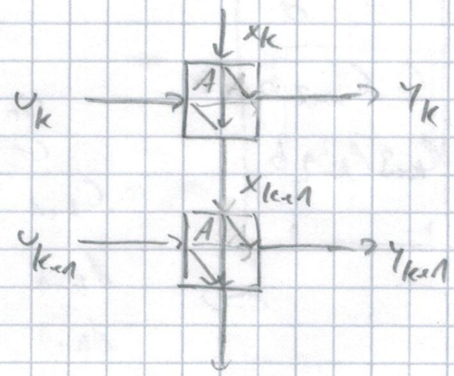
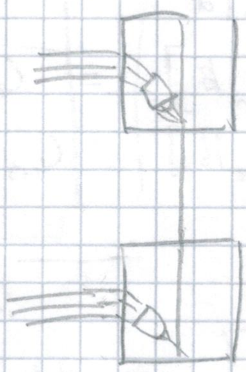
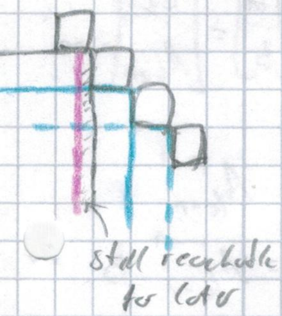




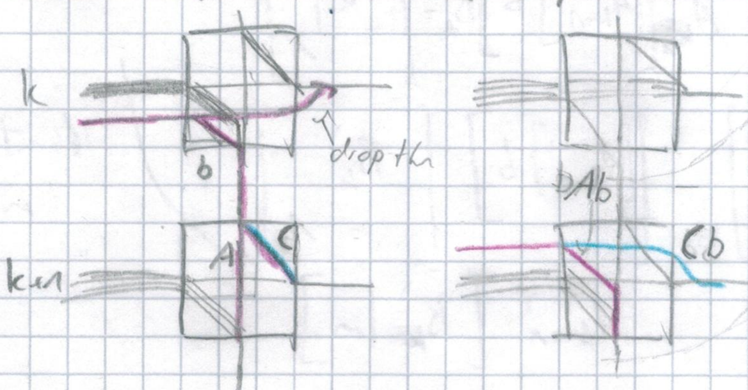
observability does not change

$$\begin{bmatrix} C_{k+1} \\ C_{k+2}A_{k+2} \\ \vdots \end{bmatrix} \begin{bmatrix} \cdots & A_{k-1} & B_{k-1} & B_k \end{bmatrix}$$

without last column

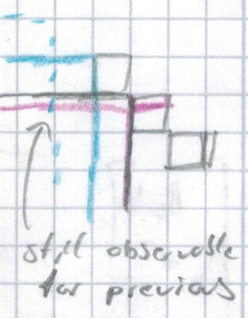


move left → change input

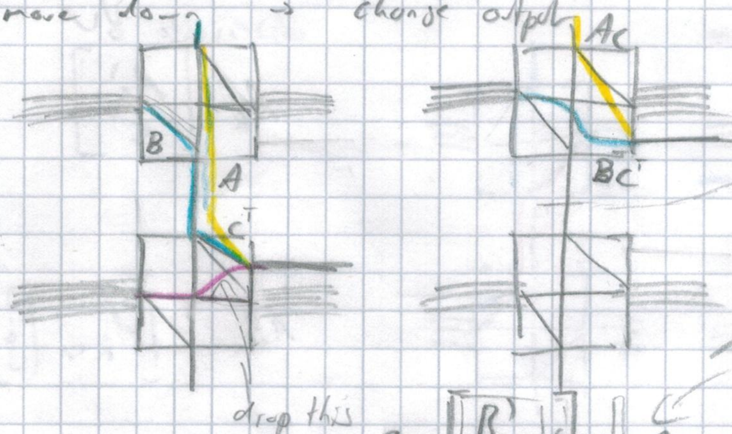


$$\begin{bmatrix} C_{k+2} \\ C_{k+3}A_{k+2} \end{bmatrix} \begin{bmatrix} \cdots A_{k+1} & B_k & A_{k+1}b & B_{k+1} \end{bmatrix}$$

