Greenberger-Horne-Zeilinger State

Team SSQRT

Problem Statements

1

Build and run a GHZ state on an Real Quantum Hardware

Apply Readout Error Mitigation

Implement Quantum Communication Scheme

2

Improve GHZ circuit with Pulse-level Calibration

Optimize Pulse Parameters

3

Perform Zero-noise Extrapolation

Calibrate DRAG pulse and Rotary term

Problem Statements

1

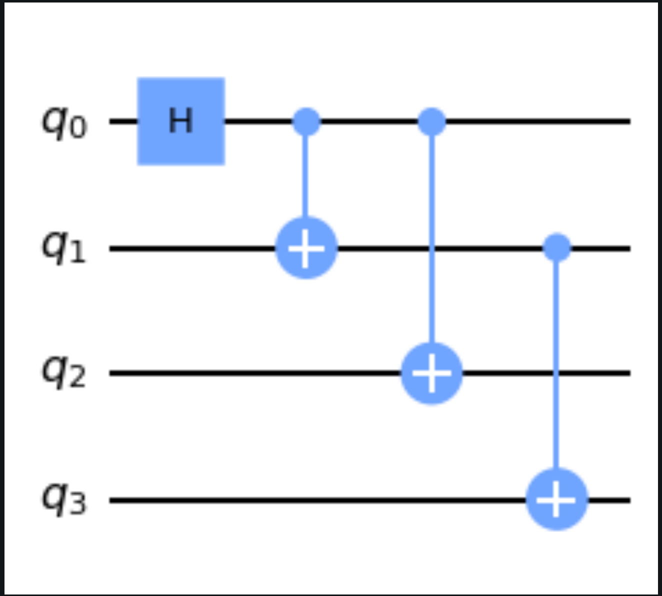
Build and run a GHZ state on an Real Quantum Hardware

Apply Readout Error Mitigation

Implement Quantum Communication Scheme

How to make a GHZ state

$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}[|0000\rangle + |1111\rangle]$$

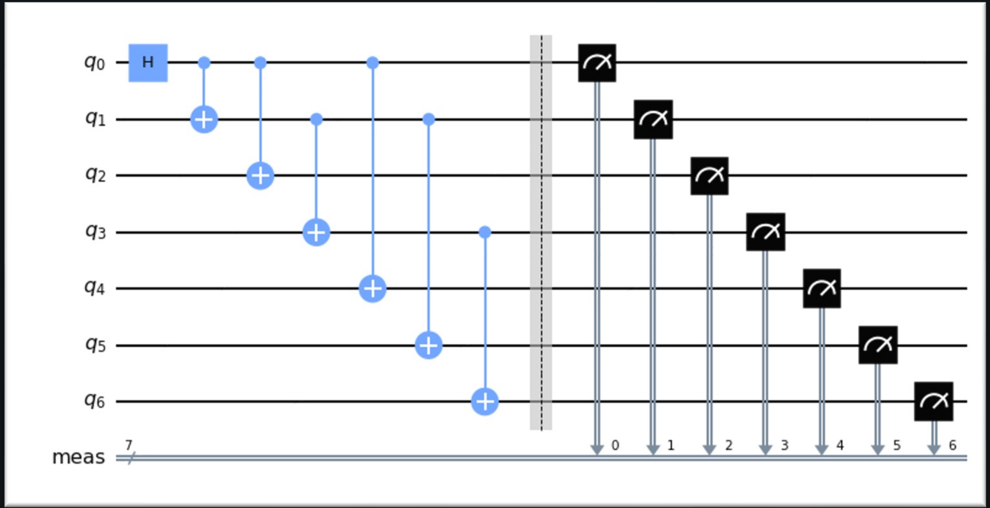


Four-qubit GHZ State

How to make a GHZ state

Qiskit

$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}[|0000\rangle + |1111\rangle]$$

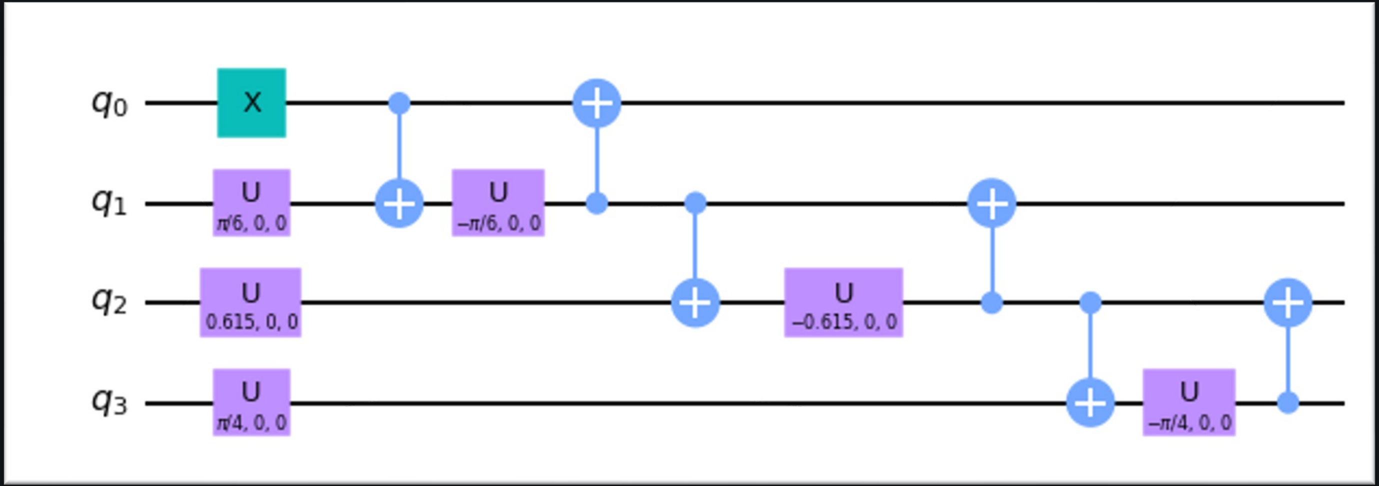


Four-qubit GHZ State including Two-qubit Repetition Code

W state

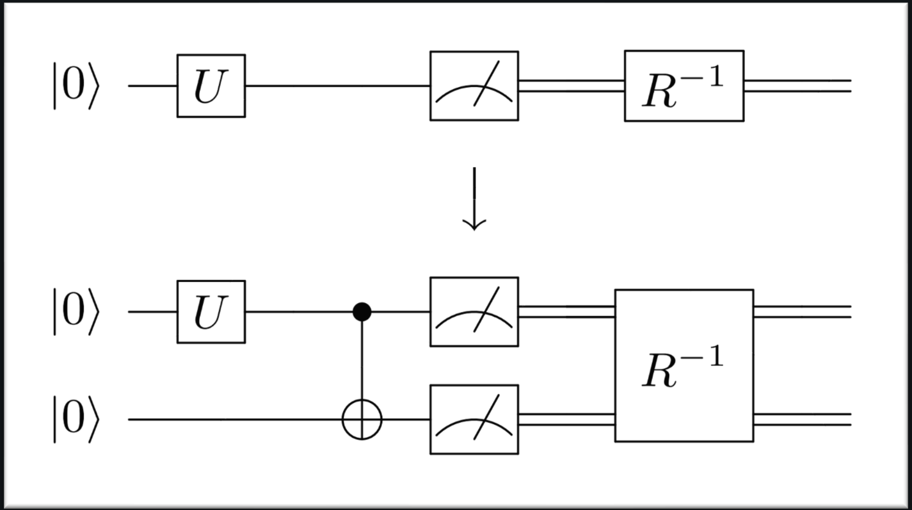
Qiskit

$$|\Psi\rangle_W = \frac{1}{2}[|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle]$$



