수학 문제 연구회

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Consider the set

$$\prod_{n=1}^{\infty} \mathbb{R} = \{(x_1, x_2, \dots) : x_i \in \mathbb{R}, \quad \text{for } i \in \mathbb{N}\}.$$

Solve following problems:

1. Show that $\prod_{n=1}^{\infty} \mathbb{R}$, with componentwise addition and scalar multiplication, forms a vector space.
2. Let $R_i := \{(\dots, 0, x_i, 0, \dots) \in \prod_{n=1}^{\infty} \mathbb{R} : x_i \in \mathbb{R}\}$. Then show that $\bigoplus_{i=1}^{\infty} R_i$ is a proper subspace of $\prod_{n=1}^{\infty} \mathbb{R}$.
3. When two elements are equal in the quotient space $\prod_{n=1}^{\infty} \mathbb{R} / \bigoplus_{i=1}^{\infty} R_i$?