Coordination with Cognitive Noise*

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Abstract

We experimentally study how cognitive noise affects behavior in coordination games. Our key testable prediction is that equilibrium behavior depends on *context*, which we define as the distribution from which games are drawn. This prediction arises from players efficiently using their limited cognitive resources. Furthermore, this prediction distinguishes *cognitive noise* from a large class of alternative behavioral game theory and learning models. Experimentally, we find that subjects coordinate more frequently when game payoffs are drawn from a narrower distribution. Nearly 50% of the variability in behavior can be attributed to cognitive noise rather than alternative sources of strategic uncertainty.

Keywords: Complexity, Efficient Coding, Context-Dependence, Stochastic Choice, Coordination Games, Laboratory Experiment

JEL Codes: C72, C92, D91, E71

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	Not Invest	Invest
Not Invest	θ, θ	θ, a
Invest	a, θ	b, b

Figure 1: **The Game**

1 Introduction

Consider two players who would like to coordinate on an investment opportunity, described by the payoff matrix in Figure 1. When $a \leq \theta \leq b$, there are multiple equilibria. If the row player believes the column player will invest, then she prefers to also invest. If the row player believes the column player will not invest, then she prefers to not invest. How can we predict which action the players will select? In this paper, we argue that *cognitive noise* prevents each player from using exact knowledge about θ to compute the value of each action. We attribute this imprecision in θ to perceptual and cognitive imperfections that arise during the process of computing each action's value. Cognitive noise, thus, generates a new source of uncertainty about the value of θ that the opponent uses in their decision process. This uncertainty eliminates multiple equilibria and gives rise to a unique prediction about game play. Our model of cognitive noise generates additional testable predictions that distinguish it from leading behavioral models of strategic behavior, including Quantal Response Equilibrium (QRE; McKelvey and Palfrey 1995, 1998) and Level-k Thinking (Nagel, 1995; Camerer, Ho and Chong, 2004).

In a pair of pre-registered experiments, we demonstrate that cognitive noise is inherent in strategic play and that it systematically affects the probability of coordination. We experimentally implement the game shown in Figure 1 and we find two main results. First, we manipulate the level of cognitive noise and find that it causally affects the frequency of coordination. Second, we conduct a decomposition analysis which reveals that cognitive noise represents a substantial source of the noise observed in strategic behavior; we estimate that roughly 50% of noise in behavior stems from an imprecise representation of payoffs.

Our analyses highlight that the particular manner in which noise is modeled has important implications for equilibrium. Thus, we are careful to ground our assumptions about the source of noise in a recent empirical literature in economics that has begun investigating imprecision in valuation and choice (Woodford, 2020). In particular, a series of recent individual decision-making experiments has shown that noise arising in the subjective valuation process exhibits clear parallels with noise in basic perceptual decisions (Polania, Woodford and Ruff, 2019; Khaw, Li and Woodford, 2021, 2024; Frydman and Jin, 2022; Enke and Graeber, 2023; Enke, Graeber and Oprea, 2025). This conceptual link between perceptual and economic decisions motivates our modeling approach: we assume each player holds a prior about the fundamental parameter θ and then observes only a noisy signal of θ — even after θ is clearly presented to the player. The noisy signal is meant to capture errors involved with encoding and retrieving θ. Each player then uses her noisy signal as an input to computing the value of an action.

Our particular model of cognitive noise generates a sharp and testable prediction about strategic behavior. In the model, while each player faces unavoidable cognitive noise, we assume that the noise distribution is optimally adapted to their environment. This assumption of *efficient coding* generates the following prediction: as a player's prior about θ becomes more dispersed, she processes information about each value of θ with less precision (Barlow, 1961; Laughlin, 1981). Intuitively, if we assume a player has a fixed budget of cognitive resources, then as the prior becomes more dispersed, she needs to allocate those cognitive resources more broadly across the state space. This, in turn, leads her to encode each value of θ with more noise. Importantly, the noisier representation of θ affects the player's probability of investing, and hence, coordination. We, therefore, test whether cognitive noise plays an important role in coordination games by experimentally manipulating the prior and testing for the impact on game outcomes.

We present the details of our two experiments in the main body of the paper, but here we preview the key aspects of the design. In our first experiment, subjects are randomly matched on each of three hundred rounds, and they play the game outlined in Figure 1. We set the values of $a = 47$ and $b = 63$, and the only object that varies across rounds is θ. On each round, we assume that a subject's prior is governed by the distribution of θ that she has experienced during the experiment. Thus, to manipulate the prior, we implement a between-subjects treatment where half of the subjects observe values of θ drawn from a high volatility distribution, and the other half observe values of θ drawn from a low volatility distribution. The key prediction is that the prior affects the manner in which players process information about θ, and this, in turn, affects the subjective valuation of investing and not investing.

In our main test, we compare the frequency that a player invests — conditional on θ — across the two experimental treatments. Consistent with our theoretical model, we find that for a given value of θ, the probability of investing depends on the prior to which the player is adapted. In both treatments, behavior is consistent with subjects playing the unique equilibrium threshold strategy. The smoking gun evidence for cognitive noise is that behavior exhibits significantly more randomness in the high volatility treatment, where our model predicts that information about θ will be processed with more noise. This result is consistent with previous work from individual decision-making experiments (Frydman and Jin, 2022).

However, a crucial difference is that here cognitive noise is key to endogenously producing the equilibrium threshold strategy. Put differently, the data we produce are consistent with a cognitive noise mechanism that endogenously generates *both* the equilibrium threshold strategy and the greater degree of randomness that subjects exhibit in the high volatility treatment. Overall, our experimental data indicate that coordination (both players investing or neither player investing) is more likely when players are adapted to the low volatility distribution and face a lower amount of cognitive noise.

We emphasize that other models of noisy strategic behavior, such as QRE, do not predict that strategic behavior depends on the player's prior. The intuition for this difference in predictions is as follows. Our model of cognitive noise assumes that the agent is unable to precisely compute the value of an action, owing to the noisy representation of θ. Thus, because the prior is informative about the value of an action, any shift in the prior will affect the subjective valuation of the action. In contrast, QRE assumes that each agent has no problem with precisely representing θ and computing the value of each action, conditional on θ. The noise in QRE arises only during the process of action selection, where the agent trembles. In this case, the prior has no bearing on behavior, as the agent is already fully confident about the precise value of each action.

Our results highlight cognitive noise as a novel and important source of *strategic uncertainty* — which refers to uncertainty about an opponent's behavior. In coordination games, strategic uncertainty typically arises from uncertainty about which of multiple equilibria an opponent will select, though other sources of strategic uncertainty also include uncertainty about an opponent's preferences, information, or rationality. Because cognitive noise corrupts a player's representation of θ, it necessarily leads the player to be uncertain about the opponent's valuation of each action. An important question, then, is how much of the noise in behavior that we observe is actually driven by cognitive noise, rather than alternative sources of strategic uncertainty?

To address this question, we conduct a second experiment that enables us to decompose the observed variability in behavior into structural uncertainty (arising from cognitive noise) and strategic uncertainty (arising from sources other than cognitive noise). The main innovation in this second experiment is that we incentivize subjects to play the same series of games as in our first experiment, except the opponent is now a computer. Crucially, we inform subjects that the computer plays a known and deterministic strategy. This design feature purges any strategic uncertainty that arises from sources other than cognitive noise. We find that, even when playing against a computer, subjects still make errors that have signature features of cognitive noise. More importantly, we estimate that roughly half of the variability in behavior from our first experiment is driven by cognitive noise. We attribute the remaining variability in behavior to alternative sources of strategic uncertainty.

Our model of cognitive noise is closely related to the literature on global games (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Goldstein and Pauzner, 2005; Angeletos and Lian, 2016). In a global game, a player is assumed to behave *as if* she receives a noisy private signal about the state of the world, θ.[1] We view cognitive noise as providing a microfoundation for the source of noise in the private signals that are assumed in global games. Importantly, this microfoundation also gives rise to two novel predictions that do not obtain in a global games model. First, our model of cognitive noise predicts that behavior should be context-dependent in equilibrium, whereas in a global game, there is no reason to expect that the signal precision depends on the player's prior. Our experimental data provide clear evidence that *signal* precision does increase in the player's *prior* precision.[2]

The second prediction that distinguishes cognitive noise from global games involves the role of public signals (Woodford, 2020). A series of papers has argued that the unique equilibrium generated in a global game will not obtain when there exists a sufficiently precise public signal, such as a market price (Atkeson, 2000; Angeletos and Werning, 2006; Hellwig, Mukherji and Tsyvinski, 2006). This precise public signal can act as a coordination device, which restores multiple equilibria. However, in our model, even public signals like a market price or government announcement should be processed with cognitive noise, which prevents coordination and sustains a unique equilibrium. Thus, a testable prediction is that the provision of public information should lead to a different equilibrium under the global games model, while equilibrium should stay fixed under our model of cognitive noise. Interestingly, Heinemann et al. (2004) show that behavior in a coordination game remains largely unchanged when exogenously manipulating the provision of public information; this result is consistent with the interpretation that even publicly available information is processed with cognitive noise. In Section 6.3, we provide more details on how our results fit into the broader literature on global games.

Our experimental results have important implications for the modeling of incomplete information games. Specifically, our results suggest that the class of games in which it is appropriate to assume agents have incomplete information is likely broader than previously thought. Even in situations where there is no explicit private information, cognitive noise will break common knowledge about the valuation of each player's action. In addition to

[1] Indeed, a majority of experimental tests of global games involve explicitly endowing subjects with noisy signals of the fundamental value (Heinemann, Nagel and Ockenfels, 2004; Cabrales, Nagel and Armenter, 2007; Van Huyck, Viriyavipart and Brown, 2018; Szkup and Trevino, 2020; Helland, Iachan, Juelsrud and Nenov, 2021; Avoyan, 2024).

[2] In Section 6.4, we describe in more detail how our treatment effect cannot be explained merely by a shift in the player's prior and why a shift in the signal distribution is needed.

providing guidance for appropriate modeling assumptions, the idea that cognitive noise arises near universally also has implications for experimental design. To see this, consider a recent experiment by Goryunov and Rigos (2022) who use a clever design to explicitly inject noise into the perception of a state variable. Subjects in their experiment observe a visual dot that represents the state, and the authors rely on the inherent difficulty of visually perceiving the exact location of the dot to generate private noise. Our results suggest that noise in valuation arises in a much broader class of games, owing to the imprecision involved with perceiving or retrieving payoffs. As we demonstrate with our experiments, even when information about the state variable is clearly communicated to subjects through symbolic numerals, we find evidence that cognitive noise is sizeable. We also note that our measurements of cognitive noise are likely to represent a *lower* bound relative to more complex strategic applications outside the lab.

Our results build on a set of papers that have begun testing whether principles of cognitive noise are active in individual economic decision-making (Polania, Woodford and Ruff, 2019; Gershman and Bhui, 2020; Khaw, Li and Woodford, 2021, 2024; Frydman and Jin, 2022; Enke and Graeber, 2023; Enke, Graeber and Oprea, 2025). In addition to testing whether similar mechanisms extend into strategic environments, our setting of a coordination game enables a novel test of the hypothesis that noise arises during action value formation, which is distinct from the hypothesis that noise arises during action selection. Sharp tests of this hypothesis are important because the distinction between the two different types of noise can also shed light on the origin of choice biases in individual decision-making (Woodford 2020).[3] Of course, one additional factor that is present in strategic environments is the need for subjects to form beliefs about opponents' behavior. In our setting, it is important for equilibrium that subjects are aware that (or at least believe that) their opponent faces cognitive noise. In the Online Appendix, we provide evidence from an additional experiment which helps to validate such an assumption. We find that subjects report beliefs that their opponent exhibits more errors in a discrimination task as the distance between states gets smaller. In related work, Enke, Graeber and Oprea (2023) demonstrate that meta-cognition of errors is important for understanding how these errors aggregate at the level of institutions.

The model we propose is also closely related to a set of recent theoretical papers that investigate endogenous information acquisition in coordination games. Yang (2015) shows that the uniqueness result from the global games literature breaks down when players endogenously acquire information about the fundamental using a mutual information cost function. Morris and Yang (2022) show that when the cost function satisfies "infeasible perfect dis-

[3]We discuss the distinction between noise during action value formation and noise during action selection in further detail in Section 2.3.

crimination" — so that signal probabilities vary continuously with the fundamental — then uniqueness is restored. Hébert and Woodford (2021) propose a set of "neighborhood-based" cost functions for rational inattention problems, which are motivated in part by evidence from perceptual experiments. These cost functions satisfy the infeasible perfect discrimination property and, thus, lead to a unique equilibrium in a coordination game. Our model of cognitive noise also gives rise to an endogenous information structure that satisfies infeasible perfect discrimination and leads to a unique equilibrium. Importantly, our experimental data provide novel support for infeasible perfect discrimination in a setting where all information is represented numerically, which complements recent work on experimental tests of the cost function in rational inattention models (Dean and Neligh, 2023).[4]

Finally, our findings connect directly with experimental studies on Stag Hunt games, which emphasize how strategic uncertainty shapes equilibrium selection. Previous research documents that participants frequently coordinate on the risk-dominant equilibrium, especially under heightened uncertainty or payoff structures that increase strategic risk (Cooper et al. 1992, Battalio et al. 2001). Dal Bó et al. (2021) further establish the critical role of the basin of attraction—defined as the largest probability of a counterpart selecting the less cooperative strategy that still incentivizes cooperation—in determining equilibrium outcomes. They show experimentally that enlarging this basin substantially raises coordination on the efficient equilibrium. Our results align closely with these findings: increasing the key payoff parameter θ systematically shrinks the basin of attraction, thereby reducing coordination on the efficient equilibrium. However, our experiment introduces an important and novel dimension: we demonstrate that equilibrium outcomes can systematically depend on cognitive adaptation to the statistical environment. This finding provides a new perspective on how cognitive mechanisms shape strategic uncertainty and equilibrium selection.

2 Model of Cognitive Noise Equilibrium

We study the game shown in Figure 1, where $b \geq a$. Our goal in this section is to derive predictions for this game from our cognitive noise model. As a benchmark, we first present the predictions from the standard model without any cognitive noise. We then introduce cognitive noise into the model and derive its implications for the game, which leads to a cognitive noise equilibrium (CNE). Finally, in order to contrast the predictions of our cognitive noise model with those of leading behavioral game theory models, we also present

[4]For a given fundamental value, we also find intriguing evidence that response times are significantly longer in the high volatility condition (see Online Appendix D for more details). This finding suggests that the implementation of strategies may be more complex (in the spirit of Oprea 2020) in the high volatility condition.

predictions from the Quantal Response Equilibrium model and the Level-k Thinking model.

In what follows, we assume that a and b are encoded without any noise by both players, and we are interested in the implications of noisy encoding of θ.[5] We further assume that each player has linear utility.

2.1 Benchmark: No Cognitive Noise

Without any cognitive noise, the game is one of complete information and its Nash equilibria depend on the true value of θ, as outlined below:

- If $\theta > b$, then Invest is a strictly dominated action for each player, and (Not Invest, Not Invest) is the unique Nash (and dominant strategy) equilibrium.
- If $\theta < a$, then Not Invest is a strictly dominated action for each player, and (Invest, Invest) is the unique Nash (and dominant strategy) equilibrium.
- If $a \leq \theta \leq b$, then there are two Nash equilibria in pure strategies: (Not Invest, Not Invest) and (Invest, Invest). There also exists one Nash equilibrium in mixed strategies.

Thus, when θ takes on values in the intermediate range $[a, b]$, there are multiple pure strategy Nash equilibria. This prediction relies on each player's ability to precisely observe θ, which generates common knowledge about θ. The common knowledge, in turn, enables coordination and gives rise to multiple self-fulfilling equilibria. The predictions change dramatically, however, when we relax the assumption that players can precisely perceive θ.

2.2 Information Processing Constraint: Cognitive Noise

Suppose now that each player's decision process is subject to cognitive noise. Such a hypothesis is motivated in part by the growing evidence from individual decision-making experiments that validates the assumption of cognitive noise in incentivized economic tasks (Gershman and Bhui, 2020; Khaw, Li and Woodford, 2021, 2024; Frydman and Jin, 2022; Enke and Graeber, 2023; Charles, Frydman and Kilic, 2024; Enke, Graeber and Oprea, 2025).

[5] Our assumption that a and b are processed without noise can be justified, for example, through a learning mechanism. In our experiment, we keep a and b constant across all rounds, so the amount of noise in processing a and b is arguably minimal. Nevertheless, if we allow for a small but positive amount of noise in perceiving a and b, the core theoretical result that we present in Section 2.2 regarding equilibrium existence and uniqueness still holds. Specifically, as noise vanishes, the risk-dominant equilibrium emerges as the unique equilibrium. Indeed, Carlsson and Van Damme (1993) show that equilibrium uniqueness via iterative elimination of strictly dominated strategies survives when all payoffs in a 2×2 coordination game are observed with noise.

In particular, we assume that each player makes decisions based on an imperfect representation of θ. The inability of each player to use the exact value of θ in their decision process could stem from noise in the perception or retrieval of θ. Regardless of exactly where noise enters the decision process, the important aspect of our assumption is that the noise corrupts a player's internal representation of θ before she computes the valuation of each action. Such an assumption is, however, still consistent with the ability of each player to repeat back the precise value of θ to the experimenter (for further discussion on this point, see Khaw et al. 2021). In order to minimally depart from the rational benchmark, we assume cognitive noise only corrupts the representation of θ. We formalize this assumption as follows:

Assumption 1 (Cognitive Noise) *Players have a common prior belief that θ is drawn from a continuously differentiable strictly positive density $f(\cdot)$ on the real line. Each player i, $i = \{1, 2\}$, observes a noisy signal of the realized value of θ, $S_i = \theta + \sigma\varepsilon_i$, where each ε_i is independently and identically drawn from a continuous and strictly positive density $g(\cdot)$ on the real line.*

The prior belief about θ, which we denote by $f(\theta)$, can represent public information or past experience in a similar environment that is common to both players. It is worth highlighting how Assumption 1 introduces uncertainty into various aspects of the decision process. To illustrate, we derive the condition under which each player chooses to invest. Player i will invest if and only if:

$$\begin{aligned} \text{EU[Not Invest } |S_i] &< \text{EU[Invest } |S_i] \\ \text{E}[\theta|S_i] &< a + [b-a]\text{E}[p(a,b,\theta)|S_i] \\ \int \theta f(\theta|S_i)d\theta &< a + [b-a]\int p(a,b,\theta)f(\theta|S_i)d\theta, \end{aligned} \tag{1}$$

where $f(\theta|S_i)$ is player i's posterior distribution of θ after observing signal S_i. The function, $p(a, b, \theta)$, maps the game payoffs into a belief about the probability that the opponent invests. In the equilibria of the game, p will be pinned down endogenously by rational expectations but, for now, it is instructive to consider p as exogenous.

In inequality (1), the noisy signal, S_i, appears on both sides of the expression. On the left-hand side, S_i induces uncertainty about player i's own payoff from not investing, which we refer to as *structural uncertainty* (Brandenburger, 1996). In our setting, structural uncertainty can arise from noisy encoding of θ. On the right-hand side, S_i induces uncertainty about the opponent's probability of investing, which we refer to as *strategic uncertainty*.[6] If, for example, player i believes the opponent uses a cutoff rule, then her belief about the

[6]Following, e.g., Morris and Shin (2002, 2004), we define strategic uncertainty as any uncertainty about

opponent investing depends on her belief about the opponent's signal. Since S_i and S_{-i} are drawn conditional on θ, player $i's$ belief about her opponent's perception of θ will depend on S_i.

Given Assumption 1, we can invoke theoretical results from the global games literature to characterize the equilibrium distribution of actions. Let $u(x, l, \theta)$ be a player's payoff when (i) he chooses action x, (ii) the probability that his opponent chooses Not Invest is l, and (iii) the state is θ. Moreover, define $\pi(l, \theta)$ as the payoff gain from choosing Not Invest rather than Invest:

$$\pi(l,\theta) \quad \equiv \quad u(\text{Not Invest}, l, \theta) - u(\text{Invest}, l, \theta) = \theta - b + l(b-a)$$

Our setup satisfies the six conditions from Section 2.2.2 in Morris and Shin (2003):

A1. Action Monotonicity: $\pi(l, \theta)$ is non-decreasing in l.