Four-qubit W State

Readout Error Mitigation



Readout Error Mitigation Scheme using CNOT gate

R. Hicks et. al, *"Active Readout Error Mitigation,"* University of California, Berkeley, 2022.

Bit flip Readout Error Probability

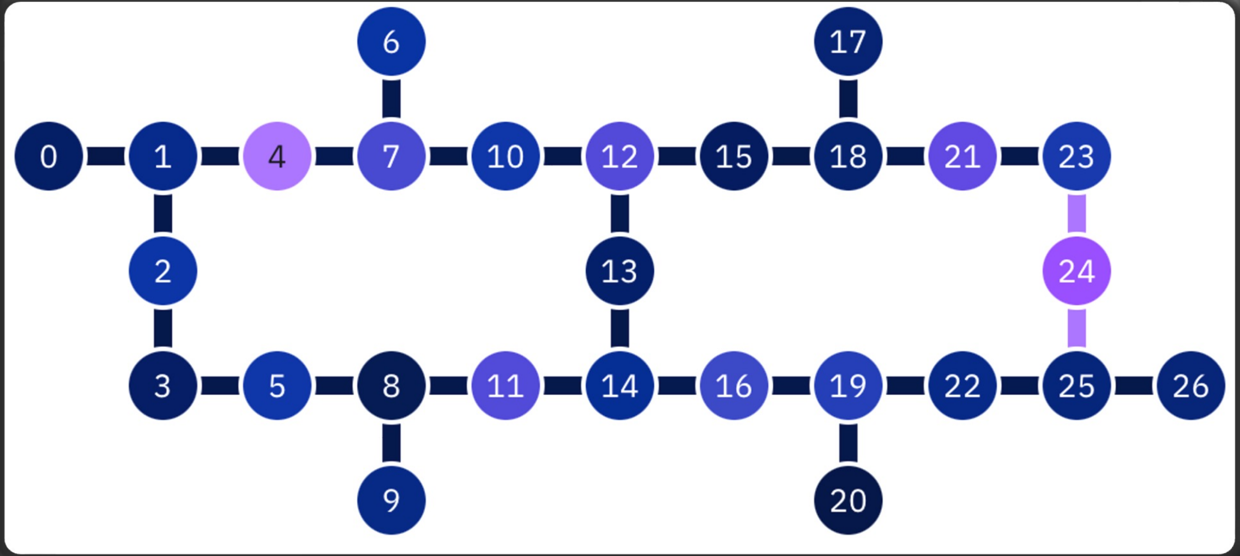
$$q_{eff,2} \approx \frac{\epsilon}{4} + q^2$$

Two-qubit readout error detection

$$q_{eff,3} \approx 3(\frac{\epsilon}{4} + q^2)$$

Three-qubit readout error correction

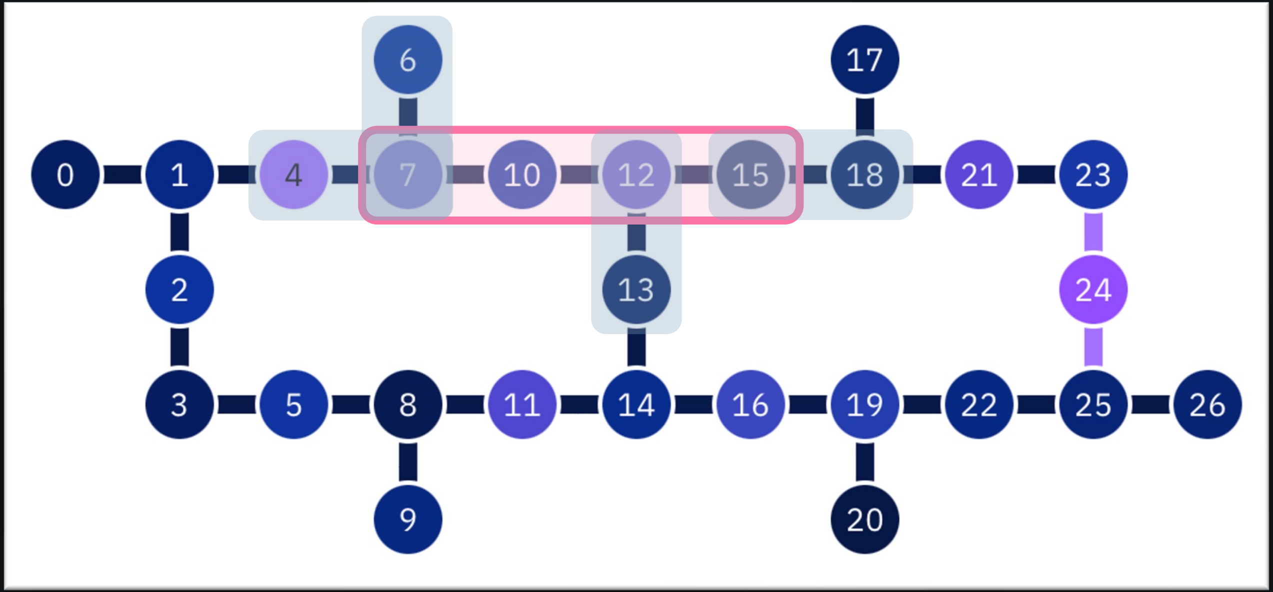
IBM Canberra



Falcon r6 Processor

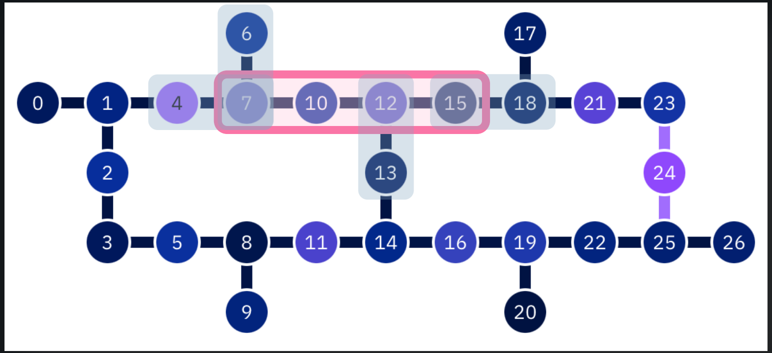
27 Qubits

Optimal Qubit Layout Algorithm



Optimal Qubit Layout Algorithm

Qiskit



Optimal Qubit Layout Algorithm

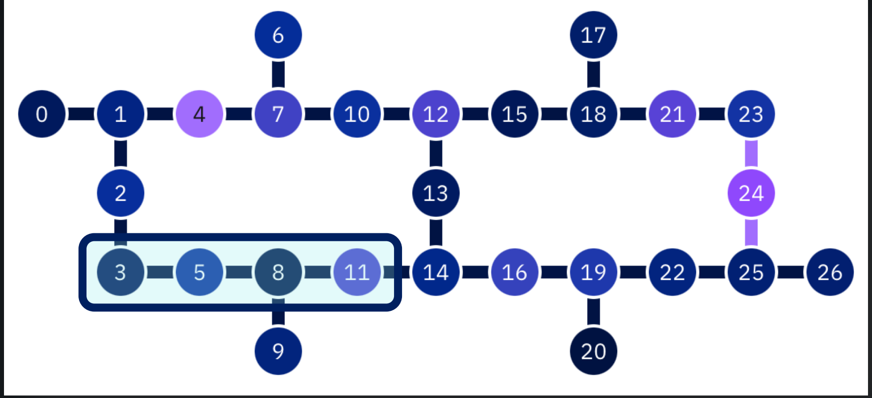
Find all possible sets of four connected qubits

Find sets of qubits capable of three CNOT connections

If a qubit is subject to two CNOT possibilities, compare the error probabilities of the two cases

Compare among the finalists for the optimal result

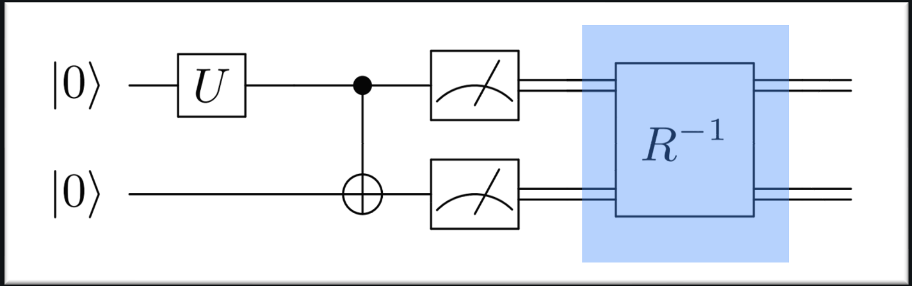
Optimal Qubit Layout Algorithm



Fidelity	Qubit Set

```
[[0.7101795149797284, [15, 18, 21, 23]],
 [0.9211295107767663, [1, 2, 4, 7]],
 [0.9334404942956175, [22, 23, 24, 25]],
 [0.9373700099975102, [19, 22, 24, 25]],
 [0.9478085901001727, [7, 10, 12, 15]],
 [0.9479723635803164, [5, 8, 11, 14]],
 [0.9480160560790225, [7, 10, 12, 13]],
 [0.9502487309366804, [11, 14, 16, 19]],
 [0.9506613203193462, [13, 14, 16, 19]],
 [0.951626455094902, [12, 13, 14, 15]],
 [0.9532287676512312, [1, 4, 7, 10]],
 [0.95455368823144, [4, 7, 10, 12]],
 [0.9592733452157228, [14, 16, 19, 22]],
 [0.9667088227140149, [1, 2, 3, 4]],
 [0.9690453653527326, [12, 13, 14, 16]],
 [0.9691544876849959, [16, 19, 22, 25]],
 [0.9705736050821081, [10, 12, 13, 14]],
 [0.9707075355230773, [8, 11, 14, 16]],
 [0.970788798420706, [11, 12, 13, 14]],
 [0.9754935406708112, [12, 15, 18, 21]],
 [0.9758126844009003, [10, 12, 15, 18]],
 [0.9778210520288816, [8, 11, 13, 14]],
 [0.97812598828705, [12, 13, 15, 18]],
 [0.9831852999039822, [3, 5, 8, 11]]]
```

