A Model of Focusing in Political Choice *

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Abstract

This paper develops a model of voters' and politicians' behavior based on the notion that voters focus disproportionately on and, hence, overweigh certain attributes of policies. We assume that policies have two attributes—resources devoted to two distinct issues (e.g., defense and education)—and that voters focus more on the attribute in which their options differ more. First, we consider exogenous policies and show that focusing polarizes the electorate. Second, we consider the endogenous supply of policies by politicians running for office and show that focusing leads to inefficiencies: voters that are more focused are more influential; distorted attention empowers social groups that are larger and more sensitive to changes on either issue; resources are channelled towards divisive issues. Finally, we show that augmenting classical models of electoral competition with focusing can contribute to explain puzzling stylized facts such as the inverse correlation between income inequality and redistribution.

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1 Introduction

Evaluating political candidates or parties is a complex, multidimensional task. This is because, in an election, a candidate typically represents a bundle of positions on multiple policy issues. For example, in the 2016 U.S. presidential election, Hillary Clinton was in favor of the Affordable Care Act and the Paris agreement on climate, while Donald Trump opposed both measures. On the other hand, both candidates proposed a plan of public investment in infrastructure and expressed skepticism about the Trans-Pacific Partnership.¹

This suggests that how citizens weigh a candidate's position on different issues is crucial for the formation of their political preferences. Even when citizens have access to detailed information on candidates' platforms, evaluating them is a complex task, associated with low stakes and no direct feedback from experience, as an individual's political choice is unlikely to be pivotal. This might lead citizens to consistently misperceive the overall value of the available alternatives.

In particular, a large body of experimental research has documented that preferences over alternatives with multiple dimensions, or attributes, are influenced by the environment.² Building on this evidence, economists have recently developed models where the choice set can distort the weights a decision-maker attaches to the attributes of an alternative (Rubinstein, 1988; Leland, 1994; Bordalo, Gennaioli, and Shleifer, 2012, 2013a,b, 2015; Kőszegi and Szeidl, 2013; Bushong, Rabin, and Schwartzstein, 2021). At the same time, the theoretical implications of this selective focus for political behavior are largely unexplored and unclear. In fact, most models of voting are based on the classic model of choice where the subjective value each alternative gives to a decision-maker is independent of the other available alternatives.

In this paper, we develop a model of voters' and politicians' behavior based on the

¹See 'Policy details lost in the din of debate' in Financial Times on Nov 8, 2016.

²Manipulating the set of available alternatives affects choice over consumer products which differ in quality and price (Huber, Payne, and Puto, 1982; Simonson, 1989; Simonson and Tversky, 1992; Heath and Chatterjee, 1995); choice over lotteries which vary in prizes and probabilities (Allais, 1953; Slovic and Lichtenstein, 1971; Herne, 1999); and choice over monetary allocations which differ in efficiency and fairness (Roth, Murnighan, and Schoumaker, 1988; Galeotti, Montero, and Poulsen, 2019).

idea that voters perceive policy issues as more or less *salient* depending on the choice environment. In line with a recent literature in economics and psychology, we assume that a voters' attention is captured by the issue in which the available candidates differ more and that, in turn, this issue is overweighed in the decision-making process. This assumption is based on the notion that our limited cognitive resources are unconsciously attracted by a subset of the available sensory data (Taylor and Thompson, 1982) and, in particular, that "our mind has a useful capability to focus on whatever is odd, different or unusual" (Kahneman, 2011; see also Baumeister and Vohs, 2007). Importantly, this assumption and its implications for choice under risk and over time have recently found validation in a number of controlled laboratory experiments (Bondi, Csaba, and Friedman, 2018, Castillo, 2020, Andersson, Carlson, and Wengström, 2021, Dertwinkel-Kalt, Gerhardt, Riener, Schwerter, and Strang, Forthcoming).

In our basic framework, a government provides two public goods to its citizens, e.g., defense and education. The government has a fixed amount of available resources and each policy has two attributes, the resources devoted to each issue, that is, to the production of each public good. The polity consists of a continuum of voters in different social groups. All voters from the same social group have the same preferences but voters in different social groups differ in their relative preference for the two public goods, that is, in the rate at which they are willing to trade resources across issues. When evaluating policies, voters focus more on the issue in which options differ more, that is, on the issue which delivers the greater range of utility.

We present three sets of results. First, we analyze the consequences of focusing for voters' preferences over an exogenous pair of policies. We show that voters focus on the relative advantage—that is, the larger spending in defense or the larger spending in education—of the policy which gives them the higher consumption utility (i.e., the utility perceived by a rational voter). To understand why, consider a citizen who receives higher consumption utility from the policy with larger spending in defense in the choice set. For this citizen, the larger utility from larger spending in defense more than compensates the lower utility from smaller spending in education. This happens precisely because the

range of utility from defense spending in the citizen's choice set is larger than the range of utility from education spending. Since voters' attention is attracted by the attribute with larger range, this leads the citizen to focus on defense and, thus, to overweight defense spending and underweight education spending in his focus-weighted utility. As a consequence, focusing does not affect what policy a voter prefers but it strengthens the intensity of preferences between this policy and the alternative—that is, it polarizes the electorate.

Second, we consider the effect of focus on the endogenous formation of voters' choice set and the aggregation of their preferences in a collective choice by introducing focusing voters into a model of electoral competition between two office-motivated parties. In the unique equilibrium of this game, the two parties offer the same policy and, thus, voters have undistorted focus. Nonetheless, any deviation from the equilibrium policies triggers voters' selective focus—on different policy issues for different voters—and, thus, focusing affects the politicians' electoral calculus. We show that equilibrium policies are generically different than the ones emerging with rational voters and do not maximize utilitarian welfare: politicians are more likely to inefficiently cater to larger groups, to groups with more distorted focus, and to groups that are more sensitive to changes in either policy issue. This last determinant of political influence, which only emerges with selective focus, can dominate size and make minority groups more important in the electoral calculus. An empirical implication of this result is that, when one issue is a pure common value but there is a conflict of preferences on the other, politicians might be overly responsive to a minority which prefers relatively more spending on the divisive issue and is prone to focus on this issue. Our model highlights that policy capture from special interests can be a consequence of the psychology of attention without relying on the coordination and costly collective action necessary for lobbying. When attention and, in turn, preferences are influenced by the choice environment, a small group which neglects one side of the trade-off but is really sensitive on the other can be overly influential in obtaining what it desires.

Third, we explore the relevance of voters' distorted attention in one important ap-

plication, fiscal policy. In particular, we consider a stylized Meltzer and Richard (1981) model where parties offer a public good funded by a proportional tax rate and show that the model helps explain facts that are puzzling from the perspective of existing political economy theories—the negative correlation between income inequality and both the support for redistribution (Ashok, Kuziemko, and Washington, 2015) and the top marginal tax rates (Piketty, Saez, and Stantcheva, 2014). Following a marginal deviation from the convergent equilibrium policies, poor voters—who prefer more redistribution focus on public good provision, while rich voters—who prefer less redistribution—focus on after-tax income. If a shock to income inequality mainly affects how revenues are raised, selective focus amplifies rich voters' sensitivity to policies more than poor voters': the former group focuses on the cost of redistribution and overweighs its higher tax bill; on the other hand, the latter group focuses on the benefit of redistribution and neglects the increased bang-for-the-buck of redistributive measures. This makes poor voters lukewarm towards more redistribution and rich voters very responsive to the promise of tax cuts. Thus, rich voters become more influential in the politicians' calculus even when they constitute a minority of the population and they do not engage in lobbying to gain the favors of political elites.

Our work is primarily related to a recent, yet rapidly growing, research program in theoretical political economy with non-standard preferences or boundedly rational agents (also known as behavioral political economy), which studies electoral competition or political agency models when voters employ decision heuristics or are prone to cognitive biases (Bendor, Diermeier, Siegel, and Ting, 2011; Bisin, Lizzeri, and Yariv, 2015; Levy and Razin, 2015; Ortoleva and Snowberg, 2015; Diermeier and Li, 2017, 2019; Lockwood, 2017; Passarelli and Tabellini, 2017; Alesina and Passarelli, 2019; Little, 2019; Lockwood and Rockey, 2020). More closely related to this paper, Callander and Wilson (2006, 2008), Balart, Casas, and Troumpounis (2019) and Gallice and Grillo (2020) introduce a theory of Downsian competition with context-dependent voting. In Callander and Wilson (2006, 2008), the propensity to turn out and vote for the preferred candidate is greater when the other candidate is more extreme. In Balart et al. (2019), voters' preferences over

political candidates depend on two attributes—their policy position and their expenditure in electoral advertising—and voters' decision depend on candidates' policies when these are sufficiently different and on candidates' advertising expenditure otherwise.³ In Gallice and Grillo (2020), citizens care about their consumption and about their social status; a citizen's social status is determined by his consumption and his social class; and the relative weights of consumption and social class in determining social status depend on how much these two attributes vary in the population.⁴

We also contribute to a large literature in economics and political science which has shown that politicians distort their platforms to target each policy to those voters who care the most about it. Incentives to do so have been attributed to the fact that voters are (exogenously or endogenously) more informed about the policies they care more about (Glaeser, Ponzetto, and Shapiro, 2005; Gavazza and Lizzeri, 2009; Prato and Wolton, 2016; Matějka and Tabellini, 2021). Another modeling strategy, however, is to assume that voters ignore information they have but care too little about, up to the point of behaving as single-issue voters (List and Sturm, 2006). The latter assumption reflects the long-standing notion of issue saliences (Converse, 1964; RePass, 1971; Rabinowitz, Prothro, and Jacoby, 1982; Niemi and Bartels, 1985). Recent theoretical studies have mostly focused on media coverage (Edwards, Mitchell, and Welch, 1995; Epstein and Segal, 2000) or political advertising (Aragonès, Castanheira, and Giani, 2015; Dragu and Fan, 2016) as a driver of salience. This paper revisits issue salience and grounds it in a known psychological bias in information processing. In our model, voters have complete information on policies. Our innovation lies in assuming that issue salience for each voter does not depend only on how much the voter cares about different policies, but also on how distant alternative proposals are for different policies.

Finally, we also contribute to the theoretical literature on context-dependent preferences in economic choice: Bordalo et al. (2012, 2013a,b, 2015); Cunningham (2013); Kőszegi and Szeidl (2013), and Bushong et al. (2021) introduce models where the choice

³This model can be seen as a special case of focusing in which only the difference in one attribute (the candidates' policy position) is relevant to assign the weights and weights take value 0 or 1.

⁴This model can be seen as a special case of focusing in which only one attribute (the citizen's consumption) varies with the candidates' policy position.

set distorts the relative weights a decision-maker attaches to the attributes of an alternative.⁵ We share with these models the notion that the main determinant of these weights is the *range* of utilities across an attribute (in our case, a policy issue). With respect to these models, we consider agents with heterogeneous preferences, the aggregation of these agents' conflicting preferences in a collective choice, and the endogenous formation of the choice set by political candidates.

2 Model

Consider a government which provides two public goods to its citizens: d (e.g., defense) and e (e.g., education). The government has a fixed amount of available resources, $W \in \mathbb{R}_{++}$. Each policy, $p = (p_d, p_e)$, has two attributes, the resources devoted to each issue, that is, to the production of each public good. If policy p is implemented, then the provided levels of education and defense are, respectively, $p_d \in [0, W]$ and $p_e = (W - p_d)$.

The polity consists of a continuum of voters who belong to $n \geq 1$ social groups. The fraction of voters in group $i \in N = \{1, ..., n\}$ is $m_i > 0$, with $\sum_{i \in N} m_i = 1$. All voters from the same social group have the same policy preferences. While devoting more resources to either issue is unambiguously better for everybody, voters in different social groups differ in their relative preference for the two issues, that is, in the rate at which they are willing to trade resources across issues. In particular, a voter in group i derives consumption utility from policy p equal to:

$$V_i(p) = \theta_{id}u(p_d) + \theta_{ie}u(p_e), \tag{1}$$

where $\theta_{id}, \theta_{ie} > 0$ for all $i \in N$, and $\frac{\theta_{id}}{\theta_{ie}} \neq \frac{\theta_{jd}}{\theta_{je}}$ if $i \neq j$.

We make the following assumptions on $u: \mathbb{R}_+ \to \mathbb{R}$. It is (i) continuous, (ii) strictly increasing, (iii) strictly concave, (iv) twice continuously differentiable on \mathbb{R}_{++} , and (v) $\lim_{x\to 0^+} u'(x) = \infty$.

⁵In earlier work, Rubinstein (1988) and Leland (1994) also propose models of context-dependent choice where the similarity of attributes affects the evaluation of an option. They focus on choice over lotteries and do not motivate their model with the cognitive psychology of attention.