A2. State Monotonicity: $\pi(l, \theta)$ is non-decreasing in θ.

A3. Strict Laplacian State Monotonicity: There is a unique $\theta^\star$ solving $\int_{l=0}^{1} \pi(l, \theta^\star) dl = 0$.

A4. Uniform Limit Dominance: There exist $\underline{\theta} \in \mathbb{R}$, $\overline{\theta} \in \mathbb{R}$, and $c \in \mathbb{R}_{++}$, such that (1) $\pi(l, \theta) \leq -c$ for all $l \in [0, 1]$ and $\theta \leq \underline{\theta}$; and (2) $\pi(l, \theta) > c$ for all $l \in [0, 1]$ and $\theta \geq \overline{\theta}$.

A5. Continuity: $\int_{l=0}^{1} h(l)\pi(l, \theta) dl$ is continuous with respect to θ and density h.

A6. Finite Moments of Signals: $\int_{z=-\infty}^{\infty} zg(z)dz$, and $\int_{z=-\infty}^{\infty} z^2 g(z)dz$ are well defined.

In the game we study, condition A1 states that the incentive to choose Not Invest is increasing in the probability the opponent chooses the same action (i.e., there are strategic complementarities between players' actions). Condition A2 states that the incentive to choose Not Invest is increasing in the state. Condition A3 introduces a further strengthening of A2 to ensure that there is at most one crossing point between the utilities from the two actions for a player who believes the opponent randomizes uniformly over the available actions. Note that, in our game, $\theta^\star = (a + b)/2$. Condition A4 requires that the payoff gain from choosing Not Invest is uniformly negative for sufficiently low values of θ, and uniformly

the opponent's choice. Thus, uncertainty about the opponent's perception of θ, even conditional on precise knowledge of the opponent's strategy, would still generate strategic uncertainty as long as the opponent's strategy is a function of their perception of θ. Another potential source of strategic uncertainty can derive from uncertainty over the opponent's preferences, e.g., risk aversion (Heinemann et al., 2009). In Section 5, we describe an experiment that can separate between sources of strategic uncertainty that do and do not originate from uncertainty about θ.

positive for sufficiently high values of θ.[7] Condition A5 is a continuity property, where continuity in h is with respect to the weak topology. Condition A6 requires the distributions of noise and squared noise to be integrable and it guarantees that the mean and the variance of noisy signals are finite. Because our game satisfies these six conditions, we can state the following proposition.

Proposition 1 (Equilibrium Existence and Uniqueness) *Let $\theta^\star$ be defined as in A3. For any $\delta > 0$, there exists $\overline{\sigma} > 0$ such that, for all $\sigma < \overline{\sigma}$, the unique strategy surviving iterative elimination of strictly dominated strategies in the game is to choose Invest if $S_i \leq \theta^\star - \delta$ and to choose Not Invest if $S_i \geq \theta^\star + \delta$.*

Proposition 1, adapted directly from Proposition 2.2 in Morris and Shin (2003), tells us that if players have cognitive noise over θ, and the distribution of this noise has a sufficiently small (but positive) variance, then there is a unique equilibrium strategy: choose Invest if the noisy signal is smaller than $\theta^\star - \delta$ and choose Not Invest if the noisy signal is larger than $\theta^\star + \delta$. The proposition from Morris and Shin (2003) does not specify the equilibrium strategy for noisy signals in the interval $(\theta^\star - \delta, \theta^\star + \delta)$. When θ is in this tight interval around the cutoff $\theta^\star$, the proposition only tells us that $\Pr(\text{Invest}|\theta)$ is between $\Pr(S < \theta^\star - \delta|\theta)$ and $\Pr(S < \theta^\star + \delta|\theta)$. However, since the statement holds for any $\delta > 0$, we can focus on small values of δ such that the indeterminate interval $(\theta^\star - \delta, \theta^\star + \delta)$ is negligible. In this case, $\Pr(\text{Invest}|\theta)$ is well-approximated by $\Pr(S < \theta^\star|\theta)$. This allows us to derive the following comparative static predictions.

Proposition 2 (Comparative Statics) *The equilibrium probability that each player invests, $Pr(Invest|\theta)$, is continuous and strictly decreasing in θ. Moreover, increasing the variance of the noisy signal, σ, decreases the sensitivity of equilibrium choices to θ (that is, the rate at which $Pr(Invest|\theta)$ decreases with θ).*

The first part of Proposition 2 indicates that, even in the absence of any explicit private signals, there exists a unique equilibrium in which the probability of investing is decreasing in θ. If we operationalize coordination as both players investing or both players not investing, then it follows that coordination will also be systematically related to θ. In particular, the model predicts that the probability of coordination is a U-shaped function of θ, which has its minimum at $\theta = (a+b)/2$. We emphasize that the prediction of a systematic relationship

[7] To see that our setup satisfies this condition, recall that $\pi(l,\theta) = \theta - b + l(b-a)$ and note that $-b + l(b-a)$ reaches its minimum at $-b$ (when $l = 0$), grows linearly in l, and reaches its maximum at $-a$ (when $l = 1$). Thus, $\pi(l,\theta) \in [\theta - b, \theta - a]$. Fix a $c \in \mathbb{R}_{++}$. Any $\underline{\theta} \leq a - c$ and any $\overline{\theta} \geq b + c$ satisfy A4.

between θ and the probability of coordination does not arise in the complete information version of the game.

The second part of Proposition 2 is not immediately testable, as we cannot directly observe the amount of cognitive noise. However, we can make an even starker prediction about equilibrium outcomes by putting more structure on S_i in a way that is grounded in the growing literature on noisy coding in economic decision-making. Specifically, there are now several pieces of empirical evidence which suggest that the structure of cognitive noise—i.e., the distribution of S_i in our model—is shaped by prior beliefs (Polania, Woodford and Ruff, 2019; Frydman and Jin, 2022; Payzan-LeNestour and Woodford, 2022). This evidence supports the hypothesis that the conditional noisy signal distribution is not fixed, but is instead malleable and shaped by the decision-maker's prior beliefs. Such an endogenous relationship between priors and signals can be micro-founded through a variety of mechanisms, one of which is called *efficient coding* (Girshick, Landy and Simoncelli, 2011; Wei and Stocker, 2015; Heng, Woodford and Polania, 2020).

A key premise of efficient coding is that, while the brain has limited cognitive resources and is thus unable to precisely discriminate between all possible values of θ, a player can optimally allocate cognitive resources based on her prior beliefs about θ. Consider a player who faces a sequence of games, each of which is characterized by a realization of θ. Suppose further that, in each instance of the game, θ is drawn from a normal distribution—as it will be in our experimental design. As the variance of the normal distribution increases (holding the mean fixed), efficient coding predicts that cognitive resources will be allocated away from the mean and toward more extreme values. Intuitively, as the variance of the prior increases, the player needs to distribute her cognitive resources more broadly across the state space, which results in noisier perception of each value of θ. Thus, for a given value of θ, the amount of cognitive noise that the player faces will depend on her prior beliefs about θ. In Appendix A, we adopt the efficient coding and normal prior model of Khaw et al. (2021) to microfound the following assumption:

Assumption 2 (Efficient Coding) *Suppose that the distributions of θ and ε_i are normal, $\theta \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$ and $\varepsilon_i \sim \mathcal{N}(0,1)$. The variance of the noisy signal distribution, σ^2, is increasing in the variance of the prior distribution, σ_θ^2.*

Note that because Proposition 1 does not depend on distributional assumptions on θ or ε_i, we obtain the same unique equilibrium under the additional conditions imposed in Assumption 2. Crucially, Assumption 2 generates a novel comparative static prediction: the rate at which the probability of investing decreases in θ is faster as the prior variance decreases.

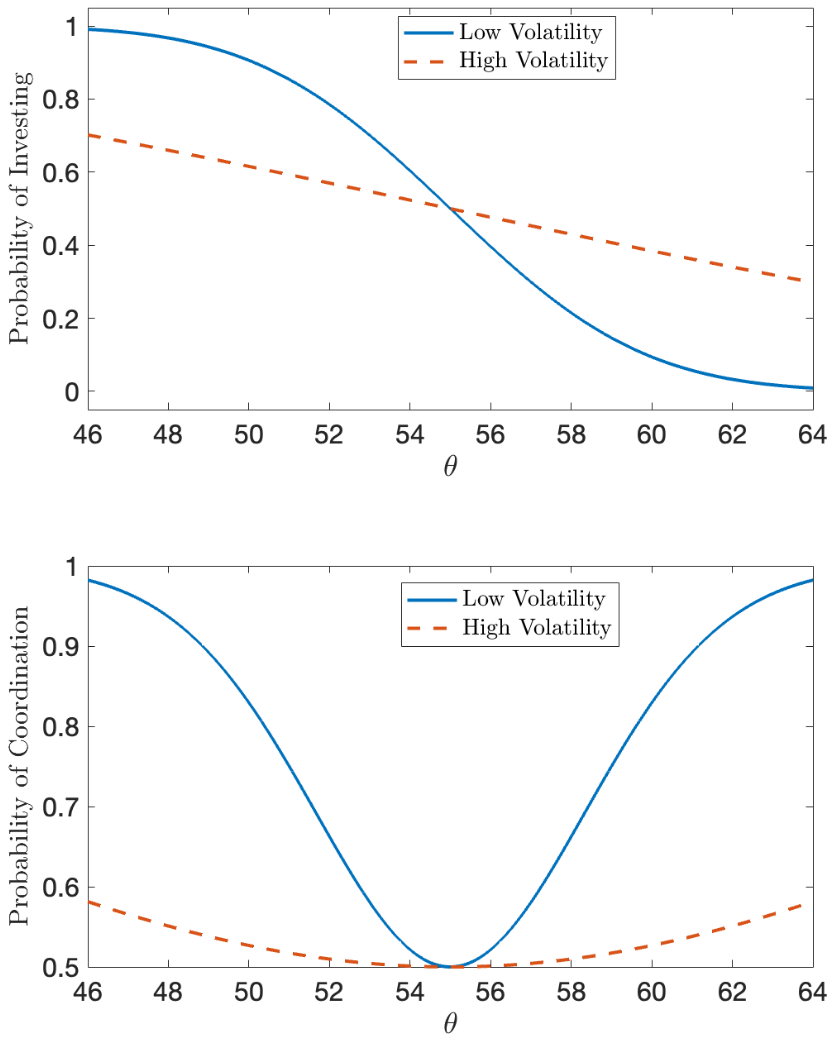

Figure 2: **Probability of Investing and Coordination as a Function of θ.** Note: The upper panel displays the predicted probability of investing. The bottom panel displays the predicted probability of coordination, where coordination is defined as both players investing or both players not investing. In both panels, the solid line denotes the prediction for a low volatility prior distribution with $\theta \sim \mathcal{N}(55, 20)$; the dashed line denotes the prediction for a high volatility prior distribution with $\theta \sim \mathcal{N}(55, 400)$; the conditional signal distribution is normal: $S_i \sim \mathcal{N}(\theta, \omega_C^2 \sigma_\theta^2)$; we set the following parameter values: $\omega_C = 0.85$, $a = 47$, $b = 63$.

Corollary 1 *Increasing the prior variance of θ, σ_θ, will decrease the sensitivity of choices to θ (that is, the rate at which $Pr(Invest|\theta)$ decreases with θ).*

The intuition for this result is simple. Each player's action is deterministic in their signal S_i, such that they invest if and only if $S_i < (a + b)/2$. Efficient coding induces the signal to become noisier as the prior becomes more dispersed. The noisier signal immediately leads to a less sensitive relationship between behavior and the true fundamental θ. In Appendix A, we provide a model with normally distributed prior beliefs and normally distributed noise, for which we characterize the distribution of equilibrium actions. This special normal-normal case is particularly relevant, as our experimental design induces a prior over θ that is approximately normally distributed.

We illustrate the theoretical predictions of our model under this special case in Figure 2. In this case of normal prior and normal signal, the equilibrium probability of investing is given by $\mathbb{P}(\text{Invest}|\theta, \omega_C, \sigma_\theta) = \Phi\left(\frac{55-\theta}{\omega_C \sigma_\theta}\right)$. In this expression (which we formally derive in Appendix A using the model from Khaw et al. 2021), we replace σ with $\omega_C \sigma_\theta$ so that the standard deviation of noisy signals, σ, is proportional to the standard deviation of the prior. The constant of proportionality is the free parameter ω_C. One can think of this free parameter as indexing the degree of the cognitive constraint: fixing the prior, a larger value of ω_C leads to a noisier signal of θ. Figure 2 shows that as we shift from a low volatility prior to a high volatility prior, behavior becomes less sensitive to θ. We note that the prediction that greater dispersion of stimuli leads behavior to become less sensitive to a change in the stimulus value is predicted by a broad class of theories, including the decision-by-sampling model from cognitive science (Stewart et al., 2006), theories of normalization from neuroscience (Rangel and Clithero, 2012; Louie et al., 2015), and alternative specifications of efficient coding (Wei and Stocker, 2015; Heng et al., 2020; Payzan-LeNestour and Woodford, 2022; Frydman and Jin, 2022). Thus, the context-dependent perception that is encoded in Assumption 2, and generates our main theoretical prediction, can arise from a variety of microfoundations. Next, we show that this theoretical prediction of context-dependence does not arise under alternative behavioral game theory models.

2.3 Predictions from Alternative Models

Behavioral game theorists have proposed a variety of alternative models which relax the standard assumptions of perfect maximization and rational beliefs. For example, Quantal Response Equilibrium assumes imperfect maximization but retains the rational beliefs assumption. Level-k Thinking relaxes the rational expectations assumption but maintains best responses. Below, we derive predictions from these two leading behavioral game theory models and demonstrate how our model of CNE differs in terms of both assumptions and predictions. As we will see, one important conclusion is that neither QRE nor Level-K Thinking predict that behavior depends on prior beliefs about θ.

Quantal Response Equilibrium

In our cognitive noise equilibrium model, noisy encoding of θ generates stochastic strategic behavior. As such, our model is related to Quantal Response Equilibrium (McKelvey and Palfrey, 1995, 1998; Goeree, Holt and Palfrey, 2016), which is a leading model of stochastic behavior in experimental game theory.[8] For some parameter values, the models of QRE and cognitive noise deliver similar predictions, in that both theories predict that the probability of investing is stochastic and decreases smoothly and monotonically in θ. However, there are fundamental differences in the assumptions of the two theories, which generate distinguishing predictions.

The key difference in assumptions comes from the stage at which noise enters the decision process.[9] In our model, noise arises early in the decision process when each player forms a noisy internal representation of θ. This noisy representation of θ is then used as an input to compute the expected utility of each action. In contrast, under QRE, noise arises late in the decision process, after each player has computed the expected utility of each action, conditional on the exact value of θ.

In the game we study in this paper, QRE predicts that a player invests if and only if:

$$\begin{aligned} \text{EU[Not Invest]} + \eta_1 &< \text{EU[Invest]} + \eta_2 \\ \theta + \eta_1 &< a + p[b-a] + \eta_2 \\ \theta &< a + p[b-a] - (\eta_1 - \eta_2), \end{aligned} \tag{2}$$

where p is the belief about the probability the opponent invests, and η_1 and η_2 are the late noise perturbations to payoffs. Before making her choice, each player receives a perfectly informative signal about η_1 and η_2 (uncorrelated with the opponent's perturbations to payoffs). If we assume that these perturbations are independently and normally distributed with mean 0 and variance $\sigma_\eta^2 > 0$, we have:

$$\mathbb{P}(\text{Invest}) = \phi(p, \theta) = \Phi\left(\frac{a + p[b-a] - \theta}{\sqrt{2}\sigma_\eta}\right).$$

A Quantal Response Equilibrium then requires that p is a fixed point, conditional on θ; i.e., a QRE is a solution to $p = \phi(p, \theta)$.

It is useful to compare the condition for investing under QRE (displayed in inequality

[8] For other models of strategic interaction with stochastic choice, see Goeree and Holt (2004), Friedman and Mezzetti (2005), Goeree and Louis (2021), and Gonçalves (2023).

[9] We are grateful to Michael Woodford for highlighting this point in an illuminating discussion of our paper.

(2) above) with the analogous condition for investing under cognitive noise (displayed in inequality (1)). Inequality (1) indicates that, with cognitive noise, players remain uncertain about the true value of θ even after θ is realized; the residual uncertainty comes from the fact that players only have access to a noisy signal of θ. As a consequence, player i believes that player j's signal about θ is centered at i's perceived value of θ (which is a function of i's signal about θ). In contrast, the true value of θ appears in inequality (2), which implies that, in QRE, the player has no uncertainty about θ. It follows that, in QRE, player i believes that player j's signal about θ is centered at the true value of θ.

The difference in assumptions about when noise enters the decision process leads to two important distinguishing predictions. The first difference is that, in QRE, each player encodes θ precisely, and thus there is no role for a prior belief over θ. The prior belief does, however, play a key role in our model of cognitive noise. Specifically, our model predicts that the prior belief affects the precision of the noisy signal, S_i, through efficient coding. Our model therefore endogenizes the noise structure and generates context dependent behavior in equilibrium.[10]

The second difference between QRE and cognitive noise involves the theoretical conditions that are sufficient to generate a unique equilibrium. As shown in Proposition 1, cognitive noise generates a unique equilibrium when the variance of noise is sufficiently small. One interpretation of this condition is that when players pay sufficient attention to the coordination game, and hence the variance of the noisy signal S_i is sufficiently small, uniqueness obtains under our theory of cognitive noise. In contrast, QRE delivers a unique equilibrium when the variance of the shock to payoffs is sufficiently *large* (Ui, 2006). While our experimental data will not enable us to test between this difference in conditions for uniqueness, one implication is that when players devote a substantial amount of attention to the coordination game, the multiplicity of equilibria is more likely to be eliminated under cognitive noise, compared with QRE.

Level-k Thinking

Another leading model in behavioral game theory is Level-k Thinking (Stahl and Wilson, 1994, 1995; Nagel, 1995; Camerer, Ho and Chong, 2004). In one prominent version of this theory, there are different types of players, and each type best responds to another type who exhibits one less degree of strategic sophistication. For example, a Level-0 type would be characterized by no strategic sophistication and, thus, would exhibit purely random behavior. A Level-1 type would then best respond to a Level-0 player, and a Level-2 player would best

[10] In QRE, the noise structure is usually taken to be exogenous. Friedman (2020) proposes a model that endogenizes the precision parameter in QRE through the set of payoffs in the current game.

respond to a Level-1 player, and so on. What are the predictions of Level-k Thinking for the game we study in Figure 1? Following the analysis in Kneeland (2016) and given that Level-0 players randomize, the expected utility of a Level-1 player from Invest is

$$EU_{L1}(\text{Invest}) = \frac{1}{2}a + \frac{1}{2}b$$

Thus, $EU_{L1}(\text{Invest}) > EU(\text{Not Invest})$ if and only if $\theta < (a+b)/2$. Next, under the assumption that Level-2 players believe they are facing a Level-1 opponent, the expected utility from Invest for a Level-2 player is

$$EU_{L2}(\text{Invest}) = \begin{cases} b \text{ if } \theta < (a+b)/2 \\ a \text{ if } \theta > (a+b)/2 \end{cases}$$

When $\theta < (a+b)/2$, then $EU_{L2}(\text{Invest}) = b > \theta$. Conversely, when $\theta > (a+b)/2$, then $EU_{L2}(\text{Invest}) = a < \theta$. Thus, Level-2 players choose Invest if and only if $\theta < (a+b)/2$. Using the same logic, we obtain the same prediction for all higher levels.