Post-Measurement Treatment



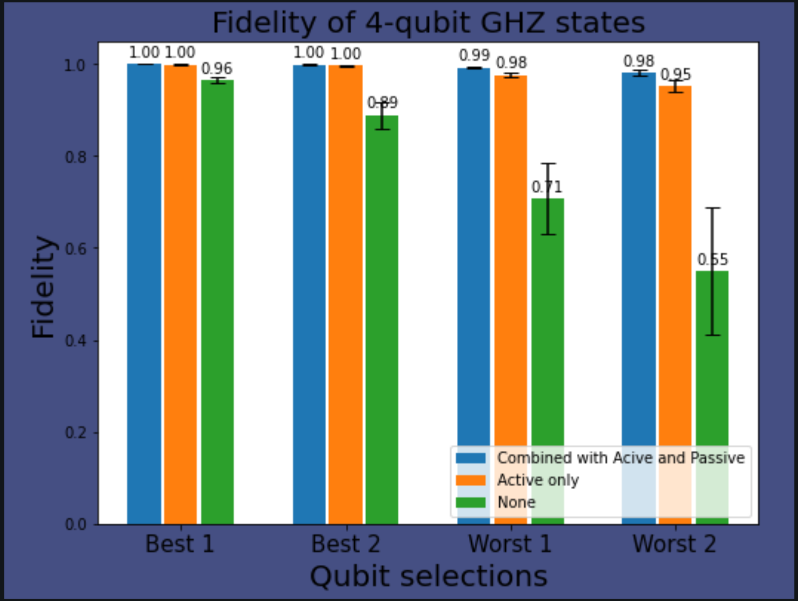
Response Matrix

Local Error Approximation

$$R \cong R_1 \otimes R_2 \otimes R_3 \otimes R_4$$

	Before Correction	After Correction
Two Best Sets		
	0.96481	0.99978
	0.88822	0.99846
Two Worst Sets		
	0.70727	0.99152
	0.54980	0.98054

Post-Measurement Treatment



	Before Correction	After Correction
Two Best Sets		
	0.96481	0.99978
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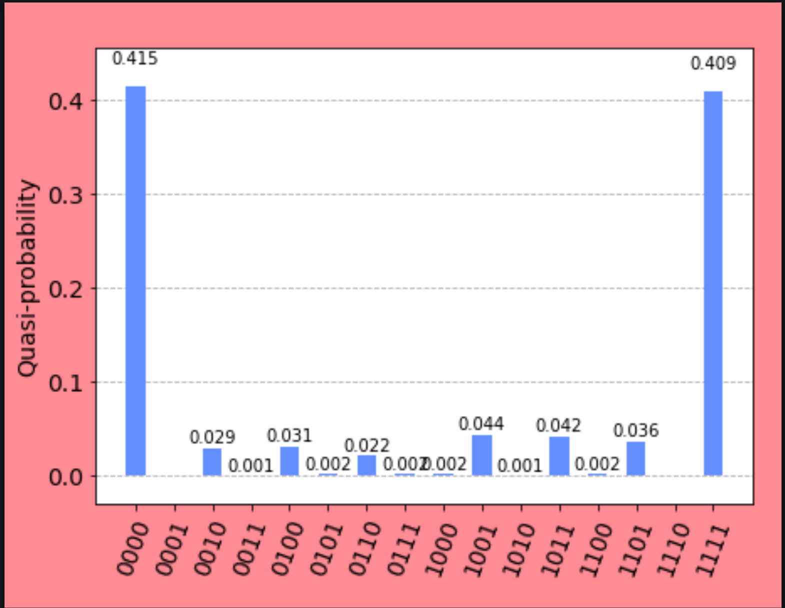
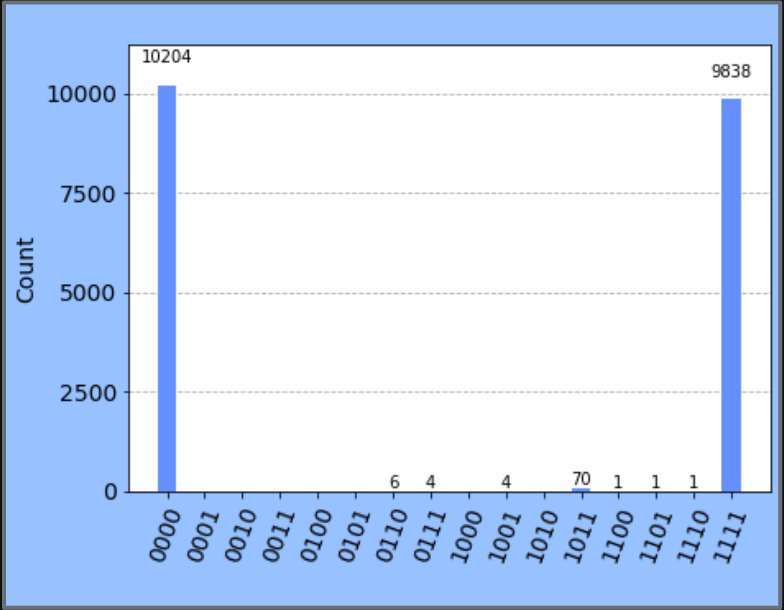
Two Worst Sets		
	0.70727	0.99152
	0.54980	0.98054

Comparison with IBM Runtime Sampler



IBM Runtime Sampler
(Resilience Level 1)



Active & Passive
Readout Error Mitigation

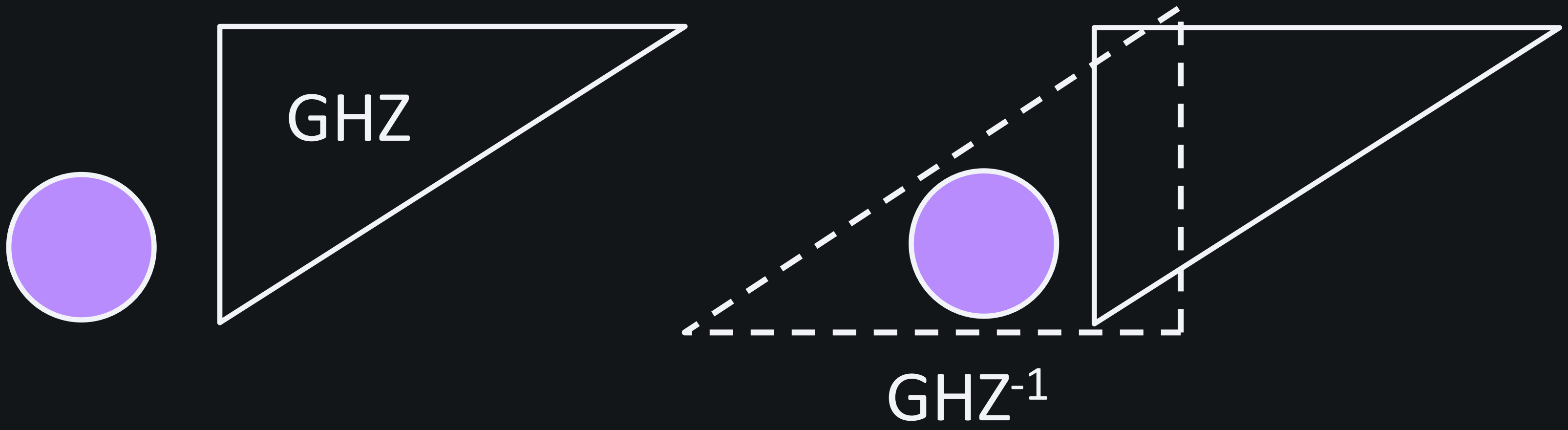
One-Hop Bidirectional Quantum Transportation

Not a Bidirectional Channel!

