, 10, x._1), x._2)

foo[String,Int]((x:String)=>x+"!",("abc",10))
```

Parametric Polymorphism: Datatypes

```scala
sealed abstract class MyOption[A]
case class MyNone[A]() extends MyOption[A]
case class MySome[A](some: A) extends MyOption[A]

sealed abstract class MyList[A]
case class MyNil[A]() extends MyList[A]
case class MyCons[A](hd: A, tl: MyList[A]) extends MyList[A]

sealed abstract class BTree[A]
case class Leaf[A]() extends BTree[A]
case class Node[A](value: A, left: BTree[A], right: BTree[A])
extends BTree[A]

def x: MyList[Int] = MyCons(3, MyNil())
def y: MyList[String] = MyCons("abc", MyNil())
```

Revisit: Solution with Tail Recursion

```
def find[A](t: BTree[A], x: A) : Boolean = {
  def findIter(ts: MyList[BTree[A]]) : Boolean =
    ts match {
      case MyNil() => false
      case MyCons(Leaf(), tl) => findIter(tl)
      case MyCons(Node(v, _, _), _) if v == x => true
      case MyCons(Node(_,l,r), tl) =>
        findIter(MyCons(l, MyCons(r, tl)))    }
  findIter(MyCons(t, MyNil()))
}

def genTree(v: Int, n: Int) : BTree[Int] = {
  def genTreeIter(t: BTree[Int], m : Int) : BTree[Int] =
    if (m == 0) t
    else genTreeIter(Node(v, t, Leaf()), m-1)
  genTreeIter(Leaf(), n)
}

find(genTree(0,10000), 1)
```

Exercise

BSTree[A] = Leaf
| Node of Int * A * BSTree[A] * BSTree[A]

```
def lookup[A](t: BSTree[A], k: Int) : MyOption[A] =
  ???

def t : BSTree[String] =
  Node(5,"My5",
    Node(4,"My4",Node(2,"My2",Leaf(),Leaf()),Leaf()),
    Node(7,"My7",Node(6,"My6",Leaf(),Leaf()),Leaf()))

lookup(t, 7)
lookup(t, 3)
```

Solution

```
sealed abstract class BSTree[A]
case class Leaf[A]() extends BSTree[A]
case class Node[A](key: Int, value: A, left: BSTree[A], right:
BSTree[A]) extends BSTree[A]
def lookup[A](t: BSTree[A], key: Int) : MyOption[A] =
  t match {
    case Leaf() => MyNone()
    case Node(k,v,lt,rt) =>
      k match {
        case _ if key == k => MySome(v)
        case _ if key < k => lookup(lt,key)
        case _ => lookup(rt, key)
      }
  }
def t : BSTree[String] =
  Node(5,"My5",
    Node(4,"My4",Node(2,"My2",Leaf(),Leaf()),Leaf()),
    Node(7,"My7",Node(6,"My6",Leaf(),Leaf()),Leaf()))
lookup(t, 7)
lookup(t, 3)
```

A Better Way

```
sealed abstract class BTree[A]
case class Leaf[A]() extends BTree[A]
case class Node[A](value: A, left: BTree[A], right: BTree[A])
  extends BTree[A]

type BSTree[A] = BTree[(Int,A)]

def lookup[A](t: BSTree[A], k: Int) : MyOption[A] =
  ???

def t : BSTree[String] =
  Node((5,"My5"),
    Node((4,"My4"),Node((2,"My2"),Leaf(),Leaf()),Leaf()),
    Node((7,"My7"),Node((6,"My6"),Leaf(),Leaf()),Leaf()))

lookup(t, 7)
```

Solution

```scala
type BSTree[A] = BTree[(Int,A)]

def lookup[A](t: BSTree[A], key: Int) : MyOption[A] =
  t match {
    case Leaf() => MyNone()
    case Node((k,v),lt,rt) =>
      k match {
        case _ if key == k => MySome(v)
        case _ if key < k => lookup(lt,key)
        case _ => lookup(rt, key)
      }
  }

def t : BSTree[String] =
  Node((5,"My5"),
    Node((4,"My4"),Node((2,"My2"),Leaf(),Leaf()),Leaf()),
    Node((7,"My7"),Node((6,"My6"),Leaf(),Leaf()),Leaf()))

lookup(t, 7)
lookup(t, 3)
```

Polymorphic Option (Library)

- Option[T]

  Intro:
  - None
  - Some(x)
  - Library functions

  Elim:
  - Pattern matching
  - Library functions

  Some(3) : Option[Int]
  Some(“abc”): Option[String]
  None: Option[Int]
  None: Option[String]

Polymorphic List (Library)

- List[T]

  Intro:
  - Nil
  - x :: L
  - Library functions

  Elim:
  - Pattern matching
  - Library functions

  “abc”::Nil : List[String]
  List(1,3,4,2,5) = 1::3::4::2::5::Nil : List[Int]

Summary: Parametric Polymorphism

- Parametric Polymorphism
  - Program against unknown datatypes
  - How is it possible?

PART 2

Object-Oriented Programming

Sub Type Polymorphism (Concept)

Motivation

We want:

```
object tom {
  val name = "Tom"
  val home = "02-880-1234"
}
object bob {
  val name = "Bob"
  val mobile = "010-1111-2222"
}
def greeting(r: ???) = "Hi " + r.name + ", How are you?"
greeting(tom)
greeting(bob)
```

Note that we have

```
tom: {val name: String; val home: String}
bob: {val name: String; val mobile: String}
```

Sub Types to the Rescue!

```scala
import reflect.Selectable.reflectiveSelectable

type NameHome = { val name: String; val home: String }
type NameMobile = { val name: String; val mobile: String}
type Name = { val name: String }
```

`NameHome <: Name` (NameHome is a sub type of Name)

`NameMobile <: Name` (NameMobile is a sub type of Name)

```scala
def greeting(r: Name) = "Hi " + r.name + ", How are you?"
greeting(tom)
greeting(bob)
```

Sub Types

- The sub type relation is kind of the subset relation.
- But they are NOT the same.

- T <: S
  Every element of T can be used as that of S.

- *Cf.* T is a subset of S.
  Every element of T is that of S.

- Why polymorphism?
  A function of type S=>R can be used as T=>R for many sub types T of S.
  Note that S=>R <: T=>R when T <: S.

Summary: Subtype Polymorphism

- Subtype Polymorphism
  - Program against known datatypes with common structures
  - How is it possible?

Two Kinds of Sub Types

- Structural Sub Types (a.k.a. Duck Typing)
  - The system implicitly determines the sub type relation by the structures of data types.
  - Structurally equivalent types are treated the same.

- Nominal Sub Types (a.k.a. Ad hoc Polymorphism)
  - The user explicitly specify the sub type relation using the names of data types.
  - Structurally equivalent types with different names may be treated differently.

Structural Sub Types

General Sub Type Rules

- Reflexivity:
  For any type T, we have:

$$T <: T$$

- Transitivity:
  For any types T, S, R, we have:

$$\frac{T <: R \qquad R <: S}{T <: S}$$

Sub Types for Special Types

- Nothing: The empty set
- Any: The set of all values

- For any type T, we have:

$$\text{Nothing} <: T <: \text{Any}$$

- Example

```
val a : Int = 3
val b : Any = a
def f(a: Nothing) : Int = a
```

Sub Types for Records

- Permutation

$$\frac{}{\{\ldots;\ x\text{: } T1;\ y\text{: } T2;\ \ldots\} <: \{\ldots;\ y\text{: } T2,\ x\text{: } T1;\ \ldots\}}$$

- Width

$$\frac{}{\{\ldots;\ x\text{: } T;\ \ldots\} <: \{\ldots;\ \ldots\}}$$

- Depth

$$\frac{T <: S}{\{\ldots;\ x\text{: } T;\ \ldots\} <: \{\ldots;\ x\text{: } S;\ \ldots\}}$$

Sub Types for Records

- Example

  {val x: { val y: Int; val z: String}, val w: Int}

  <: (by permutation)

  {val w: Int; val x: { val y: Int; val z: String}}

  <: (by depth & width)

  {val w: Int; val x: {val z: String}}

Sub Types for Tuples

- Depth

$$\frac{T <: S}{(\ldots, T, \ldots) <: (\ldots, S, \ldots)}$$

Sub Types for Functions

- Function Sub Type

$$\frac{T <: T' \qquad S <: S'}{(T' => S) <: (T => S')}$$

- Example

```
import reflect.Selectable.reflectiveSelectable
def foo(s: {val a: Int; val b: Int}) : {val x: Int; val y: Int} = {
  object tmp {
    val x = s.b
    val y = s.a
  }
  tmp
}
val gee: {val a: Int; val b: Int; val c: Int} => {val x: Int} =
  foo _
```

Classes

Class: Parameterized Record

```
import reflect.Selectable.reflectiveSelectable

type gee_type = {val name:String; val age: Int; def getPP(): String}
def gee_fun(_name: String, _age: Int) : gee_type = {
  if (!(_age >= 0 && _age < 200)) throw new Exception("Out of range")
  object tmp {
    val name : String = _name
    val age : Int = _age
    def getPP() : String = name + " of age " + age.toString() }
  tmp }
val gee : gee_type = gee_fun("David Jones",25)

gee.getPP()
```

Class: Parameterized Record

```
class foo_type(_name: String, _age: Int) {
  if (!(_age >= 0 && _age < 200)) throw new Exception("Out of range")
  val name : String = _name
  val age : Int = _age
  def getPP() : String = name + " of age " + age.toString() }
val foo : foo_type = new foo_type("David Jones",25)
foo.getPP()
```

use: `foo.name foo.age foo.getPP`

- `foo` is a value of `foo_type`
- `gee` is a value of `gee_type`

Class: No Structural Sub Typing

- Records: Structural sub-typing

```
foo_type <: gee_type
```

- Classes: Nominal sub-typing

$$\texttt{gee\_type} \not<: \texttt{foo\_type}$$

```
val v1 : gee_type = foo
val v2 : foo_type = gee  // type error

def greeting(r:{val name:String}) =
  "Hi " + r.name + ", How are you?"
greeting(foo)
```

Structural Types vs. Nominal Types

- Structural Types
  - Includes arbitrary values with the required structures as elements
  - Allows arbitrary types with the required structures as sub types
  - Cannot assume any properties on their elements

- Nominal Types
  - Includes only specific values as elements
  - Allows only specific types as sub types
  - Can assume specific properties on their elements

Class: Can be Recursive!

```
class MyList[A](v: A, nxt: Option[MyList[A]]) {
  val value : A = v
  val next : Option[MyList[A]] = nxt
}
type YourList[A] = Option[MyList[A]]

val t : YourList[Int] =
  Some(new MyList(3,Some (new MyList(4,None))))
val s : YourList[Int] =
  None
```

Note on Null value

- `null`: The special element of every class & structural type

- `null` is often used to represent None instead of using an Option type (Efficient but Not Safe)

- It is discouraged to use `null` in Scala although Scala supports `null` for compatibility with Java.

Simplification using Argument Members

```scala
class MyList[A](v: A, nxt: Option[MyList[A]]) {
  val value : A = v
  val next : Option[MyList[A]] = nxt
}
```

```scala
class MyList[A](val value:A, val next:Option[MyList[A]]) {
}
```

```scala
class MyList[A](val value:A, val next:Option[MyList[A]])
```

Simplification using Companion Object

```
class MyList[A](val value:A, val next:Option[MyList[A]])
object MyList
{ def apply[A](v: A, nxt: Option[MyList[A]]) =
    new MyList(v,nxt)
}

type YourList[A] = Option[MyList[A]]
object YourList
{ def apply[A](v: A, nxt: Option[MyList[A]]) =
    Some(new MyList(v,nxt))
}
val t0 = None
val t1 = Some(new MyList(3,Some(MyList(4,None))))
val t2 = YourList(3,(YourList(4,None)))
```

Exercise

Define a class "`MyTree[A]`" for binary trees:

```
MyTree[A] =
    (value: A) *
    (left: Option[MyTree[A]]) *
    (right: Option[MyTree[A]])
```

Solution

```
class MyTree[A](v: A,
                lt: Option[MyTree[A]],
                rt: Option[MyTree[A]]) {
  val value = v
  val left = lt
  val right = rt
}

type YourTree[A] = Option[MyTree[A]]

val t0 : YourTree[Int] = None
val t1 : YourTree[Int] = Some(new MyTree(3, None, None))
val t2 : YourTree[Int] =
  Some(new MyTree(3, Some (new MyTree(4,None,None)), None))
```

Simplified Solution

```scala
class MyTree[A](val value : A,
                val left : Option[MyTree[A]],
                val right : Option[MyTree[A]])

type YourTree[A] = Option[MyTree[A]]

object YourTree
{ def apply[A](v:A, lt:Option[MyTree[A]], rt:Option[MyTree[A]]) =
    Some(new MyTree(v,lt,rt))
}

val t0: YourTree[Int] = None
val t1: YourTree[Int] = YourTree(3,None,None)
val t2: YourTree[Int] = YourTree(3,YourTree(4,None,None),None)
```

Nominal Sub Typing for Classes

Nominal Sub Typing, a.k.a. Inheritance

```
class foo_type(x: Int, y: Int) {
  val a : Int = x
  def b : Int = a + y
  def f(z: Int) : Int = b + y + z
}

class gee_type(x: Int) extends foo_type(x+1,x+2) {
  val c : Int = f(x) + b
}
```

gee_type <: foo_type

```
(new gee_type(30)).c
def test(f: foo_type) = f.a + f.b
test(new foo_type(10,20))
test(new gee_type(30))
```

Overriding

```
class foo_type(x: Int, y: Int) {
  val a : Int = x
  def b : Int = 0
  def f(z: Int) : Int = b * z
}
class gee_type(x: Int) extends foo_type(x+1,x+2) {
 override def b = 10
 // or, override def b = super.b + 10
 val c : Int = f(x) + b
}

(new gee_type(30)).c
def test(v: foo_type) =
 println(v.f(42))
test(new foo_type(1,2))
test(new gee_type(0))
```

Overriding vs. Overloading

```
class foo_type(x: Int, y: Int) {
  val a : Int = x
  def b : Int = 0
  def f(z: Int) : Int = b * z
}
class gee_type(x: Int) extends foo_type(x+1,x+2) {
  def f(z: String) : Int = 77
}
```

Q: Can we override with a different type?

