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Section 8

ACM116 Set 1

1. P(at least 2 common birthdays) = 1 - P(no common birthdays)

P(no common birthdays) = P(2nd unique birthday)P(3rd unique birthday)... P(nth unique birthday)

P(2nd unique birthday) = $\frac{364}{365}$

P(3rd unique birthday) = $\frac{363}{365}$

P(nth unique birthday) = $\frac{366-n}{365}$

→ P(no common birthdays) = $\prod_{i=1}^{n} \frac{366-i}{365}$

⇒ $\boxed{P(\text{at least 2 common birthdays}) = 1 - \prod_{i=1}^{n} \frac{366-i}{365}}$ $n=55$ → probability is 0.9863

plot is attached.

2. P(have disease | tested positive) = $\frac{P(\text{test positive and have disease})}{P(\text{test positive})} = \frac{(0.99)(0.001)}{(0.99)(0.001)+(0.999)(0.02)}$

$= \boxed{0.04721}$

3. a) $\mathbb{E}(X)$ = (number of trials)(probability of success) = np b) Matlab - average X is 29.98, which is very close to $\mathbb{E}[X] = 30$.

4. Matlab

5. a) $X = F^{-1}(Z) \rightarrow F(X) = Z$ where $Z \sim U[0,1]$

It follows that F is the CDF of X: the only way for $F(X) \sim U[0,1]$ is for the high probability regions of X to correspond to quickly changing values of F(X) and low probability regions of X to correspond to slowly changing values of F(X), which describes the CDF of X. Therefore F is the CDF of X.

b) Matlab

6. We are guaranteed one half of the stick. The break will always occur at a random point of the "other half" of the stick. On average, the "other half" will have its break at its midpoint (of the half), so we can safely state that the expected length of the longer stick is $\underset{\text{(guaranteed half)}}{0.5} + \underset{\text{(expected length of "other half")}}{0.25} = \boxed{0.75}$

8. Suppose we only need to get 1 more. Odds of success is $\frac{1}{n}$ so expected number of tries is n. If we need 2 more, then chance of success is $\frac{2}{n}$ so expected number of tries to get another is $\frac{n}{2}$.* Following this pattern back to the beginning, the expected number of tries at any given point to get another unowned pokemon given that we already have p unique pokemon is $\frac{n}{n-p}$. Therefore the total expected number of tries is $\boxed{\sum_{i=0}^{n-1} \frac{n}{n-i}}$

* Shown on backside

(cont'd on back)

7. I simulates an unbiased coin, II does not.

Explanation: I: The game only ends when the 2 flips have different outcomes. If we look at the distribution of results for flipping the coin twice given that the 2 outcomes are different, the order of the outcomes is independent; we are equally likely to have TH as HT. This makes this algorithm simulate a fair coin flip.

II: The problem with this algorithm is that the first flip determines the outcome of the algorithm. If we first see H, then the game will end with the next T, and vice versa. All this algorithm does is effectively flip the probabilities of H and T, which does not simulate a fair coin flip.

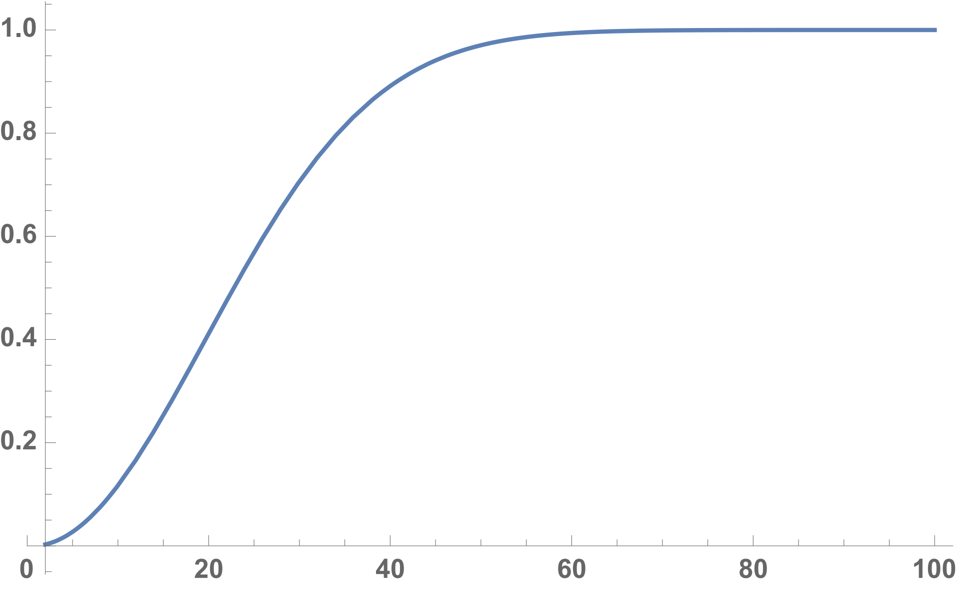
*8 addendum: proof that success probability of p corresponds to $\frac{1}{p}$ expected trials: let expected number of trials = E:

Game reset results in original game having expected trials of E+1 (recursive)

$$E = p(1) + (1-p)(E+1)$$

$$\rightarrow E = p + E + 1 - pE - p$$

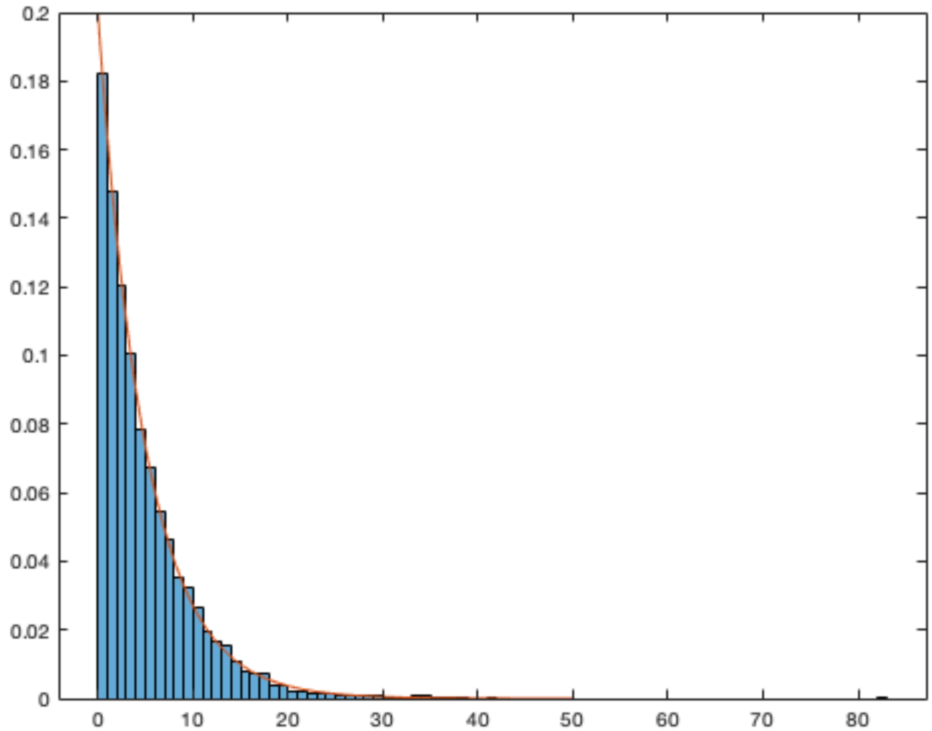
$$\rightarrow pE = 1 \rightarrow \boxed{E = \frac{1}{p}}$$



1.0
0.8
0.6
0.4
0.2
0
20
40
60
80
100

```
data = rand(10000,1);
data2 = icdf('Exponential',data,5);

histogram(data2,'Normalization','pdf');
hold on;
fplot(@(x) exppdf(x,5), [0 50]);
```



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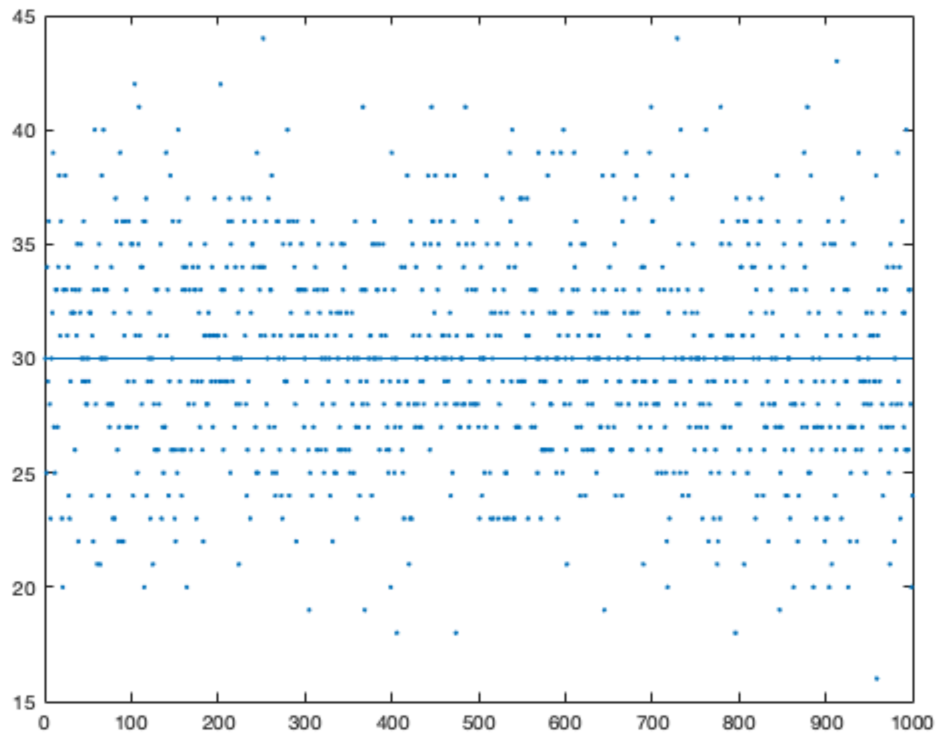
```
expvals=[];

for k=1:1000
  data = binornd(100, 0.3);
  mn = mean(data);
  expvals = [expvals mn];
end

disp(mean(expvals))

plot(expvals, '.');
hold on;
line([0, 1000],[30,30]);
```

```
    29.9540
```



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```
ab=0;
a=0;
b=0;

for k=1:1000
  data = unidrnd(6);
  if data<5
      b=b+1;
  end
  if data==2 || data==4
      a=a+1;
      ab=ab+1;
  elseif data==6
      a=a+1;
  end
end

atb=(a./1000).*(b./1000);

disp('P(AB)');
disp(ab./1000);
disp('P(A)P(B)');
disp(atb);
```

```
P(AB)
    0.3270

P(A)P(B)
    0.3284
```

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