Given these assumptions, the problem of maximizing a group's consumption utility with a policy satisfying the government's budget constraint has a unique solution and this solution lies in (0, W). Let d_i^* be group i's consumption bliss point, that is, the policy such that

$$\theta_{id}u'(d_i^*) - \theta_{ie}u'(W - d_i^*) = 0. \tag{2}$$

Note that d_i^{\star} is strictly increasing in θ_{id} and strictly decreasing in θ_{ie} . We index groups such that if j > i, then $\frac{\theta_{jd}}{\theta_{je}} > \frac{\theta_{id}}{\theta_{ie}}$ —that is, groups with a higher index have a stronger preference for spending in defense. As a consequence, groups with a higher index have a larger consumption bliss point, that is, if j > i, then $d_i^{\star} > d_i^{\star}$.

Our key assumption and main departure from the classical political economy models is that, when evaluating policies, voters use their focus-weighted utility rather than their consumption utility. Consider a choice set composed of a finite number of policies, $\mathcal{P} = \{p, q, \ldots\}$, with $|\mathcal{P}| \geq 2$. Let $\Delta_{ik}(\mathcal{P})$ be the range of utility voters in group $i \in N$ derive from issue $k \in \{d, e\}$ in choice set \mathcal{P} :

$$\Delta_{ik}(\mathcal{P}) = \max_{p \in \mathcal{P}} \theta_{ik} u(p_k) - \min_{p \in \mathcal{P}} \theta_{ik} u(p_k). \tag{3}$$

We assume that voters focus more on the attribute in which their available options differ more, that is, on the attribute which generates a greater range of consumption utility.⁶ As discussed in Section 1, this assumption is compatible with the psychology of human cognition and has been validated in laboratory experiments.

Formally, we assume that voters in group *i focus on defense* if $\Delta_{id}(\mathcal{P}) > \Delta_{ie}(\mathcal{P})$, focus on education if $\Delta_{id}(\mathcal{P}) < \Delta_{ie}(\mathcal{P})$ and have undistorted focus if $\Delta_{id}(\mathcal{P}) = \Delta_{ie}(\mathcal{P})$. For a

⁶Similarly to what we do, Bordalo et al. (2012, 2013a,b, 2015) and Kőszegi and Szeidl (2013) assume that the salience of different attributes and, thus, the decision-maker's focus is driven by contrast. In addition, Bordalo et al. (2012, 2013a,b, 2015) assume that contrast is perceived with *diminishing sensitivity*.

voter in group $i \in N$, the focus-weighted utility from $p \in \mathcal{P}$ is:

$$\widetilde{V}_{i}(p|\mathcal{P}) = \begin{cases}
(1+\delta_{i})\theta_{id}u(p_{d}) + (1-\delta_{i})\theta_{ie}u(p_{e}) & \text{if } \Delta_{id}(\mathcal{P}) > \Delta_{ie}(\mathcal{P}) \\
(1-\delta_{i})\theta_{id}u(p_{d}) + (1+\delta_{i})\theta_{ie}u(p_{e}) & \text{if } \Delta_{id}(\mathcal{P}) < \Delta_{ie}(\mathcal{P}) \\
\theta_{id}u(p_{d}) + \theta_{ie}u(p_{e}) & \text{if } \Delta_{id}(\mathcal{P}) = \Delta_{ie}(\mathcal{P})
\end{cases} \tag{4}$$

where $\delta_i \in [0, 1)$ increases in the severity of focusing.

When voters in group i focus on defense (education), the relative weight they place on defense (education) is larger than the weight used by rational voters— $1 + \delta_i \in [1, 2)$; and the weight they place on education (defense) is smaller than the weight used by rational voters— $1 - \delta_i \in (0, 1]$. The weights on defense and education change discontinuously when the object of focus changes but remain constant when focus remains on a given attribute.⁷ The weighting distortion is allowed to be heterogeneous across social groups. As δ_i goes to 0, focusing voters in group i converge to rational voters. As δ_i goes to 1, focusing voters in group i consider only the attribute that attracts their attention and completely neglect the other. Voters in group i focus on the same attribute for all policies in a given choice set.⁸ Finally, the sum of the weights is independent of δ_i and of the attribute voters focus on. This ensures that the model is not biased towards focus on any single attribute by construction.

3 Consequences of Focus on Voters' Preferences

In this section, we study how focusing changes voters' preferences over exogenous policies. Although results similar to Proposition 1 have been derived in related work (see Section III.B in Kőszegi and Szeidl 2013), we additionally trace the effect of focusing across

⁷Bordalo et al. (2012, 2013a,b, 2015) consider similar focus weights, while Kőszegi and Szeidl (2013) use weights that change continuously with the range of an attribute. We use discontinuous weights for mathematical tractability. Most of the results we present below continue to hold if we assume continuous weights.

⁸Kőszegi and Szeidl (2013) make a similar assumption. In Bordalo et al. (2012, 2013a,b, 2015), in principle, the salient attribute of different alternatives can be different. However, with binary choice sets and *homogeneity of degree zero*, as assumed in Bordalo et al. (2013b), the same attribute is salient for both alternatives.

different social groups (Proposition 2) and its consequences for the conflict of preferences in society (Corollary 1). Moreover, the results presented here form the basis for the effect of focusing on the endogenous offer of policies, which we study in Section 4.

Consider an exogenous choice set given by $\mathcal{P} = \{p, q\}$. Proposition 1 shows that focusing voters maintain the same ranking between the two policies in their choice set for any degree of focusing but that their intensity of preferences—that is, how much each voter cares about his preferred policy and, thus, the conflict of preferences between members of any two disagreeing groups—grows in the degree of focusing (that is, increases in δ_i). We present all proofs in Section A of the Appendix.

Proposition 1. Assume $\mathcal{P} = \{p, q\}$. For all social groups $i \in \mathbb{N}$ with $\delta_i > 0$:

- (a) focusing does not change the ranking of policies in voters' preferences, that is, the signs of $V_i(p) V_i(q)$ and $\widetilde{V}_i(p|\mathcal{P}) \widetilde{V}_i(q|\mathcal{P})$ coincide;
- (b) focusing increases the intensity of preferences between policies, that is, if voters in group i focus on an issue, then $\widetilde{V}_i(p|\mathcal{P}) \widetilde{V}_i(q|\mathcal{P}) = c \cdot [V_i(p) V_i(q)]$, where c > 1 is strictly increasing in δ_i .

To understand the intuition behind Proposition 1, consider $p \neq q$ and, without loss of generality, $p_d > q_d$. Policy p gives all voters larger utility from defense spending and smaller utility from education spending than policy q. In this sense, p's relative advantage lies in its larger defense spending, while q's relative advantage lies in its larger education spending. Consider a social group i that receives higher consumption utility from p, the policy with larger defense spending. For voters in this social group, the benefit from larger defense spending granted by p more than compensate its cost in terms of lower education spending. This happens if and only if the range of utility in defense spending—which measures the advantage of p in the consumption utility space—is larger than the range of utility in education spending—which measures the disadvantage of p in the same space. Given our assumption on the determinants of voters' attention, this leads voters in group i to focus on defense. These voters overweigh the relative advantage of p with respect to q, that is, the larger spending in defense, and underweigh its relative disadvantage, that is,

the smaller spending in education. As a consequence, the difference in perceived, or focusweighted, utility between the two policies is larger than the difference in consumption utility, that is, $\tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(q|\mathcal{P}) > V_i(p) - V_i(q)$.

The first part of Proposition 1 implies that distorted focus does not affect social choice when society votes over *exogeneous* binary agendas and no abstention is allowed. However, as we hope to show in the rest of this paper, this does not mean that focusing is not important in politics or collective decision making. In particular, as the second part of the proposition suggests, focusing matters whenever the intensity of preferences affects the likelihood of casting a vote (for example, with costly voting) or the likelihood of voting for a particular candidate (for example, with stochastic choice, or whenever other considerations enter the voters' decision). In turn, this can affect the *endogeneous* offer of policies. We explore this possibility in Sections 4.

Our next result shows that focusing separates the electorate into two contiguous subsets of social groups, or *factions*, with different focus and opposed intensified preferences over policies. Recall groups with higher index have stronger preference for defense.

Proposition 2. Consider any $\mathcal{P} = \{p, q\}$ such that $p_d > q_d$. Assume $\delta_i > 0$ for all $i \in N$. As a result of focusing group i^* exists such that voters in any group $i > i^*$ focus on defense and $\widetilde{V}_i(p|\mathcal{P}) - \widetilde{V}_i(q|\mathcal{P}) > V_i(p) - V_i(q) > 0$, while voters in any group $i < i^*$ focus on education and $\widetilde{V}_i(p|\mathcal{P}) - \widetilde{V}_i(q|\mathcal{P}) < V_i(p) - V_i(q) < 0$.

Proposition 2 implies that focusing polarizes the electorate. Denote by d_i^d the defensefocus bliss point of voters in group i—that is, the unique maximizer of \widetilde{V}_i when voters in
group i focus on defense; and by d_i^e the education-focus bliss point—that is, the unique
maximizer of \widetilde{V}_i when voters in group i focus on education. When $\delta_i \in (0,1)$, we have $d_i^e < d_i^* < d_i^d$, where d_i^d increases and d_i^e decreases with the degree of focusing. As shown
in Proposition 2, a subset of contiguous social groups focus on defense while a subset of
contiguous social groups focus on education. Focusing pushes the perceived bliss points of
the members of these two factions in opposite directions, exacerbating their disagreement.

Corollary 1. Focusing polarizes the electorate: groups with a relative preference for defense focus on defense, while groups with a relative preference for education focus on

education; the distance between the defense-focus bliss points of the faction focusing on defense and the education-focus bliss points of the faction focusing on education is increasing in the degree of focusing.

We have shown that focusing increases the intensity of preferences and creates factions that focus on different attributes. A natural question is whether this heightened intensity displays any pattern across social groups. To answer this question, consider choice set $\mathcal{P} = \{p,q\}$ with two distinct policies, $p = (p_d, W - p_d)$ and $q = (q_d, W - q_d)$, and suppose, without loss of generality, that $p_d > q_d$. We want to investigate how $\frac{\partial}{\partial \delta_i} \left(\tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(q|\mathcal{P}) \right)$ —a term that measures the extent to which focusing intensifies preferences—varies across social groups. By Proposition 1, this term has the same sign as $\tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(q|\mathcal{P})$ and its magnitude equals

$$\theta_{id}(u(p_d) - u(q_d)) + \theta_{ie}(u(W - q_d) - u(W - p_d)).$$
 (5)

Since $u(p_d)-u(q_d) > 0$ and $u(W-q_d)-u(W-p_d) > 0$, this magnitude is increasing in both θ_{id} and θ_{ie} . Thus, focusing has a stronger effect on distorting the perceived preferences of social groups which are more sensitive to either issue (in the sense of having a larger marginal utility from either public good). This sensitivity is independent of a group's relative preference for the two public goods: if, for instance, we double θ_{id} and θ_{ie} , this leaves group i's consumption bliss point intact but doubles the effect of focusing on group i's focus-weighted utility.

4 Electoral Competition with Focusing Voters

4.1 Modeling Electoral Competition

In the previous section, we considered the effect of focus on voters' preferences over an *exogenous* choice set. In this section, we consider the effect of voters' focus on the endogenous supply of policies by political parties or candidates.

In particular, we introduce focusing voters into a classical model of electoral compe-

tition, the probabilistic voting model à la Lindbeck and Weibull (1987). Two identical parties, A and B, simultaneously announce a binding policy, that is, the amount of available resources devoted to each public good. We denote A's policy with $a = (a_d, W - a_d)$ and B's policy with $b = (b_d, W - b_d)$, where $a_d, b_d \in [0, W]$. Voters observe parties' policies, evaluate them with their focus-weighted utility (rather than their consumption utility) and vote as if they are pivotal (or derive expressive utility from voting). The indirect utility voter v in group i receives when voting for each party is:

$$u_{v,i}(A) = \widetilde{V}_i(a|\mathcal{P})$$

$$u_{v,i}(B) = \widetilde{V}_i(b|\mathcal{P}) + \epsilon_{v,i}$$
(6)

where $\mathcal{P} = \{a, b\}$ is voters' endogenous choice set and $\epsilon_{v,i} \sim U[-\frac{1}{2\phi_i}, \frac{1}{2\phi_i}]$ is an individual-specific parameter that measures voter v's individual ideological bias toward party B. A positive value of $\epsilon_{v,i}$ implies that voter v has a bias in favor of party B. The shock to the relative popularity of party B is realized after policies are announced but before the election. Given these assumptions, voter v in group i votes for A if and only if $\widetilde{V}_i(a|\mathcal{P}) > \widetilde{V}_i(b|\mathcal{P}) + \epsilon_{v,i}$.