```
override def f(z: String): Int = 77   //No,  arg: diff type
def f(z: String): Int = 77   // Overloading, arg: diff type
override def f(z: Int): Int = 77 //Yes, arg: same type
```

Example: MyList using Inheritance

```scala
class MyList[A](v: A, nxt: Option[MyList[A]]) {
  val value : A = v
  val next : Option[MyList[A]] = nxt
}
type YourList[A] = Option[MyList[A]]
val t : YourList[Int] =
  Some(new MyList(3,Some (new MyList(4,None))))
```

```scala
class MyList[A]()
class MyNil[A]() extends MyList[A]
class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A]
val t: MyList[Int] =
    new MyCons(3, new MyCons(4, new MyNil()))
```

Simplification: MyList

```
class MyList[A]

class MyNil[A]() extends MyList[A]
object MyNil { def apply[A]() = new MyNil[A]() }

class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A]
object MyCons {
  def apply[A](hd:A, tl:MyList[A]) = new MyCons[A](hd, tl)}

val t: MyList[Int] = MyCons(3, MyNil())

def length(x: MyList[Int]) = ???
```

Example: MyList with match

```
abstract class MyList[A]() {
  def matches[R](nilE: =>R, consE: (A,MyList[A]) => R) : R
}
class MyNil[A]() extends MyList[A] {
  def matches[R](nilE: =>R, consE: (A,MyList[A]) => R) : R =
    nilE
}
class MyCons[A](val hd: A, val tl: MyList[A]) extends MyList[A] {
  def matches[R](nilE: =>R, consE: (A,MyList[A]) => R) : R =
    consE(hd,tl)
}
def length[A](l: MyList[A]) : Int =
  l.matches(0,
                (hd, tl) => 1 + length(tl))

length(new MyCons(10, new MyCons(5, new MyNil())))
```

Case Class

```
sealed abstract class MyList[A] { … }
case class MyNil[A]() extends MyList[A] { … }
~~object MyNil { def apply[A]() = new MyNil[A]() }~~
case class MyCons[A](~~val~~ hd: A, ~~val~~ tl: MyList[A])
  extends MyList[A] { … }
~~object MyCons {~~
~~  def apply[A](hd:A, tl:MyList[A]) = new MyCons[A](hd, tl)}~~
val t: MyList[Int] = MyCons(3, MyNil())
```

Allow Pattern Matching

```
def length(x: MyList[Int]): Int =
  x match {
    case MyNil() => 0
    case MyCons(hd, tl) => 1 + length(tl)
  }
```

Cf. `sealed abstract class MyList[A]`

Exercise

Define "MyTree[A]" using sub class.

```
class MyTree[A](v: A,
                lt: Option[MyTree[A]],
                rt: Option[MyTree[A]]) {
  val value = v
  val left = lt
  val right = rt
}

type YourTree[A] = Option[MyTree[A]]
```

Solution

```scala
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value:A, left:MyTree[A], right:MyTree[A])
  extends MyTree[A]

val t : MyTree[Int] =
  Node(3, Node(4,Empty(),Empty()), Empty())

t match {
  case Empty() => 0
  case Node(v,l,r) => v
}
```

Solution with Monotonicity

```
// sealed abstract class MyTree[A]
// case class Empty[A]() extends MyTree[A]
// case class Node[A](value:A, left:MyTree[A], right:MyTree[A])
//   extends MyTree[A]

// MyTree[+A]: A <: B  ⟹  MyTree[A] <: MyTree[B]
// MyTree[-A]: A <: B  ⟹  MyTree[B] <: MyTree[A]
sealed abstract class MyTree[+A]
case object Empty extends MyTree[Nothing]
case class Node[A](value:A, left:MyTree[A], right:MyTree[A])
  extends MyTree[A]

val t : MyTree[Int] = Node(3, Node(4,Empty,Empty), Empty)
t match {
  case Empty => 0
  case Node(v,l,r) => v
}
```

Solution with enum

```scala
// sealed abstract class MyTree[+A]
// case object Empty extends MyTree[Nothing]
// case class Node[A](value:A, left:MyTree[A], right:MyTree[A])
//   extends MyTree[A]

enum MyTree[+A]:
  case Empty  //: MyTree[Nothing]
  case Node(value: A, left: MyTree[A], right: MyTree[A])
import MyTree._

val t : MyTree[Int] = Node(3, Node(4,Empty,Empty), Empty)
t match {
  case Empty => 0
  case Node(v,l,r) => v
}
```

Encoding ADT using classes: Monotonicity

```
sealed abstract class MyList[+A] {
  def matches[R](nilE: =>R, consE: (A,MyList[A])=>R) : R
  def append[B>:A](l: MyList[B]) : MyList[B]
}
object MyNil extends MyList[Nothing] {
  def matches[R](nilE: =>R, consE: (Nothing,MyList[Nothing])=>R) = nilE
  def append[B](l: MyList[B]) = l
}
class MyCons[A](val hd: A, val tl: MyList[A]) extends MyList[A] {
  def matches[R](nilE: =>R, consE: (A,MyList[A])=>R) = consE(hd,tl)
  def append[B>:A](l: MyList[B]) = new MyCons[B](hd, tl.append(l))
}
object MyCons{ def apply[A](hd:A, tl:MyList[A]) = new MyCons[A](hd, tl) }
def length[A](l: MyList[A]) : Int =
  l.matches(
    0,
    (_,tl) => 1 + length(tl) )
length(MyCons(3, MyCons(2, MyNil)).append(MyCons(1,MyNil)))
```

Abstract Classes for Interface

Abstract Class: Interface

- Abstract Classes
  - Can be used to abstract away the implementation details.

Abstract classes for Interface
Concrete sub-classes for Implementation

Abstract Class: Interface

➢ Example Interface

```
// Written by Alice
// if getValue(i) returns None, you should not use i.getNext()
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =
  xs.getValue match {
    case None => 0
    case Some(n) => f(n) + sumElements(f)(xs.getNext)
  }
def sumElementsId(xs:Iter[Int]) =
  sumElements((x:Int)=>x)(xs)
```

Concrete Class: Implementation

```
// Written by Bob
sealed abstract class MyList[A] extends Iter[A]
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
}
case class MyCons[A](hd: A, tl: MyList[A])
  extends MyList[A]
{
  def getValue = Some(hd)
  def getNext = tl
}

val t1 = MyCons(3, MyCons(5, MyCons(7, MyNil())))

sumElementsId(t1)
```

Exercise

Define `IntCounter(n)` that implements the interface `Iter[A]`.

```
// Written by Catherine
class IntCounter(n: Int) extends Iter[Int] {
  def getValue = ???
  def getNext = ???
}

sumElementsId(new IntCounter(100))
```

Solution

Define `IntCounter(n)` that implements the interface `Iter[A]`.

```
// Written by Catherine
class IntCounter(n: Int) extends Iter[Int] {
  def getValue = if (n >= 0) Some(n) else None
  def getNext = new IntCounter(n-1)
}

sumElementsId(new IntCounter(100))
```

A Better Interface

```
abstract class Iter[A] {
  def get: Option[(A,Iter[A])]
}
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =
  xs.get match {
    case None => 0
    case Some(n,nxt) => f(n) + sumElements(f)(nxt)
  }
def sumElementsId(xs:Iter[Int]) = sumElements((x:Int)=>x)(xs)
sealed abstract class MyList[A] extends Iter[A]
case class MyNil[A]() extends MyList[A] {
  def get = None }
case class MyCons[A](hd: A, tl: MyList[A]) extends MyList[A] {
  def get = Some(hd,tl) }
class IntCounter(n: Int) extends Iter[Int] {
  def get = if (n >= 0) Some(n, new IntCounter(n-1)) else None }
```

More on Abstract Classes

Problem: Iter for MyTree

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

// Written by David
sealed abstract class MyTree[A]
case class Empty[A]() extends MyTree[A]
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A]
```

Q: Can MyTree[A] implement Iter[A]?

Try it, but it is not easy.

Possible Solution

```
// Written by David
sealed abstract class MyTree[A] extends Iter[A]
case class Empty[A]() extends MyTree[A] {
  def getValue = None
  def getNext = this  }
case class Node[A](value: A, left: MyTree[A], right: MyTree[A])
     extends MyTree[A] {
  def getValue = Some(value)
  def getNext: MyTree[A] = {
    def merge_right(l : MyTree[A]): MyTree[A] = l match {
      case Empty() => right
      case Node(v, lt, rt) => Node(v, lt, merge_right(rt))  }
    merge_right(left)  } }
val t1 = Node(3, Node(7, Node(2, Empty(), Empty()), Empty()),
                  Node(8, Empty(), Empty()))
sumElements[Int]((x)=>x*x)(t1)
```

Solution: Better Interface

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}
abstract class Iterable[A] {
  def iter : Iter[A]
}

def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =
  xs.getValue match {
    case None => 0
    case Some(n) => f(n) + sumElements(f)(xs.getNext)
  }
def sumElementsGen[A](f: A=>Int)(xs: Iterable[A]) : Int =
  sumElements(f)(xs.iter)
```

Let's Use MyList

```scala
sealed abstract class MyList[A] extends Iter[A]
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
}
case class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A] {
  def getValue = Some(hd)
  def getNext = tl
}
```

MyTree <: Iterable (Try)

```
sealed abstract class MyTree[A] extends Iterable[A]

case class Empty[A]() extends MyTree[A] {
  val iter = MyNil()
}

case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A] {
  // "val iter" is more specific than "def iter",
  // so it can be used in a sub type.
  // In this example, "val iter" is also
  // more efficient than "def iter".
  val iter = MyCons(value, ???(left.iter,right.iter))
}
```

Extend MyList with append

```
sealed abstract class MyList[A] extends Iter[A] {
  def append(lst: MyList[A]) : MyList[A]
}
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...")
  def append(lst: MyList[A]) = lst
}
case class MyCons[A](val hd: A, val tl: MyList[A])
  extends MyList[A]
{
  def getValue = Some(hd)
  def getNext = tl
  def append(lst: MyList[A]) = MyCons(hd,tl.append(lst))
}
```

MyTree <: Iterable

```
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : MyList[A]
  // Note:
  // def iter : Int // Type Error because not (Int <: Iter[A])
}
case class Empty[A]() extends MyTree[A] {
  val iter = MyNil()
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A] {
  def iter = MyCons(value, left.iter.append(right.iter))
  // def iter = left.iter.append(MyCons(value,right.iter))
  // def iter = left.iter.append(right.iter.append(
  //                  MyCons(value,MyNil())))
}
```

Test

```
def generateTree(n: Int) : MyTree[Int] = {
  def gen(lo:Int, hi: Int) : MyTree[Int] =
    if (lo > hi) Empty()
    else {
      val mid = (lo+hi)/2
      Node(mid, gen(lo,mid-1), gen(mid+1,hi))
    }
  gen(1,n)
}

sumElementsGen((x:Int)=>x)(generateTree(100))
```

Iter <: Iterable

```
abstract class Iterable[A] {
  def iter : Iter[A]
}

abstract class Iter[A] extends Iterable[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
  def iter = this
}

val lst : MyList[Int] =
  MyCons(3, MyCons(4, MyCons(2,MyNil())))

sumElementsGen ((x:Int)=>x)(lst)
```

Note: tail-recursive "append"

```
sealed abstract class MyList[A] extends Iter[A] {
  def append(lst: MyList[A]) : MyList[A] =
    MyList.revAppend(MyList.revAppend(this, MyNil()), lst)
}
object MyList { //  Mutual references are allowed between class T and object T
  //  Tail-recursive functions should be written in "object", or as final methods
  def revAppend[A](lst1: MyList[A], lst2: MyList[A]): MyList[A] =
    lst1 match {
      case MyNil() => lst2
      case MyCons(hd, tl) => revAppend(tl, MyCons(hd, lst2))
    }
}
case class MyNil[A]() extends MyList[A] {
  def getValue = None
  def getNext = throw new Exception("...") }
case class MyCons[A](val hd:A, val tl:MyList[A]) extends MyList[A] {
  def getValue = Some(hd)
  def getNext = tl }
```

Lazy List

Problem: Inefficiency

```
def time[R](block: => R): R = {
  val t0 = System.nanoTime()
  val result = block    // call-by-name
  val t1 = System.nanoTime()
  println("Elapsed time: " + ((t1 - t0)/1000000) + "ms"); result
}
def sumN[A](f: A=>Int)(n: Int, xs: Iterable[A]) : Int = {
  def sumIter(res : Int, n: Int, xs: Iter[A]) : Int =
    if (n <= 0) res
    else xs.getValue match {
      case None => res
      case Some(v) => sumIter(f(v) + res, n-1, xs.getNext)
    }
  sumIter(0,n,xs.iter)
}
// Problem: takes a few seconds to get a single value
{ val t: MyTree[Int] = generateTree(200000)
  time (sumN((x:Int) => x)(1, t)) }
```

Solution 1: Using Lists of Trees

