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CS156a Set 3

1. B

$$2e^{-2*0.05^2*1000} = 0.164$$
$$2e^{-2*0.05^2*1000} = 0.0135$$

Therefore 1000 is the least number of examples N among the given choices.

2. C

$$2*10e^{-2*10*0.05^2*1000} = 0.135$$
$$2*10e^{-2*10*0.05^2*1500} = 0.0111$$

Therefore 1500 is the least number of examples N among the given choices.

3. D

$$2*100e^{-2*10*0.05^2*1500} = 0.111$$
$$2*100e^{-2*10*0.05^2*2000} = 0.0098$$

Therefore 2000 is the least number of examples N among the given choices.

4. B

It is not possible to split 5 points into every possible combination of 2 and 3 points, given a configuration of 5 points. This means that we can't get every combination of splits for 5 points so the breakpoint for the Perceptron Model in $\mathbb{R}^3$ is 5.

5. B

(i), (ii), and (v) are all possible growth functions because they were shown to be valid in lecture (for ii, $1 + N + \binom{N}{2} = \binom{N+1}{2} + 1$).
(iii) are not growth functions because if they were, they would violate the key inequality from lecture 6: $m_{\mathcal{H}}(N) \leq \sum_{i=0}^{k-1}\binom{N}{i}$ where k is a break point.
Note that a breakpoint exists at 2 for (iii) and at 1 for (iv):

$$\sum_{i=1}^{\lfloor\sqrt{2}\rfloor}\binom{2}{i} = 2 \neq 2^2$$

$$2^{\lfloor\frac{1}{2}\rfloor} = 1 \neq 2^1$$

But the inequality for each is not satisfied:

$$\sum_{i=1}^{\lfloor\sqrt{4}\rfloor}\binom{4}{i} \leq \sum_{i=0}^{1}\binom{4}{i} \to 10 \leq 5 \ \ (for\ N = 4)$$

$$2^{\lfloor 1 \rfloor} \leq \sum_{i=0}^{0}\binom{2}{i} \to 2 \leq 1 \ \ (for\ N = 2)$$

Therefore (iii) and (iv) cannot be formulas for growth functions.

6. C

The smallest break point for this hypothesis set is 5. 5 points is the lowest number of points at which we cannot alternate +1 and -1 for each point (starting with +1 so that there are 3 points with +1) because we don't have enough intervals; therefore we can't shatter 5 points because we'll never have the case where we alternate on each point starting with +1.

7. C
The growth function is $\binom{N+1}{4} + \binom{N+1}{2} + 1$. The $\binom{N+1}{4}$ term is for when we choose 4 "in-between" points for the ends of the intervals to be (2 ends for each interval). We must then add the $\binom{N+1}{2}$ term to account for the case in which one interval covers no points (the ends of the interval start and end in the same "in-between"). We must then add 1 to the expression to account for the case in which both intervals cover no points. We therefore have $\binom{N+1}{4} + \binom{N+1}{2} + 1$.

8. D
As explained in problem 7, the smallest breakpoint occurs when we have too few intervals to cover every other point. Because covering every other point requires $\frac{N}{2}$ intervals, we can deduce that the first breakpoint must occur when there are $2M + 1$ intervals.

9. D
We can apply similar logic as in the convex set example presented in lecture. The N points will be placed around the circumference of a circle, as this is how we will maximize the number of dichotomies. However, we are restricted to using a triangle to shatter. Each vertex of the triangle can cover as many adjacent points as it wants. Therefore the most demanding combination would be to have each adjacent point have alternating values. This works for $N = 7$ because there are 4 points to cover but 2 of them are adjacent, so only 3 vertices are required. However, for $N = 9$, there are 5 points to cover with 2 adjacent points, but only 3 vertices (but 4 would be required). Therefore the largest N that can be shattered is $N = 7$.

10. B
This learning model works the same as the positive intervals model seen in class. The interval can be chosen by choosing a and b so that the interval is between a and b. The points are then measured based on their distance from the origin in the sense that if the distance from the origin is between a and b, then the point is in the interval and will be classified as +1. Otherwise it's outside of the interval and is classified as -1. Therefore this model has the same growth function as the one in lecture:

$$m_{\mathcal{H}}(N) = \binom{N+1}{2} + 1$$