Parties are purely office-motivated and maximize their vote shares.⁹ From the parties' perspective, the expected share of voters in group i who vote for A is:¹⁰

$$\frac{1}{2} + \phi_i \left[\widetilde{V}_i(a|\mathcal{P}) - \widetilde{V}_i(b|\mathcal{P}) \right]. \tag{7}$$

The two parties objective functions are:

$$\pi_{A}(a|\mathcal{P}) = \frac{1}{2} + \sum_{i \in N} \phi_{i} m_{i} \left[\widetilde{V}_{i}(a|\mathcal{P}) - \widetilde{V}_{i}(b|\mathcal{P}) \right]$$

$$\pi_{B}(b|\mathcal{P}) = 1 - \pi_{A}(a|\mathcal{P}).$$
(8)

⁹All results we present below are robust to parties maximizing the probability of winning.

 $^{^{10}}$ As common in probabilistic voting models, we assume that ϕ 's are small enough to guarantee that vote shares are always interior.

4.2 Benchmark: Endogenous Policies with Rational Voters

In this electoral game, parties simultaneously announce their policies. A pure strategy of party A(B) is a policy $a_d(b_d) \in [0, W]$. The solution concept we adopt is Nash equilibrium. As a benchmark, we first consider fully rational voters, that is, $\delta_i = 0$ for all $i \in N$. In this case, $\widetilde{V}_i(a|\mathcal{P}) = \theta_{id}u(a_d) + \theta_{ie}u(W - a_d)$ only depends on a, not on the entire choice set \mathcal{P} . Similarly, $\widetilde{V}_i(b|\mathcal{P}) = \theta_{id}u(b_d) + \theta_{ie}u(W - b_d)$ only depends on b.

Proposition 3. (a) The utilitarian optimum, p_d^o , is the unique solution to:

$$\sum_{i \in N} m_i \left[\theta_{id} u'(p_d^o) - \theta_{ie} u'(W - p_d^o) \right] = 0. \tag{O_o}$$

(b) Assume $\delta_i = 0$ for all $i \in N$. A Nash equilibrium in pure strategies exists and is unique. The equilibrium policies are (p_d^r, p_d^r) , where p_d^r is the unique solution to:

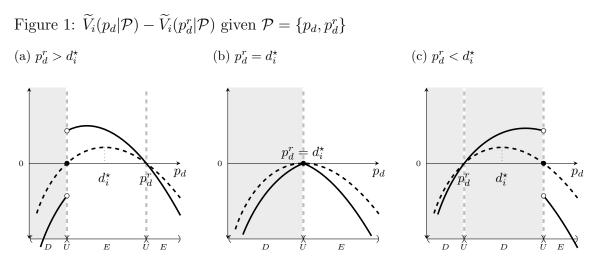
$$\sum_{i \in N} \phi_i m_i \left[\theta_{id} u'(p_d^r) - \theta_{ie} u'(W - p_d^r) \right] = 0. \tag{O_r}$$

Moreover, $p_d^r \in (d_1^{\star}, d_n^{\star})$.

Proposition 3(a) shows that, when voters do not suffer from distorted focus, equilibrium policies maximize a weighted social consumption utility function where the weight on each social group is determined by the population share, m_i , as in the utilitarian optimum, but also by the precision of the popularity shock, ϕ_i . Note that electoral competition leads to policies that are optimal in an utilitarian sense—that is, policies that maximize the sum of voters' utilities—if and only if the distribution of the popularity shock is homogeneous across social groups.

4.3 Endogenous Policies with Focusing Voters

We now introduce focusing voters. In order to convey a sharper intuition, we consider a society composed of two groups, where group 1 has a relative preference for education and group 2 has a relative preference for defense—that is, $d_1^{\star} < d_2^{\star}$. We characterize the



Note: — is $\widetilde{V}_i(p_d|\mathcal{P}) - \widetilde{V}_i(p_d^r|\mathcal{P})$ and ---- is $V_i(p_d) - V_i(p_d^r)$. Given $\mathcal{P} = \{p_d, p_d^r\}$, p_d in the gray area implies focus on defense (D), in the white area implies focus on education (E) and undistorted focus in between (U).

equilibrium for an arbitrary number of social groups in Section B of the Appendix. In the rational benchmark, the equilibrium platforms are (p_d^r, p_d^r) , where $p_d^r \in (d_1^\star, d_2^\star)$ is the unique solution to $\phi_1 m_1 V_1'(p_d) + \phi_2 m_2 V_2'(p_d) = 0$: a marginal deviation by either party results in a gain of votes from one group which is exactly offset by a loss of votes from the other group.

Focusing changes the parties' calculus. Consider a marginal deviation from (p_d^r, p_d^r) to (p_d, p_d^r) . The expected vote share in group i of the party deviating to p_d is proportional to $\widetilde{V}_i(p_d|\{p_d, p_d^r\}) - \widetilde{V}_i(p_d^r|\{p_d, p_d^r\})$. A first, important, implication of our assumptions is that a deviation by a single party changes voters' evaluation of the policies offered by both parties. Formally, a deviation to p_d changes both terms in $\widetilde{V}_i(p_d|\{p_d, p_d^r\}) - \widetilde{V}_i(p_d^r|\{p_d, p_d^r\})$.

Consider first voters in group 1, that is, voters with a lower consumption bliss point. Figure 1a shows that a marginal deviation from $p_d^r > d_1^*$ to p_d implies that voters in group 1, who are now choosing from the set $\{p_d, p_d^r\}$, derive larger consumption utility from the policy with less defense spending and, thus, focus on education. As Lemma B1 shows formally, this means that the derivative of $\widetilde{V}_1(p_d|\{p_d, p_d^r\}) - \widetilde{V}_1(p_d^r|\{p_d, p_d^r\})$ with respect to p_d evaluated at p_d^r equals:

$$(1 - \delta_1)\theta_{1d}u'(p_d^r) - (1 + \delta_1)\theta_{1e}u'(W - p_d^r). \tag{9}$$

At the margin, voters in group 1 overweigh education and underweigh defense relative to their rational counterparts. This gives parties an incentive to run on *lower* platforms (that is, platforms with less defense spending).

At the same time, this incentive is counter-balanced by a push to run on larger platforms (that is, platforms with more defense spending), which results from the focus of voters in group 2. As Figure 1c shows, a marginal deviation from $p_d^r < d_2^*$ to p_d implies that voters in group 2, who are now choosing from the set $\{p_d, p_d^r\}$, derive larger consumption utility from the policy with more defense spending and, thus, focus on defense. This implies that the derivative of $\tilde{V}_2(p_d|\{p_d, p_d^r\}) - \tilde{V}_2(p_d^r|\{p_d, p_d^r\})$ with respect to p_d evaluated at p_d^r equals:

$$(1+\delta_2)\theta_{2d}u'(p_d^r) - (1-\delta_2)\theta_{2e}u'(W-p_d^r). \tag{10}$$

At the margin, voters in group 2 overweigh defense and underweigh education, creating an incentive for parties to propose policies with more defense spending. The equilibrium platforms balance these two incentives, as characterized in Proposition 4.

Proposition 4. Consider n = 2 with $d_1^{\star} < d_2^{\star}$. A Nash equilibrium in pure strategies exists and is unique. Let:

$$\mathcal{O}_{f,2}(p_d) = \phi_1 m_1 \left[(1 - \delta_1) \theta_{1d} u'(p_d) - (1 + \delta_1) \theta_{1e} u'(W - p_d) \right]$$

$$+ \phi_2 m_2 \left[(1 + \delta_2) \theta_{2d} u'(p_d) - (1 - \delta_2) \theta_{2e} u'(W - p_d) \right].$$

$$(\mathcal{O}_{f,2})$$

The equilibrium platforms of the two parties are (p_d^f, p_d^f) , where:

(a) $p_d^f \in (d_1^{\star}, d_2^{\star})$ is the unique solution to $\mathcal{O}_{f,2}(p_d^f) = 0$ if $\mathcal{O}_{f,2}(d_1^{\star}) > 0 > \mathcal{O}_{f,2}(d_2^{\star})$;

(b)
$$p_d^f = d_1^* \text{ if } \mathcal{O}_{f,2}(d_1^*) \le 0;$$

(c)
$$p_d^f = d_2^* \text{ if } \mathcal{O}_{f,2}(d_2^*) \ge 0.$$

Proposition 4 implies that groups that are larger, have a higher precision of the relative popularity shock, and have more distorted focus are more influential in the electoral calculus. Groups with more distorted focus—that is, groups with larger δ_i —perceive

a larger utility differential between any given pair of policies (as noted in Proposition 1 and illustrated in Figure 1) and, thus, are more likely to vote on the basis of the promised allocation of resources and less likely to be swayed by popularity shocks. These comparative statics are summarized in Corollary 2.

Corollary 2. Consider the unique equilibrium policy of the electoral competition game with focusing voters and two groups, p_d^f . If $p_d^f \in (d_1^{\star}, d_2^{\star})$, p_d^f approaches d_i^{\star} when m_i , δ_i or ϕ_i increase for any $i \in \{1, 2\}$.

The effect of m_i and ϕ_i on social group i's ability to steer policy in the preferred direction is a classic result from probabilistic voting models (see, e.g., Persson and Tabellini, 2000, Section 3.4). The effect of selective attention, on the other hand, is novel. While the degree of focusing and the precision of the relative popularity shock both contribute to determine a social group's sensitivity to electoral promises, they are not pure substitutes. In order to disentangle the effect of these two parameters and highlight the peculiar consequences of focusing, it is useful to define $\zeta_i = \phi_i m_i (1 + \delta_i)$ and note that, when $p_d^f \in (d_1^\star, d_2^\star)$, the equilibrium platform solves

$$\zeta_1 \left[\theta_{1d} u'(p_d) - \theta_{1e} u'(W - p_d) - 2\delta_1 \theta_{1d} u'(p_d) \right] +
\zeta_2 \left[\theta_{2d} u'(p_d) - \theta_{2e} u'(W - p_d) + 2\delta_2 \theta_{2e} u'(W - p_d) \right] = 0.$$
(11)

Similarly to a larger ϕ_i , a larger δ_i increases the weight group *i*'s bliss point receives in the social welfare function maximized by candidates (that is, it increases ζ_i). However, in addition to this, δ_i changes a social group's perceived bliss point, with a larger δ_i leading to a larger distortion in perception. Since the perceived bliss point is more extreme than the consumption bliss point, a larger δ_i increases the group's ability to pull resources towards the preferred public good. As a consequence, when voters suffer from focusing, the weights in candidates' social welfare function are generically different from those used by a social planner (that is, population shares) even when ϕ_i 's and δ_i 's are homogeneous, while they coincide with population shares when ϕ_i 's are homogeneous and voters do not suffer from focusing. In this sense, focusing provides a micro-foundation for social groups' heterogeneous sensitivity to candidates' electoral promises: what groups

are more responsive to electoral promises is a function of the distribution of preferences in the population rather than an ad hoc assumption (e.g., exogenous heterogeneity in the distribution of the popularity shock).

In order to highlight the inefficiencies that are solely due to focusing, consider the case where $\phi_1 = \phi_2$. As noted in Proposition 3, in this case, the equilibrium policy that emerges from competition with rational voters coincides with the utilitarian optimum, $p_d^r = p_d^o$. It is interesting to compare the equilibrium policy with focusing voters, p_d^f , to p_d^r . In general, we can have both $p_d^f > p_d^r$ and $p_d^f < p_d^r$. Corollary 3 characterizes the direction of the inefficiency generated by focusing for two social groups, homogeneous distribution of the popularity shock and homogeneous degree of focusing.¹¹

Corollary 3. Assume n = 2, $\phi_1 = \phi_2$, and $\delta_1 = \delta_2$. The equilibrium policy with focusing voters is generically inefficient and entails either excessive spending in the issue preferred by the larger group (tyranny of the majority) or excessive spending in the issue preferred by the smaller group (special interest capture). We have:

(a)
$$p_d^f = p_d^r$$
 if and only if $\frac{m_2}{m_1} = \frac{\theta_{1d}u'(p_d^r) + \theta_{1e}u'(W - p_d^r)}{\theta_{2d}u'(p_d^r) + \theta_{2e}u'(W - p_d^r)}$;

(b)
$$p_d^f > p_d^r$$
 if and only if $\frac{m_2}{m_1} > \frac{\theta_{1d}u'(p_d^r) + \theta_{1e}u'(W - p_d^r)}{\theta_{2d}u'(p_d^r) + \theta_{2e}u'(W - p_d^r)}$;

- (c) with homogeneous utility from defense—that is, $\theta_{id} = \theta_d$ for all $i \in N$ — $p_d^f > p_d^r$ if and only if $\frac{m_2}{m_1} > \frac{\theta_d u'(p_d^r) + \theta_{1e} u'(W p_d^r)}{\theta_d u'(p_d^r) + \theta_{2e} u'(W p_d^r)} > 1$;
- (d) with homogeneous utility from education—that is, $\theta_{ie} = \theta_e$ for all $i \in N$ — $p_d^f < p_d^r$ if and only if $\frac{m_1}{m_2} > \frac{\theta_{2d}u'(p_d^r) + \theta_eu'(W p_d^r)}{\theta_{1d}u'(p_d^r) + \theta_eu'(W p_d^r)} > 1$.

