# Make Minimal II

In general

Premisse output normal System $\Leftrightarrow$ $O_m^T O_m = \mathbb{1} \quad \forall_{m>k}$

Loop through the stages from $k=0\ldots$

From previous stage : $S_k$

Transformed previous stages to input normal

$$\hat{R}_m \hat{R}_m^T = \mathbb{1} \quad \forall_{m<k}$$

$$H_k = O_k R_k = O_k \left[A_k \middle| B_k\right] \begin{bmatrix} R_{k-1} & \\ & 1 \end{bmatrix}$$

$$\hat{R}_{k-1} = S_k^{-1} R_{k-1}$$

$$= O_k \left[A_k S_k \middle| B_k\right] \begin{bmatrix} S\hat{R}_{k-1} & \\ & 1 \end{bmatrix}$$

$$= \underbrace{O_k \quad U_k}_{U} \quad \underbrace{S}_{S} \quad \underbrace{V_k \begin{bmatrix} R_{k-1} & \\ & 1 \end{bmatrix}}_{V} = H_k$$

$\Rightarrow$ crop $\hat{U}_k \hat{S} \hat{V}_k$

$$S_{k+1} = \hat{U}_k \hat{S} \qquad \left[\hat{A}_k \middle| \hat{B}_k\right] = \hat{V}_k$$

also needed $\hat{R}_{k+1}$ is orthogonal

$$\hat{R}_{k+1} = \underbrace{\underbrace{\left[\hat{A}_k \; \hat{B}_k\right]}_{\substack{\| \\ \hat{V}_k \text{ orthogonal}}} \underbrace{\begin{bmatrix} \hat{R}_{k-1} & \\ & 1 \end{bmatrix}}_{\text{orthogonal}}}_{\text{orthogonal}}$$

orthogonal

orthogonal

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$$H = \begin{bmatrix} C_k \\ C_{k-1} A_k \\ \vdots \end{bmatrix} \begin{bmatrix} \cdots & A_{k-1} B_{k-2} & B_{k-1} \end{bmatrix} = O R = U S V^T$$

$H_1 = [\,]$ empty

$$H_2 = \begin{bmatrix} C_2 \\ C_3 A_2 \\ \vdots \end{bmatrix} \begin{bmatrix} B_1 \end{bmatrix} = \underbrace{O\, U}_{\text{orthogonal} \Rightarrow \text{is } USV} S_1 V_1^T = O\, U_1 S_1 V^T$$

$\Rightarrow \hat{U}_1 \hat{S}_1 \hat{V}_1$ Crop

orthogonal because previously made output normal

$\hat{B}_1 = \hat{V}_1^T$ $\quad S_{k-2}\, S_2 = \hat{U}_1 \hat{S}_1$

Transform $\hat{A}_k = S_{k+1}^{-1} A_k S_k$

$$H_3 = \begin{bmatrix} C_3 \\ C_4 A_3 \\ \vdots \end{bmatrix} \begin{bmatrix} A_2 \hat{B}_1 & B_2 \end{bmatrix} = O\, U_2 S_2 V_2^T \begin{bmatrix} \hat{D}_1 & \\ & 1 \end{bmatrix}$$

$\perp$ $\qquad \perp$

$$\begin{bmatrix} A_2 & \hat{B}_2 \end{bmatrix} \begin{bmatrix} \hat{B}_1 & \\ & 1 \end{bmatrix}$$

$\perp$

$\Rightarrow \hat{U}_2 \hat{S}_2 \hat{V}_2^T$

$\sigma_s$ of $H$

$$\begin{bmatrix} \hat{A}_2 & \hat{B}_2 \end{bmatrix} = \hat{V}_2 \qquad S_3 = \hat{U}_2 \hat{S}_2$$

$$R = \begin{bmatrix} \hat{A}_2 & \hat{B}_2 \end{bmatrix} \begin{bmatrix} \hat{B}_1 & \\ & 1 \end{bmatrix} = \hat{V}_2 \underbrace{\begin{bmatrix} \hat{B}_1 & \\ & 1 \end{bmatrix}}_{\text{orthogonal}}$$