already lin independent



move down → change output



cut

$$\begin{bmatrix} C_{k+1} \\ C_{k+2}A_{k+1} \end{bmatrix} \quad B_k$$

$$A_k = \begin{bmatrix} A_k' \\ 1 - 0 - 1 \end{bmatrix}$$

$$B_k = \begin{bmatrix} B_k' & 0 \\ 0 - & 1 \end{bmatrix}$$

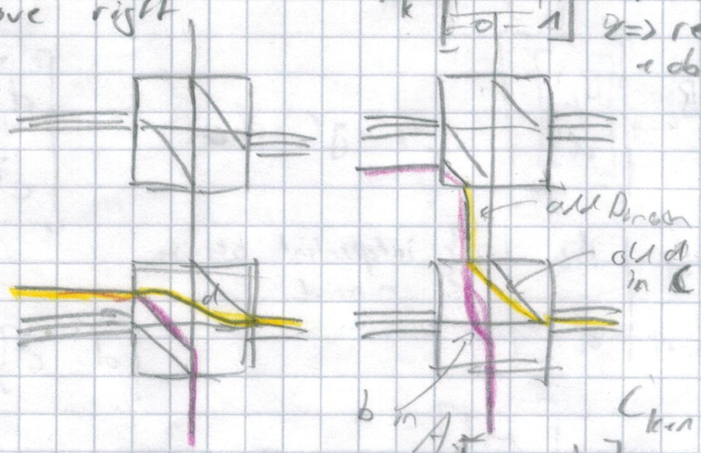
minimal ⇒ reachable + observable

always true as separate input

move right

lin independent

$$\begin{bmatrix} C_{k+1} & d \\ C_{k+2}A_{k+1} & b \\ C \cdots A \cdots & A_{k+1}b \end{bmatrix} \quad O_{k+1}$$



$$C_{k+1} = \begin{bmatrix} C_{k+1}' & d \end{bmatrix}$$

$$A_{k+1} = \begin{bmatrix} A_{k+1}' & b \end{bmatrix}$$

if $\exists A_{k+1}^{-1}$

$$B_k = \begin{bmatrix} B_k' & A_{k+1}^{-1} b \end{bmatrix}$$

obs minimal $\Leftrightarrow$ C in independent

$A\,[B|C] = [AB|AC]$

$$\begin{bmatrix} & C_{k+1} & d \\ C_{k+2} & [A_{k+1} & b] \\ C_{k+3}A_{k+2} & [A_{k+1} & b] \end{bmatrix} = \left[\begin{array}{c|c} C_{k+1} & d \\ C_{k+2}A_{k+1} & C_{k+2}\,b \\ C_{k+3}A_{k+2}A_{k+1} & C_{k+3}A_{k+2}\,b \end{array}\right] =$$

identisch

C in independent

$$\begin{matrix} C_{k+1} & d \\ A_{k+1} & b \\ A_{k+1} & b \\ \vdots & \end{matrix} \Leftrightarrow \begin{bmatrix} d \\ b \end{bmatrix} \in \operatorname{range} \begin{bmatrix} C_k \\ A_{k+1} \end{bmatrix}$$

$\Leftrightarrow$

$\begin{bmatrix} d \\ b \end{bmatrix} \in \operatorname{range} \begin{bmatrix} C_k \\ A_{k+1} \end{bmatrix}$ $\}$ state transform

$\exists\, m$ s.t. $\begin{bmatrix} d \\ b \end{bmatrix} = \begin{bmatrix} C_{k+1} \\ A_{k+1} \end{bmatrix} m$

$C_{k+1} = \begin{bmatrix} [C_{k+1}'] & d \end{bmatrix}$ $\quad A_{k+1} = \begin{bmatrix} [A_{k+1}'] & b \end{bmatrix}$ $\quad B_k = \begin{bmatrix} B_k' & \\ & 1 \end{bmatrix}$

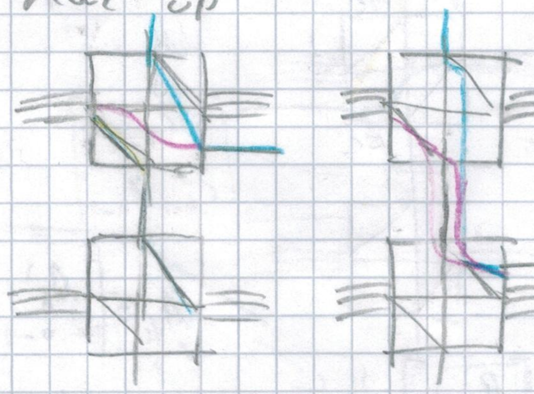
$\left[[C_{k+1}']\ d\right] \begin{bmatrix} B_k' & \\ & 1 \end{bmatrix}$ $\quad \left[[A_{k+1}']\ b\right] \begin{bmatrix} B_k' & \\ & 1 \end{bmatrix}$

if $\exists\, A_{k+1}'^{-1}$

$m = A_{k+1}'^{-1}\, b$

$C_{k+1}'\, [B_k' | m]$ $\quad A_{k+1}'\, [B_k' | m]$

move up



observable ?✓

$A_k = \begin{bmatrix} A_k' \\ c^T \end{bmatrix}$ $\quad B_k = \begin{bmatrix} B_k' \\ d^T \end{bmatrix}$

$A_{k+1} = \begin{bmatrix} A_{k+1}' & \begin{matrix} 1 \\ 0 \\ \vdots \end{matrix} \end{bmatrix}$ $\quad C_{k+1} = \begin{bmatrix} & 1 \\ C_{k+1}' & \\ & 1 \end{bmatrix}$

Recovable?