The first two statements of Corollary 3 imply that equilibrium policies are generically inefficient. Two factors determine what social group politicians inefficiently cater to: the groups' sizes and the groups' sensitivity to policy changes on the two issues. This latter factor can dominate size and make minority groups more important in the electoral calculus. Our model highlights that policy capture from special interests can be a consequence of the psychology of attention without relying on the coordination and costly collective

¹¹We omit the formal argument, which subtracts (\mathcal{O}_r) from $(\mathcal{O}_{f,2})$, both evaluated at p_d^r , and uses the fact that $\mathcal{O}_{f,2}(p_d)$ is strictly decreasing in p_d by strict concavity of u.

action necessary for lobbying. When attention and, in turn, preferences are influenced by the choice environment, a small group which is really sensitive to changes on either issue can be overly influential in obtaining what it desires.

Consider now a community where a given level of spending in issue k delivers the same utility to all social groups but citizens have heterogeneous preferences for issue $j \neq k$ (that is, citizens in different social groups derive a different level of utility from a given level of spending in issue j). In this case, policy might be captured by a minority which prefers issue j (and is prone to focus on this issue) but cannot be captured by a minority which prefers issue k (and is prone to focus on this issue). The third and fourth statements of Corollary 3 imply that the equilibrium policy allocates an excessive amount of resources to issue k if and only if the social group which prefers issue k is a sufficiently large majority; when the two social groups have similar sizes (or the social group which prefers issue k is smaller), the equilibrium policy under-provides good k.

In sum, when the electorate suffers from selective attention, we can have both special interest capture (that is, over-provision of the public good preferred by the minority) and tyranny of the majority (that is, over-provision of the public good preferred by the majority). When spending on one public good is a common value issue and spending on the other public good is a divisive issue, the government budget is more likely to be inefficiently channelled towards the divisive issue (that is, there is a larger range of relative population sizes for which this happens).

Proposition 4 also shows that the equilibrium policy can coincide with the consumption bliss point of one of the two groups, something that cannot happen with rational voters. The intuition behind this result lies in the polarization of preferences induced by focusing (see Proposition 2 and Corollary 1). As discussed above, a marginal deviation from a pair of identical policies makes voters in group 1 focus on education and voters in group 2 focus on defense. Therefore, the electoral calculus of parties facing two groups of focusing voters is similar to the electoral calculus of parties facing two groups of rational but more strongly opposed voters, one with ideal policy $d_1^e < d_1^*$ and one with ideal policy $d_2^d > d_2^*$. For this reason, focusing might lead to extreme policies. When the equilibrium

policy coincides with the consumption bliss point of one of the groups, it is *locally un*responsive to the model parameters, that is, it remains constant in some regions of the parameter space.¹² This is another feature of equilibrium policies with focusing voters which is not shared with the case of rational voters.

Corollary 4. Electoral competition with focusing voters might lead to extreme policies.

The equilibrium policy might coincide with the consumption bliss point of a group of voters and, thus, be locally unresponsive to parameter changes.

Finally, it is worth discussing the fact that, in the equilibrium of our probabilistic voting model with purely office-motivated parties, policy platforms converge. While this prediction finds little empirical support in real world elections, a number of plausible extensions of our base model—for example, adding policy motivation to parties' preferences—would easily induce divergent policy platforms (see Grofman 2004 for a survey of modeling assumptions known to generate this result). These extensions would not change the way voters' distorted attention influences parties' electoral calculus (with respect to the benchmark with rational voters) and, thus, our qualitative results.

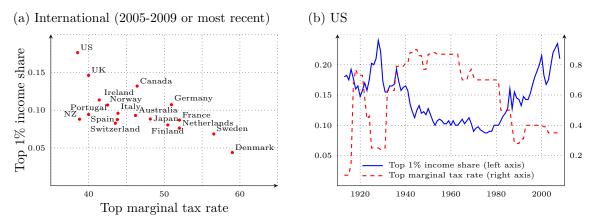
5 Application: Fiscal Policy

In the last 30 years, the U.S. (as well as other developed economies) have experienced a rapid and sustained increase in the degree of income inequality (see Figure 2b). Contrary to the predictions of standard political economy models, this trend has not been accompanied by an increased demand for redistribution (see Figure 3) or by more redistributive policies (see Figure 2b). To the contrary, the data points to an inverse correlation between these time series. We observe a similar inverse correlation between income inequality and redistribution in a cross-country perspective (see Figure 2a).

What is the impact of voters' distorted focus on voters' preferences and parties' political offer regarding taxation and public goods provision? Can selective attention help us explain the puzzling empirical patterns from Figures 2 and 3?

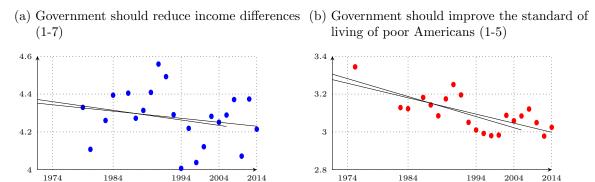
¹²This is due to a kink in the focus-weighted utility of this social group, as shown in Figure 1b.

Figure 2: Top 1% income share and top marginal tax rate



Note: Data courtesy of Piketty et al. (2014) (see their paper for original sources). Income excludes government transfers and is before individual taxes.

Figure 3: Preferences for redistribution in General Social Survey (GSS)



Note: GSS obtained from http://gss.norc.org/. Variables rescaled so that larger values correspond to stronger support for redistribution. Shorter trend ends in 2006. Left panel: Average of eqwlth variable. Both trends insignificant. Right panel: average of helppoor variable. Both trends significant at 1%. See Ashok et al. (2015) for a thorough analysis of the data.

In order to answer these questions, we introduce a basic model of fiscal policy à la Meltzer and Richard (1981). A public good, $g \in \mathbb{R}_+$, is financed by a linear income tax, $\tau \geq 0$. Society is composed of two groups of voters, R for Rich and P for Poor, with different income: $y_R > y_P \geq 0$. The measure of voters in group $i \in \{R, P\}$ is $m_i \in (0, 1)$. The average income in society is $\overline{y} = m_R y_R + m_P y_P$. Given public good g and tax τ , the consumption utility of voters in group i is:

$$(1-\tau)y_i + u(g) \tag{12}$$

where u(g) is the function mapping the level of public good provision into its benefits

and satisfies the assumptions from Section 2. The government budget is balanced—that is, $g = \tau \overline{y}$ —and, thus, the indirect consumption utility of voters in group i from public good level g is:

$$V_i(g) = \left(y_i - \frac{y_i}{\overline{y}}g\right) + u(g). \tag{13}$$

With respect to the general model we introduced above, a citizen's indirect utility from policy g is composed of two attributes: the citizen's after tax income, $\left(y_i - \frac{y_i}{\overline{y}}g\right)$, and the utility the citizen derives from public good provision, u(g). Note that the policy g gives homogeneous benefits to the two groups, u(g), while its costs, in terms of foregone income, are heterogeneous and proportional to a group's relative income. The latter implies that a group's consumption bliss point depends negatively on its relative income:

$$g_i^{\star} = u'^{-1} \left(\frac{y_i}{\overline{y}} \right). \tag{14}$$

As a benchmark, we first consider the utilitarian optimum and electoral competition between two office-motivated parties facing rational voters.

Proposition 5. (a) The utilitarian optimum, g^o , is the unique solution to:

$$m_R \left[u'(g^o) - \frac{y_R}{\overline{y}} \right] + (1 - m_R) \left[u'(g^o) - \frac{y_P}{\overline{y}} \right] = 0. \tag{A_o}$$

(b) Assume $\delta_R = \delta_P = 0$. A Nash equilibrium in pure strategies exists and is unique. Equilibrium policies are (g^r, g^r) , where g^r is the unique solution to:

$$\phi_R m_R \left[u'(g^r) - \frac{y_R}{\overline{y}} \right] + \phi_P (1 - m_R) \left[u'(g^r) - \frac{y_P}{\overline{y}} \right] = 0. \tag{A_r}$$

(c) Moreover, the equilibrium level of public good provision, g^r , is strictly decreasing in y_R if and only if $\phi_R > \phi_P$ and is insensitive to y_R if and only if $\phi_R = \phi_P$.

The equilibrium policy with rational voters balances the weighted average of marginal benefits against the weighted average of marginal costs where the weights are given by the population shares and the precisions of the relative popularity shock. When the two social groups only differ in their income and size but relative popularity shocks are drawn from the same distribution, the equilibrium policy is efficient in the sense of maximizing the sum of citizens' utilities. Moreover, since the average marginal costs are invariant to the income distribution as well as to population shares, in this case, these two variables have no impact on the equilibrium level of public good provision. Relative incomes affect the equilibrium level of redistribution only when the two groups are exogenously assumed to have a different sensitivity to the promised economic policies. Larger income inequality leads to lower redistribution when rich voters are more likely than poor voters to reward at the voting booth politicians offering a more favorable policy. While this is consistent with the stylized facts discussed above, it is the consequence of an ad hoc assumption and the model does not explain why or clarifies when a group of voters should be more or less sensitive than another to electoral promises.^{13,14}

The comparative statics, however, are different if we introduce focusing voters. When voters suffers from selective attention, the equilibrium level of redistribution is decreasing in y_R even when groups only differ in their relative income.

Proposition 6. Assume $\delta_R = \delta_P = \delta > 0$. A Nash equilibrium in pure strategies exists and is unique. Let:

$$\mathcal{A}_f(g) = \phi_R m_R \left[(1 - \delta) u'(g) - (1 + \delta) \frac{y_R}{\overline{y}} \right]$$

$$+ \phi_P (1 - m_R) \left[(1 + \delta) u'(g) - (1 - \delta) \frac{y_P}{\overline{y}} \right].$$

$$(\mathcal{A}_f)$$

The equilibrium policies are (g^f, g^f) , where:

 $^{^{13}}$ Indeed, some empirical evidence suggests low-income voters might be *more* sensitive to electoral promises than high-income voters (that is, $\phi_P > \phi_R$). For example, according to the "Political Polarization in the American Public" survey conducted by the Pew Research Center in 2014, the share of high-income voters who express consistently conservative or consistently liberal opinions is more than twice as much as the same share among poor-income voters (29% versus 13%). Moreover, in a survey conducted before the 2004 US Presidential election, Dimock, Clark, and Horowitz (2008) find that swing voters (that is, voters potentially open to persuasion) had lower incomes than voters committed to vote for a presidential candidate.

¹⁴Note that the stylized facts from Figures 2 and 3 are also inconsistent with another workhorse model of electoral competition, the median voter model (Downs, 1957). The median voter model obtains as a special case of the probabilistic voting model when $\epsilon_v = 0$ for all voters. In this case, the equilibrium policy is the consumption bliss point of the median voters who, with two social groups, belongs to the larger group. If we assume that P voters are the majority and R voters are an elite, that is, $m_R < 1/2$, the equilibrium policy coincides with g_P^* , which is increasing with income inequality, that is, with larger y_R or smaller y_P . In short, in the median voter model, larger income inequality leads to larger redistribution.

(a)
$$g^f \in (g_R^{\star}, g_P^{\star})$$
 is the unique solution to $\mathcal{A}_f(g^f) = 0$ if $\mathcal{A}_f(g_R^{\star}) > 0 > \mathcal{A}_f(g_P^{\star})$;

(b)
$$g^f = g_R^* \text{ if } A_f(g_R^*) \le 0;$$

(c)
$$g^f = g_P^*$$
 if $\mathcal{A}_f(g_P^*) \ge 0$.