```
class MyTreeIter[A](val lst: MyList[MyTree[A]]) extends Iter[A] {
  val getValue = lst match {
    case MyCons(Node(v,_,_), _) => Some(v)
    case _ => None
  }
  def getNext = {
    val remainingTrees : MyList[MyTree[A]] = lst match {
      case MyNil() => throw new Exception("...")
      case MyCons(hd,tl) => hd match {
        case Empty() => throw new Exception("...")
        case Node(_,Empty(),Empty()) => tl
        case Node(_,lt,Empty()) => MyCons(lt,tl)
        case Node(_,Empty(),rt) => MyCons(rt,tl)
        case Node(_,lt,rt) => MyCons(lt,MyCons(rt,tl))
      }
    }
    new MyTreeIter(remainingTrees)
  } }
```

Lazy Iteration using Lists of Trees

```
sealed abstract class MyTree[A] extends Iterable[A]
case class Empty[A]() extends MyTree[A] {
  val iter = new MyTreeIter(MyNil())
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A]
{
  val iter = new MyTreeIter(MyCons(this,MyNil()))
}

{ val t: MyTree[Int] = generateTree(200000)
  time (sumN((x:Int) => x)(100, t))
  time (sumN((x:Int) => x)(100000, t))
}
```

Solution 2: Lazy List

```scala
sealed abstract class LazyList[A] extends Iter[A] {
  def append(lst: LazyList[A]) : LazyList[A]
}

case class LNil[A]() extends LazyList[A] {
  def getValue = None
  def getNext = throw new Exception("")
  def append(lst: LazyList[A]) = lst
}

class LCons[A](hd: A, _tl: =>LazyList[A]) extends LazyList[A] {
  lazy val tl = _tl
  def getValue = Some(hd)
  def getNext = tl
  def append(lst: LazyList[A]) = LCons(hd, tl.append(lst)) }
object LCons {
  def apply[A](hd: A, tl: =>LazyList[A]) = new LCons(hd, tl)
}
```

Note: “append” is not recursive!!!

Lazy Iteration using LazyList

```scala
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : LazyList[A]
}
case class Empty[A]() extends MyTree[A] {
  val iter = LNil()
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A] {
  lazy val iter = LCons(value, left.iter.append(right.iter))
  // lazy val iter = left.iter.append(LCons(value,right.iter))
  // lazy val iter = left.iter.append(right.iter.append(
  //                    LCons(value,LNil())))
}
{ val t: MyTree[Int] = generateTree(200000)
  time (sumN((x:Int) => x)(100, t))
  time (sumN((x:Int) => x)(100000, t))
}
```

Note: “iter” is not recursive!!!

Wrapper for Inheritance

Using a Wrapper Class

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A] {
  def getValue = list.headOption
  def getNext = new ListIter(list.tail)
}

sumElements((x:Int)=>x)(new ListIter(List(1,2,3,4)))
```

MyTree Using ListIter

```
abstract class Iterable[A] {
  def iter : Iter[A]
}
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : ListIter[A]
}
case class Empty[A]() extends MyTree[A] {
  val iter : ListIter[A] = new ListIter(Nil)
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A])
  extends MyTree[A] {
  val iter : ListIter[A] = new ListIter(
    value::(left.iter.list ++ right.iter.list))
}
```

Test

```
val t : MyTree[Int] =
  Node(3, Node(4,Node(2,Empty(),Empty()),
    Node(3,Empty(),Empty())),
    Node(5,Empty(),Empty()))

sumElementsGen((x:Int)=>x)(t)
```

Abstract Class With Associate Types

Using an Associate Type

```
abstract class Iterable[A] {
  type iter_t
  def iter: iter_t
  def getValue(i: iter_t) : Option[A]
  def getNext(i: iter_t) : iter_t
}

def sumElements[A](f:A=>Int)(xs: Iterable[A]) : Int = {
  def sumElementsIter(i: xs.iter_t) : Int =
    xs.getValue(i) match {
      case None => 0
      case Some(n) => f(n) + sumElementsIter(xs.getNext(i))
    }
  sumElementsIter(xs.iter)
}
```

MyTree Using List

```
sealed abstract class MyTree[A] extends Iterable[A] {
  type iter_t = List[A]
  def getValue(i: List[A]): Option[A] = i.headOption
  def getNext(i: List[A]): List[A] = i.tail
}
case class Empty[A]() extends MyTree[A] {
  val iter : List[A] = Nil
}
case class Node[A](value: A,
                   left: MyTree[A], right: MyTree[A])
  extends MyTree[A] {
  val iter = value :: (left.iter ++ right.iter) //Pre-order
//val iter = left.iter ++ (value :: right.iter) // In-order
//val iter = left.iter ++ (right.iter ++ List(value))
                                              //Post-order
}
```

Test

```
val t : MyTree[Int] =
  Node(3, Node(4,Node(2,Empty(),Empty()),
    Node(3,Empty(),Empty())),
    Node(5,Empty(),Empty()))

sumElements((x:Int)=>x)(t)
```

Abstract Class with Arguments

Abstract Class with Arguments

```
abstract class IterableH[A] extends Iterable[A] {
  def hasElement(a: A) : Boolean
}
abstract class IterableHE[A](eq: (A,A) => Boolean)
  extends IterableH[A]
{
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
}
```

MyTree

```scala
sealed abstract class MyTree[A](eq:(A,A)=>Boolean)
  extends IterableHE[A](eq) {
  type iter_t = List[A]
  def getValue(i : List[A]) : Option[A] = i.headOption
  def getNext(i: List[A]) : List[A] = i.tail
}
case class Empty[A](eq: (A,A)=>Boolean)
  extends MyTree[A](eq) {
  val iter : List[A] = Nil
}
case class Node[A](eq: (A,A)=>Boolean,
               value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A](eq) {
  val iter : List[A] = value :: (left.iter ++ right.iter)
}
```

Test

```
val leq = (x:Int,y:Int) => x == y
val lEmpty = Empty(leq)
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =
  Node(leq,n,t1,t2)

val t : MyTree[Int] =
  lNode(3, lNode(4,lNode(2,lEmpty,lEmpty),
                   lNode(3,lEmpty,lEmpty)),
           lNode(5,lEmpty,lEmpty))

sumElements((x:Int)=>x)(t)
t.hasElement(5)
t.hasElement(10)
```

Alternatively, Argument Elimination

```
abstract class IterableHE[A]
  extends Iterable[A]
{
  def eq(a:A, b:A) : Boolean
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
}
```

MyTree

```scala
sealed abstract class MyTree[A] extends IterableHE[A] {
  type iter_t = List[A]
  def getValue(i : List[A]) : Option[A] = i.headOption
  def getNext(i: List[A]) : List[A] = i.tail
}
case class Empty[A](_eq:(A,A)=>Boolean) extends MyTree[A] {
  def eq(a:A, b:A) = _eq(a,b)
  val iter : List[A] = Nil
}
case class Node[A](_eq: (A,A)=>Boolean,
               value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A] {
  def eq(a:A, b:A) = _eq(a,b)
  val iter : List[A] = value :: (left.iter ++ right.iter)
}
```

Test

```
val leq = (x:Int,y:Int) => x == y
val lEmpty = Empty(leq)
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =
  Node(leq,n,t1,t2)

val t : MyTree[Int] =
  lNode(3, lNode(4,lNode(2,lEmpty,lEmpty),
                   lNode(3,lEmpty,lEmpty)),
           lNode(5,lEmpty,lEmpty))

sumElements((x:Int)=>x)(t)
t.hasElement(5)
t.hasElement(10)
```

More on Classes

Motivating Example

```scala
class Primes(val prime: Int, val primes: List[Int]) {
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n <= 0) 2 else go(new Primes(3, List(3)), n)
}
nthPrime(10000)
```

Multiple Constructors

```scala
class Primes(val prime: Int, val primes: List[Int]) {
  def this() = this(3, List(3))
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n == 0) 2 else go(new Primes, n)
}
nthPrime(10000)
```

Access Modifiers

- Access Modifiers
  - Private: Only the class can access the member.
  - Protected: Only the class and its sub classes can access the member.

Using Access Modifiers

```
class Primes private (val prime:Int, protected val primes:List[Int])
{ def this() = this(3, List(3))
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  private def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n == 0) 2 else go(new Primes, n)
}
nthPrime(10000)
```

Traits for Multiple Inheritance

Multiple Inheritance Problem

- Multiple Inheritance
  - The famous “diamond problem”

```
class A(val a: Int)
class B extends A(10)
class C extends A(20)
class D extends B, C.
```

Problem 1: What is the value of (new D).a ?

Problem 2: The constructor of A must be executed once because A may contain side effects such as sending messages over the network.

Java’s Solution: Interface

- Interface
  - An interface cannot contain any implementation but only types of its methods.
  - A class can inherit implementations from only one parent class but implement multiple interfaces.

Scala's Solution: Trait

- Traits
  - A trait can implement any of its methods,
    but should have only one constructor with no arguments.
  - An [abstract] class (resp. trait) X can
    "extends" one trait or [abstract] class with any (resp. no) arguments
    "with" multiple traits T_1 , … , T_n
    such that, for each i,
    the least superclass of T_i, if exists, should be a superclass of X
    where
    C is a superclass of T if C is an (abstract) class and T transitively "extends" C.
  - No cyclic inheritance is allowed.
- Property
  - For any ancestor class in the inheritance tree of a class:
    - Its constructor with arguments can appear at most once
    - Its constructor with no argument can appear multiple times

Example

```scala
class A(val a : Int) {
  def this () = this(0)
}
trait B {
  def f(x: Int): Int = x
}
trait C extends A with B {
  def g(x: Int): Int = x + a
}
trait D extends B {
  def h(x: Int): Int = f(x + 50)
}
class E extends A(10) with C with D {
  override def f(x: Int) = x * a
}

val e = new E
```

Algorithm for Multiple Inheritance

➢ Algorithm

- Give a linear order among all ancestors by "post-order" traversing without revisiting the same node.
- Invoke the constructors once in that order.

  Note. Post-order traversal of a class C means

  – Recursively post-order traverse C's first parent; …;

  – Recursively post-order traverse C's last parent; and

  – Visit C.

  By post-order traversing from "E" in the previous example,

  we have the order: `A(10) -> B -> C -> D -> E`

```
val e = new E
e.a // 10                     e.f(100) // 100*10
e.g(100) // 100 + 10          e.h(100) // (100 + 50) * 10
```

- A constructor with arguments is always visited before the same constructor with no arguments.
- Compile error if the same field is implemented by multiple classes

A Simple Example With Traits

Motivation

```scala
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A] {
  def getValue = list.headOption
  def getNext = new ListIter(list.tail)
}

abstract class Dict[K,V] {
  def add(k: K, v: V): Dict[K,V]
  def find(k: K): Option[V]
}
```

Q: How can we extend ListIter and implement Dict?

Interface using Traits

```
// abstract class Dict[K,V] {
//    def add(k: K, v: V): Dict[K,V]
//    def find(k: K): Option[V] }

trait Dict[K,V] {
  def add(k: K, v: V): Dict[K,V]
  def find(k: K): Option[V]
}
```

Implementing Traits

```
class ListIterDict[K,V]
      (eq: (K,K)=>Boolean, list: List[(K,V)])
      extends ListIter[(K,V)](list)
         with Dict[K,V]
{
  def add(k:K,v:V): ListIterDict[K,V] =
    new ListIterDict(eq,(k,v)::list)
  def find(k: K) : Option[V] = {
    def go(l: List[(K, V)]): Option[V] = l match {
        case Nil => None
        case (k1, v1) :: tl =>
          if (eq(k, k1)) Some(v1) else go(tl) }
    go(list) }
}
```

Test

```
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =
  xs.getValue match {
    case None => 0
    case Some(n) => f(n) + sumElements(f)(xs.getNext)
  }

def find3(d: Dict[Int,String]) = {
  d.find(3)
}

val d0 = new ListIterDict[Int,String]((x,y)=>x==y,Nil)
val d = d0.add(4,"four").add(3,"three")

sumElements[(Int,String)](x=>x._1)(d)
find3(d)
```

Mixin with Traits

Motivation: Mixin Functionality

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A]
{
  def getValue = list.headOption
  def getNext: ListIter[A] = new ListIter(list.tail)
}

trait MRIter[A] extends Iter[A] {
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = ???
}
```

Mixin Composition

```
trait MRIter[A] extends Iter[A] {
  override def getNext: MRIter[A]
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C =
    getValue match {
      case None => ival
      case Some(v) =>
        combine(f(v), getNext.mapReduce(combine, ival, f))
    }
}

class MRListIter[A](list: List[A])
  extends ListIter (list) with MRIter[A]
{
 override def getNext = new MRListIter(super.getNext.list)
                  // new MRListIter(list.tail)
}
val mr = new MRListIter[Int](List(3,4,5))
mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Mixin Composition: A Better Way

```
trait MRIter[A] extends Iter[A] {
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = {
    def loop(c: Iter[A]): C = c.getValue match {
      case None => ival
      case Some(v) => combine(f(v), loop(c.getNext))
    }
    loop(this)
  }
}

class MRListIter[A](list: List[A])
  extends ListIter (list) with MRIter[A]

val mr = new MRListIter[Int](List(3,4,5))

// or, val mr = new ListIter(List(3,4,5)) with MRIter[Int]

mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Syntactic Sugar: new A with B with C { … }

