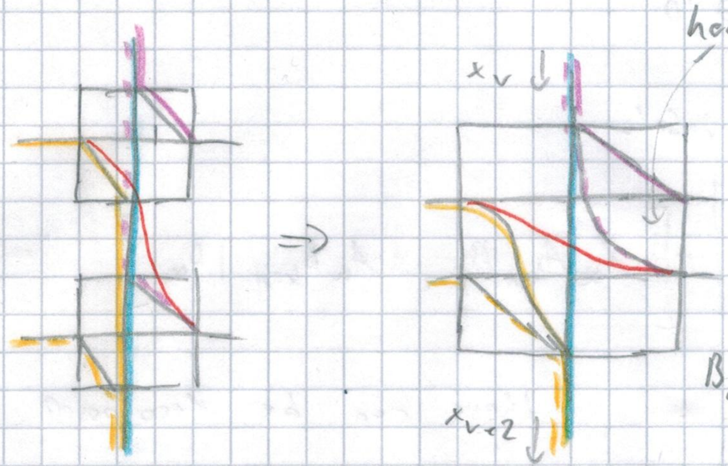
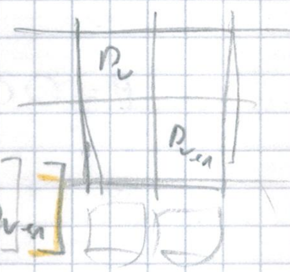
Combine 2 steps



here the D also allow an anticausal part that is not possible to the left



$$B_c = \left[A_{v+1} B_v \mid B_{v+1} \right]$$

$$A_c = \left[A_{v+1} A_v \right]$$

$$C_c = \begin{bmatrix} C_v \\ C_{v+1} A_v \end{bmatrix} \quad D_c = \begin{bmatrix} D_v & \\ C_{v+1} B_v & D_{v+1} \end{bmatrix}$$

$$\left[A_{v+1} A_v \right] B_{v+1} \left[A_{v+1} B_v \quad B_{v+1} \right]$$

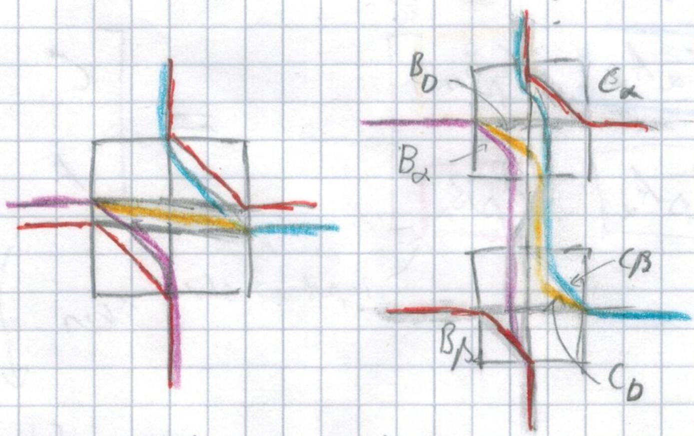
minimal ?

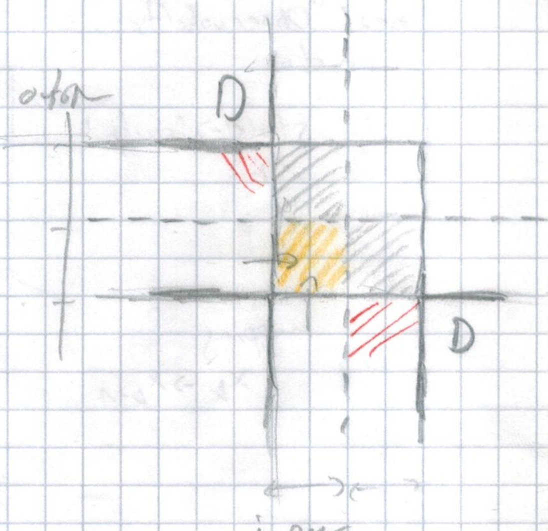
x_v

$$\begin{bmatrix} \begin{bmatrix} C_v \\ C_{v+1} A_v \end{bmatrix} \\ C_{v+2} \left[A_{v+1} A_v \right] \\ C_{v+3} A_{v+2} \left[A_{v+1} A_v \right] \end{bmatrix} \left[\cdots A_{v-1} A_{v-2} B_{v-1} \right]$$

x_{v+2}

$$\begin{bmatrix} C_{v+2} \\ C_{v+3} A_{v+2} \end{bmatrix}$$





α, β in this notation with different spaces for D

$$A' = A_\beta A_\alpha$$

Bold ≙ known a-priori

$$D' = \begin{bmatrix} \mathbf{D_\alpha} & \\ \underbrace{C_\beta B_\alpha + C_D B_D}_{D^*} & \mathbf{D_\beta} \end{bmatrix} \qquad B' = \left[\underbrace{A_\beta B_\alpha}_{B^*} \mid \mathbf{B_\beta} \right] \qquad C' = \underbrace{\begin{bmatrix} \mathbf{C_\alpha} \\ C_\beta A_\alpha \end{bmatrix}}_{C^*}$$

$A = A_\beta A_\alpha \qquad B^* = A_\beta B_\alpha \qquad C^* = C_\beta A_\alpha$

$D^* = C_\beta B_\alpha + C_D B_D$

Hankels of new state inbetween

$$H = OC = \begin{bmatrix} C_\beta \\ C_{k+1} A_\beta \\ C_{k+2} A_{k+1} A_\beta \\ \vdots \end{bmatrix} \cdot \begin{bmatrix} \cdots A_\alpha A_{k-1} B_{k-2} & A_\alpha B_{k-1} & B_\alpha \end{bmatrix}$$

these can be decomposed

Previous reachability matrix

$$\begin{bmatrix} \mathbb{1} & 0 \\ 0 & C_{k+1} \\ & C_{k+2} A_{k+1} \\ & \vdots \end{bmatrix} \overset{C}{\begin{bmatrix} C_\beta \\ A_\beta \end{bmatrix}} \begin{bmatrix} A_\alpha & B_\alpha \end{bmatrix} \begin{bmatrix} \cdots A_{k-1} B_{k-2} & B_{k-1} & 0 \\ \leftarrow 0 & & \mathbb{1} \end{bmatrix}$$

next Observability matrix

input → output

state to output x_k

$$\begin{bmatrix} \boxed{C_\alpha A_\alpha} & \boxed{C_\beta B_\alpha} \\ \boxed{A_\beta A_\alpha} & \boxed{A_\beta B_\alpha} \end{bmatrix} = \begin{bmatrix} C^* & D \\ A & B^+ \end{bmatrix}$$

mapping $x_k \to x_{k+1}$

input → state x_{k+1}

SVD

reduce

$U \; \boxed{S} \; V^T$

S_k

$$\begin{bmatrix} U_C \\ U_A \end{bmatrix} S \begin{bmatrix} V_A & V_B \end{bmatrix}$$

split

split according to shapes