Moreover, in case (a), the equilibrium level of public good provision, g^f , is strictly decreasing in y_R if and only if $\phi_R > \phi_P \frac{1-\delta}{1+\delta}$, which is always satisfied if $\phi_R = \phi_P$.¹⁵

The equilibrium characterization and its uniqueness are a direct consequence of Proposition 4, adapted to the application at hand. At the margin, voters in group R—who prefer less redistribution than g^f —focus on after-tax income; and voters in group P—who prefer more redistribution than g^f —focus on public good provision. In their quest for electoral support, parties generically prefer to offer a policy different than the efficient level of redistribution. Proposition 6 shows that, when it lies between the groups' bliss points, the equilibrium level of public good is decreasing with income inequality as long as poor voters' sensitivity to electoral promises is not too large relative to the sensitivity of rich voters. This condition is always satisfied when voters from the two social groups have the same propensity to reward politicians for favorable policies and the range of values of ϕ_R, ϕ_P for which this condition is satisfied grows with δ . To understand the intuition behind this result, consider the condition that defines g^f in Proposition 6: in this expression, income inequality only affects marginal after-tax incomes. Consider an increase in y_R . With rational voters, an increase in y_R by dy_R increases the marginal cost of redistribution of R voters by $\frac{y_P m_P}{\bar{y}^2} dy_R$ and decreases the marginal cost of redistribution of P voters by $\frac{y_P m_R}{\overline{v}^2} dy_R$. In the politicians' calculus, the former increase and the latter decrease are weighted, respectively, by $\phi_R m_R$ and $\phi_P m_P$. Thus, when $\phi_P = \phi_R$, the two effects perfectly offset each other, making g^r invariant to the income distribution. With focusing voters, a higher y_R still increases the marginal cost of redistribution of R voters and decreases the marginal cost of redistribution of P voters. However, since P voters focus on public good provision and neglect after-tax income, they underweigh

¹⁵When $g^f \in \{g_R^{\star}, g_P^{\star}\}$, the effect of income inequality on g^f is given by its impact on the consumption bliss points of the two groups. In this case, g^f can both decrease and increase with income inequality.

¹⁶When $\delta = 0$, the condition reduces to $\phi_R > \phi_P$ as in Proposition 5(c). On the other hand, as δ approaches 1, the condition is satisfied for any $\phi_R, \phi_P > 0$.

the decrease in their marginal cost of redistribution induced by an upward shock to income inequality. Conversely, since R voters focus on after-tax income and neglect public good provision, they overweigh the increase in their marginal cost of redistribution due to the same shock. An increase in y_R (or, analogously, a decrease in y_P), thus, leads to an increase in the average perceived marginal cost from redistribution (without affecting the perceived marginal benefit from public good provision) and to a decrease in the demand for redistribution. In sum, an increase in income inequality amplifies rich voters' marginal sensitivity to policies more than poor voters' and, in turn, rich voters become more influential regardless of their relative size. The psychology of voters' attention can, thus, explain why increased income inequality is associated with constant or decreasing demand for redistribution and, hence, with constant or decreasing implemented levels of redistribution.¹⁷

6 Conclusion

How voters allocate their attention is fundamental for understanding political preferences and public policies. Cognitive psychology has pointed to two complementary mechanisms: a goal-driven and ex-ante allocation of attention that is driven by preferences (also called "top-down attention" or "rational inattention") and a stimulus-driven and ex-post allocation of attention that shapes preferences (also called "bottom-up attention"). While the existing literature in political economy has centered on the former, this is the first paper to explore the latter.

We introduce bottom-up attention in a model of electoral competition by assuming that, in forming their perception of policies, voters' attention is attracted by the attribute in which their options differ more and that, in turn, they weigh disproportionately the attribute they focus on. We show that this selective focus leads to a polarized electorate;

¹⁷The main alternative explanations for the observed correlations (or lack thereof) between income inequality and redistribution are stronger political participation or lobbying by the wealthy (Benabou 2000), the prospect of upward mobility (Benabou and Ok 2001), other-regarding preferences (Galasso 2003), and changes in patterns of social identity (Shayo 2009). Most of these explanations attenuate the positive relationship between redistribution and income inequality predicted by Meltzer and Richard (1981), rather than reversing it. See Borck (2007) for a survey.

that politicians facing focusing voters offer inefficient policies, with resources more likely to be channelled to divisive issues; that social groups that have more distorted focus are more influential; and that selective focus can contribute to explain puzzling empirical patterns, as the inverse correlation between income inequality and redistribution.

Our simple framework can deliver many other interesting results that we have not explored in this paper: for example, voters with distorted focus have stronger preferences and this makes them more likely to turn out to vote, make financial contributions, actively participate to a candidate's campaign or engage in other forms of collective action. We believe that there are many possible directions for the next steps in this research. Regarding the model we introduced, it would be interesting to introduce heterogeneous parties (for example, policy motivated parties) or allow policies to have uncorrelated attributes (for example, electoral platforms which offer an ideological position on—rather than the amount of available resources to be devoted to—different issues). While we explored the consequences of focusing on electoral competition, incorporating the psychology of voters' attention in models of campaign rhetoric and agenda setting is likely to generate novel insights. For example, in a monopolistic agenda setting model à la Romer and Rosenthal (1978, 1979), the agenda setter could propose multiple reforms of the status quo in order to affect the focus of its bargaining counterpart (e.g., the median voter or a veto holder) and expand the set of acceptable reforms to his advantage. More generally, there are many exciting open questions, as what exact features of the political environment trigger voters' attention and how focusing interacts with the conscious research for information by poorly informed voters.

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A Proofs

Proof of Proposition 1

Fix $\mathcal{P} = \{p, q\}$ and $i \in N$. We have $p = (p_d, W - p_d)$ with $p_d \in [0, W]$ and $q = (q_d, W - q_d)$ with $q_d \in [0, W]$. Suppose, without loss of generality, that $p_d \geq q_d$. Because $p_d \geq q_d$, we have $\Delta_{id}(\mathcal{P}) = \theta_{id}(u(p_d) - u(q_d)) \geq 0$ and $\Delta_{ie}(\mathcal{P}) = \theta_{ie}(u(W - q_d) - u(W - q_d))$

 $(p_d) \geq 0$, and hence $V_i(p) - V_i(q) = \Delta_{id}(\mathcal{P}) - \Delta_{ie}(\mathcal{P})$. Moreover, from (4), we have

$$\widetilde{V}_{i}(p|\mathcal{P}) - \widetilde{V}_{i}(q|\mathcal{P}) = V_{i}(p) - V_{i}(q)$$

$$+ \begin{cases}
+\delta_{i} \left[\Delta_{id}(\mathcal{P}) + \Delta_{ie}(\mathcal{P})\right] & \text{if } \Delta_{id}(\mathcal{P}) > \Delta_{ie}(\mathcal{P}) \\
-\delta_{i} \left[\Delta_{id}(\mathcal{P}) + \Delta_{ie}(\mathcal{P})\right] & \text{if } \Delta_{id}(\mathcal{P}) < \Delta_{ie}(\mathcal{P}) \\
0 & \text{if } \Delta_{id}(\mathcal{P}) = \Delta_{ie}(\mathcal{P}).
\end{cases} (A1)$$

We need to consider three cases. If $V_i(p) = V_i(q)$, then $\Delta_{id}(\mathcal{P}) = \Delta_{ie}(\mathcal{P})$ and part (a) follows from (A1) and part (b) does not apply. If $V_i(p) > V_i(q)$, then $\Delta_{id}(\mathcal{P}) > \Delta_{ie}(\mathcal{P})$ and both parts follow from (A1). If $V_i(p) < V_i(q)$, then $\Delta_{id}(\mathcal{P}) < \Delta_{ie}(\mathcal{P})$ and both parts follow from (A1).

Proof of Proposition 2

Fix $\mathcal{P} = \{p,q\}$ such that $p_d > q_d$. Suppose that $\delta_i > 0$ for all $i \in N$. For any $i \in N$, $p_d > q_d$ implies that $\Delta_{id}(\mathcal{P}) = \theta_{id}(u(p_d) - u(q_d))$ and $\Delta_{ie}(\mathcal{P}) = \theta_{id}(u(W - q_d) - u(W - p_d))$, and hence $V_i(p) - V_i(q) = \Delta_{id}(\mathcal{P}) - \Delta_{ie}(\mathcal{P})$. Thus, if $V_i(p) - V_i(q) > 0$ for some $i \in N$, then voters in group i focus on defense and $\widetilde{V}_i(p|\mathcal{P}) - \widetilde{V}_i(q|\mathcal{P}) > V_i(p) - V_i(q)$ by Proposition 1, and if $V_i(p) - V_i(q) < 0$ for some $i \in N$, then voters in group i focus on education and $\widetilde{V}_i(p|\mathcal{P}) - \widetilde{V}_i(q|\mathcal{P}) < V_i(p) - V_i(q)$ by Proposition 1. To see that group i^* exists, note that for any $i \in N$, we have $\frac{V_i(p) - V_i(q)}{\theta_{ie}} = \frac{\theta_{id}}{\theta_{ie}}(u(p_d) - u(q_d)) + (u(W - p_d) - u(W - q_d))$. Because $p_d > q_d$, we have $u(p_d) - u(q_d) > 0$ and $u(W - p_d) - u(W - q_d) < 0$. Thus $V_i(p) - V_i(q) \geq 0$ for some $i \in N$ implies that $V_j(p) - V_j(q) > 0$ for any j > i and $V_i(p) - V_i(q) \leq 0$ for some $i \in N$ implies that $V_j(p) - V_j(q) < 0$ for any j < i.

Proof of Proposition 3

In Appendix B, where we study electoral competition model that includes the models from Sections 4 and 5 as special cases, we state and prove Propositions B1 and B3. The former proposition implies equilibrium existence and uniqueness from part (b) and the latter proposition implies the remaining claims. Translating the model from Appendix

B to the model in Section 4 entails a straightforward change in notation where $b_i(x)$ becomes $\theta_{id}u(x)$ and $c_i(x)$ becomes $\theta_{ie}u(W-x)$.

Proof of Proposition 4

Equilibrium existence and uniqueness is a consequence of Proposition B1. The remaining claims are a consequence of Proposition B2. Both of these propositions are stated and proven in Appendix B. Translating the model from Appendix B to the model in Section 4 entails a change in notation, as described in the proof of Proposition 3.

Proof of Proposition 5

Equilibrium existence and uniqueness in part (b) is a consequence of Proposition B1. The remaining claims in parts (a) and (b) are a consequence of Proposition B3. Both of these propositions are stated and proven in Appendix B. Translating the model from Appendix B to the model in Section 5 entails a straightforward change in notation where $b_i(x)$ becomes u(x), $c_i(x)$ becomes $-\frac{y_i}{\bar{y}}x$, group 1 corresponds to group R and group 2 corresponds to group P.

Part (c), the comparative static with respect to y_R , follows by the implicit function theorem, which implies that the sign of $\frac{\partial g^r}{\partial y_R}$ is determined by the sign of $\frac{\partial}{\partial y_R} [-\phi_R m_R \frac{y_R}{\overline{y}} - \phi_R (1-m_R) \frac{y_P}{\overline{y}}] = (\phi_P - \phi_R) \frac{m_R (1-m_R) y_P}{\overline{y}^2}$.

Proof of Proposition 6

Equilibrium existence and uniqueness is a consequence of Proposition B1. The remaining claims, except for the comparative static, are a consequence of Proposition B2. Both of these propositions are stated and proven in Appendix B. Translating the model from Appendix B to the model in Section 5 entails a change in notation, as described in the proof of Proposition 5. The comparative static with respect to y_R follows by the implicit function theorem, which implies that the sign of $\frac{\partial g^f}{\partial y_R}$ is determined by the sign of $\frac{\partial}{\partial y_R}[-\phi_R m_R(1+\delta)\frac{y_R}{\bar{y}}-\phi_P(1-m_R)(1-\delta)\frac{y_P}{\bar{y}}] = (\phi_P(1-\delta)-\phi_R(1+\delta))\frac{m_R(1-m_R)y_P}{\bar{y}^2}$. \square

B Electoral Equilibrium, $n \ge 2$

This section provides characterization of the electoral equilibrium with n social groups for a model that includes both the general model from Section 4 and the fiscal policy application from Section 5 as special cases.

Let $X = [0, \bar{x}]$ for some $\bar{x} > 0$ be the policy space and $X^o = (0, \bar{x})$ its interior. Let \mathcal{X} be all nonempty finite subsets of X.

The consumption utility of voter $i \in N$ from policy $x \in X$ is

$$U_i(x) = b_i(x) + c_i(x). \tag{B1}$$

Suppose that, for any $i \in N$, $b_i : X \to \mathbb{R}$ is (i) continuous and (ii) strictly increasing; and $c_i : X \to \mathbb{R}$ is (i) continuous and (ii) strictly decreasing. Moreover, suppose that, for any $i \in N$, $U_i : X \to \mathbb{R}$ is (i) strictly concave, (ii) twice continuously differentiable; and (iii) admits a unique maximizer, $x_i \in X^o$. Suppose no two groups $i, j \in N$ with $x_i = x_j$ exist and index groups such that $x_i < x_{i+1}$ for $i \in N \setminus \{n\}$.