```
new A(...) with B1 ··· with Bm {
  code
}
```

is equivalent to

```
{
  class _tmp_(args) extends A(args) with B1 ··· with Bm {
    code
  }
  new _tmp_(...)
}
```

Intersection Types

Intersection Types

- Typing Rule

$$\frac{t : T1 \qquad t : T2}{t : T1 \text{ with } T2}$$

- Example

```
trait A { val a: Int = 0 }
trait B { val b: Int = 0 }
class C extends A with B {
  override val a = 10
  override val b = 20
  val c = 30
}
val x = new C
val y: A with B = x
y.a // 10
y.b // 20
y.c // type error
```

Subtype Relation for “with”

The subtype relation for “with” is structural.

- Permutation

$$\frac{}{\ldots \text{ with } T1 \text{ with } T2 \ldots <: \ldots \text{ with } T2 \text{ with } T1 \ldots}$$

- Width

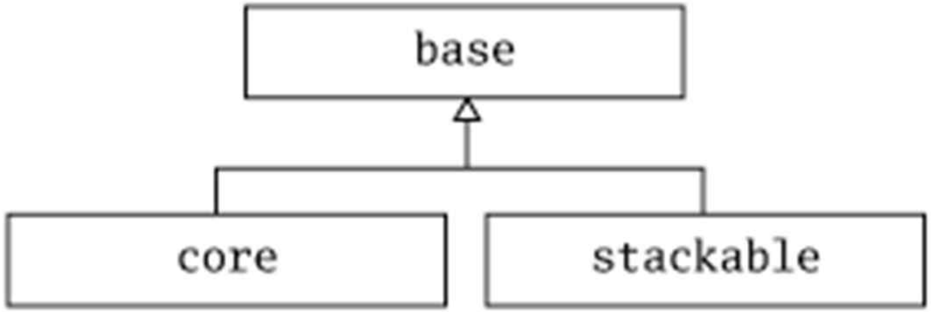
$$\frac{}{\ldots \text{ with } T \ldots <: \ldots \ldots}$$

- Depth

$$\frac{T <: S}{\ldots \text{ with } T \ldots <: \ldots \text{ with } S \ldots}$$

Stacking with Traits

Typical Hierarchy in Scala



- **BASE**
  Interface (trait or abstract class)

- **CORE**
  Functionality (trait or concrete class)

- **CUSTOM**
  Modifications (each in a separate, composable trait)

IntStack: Base

- **BASE**

```
trait Stack[A] {
  def get(): (A, Stack[A])
  def put(x: A): Stack[A]
}
```

IntStack: Core

- **CORE**

```
class BasicIntStack protected (xs: List[Int]) extends Stack[Int]
{
  override val toString = "Stack:" + xs.toString
  def this() = this(Nil)

  def get():(Int,Stack[Int]) = (xs.head,new BasicIntStack(xs.tail))
  def put(x:Int): Stack[Int] = new BasicIntStack(x :: xs)
}

val s0 = new BasicIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

IntStack: Custom Modifications

- CUSOM

```
trait Doubling extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] = super.put(2 * x)
}

trait Incrementing extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] = super.put(x + 1)
}

trait Filtering extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] =
    if (x >= 0) super.put(x) else this
}
```

IntStack: Stacking

- **Stacking**

```
class DIFIntStack protected (xs: List[Int])
  extends BasicIntStack(xs)
  with Doubling with Incrementing with Filtering
{
  def this() = this(Nil)
}

val s0 = new DIFIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
val (v2,s6) = s5.get()
```

IntStack: Core (Correct)

- **CORE**

```
class BasicIntStack protected (xs: List[Int]) extends Stack[Int]
{
  override val toString = "Stack:" + xs.toString
  def this() = this(Nil)
  protected def mkStack(xs: List[Int]): Stack[Int] =
    new BasicIntStack(xs)
  def get(): (Int,Stack[Int]) = (xs.head, mkStack(xs.tail))
  def put(x: Int): Stack[Int] = mkStack(x :: xs)
}

val s0 = new BasicIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

IntStack: Stacking (Correct)

➢ **Stacking**

```
class DIFIntStack protected (xs: List[Int])
  extends BasicIntStack(xs)
  with Doubling with Incrementing with Filtering
{
  def this() = this(Nil)
  override def mkStack(xs: List[Int]): Stack[Int] =
    new DIFIntStack(xs)
}

val s0 = new DIFIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

Additional Resources

- Traits
  - http://www.scala-lang.org/old/node/126
- Mixin Composition
  - http://www.scala-lang.org/old/node/117
- Stackable Trait Pattern
  - http://www.artima.com/scalazine/articles/stackable_trait_pattern.html
- Multiple Inheritance via Traits
  - https://www.safaribooksonline.com/blog/2013/05/30/traits-how-scala-tames-multiple-inheritance/
- UCSD CSE 130
  - http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/02-classes.html

PART 3

Type Classes for Interfaces

Problems with OOP

Subtype Polymorphism

```
trait Ord {
  // this cmp that < 0  iff this < that
  // this cmp that > 0  iff this > that
  // this cmp that == 0 iff this == that
  def cmp(that: Ord): Int

  def ===(that: Ord): Boolean = (this.cmp(that)) == 0
  def <  (that: Ord): Boolean = (this cmp that) < 0
  def >  (that: Ord): Boolean = (this cmp that) > 0
  def <= (that: Ord): Boolean = (this cmp that) <= 0
  def >= (that: Ord): Boolean = (this cmp that) >= 0
}
def max3(a: Ord, b: Ord, c: Ord) : Ord =
  if (a <= b) { if (b <= c) c else b }
  else        { if (a <= c) c else a }
```

* Problem: hard (almost impossible) to implement Ord (e.g., using Int)

Interface over Parameter Types

```
trait Ord[A] {
  def cmp(that: A): Int

  def ===(that: A): Boolean = (this.cmp(that)) == 0
  def <  (that: A): Boolean = (this cmp that) < 0
  def >  (that: A): Boolean = (this cmp that) > 0
  def <= (that: A): Boolean = (this cmp that) <= 0
  def >= (that: A): Boolean = (this cmp that) >= 0
}
def max3[A <: Ord[A]](a: A, b: A, c: A) : A =
  if (a <= b) {if (b <= c) c else b }
  else        {if (a <= c) c else a }

class OInt(val value : Int) extends Ord[OInt] {
  def cmp(that: OInt) = value - that.value
}
max3(new OInt(3), new OInt(2), new OInt(10)).value
```

Further example: Ordered Bag

```
class Bag[U <: Ord[U]] protected (val toList: List[U]) {
  def this() = this(Nil)
  def add(x: U) : Bag[U] = {
    def go(elmts: List[U]): List[U] =
      elmts match {
        case Nil => x :: Nil
        case e :: _ if (x < e) => x :: elmts
        case e :: _ if (x === e) => elmts
        case e :: rest => e :: go(rest)
      }
    new Bag(go(toList))
  }
}
val emp = new Bag[OInt]()
val b = emp.add(new OInt(3)).add(new OInt(2)).
            add(new OInt(10)).add(new OInt(2))
b.toList.map((x)=>x.value)
```

Problems with OOP

1. Needs "subtyping" like "OInt <: Ord[OInt]", which is quite complex as we have seen (and moreover, involves more complex concepts like variance).

2. Needs a wrapper class like "OInt" in order to add a new interface to an existing type like "Int".

3. Interface only contains only "elimination" functions, not "introduction" functions.

4. No canonical operator

5. …

Type Classes

Separating Functions from Data

```
trait Ord[A] {
  def cmp(self: A)(a: A): Int

  def ===(self: A)(a: A) = cmp(self)(a) == 0
  def <  (self: A)(a: A) = cmp(self)(a) < 0
  def >  (self: A)(a: A) = cmp(self)(a) > 0
  def <= (self: A)(a: A) = cmp(self)(a) <= 0
  def >= (self: A)(a: A) = cmp(self)(a) >= 0
}

def max3[A](a: A, b: A, c: A)(implicit ORD: Ord[A]) : A =
  if (ORD.<=(a)(b)) {if (ORD.<=(b)(c)) c else b }
  else              {if (ORD.<=(a)(c)) c else a }

// behaves like Int <: Ord in OOP
implicit val intOrd : Ord[Int] = new {
  def cmp(self: Int)(a: Int) = self - a }
max3(3,2,10) // 10
```

Implicit

- Implicit
  - An argument is given “implicitly”

```scala
def foo(s: String)(implicit t: String) = s + t

implicit val exclamation : String = "!!!!!!"

foo("Hi")
foo("Hi")("???") // can give it explicitly
```

Syntax for type class: syntactic sugar

```
trait Ord[A]:
  extension (self: A)
    def cmp(a: A): Int
    def ===(a: A) = self.cmp(a) == 0
    def <  (a: A) = self.cmp(a) < 0
    def >  (a: A) = self.cmp(a) > 0
    def <= (a: A) = self.cmp(a) <= 0
    def >= (a: A) = self.cmp(a) >= 0

def max3[A: Ord](a: A, b: A, c: A) : A =
  if (a <= b) { if (b <= c) c else b }
  else        { if (a <= c) c else a }

given intOrd : Ord[Int] with
  extension (self: Int)
    def cmp(a: Int) = self - a


max3(3,2,10) // 10
```

Syntax for type class: syntactic sugar

```
trait Ord[A]:

  def cmp(self: A)(a: A): Int
  def ===(self: A)(a: A) = cmp(self)(a) == 0
  def < (self: A)(a: A) = cmp(self)(a) < 0
  def > (self: A)(a: A) = cmp(self)(a) > 0
  def <= (self: A)(a: A) = cmp(self)(a) <= 0
  def >= (self: A)(a: A) = cmp(self)(a) >= 0

def max3[A](a: A, b: A, c: A)(implicit ORD: Ord[A]) : A =
  if (ORD.<=(a)(b)) { if (ORD.<=(b)(c)) c else b }
  else              { if (ORD.<=(a)(c)) c else a }

implicit def intOrd : Ord[Int] = new {
  def cmp(self:Int)(a: Int) = self - a
}

max3(3,2,10) // 10
```

Bag Example using type class

```
class Bag[A: Ord] protected (val toList: List[A])
{ def this() = this(Nil)
  def add(x: A) : Bag[A] = {
    def loop(elmts: List[A]) : List[A] =
      elmts match {
        case Nil => x :: Nil
        case e :: _ if (x < e) => x :: elmts
        case e :: _ if (x === e) => elmts
        case e :: rest => e :: loop(rest)
      }
    new Bag(loop(toList))
  }
}

(new Bag[Int]()).add(3).add(2).add(3).add(10).toList
```

Bag Example using type class

```
class Bag[A] protected (val toList: List[A])(implicit ORD: Ord[A])
{ def this()(implicit ORD: Ord[A]) = this(Nil)
  def add(x: A) : Bag[A] = {
    def loop(elmts: List[A]) : List[A] =
      elmts match {
        case Nil => x :: Nil
        case e :: _ if (ORD.<(x)(e)) => x :: elmts
        case e :: _ if (ORD.===(x)(e)) => elmts
        case e :: rest => e :: loop(rest)
      }
    new Bag(loop(toList))
  }
}

(new Bag[Int]()).add(3).add(2).add(3).add(10).toList
```

Bootstrapping Implicits

```
// lexicographic order
given tupOrd[A, B](using Ord[A], Ord[B]): Ord[(A,B)] with
  extension (self: (A,B))
    def cmp(a: (A, B)) : Int = {
      val c1 = self._1.cmp(a._1)
      if (c1 != 0) c1
      else { self._2.cmp(a._2) }
    }

val b = new Bag[(Int,(Int,Int))]
b.add((3,(3,4))).add((3,(2,7))).add((4,(0,0))).toList
```

Bootstrapping Implicits

```
// lexicographic order
implicit def tupOrd[A, B](implicit ORDA: Ord[A], ORDB: Ord[B]): Ord[(A,B)] =
new {
  def cmp(self:(A,B))(a: (A, B)) : Int = {
    val c1 = ORDA.cmp(self._1)(a._1)
    if (c1 != 0) c1
    else { ORDB.cmp(self._2)(a._2) }
  }
}

val b = new Bag[(Int,(Int,Int))]
b.add((3,(3,4))).add((3,(2,7))).add((4,(0,0))).toList
```

With Different Orders

```
def intOrdRev : Ord[Int] = new {
  extension (self: Int)
    def cmp(a: Int) = a - self
}

(new Bag[Int]()).add(3).add(2).add(10).toList
(new Bag[Int]()(intOrdRev)).add(3).add(2).add(10).toList
```

With Different Orders

```
def intOrdRev : Ord[Int] = new {
  def cmp(self: Int)(a: Int) = a - self
}

(new Bag[Int]()).add(3).add(2).add(10).toList
(new Bag[Int]()(intOrdRev)).add(3).add(2).add(10).toList
```

Type Classes: Abstraction

Interfaces I: elimination

```
trait Iter[I,A]:
  extension (self: I)
    def getValue: Option[A]
    def getNext: I

trait Iterable[I,A]:
  type Itr
  given ITR: Iter[Itr,A]
  extension (self: I)
    def iter: Itr

// behaves like Iter[A] <: Iterable[A] in OOP
given iter2iterable[I,A](using _ITR: Iter[I,A]): Iterable[I,A] with
  type Itr = I
  def ITR = _ITR
  extension (self: I)
    def iter = self
```

