Derivatives with Respect to the Parameters

define a function $T : \mathbb{R}^{r\times r} \times \dots \mathbb{R}^{r\times r} \mapsto \underset{\text{Matrix}}{\mathbb{R}^{n\times p}}$

$$\underset{\left[\begin{array}{c|c} A & B \\ C & D \end{array}\right] \;\dots}{}$$

$$X = \begin{bmatrix} D_1 & & \\ C_2 B_1 & D_2 & \\ \vdots & & \ddots \\ C_n \cdots A \cdots B_1 & \cdots & D_n \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]_1}_{T_1} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]_2 \cdots \underbrace{\left[\begin{array}{c|c} A & D \\ \hline C & D \end{array}\right]_n}_{T_n}$$

$$D_{k,V}\, T_1 \cdots T_k\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right) \cdots T_n = \frac{d}{dt}\Big|_{t=0} T_1 \cdots T_k\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix} + t \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\right) \cdots T_n$$

Derivative k-th stage

$$\left[\begin{array}{c|c} A+tA' & B+tB' \\ \hline C+tC' & D+tD' \end{array}\right]_1$$

$$= \Big|_{t=0} T_1 \cdots T_k\left(\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\right) \cdots T_n$$

$$= T_1 \cdots T_k\left(\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\right) \cdots T_n$$

something like $\boxed{}\ \mathrm{vec}\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$

$$\mathrm{vec}\left(\underbrace{T_1 \cdots}_{A} T_k\left(\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\right) \underbrace{\cdots T_n}_{B}\right)$$

$$= \underbrace{\left(T_n^T \cdots T_{k+1}^T \otimes T_1 \cdots T_{k-1}\right)}_{D_k} \mathrm{vec}\left(\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\right)$$

$$D(\square) = \underbrace{\begin{bmatrix} D_1 & \cdots & D_n \end{bmatrix}}_{\text{Kernel of this}} \begin{bmatrix} \mathrm{vec}\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}_1 \\ \vdots \\ \mathrm{vec}\begin{bmatrix} A & B \\ C & D \end{bmatrix}_n \end{bmatrix}$$

Kernel of this

dim Manifold with $T(\dots) = T_{ref}$

$\Rightarrow$ all $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_s$ that are equivalent