Given a choice set $\mathcal{P} \subseteq \mathcal{X}$, let

$$\Delta_{ib}(\mathcal{P}) = \max_{x \in \mathcal{P}} b_i(x) - \min_{x \in \mathcal{P}} b_i(x)$$

$$\Delta_{ic}(\mathcal{P}) = \max_{x \in \mathcal{P}} c_i(x) - \min_{x \in \mathcal{P}} c_i(x)$$
(B2)

be the utility ranges of voters in group $i \in N$ from choice set \mathcal{P} for the first and the second issue, respectively. Because b_i is strictly increasing $\Delta_{ib}(\mathcal{P}) = b_i(\max \mathcal{P}) - b_i(\min \mathcal{P})$ and because c_i is strictly decreasing $\Delta_{ic}(\mathcal{P}) = c_i(\min \mathcal{P}) - c_i(\max \mathcal{P})$. Note that for any $i \in N$ and $\mathcal{P} \subseteq \mathcal{X}$, we have $\Delta_{ib}(\mathcal{P}) - \Delta_{ic}(\mathcal{P}) = U_i(\max \mathcal{P}) - U_i(\min \mathcal{P})$.

The focus weighted utility of $i \in N$ from $x \in X$ given choice set $P \in \mathcal{X}$ is

$$\widetilde{U}_{i}(x|\mathcal{P}) = \begin{cases}
(1+\delta_{i})b_{i}(x) + (1-\delta_{i})c_{i}(x) & \text{if } \Delta_{ib}(\mathcal{P}) > \Delta_{ic}(\mathcal{P}) \\
(1-\delta_{i})b_{i}(x) + (1+\delta_{i})c_{i}(x) & \text{if } \Delta_{ib}(\mathcal{P}) < \Delta_{ic}(\mathcal{P}) \\
b_{i}(x) + c_{i}(x) & \text{if } \Delta_{ib}(\mathcal{P}) = \Delta_{ic}(\mathcal{P})
\end{cases}$$
(B3)

where $\delta \in [0, 1)$.

The electoral competition proceeds as follows. First, parties simultaneously announce $a, b \in X$. Second, each voter v in group i observes a and b, draws her ideological shock $\epsilon_{v,i}$, compares $\widetilde{U}_i(a|\{a,b\})$ with $\widetilde{U}_i(b|\{a,b\}) + \varepsilon_{v,i}$ and casts her vote for the party that provides her with higher utility. The objective function of party A and B is, respectively,

$$\pi_{A}(a|\{a,b\}) = \frac{1}{2} + \sum_{i \in N} \phi_{i} m_{i} \left[\widetilde{U}_{i}(a|\{a,b\}) - \widetilde{U}_{i}(b|\{a,b\}) \right]$$

$$\pi_{B}(b|\{a,b\}) = \frac{1}{2} + \sum_{i \in N} \phi_{i} m_{i} \left[\widetilde{U}_{i}(b|\{a,b\}) - \widetilde{U}_{i}(a|\{a,b\}) \right].$$
(B4)

We use the following notation throughout. First, for any $i \in N$ and $x \in X$, $L_i(x) = \{z \in X | z < x_i, U_i(z) = U_i(x)\}$, $H_i(x) = \{z \in X | z > x_i, U_i(z) = U_i(x)\}$, and define

$$e_{i}(x) = \begin{cases} -1 & \text{if } x < x_{i}, H_{i}(x) = \emptyset \\ \in H_{i}(x) & \text{if } x < x_{i}, H_{i}(x) \neq \emptyset \end{cases}$$

$$x_{i} & \text{if } x = x_{i}$$

$$\in L_{i}(x) & \text{if } x > x_{i}, L_{i}(x) \neq \emptyset$$

$$\bar{x} + 1 & \text{if } x > x_{i}, L_{i}(x) = \emptyset.$$
(B5)

The construction of e_i searches for the policy p located on the opposite side of x_i relative to $x \in X$ such that $U_i(p) = U_i(x)$. When such a policy exists, it is unique and becomes $e_i(x)$. When such a policy does not exist, then either $x = x_i$, in which case $e_i(x) = x_i$, or $x \neq x_i$, in which case $e_i(x) \notin X$. Note that, for any $i \in N$ and $x \in X$, we have $e_i(x) > x_i$ when $x < x_i$ and $e_i(x) < x_i$ when $x > x_i$. Moreover, for any $i \in N$ and $x \in X$ such that $e_i(x) \in X$, we have $\widetilde{U}_i(x|\{x,e_i(x)\}) - \widetilde{U}_i(e_i(x)|\{x,e_i(x)\}) = 0$. This follows because $U_i(x) = U_i(e_i(x))$ when $e_i(x) \in X$, which implies that voters in group i have undistorted focus given a choice set $\{x,e_i(x)\}$. Second, for any $i \in N$ and $x,x' \in X$, let $\widetilde{D}_i(x|\{x,x'\}) = \widetilde{U}_i(x|\{x,x'\}) - \widetilde{U}_i(x'|\{x,x'\})$. The derivative of \widetilde{D}_i with respect to x is $\widetilde{D}_i'(x|\{x,x'\})$. Note that x affects $\widetilde{D}_i(x|\{x,x'\})$ both directly and indirectly through the choice set. Third, for a real valued function f, denote by f'^- and f'^+ the left and right derivative of f respectively.

Proposition B1. Let x^* be the unique solution to

$$\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^{-}(x | \{x, x\}) \ge 0 \qquad \sum_{i \in N} \phi_i m_i \widetilde{D}_i'^{+}(x | \{x, x\}) \le 0. \tag{O}$$

A Nash equilibrium in pure strategies exists and is unique. The equilibrium platforms of the two parties are (x^*, x^*) . Moreover, $x^* \in [x_1, x_n]$.

Proof. The proof of Proposition B1 relies on Lemmas B3, B4, and B5. These lemmas rely on Lemmas B1 and B2. We state and prove all the lemmas first.

Lemma B1. For all $i \in N$ and $x, x' \in X$,

- 1. if $x' = x_i$, voters in group i focus on the first issue when $x < x_i$ and focus on the second issue when $x > x_i$; $\widetilde{U}_i(x|\{x,x'\}) \widetilde{U}_i(x'|\{x,x'\})$ is continuous in x and is differentiable in x except at $x = x_i$;
- 2. if $x' < x_i$, voters in group i focus on the first issue when $x \in [0, x') \cup (x', e_i(x'))$ and focus on the second issue when $x > e_i(x')$; $\widetilde{U}_i(x|\{x,x'\}) \widetilde{U}_i(x'|\{x,x'\})$ is continuous and differentiable in x except at $x = e_i(x')$ and

$$\lim_{x \to e_i(x')^-} \widetilde{U}_i(x|\{x, x'\}) - \widetilde{U}_i(x'|\{x, x'\}) \ge 0 \text{ when } e_i(x) \in X$$
$$\lim_{x \to e_i(x')^+} \widetilde{U}_i(x|\{x, x'\}) - \widetilde{U}_i(x'|\{x, x'\}) \le 0 \text{ when } e_i(x) \in X^o;$$

3. if $x' > x_i$, voters in group i focus on the first issue when $x < e_i(x')$ and focus on the second issue when $x \in (e_i(x'), x') \cup (x', \bar{x}]$; $\widetilde{U}_i(x|\{x, x'\}) - \widetilde{U}_i(x'|\{x, x'\})$ is continuous and differentiable in x except at $x = e_i(x')$ and

$$\lim_{x \to e_i(x')^-} \widetilde{U}_i(x | \{x, x'\}) - \widetilde{U}_i(x' | \{x, x'\}) \le 0 \text{ when } e_i(x') \in X^o$$
$$\lim_{x \to e_i(x')^+} \widetilde{U}_i(x | \{x, x'\}) - \widetilde{U}_i(x' | \{x, x'\}) \ge 0 \text{ when } e_i(x') \in X;$$

4. $\widetilde{D}'_i(x|\{x,x'\})$ equals

$$(1 + \delta_i)b'_i(x) + (1 - \delta_i)c'_i(x) \text{ if } x < z$$
$$(1 - \delta_i)b'_i(x) + (1 + \delta_i)c'_i(x) \text{ if } x > z$$

where $z = x_i$ if $x' = x_i$ and $z = e_i(x')$ if $x' \neq x_i$;

5. we have

$$\widetilde{D}_{i}^{\prime-}(x_{i}|\{x_{i},x_{i}\}) = (1+\delta_{i})b_{i}^{\prime}(x_{i}) + (1-\delta_{i})c_{i}^{\prime}(x_{i})$$

$$\widetilde{D}_{i}^{\prime+}(x_{i}|\{x_{i},x_{i}\}) = (1-\delta_{i})b_{i}^{\prime}(x_{i}) + (1+\delta_{i})c_{i}^{\prime}(x_{i}).$$

Proof. Fix any $i \in N$ and $x, x' \in X$. Note that because $\Delta_{ib}(\{x, x'\}) - \Delta_{ic}(\{x, x'\}) = U_i(\max\{x, x'\}) - U_i(\min\{x, x'\})$, voters in group i focus on the first and second issue when $U_i(\max\{x, x'\}) > U_i(\min\{x, x'\})$ and $U_i(\max\{x, x'\}) < U_i(\min\{x, x'\})$ respectively.

Part 1: Because $x' = x_i$, $U_i(x) < U_i(x')$ if $x \neq x'$. Hence voters in group i focus on the first issue when x < x', focus on the second issue when x > x' and have have undistorted focus when $x = x_i$. $\widetilde{D}_i(x|\{x,x'\})$ thus equals

$$(1 + \delta_i)[b_i(x) - b_i(x_i)] + (1 - \delta_i)[c_i(x) - c_i(x_i)] \text{ if } x > x_i$$

$$(1 - \delta_i)[b_i(x) - b_i(x_i)] + (1 + \delta_i)[c_i(x) - c_i(x_i)] \text{ if } x < x_i$$

$$[b_i(x) - b_i(x_i)] + [c_i(x) - c_i(x_i)] \text{ if } x = x_i.$$
(B6)

 $\widetilde{D}_i(x|\{x,x'\})$ is continuous in x at any $x \neq x_i$ because b_i and c_i are continuous. At $x = x_i$, $\lim_{x \to x_i^-} \widetilde{D}_i(x|\{x,x'\}) = \widetilde{D}_i(x_i|\{x_i,x'\}) = \lim_{x \to x_i^+} \widetilde{D}_i(x|\{x,x'\}) = 0$. $\widetilde{D}_i(x|\{x,x'\})$ is differentiable in x at any $x \neq x_i$ because b_i and c_i are differentiable.

Part 2: Because $x' < x_i$, we have $x' < x_i < e_i(x')$. When x < x', we have $U_i(x) < U_i(x')$ and hence voters in group i focus on the first issue. When x > x', voters in group i focus on the first issue when $U_i(x) > U_i(x')$, or, equivalently, when $x \in (x', e_i(x'))$, and focus on the second issue when $U_i(x) < U_i(x')$, or, equivalently, when $x > e_i(x')$. Voters in group i have undistorted focus when $x \in \{x', e_i(x')\}$. $\widetilde{D}_i(x|\{x, x'\})$ thus equals

$$(1 + \delta_i)[b_i(x) - b_i(x')] + (1 - \delta_i)[c_i(x) - c_i(x')] \text{ if } x \in [0, x') \cup (x', e_i(x'))$$

$$(1 - \delta_i)[b_i(x) - b_i(x')] + (1 + \delta_i)[c_i(x) - c_i(x')] \text{ if } x \in (e_i(x'), \bar{x}]$$

$$[b_i(x) - b_i(x')] + [c_i(x) - c_i(x')] \text{ if } x \in \{x', e_i(x')\}.$$
(B7)

 $\widetilde{D}_i(x|\{x,x'\})$ is continuous in x at any $x \notin \{x',e_i(x')\}$ because b_i and c_i are continuous.