Interfaces I: elimination

```
trait Iter[I,A]:
  def getValue(self: I): Option[A]
  def getNext(self: I): I

trait Iterable[I,A]:
  type Itr
  implicit def ITR: Iter[Itr,A]
  def iter(self: I): Itr

// behaves like Iter[A] <: Iterable[A] in OOP
implicit def iter2iterable[I,A](implicit _ITR: Iter[I,A]): Iterable[I,A] = new {
  type Itr = I
  def ITR = _ITR
  def iter(self: I) = self
}
```

Programs for Testing: use Iter, Iterable

```
def sumElements[I](xs: I)(implicit ITRA:Iterable[I,Int]) = {
  def loop(i: ITRA.Itr): Int =
    i.getValue match {
      case None => 0
      case Some(n) => n + loop(i.getNext)
    }
  loop(xs.iter)
}

def printElements[I,A](xs: I)(implicit ITRA: Iterable[I,A]) = {
  def loop(i: ITRA.Itr): Unit =
    i.getValue match {
      case None =>
      case Some(a) => {println(a); loop(i.getNext)}
    }
  loop(xs.iter)
}
```

Programs for Testing: use Iter, Iterable

```
def sumElements[I](xs: I)(implicit ITRA:Iterable[I,Int]) = {
 def loop(i: ITRA.Itr): Int =
  ITRA.ITR.getValue(i) match {
   case None => 0
   case Some(n) => n + loop(ITRA.ITR.getNext(i))
  }
 loop(ITRA.iter(xs))
}

def printElements[I,A](xs: I)(implicit ITRA: Iterable[I,A]) = {
 def loop(i: ITRA.Itr): Unit =
  ITRA.ITR.getValue(i) match {
   case None =>
   case Some(a) => {println(a); loop(ITRA.ITR.getNext(i))}
  }
 loop(ITRA.iter(xs))
}
```

Interfaces II: introduction + elimination

```
trait Listlike[L,A]:
  extension(u:Unit)
    def unary_! : L
  extension(elem:A)
    def ::(l: =>L): L
  extension(l: L)
    def head: Option[A]
    def tail: L
    def ++(l2: L): L

trait Treelike[T,A]:
  extension(u:Unit)
    def unary_! : T
  extension(a:A)
    def has(lt: T, rt: T): T
  extension(t: T)
    def root : Option[A]
    def left : T
    def right : T
```

Interfaces II: introduction + elimination

```
trait Listlike[L,A]:
  def ! : L
  def ::(elem:A)(l: =>L): L
  def head(l: L): Option[A]
  def tail(l: L): L
  def ++(l: L)(l2: L): L

trait Treelike[T,A]:
  def ! : T
  def has(a:A)(lt: T, rt: T): T
  def root(t: T) : Option[A]
  def left(t: T) : T
  def right(t: T) : T
```

Programs for Testing: use All

```
def testList[L](implicit LL: Listlike[L,Int], ITRA: Iterable[L,Int]) = {
  val l = (3 :: !()) ++ (1 :: 2 :: !())
  println(sumElements(l))
  printElements(l)
}

def testTree[T](implicit TL: Treelike[T,Int], ITRA: Iterable[T,Int]) = {
  val t = 3.has(4.has(!(), !()), 2.has(!(),!()))
  println(sumElements(t))
  printElements(t)
}
```

Programs for Testing: use All

```
def testList[L](implicit LL: Listlike[L,Int], ITRA: Iterable[L,Int]) = {
  val l = LL.++(LL.::(3)(LL.!))(LL.::(1)(LL.::(2)(LL.!)))
  println(sumElements(l))
  printElements(l)
}

def testTree[T](implicit TL: Treelike[T,Int], ITRA: Iterable[T,Int]) = {
  val t = TL.has(3)(TL.has(4)(TL.!, TL.!), TL.has(2)(TL.!,TL.!))
  println(sumElements(t))
  printElements(t)
}
```

Implement Iter and Listlike for List

```
// behaves like Listlike[A] <: Iter[A] in OOP
given listIter[L,A](using LL: Listlike[L,A]): Iter[L,A] with
  extension (l: L)
    def getValue = l.head
    def getNext = l.tail

// behaves like List[A] <: Listlike[A] in OOP
given listListlike[A]: Listlike[List[A],A] with
  extension (u: Unit)
    def unary_! = Nil
  extension (a: A)
    def ::(l: =>List[A]) = a::l
  extension (l: List[A])
    def head = l.headOption
    def tail = l.tail
    def ++(l2: List[A]) = l ::: l2
```

Implement Iter and Listlike for List

```
// behaves like Listlike[A] <: Iter[A] in OOP
implicit def listIter[L,A](implicit LL: Listlike[L,A]): Iter[L,A] = new {
  def getValue(l: L) = LL.head(l)
  def getNext(l: L) = LL.tail(l)
}

// behaves like List[A] <: Listlike[A] in OOP
implicit def listListlike[A]: Listlike[List[A],A] = new {
  def ! = Nil
  def ::(a: A)(l: => List[A]) = a :: l
  def head(l: List[A]) = l.headOption
  def tail(l: List[A]) = l.tail
  def ++(l: List[A])(l2: List[A]) = l ::: l2
}
```

Implement Iterable for MyTree using Listlike,Iter

```
enum MyTree[+A]:
  case Leaf
  case Node(value: A, left: MyTree[A], right: MyTree[A])
import MyTree._

// behaves like MyTree[A] <: Iterable[A], but clumsy in OOP
given treeIterable[L,A](using LL: Listlike[L,A], _ITR: Iter[L,A])
  : Iterable[MyTree[A], A] with
  type Itr = L
  def ITR = _ITR
  extension (t: MyTree[A])
    def iter: L = t match {
      case Leaf => !()
      case Node(v, lt, rt) => v :: (lt.iter ++ rt.iter)
    }
```

Implement Iterable for MyTree using Listlike,Iter

```
enum MyTree[+A]:
  case Leaf
  case Node(value: A, left: MyTree[A], right: MyTree[A])
import MyTree._

// behaves like MyTree[A] <: Iterable[A], but clumsy in OOP
implicit def treeIterable[L,A](implicit LL: Listlike[L,A], _ITR: Iter[L,A])
  : Iterable[MyTree[A], A] = new {
  type Itr = L
  def ITR = _ITR
  def iter(t: MyTree[A]): L = t match {
    case Leaf => LL.!
    case Node(v, lt, rt) => LL.::(v)(LL.++(iter(lt))(iter(rt)))
  }
}
```

Implement Treelike for MyTree

```
// behaves like MyTree[A] <: Treelike[A] in OOP
given mytreeTreelike[A] : Treelike[MyTree[A],A] with
  extension (u: Unit)
    def unary_! = Leaf
  extension (a: A)
    def has(l: MyTree[A], r: MyTree[A]) = Node(a,l,r)
  extension (t: MyTree[A])
    def root = t match {
      case Leaf => None
      case Node(v,_,_) => Some(v)
    }
    def left = t match {
      case Leaf => t
      case Node(_,lt,_) => lt
    }
    def right = t match {
      case Leaf => t
      case Node(_,_,rt) => rt }
```

Implement Treelike for MyTree

```
// behaves like MyTree[A] <: Treelike[A] in OOP
implicit def mytreeTreelike[A] : Treelike[MyTree[A],A] = new {
  def ! = Leaf
  def has(a: A)(l: MyTree[A], r: MyTree[A]) = Node(a, l, r)
  def root(t: MyTree[A]) = t match {
    case Leaf => None
    case Node(v, _, _) => Some(v)
  }
  def left(t: MyTree[A]) = t match {
    case Leaf => t
    case Node(_, lt, _) => lt
  }
  def right(t: MyTree[A]) = t match {
    case Leaf => t
    case Node(_, _, rt) => rt
  }
}
```

Linking Modules

```
testList[List[Int]]

testTree[MyTree[Int]]
```

Test for Lazy List

```
def time[R](block: => R): R = {
  val t0 = System.nanoTime()
  val result = block    // call-by-name
  val t1 = System.nanoTime()
  println("Elapsed time: " + ((t1 - t0)/1000000) + "ms"); result
}
def sumN[I](n: Int, t: I)(implicit ITRA: Iterable[I,Int]): Int = {
  def go(res: Int, n: Int, itr: ITRA.Itr): Int =
    if (n <= 0) res
    else itr.getValue match {
      case None => res
      case Some(v) => go(v + res, n - 1, itr.getNext)
    }
  go(0, n, t.iter)
}
```

Test for Lazy List

```
def time[R](block: => R): R = {
  val t0 = System.nanoTime()
  val result = block    // call-by-name
  val t1 = System.nanoTime()
  println("Elapsed time: " + ((t1 - t0)/1000000) + "ms"); result
}
def sumN[I](n: Int, t: I)(implicit ITRA: Iterable[I,Int]): Int = {
  def go(res: Int, n: Int, itr: ITRA.Itr): Int =
    if (n <= 0) res
    else ITRA.ITR.getValue(itr) match {
      case None => res
      case Some(v) => go(v + res, n - 1, ITRA.ITR.getNext(itr))
    }
  go(0, n, ITRA.iter(t))
}
```

Test for Lazy List

```scala
def testTree2[T](implicit TL: Treelike[T,Int], ITRA: Iterable[T,Int]) = {
  def generateTree(n: Int): T = {
    def gen(lo: Int, hi: Int): T = {
      if (lo > hi) !()
      else {
        val mid = (lo + hi) / 2
        mid.has(gen(lo, mid - 1), gen(mid + 1, hi))
      }
    }
    gen(1, n)
  }

  // Problem: takes a few seconds to get a single value
  { val t = generateTree(200000)
    time (sumN(2, t)) }
}
```

Test for Lazy List

```
def testTree2[T](implicit TL: Treelike[T,Int], ITRA: Iterable[T,Int]) = {
  def generateTree(n: Int): T = {
    def gen(lo: Int, hi: Int): T = {
      if (lo > hi) TL.!
      else {
        val mid = (lo + hi) / 2
        TL.has(mid)(gen(lo, mid - 1), gen(mid + 1, hi))
      }
    }
    gen(1, n)
  }

  // Problem: takes a few seconds to get a single value
  { val t = generateTree(200000)
    time (sumN(2, t)) }
}
```

Lazy List

```
sealed abstract class LazyList[+A] {
  def matches[R](caseNil: =>R, caseCons: (A,LazyList[A])=>R) : R
}
case object LNil extends LazyList[Nothing] {
  def matches[R](caseNil: =>R, _u: (Nothing,LazyList[Nothing])=>R) =
    caseNil
}
class LCons[A](hd: A, _tl: =>LazyList[A]) extends LazyList[A] {
  lazy val tl = _tl
  def matches[R](_u: =>R, caseCons: (A, LazyList[A])=>R) =
    caseCons(hd, tl)
}
object LazyList {
  extension [A](l: LazyList[A])
    def append(l2: LazyList[A]) : LazyList[A] =
      l.matches(l2, (hd,tl) => LCons(hd, tl.append(l2)))
}
import LazyList.*
```

Lazy List

```
sealed abstract class LazyList[+A] {
  def matches[R](caseNil: =>R, caseCons: (A,LazyList[A])=>R) : R
}
case object LNil extends LazyList[Nothing] {
  def matches[R](caseNil: =>R, _u: (Nothing,LazyList[Nothing])=>R) =
    caseNil
}
class LCons[A](hd: A, _tl: =>LazyList[A]) extends LazyList[A] {
  lazy val tl = _tl
  def matches[R](_u: =>R, caseCons: (A, LazyList[A])=>R) =
    caseCons(hd, tl)
}
object LazyList {
  def append[A](l: LazyList[A])(l2: LazyList[A]) : LazyList[A] =
    l.matches(l2, (hd,tl) => LCons(hd, append(tl)(l2)))
}
import LazyList.*
```

Lazy List

```
given lazylistListlike[A]: Listlike[LazyList[A],A] with
  extension (u: Unit)
    def unary_! = LNil
  extension (a: A)
    def ::(l: =>LazyList[A]) = LCons(a,l)
  extension (l: LazyList[A])
    def head = l.matches(None, (hd,tl) => Some(hd))
    def tail = l.matches(LNil, (hd,tl)=>tl)
    def ++(l2: LazyList[A]) = l.append(l2)


testList[LazyList[Int]]
testTree[MyTree[Int]]
testTree2[MyTree[Int]]
```

Lazy List

```
implicit def lazylistListlike[A]: Listlike[LazyList[A],A] = new {
  def ! = LNil
  def ::(a: A)(l: => LazyList[A]) = LCons(a, l)
  def head(l: LazyList[A]) = l.matches(None, (hd, tl) => Some(hd))
  def tail(l: LazyList[A]) = l.matches(LNil, (hd, tl) => tl)
  def ++(l: LazyList[A])(l2: LazyList[A]) = LazyList.append(l)(l2)
}

testList[LazyList[Int]]
testTree[MyTree[Int]]
testTree2[MyTree[Int]]
```

Type class: Code Reuse

IntStack Spec

```
trait Stack[S,A]:
  extension (u: Unit)
    def empty : S
  extension (s: S)
    def get: (A,S)
    def put(a: A): S

def testStack[S](implicit STK: Stack[S,Int]) = {
  val s = ().empty.put(3).put(-2).put(4)
  val (v1,s1) = s.get
  val (v2,s2) = s1.get
  (v1,v2)
}
```

IntStack Spec

```
trait Stack[S,A]:
  def empty : S
  def get(s: S): (A,S)
  def put(s: S)(a: A): S

def testStack[S](implicit STK: Stack[S,Int]) = {
  val s = STK.put(STK.put(STK.put(STK.empty)(3))(-2))(4)
  val (v1,s1) = STK.get(s)
  val (v2,s2) = STK.get(s1)
  (v1,v2)
}
```