In sum, the fraction of subjects who choose Invest is:

$$\Pr[\text{Invest}] = \begin{cases} \Pr[L_0]\frac{1}{2} + (1 - \Pr[L_0]) & \text{if } \theta < (a+b)/2 \\ \Pr[L_0]\frac{1}{2} & \text{if } \theta > (a+b)/2 \end{cases}$$

where $\Pr[L_0]$ is the fraction of Level-0 players in the population. The theory therefore predicts that, in the aggregate, the probability of investing is monotone in θ and exhibits a sharp decrease at $\theta = (a+b)/2$. In contrast, both our model of cognitive noise and QRE predict that the probability of investing declines continuously in θ. The more important difference with respect to cognitive noise is that Level-k Thinking does not predict that behavior depends on prior beliefs.

In summary, cognitive noise equilibrium predicts that conditional on θ, behavior will depend systematically on prior beliefs. In contrast, both QRE and Level-k do not generate any context-dependence. It is this unique prediction of the cognitive noise model that motivates our experimental design.

3 Experimental Design of Coordination Game

We test the cognitive noise model by incentivizing subjects to play a simultaneous move game, and we manipulate the distribution that generates the fundamental payoff, θ. We preregister the experiment and recruit 300 subjects from the online data collection platform, Prolific.[11] We restrict our sample to subjects who, at the time of data collection, (i) were UK nationals and residents, (ii) did not have any previous "rejected" submissions on Prolific, and (iii) answered all comprehension quiz questions correctly.[12] Subjects are paid 2 GBP (~ 2.8 USD) for completing the experiment, and they have the opportunity to receive additional earnings based on their choices and the choices of other participants.

The experiment consists of 300 rounds, and each subject participates in all rounds. In each round, a subject is randomly matched with another subject and, together, they play the simultaneous move game in Figure 1. We hold constant the payoff parameters $a = 47$ and $b = 63$ across all rounds. The only feature of the game that varies across rounds is the value of θ, which is drawn from the condition-specific distribution $f(\theta)$. In each round, both subjects observe the same realization of θ. In order to shut down learning about other participants' behavior, we choose not to provide subjects with feedback about their earnings or their opponent's choice in a given round. At the end of the experiment, one round is selected at random, and subjects are paid according to the number of points they earned in that round, which in turn, depends on their action, their opponent's action, and the (round-specific) value of θ. Points are converted to GBPs using the rate 20:1. The average duration of the experiment was ~ 25 minutes and average earnings, including the participation fee, were ~ 5.5 GBP (~ 7.7 USD).

Subjects are randomized into one of two experimental conditions: a *high volatility* condition or a *low volatility* condition, which differ only based on the distribution of θ. In the high volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 400. In the low volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 20. In both conditions, after drawing θ from its respective distribution, we round θ to the nearest integer, and we re-draw θ if the rounded value is less than 11 or greater than 99. We implement these modifications to the normal distribution to control complexity and ensure that θ is a two-digit number on each round.

We do not give subjects any explicit information about $f(\theta)$ in the instructions, as our intention is to test whether a subject can adapt to the statistical properties of the environment without explicit top-down information. Moreover, we believe that such a design is more natural than explicitly telling subjects the distribution of parameters they will experience, as this could artificially direct their attention to the distribution and potentially generate an

[11] The pre-registration document is available at `https://aspredicted.org/IHU_KCE`.

[12] After reading the instructions, participants were asked three questions to test comprehension of the instructions. Only participants who answered all three questions correctly were allowed to continue with the experiment. In each question, participants were presented with an example game they might face during the experiment (that is, they were shown a game with a realized value of θ) and asked to compute the earnings resulting from a hypothetical configuration of own and other's actions.

experimenter demand effect. Because we do not explicitly endow subjects with the prior, our design enables us to test for learning effects (i.e., is the predicted treatment effect stronger towards the end of the experiment?) We note that in previous experimental work on efficient coding, full adaptation can take roughly 200 rounds, which is why we choose a relatively large number of rounds in our design (Heng et al., 2020). Each condition contains an identical set of instructions and comprehension quiz.[13] As outlined in our pre-registration, we exclude the first 30 rounds from our analyses, in order to allow subjects time to adapt to the distribution of θ.[14]

Recall that, in the complete information version of the game, there are multiple equilibria when θ is in the range $[47, 63]$. We therefore focus our analyses on games for which θ lies in this range. We pre-register that our main analyses are restricted to those rounds for which $\theta \in [47, 63]$ and we call these "common rounds." This is a crucial feature of our design, because it allows us to compare behavior across conditions using the exact same set of games and varying only the context, that is, the distribution of past games.

In choosing the parameters for our design (a, b and the two condition-specific values of σ_θ), we strike a balance among three competing objectives: (i) generating a substantial number of common rounds to analyze, (ii) creating a large predicted treatment effect, and (iii) guaranteeing the empirical distributions of θ approximate the distributions that we assume in the theory. There is a tension between the first objective and each of the latter two. First, a natural way to create a large predicted treatment effect is to set a large value of σ_θ in the high volatility condition. However, if this parameter is too large, there will be relatively few draws for which $\theta \in [47, 63]$ and, thus, few common rounds to analyze in this condition. Second, theory requires us to choose an $[a, b]$ range which is not too large. Specifically, equilibrium uniqueness requires that, in both conditions, subjects believe there is some chance of observing games with dominant strategies, that is, games with $\theta < a$ and games with $\theta > b$. At the same time, reducing the distance between a and b — e.g., choosing $a = 50$ and $b = 60$ — would reduce the number of common rounds to analyze.

Figure 3 provides a screenshot of a single round shown to subjects. In order to avoid framing effects, we label the two options "Option A" and "Option B", and the left-right location of each option is randomized across rounds. The number "45" is the realized value of θ on the specific round shown in Figure 3. We emphasize that — while the number is clearly displayed to all subjects and, thus, would traditionally be interpreted as public

[13]The experimental instructions are available in Online Appendix C.

[14]While previous work has shown that full adaptation can take roughly 200 rounds (Heng et al., 2020), our analysis does not require complete adaptation. Our pre-registered choice of excluding only the first 30 rounds in the empirical analyses reflects a balance between allowing sufficient adaptation time and statistical power for detecting a treatment effect.

Option A	Option B
45	**47** if other participant chooses A **63** if other participant chooses B

Figure 3: **Sample Screenshot Shown to Participants in Experiment 1.** Note: In this round, the realized value of θ is 45, which is clearly and explicitly displayed to both subjects. Subjects choose “Option A” or “Option B” by pressing one of two keys on the keyboard.

information — here we rely on cognitive noise to transform the fundamental value into private information. In other words, we assume that cognitive constraints prevent each player from precisely perceiving and retrieving the fundamental value in order to compute the value of each action.

Finally, we intentionally choose the visual display of the experiment to be as simple as possible, so that we only present the values of a, b, and θ once on each experimental screen. An alternative approach would be to display the game in matrix form, similar to the display in Figure 1. While the matrix approach is more standard in experimental economics, it may also be interpreted by subjects as more complex compared to our design in Figure 3. Importantly, the complexity of how information is presented has recently been shown to affect the level of cognitive noise (Enke and Graeber, 2023). Thus, we do not believe one display strictly dominates another. On the contrary, differences in display may systematically affect cognitive noise which could motivate modifications of our design to assess the impact on coordination.

4 Experimental Results from the Coordination Game

4.1 Choice Behavior

Following our pre-registration, we restrict our analysis to common rounds after the initial 30-round adaptation period. We also exclude observations for which subjects execute a decision with a response time of less than 0.5 seconds, which generates a final sample of

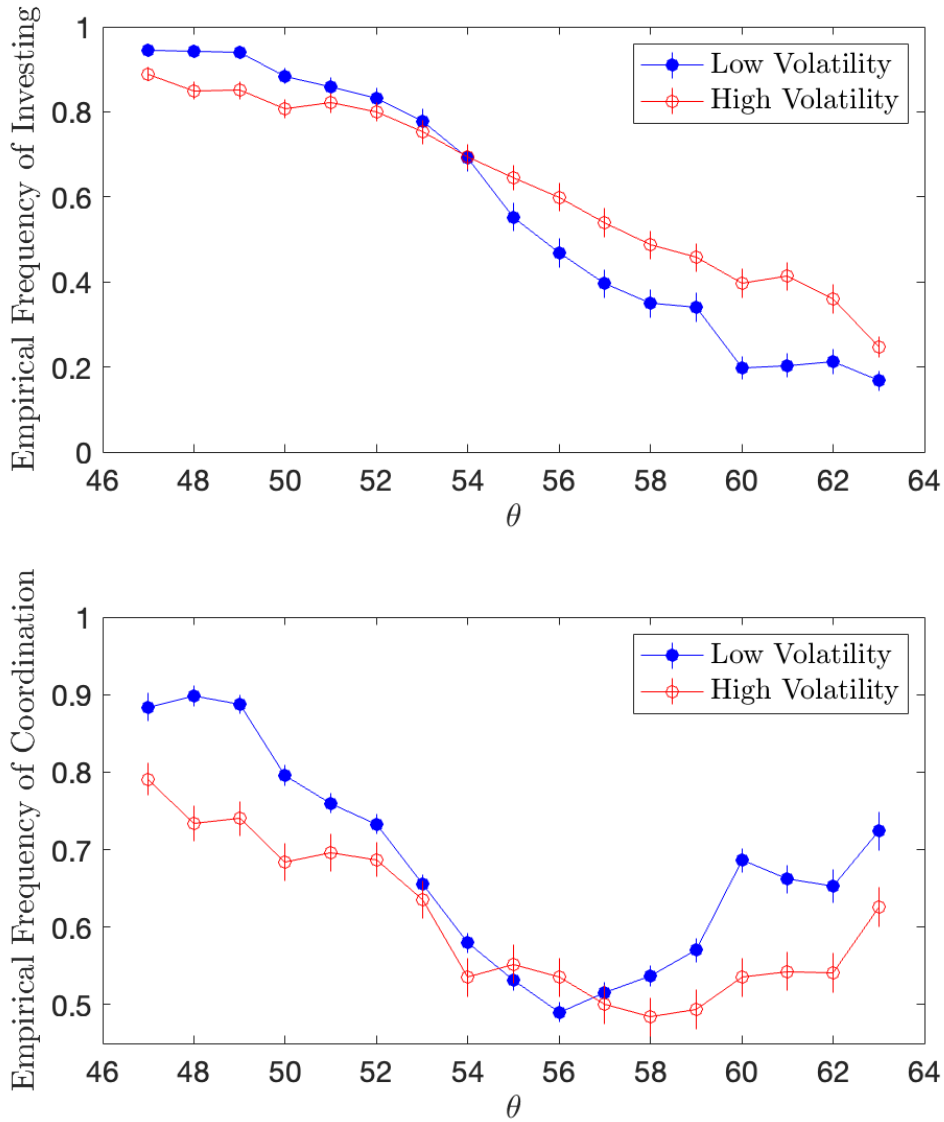

Figure 4: **Empirical Frequency of Investing and Coordination as a Function θ.** Note: In the Upper Panel, for each value of θ between 47 and 63, we plot the proportion of rounds on which a subject chooses to invest, separately for each of the two conditions. Data are pooled across subjects and are shown for rounds 31-300, after an initial 30-round adaptation period. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject. In the Lower Panel, we plot the proportion of games for which the pair of subjects coordinate (both subjects invest or neither subject invests). Data are shown for rounds 31-300. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject pair.

50,129 decisions (36,580 decisions in the low volatility condition and 13,549 decisions in the high volatility condition).[15] Across both conditions, subjects choose to invest on 58.9% of rounds.

In the upper panel of Figure 4, we plot the probability of investing as a function of θ, separately for the two experimental conditions. One can see that, in both conditions, the aggregate data are consistent with Proposition 2: the frequency with which subjects invest is monotone in θ. In the bottom panel of Figure 4, we plot the frequency of coordination outcomes as a function of θ. In both conditions, we observe a systematic relationship between the likelihood of coordination and θ: coordination is more likely as θ becomes farther from 55.

In order to provide a more targeted test of cognitive noise, we focus on the prediction from Corollary 1, which implies that the distribution of noisy signals should vary systematically across our two experimental conditions. Specifically, efficient coding predicts context-dependent behavior, where subjects in the low volatility condition can more precisely detect whether the fundamental crosses the unique equilibrium threshold. The upper panel of Figure 4 provides evidence consistent with this prediction: the frequency of investing is more sensitive to the fundamental in the low volatility condition. The differential slopes shown in the upper panel of Figure 4 represent our main experimental result, which separates cognitive noise equilibrium from a broad class of alternative game-theoretic models, such as QRE and Level-k thinking.

To formally test the difference in slope, we estimate a series of mixed effects logistic regressions which account for the fact that each subject contributes more than one observation to the dataset. Column (1) of Table 1 confirms our main result: the coefficient on the interaction term $(\theta - 55)$ x *Low* is significantly negative ($p < 0.001$), indicating that the probability of investing decreases in the fundamental more rapidly when a subject is adapted to the low volatility condition.[16] Columns (2) and (3) show that this result holds in both

[15] We impose the pre-registered cutoff of 0.5 seconds to avoid analyzing decisions that are made excessively fast. This restriction excludes 1,916 decisions; our results are robust to including these excessively fast decisions in our analyses. Note also that our final sample contains substantially more observations from the low volatility condition, which is driven by the fact that the realized value of θ on each round is more likely to fall in the range [47, 63] in the low volatility condition, compared to the high volatility condition.

[16] The results in Table 1 are restricted to those rounds where $46 < \theta < 64$. One reason for this restriction is because theory predicts that behavior will be more sensitive to θ in the low volatility condition only for values of θ sufficiently close to 55 (see upper panel of Figure 2). Thus, restricting our tests to those values of θ close to 55 provides a more targeted test of the theory. Also note that by design, we have many fewer rounds for which $\theta < 47$ or $\theta > 63$, which comprises the "dominance region". Our theory of cognitive noise predicts that even among these games, subjects will choose a dominated action with positive probability. To test this prediction, we first pool games across conditions because of the limited sample size. We find that subjects choose the dominated action of "not invest" on 8.4% of rounds where $\theta < 47$. When regressing the probability of investing on θ for rounds when $\theta < 47$, we find a negative, though statistically insignificant

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.458***	-0.463***	-0.568***	-0.482***
	(0.033)	(0.041)	(0.058)	(0.039)
$(\theta - 55)$ x Low	-0.499***	-0.317***	-0.477***	-0.340***
	(0.063)	(0.058)	(0.079)	(0.060)
Low	-0.182	-0.316	-0.215	-0.268
	(0.386)	(0.335)	(0.419)	(0.351)
Late				-0.064
				(0.140)
$(\theta - 55)$ x Late				0.024
				(0.024)
Low x Late				0.115
				(0.173)
Low x $(\theta - 55)$ x Late				-0.083**
				(0.039)
Constant	1.351***	1.260***	1.470***	1.263***
	(0.221)	(0.223)	(0.295)	(0.228)
Observations	50,129	9,425	9,201	18,626
Rounds	31-300	31-80	251-300	(31-80)
				& (251-300)

Table 1: **Treatment Effect Estimates.** Note: Table displays results from mixed effects logistic regressions. Observations are at the subject-round level. The dependent variable takes the value 1 if the subject chooses to Invest and 0 otherwise. The variable *Low* takes the value 1 if the round belongs to the low volatility condition and 0 otherwise. The variable *Late* takes the value 1 if the round number is 251 or greater and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included in the regressions. There are random effects on $(\theta - 55)$ and the intercept. Standard errors are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

early (first 50 trials after adaptation) and late (last 50 rounds of the session) subsamples (both with $p < 0.001$). Column (4) indicates that the treatment effect becomes moderately stronger over the course of the experiment, as the coefficient on the triple interaction is negative ($p = 0.034$). The strengthening of the treatment effect over the course of the experiment suggests that subjects have not fully adapted to the distribution by round 80 and that additional rounds of play provide the opportunity for further adaptation.

The bottom panel of Figure 4 shows that coordination also exhibits a strong degree of context-dependence. Subjects in the low volatility condition are significantly more likely to coordinate their behavior than subjects in the high volatility condition (63.8% vs. 60.5%; $p < 0.001$ for a difference in means). Moreover, this difference in coordination frequency is more pronounced for games where θ is farther from 55, consistent with the theoretical prediction shown in the bottom panel of Figure 2. The difference in coordination frequency across conditions also holds (and becomes moderately stronger) when we control for θ. In sum, our main results in Figure 4 demonstrate that (i) coordination frequency depends systematically on θ and that (ii) increasing the precision with which subjects process information about θ increases the likelihood of coordination.

4.2 Structural Estimation

In this subsection we quantitatively compare the fits of each of the three different models discussed in Section 2: cognitive noise equilibrium, QRE, and Level-k. As with the reduced form analyses reported in the previous subsection, here we exclude rounds 1-30 and any rounds for which the subject executed a decision in less than 0.5 seconds. We use all remaining data in our structural estimation and we do not place any restriction on the value of θ.[17]

We begin by estimating the cognitive noise equilibrium model, assuming that the cognitive noise parameter, ω_C, is homogenous across subjects. In equilibrium, the probability of investing is given by the following equation:

coefficient. When $\theta > 63$, subjects choose the dominated action of "invest" on 12.8% of rounds. When regressing the probability of investing on θ for rounds where $\theta > 63$, we find a negative and statistically significant relationship. These results are therefore consistent with our model of cognitive noise, whereby cognitive noise is apparent in the dominance region, and its impact on choice becomes more consequential as θ moves closer to the cutoff of 55.

[17] The cognitive noise model predicts that choice sensitivity to θ is greater in the low volatility condition when θ is sufficiently close to 55. Hence in the reduced form regressions that we report in the previous subsection, we restricted the analysis to $\theta \in (46, 64)$. This restriction is not necessary in our structural estimation, and thus we additionally include those rounds for which the value of θ is inside the dominance regions. Moreover, including all values of θ in the structural estimation allows for a cleaner comparison of model fits with the Level-k and QRE models.

$$\mathbb{P}(\text{Invest}|\theta, \omega_C, \sigma_\theta) = \Phi\left(\frac{55-\theta}{\omega_C \sigma_\theta}\right), \tag{3}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal.[18] We estimate the one free parameter in the model, ω_C, using maximum likelihood estimation. We maximize the following log-likelihood function over ω_C, using data from all 300 subjects:

$$LL(\omega_C) = \sum_{i=1}^{300}\sum_{t=31}^{300} y_{it} \cdot \log\left(\mathbb{P}(\text{Invest}|\theta_t, \omega_C, \sigma_{\theta,i})\right) + (1-y_{it}) \cdot \log(1 - \mathbb{P}(\text{Invest}|\theta_t, \omega_C, \sigma_{\theta,i})), \tag{4}$$

where y_{it} denotes subject i's choice in round t, with $y_{it} = 1$ if the subject chooses to invest and $y_{it} = 0$ if the subject chooses not to invest. We maximize Equation (4) using both the Optim package in Julia and a grid search method to ensure the best fitting parameter does not depend on the maximization algorithm. We find that the log-likelihood is maximized at the parameter value $\omega_C = 1.057$, and the value of the maximized log-likelihood is -40,269. Note that even though we restrict ω_C to be the same across each of our two experimental conditions, the estimated model still predicts different behavior across the two conditions as the condition-specific volatility parameter σ_θ appears in equation (3).

We use the same estimation procedure for the probit QRE model, which predicts that the probability of investing is given by:

$$\mathbb{P}(\text{Invest}|\theta, \sigma_Q) = \phi(p, \theta) = \Phi\left(\frac{47 + p[16] - \theta}{\sqrt{2}\sigma_Q}\right), \tag{5}$$

where p is the player's belief about the probability that his opponent invests and $\Phi(\cdot)$ is the cumulative density function of the standard normal. A QRE then requires that p is a fixed point, conditional on θ; i.e., a QRE is a solution to $p = \phi(p, \theta)$. We again use maximum likelihood to estimate the model, and estimate the parameter σ_Q, assuming it is homogenous across the subject population. In particular, we substitute equation (5) into the right-hand side of equation (4). The log-likelihood is maximized at the parameter value $\sigma_Q = 12.574$, and the value of the maximized log-likelihood is -41,256.[19]

[18]When ω_C is sufficiently high, the CNE model can generate multiple equilibria for some values of θ. In particular, the multiple equilibria are characterized by different values of k^* in the equation, $\mathbb{P}(\text{Invest}|\theta, \omega_C, \sigma_\theta) = \Phi\left(\frac{k^*-\theta}{\omega_C\sigma_\theta}\right)$. To provide a conservative estimate of the in-sample fit of the CNE model, when there are multiple equilibria for a given (ω_C, θ) pair, we select the equilibrium k^* threshold that minimizes the likelihood of the observed choice.

