수학 문제 연구회

Taeyoung Kim

26, January 2025

Consider matrix

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

solve following problems:

1. Show that there exists no real 2×2 matrix R such that RAR^{-1} is diagonal matrix.
2. Show that there exists complex 2×2 matrix C such that $D = CAC^{-1}$ is diagonal matrix by finding C, D explicitly. And show that there is no such C for the case of matrix B.
3. Show that the set of 2×2 diagonalizable matrices is not subspace of $\mathcal{M}_{2\times 2}(\mathbb{C})$.

Here's an additional question: Is XY diagonalizable for diagonalizable matrices X, Y?