Two independent path of channel each consuming one GHZ pair

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|000\rangle + |111\rangle}{\sqrt{2}} = \frac{\alpha|0000\rangle + \alpha|0111\rangle + \beta|1000\rangle + \beta|1111\rangle}{\sqrt{2}}$$

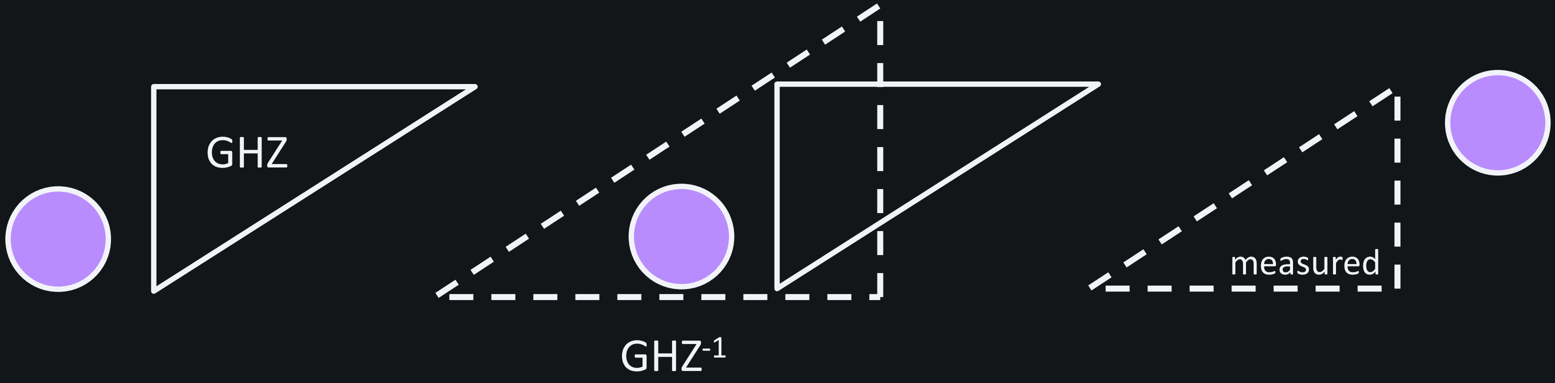
One-Hop Bidirectional Quantum Transportation



$$\frac{\alpha|{+}000\rangle + \alpha|{+}111\rangle + \beta|{-}110\rangle + \beta|{-}001\rangle}{\sqrt{2}}$$

$$= \frac{1}{2}\left[|000\rangle(\alpha|0\rangle + \beta|1\rangle) + |001\rangle(\alpha|0\rangle - \beta|1\rangle) + |110\rangle(\alpha|1\rangle + \beta|0\rangle) + |111\rangle(\alpha|1\rangle - \beta|0\rangle)\right]$$

One-Hop Bidirectional Quantum Transportation

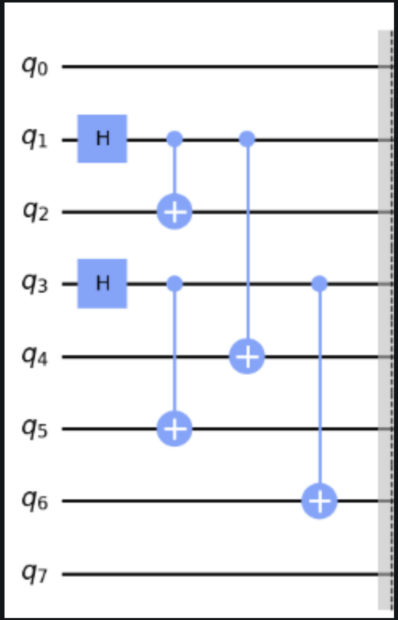


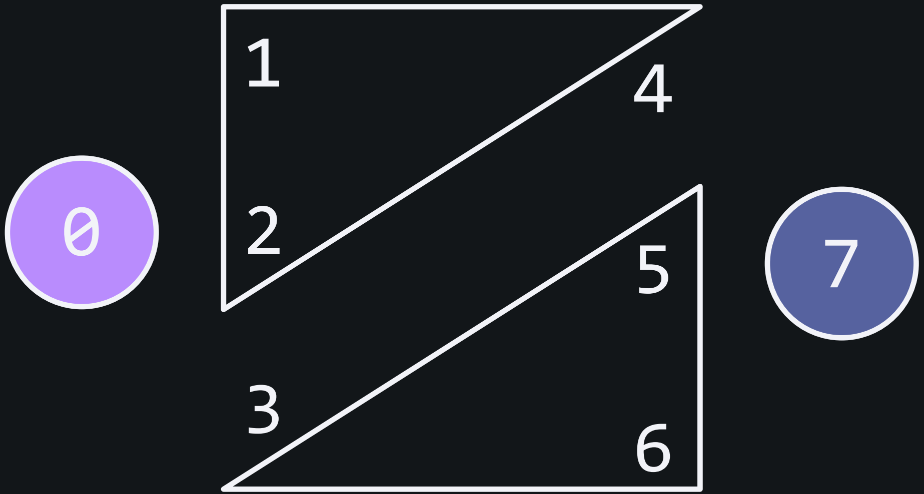
$$\frac{|000\rangle(\alpha|0\rangle+\beta|1\rangle)+|001\rangle(\alpha|0\rangle-\beta|1\rangle)+|110\rangle(\alpha|1\rangle+\beta|0\rangle)+|111\rangle(\alpha|1\rangle-\beta|0\rangle)}{2}$$

$\|000\rangle$	$\|001\rangle$	$\|011\rangle$	$\|111\rangle$
No Process	Z Gate	X Gate	X – Z Gates

Procedures

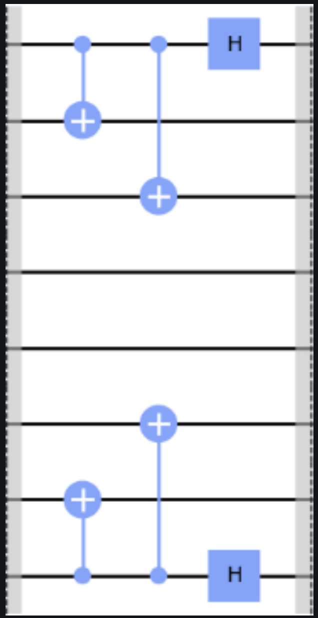
1 Prepare GHZ States

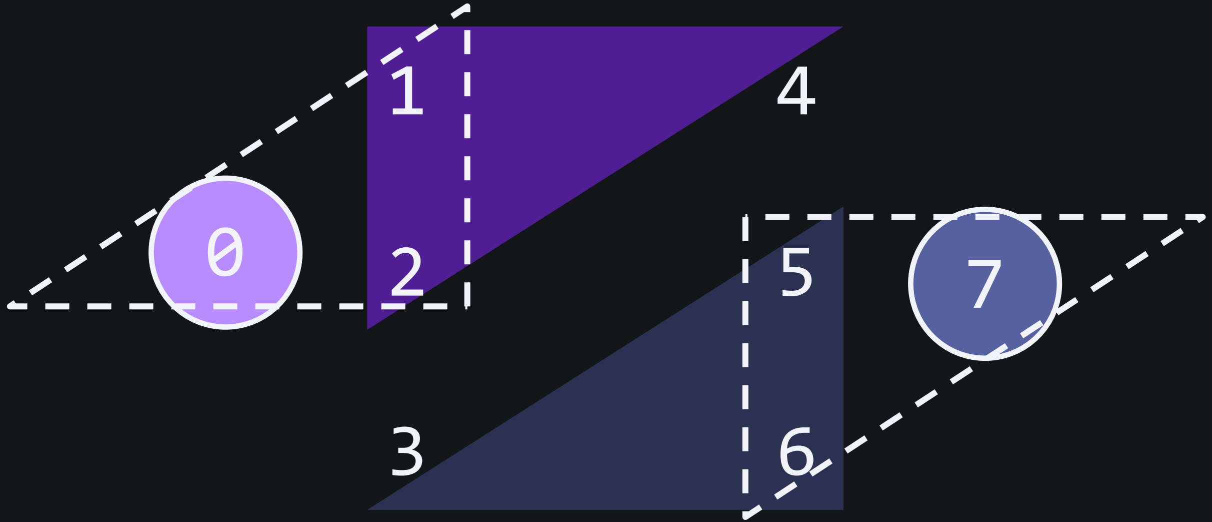




Procedures

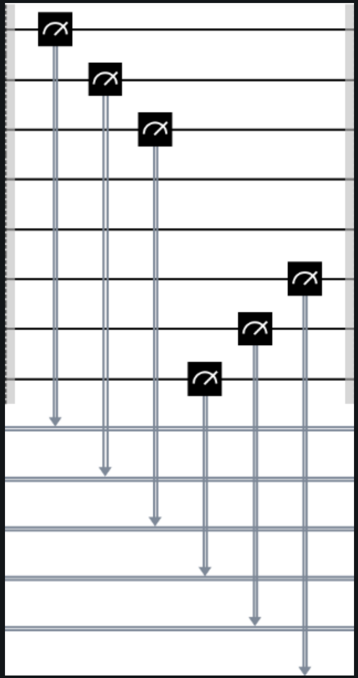
2 Transfer Information to the Next Qubit with GHZ^{-1} Gate