At x = x', if x' > 0, then $\lim_{x \to x'^-} \widetilde{D}_i(x | \{x, x'\}) = 0$, and if $x' \ge 0$, then $\widetilde{D}_i(x' | \{x', x'\}) = \lim_{x \to x'^+} \widetilde{D}_i(x | \{x, x'\}) = 0$. At $x = e_i(x')$, if $e_i(x') \le \bar{x}$, then $\lim_{x \to e_i(x')^-} \widetilde{D}_i(x | \{x, x'\})$ equals

$$(1 + \delta_i)[b_i(e_i(x')) - b_i(x')] + (1 - \delta_i)[c_i(e_i(x')) - c_i(x')]$$

$$= U_i(e_i(x')) - U_i(x') + \delta_i[b_i(e_i(x')) - b_i(x')] - \delta_i[c_i(e_i(x')) - c_i(x')] \ge 0$$
(B8)

where the inequality follows because $U_i(e_i(x')) = U_i(x')$, because $e_i(x') > x'$ and because b_i and c_i are, respectively, strictly increasing and strictly decreasing. If $e_i(x') < \bar{x}$, then $\lim_{x \to e_i(x')^+} \widetilde{D}_i(x|\{x,x'\})$ equals

$$(1 - \delta_i)[b_i(e_i(x')) - b_i(x')] + (1 + \delta_i)[c_i(e_i(x')) - c_i(x')]$$

$$= U_i(e_i(x')) - U_i(x') - \delta_i[b_i(e_i(x')) - b_i(x')] + \delta_i[c_i(e_i(x')) - c_i(x')] \le 0.$$
(B9)

 $\widetilde{D}_i(x|\{x,x'\})$ is differentiable in x at any $x \notin \{x',e_i(x')\}$ because b_i and c_i are differentiable. At x=x', using $\widetilde{D}'_i(x'|\{x',x'\}) = \lim_{x\to x'} \frac{\widetilde{D}_i(x|\{x,x'\}) - \widetilde{D}_i(x'|\{x',x'\})}{x'-x}$ and (B7), we have $\widetilde{D}'_i(x'|\{x',x'\}) = (1+\delta_i)b'_i(x') + (1-\delta_i)c'_i(x')$.

Part 3: Because $x' > x_i$, we have $e_i(x') < x_i < x'$. When x > x', we have $U_i(x) < U_i(x')$ and hence voters in group i focus on the second issue. When x < x', voters in group i focus on the second issue when $U_i(x) > U_i(x')$, or, equivalently, when $x \in (e_i(x'), x')$, and focus on the first issue when $U_i(x) < U_i(x')$, or, equivalently, when $x < e_i(x')$. Voters in group i have undistorted focus when $x \in \{x', e_i(x')\}$. $\widetilde{D}_i(x|\{x, x'\})$ thus equals

$$(1 + \delta_i)[b_i(x) - b_i(x')] + (1 - \delta_i)[c_i(x) - c_i(x')] \text{ if } x \in [0, e_i(x'))$$

$$(1 - \delta_i)[b_i(x) - b_i(x')] + (1 + \delta_i)[c_i(x) - c_i(x')] \text{ if } x \in (e_i(x'), x') \cup (x', \bar{x}]$$

$$[b_i(x) - b_i(x')] + [c_i(x) - c_i(x')] \text{ if } x \in \{x', e_i(x')\}.$$
(B10)

 $\widetilde{D}_i(x|\{x,x'\})$ is continuous in x at any $x \notin \{x',e_i(x')\}$ because b_i and c_i are continuous. At x=x', if $x' \leq \overline{x}$, then $\lim_{x \to x'^-} \widetilde{D}_i(x|\{x,x'\}) = \widetilde{D}_i(x'|\{x',x'\}) = 0$, and if $x' < \overline{x}$, then $\lim_{x \to x'^+} \widetilde{D}_i(x|\{x,x'\}) = 0$. At $x=e_i(x')$, if $e_i(x')>0$, then $\lim_{x \to e_i(x')^-} \widetilde{D}_i(x|\{x,x'\})$

equals

$$(1 + \delta_i)[b_i(e_i(x')) - b_i(x')] + (1 - \delta_i)[c_i(e_i(x')) - c_i(x')]$$

$$= U_i(e_i(x')) - U_i(x') + \delta_i[b_i(e_i(x')) - b_i(x')] - \delta_i[c_i(e_i(x')) - c_i(x')] \le 0$$
(B11)

where the inequality follows because $U_i(e_i(x')) = U_i(x')$, because $e_i(x') < x'$ and because b_i and c_i are, respectively, strictly increasing and strictly decreasing. If $e_i(x') \geq 0$, then $\lim_{x \to e_i(x')^+} \widetilde{D}_i(x|\{x,x'\})$ equals

$$(1 - \delta_i)[b_i(e_i(x')) - b_i(x')] + (1 + \delta_i)[c_i(e_i(x')) - c_i(x')]$$

$$= U_i(e_i(x')) - U_i(x') - \delta_i[b_i(e_i(x')) - b_i(x')] + \delta_i[c_i(e_i(x')) - c_i(x')] \ge 0.$$
(B12)

 $\widetilde{D}_i(x|\{x,x'\})$ is differentiable in x at any $x \notin \{x',e_i(x')\}$ because b_i and c_i are differentiable. At x=x', using $\widetilde{D}_i'(x'|\{x',x'\})=\lim_{x\to x'}\frac{\widetilde{D}_i(x|\{x,x'\})-\widetilde{D}_i(x'|\{x',x'\})}{x'-x}$ and (B10), we have $\widetilde{D}_i'(x'|\{x',x'\})=(1-\delta_i)b_i'(x')+(1+\delta_i)c_i'(x')$.

Part 4 for $x = x_i$ follows from (B6), for $x' < x_i$ follows from (B7) and for $x' > x_i$ follows from (B10). Part 5 follows from (B6).

To state Lemma B2, for any $k \in \{0, ..., n\}$ and $x \in X$ define T(x, k) as

$$T(x,k) = \sum_{i=1}^{k} \phi_i m_i \left[(1 - \delta_i) b_i'(x) + (1 + \delta_i) c_i'(x) \right]$$

$$+ \sum_{i=k+1}^{n} \phi_i m_i \left[(1 + \delta_i) b_i'(x) + (1 - \delta_i) c_i'(x) \right].$$
(B13)

T(x,k) is the derivative, if it exists, of $\sum_{i\in N} \phi_i m_i \widetilde{D}_i(x|\{x,x\})$ when groups $i\leq k$ focus on the second issue and groups $i\geq k+1$ focus on the first issue in case of a marginal deviation from (x,x). Lemma B2 proves several properties of T(x,k), where T'(x,k) denotes the derivative of T(x,k) with respect to x.

Lemma B2.

1.
$$\forall x \in X \text{ and } \forall k \in \{0, \dots, n-1\}, T(x,k) \ge T(x,k+1)$$
;

2.
$$\forall x \in X \text{ and } \forall k \in \{0, ..., n\}, T'(x, k) < 0;$$

3. $T(x,0) > 0 \ \forall x \leq x_1 \ and \ T(x,n) < 0 \ \forall x \geq x_n$.

Proof. For part 1, $\forall x \in X$ and $\forall k \in \{0, ..., n-1\}$:

$$T(x,k) - T(x,k+1) = \phi_{k+1} m_{k+1} 2\delta_{k+1} \left[b'_{k+1}(x) - c'_{k+1}(x) \right] \ge 0$$
 (B14)

where the inequality follows because, $\forall x \in X, b'(x) > 0$ and c'(x) < 0.

Part 2 is immediate because, $\forall i \in N$ and $\forall x \in X$, $U_i''(x) < 0$ and hence $b_i''(x) \leq 0$ and $c_i''(x) \leq 0$ with at least one strict inequality.

For part 3,

$$T(x,0) = \sum_{i \in N} \phi_i m_i U_i'(x) + \phi_i m_i \delta_i \left[b_i'(x) - c_i'(x) \right] > 0$$
 (B15)

where the inequality follows from $x \leq x_1$, and

$$T(x,n) = \sum_{i \in N} \phi_i m_i U_i'(x) - \phi_i m_i \delta_i [b_i'(x) - c_i'(x)] < 0$$
 (B16)

where the inequality follows from $x \geq x_n$.

Lemma B3. If (x_A^*, x_B^*) constitutes a pure strategy NE, then, $\forall j \in \{A, B\}$,

$$\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^{-}(x_j^* | \{x_j^*, x_j^*\}) \ge 0$$

$$\sum_{i \in N} \phi_i m_i \widetilde{D}_i^{\prime +}(x_j^* | \{x_j^*, x_j^*\}) \le 0.$$

Proof. Suppose (x_A^*, x_B^*) constitutes a NE. Because (x_A^*, x_B^*) constitutes a NE in a constantsum game, $\pi_A(x_A^*|\{x_A^*, x_B^*\}) = \pi_B(x_B^*|\{x_A^*, x_B^*\}) = \frac{1}{2}$. Moreover, $\forall j \in \{A, B\}, \sum_{i \in N} \phi_i m_i \widetilde{D}_i(x_j^*|\{x_j^*, x_j^*\}) = 0$ and hence $\pi_{-j}(x_j^*|\{x_j^*, x_j^*\}) = \frac{1}{2}$. Finally, $\forall j \in \{A, B\}, x_j^* \in X^o$; if $x_j^* \in \{0, \bar{x}\}$ for party $j \in \{A, B\}$, then party -j has profitable deviation because for sufficiently small $\varepsilon > 0$, we have $\widetilde{D}_i(\varepsilon|\{0, \varepsilon\}) > 0$ and $\widetilde{D}_i(\bar{x} - \varepsilon|\{\bar{x}, \bar{x} - \varepsilon\}) > 0$ $\forall i \in N$.

By Lemma B1, $\forall x \in X^o$, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^-(x|\{x,x\})$ and $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^+(x|\{x,x\})$ exist. Suppose, towards a contradiction, that either $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^-(x_j^*|\{x_j^*, x_j^*\}) < 0$ or $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^+(x_j^*|\{x_j^*, x_j^*\}) > 0$ for some $j \in \{A, B\}$. Then there exists either $x < x_j^*$

or $x > x_j^*$ such that $\pi_{-j}(x|\{x, x_j^*\}) > \frac{1}{2}$, a contradiction because (x_A^*, x_B^*) constitutes a NE.

Lemma B4. A solution to (\mathcal{O}) , x^* , exists, is unique and satisfies $x^* \in [x_1, x_n]$.

Proof. Let $x_0 = 0$ and $x_{n+1} = \bar{x}$. Because $0 < x_i < x_{i+1} < \bar{x} \ \forall i \in \{1, \dots, n-1\}$, $x_i < x_{i+1} \ \forall i \in \{0, \dots, n\}$. Notice that, $\forall k \in \{0, \dots, n\}$, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'(x | \{x, x\}) = T(x, k)$ if $x \in (x_k, x_{k+1})$ and, $\forall k \in \{1, \dots, n\}$, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^-(x | \{x, x\}) = T(x, k-1)$ and $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^+(x | \{x, x\}) = T(x, k)$ if $x = x_k$. The former by Lemma B1 part 4 and the latter by Lemma B1 parts 4 and 5. Therefore, if x^* solves (\mathcal{O}), then either $T(x^*, k) = 0$ and $x^* \in (x_k, x_{k+1})$ for some $k \in \{0, \dots, n\}$ or $T(x^*, k-1) \geq 0$, $T(x^*, k) \leq 0$ and $x^* = x_k$ for some $k \in \{1, \dots, n\}$. Conversely, any $x' \in X$ such that either T(x', k) = 0 and $x' \in (x_k, x_{k+1})$ for some $k \in \{0, \dots, n\}$ or $T(x', k-1) \geq 0$, $T(x', k) \leq 0$ and $x' = x_k$ for some $k \in \{1, \dots, n\}$ solves (\mathcal{O}). To prove the lemma, it thus suffices to show that x' exists, is unique and $x' \in [x_1, x_n]$.

For existence, we will show that if $x' \in X$ such that T(x',k) = 0 and $x' \in (x_k, x_{k+1})$ for some $k \in \{0, \ldots, n\}$ does not exist, then there exists $x' \in X$ such that $T(x', k-1) \geq 0$, $T(x',k) \leq 0$ and $x' = x_k$ for some $k \in \{1,\ldots,n\}$. Because x' such that T(x',k) = 0 and $x' \in (x_k, x_{k+1})$ for some $k \in \{0,\ldots,n\}$ does not exist and because T(x,k) is continuous in $x \, \forall k \in \{0,\ldots,n\}$, we have, $\forall k \in \{0,\ldots,n\}$, either $T(x,k) > 0 \, \forall x \in (x_k, x_{k+1})$ or $T(x,k) < 0 \, \forall x \in (x_k, x_{k+1})$. By Lemma B2 part 3, $T(x,0) > 0 \, \forall x \in (x_0, x_1)$ and $T(x,n) < 0 \, \forall x \in (x_n, x_{n+1})$. Because $T(x,k) > T(x'',k+1) \, \forall x \in X$, $\forall k \in \{0,\ldots,n-1\}$ and $\forall x'' > x$ by Lemma B2 parts 1 and 2, there exist $k' \in \{1,\ldots,n\}$ such that, $\forall k'' \leq k'$, $T(x,k''-1) > 0 \, \forall x \in (x_{k''-1}, x_{k''})$ and, $\forall k'' \geq k'$, $T(x,k'') < 0 \, \forall x \in (x_{k''}, x_{k''+1})$. By continuity of T(x,k) in $x \, \forall k \in \{0,\ldots,n\}$, we thus have $T(x_{k'},k'-1) \geq 0$ and $T(x_{k'},k') \leq 0$.

