Seoul National Uiniversity

M1522.000900 Data Structure

Homework 4: Binary Trees (Chapter 5)

Computer Science & Engineering

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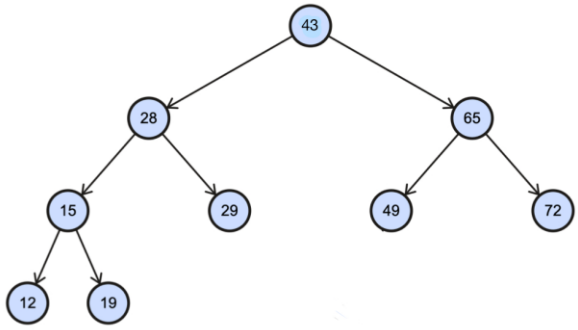
Question 1

The number of 2-degree nodes in any binary tree is one less than the number of leaves.

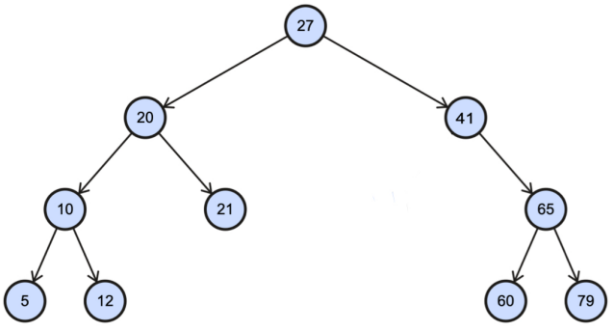
If the number of leaves in a binary tree is n, then the number of 2-degree nodes is n-1.

Question 2

(a) deleting value 32

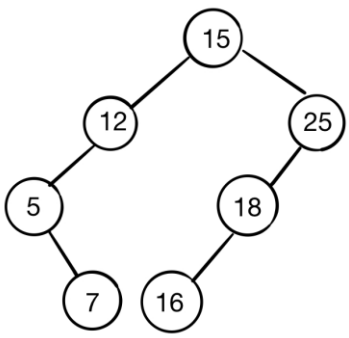


(b) deleting value 44



Question 3

(a)



(b)

A. preorder: 15-12-5-7-25-18-16

B. inorder: 5-7-12-15-16-18-25

C. postorder: 7-5-12-16-18-25-15

Question 4

(1) find max-heap

(a) node that has value 9 is children of node that has value 8. This binary tree is not a max-heap because the value of parent is less than value of its children.

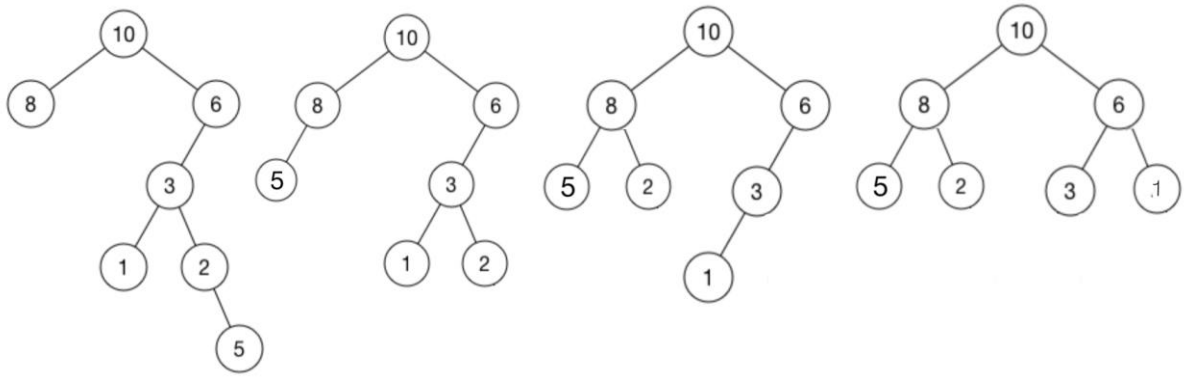
(b) This binary tree is not a max-heap because the value of parent is less than values of its children.

(c) This binary tree is not a max-heap because it is not a complete binary tree. (the bottom level doesn't have nodes to the left side.)

(d) This binary tree is a max-heap. It is a complete binary tree where the value of each parent is greater than or equal to the values of its children. (all levels except the last level are completely full and the bottom level has all nodes to the left side.)

(2)

0 1 2 3



The minimum number of nodes that should be moved is 3. The output is complete binary tree and the value of each parent is greater than to the values of its children.

Question 5

Base case: the sum of depth of each node in a perfect binary tree of height 1 is 0.

Induction Hypothesis: Assume that the sum of depth of each node in a perfect binary tree of height h-1 is ((h-1)-2)($2^{h-1} - 1$) + (h-1).

Induction step: the sum of depth of each node in a perfect binary tree of height h is (the sum of depth of each node in a perfect binary tree of height h-1) + (the sum of depth of nodes that depth is h-1) By induction hypothesis, the sum of depth of each node in a perfect binary tree of height h-1 is ((h-1)-2)($2^{h-1} - 1$) + (h-1). And the sum of depth of nodes that depth is h-1 is (h-1) 2^{h-1}. 2^{h-1} is the number of leaf nodes.

((h-1)-2)($2^{h-1} - 1$) + (h-1) + (h-1) 2^{h-1}

= (h-2) 2^h + 2

= (h-2) (2^h -1) + h

Thus, the sum of depth of each node in a perfect binary tree of height h is (h-2) (2^h -1) + h.