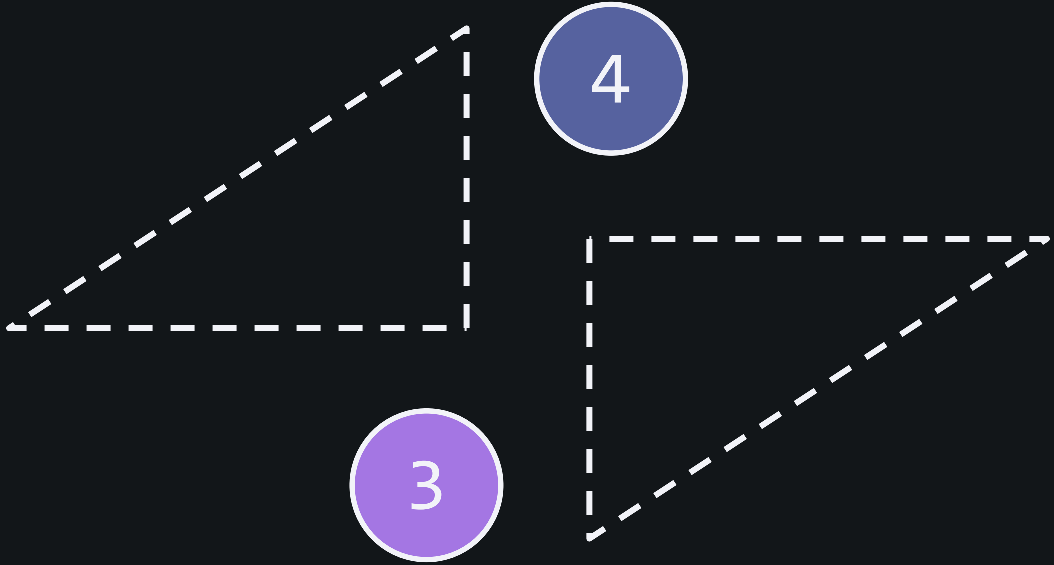




Procedures

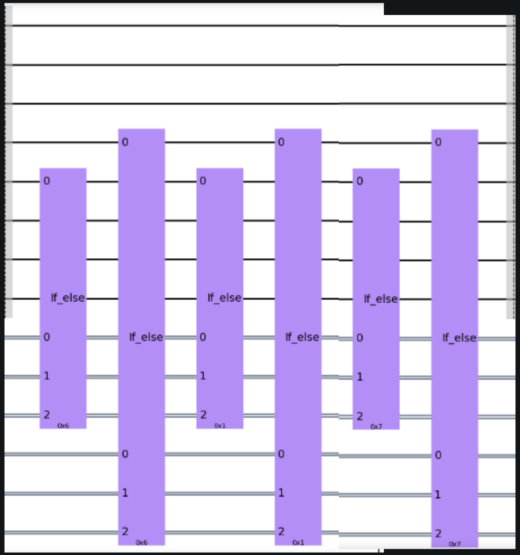
3 Measurement: Transfer the Entire Information





