Ordered Realization:

| Get scaling from O/R

$O^T O = 1 \rightarrow$ Orth Norm

$R R^T = 1 \rightarrow$ Inp Norm

-> generalization of balanced

-> also possible to have different scalings

$O = U\sqrt{\Sigma_O} \quad R = \sqrt{\Sigma_R} V^T$

$O^T O = \sqrt{\Sigma_O} \underbrace{U^T U}_{1} \sqrt{\Sigma_O} = \sqrt{\Sigma_O}\sqrt{\Sigma_O} = \Sigma_O$

$R R^T = \sqrt{\Sigma_R} \underbrace{V^T V}_{1} \sqrt{\Sigma_R} = \sqrt{\Sigma_R}\sqrt{\Sigma_R} = \Sigma_R$

$\sigma_{O_i} \geq \sigma_{O_{i+1}} \;\&\; \sigma_{R_i} \geq \sigma_{R_{i+1}} \Rightarrow \sigma_{O_i} \cdot \sigma_{R_i} \geq \sigma_{O_{i+1}} \cdot \sigma_{R_{i+1}}$

True $\sigma \geq 0 \;\forall \sigma$

| Proof

$\sigma_{O_i} \cdot \sigma_{R_i} \geq \sigma_{O_{i+1}} \cdot \sigma_{R_i} \geq \sigma_{O_{i+1}} \sigma_{R_{i+1}}$ D

$a \cdot b - a' \cdot b = \underbrace{(a-a')}\underbrace{b}_{\geq 0} \geq 0$

$a \geq a' \Leftrightarrow (a-a') \geq 0$ D