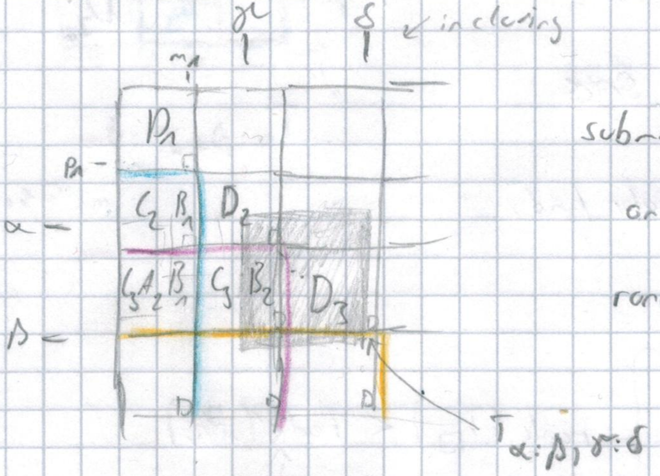
local properties

Bounds on Ranks for Submatrices



submatrix S with size $(\beta-\alpha+1)\times(\delta-\gamma+1)$

only in one Hankel matrix

$\text{rank}(S) \leq \min(u,v)$

$\leq$ Hankelrank

Can a submatrix can only contain C, A, Bs

while not being included into one Hankel operator

included into one hankel operator v $\qquad$ $\alpha \leq \beta$ ← no empty selection

$\Updownarrow$ $\qquad$ $\gamma \leq \delta$

$\delta \leq \sum_{k=1}^{v} m_k$ ← input dim

$\wedge \quad \alpha \geq \sum_{k=1}^{v} p_k$ ← output dim

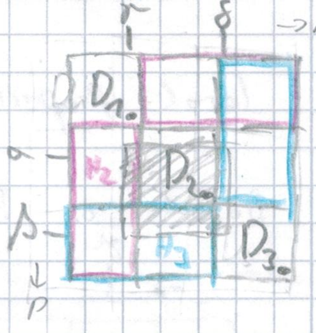
Does not include D matrix

$\Leftrightarrow \forall_v \quad \alpha \leq \sum_{k=1}^{v+1} p_k \rightarrow \delta \leq \sum_{k=1}^{v} m_k$

$\Leftrightarrow \forall_v \quad \delta > \sum_{k=1}^{v} m_k \rightarrow \alpha \geq \sum_{k=1}^{v+1} p_k$

except last ? ↗

Rank limits



$T_{\alpha:\beta,\ \gamma:\delta}$ ← inclusive

General → $\operatorname{rank}(T_{\square}) \le \min(\beta-\alpha+1,\ \delta-\gamma+1)$

↑ simply size

① included in one Hankel operator / index

Kausal part

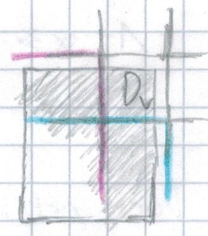
$$\left.\begin{aligned} &\delta \le \sum_{k=1}^{v} m_k \\ \wedge\ &\alpha > \sum_{k=1}^{v} p_k \end{aligned}\right\} \operatorname{rank}(T_{\square}) < \operatorname{rank}(H_v)$$

↑ Hankelrank v

Antikausal part

$$\left.\begin{aligned} &\beta \le \sum_{k=1}^{v} p_k \\ \wedge\ &\gamma > \sum_{k=1}^{v} m_k \end{aligned}\right\} \operatorname{rank}(T^a_{\square}) < \operatorname{rank}(H^a_v)$$

② Combination of Hankel operator and one D

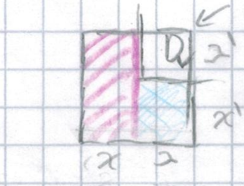


→ can be split up in two parts

can be full rank due to no constraints on D

kausal

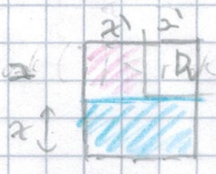
$\alpha > \sum_{k=1}^{v} p_k$



$$\operatorname{rank}(T_D) \le \min(\operatorname{rank}(H_v), x) + \min(x,\ x' + \min(x', \operatorname{rank}(H_{v+1})))$$

$\le x + x$ and $\le x' + x'$

$\delta \le \sum_{k=1}^{v+1} m_k$



$$\operatorname{rank}(T_D) \le \min(\operatorname{rank}(H_{v+1}), x) + \min(x,\ x' + \min(x', \operatorname{rank}(H_v)))$$

↑ no condition on D

Antikausal

$\beta \le \sum_{k=1}^{v+1} p_k$

$\gamma > \sum_{k=1}^{v} m_k$

$\operatorname{rank}(▯▯) \le \min(\operatorname{rank}(▯) + \operatorname{rank}(▯))$

Proof $\operatorname{rang}(▯0) = \operatorname{rang}(▯)$ $\operatorname{rang}(0▯) = \operatorname{rang}(▯)$

$\operatorname{rank}(▯▯) = \operatorname{rank}(▯0 + 0▯) \le \operatorname{rank}(▯) + \operatorname{rank}(▯)$

Bronstein 16 ↑ (4.26c)