Implementation using List

```
given BasicStack[A] : Stack[List[A],A] with
  extension (u: Unit)
    def empty = List()
  extension (s: List[A])
    def get = (s.head, s.tail)
    def put(a: A) = a :: s
```

Implementation using List

```
implicit def BasicStack[A] : Stack[List[A],A] = new {
  def empty = List()
  def get(s: List[A]) = (s.head, s.tail)
  def put(s: List[A])(a: A) = a :: s
}
```

Modifying Traits

```
def StackOverridePut[S,A](newPut: (S,A)=>S)(implicit STK: Stack[S,A])
: Stack[S,A] = new {
  extension (u: Unit)
    def empty = STK.empty(u)
  extension (s: S)
    def get = STK.get(s)
    def put(a: A) = newPut(s,a)
}

def Doubling[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => s.put(2 * a))

def Incrementing[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => s.put(a + 1))

def Filtering[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => if (a >= 0) s.put(a) else s)
```

Modifying Traits

```
def StackOverridePut[S,A](newPut: (S,A)=>S)(implicit STK: Stack[S,A])
: Stack[S,A] = new {
  def empty = STK.empty
  def get(s: S) = STK.get(s)
  def put(s: S)(a: A) = newPut(s,a)
}

def Doubling[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => STK.put(s)(2 * a))

def Incrementing[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => STK.put(s)(a + 1))

def Filtering[S](implicit STK: Stack[S,Int]) : Stack[S,Int] =
  StackOverridePut((s,a) => if (a >= 0) STK.put(s)(a) else s)
```

Linking

```
// testStack(BasicStack)
testStack

// testStack(Filtering(Incrementing (Doubling(BasicStack))))
testStack(Filtering (Incrementing (Doubling)))

// testStack(Filtering(Incrementing(Incrementing(Doubling(BasicStack)))))
testStack(Filtering (Incrementing (Incrementing (Doubling))))
```

Implementation: Sorted Stack

```
def SortedStack : Stack[List[Int],Int] = new {
  extension (u: Unit)
    def empty = List()
  extension (s: List[Int])
    def get = (s.head, s.tail)
    def put(a: Int) : List[Int] = {
      def loop(l: List[Int]) : List[Int] = l match {
        case Nil => a :: Nil
        case hd :: tl => if (a <= hd) a :: l else hd :: loop(tl)
      }
      loop(s)
    }
  }

testStack(Filtering(Incrementing(Doubling(SortedStack))))
```

Implementation: Sorted Stack

```
def SortedStack : Stack[List[Int],Int] = new {
  def empty = List()
  def get(s: List[Int]) = (s.head, s.tail)
  def put(s: List[Int])(a: Int) : List[Int] = {
    def loop(l: List[Int]) : List[Int] = l match {
      case Nil => a :: Nil
      case hd :: tl => if (a <= hd) a :: l else hd :: loop(tl)
    }
    loop(s)
  }
}

testStack(Filtering(Incrementing(Doubling(SortedStack))))
```

Higher Type Classes

Interfaces I

```
// eg. Iter[List]
trait Iter[I[_]]:
  extension [A](i: I[A])
    def getValue: Option[A]
    def getNext: I[A]

// trait Iter[I,A]:
//   extension (i: I)
//     def getValue: Option[A]
//     def getNext: I

// eg. Iterable[MyTree]
trait Iterable[I[_]]:
  type Itr[_]
  given ITR: Iter[Itr]
  extension [A](i: I[A])
    def iter: Itr[A]

// trait Iterable[I,A]:
//   type Itr
//   given ItrI: Iter[Itr,A]
//   extension (i: I)
//     def iter: Itr

given iter2iterable[I[_]](using _ITR: Iter[I]): Iterable[I] with
  type Itr[A] = I[A]
  def ITR = _ITR
  extension [A](i:I[A])
    def iter = i
```

Interfaces I

```
// eg. Iter[List]
trait Iter[I[_]]:
  def getValue[A](i: I[A]): Option[A]
  def getNext[A](i: I[A]): I[A]

// eg. Iterable[MyTree]
trait Iterable[I[_]]:
  type Itr[_]
  implicit def ITR: Iter[Itr]
  def iter[A](i: I[A]): Itr[A]

implicit def iter2iterable[I[_]](using _ITR: Iter[I]): Iterable[I] = new {
  type Itr[A] = I[A]
  def ITR = _ITR
  def iter[A](i:I[A]) = i
}
```

Programs for Testing: use Iter, Iterable

```
def sumElements[I[_]](xs: I[Int])(implicit ITRA:Iterable[I]) = {
  def loop(i: ITRA.Itr[Int]): Int =
    i.getValue match {
      case None => 0
      case Some(n) => n + loop(i.getNext)
    }
  loop(xs.iter)
}

def printElements[I[_],A](xs: I[A])(implicit ITRA: Iterable[I]) = {
  def loop(i: ITRA.Itr[A]): Unit =
    i.getValue match {
      case None =>
      case Some(a) => {println(a); loop(i.getNext)}
    }
  loop(xs.iter)
}
```

Programs for Testing: use Iter, Iterable

```
def sumElements[I[_]](xs: I[Int])(implicit ITRA:Iterable[I]) = {
  def loop(i: ITRA.Itr[Int]): Int =
    ITRA.ITR.getValue(i) match {
      case None => 0
      case Some(n) => n + loop(ITRA.ITR.getNext(i))
    }
  loop(ITRA.iter(xs))
}

def printElements[I[_],A](xs: I[A])(implicit ITRA: Iterable[I]) = {
  def loop(i: ITRA.Itr[A]): Unit =
    ITRA.ITR.getValue(i) match {
      case None =>
      case Some(a) => {println(a); loop(ITRA.ITR.getNext(i))}
    }
  loop(ITRA.iter(xs))
}
```

Interfaces II

```
trait Listlike[L[_]]:
  extension[A](u:Unit)
    def unary_! : L[A]
  extension[A](elem:A)
    def ::(l: =>L[A]): L[A]
  extension[A](l: L[A])
    def head: Option[A]
    def tail: L[A]
    def ++(l2: L[A]): L[A]

trait Treelike[T[_]]:
  extension[A](u:Unit)
    def unary_! : T[A]
  extension[A](a:A)
    def has(lt: T[A], rt: T[A]): T[A]
  extension[A](t: T[A])
    def root : Option[A]
    def left : T[A]
    def right : T[A]
```

Interfaces II

```
trait Listlike[L[_]]:
  def ![A] : L[A]
  def ::[A](elem:A)(l: =>L[A]): L[A]
  def head[A](l: L[A]): Option[A]
  def tail[A](l: L[A]): L[A]
  def ++[A](l: L[A])(l2: L[A]): L[A]

trait Treelike[T[_]]:
  def ![A] : T[A]
  def has[A](a:A)(lt: T[A], rt: T[A]): T[A]
  def root[A](t: T[A]) : Option[A]
  def left[A](t: T[A]) : T[A]
  def right[A](t: T[A]) : T[A]
```

Programs for Testing: use All

```
def testList[L[_]](implicit LL: Listlike[L], ITRA: Iterable[L]) = {
  val l = (3 :: !()) ++ (1 :: 2 :: !())
  println(sumElements(l))
  printElements(l)
}

def testTree[T[_]](implicit TL: Treelike[T], ITRA: Iterable[T]) = {
  val t = 3.has(4.has(!(), !()), 2.has(!(),!()))
  println(sumElements(t))
  printElements(t)
}
```

Programs for Testing: use All

```
def testList[L[_]](implicit LL: Listlike[L], ITRA: Iterable[L]) = {
  val l = LL.++(LL.::(3)(LL.!))(LL.::(1)(LL.::(2)(LL.!)))
  println(sumElements(l))
  printElements(l)
}

def testTree[T[_]](implicit TL: Treelike[T], ITRA: Iterable[T]) = {
  val t = TL.has(3)(TL.has(4)(TL.!, TL.!), TL.has(2)(TL.!,TL.!))
  println(sumElements(t))
  printElements(t)
}
```

List: provide Iter, ListIF

```
// behaves like List[A] <: Iter[A] in OOP
given listIter: Iter[List] with
  extension [A](l: List[A])
    def getValue = l.headOption
    def getNext = l.tail

// behaves like List[A] <: Listlike[A] in OOP
given listListlike: Listlike[List] with
  extension [A](u: Unit)
    def unary_! = Nil
  extension [A](a: A)
    def ::(l: =>List[A]) = a::l
  extension [A](l: List[A])
    def head = l.headOption
    def tail = l.tail
    def ++(l2: List[A]) = l ::: l2
```

List: provide Iter, ListIF

```
// behaves like List[A] <: Iter[A] in OOP
implicit def listIter: Iter[List] = new {
  def getValue[A] (l: List[A]) = l.headOption
  def getNext[A] (l: List[A]) = l.tail
}
// behaves like List[A] <: Listlike[A] in OOP
implicit def listListlike: Listlike[List] = new {
  def ![A] = Nil
  def ::[A](a: A)(l: => List[A]) = a :: l
  def head[A](l: List[A]) = l.headOption
  def tail[A](l: List[A]) = l.tail
  def ++[A](l: List[A])(l2: List[A]) = l ::: l2
}
```

MyTree: use Iter, ListIF, provide Iterable, TreeIF

```
enum MyTree[+A]:
  case Leaf
  case Node(value: A, left: MyTree[A], right: MyTree[A])
import MyTree._

// behaves like MyTree[A] <: Iterable[A], but clumsy in OOP
given treeIterable[L[_]](using LL: Listlike[L], _ITR: Iter[L]): Iterable[MyTree]
with
  type Itr[A] = L[A]
  def ITR = _ITR
  extension [A](t: MyTree[A])
    def iter: L[A] = t match {
      case Leaf => !()
      case Node(v, lt, rt) => v :: (lt.iter ++ rt.iter)
    }
```

MyTree: use Iter, ListIF, provide Iterable, TreeIF

```
enum MyTree[+A]:
  case Leaf
  case Node(value: A, left: MyTree[A], right: MyTree[A])
import MyTree._

// behaves like MyTree[A] <: Iterable[A], but clumsy in OOP
implicit def treeIterable[L[_]](using LL: Listlike[L], _ITR: Iter[L]):
Iterable[MyTree] = new {
  type Itr[A] = L[A]
  def ITR = _ITR
  def iter[A] (t: MyTree[A]): L[A] = t match {
    case Leaf => LL.!
    case Node(v, lt, rt) => LL.::(v)(LL.++(iter(lt))(iter(rt)))
  }
}
```

MyTree: use Iter, ListIF, provide Iterable, TreeIF

```
// behaves like MyTree[A] <: Treelike[A] in OOP
given mytreeTreelike: Treelike[MyTree] with
  extension [A](u: Unit)
    def unary_! = Leaf
  extension [A](a: A)
    def has(l: MyTree[A], r: MyTree[A]) = Node(a,l,r)
  extension [A](t: MyTree[A])
    def root = t match {
      case Leaf => None
      case Node(v,_,_) => Some(v)
    }
    def left = t match {
      case Leaf => t
      case Node(_,lt,_) => lt
    }
    def right = t match {
      case Leaf => t
      case Node(_,_,rt) => rt }
```

MyTree: use Iter, ListIF, provide Iterable, TreeIF

```
// behaves like MyTree[A] <: Treelike[A] in OOP
implicit def mytreeTreelike: Treelike[MyTree] = new {
  def ![A] = Leaf
  def has[A] (a: A)(l: MyTree[A], r: MyTree[A]) = Node(a, l, r)
  def root[A](t: MyTree[A]) = t match {
    case Leaf => None
    case Node(v, _, _) => Some(v)
  }
  def left[A](t: MyTree[A]) = t match {
    case Leaf => t
    case Node(_, lt, _) => lt
  }
  def right[A](t: MyTree[A]) = t match {
    case Leaf => t
    case Node(_, _, rt) => rt
  }
}
```

Linking Modules

```
testList[List]

testTree[MyTree]
```

List with Map

```
trait Maplike[L[_]]:
  extension[A](l: L[A])
    def map[B](f: A => B): L[B]

def testMapList[L[_]](implicit LL: Listlike[L], ML: Maplike[L], ITR: Iter[L]) = {
  val l1 = 3.3 :: 2.2 :: 1.5 :: !()
  val l2 = l1.map((n:Double)=>n.toInt)
  val l3 = l2.map((n:Int)=>n.toString)
  printElements(l3)
}
```

List with Map

```
trait Maplike[L[_]]:
  def map[A,B](l: L[A])(f: A => B): L[B]

def testMapList[L[_]] (implicit LL: Listlike[L], ML: Maplike[L], ITR: Iter[L]) = {
  val l1 = LL.::(3.3)(LL.::(2.2)(LL.::(1.5)(LL.!)))
  val l2 = ML.map(l1)((n:Double)=>n.toInt)
  val l3 = ML.map(l2)((n:Int)=>n.toString)
  printElements(l3)
}
```

List with Map

```
given listMaplike: Maplike[List] with
extension [A](l: List[A])
   def map[B](f: A => B) = l.map(f)

testMapList[List]
```

List with Map

```
implicit def listMaplike: Maplike[List] = new {
  def map[A,B](l: List[A])(f: A => B) = l.map(f)
}

testMapList[List]
```

Turning Type Classes into OO Classes

Interfaces

```
trait DataProcessor[D]:
  extension (d: D)
    def input(s: String) : D
    def output : String

trait DPFactory:
  extension (u: Unit)
    def getTypes: List[String]
    def makeDP(dptype: String) : ???

def run(implicit factory: DPFactory) : Unit
```

How to return data with associated functions like OOP?