Procedures

4 Post-Processing



4

3

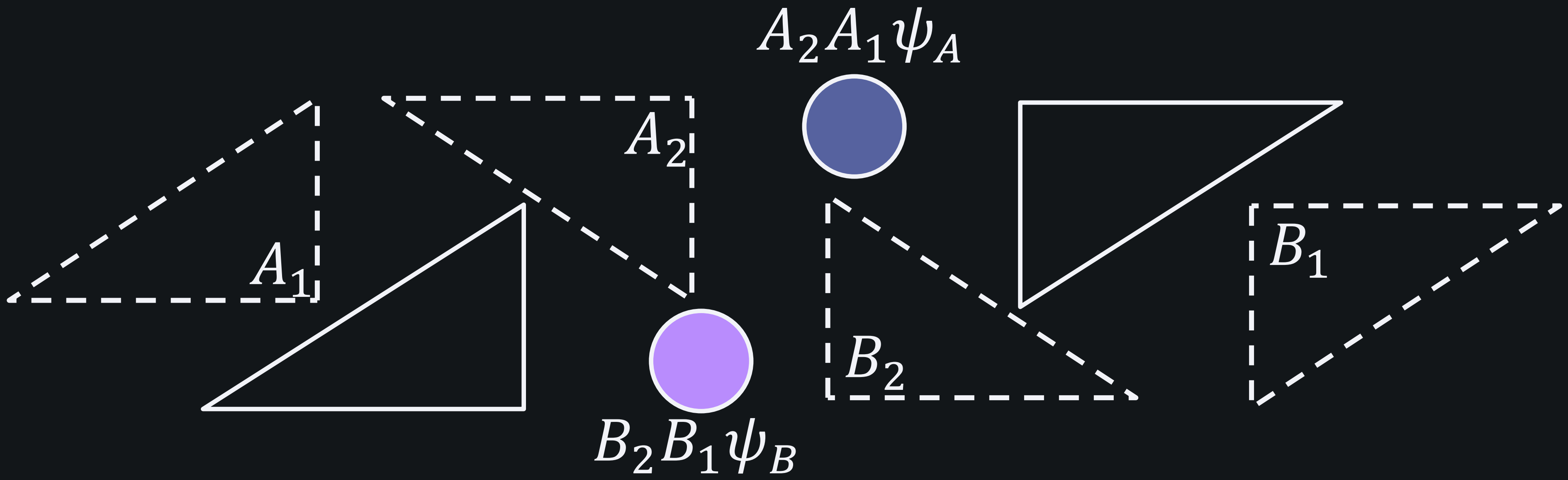
Multi-Hop

Multi-Hop

A_1, B_1 :one of I, X, Z, ZX

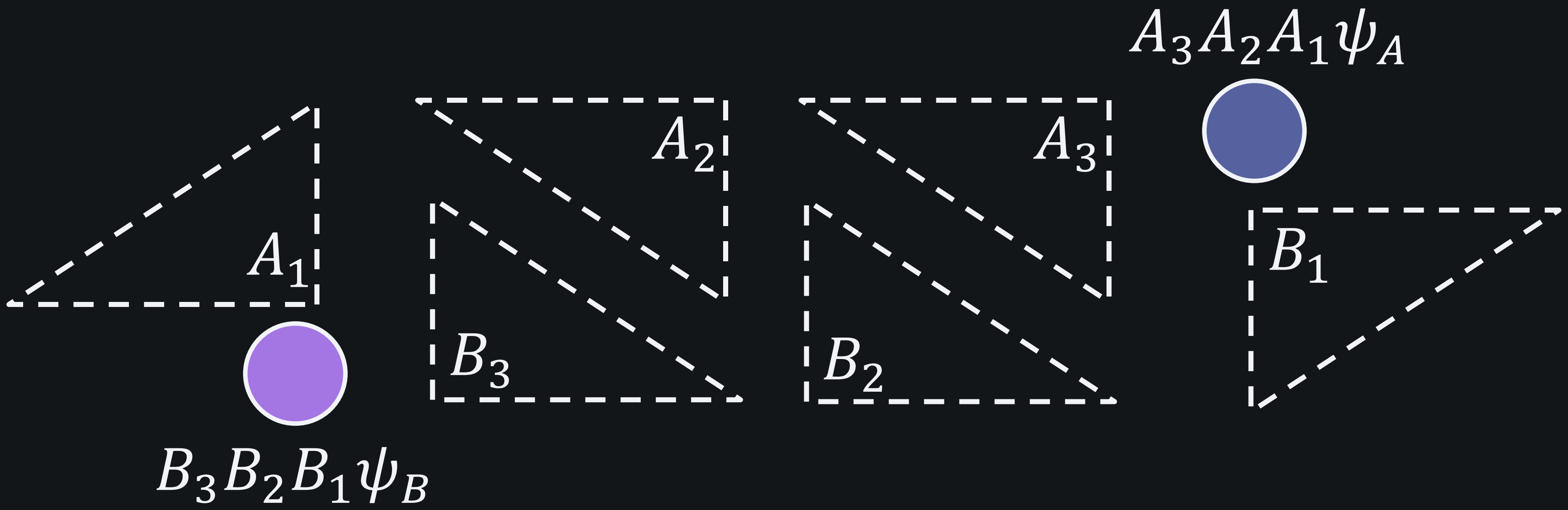
Multi-Hop

A_1, B_1, A_2, B_2 :one of I, X, Z, ZX



Multi-Hop

$A_1, B_1, A_2, B_2, A_3, B_3$:one of I, X, Z, ZX



Multi-Hop

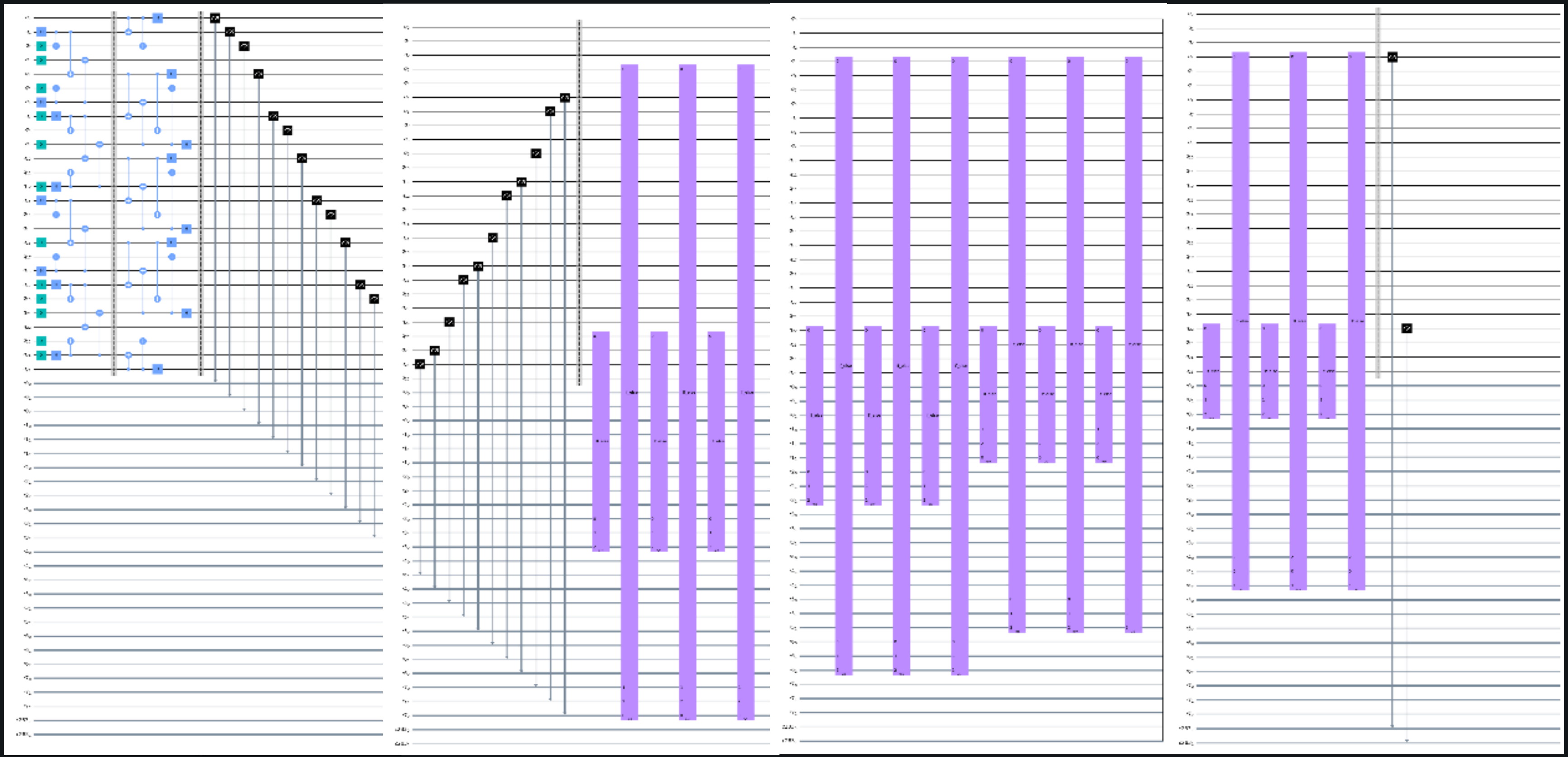
Qiskit

$A_1, B_1, A_2, B_2, A_3, B_3$:one of I, X, Z, ZX

$$\psi_A = A_1^{-1} A_2^{-1} A_3^{-1} A_3 A_2 A_1 \psi_A$$
$$\psi_B = B_1^{-1} B_2^{-1} B_3^{-1} B_3 B_2 B_1 \psi_B$$

```
ch = Multinode(0, 25, [([1, 2, 4], [3, 5, 6]),
                       ([7, 8, 10], [9, 11, 12]),
                       ([13, 14, 16], [15, 17, 18]),
                       ([19, 20, 22], [21, 23, 24])],
                       [(2, 3), (4, 5), (1, 0), (6, 7)])
```

