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ACM157 Set 5

1\. a)

$$\beta(\theta) = \mathbb{P}(\{X_1,\dots,X_n\} \in R \mid \theta) = \mathbb{P}(X_{(n)} > c \mid \theta)$$

If $\theta < c$, $\beta(\theta) = 0$

• If $\theta \geq c$, $\mathbb{P}(X_{(n)} > c \mid \theta) = 1 - \left(\frac{c}{\theta}\right)^n$

$$\rightarrow \beta(\theta) = \begin{cases} 0 & \theta < c \\ 1 - \left(\frac{c}{\theta}\right)^n & \theta \geq c \end{cases}$$

b) $$\alpha = \sup_{\theta \in H_0} \beta(\theta) = \beta(1/2) = \begin{cases} 0 & 1/2 < c \\ 1-(2c)^n & 1/2 \geq c \end{cases}$$

$$\rightarrow \alpha = 1 - (2c)^n \rightarrow 2c = [1-\alpha]^{1/n} \rightarrow \boxed{c = \tfrac{1}{2}(1-\alpha)^{1/n}, \quad c \leq 1/2}$$

c) let $c = 0.48 \rightarrow 0.48 = \frac{1}{2}(1-\alpha)^{1/20} \rightarrow 0.96^{20} = 1-\alpha$

$$\rightarrow p_{val} = 1 - 0.96^{20} = \boxed{0.558}$$

2\. a) $\beta(\theta) = \mathbb{P}(\{X_1,\dots,X_n\} \in R \mid \theta) = \mathbb{P}(\bar{X}_n > c \mid \theta)$

$$\alpha = \sup_{\theta \in H_0} \beta(\theta) = P(\bar{X}_n > c \mid \theta = 0) = 1 - \Phi(\sqrt{n}\, c)$$

$$\rightarrow \sqrt{n}\, c = \Phi^{-1}(1-\alpha)$$

$$\rightarrow \boxed{c = \frac{\Phi^{-1}(1-\alpha)}{\sqrt{n}}}$$

b) $\beta(1) = \mathbb{P}(\bar{X}_n > c \mid \theta = 1) = \boxed{1 - \Phi(\sqrt{n}(c-1))}$ c defined above

c) $$\lim_{n\to\infty} \beta(\theta) = 1 - \lim_{n\to\infty} \Phi\left(\frac{\sqrt{n}(c-\mu)}{1}\right) = 1 - 0 = 1$$

3\. a) $c = -z_{\frac{\alpha}{2}}$

Note that $\hat{\lambda}_{MLE} = \bar{X}_n$, $\hat{se}_{MLE} = (n I(\bar{X}_n))^{-1/2}$

$$\rightarrow \text{reject } H_0 \text{ if } \left| (\bar{X}_n - \lambda_0)(n I(\bar{X}_n))^{1/2} \right| > -z_{\frac{\alpha}{2}}$$