

ACM104 Set 6

3. $F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Find eigenvalues:

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = -\lambda(1-\lambda) - 1 = \lambda^2 - \lambda - 1 \rightarrow \text{roots are } \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

eigenvalues: $\frac{1 \pm \sqrt{5}}{2}$

Find eigenvectors:

For $\lambda = \frac{1+\sqrt{5}}{2}$:

$$\begin{bmatrix} 1 - \frac{1+\sqrt{5}}{2} & 1 \\ 1 & -\frac{1+\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{aligned} \frac{1-\sqrt{5}}{2} v_1 + v_2 &= 0 \\ v_1 - \frac{1+\sqrt{5}}{2} v_2 &= 0 \Rightarrow v_1 = \frac{1+\sqrt{5}}{2} v_2 \Rightarrow \begin{bmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{bmatrix} \end{aligned}$$

For $\lambda = \frac{1-\sqrt{5}}{2}$:

$$\begin{bmatrix} 1 - \frac{1-\sqrt{5}}{2} & 1 \\ 1 & -\frac{1-\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{aligned} \frac{1+\sqrt{5}}{2} v_1 + v_2 &= 0 \\ v_1 - \frac{1-\sqrt{5}}{2} v_2 &= 0 \Rightarrow v_1 = \frac{1-\sqrt{5}}{2} v_2 \Rightarrow \begin{bmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{bmatrix} \end{aligned}$$

Therefore, $S = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{bmatrix}$

Therefore we have $S^{-1}FS = D$