Multi-Hop



3-Hop Circuit, 27 qubits

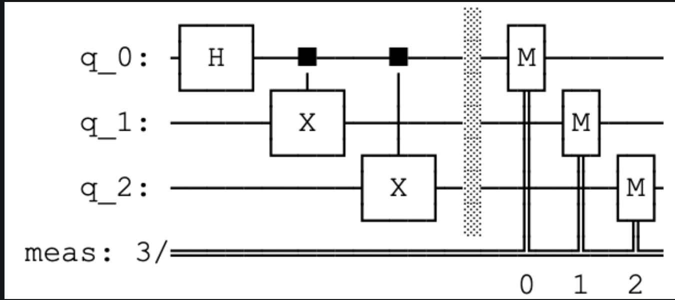
Problem Statements

2

Improve GHZ circuit with Pulse-level Calibration

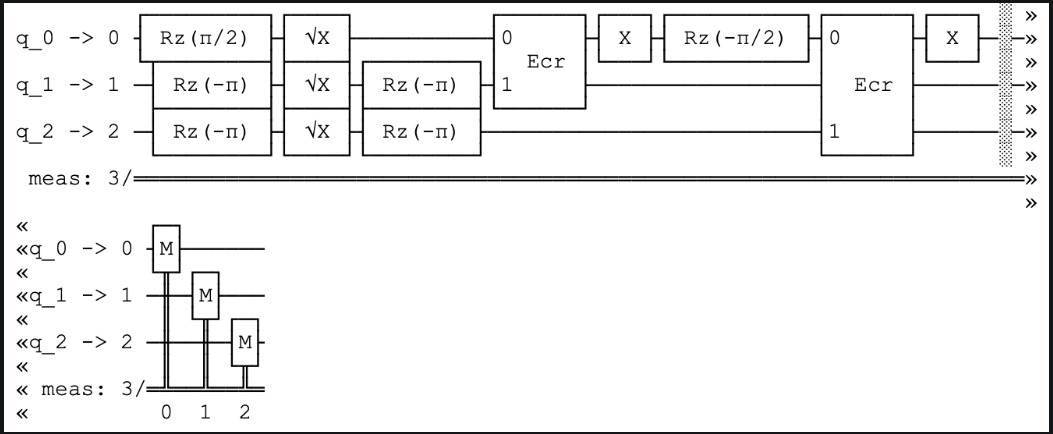
Optimize Pulse Parameters

Single Qubit Calibration



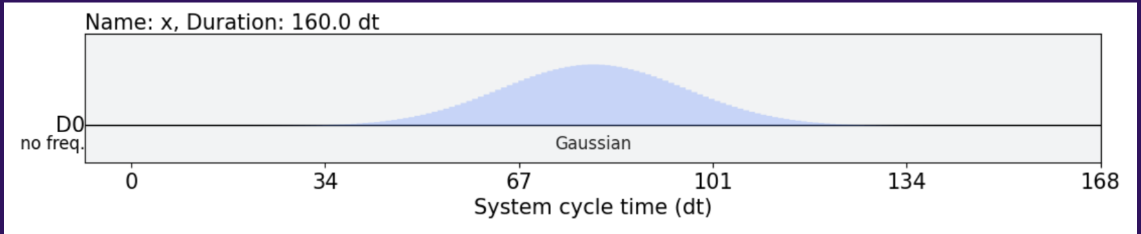
3-qubit GHZ Gate Abstraction

Calibrated 3-qubit GHZ Gate

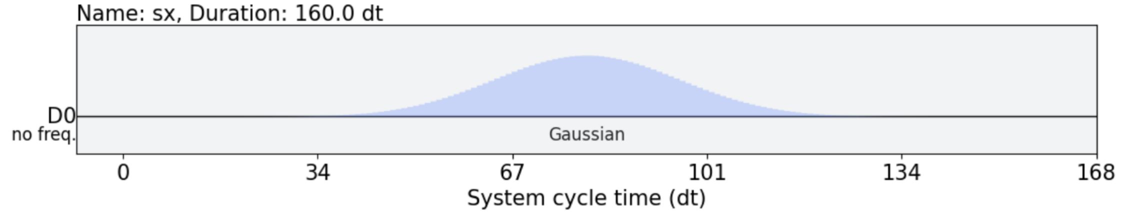


Single Qubit Calibration

X Gate



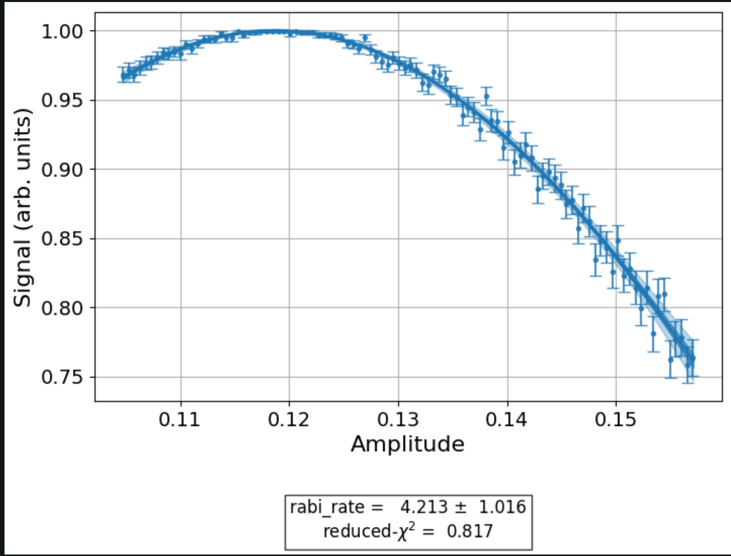
SX Gate



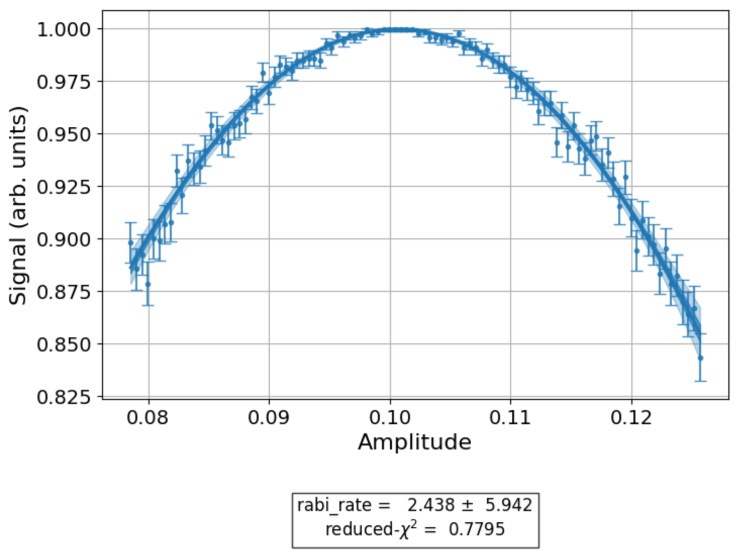
Single Qubit Calibration

Qiskit

Amplitude Optimization



Qubit 1



Qubit 2

Single Qubit Calibration

Qiskit

SX Gate Optimal Values

In [173]:
```python
from qiskit import QuantumCircuit
from qiskit_experiments.library import ProcessTomography
qc = QuantumCircuit(2)
qc.sx(0)

exp = ProcessTomography(qc, physical_qubits=(0, 1), backend=backend)
exp.analysis.set_options(fitter="cvxpy_linear_lstsq")
exp_data = exp.run().block_for_results()
exp_data.analysis_results("process_fidelity").value
```
Out[173]: 0.9866742495096311

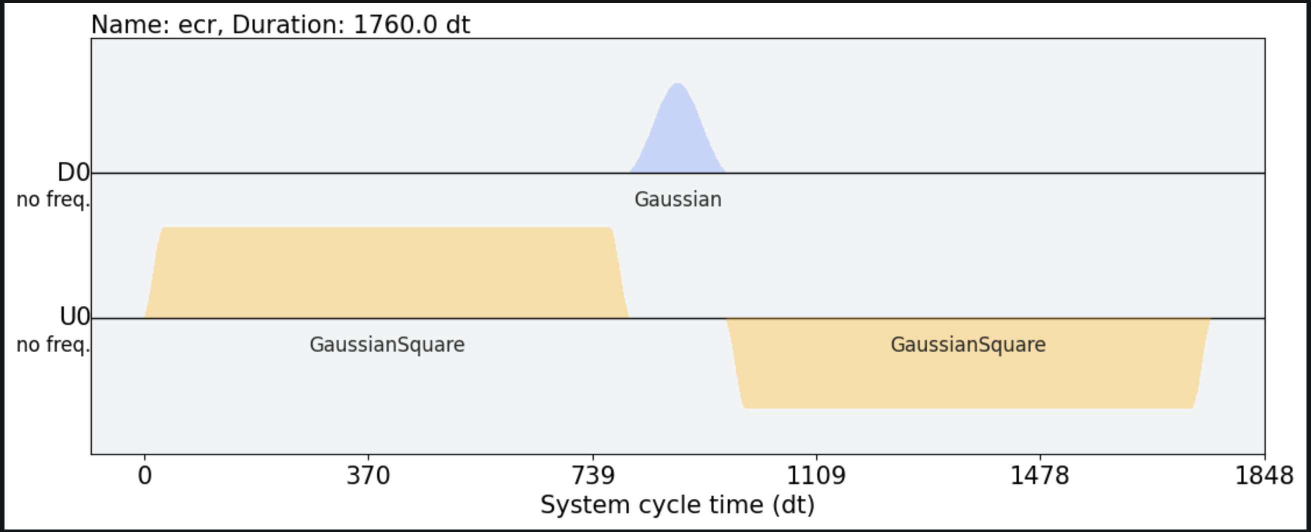
In [174]:
```python
from qiskit import QuantumCircuit
from qiskit_experiments.library import ProcessTomography
qc = QuantumCircuit(2)
qc.sx(1)

exp = ProcessTomography(qc, physical_qubits=(0, 1), backend=backend)
exp.analysis.set_options(fitter="cvxpy_linear_lstsq")
exp_data = exp.run().block_for_results()
exp_data.analysis_results("process_fidelity").value
```
Out[174]: 0.9851802814896147

X Gate Optimal Values

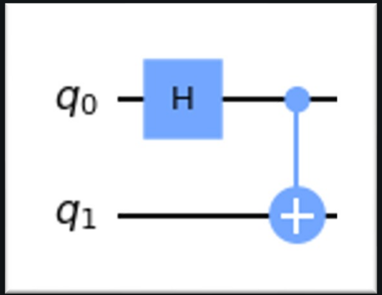
In [175]:
```python
from qiskit import QuantumCircuit
from qiskit_experiments.library import ProcessTomography
qc = QuantumCircuit(2)
qc.x(0)

exp = ProcessTomography(qc, physical_qubits=(0, 1), backend=backend)
exp.analysis.set_options(fitter="cvxpy_linear_lstsq")
exp_data = exp.run().block_for_results()
exp_data.analysis_results("process_fidelity").value
```
Out[175]: 0.9826479135791473

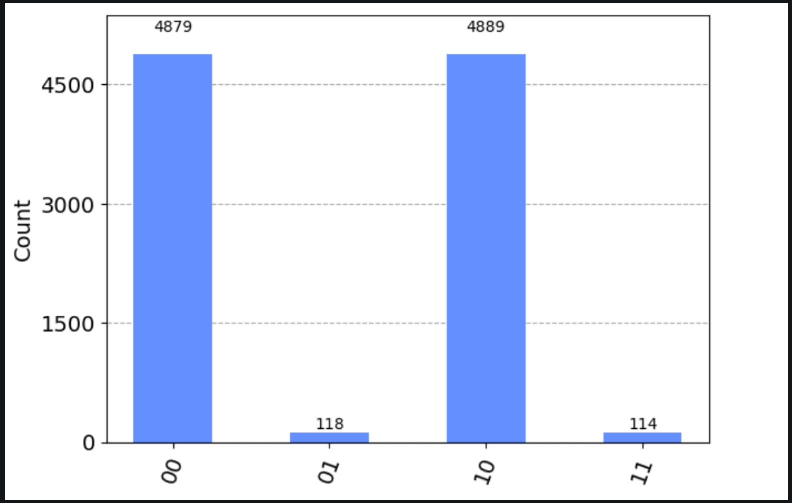
In [176]:
```python
from qiskit import QuantumCircuit
from qiskit_experiments.library import ProcessTomography
qc = QuantumCircuit(2)
qc.x(1)

exp = ProcessTomography(qc, physical_qubits=(0, 1), backend=backend)
exp.analysis.set_options(fitter="cvxpy_linear_lstsq")
exp_data = exp.run().block_for_results()
exp_data.analysis_results("process_fidelity").value
```
Out[176]: 0.98157197918988

