수학 문제 연구회

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Let N_e be the number of edges and N_v be the number of vertices for some 2-dimensional n-gon (e.g. triangle, square, ...) then it is quite trivial that $1 - N_e + N_v = 1$. Motivated by this fact, solve the following statements:

1. Let N_f, N_e, N_v be the number of faces, edges, and vertices of a cube, respectively. Then show that $-1 + N_f - N_e + N_v = 1$.
2. Calculate $-1 + N_f - N_e + N_v$ similarly for pyramid, cylinder, and sphere.
3. Calculate $-1 + N_f - N_e + N_v$ for a torus.

Here is an additional question: Let N_i be the number of i-dimensional faces for the n-dimensional (filled) polytope. What is the value of $\sum_{i=0}^{n}(-1)^i N_i$?