

In[]:= D[-1/4 * Log[q[x]], x]

$$\text{Out[]:= } -\frac{q'[x]}{4q[x]}$$

In[]:= D[-q'[x]/(4*q[x]), x]

$$\text{Out[]:= } \frac{q'[x]^2}{4q[x]^2} - \frac{q''[x]}{4q[x]}$$

In[]:= D[Sqrt[q[x]], x]

$$\text{Out[]:= } \frac{q'[x]}{2\sqrt{q[x]}}$$

In[]:= Integrate[q''[x]/(2*q[x]^(3/2)) + q'[x]/(2*q[x]) - q'[x]^2/(2*q[x]^(5/2)), x]

$$\text{Out[]:= } \int \left(\frac{q'[x]}{2q[x]} - \frac{q'[x]^2}{q[x]^5} + \frac{q''[x]}{2q[x]^{3/2}} \right) dx$$

In[]:= D[q''[x]/(8*q[x]^(3/2)) - 5*q'[x]^2/(32*q[x]^(5/2)), x]

$$\text{Out[]:= } \frac{25q'[x]^3}{64q[x]^{7/2}} - \frac{q'[x]q''[x]}{2q[x]^{5/2}} + \frac{q^{(3)}[x]}{8q[x]^{3/2}}$$

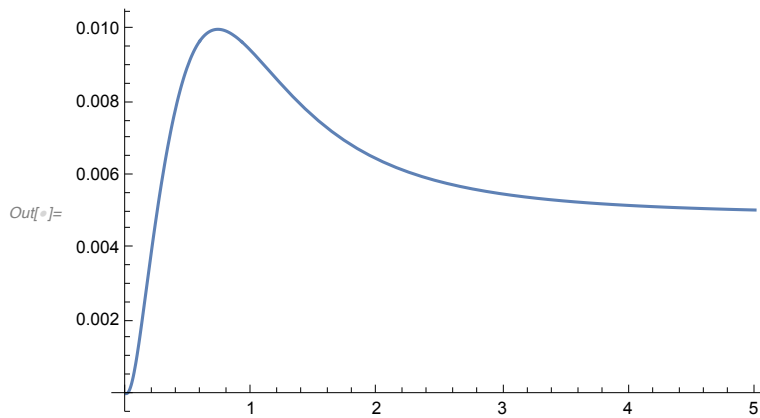
In[]:= Integrate[9*q'[x]*q''[x]/(32*q[x]^3) - 15*q'[x]^3/(64*q[x]^4) - q'''[x]/(16*q[x]^2), x]

$$\text{Out[]:= } \frac{5q'[x]^2}{64q[x]^3} - \frac{q''[x]}{16q[x]^2}$$

In[]:= sol = DSolve[{epsilon^2*y''[x] == (1+x^2)^2*y[x], y[0] == 0, y'[0] == 1}, y[x], x]

$$\text{Out[]:= } \left\{ \left\{ y[x] \rightarrow \text{DifferentialRoot} \left[\text{Function} \left[\{y, x\}, \left\{ -\left(1+x^2\right)^2 y[x] + \epsilon^2 y''[x] == 0, y[0] == 0, y'[0] == 1 \right\} \right] [x] \right\} \right\}$$

```
In[ ]:= eru = {ε → 1 / 17};
Plot[(First[y[x] /. sol /. eru] - (ε /. eru) / Sqrt[1 + x^2] *
      Sinh[1 / (ε /. eru) * (x + x^3 / 3)]) / First[y[x] /. sol /. eru], {x, 0, 5}]
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```
In[ ]:= eru = {ε → 2 / 35};
N@MaxValue[(First[y[x] /. sol /. eru] - (ε /. eru) / Sqrt[1 + x^2] *
            Sinh[1 / (ε /. eru) * (x + x^3 / 3)]) / First[y[x] /. sol /. eru], x]
```

Out[]:= 0.00975005

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In[ ]:= N@2 / 35
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Out[]:= 0.0571429

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In[ ]:= ClearAll[y1, y2, y1p, y2p, y1pp, y2pp, u1, u2, u1p, u2p]
```

