Jacob Snyder

ACM106B Set 4 Problem 1

1. $-(a(x)u'(x))' = f(x)$

$\rightarrow -[a'(x)u'(x) + a(x)u''(x)] = f(x)$

$\rightarrow -[a'(x)u'(x)v(x) + a(x)u''(x)v(x)] = f(x)v(x)$

$\rightarrow -\int_0^1 a'(x)u'(x)v(x)\,dx - \underbrace{\int_0^1 a(x)u''(x)v(x)\,dx}_{\text{applying integration by parts to this term yields:}} = \int_0^1 f(x)v(x)\,dx$

$\rightarrow -\int_0^1 a'(x)u'(x)v(x)\,dx - \underbrace{[a(x)v(x)u'(x)]_0^1}_{\text{This term goes to zero if we impose that } v(x) \text{ be periodic on } [0,1].} + \int_0^1 u'(x)a'(x)v(x) + u'(x)a(x)v'(x)\,dx = \int_0^1 f(x)v(x)\,dx$

$\rightarrow \int_0^1 u'(x)a(x)v'(x)\,dx = \int_0^1 f(x)v(x)\,dx$

$\rightarrow (au', v') = (f, v)$

In V_h, we can write $u(x)$ as follows:

$u_h(x) = \sum_{j=1}^{N} u_j \varphi_j(x)$ where $\varphi_j(x) = \sin(2\pi j x)$ for $1 \le j \le N/2$

$\varphi_j(x) = \cos(2\pi(j - N/2)x)$ for $N/2 + 1 \le j \le N$

$\rightarrow u_h'(x) = \sum_{j=1}^{N} u_j \varphi_j'(x)$

$\rightarrow \sum_{j=1}^{N} u_j (a\varphi_j', \varphi_i') = (f, \varphi_i)$ (Let $v = \varphi_i$. Note that φ_i is periodic on $[0,1]$ so our earlier imposition stands)

$\rightarrow Au = b$

where $a_{ij} = (a\varphi_j', \varphi_i')$, $b_i = (f, \varphi_i)$

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ACM106B Set 4 Problem 2

1. $-\nabla \cdot (k(x) \nabla u) = f$

$= -k(x)\nabla \cdot \nabla u + (\nabla k(x)) \cdot \nabla u = f$

$= -k(x)\Delta u + \nabla k \cdot \nabla u = f$

$\rightarrow (-k\Delta u)v + (\nabla k \cdot \nabla u)v = fv$

$\rightarrow \int_\Omega -k[\nabla u \cdot \nabla v]\, dx + \int_\Omega (\nabla k \cdot \nabla u)v\, dx = \int fv\, dx$

So $a(u,v) = \int_\Omega -k[\nabla u \cdot \nabla v] + [\nabla k \cdot \nabla u]v\, dx$

and $F(v) = \int_\Omega fv\, dx$

2. $a(u-u_h, v)$

$= \int_\Omega -k[\nabla(u-u_h)\cdot\nabla v] + v[\nabla k \cdot \nabla(u-u_h)]\, dx$

$= \int_\Omega -k[\nabla u \cdot \nabla v - \nabla u_h \cdot \nabla v] + v[\nabla k \cdot \nabla u - \nabla k \cdot \nabla u_h]\, dx$

$= \int_\Omega -k[\nabla u \cdot \nabla v] + v[\nabla u \cdot \nabla k]\, dx - \int -k[\nabla u_h \cdot \nabla v] + v[\nabla u_h \cdot \nabla k]\, dx$

$= F(v) - F(v)$

$= 0$ as desired.

3. No submission

4. No submission

5. No submission