Turning Type Classes into OO Classes

```
import scala.language.implicitConversions
type curry1[F[_,_],A1] = ([X] =>> F[X,A1])
type curry2[F[_,_,_],A1,A2] = ([X] =>> F[X,A1,A2])
type curry3[F[_,_,_,_],A1,A2,A3] = ([X] =>> F[X,A1,A2,A3])

trait dyn[S[_]]:
  type Data
  val * : Data
  given DI: S[Data]

object dyn {
  implicit // needed for implicit conversion of D into dyn[S]
  def apply[S[_],D](d: D)(implicit i: S[D]): dyn[S] = new {
      type Data = D
      val * = d
      val DI = i
    }
}
```

Turning Type Classes into OO Classes

```
import scala.language.implicitConversions
type curry1[F[_,_],A1] = ([X] =>> F[X,A1])
type curry2[F[_,_,_],A1,A2] = ([X] =>> F[X,A1,A2])
type curry3[F[_,_,_,_],A1,A2,A3] = ([X] =>> F[X,A1,A2,A3])

trait dyn[S[_]]:
  type Data
  val * : Data
  implicit def DI: S[Data]

object dyn {
  implicit // needed for implicit conversion of D into dyn[S]
  def apply[S[_],D](d: D)(implicit i: S[D]): dyn[S] = new {
    type Data = D
    val * = d
    val DI = i
  }
}
```

Interfaces

```
trait DataProcessor[D]:
  extension (d: D)
    def input(s: String): D
    def output: String

trait DPFactory:
  extension (u: Unit)
    def getTypes: List[String]
    def makeDP(dptype: String): dyn[DataProcessor]
```

Interfaces

```
trait DataProcessor[D]:
  def input(d: D)(s: String): D
  def output(d: D): String

trait DPFactory:
  def getTypes: List[String]
  def makeDP(dptype: String): dyn[DataProcessor]
```

Test

```
def test(implicit DF: DPFactory) = {
  def go(types: List[String]) : Unit =
    types match {
      case Nil => ()
      case ty :: rest => {
        val dp = ().makeDP(ty)
        println(dp.*.input("10").input("20").output)
        go(rest)
      }
    }
  val types = ().getTypes
  println(types)
  go(types)
}
```

Test

```
def test(implicit DF: DPFactory) = {
  def go(types: List[String]) : Unit =
    types match {
      case Nil => ()
      case ty :: rest => {
        val dp : dyn = DF.makeDP(ty)
        println(dp.DI.output(dp.DI.input(dp.DI.input(dp.*)("10"))("20")))
        go(rest)
      }
    }
  val types = DF.getTypes
  println(types)
  go(types)
}
```

Data Processor

```
given dpfactory: DPFactory with
  extension (u: Unit)
    def getTypes = List("sum", "mult")
    def makeDP(dptype: String) = {
      if (dptype == "sum")
        makeProc(0, (x, y) => x + y)
      else
        makeProc(1, (x, y) => x * y)
    }

    def makeProc(init: Int, op: (Int, Int) => Int): dyn[DataProcessor] = {
      given dp: DataProcessor[Int] with
        extension (d: Int)
          def input(s: String) = op(d, s.toInt)
          def output = d.toString()
      init       // dyn(init) // dyn.apply[Int,DataProcessor](init)(dp)
    }
```

Data Processor

```
implicit val dpfactory: DPFactory = new {
  def getTypes = List("sum", "mult")
  def makeDP(dptype: String) = {
    if (dptype == "sum")
      makeProc(0, (x, y) => x + y)
    else
      makeProc(1, (x, y) => x * y)
  }

  def makeProc(init: Int, op: (Int, Int) => Int): dyn[DataProcessor] = {
    implicit def dp: DataProcessor[Int] = new {
      def input(d: Int)(s: String) = op(d, s.toInt)
      def output(d: Int) = d.toString()
    }
    init // dyn(init)(dp) // dyn.apply[Int,DataProcessor](init)(dp)
  }
}
```

Linking

test

Heterogeneous List of Iter

```
trait Iter[I,A]:
  extension (i: I)
    def getValue: Option[A]
    def getNext: I

def sumElements[I](xs: I)(implicit ITR:Iter[I,Int]) : Int = {
  xs.getValue match {
    case None => 0
    case Some(n) => n + sumElements(xs.getNext)
  }
}

def sumElementsList(xs: List[dyn[curry1[Iter,Int]]]) : Int =
  xs match {
    case Nil => 0
    case hd :: tl => sumElements(hd.*) + sumElementsList(tl)
  }
```

Heterogeneous List of Iter

```
trait Iter[I,A]:
  def getValue(i: I): Option[A]
  def getNext(i: I): I

def sumElements[I](xs: I)(implicit ITR:Iter[I,Int]) : Int = {
  ITR.getValue(xs) match {
    case None => 0
    case Some(n) => n + sumElements(ITR.getNext(xs))
  }
}

def sumElementsList(xs: List[dyn[curry1[Iter,Int]]]) : Int =
  xs match {
    case Nil => 0
    case hd :: tl => sumElements(hd.*) + sumElementsList(tl)
  }
```

Test

```scala
given listIter[A]: Iter[List[A],A] with
  extension (l: List[A])
    def getValue = l.headOption
    def getNext = l.tail

given decIter : Iter[Int,Int] with
  extension (i: Int)
    def getValue = if (i >= 0) Some(i) else None
    def getNext = i - 1

sumElementsList(List(
  100,
  List(1,2,3),
  10))
```

Test

```scala
implicit def listIter[A]: Iter[List[A],A] = new {
  def getValue(l: List[A]) = l.headOption
  def getNext(l: List[A]) = l.tail
}

implicit val decIter : Iter[Int,Int] = new {
  def getValue(i: Int) = if (i >= 0) Some(i) else None
  def getNext(i: Int) = i - 1
}

sumElementsList(List(
  100,
  List(1,2,3),
  10))
```

Additional Resources

- UCSD CSE 130
  - http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/00-crash.html
  - http://cseweb.ucsd.edu/classes/wi14/cse130-a/lectures/scala/01-iterators.html

PART 4

Imperative Programming with Memory Updates

Mutable Update in Scala

Mutable Variables

- Mutable Variables
  - Use “var” instead of “val” and “def”
  - We can update the value stored in a variable.

```
class Main(i: Int) {
  var a = i
}

val m = new Main(10)
m.a  // 10
m.a = 20
m.a  // 20
m.a += 5  // m.a = m.a + 5
m.a  // 25
```

While loop

- While loop
  - Syntax: `while (`*cond*`)` *body*
    Executes *body* while *cond* holds.
  - It is equivalent to:

```
def mywhile(cond: =>Boolean)(body: =>Unit) : Unit =
  if (cond) { body; mywhile(cond)(body) } else ()
```

- Example

```
var i = 0
var sum = 0
while (i <= 100) { // mywhile (i <= 100) {
  sum += i
  i += 2
}
sum // 2550
```

For loop

- For loop
  - Syntax: `for (i <- collection) body`
    Executes *body* for each `i` in *collection*.
  - It is equivalent to:

```
def myfor[A](xs: Traversable[A])(f: A => Unit) : Unit =
  xs.foreach(f)
```

- Example

```
var sum = 0
for (i <- 0 to 100 by 2) { // myfor (0 to 100 by 2) { i =>
  sum += i
}
sum // 2550
```

Immutability, Mutability & Ownership

Immutability for Guarantee & Mutability for Efficiency

- Immutable array

  arr = Array.new([1;2;3])

  … using arr …

  arr’ = Array.set(arr, 0, 42)

  f(arr’)

  … using arr’ …

  g()

  … using arr’ …

- Mutable array

  arr = Array.new([1;2;3])

  … using arr …

  Array.set(arr, 0, 42);

  f(arr)

  … using arr …

  g()

  … using arr …

Can we not have both? Mutability with Ownership!

- Immutable array

```
arr = Array.new([1;2;3])

… using arr …

arr' = Array.set(arr, 0, 42)

f(arr')

… using arr' …

g()

… using arr' …
```

- Mutation with Ownership

```
// own
arr = Array.new([1;2;3])

… using arr …
// mutable borrow
Array.set(&mut arr, 0, 42)
// immutable borrow
f(&arr)

… using arr …

g()

… using arr …
```

Types with Ownership

- Ownership Types

  - expresses and guarantees immutability
    - making code behavior more predictable

  - automatically deallocates memory when its ownership has gone
    - guaranteeing absence of use-after-free, double-free, memory-leak

  - disallows mutating the same value at the same time
    - guaranteeing data-race freedom in concurrent code

Programming with Mutation in a Principled Way!

The Rust Programming Language Book

https://doc.rust-lang.org/book/

Types, Values & Memory

Values and Types

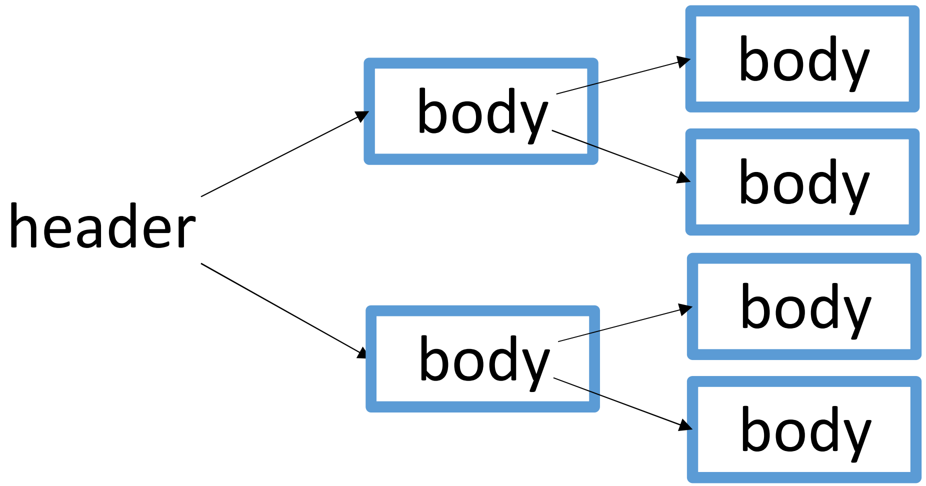
A type defines a set of values.

- For functionality, we only need to know the denotation of values.
- For efficiency, we also need to know the shape of values.

A value has a tree structure and consists of a header and bodies.

- A header is data that may contain scalar values and pointers to bodies.
- A body is data that must be stored in memory and may contain scalar values and pointers to other bodies.
- When passing or storing a value, only its header is passed or stored.

Types in Rust: i64, u64, (T1,T2), [T; 5], String, Vec<T>, ...



Stack memory with static size and static lifetime

Stack memory consists of variables of sized types (ie, static header sizes) with scopes (ie, static lifetime).

A stack variable of a type T stores a header of the type T.

```
// variable arg of type i64 (8-byte header) with Scope 1
fn foo(arg: i64) -> i64 {
    let mut x = 10;   // variable x of type i64 with Scope 1
    {
        let y = arg + x;   // variable y of type i64 with Scope 2
        x = x + y;
    }  // Scope 2
    let x2 = x; // variable x2 of type i64 with Scope 1
    let z = arg + x2;   // variable z of type i64 with Scope 1
    return z;
}   // Scope 1
```

Heap memory with dynamic size and dynamic lifetime

Heap memory consists of blocks of all types (ie, dynamic sizes) with no scopes (ie, dynamic lifetime).
A heap block can be a body of a value, which can also store headers of other values.

```
// variable sz of type usize (8-byte header) with Scope 1
fn gee(sz: usize) -> Vec<i64> {
// variable v of type Vec<i64> (24-byte header) with Scope 1
// The header points to a heap block of (sz * 8) bytes, filled with zeros
    let v : Vec<i64> = vec![0; sz];
// The header goes out of Scope 1 and the heap block is still alive
    return v;
}   // Scope 1
```

Principles of Ownership & Lifetime

Issues with mutation and deallocation for memory

<Problematic mutations>
Multiple parties modify a value logically simultaneously.

- Problem
  The value may become inconsistent despite valid individual updates.

A value is read logically while being modified.

- Problem
  An inconsistent intermediate state may be read.

<Problematic deallocation>
A value's body is deallocated while its header remains accessible.

- Problem
  The deallocated body may be accessed via the header (use-after-free bug).

Principles of Ownership

<Ownership rules>

- A header must be stored in exactly one location, which owns the header.
- Every heap block must be owned by exactly one header that directly points to it.
- Headers can be moved between locations.
- Headers are dropped when their owner is freed or overwritten.
- When dropped, a header's owned heap blocks are also freed.
- A stack variable is freed when it goes out of scope.

<Borrowing rules> (known as, "mutation-xor-sharing" checking)

- Mutable: single borrower has exclusive read/write access.
- Immutable: multiple borrowers and owner can read.

Principles of Lifetime

<Lifetime definition>

- Stack variables have a static lifetime bound by their scope.
- Heap blocks have a dynamic lifetime that ends when their owner is dropped.
- Headers have a dynamic lifetime that ends when their owner is freed or overwritten.

<Lifetime rules for borrowing> (known as, "borrow lifetime checking")

- A location's lifetime must be longer than any of its borrowers' lifetimes.

