

# 수학 문제 연구회

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Let  $x_{N,m}^{(0)}(n) := \left(\frac{n}{N}\right)^m$  and  $x_{N,m}^{(i+1)}(n) = \frac{x_{N,m}^{(i)}(n+1) - x_{N,m}^{(i)}(n)}{\frac{1}{N}}$  for  $i = 0, 1, \dots$ .

Here,  $x_{N,m}^{(i)}(n)$  can be understood as a scaled difference sequence. For example, when  $N = 1$  and  $m = 2$ ,  $x_{1,2}^{(0)}(n)$  is  $n^2$ . The first difference sequence of this is  $2n + 1$ , and the second difference sequence is 2. Keep this in mind and solve the following statements:

1. Calculate  $x_{N,l}^{(3)}(n)$  for each  $N, l, n$ .
2. Prove that for each  $m$ ,  $x_{N,m}^{(m)}(n)$  is independent of the parameters  $N$  and  $n$ .
3. Find the value of  $x_{N,m}^{(m)}(n)$ .

Here is an additional question: Are there any sequences that satisfy the property shown in Problem 2 other than polynomials?