For uniqueness, suppose either T(x',k')=0 and $x'\in(x_{k'},x_{k'+1})$ for some $k'\in\{0,\ldots,n\}$ or $T(x',k'-1)\geq0$, $T(x',k')\leq0$ and $x'=x_{k'}$ for some $k'\in\{1,\ldots,n\}$.

If $x' \in (x_{k'}, x_{k'+1})$ so that T(x', k') = 0, then $T(x'', k'') < 0 \ \forall x'' > x'$ and $\forall k'' \ge k'$ by Lemma B2 parts 1 and 2. Hence $T(x'', k') < 0 \ \forall x'' \in (x', x_{k'+1}), \ T(x'', k'') < 0 \ \forall k'' > k'$ and $\forall x'' \in (x_{k''}, x_{k''+1}), \ \text{and} \ T(x_{k''+1}, k'') < 0 \ \text{and} \ T(x_{k''+1}, k''+1) < 0 \ \forall k'' \ge k'$. Similarly,

 $T(x'', k'') > 0 \ \forall x'' < x' \ \text{and} \ \forall k'' \le k' \ \text{by Lemma B2 parts 1 and 2. Hence } T(x'', k') > 0$ $\forall x'' \in (x_{k'}, x'), \ T(x'', k'') > 0 \ \forall k'' < k' \ \text{and} \ \forall x'' \in (x_{k''}, x_{k''+1}), \ \text{and} \ T(x_{k''}, k'' - 1) > 0$ and $T(x_{k''}, k'') > 0 \ \forall k'' \le k'.$

If $x' = x_{k'}$ so that $T(x_{k'}, k' - 1) \ge 0$ and $T(x_{k'}, k') \le 0$, then, by Lemma B2 parts 1 and 2, $T(x'', k'') < 0 \ \forall k'' \ge k'$ and $\forall x'' \in (x_{k''}, x_{k''+1})$, and $T(x_{k''+1}, k'') < 0$ and $T(x_{k''+1}, k'' + 1) < 0 \ \forall k'' \ge k'$. Similarly, by Lemma B2 parts 1 and 2, T(x'', k'' - 1) > 0 $\forall k'' \le k'$ and $\forall x'' \in (x_{k''-1}, x_{k''})$, and $T(x_{k''-1}, k'' - 2) > 0$ and $T(x_{k''-1}, k'' - 1) > 0$ $\forall k'' \le k'$.

That $x' \in [x_1, x_n]$ if T(x', k') = 0 and $x' \in (x_{k'}, x_{k'+1})$ for some $k' \in \{0, \dots, n\}$ or if $T(x', k'-1) \ge 0$, $T(x', k') \le 0$ and $x' = x_{k'}$ for some $k' \in \{1, \dots, n\}$ follows because by Lemma B2 part 3, we have $T(x, 0) > 0 \ \forall x < x_1 \ \text{and} \ T(x, n) < 0 \ \forall x > x_n$.

Lemma B5. Platforms (x^*, x^*) constitute a Nash equilibrium.

Proof. It suffices to prove that $\sum_{i\in N} \phi_i m_i \widetilde{D}_i(x|\{x,x^*\})$ has a (unique) maximum at x^* as a function of x. Suppose first that there exists $k'\in N$ such that $x^*\in (x_{k'},x_{k'+1})$ and let k''=k'+1. By Lemma B1 parts 2 and 3, $\widetilde{D}_i'(x^*|\{x^*,x^*\})$ exists $\forall i\in N$, and hence, because x^* solves (\mathcal{O}) , $\sum_{i\in N} \phi_i m_i \widetilde{D}_i'(x^*|\{x^*,x^*\}) = 0$. By Lemma B1 part 4, $\forall i\in N$ and $\forall x\in X\setminus\{e_i(x^*)\}$, $\widetilde{D}_i'(x|\{x,x^*\})$ exists and $\widetilde{D}_i''(x|\{x,x^*\})<0$. Moreover, $\forall i\in N$, whenever $e_i(x^*)\in X^o$,

$$\lim_{x \to e_i(x^*)^-} \widetilde{D}'_i(x|\{x, x^*\}) = (1 + \delta_i)b'_i(e_i(x^*)) + (1 - \delta_i)c'_i(e_i(x^*))$$

$$\lim_{x \to e_i(x^*)^+} \widetilde{D}'_i(x|\{x, x^*\}) = (1 - \delta_i)b'_i(e_i(x^*)) + (1 + \delta_i)c'_i(e_i(x^*))$$
(B17)

so that $\lim_{x \to e_i(x^*)^-} \widetilde{D}_i'(x|\{x,x^*\}) \ge \lim_{x \to e_i(x^*)^+} \widetilde{D}_i'(x|\{x,x^*\})$, because the difference of the limits equals $2\delta_i \, [b_i'(e_i(x^*)) - c_i'(e_i(x^*))] \ge 0$. Therefore, $\forall i \in N$ and $\forall x \in X \setminus \{e_i(x^*)\}$, $\widetilde{D}_i'(x|\{x,x^*\}) > \widetilde{D}_i'(x^*|\{x^*,x^*\})$ when $x < x^*$ and $\widetilde{D}_i'(x|\{x,x^*\}) < \widetilde{D}_i'(x^*|\{x^*,x^*\})$ when $x > x^*$. Hence, $\forall x \in X \setminus \{e_i(x^*)|i \in N\}$, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'(x|\{x,x^*\}) > 0$ when $x < x^*$ and $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'(x|\{x,x^*\}) < 0$ when $x > x^*$. Now consider $e_i(x^*)$. If $i \ge k''$, so that $x^* < x_i$, then $e_i(x^*) > x_i > x^*$ and, by Lemma B1 part 2, $\lim_{x \to e_i(x^*)^-} \widetilde{D}_i(x|\{x,x^*\}) \ge 0 = \widetilde{D}_i(e_i(x^*)|\{e_i(x^*),x^*\}) \ge \lim_{x \to e_i(x^*)^+} \widetilde{D}_i(x|\{x,x^*\})$ (if $e_i(x^*) = \bar{x}$ only the first inequality

is relevant and if $e_i(x^*) > \bar{x}$ none are). If $i \leq k'$, so that $x^* > x_i$, then $e_i(x^*) < x_i < x^*$ and, by Lemma B1 part 3, $\lim_{x \to e_i(x^*)^-} \widetilde{D}_i(x|\{x,x^*\}) \leq 0 = \widetilde{D}_i(e_i(x^*)|\{e_i(x^*),x^*\}) \leq \lim_{x \to e_i(x^*)^+} \widetilde{D}_i(x|\{x,x^*\})$ (if $e_i(x^*) = 0$ only the second inequality is relevant and if $e_i(x^*) < 0$ none are). In summary, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i(x|\{x,x^*\})$ is strictly increasing in x on $[0,x^*)$ and strictly decreasing in x on $(x^*,\bar{x}]$.

Suppose now that $x^* = x_{k^*}$ for some $k^* \in N$. Because x^* solves (\mathcal{O}) , we have $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^-(x^*|\{x^*, x^*\}) \geq 0$ and $\sum_{i \in N} \phi_i m_i \widetilde{D}_i'^+(x^*|\{x^*, x^*\}) \leq 0$. The argument in the preceding paragraph applies to all $i \in N \setminus \{k^*\}$ using $k' = k^* - 1$ and $k'' = k^* + 1$. For group k^* , by Lemma B1 part 1, $\widetilde{D}_{k^*}(x|\{x,x^*\})$ is continuous and is differentiable except at x^* , and, by part 4, $\widetilde{D}_{k^*}'(x|\{x,x^*\})$ equals

$$(1 + \delta_{k^*})b'_{k^*}(x) + (1 - \delta_{k^*})c'_{k^*}(x) = U'_{k^*}(x) + \delta_i \left[b'_{k^*}(x) - c'_{k^*}(x)\right] > 0$$

$$(1 - \delta_{k^*})b'_{k^*}(x) + (1 + \delta_{k^*})c'_{k^*}(x) = U'_{k^*}(x) - \delta_i \left[b'_{k^*}(x) - c'_{k^*}(x)\right] < 0$$
(B18)

when $x < x^*$ and $x > x^*$ respectively, where the inequalities come from $x^* = x_{k^*}$. Therefore, $\sum_{i \in N} \phi_i m_i \widetilde{D}_i(x|\{x,x^*\})$ is strictly increasing in x on $[0,x^*)$ and strictly decreasing in x on $[0,x^*]$.

By Lemmas B3 and B4, any pair of platforms different than (x^*, x^*) does not constitute a Nash equilibrium. By Lemma B5, (x^*, x^*) constitutes a Nash equilibrium. Lemma B4 shows that $x^* \in [x_1, x_n]$.

Following propositions characterize the equilibrium platforms of the two parties for two special cases: for two social groups and for rational voters.

Proposition B2. Assume n = 2. For any $x \in X$, let

$$\mathcal{O}_2(x) = \phi_1 m_1 [(1 - \delta_1) b_1'(x) + (1 + \delta_1) c_1'(x)] + \phi_2 m_2 [(1 + \delta_2) b_2'(x) + (1 - \delta_2) c_2'(x)].$$
 (\mathcal{O}_2)

The equilibrium platform of the two parties in the unique pure strategy Nash equilibrium from Proposition B1 is

(a)
$$x^* = x_1$$
 if $\mathcal{O}_2(x_1) < 0$;

- (b) $x^* = x_2 \text{ if } \mathcal{O}_2(x_2) \ge 0;$
- (c) the unique solution to $\mathcal{O}_2(x) = 0$ if $\mathcal{O}_2(x_1) > 0 > \mathcal{O}_2(x_2)$.

Proof. Suppose that n=2. From Proposition B1, x^* is the unique solution to (\mathcal{O}) . Note $\sum_{i=1}^2 \phi_i m_i \widetilde{D}_i'^-(x_1 | \{x_1, x_1\}) = T(x_1, 0) > 0$ and $\sum_{i=1}^2 \phi_i m_i \widetilde{D}_i'^+(x_2 | \{x_2, x_2\}) = T(x_2, 2) < 0$, where the inequalities follow from Lemma B2 part 3. Moreover, $\forall x \in X$, $\mathcal{O}_2(x) = T(x, 1)$. Therefore, we have either $\sum_{i=1}^2 \phi_i m_i \widetilde{D}_i'^+(x_1 | \{x_1, x_1\}) = T(x_1, 1) \leq 0$, in which case $x^* = x_1$, or $\sum_{i=1}^2 \phi_i m_i \widetilde{D}_i'^-(x_2 | \{x_2, x_2\}) = T(x_2, 1) \geq 0$, in which case $x^* = x_2$, or $T(x_1, 1) > 0 > T(x_2, 1)$, in which case x^* is the unique solution to T(x, 1) = 0, where the solution is unique by Lemma B2 part 2.

Proposition B3. Assume $\delta_i = 0$ for all $i \in \mathbb{N}$. For all $x \in X$, let

$$\mathcal{O}_r(x) = \sum_{i \in N} \phi_i m_i [b_i'(x) + c_i'(x)]. \tag{O_r}$$

The equilibrium platform of the two parties in the unique pure strategy Nash equilibrium from Proposition B1, x^* , is the unique solution to $\mathcal{O}_r(x) = 0$. Moreover, $x^* \in (x_1, x_n)$. Finally, x^o , the unique solution to $\mathcal{O}_r(x) = 0$ when $\phi_i = 1$ for all $i \in N$, is the unique solution to the utilitarian problem $\max_{x \in X} \sum_{i \in N} U_i(x)$.

Proof. Suppose that $\delta_i = 0$ for all $i \in N$. Then, for any $i \in N$, $x \in X$ and $\mathcal{P} \in \mathcal{X}$, $\widetilde{U}_i(x|\mathcal{P}) = U_i(x)$ and hence $\widetilde{D}'_i(x|\mathcal{P}) = \widetilde{D}'_i(x|\mathcal{P}) = U'_i(x)$. (\mathcal{O}) thus becomes $\sum_{i \in N} \phi_i m_i U'_i(x) = 0$, or, equivalently, $\mathcal{O}_r(x) = 0$. $\mathcal{O}_r(x) = 0$ has unique solution by strict concavity of U_i for all $i \in N$. Moreover, $U'_i(x_1) \geq 0$ with strict inequality except for i = 1 and $U'_i(x_n) \leq 0$ with strict inequality except for i = n and hence $x^* \in (x_1, x_n)$.

The objective function in the utilitarian problem $\max_{x \in X} \sum_{i \in N} U_i(x)$ is strictly concave and $x_i \in X^o$ for all $i \in N$, and hence $\mathcal{O}_r(x) = 0$ evaluated at $\phi_i = 1$ for all $i \in N$ is both sufficient and necessary.