[19]For some values of the parameter pair (σ_Q, θ), the QRE model predicts multiple equilibria. In these instances, we select the equilibrium that maximizes the likelihood given the subject's choice. While this equilibrium selection procedure is ad-hoc, it clearly gives QRE the best shot of attaining a maximized

The third behavioral model we estimate is the Level-k model. This model also has one free parameter, f_0, which represents the fraction of "level-0" types in the population. Specifically, the model predicts that the probability of investing is given by:

$$\mathbb{P}(\text{Invest}|\theta, f_0) = \begin{cases} f_0\frac{1}{2} + (1 - f_0), & \text{if } \theta < 55 \\ \frac{1}{2}, & \text{if } \theta = 55 \\ f_0\frac{1}{2}, & \text{if } \theta > 55, \end{cases} \tag{6}$$

We proceed by substituting equation (6) into the right-hand side of equation (4) and maximizing over f_0. We find that the log-likelihood is maximized at the parameter value $f_0 = 0.418$, and the value of the maximized log-likelihood is -40,971.

Because each of the three models we estimate in this section has a single free parameter, one can assess the model fits by simply ranking the maximized log-likelihood values. Table 2 summarizes the best-fitting parameter for each model and the associated maximized log-likelihood value. We find that the CNE model fits best, followed by the Level-k model, and then finally the QRE model. One intuition for why the CNE model attains the best in-sample fit, is because it is the only model of the three that allows behavior to depend directly on the experimental condition (through the volatility parameter σ_θ). Figure 4 clearly shows that conditional on θ, behavior does depend systematically on the experimental condition.

In principle, one could allow the single free parameter in each of the three models to depend on the experimental condition. This would lead to better fits for the QRE and Level-k models. However, we emphasize that this flexibility is not motivated by any of the three theories, including CNE. The only reason that CNE predicts that choice probabilities should vary across conditions is because the condition-specific prior contains valuable information about the perception of θ. This dependence on the prior is not predicted by the QRE or Level-k models, and hence σ_θ does not appear in equations (5) or (6) (while it does appear in equation (4)). In sum, we adhere to the restrictions imposed by equations (4), (5), and (6) where the single free parameter in each of the three models is constrained to be the same across experimental conditions.

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log-likelihood value that exceeds that of the competing models.

Table 2: Structural Estimates

Model	**Parameter**	**Log $\hat{L}$**	**AIC**	**BIC**
Cognitive Noise Equilibrium	$\omega_C = 1.057$	-40269	80540	80550
Quantal Response Equilibrium	$\sigma_Q = 12.574$	-41256	82514	82523
Level-k Thinking	$f_0 = 0.418$	-40971	81945	81954

5 Experiment on Decomposing Structural Uncertainty and Strategic Uncertainty

In our model, the only source of strategic uncertainty is cognitive noise. That is, the only reason a player is uncertain about her opponent's behavior is because of uncertainty about the opponent's representation of θ (see inequality (1)). In reality, there are surely other sources of strategic uncertainty besides cognitive noise. For example, there may be uncertainty about which of multiple equilibria (of the complete information version of the game) the opponent is playing. Other possible sources include uncertainty about the opponent's degree of rationality or preferences. The stochastic behavior we observe in Figure 4 can therefore be a consequence of noise in processing θ or alternative sources of strategic uncertainty. Our objective in this section is to quantitatively assess how much of the observed noise in behavior can be attributed to cognitive noise and how much is driven, instead, by other sources of strategic uncertainty.

To address this question, we conduct a second experiment in which a new sample of subjects plays the same simultaneous move game as in the previous experiment. The only difference is that, here, subjects are told that their opponent is a computer that plays a known and deterministic strategy. In particular, we tell subjects that the computerized opponent chooses to invest if and only if $\theta < 55$. Thus, the computerized opponent's strategy coincides with the unique equilibrium strategy in the game where each player has a small amount of cognitive noise about θ.[20] This treatment should, therefore, eliminate strategic uncertainty

[20] In the game where each human player has cognitive noise, player i is indifferent between investing and not investing when (a) $E[\theta|S_i] = 55$ and (b) player i believes his human opponent follows the strategy prescribed by the unique equilibrium from Proposition 1. Because our goal here is to completely remove any uncertainty about the opponent's strategy that is not induced by noisy perception of θ, we design the computerized opponent to play a deterministic strategy when $\theta = 55$, namely, not invest with probability 1. As a consequence, in the game where $\theta = 55$, the best response of a human subject who perceives θ without

— except for the strategic uncertainty that is induced by a subject's own imprecision of θ.

5.1 Experimental Design and Procedures

As in the previous experiment, we incentivize subjects to play the simultaneous move game described in Figure 1. In the previous experiment, we manipulated the distribution from which θ is drawn in each round. Here, we use the distribution from the high volatility condition in the previous experiment, where $\theta \sim N(55, 400)$, but we tell subjects that their opponent is a computer. Subjects play three hundred rounds of the game, where the only difference across games is the random value of θ. Because we tell subjects that the computerized opponent will invest if and only if $\theta < 55$, the subject has a dominant strategy for all θ: invest if and only if her noisy signal of θ is less than 55.[21]

We pre-register the experiment and recruit 100 subjects from Prolific.[22] We apply the same recruitment restrictions as in the previous experiment. The experimental instructions are in Online Appendix C. Subjects are paid 2 GBPs for completing the experiment and are also paid according to the outcome on one randomly drawn round. Unlike in the previous experiment, here, the outcome depends exclusively on the subject's own decision since the computerized opponent plays a known and deterministic strategy. The median duration of the experiment was around 21 minutes and the average earnings, including the participation fee, were 6.30 GBPs.

5.2 Experimental Results

Following our pre-registration, we restrict our analysis to rounds where $\theta \in [47, 63]$ and where the subject executes a decision with a response time greater than 0.5 seconds. Our focus is on comparing behavior when subjects play against a computerized opponent (Algorithm) with behavior from the high volatility condition from the previous experiment (Human). By fixing the prior distribution across conditions, we control for any efficient coding effects.

Figure 5 plots the data from both the Algorithm and Human conditions. In the Algorithm condition, if subjects were implementing the optimal best response without noise, then we should observe a step function around $\theta = 55$. Instead, one can see that there is obviously noise in the Algorithm condition. However, behavior appears less noisy (that is, the data

noise is not to invest. This is consistent with the (indeterminate) best response to equilibrium beliefs in the game where each human player has cognitive noise. Because our design choice for the computer strategy when $\theta = 55$ is arbitrary, we show below that our results are robust to removing games for which $\theta = 55$.

[21] For other experiments where a game is reduced to an individual decision problem by using computerized opponents, see Roth and Murnighan (1978), Fehr and Tyran (2001), Esponda and Vespa (2014), and Koch and Penczynski (2018).

[22] The pre-registration document is available at: https://aspredicted.org/339_B5N.

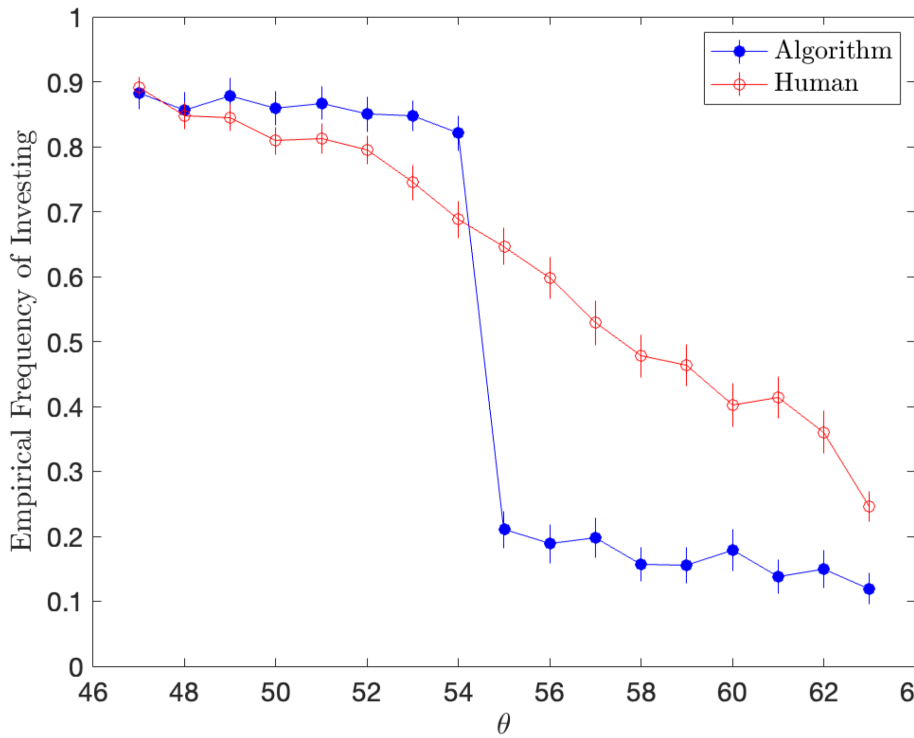

Figure 5: **Empirical Frequency of Investing as a Function of Opponent Type** Note: For each value of θ between 47 and 63, we plot the proportion of rounds on which a subject chooses to invest. The Algorithm condition denotes the data collected in the additional experiment where the subject plays against a computerized opponent. The Human condition denotes the data collected in the high volatility condition from our main experiment. Data are pooled across subjects for all rounds 1-300. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject.

more closely resemble a step function) in the Algorithm condition compared to the Human condition (in the Human condition, equilibrium behavior approximates a step function as noise vanishes).

To formally investigate the difference in noise across conditions, we run a mixed effects logistic regression where the dependent variable is a dummy that takes on the value 1 if the subject invests and is 0 otherwise. The independent variables are $(\theta - 55)$, the dummy variable $Human$ which indicates whether the observation is in the Human condition, and the interaction between $(\theta - 55)$ and $Human$. There are random effects on the intercept and on $(\theta - 55)$. Column (1) of Table 3 shows that the estimated coefficient on θ is significantly negative while the coefficient on the interaction term is significantly positive. These results indicate that the probability of investing declines with θ in both conditions, but also that this probability declines more rapidly in the Algorithm condition.

One concern with the previous test about existence of noise in the Algorithm condition is that, even under the null hypothesis of zero noise in the Algorithm condition, the estimated coefficient on θ would be negative (as long as there is some measurement error). This is because the probability of investing drops from 1 to 0 when θ crosses 55. However, continuing under the null hypothesis of zero noise, there should be no variation in behavior when conditioning on values of $\theta > 55$; similarly, there should be no variation in behavior when conditioning on values of $\theta < 55$. In columns (2) and (3) of Table 3, we show that the coefficient on θ remains significantly negative in both subsamples. Therefore, in the Algorithm condition, the probability of investing declines for $\theta \in [47, 54]$ and it also declines for $\theta \in [56, 63]$. This is consistent with the predictions of our model of cognitive noise.

In sum, there are two main takeaways from Table 3: when subjects play a simultaneous move game against a computerized opponent, (i) we continue to detect substantial noise in behavior (and the pattern of noise is consistent with our model)[23] and (ii) the noise is smaller compared to when subjects play against a human opponent. We attribute the reduction of noise to alternative sources of strategic uncertainty that are present in our original experiment and are not driven by imprecision over θ.

To quantitatively assess how much noise in behavior can be attributed to an imperfect representation of θ compared to other sources of noise, we estimate the amount of noise in each condition non-parametrically. For each subject and each round, we code behavior as "consistent" if and only if the subject chooses the action prescribed by the threshold strategy of "choose invest if and only if $\theta < 55$."[24] If a decision is not coded as consistent, we attribute the decision to noise.[25] We find that, in the Human condition, 31.8% of decisions are driven by noise. In the Algorithm condition, noisy behavior drops significantly to 15.2% of decisions (and the difference is statistically significant at the 0.1% level).[26] Thus, about half of the noise from the Human condition appears to be driven by imprecision in θ while the other half is driven by alternative factors outside our model. Our interpretation is that cognitive noise drives a substantial portion of the variability of behavior in the Human condition, but that there are clearly other important sources of noise that reflect uncertainty about the human opponent's strategy, preferences, or information. These latter sources of uncertainty are shut

[23]We provide a more detailed analysis of noise at the individual subject level in Online Appendix E.

[24]We note that the unique equilibrium threshold strategy in our theory of the Human treatment is "choose invest if and only if $S_i < 55$." However, since S_i is not observable to the econometrician, we proxy for it with θ, since S_i is assumed to be an unbiased signal of θ.

[25]For the remaining analyses in this section, we discard observations for which $\theta = 55$. This restriction is outlined in our pre-registration and is due to the fact that there is no way to unambiguously code behavior in the human condition (because subjects should be indifferent when $\theta = 55$).

[26]When lifting the restriction that $\theta \in [47, 63]$, we find that, in the Human condition, 17.7% of decisions are driven by noise, compared to 11.8% in the Algorithm condition and the difference remains statistically significant ($p = 0.013$).

down by design in our Algorithm condition. We note that the approximately 50% reduction in noise observed when moving from the Human to the Algorithm condition depends on the baseline level of noise in the Human condition. Therefore, the exact magnitude of this reduction may not generalize beyond our setting.

Dependent Variable: Pr(Invest)	(1) ($46 < \theta < 64$)	(2) ($55 < \theta < 64$)	(3) ($46 < \theta < 55$)
$(\theta - 55)$	-0.744***	-0.137***	-0.176***
	(0.064)	(0.033)	(0.043)
$(\theta - 55)$ x Human	0.359***		
	(0.068)		
Human	1.266***		
	(0.191)		
Constant	-0.207***	-2.115***	2.054***
	(0.076)	(0.236)	(0.205)
Observations	24,966	4,639	4,717

Table 3: **Comparing Behavior Across Human and Algorithm Condition** Note: Table displays results from mixed effects logistic regressions. Observations are at the subject-round level. The dependent variable takes value 1 if the subject chooses to Invest and 0 otherwise. The variable *Human* takes value 1 if the round belongs to the Human condition and 0 otherwise. Column (1) includes data from both the Human and Algorithm conditions and results are robust to excluding games where $\theta = 55$. Columns (2) and (3) include data only from the Algorithm condition. There are random effects on $(\theta - 55)$ and the intercept. Standard errors are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

6 Discussion

6.1 A Potential Alternative Explanation: Learning

When presenting our main experimental results in Table 1, we showed that the treatment effect becomes stronger as the experiment progresses. We attribute this result to subjects learning about the prior over the course of the experiment. Here, we examine whether a learning mechanism on its own can also generate our main treatment effect, rather than just the strengthening of the treatment effect across rounds of the experiment. To address this question, we consider two broad classes of learning models. The first class of models involves learning with feedback. Camerer (2003) reviews several models of learning with feedback, including reinforcement learning, belief-based learning, direction learning, experience-weighted attraction, anticipatory learning, and imitation. Each theory takes as input some form of feedback from past play, whether it be about a player's own choice or (foregone) payoff, or an opponent's choice or payoff. These theories provide rich predictions about dynamics. However, none of these theories is applicable to our experimental setting because we do not provide any feedback to subjects. Indeed, we designed our experiment intentionally to shut down these channels by construction, which enables a clean test of our cognitive noise model.

We next consider frameworks in which a player can learn without explicit feedback. In contrast to the models mentioned in the previous paragraph, learning without feedback can operate through introspection. For example, Weber (2003) finds that subjects' behavior in a competitive guessing game converges towards the predicted equilibrium even when no feedback is given. Rick and Weber (2010) demonstrate that (especially) in the absence of feedback, subjects can learn a strategic principle (e.g., iterated dominance) in one game and they can transfer this knowledge to a different game. If learning without feedback is to explain the pattern in Figure 4, then it needs to explain (i) the continuous and monotonic relationship between the probability of investing and θ, and (ii) the fact that the relationship is steeper in the low volatility condition. This is a high bar for a model of learning without feedback, since it needs to generate both the unique threshold equilibrium and the context-dependent equilibrium outcomes.

Nonetheless, to make some progress in empirically testing the hypothesis that subjects are learning without feedback, let us assume that a learning without feedback model *can* deliver the prediction that players exhibit a decreasing relationship between the probability of investing and θ. We can then test whether learning without feedback can generate the heightened sensitivity of behavior to θ in the low volatility condition compared to the high volatility condition. Table 4 presents results where we restrict the analysis to subsamples based on how many times a subject has previously observed and responded to the exact same game (characterized by the identical value of θ).

The first column restricts to those rounds on which subjects in the low and high volatility conditions have previously observed 3 games with the same value of θ as in the current round. Columns (2) – (4) further restrict the data based on more and more experience with a given game. This approach assumes that learning about how to play a given game occurs through introspection and depends solely on prior experience with that exact game. While restrictive, this assumption provides a simple way to examine whether parameter-specific experience can explain observed treatment effects. The regression results indicate that our treatment effect obtains among each of the different subsamples (at the 1% significance level). Thus, this specification of learning without feedback cannot explain the entire treatment effect we observe.

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.365***	-0.421***	-0.407***	-0.497***
	(0.034)	(0.042)	(0.042)	(0.057)
$(\theta - 55)$ x Low	-0.192***	-0.239***	-0.220***	-0.291***
	(0.044)	(0.055)	(0.052)	(0.070)
Low	-0.265	-0.383	-0.350	-0.553
	(0.256)	(0.307)	(0.281)	(0.337)
Constant	1.059***	1.162***	1.127***	1.322***
	(0.178)	(0.217)	(0.197)	(0.248)
Observations	4,249	4,001	3,582	3,065
Rounds of Experience with Game (θ)	3	4	5	6

Table 4: **Controlling for Experience with θ.** Note: Table displays results from mixed effects logistic regressions. Observations are at the subject-round level. The dependent variable takes value 1 if the subject chooses to Invest and 0 otherwise. The variable *Low* takes value 1 if the round belongs to the low volatility condition and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included in the regressions. There are random effects on $(\theta - 55)$ and the intercept. Standard errors are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

6.2 Connection with Rational Inattention

The theory of rational inattention from Sims (2003) provides an alternative approach to modeling an imprecise awareness of the fundamental parameter θ. Each player maximizes utility over possible actions, conditional on a noisy signal of the state and net of the cost of information acquisition. Typically, the literature has specified the cost function to be proportional to the mutual information between the state and the noisy signal (Maćkowiak et al., 2023). In our game, this would imply that information cost is proportional to the mutual information between the random state θ and the noisy signal S_i. Each player then endogenously chooses an information structure, balancing the benefit of a higher utility from better-informed action selection and the cost of gathering more precise information.

Here we describe a key prediction that differs between rational inattention theory and cognitive noise. We draw on the theoretical analysis from Aridor et al. (2025), who derive rational inattention predictions about our specific experiment. Specifically, Aridor et al. (2025) analyze symmetric equilibria of the game shown in Figure 1 (i.e., any equilibrium in which both players adopt the same strategy). They show that when the prior distribution is symmetric (as it is in both of our experimental conditions), then rational inattention predicts that equilibrium behavior will not depend on the variance of the prior. Empirically, we find that the frequency of investing is more sensitive to θ in the low volatility condition. Therefore, rational inattention with a mutual information cost function cannot accommodate the observed dependence of behavior on the prior variance.

6.3 Connection with Global Games Experiments

Our experimental results suggest that the noise in global games models can, in part, be interpreted as errors stemming from cognitive constraints. That is, while the starting point of the global games literature is that players behave "as if" they observe a noisy private signal of a fundamental parameter, the interpretation through the lens of the cognitive noise model is that such noisy signals are literally generated during the decision process. Thus, the private signal assumption from the global games literature can be microfounded with cognitive noise. Under the additional assumption of efficient coding, the cognitive noise and global games models make different predictions about the effect of the prior on behavior. The global games model does not predict that the signal structure will endogenously change with the prior, whereas our model and experimental results are consistent with such an endogenous relationship.

