1.a)
$$\sigma^{2} = \frac{1}{N} \sum_{i=1}^{N} (x_{i} - M)^{2}$$
 by definition

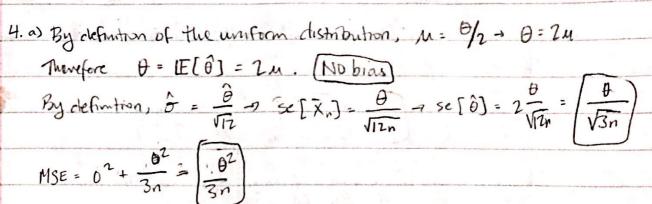
$$= \frac{1}{N} \sum_{i=1}^{N} [x_{i}^{2} - 2x_{i} + n^{2}] = \frac{1}{N} (1 - M)$$

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$$= \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N} \sum_{i=1}$$



b)
$$M = \sqrt[9]{2} \rightarrow \theta = 2M \rightarrow Bins = 2M - F[\hat{\theta}]$$

$$E[\hat{\theta}] = \int_{0}^{\theta} \frac{x}{\theta} (n) (\frac{x}{\theta}) dx = \frac{\theta n}{n+1} \rightarrow bins = \theta - \frac{\theta n}{n+1} = \theta (1 - \frac{n}{n+1})$$

By definition of the number of ensirabilities, we have

$$E[\hat{\theta}] = \int_{0}^{\theta} \frac{x^{2}}{\theta} (n) (\frac{x}{\theta})^{n-1} dx = \frac{\theta^{2} n}{n+2}$$

$$V[\hat{\theta}] = E[\hat{\theta}^{2}] - E[\hat{\theta}]^{2} = \theta^{2}n - \frac{\theta^{2}n^{2}}{n+2}$$

$$M+2 = \frac{\theta^{2}n}{n+2} - \frac{\theta^{2}n^{2}}{n+2}$$

$$M+2 = \frac{\theta^{2}n}{n+2} - \frac{\theta^{2}n^{2}}{n+2}$$

c) We see that the MSE of Dur & in (b) has n2 terms in the denomicator, whereas & in (a) has only n. Therefore, for large n we can conclude that ar estimate used in part (b) is more efficient.