```
In[ ]:= y1[x_] := q[x] ^ (-1 / 4) * Exp[1 / ε * Integrate[Sqrt[q[r]], {r, s, x}]]
y1[x]
```

Out[]:=
$$\frac{e^{\frac{\int_s^x \sqrt{q[r]} dr}{\epsilon}}}{q[x]^{1/4}}$$

```
In[ ]:= y2[x_] := q[x] ^ (-1 / 4) * Exp[-1 / ε * Integrate[Sqrt[q[r]], {r, s, x}]]
y2[x]
```

Out[]:=
$$\frac{e^{-\frac{\int_s^x \sqrt{q[r]} dr}{\epsilon}}}{q[x]^{1/4}}$$

```
In[ ]:= u1p[x_] := Simplify[-y2[x] * f[x] * (1 / ε^2) / (y1[x] * y2'[x] - y2[x] * y1'[x])]
u1p[x]
```

Out[]:=
$$\frac{e^{-\frac{\int_s^x \sqrt{q[r]} dr}{\epsilon}} f[x]}{2 \epsilon q[x]^{1/4}}$$

In[]:= **u2p[x_] := Simplify[y1[x] * f[x] * (1 / e^2) / (y1[x] * y2'[x] - y2[x] * y1'[x])]**
u2p[x]

$$\text{Out[]:= } -\frac{e^{\frac{\int_0^x \sqrt{q[r]} \, dr}{e}} f[x]}{2 e q[x]^{1/4}}$$

In[]:= **u1[x_] := Integrate[u1p[s], {s, 0, x}]**
u1[x]

$$\text{Out[]:= } \int_0^x \frac{f[s]}{2 e q[s]^{1/4}} \, ds$$

In[]:= **u2[x_] := Integrate[u2p[s], {s, 0, x}]**
u2[x]

$$\text{Out[]:= } \int_0^x -\frac{f[s]}{2 e q[s]^{1/4}} \, ds$$

In[]:= **u1[x] * y1[x] + u2[x] * y2[x]**

$$\text{Out[]:= } \frac{e^{-\frac{\int_0^x \sqrt{q[r]} \, dr}{e}} \int_0^x -\frac{f[s]}{2 e q[s]^{1/4}} \, ds}{q[x]^{1/4}} + \frac{e^{\frac{\int_0^x \sqrt{q[r]} \, dr}{e}} \int_0^x \frac{f[s]}{2 e q[s]^{1/4}} \, ds}{q[x]^{1/4}}$$

In[]:= **Integrate[(a * (x - x0) ^ beta) ^ (1 / 2), x]**

$$\text{Out[]:= } \frac{2 (x - x0) \sqrt{a (x - x0)^\beta}}{2 + \beta}$$

In[]:= **Integrate[-5 * a^2 / (32 * (a * (x)) ^ (5 / 2)), x]**

$$\text{Out[]:= } \frac{5 a^2 x}{48 (a x)^{5/2}}$$

In[]:= **q[x_] := a * (x - x0) ^ beta**

In[]:= **Integrate[q'[x] / (8 * q[x] ^ (3 / 2)) - 5 * q'[x]^2 / (32 * q[x] ^ (5 / 2)), x]**

$$\text{Out[]:= } \frac{\beta (4 + \beta)}{16 (x - x0) \sqrt{a (x - x0)^\beta} (2 + \beta)}$$

In[]:= **DSolve[{y'[x] + y[x] == 0, y[0] == 0, y'[0] == 1}, y[x], x]**

$$\text{Out[]:= } \{ \{y[x] \rightarrow \sin[x]\} \}$$

In[]:= **DSolve[{y'[x] + y[x] == -Sin[x]^3, y[0] == 0, y'[0] == 0}, y[x], x]**

$$\text{Out[]:= } \left\{ \left\{ y[x] \rightarrow \frac{1}{32} \left(12 x \cos[x] - 8 \sin[x]^5 - 8 \cos[x] \sin[2 x] + \cos[x] \sin[4 x] \right) \right\} \right\}$$