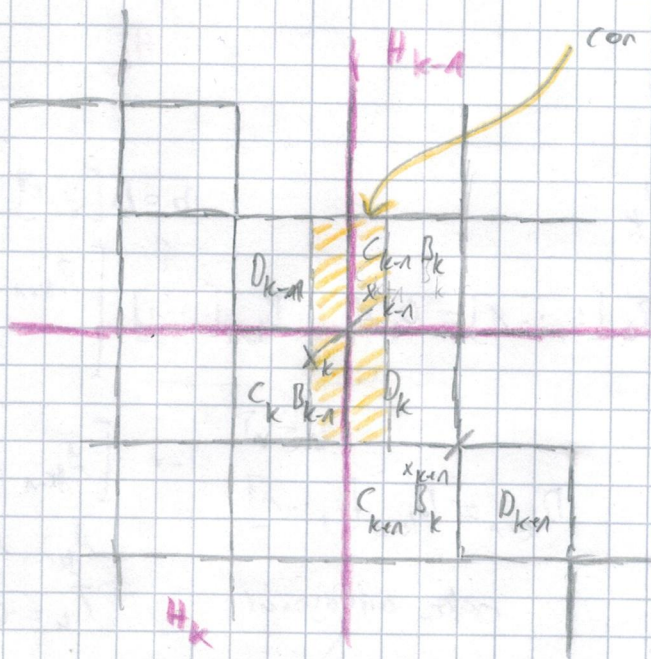


x_k



can change causality

in code as odd

move one boundary

→ work on

H_k for causal

H_{k-1} for anticausal

indices based on causal system \leftrightarrow correlate with x_k

loop $k = 2, \dots, N$ (i.e. $\text{range}(1, \text{len}(\text{stages}))$)

causal system \rightarrow output normal at start

\rightarrow convert to input normal

anticausal system \rightarrow input normal at start

\rightarrow convert to output normal

move right

$$H_k^R = \begin{bmatrix} 1 \\ 0_{k+1:n} \end{bmatrix} \begin{bmatrix} A_k & b \\ C_k & d \end{bmatrix} \begin{bmatrix} A_{k-1} & B_{k-1} \begin{smallmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{smallmatrix} \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} R_{k-1} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0_{k+1:n} \end{bmatrix} \begin{bmatrix} A_k & b \\ C_k & d \end{bmatrix} \begin{bmatrix} R_k \\ 1 \end{bmatrix} \quad \begin{matrix} b = B_k \begin{bmatrix} : \\ 1 \end{bmatrix} \\ d = D_k \begin{bmatrix} : \\ 1 \end{bmatrix} \end{matrix}$$

$$U \Sigma V^T = \begin{bmatrix} A_k & b \\ C_k & d \end{bmatrix}$$

needed to keep $R_k^T R_k = 1$ on n ordered

$$\begin{bmatrix} A_k^R \\ C_k^R \end{bmatrix} = U \Sigma$$

$$B_k^R = B_k \begin{bmatrix} : \\ 1 \end{bmatrix}$$

$$D_k^R = D_k \begin{bmatrix} : \\ 1 \end{bmatrix}$$

$$A_{k-1}^R = V^T \begin{bmatrix} A_{k-1} \\ \vdots & 0 & 1 \end{bmatrix}$$

$$B_{k-1}^R = V^T \begin{bmatrix} B_{k-1} \begin{smallmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{smallmatrix} \\ \vdots & \vdots \end{bmatrix}$$

$$C_{k-1}^R = C_{k-1}^R$$

$$D_{k-1}^R = \begin{bmatrix} D_{k-1} & d_{\text{offset}} \end{bmatrix}$$

Σ is Σ of H_k if $O_{k+1}^T O_{k+1} = 1$ on n $R_k R_k^T = 1$

$$\underbrace{\begin{bmatrix} 1 \\ 0_{k+1:n} \end{bmatrix}}_U \underbrace{U \Sigma V^T}_{V^T} \begin{bmatrix} R_k \\ 1 \end{bmatrix}$$

$$R_k^R = V R_k$$

$$\Rightarrow R_k^R R_k^{R^T} = V R_k R_k^T V^T = V V^T = 1$$

Now bounds of H_k

now left

$$H_k^L = O_k R_k \begin{bmatrix} :, : -1 \end{bmatrix} = O_k \begin{bmatrix} A_{k-1} & B_{k-1} \end{bmatrix} \begin{bmatrix} :, : -1 \end{bmatrix} \begin{bmatrix} R_{k-1} \\ 1 \end{bmatrix}$$

$$b = B_{k-1} \begin{bmatrix} :, : -1 \end{bmatrix}$$

$$U \Sigma V^T = \begin{bmatrix} A_{k-1} & B_{k-1} \end{bmatrix} \begin{bmatrix} :, : -1 \end{bmatrix} \rightarrow \text{state transition for } x_k$$

$$\begin{bmatrix} A_{k-1}^L & B_{k-1}^L \end{bmatrix} := V^T$$

$$C_{k-1}^L = C_{k-1}$$

$$D_{k-1}^L = D_{k-1} \begin{bmatrix} :, : -1 \end{bmatrix}$$

$$A_k^L = A_k U \Sigma$$

$$B_k^L = \begin{bmatrix} A_k b & B_k \end{bmatrix}$$

$$C_k^L = C_k U \Sigma$$

$$D_k = \begin{bmatrix} C_k b & D_k \end{bmatrix}$$

B old has as x_{k-1} is not transformed

Σ is Σ of H_k if $O_k O_k^T = 1$ and $R_{k-1}^T R_{k-1} = 1$

$$\text{then } \underbrace{O_k U \Sigma}_{U^L} \underbrace{V^T \begin{bmatrix} R_{k-1} \\ 1 \end{bmatrix}}_{V^L}$$

following: $R_k^T R_k = 1$

Move r/c on causal part

→ due to interting consistency we will work on H_{k-1}

Prerequisite: $O_{k-2}^T O_{k-2} = I \Rightarrow \text{also } O_{k-1}^T O_{k-1} = I$
 $R_{k-1} R_{k-1}^T = I$

Move left → add column

$$\hat{H}_{k-1} = \hat{O}_{k-1} \hat{R}_{k-1} = \begin{bmatrix} O_{k-2} & 1 \\ & 1 \end{bmatrix} \begin{bmatrix} b & A_{k-1} \\ d & C_{k-1} \end{bmatrix} \begin{bmatrix} 1 \\ R_{k-1} \end{bmatrix}$$

↖ add input to added state
 ↘ old input old state

$b = B_{i-1}[:, 0]$
 $d = D_{i-1}[:, 0]$
 $\hat{O} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\underbrace{\quad}_{U \Sigma V^T}$

No move

No move → basically make output normal

$$H_{k-1} = O_{k-1} R_{k-1} = \begin{bmatrix} O_{k-2} & 1 \\ & 1 \end{bmatrix} \begin{bmatrix} A_{k-1} \\ C_{k-1} \end{bmatrix} R_{k-1}$$

$\underbrace{\quad}_{U \Sigma V^T}$

Move right → base on no move as we need $O_{k-1}^T O_{k-1} = I$

$$\hat{H}_{k-1} = \hat{O}_{k-1} \hat{R}_{k-1} = O_{k-1} R_{k-1} \begin{bmatrix} :, 1: \end{bmatrix} = O_{k-1} \begin{bmatrix} B_{k-1} & A_k \end{bmatrix} \begin{bmatrix} 1 \\ R_k \end{bmatrix}$$

$\underbrace{\quad}_{U \Sigma V^T}$