There is also a close connection between our cognitive noise model and the idea from Heinemann, Nagel and Ockenfels (2009) that behavior in a complete information coordination game can be interpreted *as if* players are observing a fundamental parameter with noise. Like us, Heinemann, Nagel and Ockenfels (2009) conduct an experiment on coordination games and find behavior that is consistent with the unique equilibrium prediction from global games, despite the fact that subjects are not given any explicit private signals.[27] Those authors also structurally estimate a global games model and find a sizable standard deviation of private signals.

However, Heinemann, Nagel and Ockenfels (2009) argue that the only source of the estimated standard deviation of private signals is strategic uncertainty – and specifically, strategic uncertainty that does not arise directly from structural uncertainty.[28] Our second experiment demonstrates that cognitive noise, and the induced structural uncertainty, plays a sizeable role in explaining the observed amount of noisy behavior in the coordination game with two human subjects. That is, we show that a large portion of observed variability in behavior cannot be attributed to strategic uncertainty.

Another difference with respect to Heinemann et al. (2009) is that those authors empha-

[27]In contrast, many other experiments on global games explicitly endow subjects with private signals, so as to literally implement the assumption in the global games model (e.g., Schotter and Trevino (2021)).

[28]Heinemann, Nagel and Ockenfels (2009) argue that "Of course, players know the true payoff. Their uncertainty about others' behavior makes them behave *as if* they are uncertain about payoffs" (p. 203).

size the role of risk aversion in their model, whereas our model does not rely on risk aversion at all. One reason for this different emphasis on risk aversion is because Heinemann et al. (2009) appeal to the notion that each player's degree of risk aversion is private information. This implies that player i is uncertain about player j's degree of risk aversion, and this generates strategic uncertainty, which in turn, is key to eliminating the multiple equilibria. Our model instead assumes that players have irreducible uncertainty in representing θ due to cognitive noise, and it is this information asymmetry that generates strategic uncertainty. Heinemann et al. (2009) do also consider the alternative that players have common knowledge about their opponents' degree of risk aversion, but behave "as if" they receive private signals about monetary payoffs. By adopting an "as is" interpretation of noise in private signals, we are able to generate and test novel hypotheses about how the standard deviation of private signals varies across environments.

At a conceptual level, our results also suggest an important implication for the global games literature that has to do with the role of public vs. private signals. A series of papers has argued that when an institution like the government or a financial market can generate public signals, then a unique equilibrium may no longer obtain in a global games model (Atkeson, 2000; Angeletos and Werning, 2006; Hellwig, Mukherji and Tsyvinski, 2006). The argument is that a sufficiently precise public signal can act as a coordination device, and thus restore multiple equilibria. However, our theory and experimental results suggest that there is an important difference between *access* to a public signal and precise processing of a public signal. Specifically, even if all players have access to the public signal, each player may encode the same public signal with noise and thus interpret it slightly differently. This friction, driven by constraints that arise internally in the agent's mind, transforms the public signal into private information and makes it difficult to use the public signal as a coordination device. Our results, therefore, imply that the provision of a public signal is not enough to overturn the classic global games result. The ability to precisely perceive and process public information is also necessary and, as we have shown, this cannot be taken for granted.

Such an interpretation is consistent with earlier experimental work on global games, which finds that removing private information from the strategic environment still yields behavior that is consistent with the unique global games equilibrium (Heinemann, Nagel and Ockenfels, 2004). In the same vein, Szkup and Trevino (2020) conduct a global games experiment in which they manipulate the standard deviation of noise in private signals, and they include a treatment in which the standard deviation is 0, corresponding to a game of complete information. Those authors find that all subjects in their complete information treatment use threshold strategies, and a majority use the efficient threshold level. This result obtains even though standard theory predicts multiple equilibria. If one departs from standard theory and instead assumes that subjects form an internal representation of the fundamental with cognitive noise, then this can generate a unique equilibrium where all players use the same threshold strategy, which is largely consistent with the experimental results in Szkup and Trevino (2020).

6.4 Noisy Coding vs. Efficient Coding

Our model assumes that each player is subject to noisy coding and efficient coding. The noisy coding assumption implies that each player codes θ with noise. The efficient coding assumption implies that the conditional noisy signal distribution of θ depends on the player's prior beliefs about θ. Here we analyze whether noisy coding on its own is sufficient to generate our observed treatment effect. In particular, suppose that each player has a prior over θ that reflects past experience. Further, suppose that in both experimental conditions, we set the conditional distribution of S_i to an arbitrary distribution. Specifically, suppose it is the distribution that arises under efficient coding in the high volatility condition. How does the model prediction of this noisy coding model compare with the prediction from our model summarized in Figure 2?

It turns out that in this model of noisy coding, the predictions for behavior in equilibrium *will be identical across the high and low volatility conditions in Experiment 1.* The solid curve in Figure 2 will become flatter and lie directly on top of the dashed curve. The reason why the predicted treatment effect vanishes is due to a key feature of our experimental design: the prior mean in each experimental condition coincides with the equilibrium threshold of 55. To understand the intuition, note that, in equilibrium, a player's choice of whether to invest depends only on whether the posterior mean, $E[\theta|S_i]$, is below the equilibrium threshold. Each player will invest if and only if $E[\theta|S_i] < 55$. Behavior does not depend on how far the posterior mean is from the equilibrium threshold. Crucially, because (i) the posterior mean is a weighted average of the prior mean and S_i, and (ii) the prior mean is 55 in both experimental conditions, it follows that the posterior mean is below 55 if and only if the signal is below 55. Hence, each player invests with probability $Pr(S_i < 55|\theta)$.

Under the noisy coding model sketched above, the signal distribution does not depend on the prior (by assumption, the signal distribution in each experimental condition is set to the distribution that arises under efficient coding in the high volatility condition.) In this case, $Pr(S_i < 55|\theta)$ is identical across conditions, and hence, this model leads to identical predictions about behavior. In other words, noisy coding on its own does not generate the treatment effect. In contrast, if we assume that coding is also efficient, the conditional signal distribution is endogenously different under each prior, and thus behavior will systematically be different, which we find to be the case in our data (Figure 4).

6.5 Awareness of Cognitive Noise

Proposition 1 assumes common knowledge of cognitive noise. The assumption that the distribution of noise is common knowledge is standard in the global-games literature, primarily because it ensures equilibrium uniqueness through limit arguments as the noise vanishes. Relaxing this assumption by, for example, introducing higher-order uncertainty about σ can potentially restore equilibrium multiplicity (Weinstein and Yildiz, 2007). However, provided that uncertainty remains minimal at higher orders, the equilibrium predictions remain robust (Morris and Shin, 2003). Moreover, precise knowledge of the underlying information structure is not necessary for the unique equilibrium to arise. As evident from the statement of Proposition 1, the equilibrium exists regardless of the exact functional forms of the prior and noisy signal distributions. It follows that the equilibrium exists even when players have incorrect beliefs about the exact information structure (maintaining the assumption of common knowledge of noise). This is important considering that, while we manipulate the distribution of the prior in the laboratory, we do not control or measure the distribution of the noisy signal.

Common knowledge of the noisy signal distribution requires that subjects know they are imprecise and that others are imprecise. To investigate the validity of this assumption, we conduct a third experiment, where subjects are asked to classify whether a two-digit number is greater than a reference level of 55 (which we choose to be the same as the threshold in the unique equilibrium of the game in our main experiment). We incentivize subjects to report their beliefs about (i) the average accuracy of all other subjects in the experiment and (ii) their own accuracy. We find that subjects are aware of their own errors and of others' errors in the classification task. Additionally, we find interesting evidence that subjects are aware that other subjects make more errors when the fundamental is drawn from a high volatility distribution compared to a low volatility distribution. We refer the reader to Online Appendix B for more details from this additional experiment.

7 Conclusion

In this paper, we have experimentally investigated the mechanism that generates context-dependent behavior in coordination games. In our first experiment, we find that the probability of investing is monotonically declining in the fundamental parameter. This result is not readily predicted by standard theory, which generates multiple equilibria and hence no systematic relationship between the probability of investing and fundamentals. Instead, our data is well-explained by a model in which each player holds an imperfect representation of the fundamental parameter due to unavoidable cognitive noise. The second and critical pattern we observe in the data is that the sensitivity of behavior to fundamentals depends systematically on the prior distribution from which the fundamental is drawn. Specifically, for a given coordination game, behavior is noisier and coordination is less likely when subjects are adapted to a high volatility distribution compared to a low volatility distribution. This pattern is predicted by our model of cognitive noise equilibrium, under the assumption that subjects efficiently code the fundamental. Importantly, we show that alternative theories such as QRE and Level-k thinking cannot generate this context-dependent strategic behavior.

After establishing that cognitive noise is an important driver of behavior in the coordination game, we conduct a second experiment to quantify how much of the observed randomness in behavior can be attributed to cognitive noise. This second experiment mimics our first experiment, except that we replace the human opponent with a computerized opponent whose strategy is known and deterministic. Such a design enables us to shut down any strategic uncertainty, and we argue that any remaining variability in behavior is likely to come from imprecise perception and retrieval of fundamentals. We find that when subjects play a computerized opponent, there is still substantial randomness in behavior, but the amount of randomness is reduced by 50% relative to behavior when subjects play against a human opponent. Our interpretation is that cognitive noise is a substantial driver of behavior in coordination games, but there are clearly additional sources of strategic uncertainty that are just as important in explaining behavior.

We believe our analysis paves the way for at least two directions of future work on cognitive noise in games. First, there are additional theory-guided manipulations of cognitive noise which have recently been deployed in individual decision-making experiments, that could be explored in a strategic environment. For example, Polania, Woodford and Ruff (2019) show that cognitive noise can be amplified by imposing time pressure on decisions, and Enke and Graeber (2023) ramp up cognitive noise by increasing the complexity of an action. In our setting, a clear untested prediction is that imposing time pressure should lead the distribution of actions in equilibrium to be compressed towards 50-50, so that the probability of coordination can be modulated by the experimenter. The second direction is along a more theoretical route. Our current framework is confined to a stylized 2×2 coordination game, but we believe there may be much richer implications of cognitive noise in more general strategic environments. In particular, the idea that public signals are universally processed with noise due to cognitive errors is likely to have important implications for strategic behavior in a much broader class of games.

Appendix

A Model with Endogenous Noisy Coding

In this Appendix, we present a model that endogenously delivers the relationship between prior variance and signal variance that is captured in Assumption 2 from Section 2.2. We begin by first introducing an encoding function, $m(\theta)$, which maps θ into a real-valued quantity, which in turn, is used to generate the noisy signal, S_i:

Assumption 3 (Normally-Distributed Cognitive Noise) *Each player i, $i = \{1, 2\}$, has a common prior belief that θ is distributed normally, $\theta \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. Conditional on the realized value of θ, each player observes a noisy signal, $S_i = m(\theta) + \varepsilon_i$, where each ε_i is independently and normally distributed: $\varepsilon_i \sim \mathcal{N}(0, \sigma_S^2)$ with $\sigma_S^2 > 0$.*

We now draw on principles from psychology to put further structure on $m(\theta)$, which will have direct implications for the distribution of S_i. Following Khaw, Li and Woodford (2021), we constrain the encoding function, $m(\theta)$, to be linear in θ and have a bounded variance:[29]

Assumption 4 (Encoding Function) *The encoding function is linear: $m(\theta) = \xi + \psi\theta$. In addition, there is a power constraint, $E[m^2] \leq \Omega^2 < \infty$.*

The power constraint captures the idea that the brain cannot encode an arbitrarily large set of values. Without the power constraint, the player could choose the noisy signal, $S_i = m(\theta)+\varepsilon_i$, to be arbitrarily precise by making the variance of $m(\theta)$ as large as needed. By introducing the power constraint, it becomes harder for a player to discriminate between two fundamental values as they become closer together. Specifically, for any two fundamental values $\theta_1 < \theta_2$, it is more difficult for the player to discriminate between the two values as $|\theta_1 - \theta_2|$ approaches zero. This assumption is in the spirit of the cost functions proposed by Hébert and Woodford (2021) and Morris and Yang (2022). Given the cognitive constraints summarized by Assumption 4, we allow the player to choose the encoding function parameters, (ξ, ψ). In this manner, the player can *efficiently code* information about the fundamental to achieve a performance objective. To close the model, we need to specify the performance objective which drives the players' optimal choice of the encoding function parameters.[30]

[29]Khaw, Li and Woodford (2021) assume a slightly different specification of the encoding function, which is linear in the logarithm of a payoff value. See their Appendix C for details.

[30]Online Appendix A shows robustness to different assumptions about the players' performance objective.

Assumption 5 (Performance Objective) *Players choose the encoding function which minimizes the mean squared error between θ and its conditional mean, $E[\theta|S_i]$.*

With the player's performance objective in hand, we can now derive the efficient coding function that each player optimally chooses, given her cognitive constraints.

Proposition 3 (Endogenous Efficient Coding) *Given Assumptions 3–5, the optimal encoding function features $\xi^\star = -\frac{\Omega}{\sigma_\theta}\mu_\theta$ and $\psi^\star = \frac{\Omega}{\sigma_\theta}$. Consider the transformed internal representation, $Z_i \equiv (S_i - \xi^\star)/\psi^\star$. The conditional distribution of Z_i is $N(\theta, \omega^2\sigma_\theta^2)$, where $\omega = \sigma_S/\Omega$. The variance of Z_i is proportional to the variance of θ.*

Proposition 3 says that the player chooses the slope of the encoding function, $\psi^\star$, such that it becomes steeper as the variance of the prior shrinks. Intuitively, for a given change in θ, a good encoding function is one that exhibits a large change in signal. As the variance of the prior shrinks, signals can become more sensitive to a change in θ while still satisfying the power constraint. Indeed, the important implication of Proposition 3 for our purposes is that the noisy signal distribution is normalized by the prior variance. While this "normalization" result is derived from our assumptions of the power constraint and the linear encoding function, it is a robust implication of efficient coding that arises in a more general class of models (Polania, Woodford and Ruff, 2019; Khaw, Li and Woodford, 2021; Frydman and Jin, 2022; Payzan-LeNestour and Woodford, 2022).

Given the optimal encoding function in Proposition 3, we can now solve for the equilibria of the game. We restrict our analyses to monotone equilibria of the incomplete information game, that is, equilibria in which actions are monotonic in the transformed internal representation, Z_i. In such a monotone equilibrium, a player's mutual best response is to choose Invest if and only if her transformed internal representation is below a threshold $k^\star$. To derive the equilibrium, we adapt results from the global games literature (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Morris, 2010) to the game in Figure 1, with the further assumption that $\mu_\theta = (a + b)/2$ (as in our experiments). We can then establish there exists a monotone equilibrium such that player i invests if and only if $Z_i \leq \mu_\theta$, for any value of σ_θ, σ_S and Ω. Furthermore, if the noise in the transformed internal representation is sufficiently small, this is the unique monotone equilibrium.

Proposition 4 (Equilibrium Existence and Uniqueness) *Suppose Assumptions 3–5 and $\mu_\theta = (a + b)/2$. There exists an equilibrium of the game where each player invests if and only if $Z_i \leq \mu_\theta$ (or, equivalently, $E[\theta|Z_i] \leq \mu_\theta$). Moreover, if $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$, this is the unique monotone equilibrium of the game.*

Proposition 4 implies a rich set of comparative statics with respect to θ. The probability of investing is pinned down by the distribution of the transformed internal representation: $Pr[\text{Invest}|\theta] = Pr\left[Z_i \leq \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta - \theta}{\omega\sigma_\theta}\right)$, where $\Phi(\cdot)$ is the cumulative density function of the standard normal. This result indicates that, in the unique monotone equilibrium, the probability of investing is continuous and monotonically decreasing in θ.

We can make an even starker prediction about equilibrium outcomes by exploiting the malleability of the encoding function. The probability of investing depends not only on θ, but also on the prior distribution from which θ is drawn. Specifically, σ_θ modulates the optimal encoding function and, therefore, the precision with which a player detects whether a fundamental crosses the equilibrium threshold. It follows that the probability of investing declines more rapidly in θ as the prior volatility decreases. This prediction is summarized in the following proposition.

Proposition 5 (Comparative Statics) *Suppose Assumptions 3–5,* $\mu_\theta = (a+b)/2$, *and* $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$. *In the unique monotone equilibrium of the game, the probability that each player invests for a given value of* θ *is* $Pr[Invest|\theta] = \Phi\left(\frac{\mu_\theta - \theta}{\omega\sigma_\theta}\right)$. *Decreasing the variance of* θ *will increase the sensitivity of choices to* θ *(that is, the rate at which* $Pr[Invest|\theta]$ *decreases with* θ*) for values of* θ *close to* μ_θ.

B Proofs

Proof of Proposition 2

As discussed above the statement of the proposition, when σ is sufficiently small, $\Pr(\text{Invest}|\theta)$ is well approximated by $\Pr(S_i < \theta^\star|\theta)$. Since $S_i = \theta + \sigma\varepsilon_i$ and $\theta^\star = (a+b)/2$, we have:

$$\begin{aligned}
\Pr(\text{Invest}|\theta) \approx \Pr(S_i < \theta^\star|\theta) &= \Pr(\theta + \sigma\varepsilon_i < (a+b)/2) \\
&= \Pr\left(\varepsilon_i < \frac{(a+b)/2 - \theta}{\sigma}\right) \\
&= G\left(\frac{(a+b)/2 - \theta}{\sigma}\right),
\end{aligned}$$

where $G(\cdot)$ is the cumulative density function of ε_i. By Assumption 1, $G(\cdot)$ is continuous and strictly increasing in its argument, $h(\theta, \sigma) = \frac{(a+b)/2 - \theta}{\sigma}$. Since $h(\theta, \sigma)$ is continuous in θ, it follows that $\Pr(\text{Invest}|\theta)$ is continuous in θ. Moreover, we have

$$\frac{\partial h(\theta, \sigma)}{\partial \theta} = -\frac{1}{\sigma}$$

$$\frac{\partial h(\theta,\sigma)}{\partial\theta\partial\sigma} = \frac{1}{\sigma^2}$$

The first line means that $h(\theta,\sigma)$ is strictly decreasing in θ and, thus, $\Pr(\text{Invest}|\theta) \approx G\left(h(\theta,\sigma)\right)$ is strictly decreasing in θ. The second line means that the rate at which $h(\theta,\sigma)$ decreases in θ is decreasing in σ (note that $\frac{\partial h(\theta,\sigma)}{\partial\theta}$ is negative and, thus, the positive sign of $\frac{\partial h(\theta,\sigma)}{\partial\theta\sigma}$ means that $\frac{\partial h(\theta,\sigma)}{\partial\theta}$ increases in σ while still remaining negative). Therefore, the rate at which $\Pr(\text{Invest}|\theta) \approx G\left(h(\theta,\sigma)\right)$ decreases in θ is decreasing in σ.

Proof of Proposition 3

Here we adapt the theoretical derivation of efficient coding from Khaw, Li and Woodford (2021) to our framework where the distribution of θ is normal rather than lognormal. According to Assumption 3, the internal representation S of θ is drawn from

$$S|\theta \sim N(m(\theta), \sigma_S^2)$$

where the *encoding rule*, $m(\theta)$, is a linear transformation of θ, $m(\theta) = \xi + \psi\theta$, which satisfies the power constraint in Assumption 4. Parameters ξ and ψ are endogenous while the precision parameter σ_S is exogenous. The efficient coding hypothesis requires that the encoding rule $m(\theta)$ is chosen (among all linear functions satisfying the constraint) so as to maximize the system's objective function, for a given prior distribution of θ. As in Khaw, Li and Woodford (2021), we assume that the system produces an estimate of θ on the basis of S, $\tilde{\theta}(S)$, and that the goal of the design problem is to have a system that achieves as low as possible a mean squared error of this estimate. Given a noisy internal representation, the estimate which minimizes the mean squared error is $E[\theta|S]$ for all S. The goal of the design problem is, thus, to minimize the variance of the posterior distribution of θ.