An example with ownership and borrowing

An example showing how ownership and borrowing work.

```
// arg owns a string, say ARG
fn gee(arg: String) -> String {
    let mut x = String::from("abc"); // x owns the string "abc"
    {
        let y = x + &arg;  // y owns the string "abc"+ARG, x not accessible
        x = y + &arg;  // x owns the string "abc"+ARG+ARG, y not accessible
    } // y is deallocated
    let z = x; // z owns the string "abc"+ARG+ARG, x not accessible
    return z; // the string "abc"+ARG+ARG is returned, z not accessible
} // arg, x, z are dallocated, the string ARG in arg is deallocated
```

Rust's approach

- References to a location of type T:
  - &T: immutable reference
  - &mut T: mutable reference

- Function signatures specify ownership/lifetime conditions

- Compiler checks ownership/lifetime in Safe Rust at two levels:
  - Stack variables (assuming function signatures)
  - Function implementations (verifying against signatures)

- Heap Memory Management:
  - Experts implement primitive heap types in Unsafe Rust with ownership/lifetime guarantees specified in function signatures
  - Users compose these primitives in Safe Rust to build complex data structures

Datatypes with Ownership & Lifetime

Scalar Types

```
fn main() {
    let mut x = [1,2,3,4];
    println!("{}", x[2]);
    x[2] = 42;
    println!("{}", x[2]);
    let y = x;
    println!("{}", x[0]);
    println!("{}", y[0]);
}
```

String and str types

```
fn main() {
    let mut s : &str = "abc";
    {
        let mut st : String = String::from(s);
        st.push_str("def");   // String::push_str(&mut st, "def");
        println!("{}", st);
        let s2 : &str = &st[1..4];
        // st.push_str("ghi");
        println!("{}", s2);
        // s = s2;
    }
    // println!("{}", s);
}
```

Function Types

```
fn foo(s: &String) -> &str {
    return &s[0..2];
}
fn gee(f: fn(&String) -> &str) {
    let s;
    // {
        let a = String::from("test");
        s = f(&a);
    // }
    println!("{}", s);
}
fn main() {
    gee(foo);
}
```

Function Types with lifetime annotation

```
fn foo<'a>(s: &'a String) -> &'a str {
    return &s[0..2];
}
fn gee (f: for <'a> fn(&'a String) -> &'a str) {
    let s;
    // {
        let a = String::from("test");
        s = foo(&a);
    // }
    println!("{}", s);
}
fn main() {
    gee(foo);
}
```

General lifetime annotation

```
fn foo1<'a>(s1: &'a String, s2: &'a String) -> &'a str {
    …
}
fn foo2<'a,'b>(s1: &'a String, s2: &'b String) -> &'a str {
    …
}
fn foo3<'a,'b,'c>(s1: &'a String, s2: &'b String) -> &'c str {
    …
}
fn foo4<'a,'b,'c, 'd, 'e>(s1: &'a String, s2: &'b String, s3: &'c String)
     -> (&'d str, &'e str)
where 'a: 'd, 'b:'d, 'b:'e, 'c:'e {
    …
}
```

General lifetime annotation

```
fn foo1<'a>(s1: &'a String, s2: &'a String) -> &'a str {
    return if rand::random() { &s1[0..1] } else { &s2[0..2] }; }
fn foo2<'a,'b>(s1: &'a String, s2: &'b String) -> &'a str {
    return &s1[0..1]; }
fn foo3<'a,'b,'c>(s1: &'a String, s2: &'b String) -> &'c str {
    return "abc"; }
fn foo4<'a,'b,'c, 'd, 'e>(s1: &'a String, s2: &'b String, s3: &'c String)
     -> (&'d str, &'e str)
where 'a: 'd, 'b:'d, 'b:'e, 'c:'e {
    let r1 = if rand::random() { &s1[0..1] } else { &s2[0..1] };
    let r2 = if rand::random() { &s2[0..1] } else { &s3[0..1] };
    (r1,r2)
}
```

Struct

```
struct User {
    active: bool,
    name: String,
}
fn main() {
    println!("Size: {} bytes", std::mem::size_of::<User>());
    let mut user_name = String::from("gil");
    let mut u = User { active: true, name: user_name };
    u.name.push_str(" hur");
    println!("{}, {}", u.active, u.name);
    user_name = u.name;
    println!("{}, {}", u.active, user_name);
    // println!("{} {}", u.active, u.name);
}
```

Struct with lifetime annotation

```
// Key Idea: Track the lifetime of each field of a struct separately.
struct User<'a,'b> { name: &'a mut String, email: &'b String }
fn main() {
    println!("Size: {} bytes", std::mem::size_of::<User>());
    let mut user_name = String::from("gil");
    let temp : &String;
    {   let mut user_email = String::from("gil.hur@sf.snu.ac.kr");
        let mut u = User { name: &mut user_name, email: &user_email };
        u.name.push_str(" hur");
        println!("{}, {}", u.name, u.email);
        temp = u.name; // temp = u.email;
        // u.name.push_str("xxx");
    }
    println!("{}", temp);
}
```

Struct with lifetime annotation (Variations)

```
// Key Idea: Track the lifetime of each field of a struct separately.
// Easy to understand if you inline the definition of a struct.
struct User1<'a> {
  name: &'a String,
  email: &'a String }
// User1 : for <'a> fn(&'a String, &'a String) : User1<'a>
// User1::name : for <'a> fn(User1<'a>) : &'a String
// User1::email : for <'a> fn(User1<'a>) : &'a String

struct User2<'a,'b> {
  user: User1<'a>,
  addr: &'b String }
// User2 : for <'a,'b> fn(User1<'a>, &’b String) : User2<'a,'b>
// User2::user : for <'a,'b> fn(User2<'a,'b>) : User1<'a>
// User2::addr : for <'a,'b> fn(User2<'a,'b>) : &'b String
```

Struct with lifetime annotation (Variations)

```
struct Foo<'a,'b> {
    x: &'a i32,
    y: &'b i32,
}

fn main() {
    let x = 1;
    let v;
    {
        let y = 2;
        let f = Foo { x: &x, y: &y };
        v = f.x;
    }
    println!("{}", *v);
}
```

Enum

```
enum IpAddr {
    V4(u8, u8, u8, u8),
    V6(String),
}
use IpAddr::*;
fn main() {
    let ip1 = V4(147,36,52,255);
    match ip1 {  // match &ip1 // match &mut ip1
        V4(a1,a2,a3,a4) => println!("{a1:?} {a2:?} {a3:?} {a4:?}"),
        V6(addr) => println!("{addr:?}")   }
    let ip2 = V6(String::from("123.123.123.123.123.123"));
    match &ip2 {
        V4(a1,a2,a3,a4) => println!("{a1:?} {a2:?} {a3:?} {a4:?}"),
        V6(addr) => println!("{addr:?}")   }
}
```

Recursive Enum with Box

```
enum List<T> { Nil, Cons(T, Box<List<T>>) }
impl <T> List<T> {
    fn new() -> List<T> { List::Nil }
    fn cons(hd: T, tl: List<T>) -> List<T> {  List::Cons(hd, Box::new(tl)) }
}
fn mylen<T>(l: &List<T>) -> u64 {
    match l {
        List::Nil => 0,
        List::Cons(_, tl) => 1+mylen(tl) // 1+mylen(tl.as_ref())
    }
}
fn main() {
    let l = List::cons(0, List::cons(1, List::cons(2, List::new())));
    println!("{}", mylen(&l));
}
```

Update via a mutable reference

```
use std::mem;
fn test(x: &mut List<i32>) {
    // *x = List::cons(0, *x)
    let old = mem::replace(x, List::new());
    *x = List::cons(0, old)
}

fn main() {
    let mut l = List::cons(0, List::cons(1, List::cons(2, List::new())));
    println!("{}", mylen(&l));
    test(&mut l);
    println!("{}", mylen(&l));
}
```

Caveat: subtle immutability in let (seems a design bug)

- “let x : T” only guarantees its owned parts are unchanged
- “&x” guarantees that all data reachable from x are unchanged

```
struct Foo<'a> {
    x: i64,
    s: &'a mut String
}
fn main() {
    let mut str = "abc".to_string();
    let foo = Foo {x: 42, s: &mut str};
    println!("{} {}", foo.x, foo.s);
    foo.s.push_str("def");
    // (&foo).s.push_str("def");
    // foo.x = 37;
    println!("{} {}", foo.x, foo.s);
}
```

Smart Pointers

Box type

```
enum List<T> {
  Nil,
  Cons(T, Box<List<T>>)
}
```

Rc type

Use Rc type to replace static borrow lifetime checking with dynamic checking.

<Problem>

```
enum List<T> {
    Cons(T, Rc<List<T>>),
    Nil,
}
use crate::List::*;
fn test() -> (List<i64>,List<i64>) {
    let a = Cons(5, Box::new(Nil));
    let b = Cons(3, Box::new(a));
    let c = Cons(4, Box::new(a));
    (b,c)
}
```

Rc type

```
use std::rc::Rc;
enum List<T> { Cons(T, Rc<List<T>>), Nil, }
use crate::List::*;
impl <T> List<T> {
    fn unfold(&self) -> Option<(&T,&List<T>)> {
        if let Cons(hd,tl) = self { Some((hd, tl.as_ref())) } else { None }
    } }
fn test() -> (List<i64>,List<i64>) {
    let a = Rc::new(Cons(5, Rc::new(Nil)));
    let b = Cons(3, Rc::clone(&a));
    let c = Cons(4, Rc::clone(&a));
    (b,c)   }
fn main() {
    let (l1, l2) = test();   { let _t = l1; }
    println!("{}", l2.unfold().unwrap().1.unfold().unwrap().0)   }
```

RefCell type

Use RefCell type to replace static mutation-xor-sharing checking with dynamic checking.

<Problem>

Want to mutate the shared list in the previous example.

RefCell type

```
use std::rc::Rc; use std::cell::{RefCell, Ref, RefMut};
enum List<T> {
    Cons(T, Rc<RefCell<List<T>>>),
    Nil,
}
use crate::List::{Cons, Nil};
impl <T> List<T> {
    fn unfold<'a>(&'a self) -> Option<(&'a T, Ref<'a, List<T>>)> {
        if let Cons(hd,tl) = self { Some((hd, tl.borrow())) } else { None }
                                                  // tl.as_ref().borrow()
    }
    fn unfold_mut<'a>(&'a mut self) -> Option<(&'a mut T, RefMut<'a,
List<T>>)> {
        if let Cons(hd,tl) = self { Some((hd,tl.borrow_mut())) } else { None }
    } }
```

RefCell type

```
fn test() -> (List<i64>,List<i64>) {
    let a = Rc::new(RefCell::new(Cons(5, Rc::new(RefCell::new(Nil)))));
    let b = Cons(3, Rc::clone(&a));
    let c = Cons(4, Rc::clone(&a));
    (b,c)
}
fn main() {
    let (mut l1, l2) = test();
    {
        let mut r = l1.unfold_mut().unwrap().1;
        *r = Cons(42, Rc::new(RefCell::new(Nil)));
    }
    println!("{}", l2.unfold().unwrap().1.unfold().unwrap().0)
}
```

Type class and Closure

Type class

```
trait Describable {
 fn describe(&self) -> String;
}
struct Book { title: String, author: String, }
impl Describable for Book {
  fn describe(&self) -> String {
    format!("{} by {}", self.title, self.author)
  } }

//fn print_desc(a: &impl Describable) -> ()
fn print_desc<T>(a: &T) -> () where T : Describable
{ println!("{}", a.describe()) }
fn main() {
  print_desc(&Book{title: "PP".to_string(), author: "Gil Hur".to_string()})
}
```

Dyn: dynamic dispatch

```
impl Describable for i64 {
    fn describe(&self) -> String {
        format!("64-bit-signed-integer: {}", *self)
    }
  }

fn main() {
    let book = Book{title: "PP".to_string(), author: "Gil Hur".to_string()};
    let ds : Vec<Box<dyn Describable>> =
        vec![Box::new(42), Box::new(book)];
    for d in &ds {
        println!("{}", d.describe())
        // cannot use print_desc : why?
    }
}
```

Closure type: Fn

```
fn test1<F>(f: F) -> usize
where F: Fn(usize)->usize {
    f(10) + f(20)
}
fn main() {
    let mut s = "abc".to_string();
    println!("{}", test1(|n|n+s.len()));
}
```

Closure type: FnMut

```
fn test2<F>(mut f: F)
where F: FnMut(&str)->() {
    f("gil");
    f("hur");
}
fn main() {
    let mut s = "abc".to_string();
    println!("{}", test1(|n|n+s.len()));
    test2(|x|s.push_str(x));
    println!("{}", test1(|n|n+s.len()));
}
```

Closure type: FnOnce

```
fn test3<F>(f: F)->String
where F: FnOnce(&str)->String {
    f("test")
}
fn main() {
    let mut s = "abc".to_string();
    println!("{}", test1(|n|n+s.len()));
    test2(|x|s.push_str(x));
    println!("{}", test1(|n|n+s.len()));
    let s2 = test3(|x|{s.push_str(x); s});
    println!("{}", s2);
}
```

Closure type: move

```
fn main() {
    let clo;
    {
        let mut s = "abc".to_string();
        println!("{}", test1(|n|n+s.len()));
        test2(|x|s.push_str(x));
        println!("{}", test1(|n|n+s.len()));
        let s2 = test3(|x|{s.push_str(x); s});
        println!("{}", s2);
        clo = move |n|n+s2.len();
    }
    println!("{}", test1(clo));
}
```

Thanks for your hard work!