Bo Green

$$\hat{O}\,\hat{R} = U S^{\frac{1}{2}} S^{\frac{1}{2}} V^T$$

$$U_O S_O V_O^T = O$$

$$U_R S_R V_R^T = R$$

$$OR = U_O \underbrace{S_O V_O^T U_R S_R}_{U S V^T} V_R^T$$

$$OR = \underbrace{U_O\, U\, S}\ \underbrace{V^T V_R^T}$$

$$\hat{O} = U_O U S^{\frac{1}{2}} \qquad \hat{R} = V^T V_R T$$

$OA = \hat{O}$

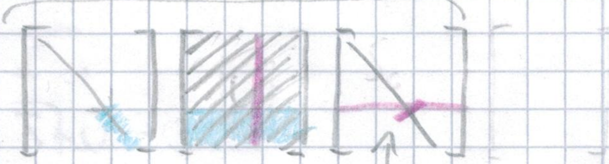
Operator

$BR = \hat{R}$

$$U_O S_O V_O^T A = U_O U S^{\frac{1}{2}} \qquad | \; U_O^T \cdot$$

$$S_O V_O^T A = U S^{\frac{1}{2}} \qquad | \; \cdot S^{-1} \cdot \; | V_O \cdot$$

$$A = V_O \underbrace{S_O^{-1} U S^{\frac{1}{2}}}$$



$$\operatorname{rank}(S_O V_O^T U_R S_R)$$

$$\leq \operatorname{rank}(S_O)\operatorname{rank}(V_O^T U_R)\operatorname{rank}(S_R)$$

Range of O

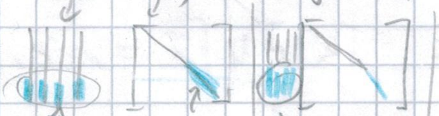
$U_O \,|\, U \,|\, S^{\frac{1}{2}}$

$\hat{=}\ \sigma_O$ of O

σ_s of H

Quadratic case

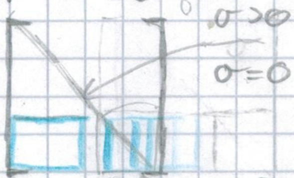
$\operatorname{Range}(U) = \operatorname{Range}(S_O) + \operatorname{Range}^{\perp}(S_O)$



$=0 \leftarrow\ =0$ $\quad \neq 0$

(not nec.) associated with $\sigma > 0$ or $\sigma = 0$

associated with $\sigma = 0$



$\sigma > 0$ ← might be fewer than $\sigma_O > 0$

$\sigma = 0$

$=0$

these not necessarily

case $O = \square$ or $R = ▯$

$\square \setminus ▭$ $\qquad$ $▯ \setminus \square$