Consider the transformed internal representation, $Z \equiv (S - \xi)/\psi$. The distribution of the transformed internal representation conditional on θ is $Z|\theta \sim N(\theta, \sigma_S^2/\psi^2)$. Thus, the distribution of θ given the transformed internal representation is

$$\theta|Z \sim N\left(\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}(Z - \mu_\theta), \frac{\sigma_\theta^2(\sigma_S^2/\psi^2)}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}\right) \tag{7}$$

The variance of the posterior distribution of θ is strictly increasing in the variance of Z, σ_S^2/ψ^2. Thus, it is desirable to make ψ as large as possible (in order to make the mean squared error of the estimate as small as possible) consistent with the power constraint.

When the distribution of θ is normal, we have

$$E[m^2] = \xi^2 + \psi^2 E[\theta^2] + 2\xi\psi E[\theta] = (\xi + \psi\mu_\theta)^2 + \psi^2\sigma_\theta^2 \leq \Omega \tag{8}$$

The largest value of ψ consistent with this constraint is achieved when

$$\xi = -\psi\mu_\theta \ , \ \psi = \frac{\Omega}{\sigma_\theta} \tag{9}$$

Thus, $m^\star(\theta) = -\frac{\Omega}{\sigma_\theta}\mu_\theta + \frac{\Omega}{\sigma_\theta}\theta$ and

$$Z|\theta \ \sim \ N\left(\theta, \frac{\sigma_S^2}{\Omega^2}\sigma_\theta^2\right) \tag{10}$$

The same optimal coding rule obtains under an alternative goal of the system. Consider the more conventional hypothesis from sensory perception literature, whereby the encoding rule is assumed to maximize the Shannon mutual information between the objective state θ and its subjective representation S. Denote with ρ_θ the precision of θ and with ρ_S the precision of S. We have $\theta \sim N\left(\mu_x, \frac{1}{\rho_\theta}\right)$, $S|\theta \sim N\left(\xi + \psi\theta, \frac{1}{\rho_S}\right)$, $Z|\theta \sim \left(\theta, \frac{1}{\rho_Z}\right)$, and $\theta|Z \sim N\left(\frac{\rho_\theta\mu_\theta+\rho_Z Z}{\rho_\theta+\rho_Z}, \frac{1}{\rho_\theta+\rho_Z}\right)$, where $Z = \frac{S-\xi}{\psi}$ and $\rho_Z = \psi^2/\sigma_S^2$. The Shannon mutual information between θ and Z is

$$I(\theta, Z) = \frac{1}{2}\log_2\left(\frac{\sigma_\theta^2}{\sigma_{\theta|Z}^2}\right) = \frac{1}{2}\log_2\left(1 + \frac{\rho_Z}{\rho_\theta}\right) \tag{11}$$

which is strictly increasing in ρ_Z and, thus, strictly decreasing in σ_Z^2. This means that, as for the previous goal, it is desirable to make ψ as large as possible (consistent with the power constraint).

Proof of Proposition 4

First, we show that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game. Remember that $Z_i \sim N(\theta, \sigma_Z^2)$, where $\sigma_Z^2 = \omega^2\sigma_\theta^2 = (\sigma_S^2/\Omega^2)\sigma_\theta^2$. Thus, player 1's posterior distribution of θ given Z_1 is

$$\theta|Z_1 \sim \mathcal{N}\left(\frac{\sigma_Z^2}{\sigma_\theta^2+\sigma_Z^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_Z^2}Z_1, \frac{\sigma_\theta^2\sigma_Z^2}{\sigma_\theta^2+\sigma_Z^2}\right)$$

Therefore, we have:

$$EU[\text{Not Invest}|Z_1] = E[\theta|Z_1] = \frac{\sigma_Z^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2+\sigma_Z^2}$$

On the other hand, player 1's expected utility from investing is

$$EU[\text{Invest}|Z_1] = a + (b-a)\Pr[\text{Opponent Invests}|Z_1]$$

Assume player 1 believes his opponent uses a monotone strategy with threshold k. In this case, player 1's expectation that the opponent invests is $\Pr[Z_2 \leq k|Z_1]$. Player 1's belief about the distribution of Z_2 given Z_1 is:

$$Z_2|Z_1 \sim \mathcal{N}\left(E[\theta|Z_1] = \frac{\sigma_Z^2}{\sigma_\theta^2+\sigma_Z^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_Z^2}Z_1, \frac{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}{\sigma_\theta^2+\sigma_Z^2}\right)$$

Thus, we have:

$$\Pr[Z_2 \leq k|Z_1] = \Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Player 1's best response is to invest if and only if

$$\frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2 Z_1}{\sigma_\theta^2+\sigma_Z^2} \quad \leq \quad a + (b-a)\Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)$$

If we write $\overline{Z}(k)$ for the unique value of Z_1 such that player 1 is indifferent between investing and not investing (this is well defined since player 1's expected payoff from not investing is strictly increasing in Z_1 and player 1's expected payoff from investing is strictly decreasing in Z_1), the best response of player 1 is to follow a monotone strategy with threshold equal to $\overline{Z}(k)$, that is, to invest if and only if $Z_1 \leq \overline{Z}(k)$.

Observe that as $k \to -\infty$ (that is, player 2 never invests), $EU[\text{Invest}|Z_1, k]$ tends to a, so $\overline{Z}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)a-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}$. As $k \to \infty$ (that is, player 2 always invests), $EU[\text{Invest}|Z_1]$ tends to b, so $\overline{Z}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)b-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}$. A fixed point of $\overline{Z}(k)$ — that is a value $k^\star$ such that $\overline{Z}(k^\star) = k^\star$ — is a monotone equilibrium of the game where each player invests if and only if his signal is below $k^\star$. Since $\overline{Z}(k)$ is a mapping from $\mathbb{R}$ to itself and is continuous in k, there exists $k \in \left[\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)a-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}, \frac{\left(\sigma_\theta^2+\sigma_Z^2\right)b-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}\right]$, such that $\overline{Z}(k) = k$ and a threshold equilibrium of this game exists.

When is there a unique equilibrium? Define $W(\overline{Z}(k), k)$ as

$$W(\overline{Z}(k), k) = \frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2\overline{Z}(k)}{\sigma_\theta^2+\sigma_Z^2} - a - (b-a)\Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2\overline{Z}(k)}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)$$

At a fixed point, $\overline{Z}(k^\star) = k^\star$. Thus, we have:

$$W(k^\star) = \frac{\sigma_Z^2 \mu_\theta + \sigma_\theta^2 k^\star}{\sigma_\theta^2 + \sigma_Z^2} - a - (b-a)\Phi\left(\frac{\sigma_Z^2}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}}\left(k^\star - \mu_\theta\right)\right)$$

Then,

$$\frac{\partial W(k^\star)}{\partial k^\star} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_Z^2} - \phi\left(\frac{\sigma_Z^2}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}}\left(k^\star - \mu_\theta\right)\right)\frac{\sigma_Z^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}}$$

And there is a unique fixed point if and only if $\frac{\partial W(k^\star)}{\partial k^\star} > 0$ at the fixed point. When $\frac{\partial W(k^\star)}{\partial k^\star} < 0$, there are at least three fixed points. Since $\phi(y) \leq \frac{1}{\sqrt{2\pi}}$, this is a sufficient condition for $\frac{\partial W(k^\star)}{\partial k^\star} > 0$:

$$\begin{aligned}
\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_Z^2} &> \frac{1}{\sqrt{2\pi}}\frac{\sigma_Z^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}} \\
\frac{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}}{(b-a)\sigma_Z^2\sqrt{\sigma_\theta^2 + \sigma_Z^2}} &> \frac{1}{\sqrt{2\pi}} \\
\sqrt{2\pi} &> \frac{(b-a)\sigma_Z^2\sqrt{\sigma_\theta^2 + \sigma_Z^2}}{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}}
\end{aligned}$$

The condition $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$ is obtained by replacing $\sigma_Z = \omega\sigma_\theta$ in the condition above and re-arranging terms. Thus, this shows that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game.

Second, we show that, when $\mu_\theta = \frac{(a+b)}{2}$, there exists a monotone equilibrium of the game where $k^\star = \mu_\theta$ for any value of σ_θ, σ_S and ω (or, equivalently, for any value of σ_θ and σ_Z). Assume player 2 uses a threshold strategy where he invests if and only if $Z_2 \leq k = \mu_\theta$. Is this an equilibrium, that is, is $\overline{Z}(\mu_\theta) = \mu_\theta$? $\overline{Z}(\mu_\theta)$ is the value of Z_1 such that the following equation is satisfied with equality:

$$\begin{aligned}
\frac{\sigma_Z^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \sigma_Z^2} &= a + (b-a)\Phi\left(\frac{\left(\sigma_\theta^2 + \sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}}\right) \\
\frac{\sigma_Z^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \sigma_Z^2} &= a + (b-a)\Phi\left(\frac{\sigma_\theta^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2 + \sigma_Z^4}\sqrt{\sigma_\theta^2 + \sigma_Z^2}}\right)
\end{aligned}$$

If we set $Z_1 = \mu_\theta$, we get:

$$\mu_\theta = a + (b-a)\Phi(0)$$
$$\mu_\theta = \frac{(a+b)}{2}$$

which is true by one of the assumptions in the statement of the Proposition.

Proof of Proposition 5

From Proposition 4 and the condition in the statement of Proposition 5, we know that there exists a unique monotone equilibrium of the game where each player invests if and only if his transformed internal representation is smaller than μ_θ. In this equilibrium, $Pr[\text{Invest}|\theta] = Pr[Z_i \leq \mu_\theta|\theta] = \Phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)$ and $\frac{\partial Pr[\text{Invest}|\theta]}{\partial\theta} = -\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta}\right)$. Thus, $Pr[\text{Invest}|\theta]$ grows with σ_θ if $\theta < \mu_\theta$ and it decreases with σ_θ is $\theta > \mu_\theta$. Moreover, the sensitivity of choices to θ decreses with σ_θ for values of θ around the cutoff.

Indeed, we have

$$\begin{aligned}
\frac{\partial Pr[\text{Invest}|\theta]}{\partial\theta\partial\sigma_\theta} &= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) + \phi'\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\omega^2\sigma_\theta^2-(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right)
\end{aligned}$$

which is positive if and only if $(\mu_\theta-\theta)^2 < \omega^2\sigma_\theta^2$.

(In the second line, we used the fact that $\phi'(x) = -x\phi(x)$.)

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Online Appendix

A Alternative Performance Objective in Model with Endogenous Efficient Coding from Appendix A

Here we revisit the assumption about efficient coding in the model from Appendix A. The specific performance objective that we assume there is only one of several plausible specifications (Ma and Woodford, 2020). In particular, there are other possible objective functions that players may have, besides minimizing the mean squared error of the estimate of θ. For example, a prominent alternative efficient coding objective from the literature on sensory perception is to maximize the mutual information between the state and its noisy internal representation. In the proof of Proposition 3, we confirm that the coding rule we use in our model is robust to this alternative objective.

Yet another alternative objective that has been examined in the economics literature is maximization of expected reward. In this section, we show that the result in Proposition 3 is robust to using this alternative objective function. Specifically, we maintain the constraints in Assumption 4 and we analyze a two-stage game. In the first stage, each player optimally chooses, simultaneously and independently, the parameters of the encoding function. In the second stage, players choose strategies in the simultaneous move game, conditional on their chosen encoding function from the first stage. We show that the optimal encoding function still takes the form characterized in Proposition 3. Thus, our theoretical predictions from Appendix sec:normal˙model are robust to three performance objectives: (i) minimizing mean squared error of the estimate of θ, (ii) maximizing mutual information between the noisy internal representation and θ and (iii) maximizing expected reward.

Assumption 6 (Alternative Performance Objective) *Players choose the encoding function which maximizes their expected reward in the simultaneous move game.*

Consider the following two-stage game: in stage 1, each player $i = \{1, 2\}$ chooses simultaneously and independently the parameters of his encoding function, (ξ_i, ψ_i), to maximize the performance objective in Assumption 6 under the constraints in Assumption 4; in stage 2, players participate to the simultaneous move game endowed with the encoding functions chosen in the previous stage. We solve this game by backward induction.

Stage 2: Simultaneous move Game (with Exogeneous Encoding Functions)

For each player $i = \{1, 2\}$, we have $S_i|\theta \sim N(m_i(\theta), \sigma_S^2)$, where $m_i(\theta) = \xi_i + \psi_i\theta$.

Consider the transformed internal representation $Z_i = (S_i - \xi_i)/\psi_i$. We have:

$$Z_i|\theta \sim N\left(\theta, \beta_i^2\right)$$

where $\beta_i = (\sigma_S/\psi_i)$.

Proposition 6 *Suppose Assumptions 1, 2, 4 and $\mu_\theta = (a+b)/2$. Regardless of σ_θ, σ_S, (ξ_1,ψ_1), and (ξ_2,ψ_2), there exists an equilibrium of the game where each player invests if and only if $Z_i \leq \mu_\theta$. Moreover, if $\frac{\sigma_\theta^2\sqrt{\beta_i^2(2\sigma_\theta^2+\beta_i^2)}}{(b-a)\beta_i^2\sqrt{\sigma_\theta^2+\beta_i^2}} > \frac{1}{\sqrt{2\pi}}$ for all $i = \{1,2\}$, this is the unique monotone equilibrium of the game.*

Proof. Since the likelihood function of Z_i is conjugate to the prior distribution of θ, we have a closed form solution for the distribution of player i's posterior beliefs over θ. In particular, player 1's posterior distribution of θ given Z_1 is

$$\theta|Z_1 \sim \mathcal{N}\left(\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2+\beta_1^2}, \frac{\sigma_\theta^2\beta_1^2}{\sigma_\theta^2+\beta_1^2}\right)$$

Thus, we have:

$$EU[\text{Not Invest}|Z_1] = E[\theta|Z_1] = \frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2+\beta_1^2}$$

On the other hand, player 1's expected utility from investing is

$$EU[\text{Invest}|Z_1] = a + (b-a)\Pr[\text{Opponent Invests}|Z_1]$$

Assume player 1 believes his opponent uses a monotone strategy with threshold k_2. In this case, player 1's expectation that the opponent invests is $\Pr[Z_2 \leq k_2|Z_1]$. Player 1's belief over the distribution of Z_2 conditional on Z_1 is:

$$Z_2|Z_1 \sim \mathcal{N}\left(\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2+\beta_1^2}, \frac{\sigma_\theta^2\left(\beta_1^2+\beta_2^2\right)+\beta_1^2\beta_2^2}{\sigma_\theta^2+\beta_1^2}\right)$$

Thus, we have:

$$\Pr[Z_2 \leq k_2|Z_1] = \Phi\left(\frac{k_2\left(\sigma_\theta^2+\beta_1^2\right) - \beta_1^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{\sigma_\theta^2+\beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2+\beta_2^2\right)+\beta_1^2\beta_2^2}}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Player 1's best response is to invest if and only if

$$\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2+\beta_1^2} \quad \leq \quad a + (b-a)\Phi\left(\frac{k_2\left(\sigma_\theta^2+\beta_1^2\right) - \beta_1^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{\sigma_\theta^2+\beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2+\beta_2^2\right)+\beta_1^2\beta_2^2}}\right)$$

Assume $k_2 = \mu_\theta$. We want to show that player's best response is to use the same cutoff. In this case, player 1's best response is to invest if and only if

$$E\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2} \quad \leq \quad a + (b-a)\Phi\left(\frac{\sigma_\theta^2\left(\mu_\theta - Z_1\right)}{\sqrt{\sigma_\theta^2+\beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2+\beta_2^2\right)+\beta_1^2\beta_2^2}}\right)$$

First, note that the LHS is a convex combination of μ_θ and Z_1 and that, thus, it is a) equal to μ_θ when $Z_1 = \mu_\theta$, b) smaller than μ_θ when $Z_1 < \mu_\theta$, and c) larger than μ_θ when $Z_1 > \mu_\theta$. Second, remember that $\mu_\theta = (a+b)/2$ and note that the RHS is a) equal to μ_θ when the argument of $\Phi(\cdot)$ is 0 (that is, when $Z_1 = \mu_\theta$, since the denominator is strictly positive); b) larger than μ_θ when the argument of $\Phi(\cdot)$ is strictly positive (that is, when $Z_1 < \mu_\theta$), and c) smaller than μ_θ when the argument of $\Phi(\cdot)$ is strictly negative (that is, when $Z_1 > \mu_\theta$). This means that, when player 2 invests if and only if $Z_2 \leq k_2 = \mu_\theta$, then player 1's best response is to invest if and only if $Z_1 \leq \mu_\theta$. This proves that there exists an equilibrium where both players use a monotone strategy with cutoff equal to μ_θ for any value of (ξ_1, ψ_1), (ξ_2, ψ_2), σ_S and σ_θ. Finally, to show that, when the condition in the statement of the proposition is satisfied, this is the unique equilibrium of the game, we can use the same steps in the proof of Proposition 2 to show that the best response mapping is a contraction (and that, thus, we can apply the contraction mapping theorem). In particular, it is sufficient to show that the derivative of the best response function of player 1 with respect to k_2 and the derivative of the best response function of player 2 with respect to k_1 have both an absolute value strictly smaller than 1. ■

Stage 1: Encoding Function Choice

When deriving the optimal choice of the encoding function in stage 1, we assume that, in stage 2, players use the cutoff strategy in the (unique) equilibrium from Proposition 6.

Proposition 7 *Suppose Assumptions 1, 2, 4, and $\mu_\theta = (a+b)/2$. The optimal encoding function is the same for both players and is given by $m^\star(\theta) = \xi^\star + \psi^\star\theta = -\frac{\Omega\mu_\theta}{\sigma_\theta} + \frac{\Omega}{\sigma_\theta}\theta$.*

Proof. In stage 2, each player $i = \{1, 2\}$ invests if and only if $Z_i \leq \mu_\theta$. Given the conditional distribution of Z_i, the probability player i invests for a given θ and encoding function is

$$\mathbb{P}_i(\text{Invest}|\theta, \psi_i) = \Phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_i}\right)$$

Thus, the expected utility player i gets from the game with a given value of θ is

$$\begin{aligned} EU_i(\theta,\psi_i) &= \mathbb{P}_i(\text{Invest}|\theta,\psi_i)\,(a+\mathbb{P}_{-i}(\text{Invest}|\theta,\psi_{-i})(b-a))+(1-\mathbb{P}_i(\text{Invest}|\theta,\psi_i))\theta \\ &= \theta+\Phi\left(\frac{\mu_\theta-\theta}{\sigma_S/\psi_i}\right)\left(a+\Phi\left(\frac{\mu_\theta-\theta}{\sigma_S/\psi_{-i}}\right)(b-a)-\theta\right) \end{aligned}$$

where we use $-i$ to denote i's opponent. How does this expected utility change with ψ_i (taking ψ_{-i} as given)?

$$\frac{\partial EU_i(\theta,\psi_i)}{\partial\psi_i}=\phi\left(\frac{\mu_\theta-\theta}{\sigma_S/\psi_i}\right)\left(\frac{\mu_\theta-\theta}{\sigma_S}\right)\left(a+\Phi\left(\frac{\mu_\theta-\theta}{\sigma_S/\psi_{-i}}\right)(b-a)-\theta\right) \tag{12}$$

Since $\phi(\cdot)$ is strictly positive for any argument, the sign of equation (12) is determined by the product of its second and third term. First, note that the second term is a) equal to 0 when $\theta=\mu_\theta$, b) strictly positive when $\theta<\mu_\theta$ and c) strictly negative when $\theta>\mu_\theta$. Second, note that — since $\mathbb{P}_{-i}(\text{Invest}|\theta,\psi_{-i})$ is greater than 1/2 if and only if $\theta<\mu_\theta$ and $\mu_\theta=(a+b)/2)$ — the third term is a) strictly positive when $\theta<\mu_\theta$ and b) strictly negative when $\theta>\mu_\theta$. This means that the product of the second and third term of equation (12) is always positive, with the exception of the case when $\theta=\mu_\theta$, in which case it is 0.