$\dots \begin{bmatrix} A_k \\ c^T \end{bmatrix} A_{k-1} B_{k-2} \left| \begin{bmatrix} A_k \\ c^T \end{bmatrix} B_{k-1} \right| \begin{bmatrix} B_k' \\ d^T \end{bmatrix}$ $\rightarrow$ vgl. oben

$\begin{bmatrix} d^T \\ c^T \end{bmatrix} = \begin{bmatrix} B_k^T \\ A_k^T \end{bmatrix} m$

$$\left[\begin{array}{c|c|c} A_k\ A_{k-1}B_{k-2} & A_k B_{k-1} & B_k \\ \hline c\ A_{k-1}B_{k-2} & c\ B_{k-1} & d \end{array}\right]$$ $\}$ independent because minimal

$[d^T\ c^T] = m^T [B_k\ A_k]$

$\begin{bmatrix} m^T \\ C_k \end{bmatrix} B_k$ $\quad \begin{bmatrix} m^T \\ C_k \end{bmatrix} A_k$

Move down revisited

state after $k \rightarrow \begin{bmatrix} C_{k+1} \\ C_{k+2} A_{k+1} \\ C_{\ldots} A_{\ldots} A_{k+1} \\ \vdots \end{bmatrix}$ c^T $\cdots \left[A_{\ldots} A_{\ldots} B_{k-2} \,|\, A_k B_{k-1} \,|\, B_k \right]$

still reachable
$\Rightarrow$ no change

but maybe not observable ?

$$O_{k+1} = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & \begin{matrix} C_{k+2} A_{k+1} \\ C_{k+3} A_{k+2} \\ \vdots \end{matrix} \end{bmatrix} \begin{bmatrix} C^{cut}_{k+1} \\ A_{k+1} \end{bmatrix}$$

$$\begin{bmatrix} C^{cut}_{k+1} \\ A_{k+1} \end{bmatrix} = U S V^T$$

reduce

U_c U_A

new $\begin{bmatrix} C_{k+1} \\ A_{k+1} \end{bmatrix}$

lin independent due to observability

multiply previous A_k and B_k with this

$\Rightarrow$ again Observable

Reachability ?

$$\boxed{V^T} \cdot \left[\cdots A_{\ldots} A_{\ldots} B_{k-2} \,|\, A_k B_{k-1} \,|\, B_k \right]$$

$\Rightarrow$ Transposed $\begin{bmatrix} B_k \\ \vdots \\ \vdots \end{bmatrix} V$ lin independent $\square$

for state $k+1$

Observable for previous state $O_k \cdots$

$$\begin{bmatrix} C_k \\ c^T A_k \\ \underbrace{U_c S}_{C_{k+1}\,new} V^T A_k \\ C_{k+2} \underbrace{U_A S}_{A_{k+1}\,new} V^T A_k \end{bmatrix} = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & \begin{matrix} c^T \\ U_c S V^T \\ C_{k+2} U_A S V^T \\ \vdots \end{matrix} \end{bmatrix} \begin{bmatrix} C_k \\ A_k \end{bmatrix} =$$

$$\begin{bmatrix} \mathbb{1} & c \\ & C_{k+2} \\ & C_{k+3} A_{k+2} \end{bmatrix} \begin{bmatrix} U_c S V^T \\ U_A S V^T \end{bmatrix}$$

back to original

$\square$

Reachable for late state ?

$$USV^T A_k A_{k-1} B_{k-2} \quad US\underbrace{V^T A_k}_{A_{knew}} B_{k-1} \quad \underbrace{US}_{A_{new}} \underbrace{V^T B_k}_{B_{ne}} \quad B_{k+1}$$

$$\left[\begin{array}{c|c} USV^T & \mathbb{1} \\ \hline & \mathbb{1} \end{array}\right] \left[\begin{array}{c|c} \dots A_k A_{k-1} B_{k-2} \quad A_k B_{k-1} \quad B_k & 0 \\ \hline \longleftarrow 0 & B_{k+1} \end{array}\right]$$