Cross Resonance Pulse Calibration

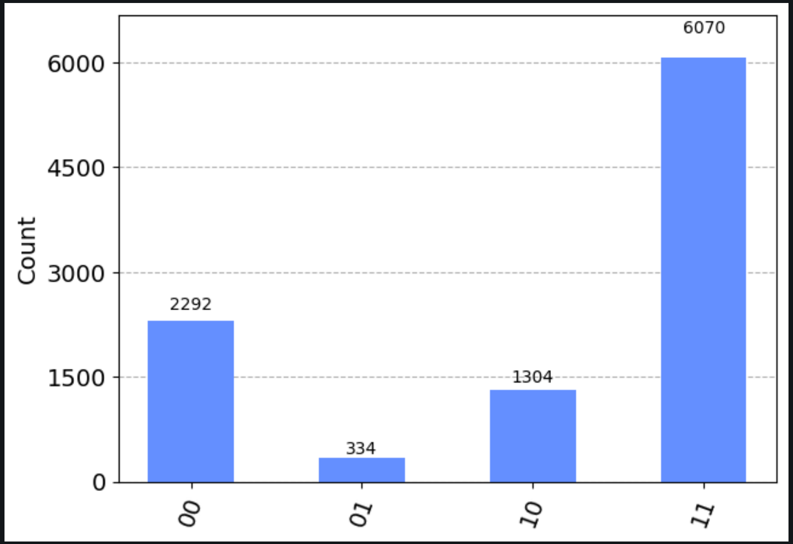


Cross Resonance Pulse Calibration



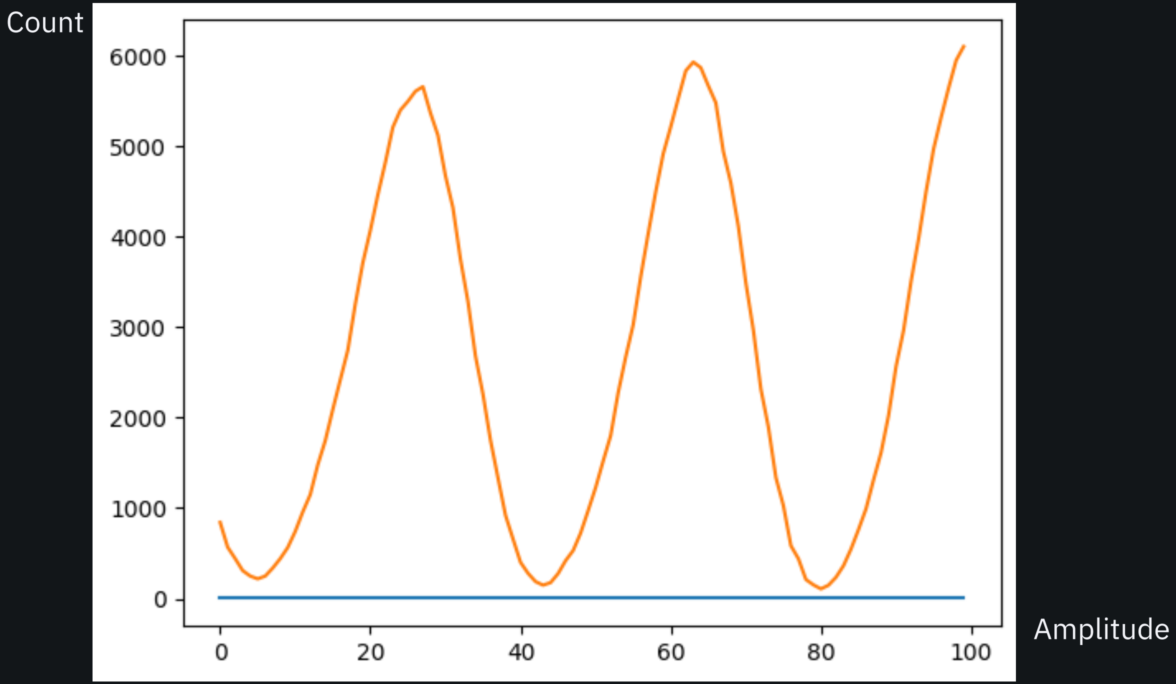


Poor Calibration



Optimized Calibration

Cross Resonance Pulse Calibration



CNOT Gate Calibration by Optimizing Amplitude

J. Jang, et. al

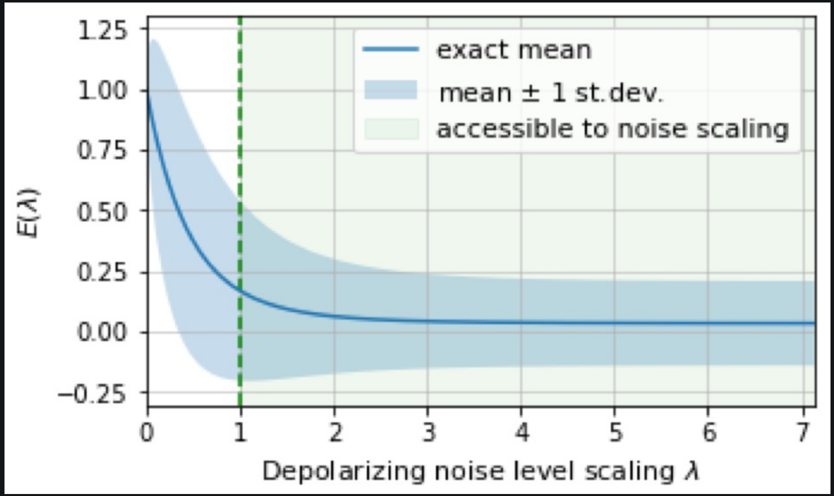
Problem Statements

3

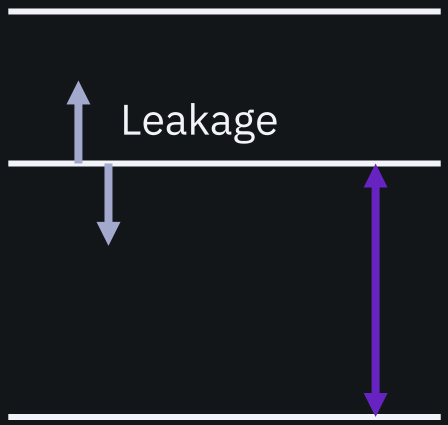
Zero-noise Extrapolation

DRAG pulse and Rotary term

Zero-Noise Extrapolation

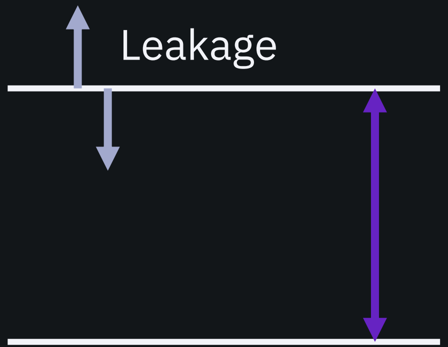


T. Girugica-Tiron, et. al, "Digital Zero noise extrapolation for quantum error mitigation," Stanford University, 2021.



Pulse Calibration: DRAG

Qiskit



$$DRAG(x, x_0, A, \sigma, \beta) = A \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right) + i\beta \frac{d}{dx}[A \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)]$$

Limits Leakage to higher energy levels

Pulse Calibration: Rotary Term

Qiskit

$$H = \sum\{\omega_j b_j^\dagger b_j + \frac{\delta_j}{2} b_j^\dagger b_j \left(b_j^\dagger b_j - I\right)\} + J\left(b_0^\dagger b_1 + b_1^\dagger b_0\right) + \Omega\cos(\omega_d t + \phi_c)(b_0^\dagger + b_0)$$

Block-Diagonalization

$$H(\Omega) = \nu_{IX}\frac{IX}{2} + \nu_{IZ}\frac{IZ}{2} + \nu_{ZI}\frac{ZI}{2} + \nu_{ZX}\frac{ZX}{2} + \nu_{ZZ}\frac{ZZ}{2},$$

Echoed Cross-Resonance Gate

$$ZX_{\pi/2} = XI \cdot ZX_{-\pi/4} \cdot XI \cdot ZX_{\pi/4}$$