We have shown that the expected payoff in a game with a given θ is strictly increasing in ψ_i for any value of $\theta\neq\mu_\theta$ and it is constant in ψ_i for $\theta=\mu_\theta$. This means that, from an ex-ante perspective (that is, when a player knows the distribution of θ but does not know its actual realization), each player's expected reward from the simultaneous move game — that is, $EU_i(\psi_i)=\int EU_i(\theta,\psi_i)f(\theta)d\theta$ — is strictly increasing in ψ_i. Therefore, it is desirable to make ψ_i as large as possible consistent with the power constraint. When the distribution of θ is normal, we have

$$E[m^2]=\xi^2+\psi^2E[\theta^2]+2\xi\psi E[\theta]=(\xi+\psi\mu_\theta)^2+\psi^2\sigma_\theta^2\leq\Omega$$

The largest value of ψ consistent with this constraint in Assumption 2 is achieved when

$$\xi=-\psi\mu_\theta\ ,\ \psi=\frac{\Omega}{\sigma_\theta}$$

Thus, $m^\star(\theta)=-\frac{\Omega}{\sigma_\theta}\mu_\theta+\frac{\Omega}{\sigma_\theta}\theta$. ■

B Experiment on Awareness of Cognitive Noise

Here we report results from an additional experiment that is designed to investigate whether subjects are aware of their own imprecision and the imprecision of others. If subjects are not aware of the cognitive noise of others, then this would shut down the channel that generates strategic uncertainty in our model, which is key to generating the unique threshold equilibrium.

Experimental Design

Our method for studying awareness of imprecision is to create a simplified version of the coordination game experiment in the main text, but one that retains the core individual decision-making prediction that subjects play a threshold strategy. We employ a task from the numerical cognition literature where subjects are incentivized to quickly and accurately classify whether a two-digit number is larger or smaller than the number 55. Note that this threshold strategy is identical to the equilibrium strategy in Experiment 1; the important difference is that here, we exogenously impose the strategy on subjects without any strategic considerations or equilibrium requirements. We then incentivize subjects to report beliefs about errors in their own classification and in the classification of others. These beliefs are the main object of study in this experiment.

We recruit 300 subjects from Prolific who did not participate in Experiment 1 or Experiment 2. We pay subjects 1 GBP for completing the study, in addition to earnings from three phases of the experiment. In Phase 1, on each of 150 rounds, subjects are incentivized to quickly and accurately classify whether a two-digit Arabic numeral on the experimental display screen is larger or smaller than 55. Subjects earn ($1.5 \times$ accuracy $- 1 \times$ speed) GBPs, where 'accuracy' is the percentage of trials where the subject classifies the number correctly, and 'speed' is the average response time in seconds.[31] As in Experiment 1, there are two conditions, and the only difference across conditions is the distribution from which the two-digit Arabic numeral (which we again denote by θ) is drawn. We use the same two distributions as in Experiment 1: in the high volatility condition, $\theta \sim \mathcal{N}(55, 400)$, and in the low volatility condition, $\theta \sim \mathcal{N}(55, 20)$. We then round each value of θ to the nearest integer and re-draw if the rounded integer is less than 11 or greater than 99 (again, to ensure that each number contains exactly two digits).

We note that one difference in incentives compared to those in Experiment 1 involves decision speed. Here, we penalize subjects for the time it takes them to respond. The reason we impose the speed incentive comes from the well known "speed-accuracy tradeoff"

[31] The experimental instructions are available in Online Appendix C.

in perceptual decision-making: one can obtain higher accuracy in classification as decision speed slows down. Thus, in order to increase statistical power to detect how accuracy differs for values of θ close and far from the threshold, we jointly reward speed and accuracy.

In Phase 2 of the experiment, we incentivize subjects to report beliefs about others' performance in the task. Furthermore, we collect data on whether subjects believe that others are more imprecise when the number on screen is closer to the reference level of 55, compared to when the number is farther from the reference level. This feature of beliefs is important because the equilibrium predictions from our previous experiment depend on the noise structure in perception. In particular, recent theoretical work has shown that an important property of the noise structure for determining equilibrium is that discriminating between nearby states is harder than discriminating between far away states (Morris and Yang, 2022; Hébert and Woodford, 2021). We ask subjects to consider the 149 other participants in their experimental condition of the study, who also just completed Phase 1. We then ask subjects the following two questions:

1. Consider only trials where the number on screen was equal to 47. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

2. Consider only trials where the number on screen was equal to 54. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

For each of the two questions, we pay the subject 0.5 GBP if their forecast is within 1% of the true percentage.[32] Question 1 elicits beliefs about others' imprecision when the distance between the number is far from the threshold (47 vs. 55), whereas Question 2 elicits beliefs about others' imprecision when the distance is close (54 vs. 55). While we could have asked subjects about their beliefs about others' imprecision for a range of numbers — rather than the single numbers 47 and 55 — this would have introduced a confound, since the distribution of numbers is different across conditions.

In Phase 3, we ask subjects about their own performance on the number classification task (that they completed in Phase 1). This question is not trivial because we do not provide subjects with feedback after any round in Phase 1 (nor after the end of Phase 1). Here, we are also interested in subjects' awareness of their own imprecision for numbers that are close and far from the threshold. Specifically, we ask subjects the following two questions:

[32]Following Hartzmark, Hirshman and Imas (2021), we choose this elicitation procedure as opposed to a more complex mechanism such as the Binarized Scoring Rule (BSR) due to recent evidence showing that the BSR can systematically bias truthful reporting (Danz, Vesterlund and Wilson, 2022).

1. Consider only trials where the number on screen was between 52 and 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

2. Consider only trials where the number on screen was less than 52 or greater than 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

For each of these two questions, we again reward subjects with 0.50 GBP if they provide an answer that is within 1% of their true accuracy. All subjects first go through Phase 1, and the order of Phase 2 and Phase 3 is randomized across subjects. We note that one potential concern with our design, is that when asking subjects about their performance in Phase 1, we are testing memory, not *ex-ante* beliefs. This is a reasonable concern, and an alternative is to have subjects forecast their performance before undertaking the classification task. However, under this alternative design, subjects' classification performance would be endogenous to their beliefs, and would invalidate the incentive compatibility of our belief elicitation procedure. For this reason, we opt to implement Phase 1 first for all subjects.

Experimental Results

The upper panel of Figure A1 replicates the classic result from previous experiments on number discrimination, whereby subjects exhibit errors, and these errors increase as the number on screen approaches the threshold (Dehaene, Dupoux and Mehler, 1990). Moreover, we see that, for numbers between 47 and 63, errors are systematically higher in the high volatility condition (Frydman and Jin, 2022). Similar patterns are reflected in the response times shown in the lower panel of Figure A1: response times increase as the number approaches the threshold of 55, and response times are systematically longer in the high volatility condition.

The purpose of Phase 1 is to create a dataset about performance, over which we can ask subjects about their beliefs in Phases 2 and 3. In the left panel of Figure A2, we see that subjects believe their behavior in the classification task exhibits imprecision (that is, beliefs about accuracy are less than 100%). Moreover, we see that subjects are aware that mistakes are more likely for numbers closer to the threshold (greater than 52 and less than 58) than for numbers farther from the threshold (less than 52 or greater than 58; $p < 0.001$).

The results in the middle panel of Figure A2 help validate a crucial assumption in our model. Specifically, we see that subjects are aware of other subjects' imprecision. Moreover, subjects believe that others are less accurate when discriminating 54 vs. 55 compared with discriminating 47 vs. 55 ($p < 0.001$). When embedded in a game, these beliefs are sufficient

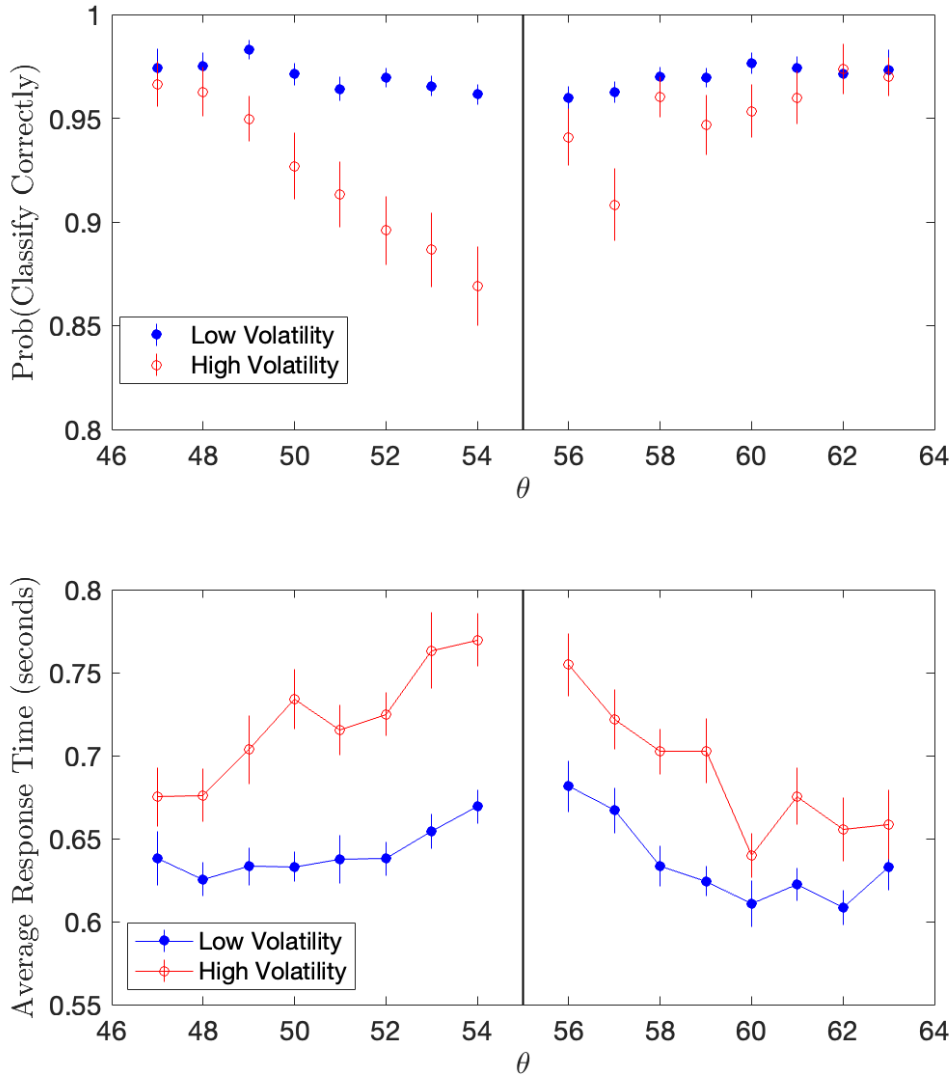


Figure A1: **Accuracy and Response Times in the Classification Task.** Note: Upper panel shows the proportion of rounds on which subjects correctly classify θ as greater than or less than the reference level of 55. Lower panel shows the average response time on rounds where subjects correctly classify θ. In both panels, the vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

to generate strategic uncertainty: if player i believes that player j perceives θ with error, then player i is uncertain about player j's perception. The data in the middle panel of Figure A2 therefore provide support for the mechanism that generates strategic uncertainty in our model.

Finally, our data also enable us to test one other feature of beliefs about others' impre-

Figure A2: **Beliefs about Own and Others' Accuracy in the Classification Task.** Note: Left panel shows the average belief about own accuracy for values of θ that are far ($\theta < 52$ or $\theta > 58$) and close ($51 < \theta < 59$) to the threshold 55. Middle panel shows the average belief about others' accuracy for values of θ that are far ($\theta = 47$) and close ($\theta = 54$) to the threshold 55. Right panel shows the average belief about others' accuracy when $\theta = 54$, split by experimental condition. In all panels, vertical bars denote two standard errors of the mean.

cision. As outlined in our pre-registration, we test whether beliefs about others' accuracy on rounds when $\theta = 54$ is higher for those subjects who experience the low volatility distribution in Phase 1.[33] Such a test investigates the hypothesis that subjects are aware that others' perception of a given number varies as a function of the experienced distribution. Indeed, the right panel of Figure A2 shows that, for $\theta = 54$, subjects who experience the high volatility distribution in Phase 1 report that others make more errors, compared to those subjects who experience the low volatility distribution in Phase 1 ($p = 0.048$).

[33] Pre-registration document is available at `https://aspredicted.org/OGG_XNK`.

C Experimental Instructions

Experiment 1 (Coordination Game)

Welcome!

You will earn £2 for completing this study and will have the opportunity to **earn more money** depending on your decisions during the study.

Specifically, at the end of the study, the computer will randomly select one question. You will receive points from the randomly selected question and the number of points depends on your decision and the decision of another participant. Points will be converted to pounds using the rate 20 points = £1. For example, if you earned 60 points for the selected question, you would then earn 60/20 = £3 (in addition to the completion fee).

All questions are equally likely to be selected so **make all choices carefully**.

The next pages give detailed instructions. Following the instructions, you will take a quiz on them. You will be allowed to continue and will be entitled to payment **only if you answer all questions on the quiz correctly**.

Instructions (1/2)

The study is separated into 6 parts of 50 rounds each.

In each round, you are randomly matched with another participant, who we call your **opponent**.

In each round, both you and your opponent will be asked to choose between **two options**:

"Option A" or "Option B"

Here is how to earn points:

- If you choose Option A, the number of points you receive does not depend on whether your opponent chooses Option A or B. The amount of points you receive for choosing Option A can be different in different rounds and will be displayed on your screen.
- If you choose Option B, the number of points you receive depends on your opponent's decision: if your opponent chooses Option A, you will receive 47 points; if your opponent also chooses Option B you will receive 63 points.

Importantly, your opponent is reading these same exaxt instructions. This means that:

- If your opponent chooses Option A, his/her payoff does not depend on your decision and the number of points he/she earns are those given by Option A.
- If your opponent chooses Option B, the number of points he/she receives depends on your decision: if you choose Option A, your opponent will receive 47 points; if you also choose Option B, your opponent will receive 63 points.

Instructions (2/2)

Below is an example screen from the study:

Option A	Option B
53	**47** if other participant chooses A **63** if other participant chooses B

In this example, Option A is on the LEFT side of the screen and Option B is on the RIGHT.

In each round, you will choose one of the two options by pressing either the "A" key on your keyboard for the LEFT option or the "L" key on your keyboard for the RIGHT option. On some rounds, Option A will be on the LEFT, and in other rounds it will be on the RIGHT.

In the example above:

- Option A pays you 53 points regardless of your opponent's decision, while Option B pays you 47 points if your opponent chooses Option A and 63 points if your opponent chooses Option B.
- Note also that, if your opponent chooses Option A, he/she earns 53 points regardless of your decision. If your opponent, instead, chooses Option B, he/she earns 47 points if you choose Option A and 63 points if you choose Option B.

Experiment 2 (Human vs. Algorithm)

Welcome!

You will earn £2 for completing this study.

You will also have the opportunity to **earn more money** depending on your decisions.

Specifically, at the end of the study, the computer will randomly select one question. You will receive points from the randomly selected question and the number of points depends on your decision and the decision of an opponent. Points will be converted to pounds using the rate 20 points = £1. For example, if you earned 60 points for the selected question, you would then earn 60/20 = £3 (in addition to the completion fee).

All questions are equally likely to be selected so **make all choices carefully**.

The next pages give detailed instructions.

Following the instructions, you will take a quiz on them. You will be allowed to continue and will be entitled to payment **only if you answer all questions on the quiz correctly**.

Instructions (1/2)

The study is separated into 6 parts of 50 rounds each.

In each round, you will play a game against the computer, who we call your **opponent**.

In each round, both you and your opponent will choose between two options:

"Option A" or "Option B"

Importantly, **your computerized opponent chooses according to a rule** that you will learn below.

Here is how to earn points:

- If you choose Option A, the number of points you receive does not depend on whether your opponent chooses Option A or B. The amount of points you receive for choosing Option A can be different in different rounds and will be displayed on your screen.
- If you choose Option B, the number of points you receive depends on your opponent's decision: if your opponent chooses Option A, you will receive 47 points; if your opponent also chooses Option B you will receive 63 points.

Your Opponent's Rule

You will not have to guess how your computerized opponent will behave.

This is because **your opponent is programmed to choose according to the following rule**:

- If Option A delivers 55 points or more, your opponent chooses Option A.
- If Option A delivers less than 55 points, your opponent chooses Option B.

To be clear: your computerized opponent follows the rule above without exceptions. So, you can be certain about how your opponent's choice depends on the number of points that Option A delivers.

Instructions (2/2)

Below is an example screen from the study:

Option A	**Option B**
53	**47** if opponent chooses A **63** if opponent chooses B

In this example, Option A is on the LEFT side of the screen and Option B is on the RIGHT.

In each round, you will choose one of the two options by pressing either the "A" key on your keyboard for the LEFT option or the "L" key on your keyboard for the RIGHT option. On some rounds, Option A will be on the LEFT, and in other rounds it will be on the RIGHT.

In the example above:

- Since Option A pays 53 points, the rule says your opponent chooses Option B.
- Therefore, if you choose Option A, you get 53 points.
- If, instead, you choose Option B, you get 63 points.

Experiment 3 (Awareness of Cognitive Noise)

Thank you for participating in this study!

Before we begin, please close all other applications on your computer and put away your cell phone. This study will last approximately **10 minutes**. During this time, we ask your complete and undistracted attention. You will earn £1 for completing the study and you will have the opportunity to **earn more money** depending on your answers during the study.

This study consists of **two phases**. The instructions for Phase 1 are given in the next page. After you go through Phase 1, you will be given a new set of instructions for Phase 2.

When you are ready to continue, press ENTER.

In **Phase 1**, you will see a series of numbers and will be asked to classify whether each number is **larger or smaller than 55**. If the number displayed is smaller than 55, press the "A" key on your keyboard. If the number displayed is larger than 55, press the "L" key.

Your bonus payment will depend on the speed and accuracy of your classification. Specifically:

Bonus Payment = £ (1.5 x accuracy – 1 x speed)

where "accuracy" is the percentage of trials where you correctly classified the number as larger or smaller than 55, and "speed" is the average amount of time it takes you to classify the number on all trials throughout the study, in seconds.

