수학 문제 연구회

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Let $\mathbb{A} := \{(a_n)_{n=0,\ldots} : a_n \in \mathbb{C}, n \in \mathbb{N}\}$ be a set of complex-valued sequences. Solve following statements:

1. Dual space of $l^2(\mathbb{C}) := \{(a_n)_{n=0,\ldots} \in \mathbb{A} : \sum_n |a_n|^2 < \infty\}$ is isomorphic to itself. (Hint: consider inner-product).

2. Let $\mathbb{A}_\infty := \{(a_n)_n \in \mathbb{A} : \lim_{n\to\infty} a_n \text{ exist}\}$ be set of convergent sequences, which is subspace of $\mathbb{A}$. Then $(b_n) \in l^1(\mathbb{C}) := \{(a_n)_{n=0,\ldots} \in \mathbb{A} : \sum_n |a_n| < \infty\}$ defines a linear functional $T_{(b_n)}$ on $\mathbb{A}_\infty$ as following:

$$T_{(b_n)}(a_n) := \sum_n a_n \overline{b_n}.$$

   Find linear functional on $\mathbb{A}_\infty$ which can't be represented by $T_{(b_n)}$.

3. Find dual space of $\mathbb{A}_\infty$.

Here's an additional question: What is dual space of $\mathbb{A}$?