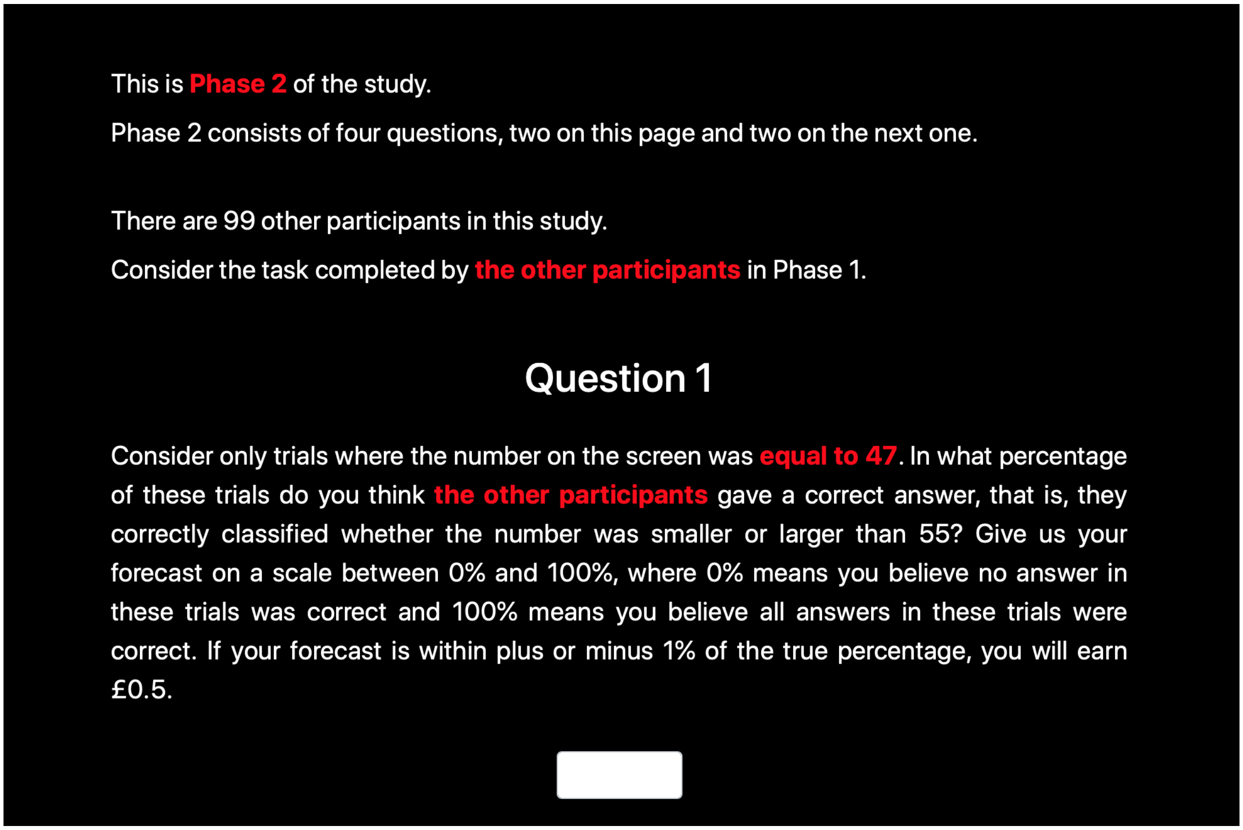
Thus, you make the most money by answering as **quickly** and as **accurately** as possible.

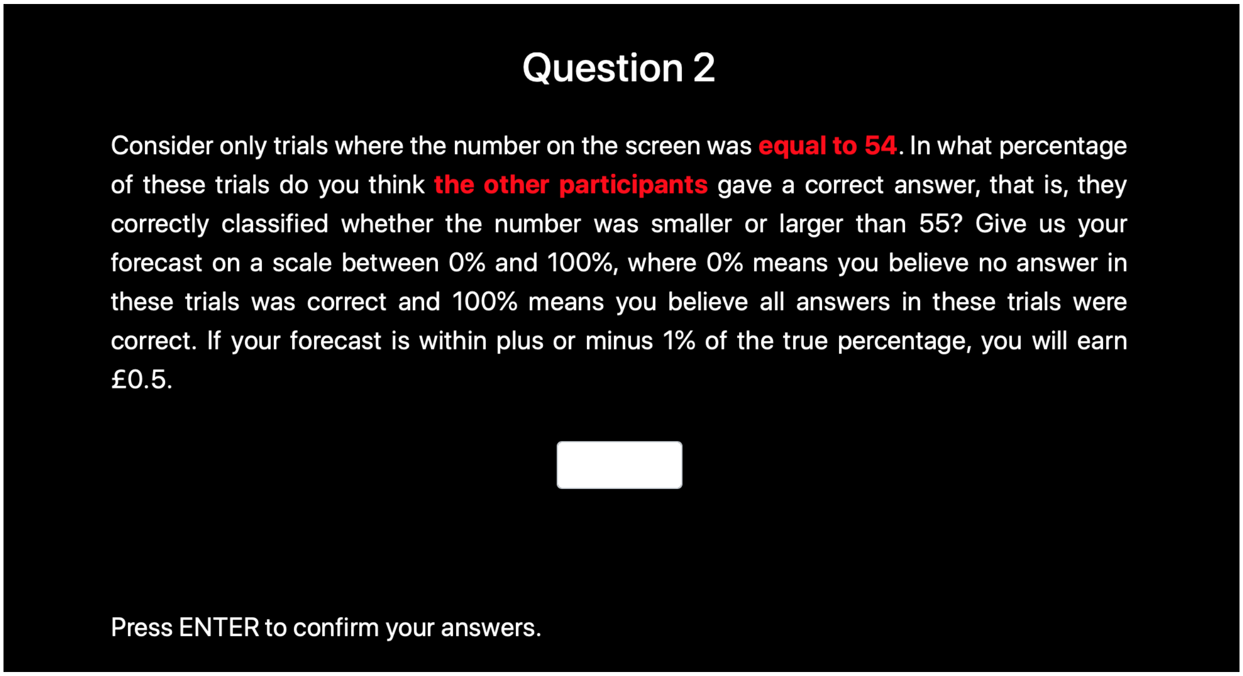
For example, if you correctly classified the number on all trials and it took you 0.3 seconds to respond to each question, you would earn £(1.5 x 100% - 10 x 0.3) = £1.20. If instead you only classified 70% of the numbers correctly and took 0.8 seconds to respond to each question, you would earn £(1.5 x 70% - 10 x 0.8) = £0.25.

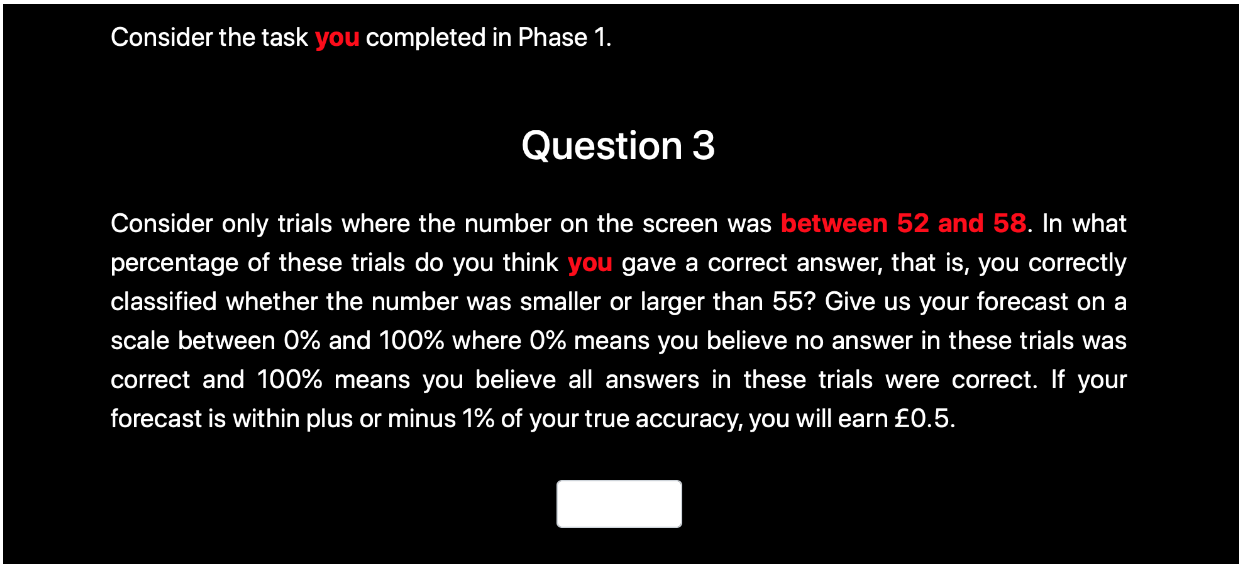
Phase 1 will be separated into 3 parts of 50 trials each. In between, you can take a short break.

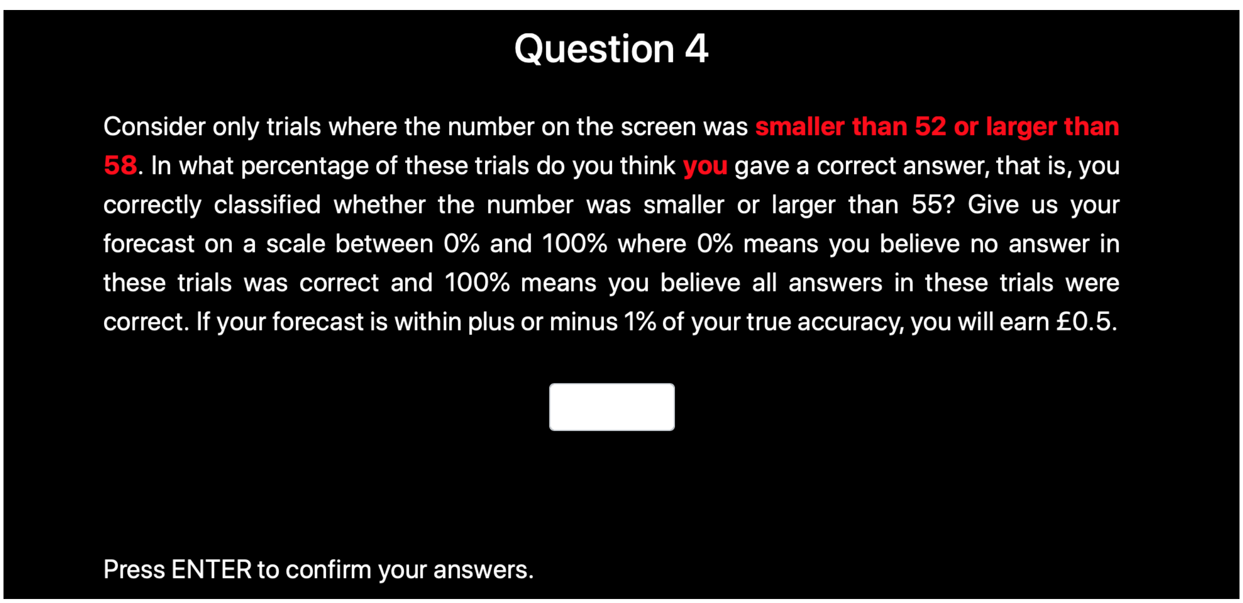
Before starting with the classification task, you will be asked a question to check your understanding of the instructions. You will be allowed to continue **only if you answer this question correctly**.

When you are ready to continue with the comprehension question, press ENTER.


This is Phase 2 of the study.
Phase 2 consists of four questions, two on this page and two on the next one.
There are 99 other participants in this study.
Consider the task completed by the other participants in Phase 1.
Question 1
Consider only trials where the number on the screen was equal to 47. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.


Question 2
Consider only trials where the number on the screen was equal to 54. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.
Press ENTER to confirm your answers.


Consider the task you completed in Phase 1.
Question 3
Consider only trials where the number on the screen was between 52 and 58. In what percentage of these trials do you think you gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.


Question 4
Consider only trials where the number on the screen was smaller than 52 or larger than 58. In what percentage of these trials do you think you gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.
Press ENTER to confirm your answers.

D Response Times from Experiment 1

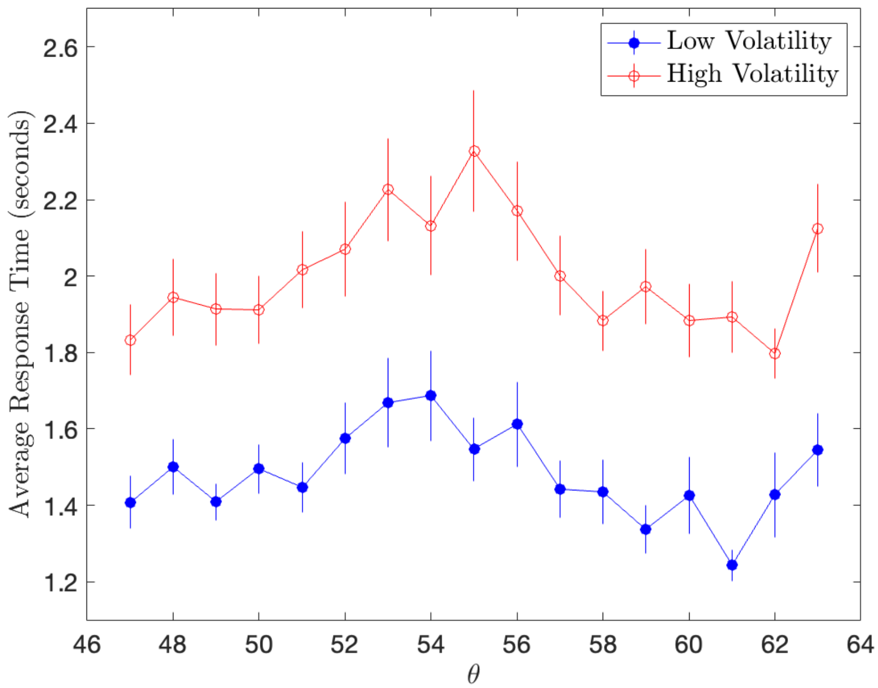


Figure A3: **Average Response Time as a Function of θ.** Note: Response times are averaged across subjects and across rounds. Vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

Here we analyze the distribution of response times in both the high volatility and low volatility conditions from Experiment 1. The response time variable is defined at the round level, and measures how long it takes a player to execute a decision after the game is presented on the screen. As outlined in our pre-registration, we test two hypotheses regarding the distribution of response times. First, response times should peak at the unique equilibrium cutoff level of 55. Second, conditional on θ, response times should be longer in the high volatility condition. Our hypotheses are motivated by the literature on sequential sampling models (Ratcliff, 1978; Bogacz, Brown, Moehlis, Holmes and Cohen, 2006), which robustly predict that response times become longer as the values of two items under comparison become closer together. Thus, the tests we present in this section are joint tests of cognitive noise, which predicts that subjects use a unique threshold strategy, and sequential sampling models, which predicts how long it takes to implement the threshold strategy on each round.

In many sequential sampling models (see, e.g., Krajbich, Armel and Rangel 2010), the agent will execute a decision as soon as a stream of incoming signals has reached a pre-defined reliability threshold. Because signals are sampled sequentially, response times increase with the number of signals drawn. While the model we present in Section 2 only allows the agent to draw a single noisy signal, S_i, one could generalize the model to allow a sequence of independent noisy signals. For every additional noisy signal that the player collects, her posterior will become narrower, and, thus, the entire stream of signals provides more reliable evidence about whether θ is less than 55. As signals become more informative about whether θ is below the (equilibrium) threshold, the agent will reach the pre-defined reliability threshold with fewer signals, and thus response times will be shorter.

In our setting, there are two particular ways in which a signal can provide more information about whether θ is less than 55. First, recall that in our model, the mean of S_i varies monotonically with θ. Thus, S_i provides cardinal information about θ, and not just ordinal information about whether θ is below 55. It follows that as $|\theta - 55|$ increases, S_i provides a more informative signal about whether $\theta < 55$. Second, as the precision of S_i increases, this naturally provides more information about whether $\theta < 55$. Taken together, sequential sampling models predict that, when a player is tasked with implementing a cutoff strategy (which is derived as the equilibrium strategy under cognitive noise), response times should decrease as (i) $|\theta - 55|$ increases and (ii) the precision of S_i increases. We can test the first prediction by relying on variation in θ within an experimental condition. We can test the second prediction by relying on the variation in signal precision across conditions, which is endogenously generated by efficient coding.

Figure A3 plots the average response time, conditional on θ, for each of the two experimental conditions. We highlight two features of the figure. First, in the high volatility condition, the peak response time is at $\theta = 55$; in the low volatility condition, the peak is not far away, at $\theta = 54$. Moreover, response times fall almost monotonically as θ moves away from the equilibrium threshold of 55 ($p = 0.001$ in a mixed-effects regression of response time on $|\theta - 55|$ for each of the two conditions). Second, there is a clear separation of the curves across conditions: conditional on θ, response times are longer in the high volatility condition compared to the low volatility condition (unconditionally, the average response time is significantly longer in the high volatility condition, $p < 0.001$). These two features of the data are roughly consistent with the predictions outlined above.

One caveat to this analysis is that the player chooses the precision of S_i according to efficient coding, but under the assumption that she can only draw one signal. Predictions may change if we endogenized the signal precision and the number of signals to be drawn (or the reliability threshold). That said, the data from Figure A3 provide suggestive evidence that subjects are implementing threshold strategies in a manner that is consistent with core predictions of sequential sampling models. Thus, the response time data help validate our assumptions about the cognitive constraints that subjects face when playing the game.

E Heterogeneity in Experiment 2

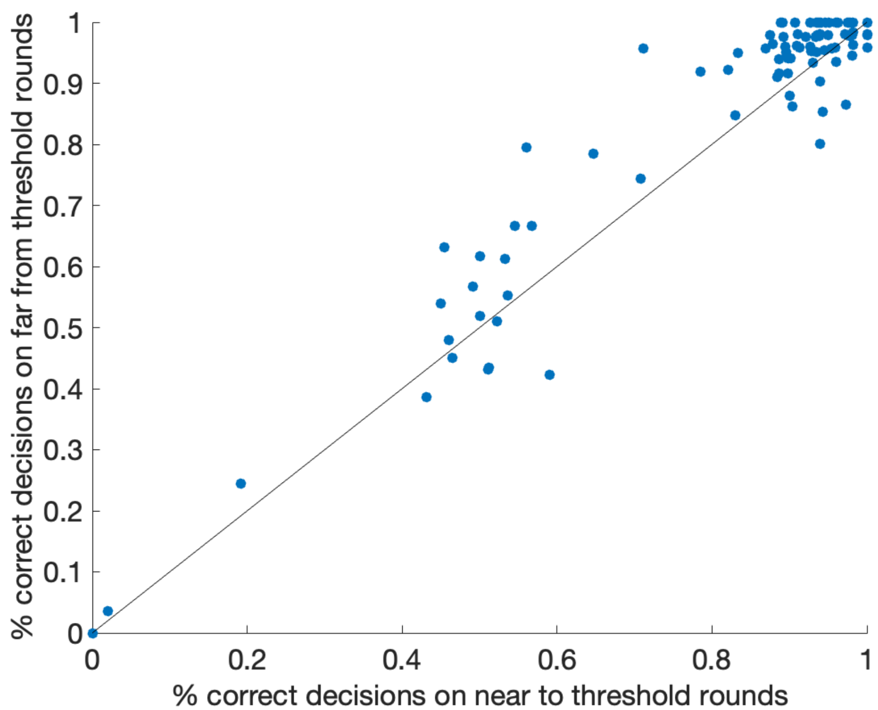


Figure A4: **Heterogeneity in % Correct Decisions in Experiment 2.** Note: Each point represents a single subject from the Algorithm condition. A "near to threshold round" is one where θ takes on one of the following values: 51, 52, 53, 54, 56, 57, 58, or 59. A "far from threshold round" is one where θ takes on one of the following values: 47, 48, 49, 50, 60, 61, 62, or 63.

Figure 5 in the main text shows that, on average, subjects in the Algorithm condition from Experiment 2 exhibit noise in their behavior. However, Figure 5 only shows the data at the aggregate level. Here, we investigate individual-level data. We demonstrate that a majority of subjects exhibit noise in their behavior, and that the noise structure is consistent with our model assumptions.

For each of the 100 subjects in the Algorithm condition, we compute the proportion of decisions that are "correct" in the sense that subjects best respond to the computer's exogenous strategy. In particular, we define a decision as "correct" if the subject (chooses not invest and $\theta < 55$) or (chooses invest and $\theta > 55$). Otherwise, we define a decision as "incorrect". We analyze the sample used to create Figure 5: all rounds for which $46 < \theta < 64$ and subjects exhibit a response time greater than 0.5 seconds.

Before presenting the results, note that our model of cognitive noise induces incorrect decisions because the subject does not observe θ, and instead only observes $S_i = \theta + \sigma\varepsilon_i$.

Thus, even if a subject perfectly follows the optimal threshold strategy of choosing invest if and only if $S_i < 55$, there may still be "incorrect" decisions, and the frequency of these incorrect decisions is increasing in σ.

Figure A4 shows that nearly all subjects (91%) exhibit incorrect decisions, consistent with them using a noisy signal to implement a threshold strategy. We also note that a majority of subjects (78%) are located above the 45-degree line, indicating that the frequency of incorrect decisions is higher when θ is near the threshold of 55, compared to when it is far from the threshold of 55. Taken together, these results are consistent with: 1) subjects exhibiting noise at the individual level and 2) our assumption of the noise specification, whereby incorrect decisions become more likely as θ approaches the threshold of 55.