Time Series Analysis for Statistical Arbitrage: A Cross-Asset Approach

Sharvil Oza and Ashutosh Singarwal

Abstract

This research develops a framework for time series modeling to identify statistical arbitrage opportunities across diverse asset classes. We leverage cointegration relationships between financial instrument pairs, using ARIMA processes to model conditional mean dynamics integrated with GARCH specifications for time-varying volatility. Our methodology enables precise decomposition of time series components while accounting for volatility clustering and employs spectral analysis for seasonality detection. Using daily price data spanning multiple market cycles, we focus on pairs such as junior gold miners (GDXJ) and natural gas (UNG). This framework identifies stationary relationships between non-stationary financial time series, providing insights into cross-asset dynamics and effectively identifying temporary pricing inefficiencies.

1 Introduction

Time series analysis offers powerful tools for investigating the dynamic behavior of financial instruments and identifying potential statistical arbitrage opportunities. This paper presents a comprehensive time series framework for detecting and modeling temporary mispricings between related financial instruments that exhibit mean-reverting behavior across multiple asset classes.

Our approach is grounded in cointegration analysis, which identifies long-term equilibrium relationships between financial time series despite individual non-stationarity. We extend traditional time series analysis by incorporating cross-asset analysis, conditional mean modeling through ARIMA processes, volatility modeling through GARCH specifications, and seasonality detection . This integrated approach enables more robust identification and characterization of exploitable patterns in financial time series.

2 Methodology

2.1 Data Collection and Preprocessing

Our analysis utilizes daily price data for various ETFs representing different asset classes and sectors. The selected assets include:

- US Treasury bonds of different durations (TLT, IEF, SHY, SPTL, VGIT)
- Corporate bonds (LQD, HYG)
- US Equity indices (SPY, VOO, IVV, QQQ, DIA)
- Sector ETFs (XLE, XOP, OIH)
- Gold and precious metals (GLD, IAU, GDX, GDXJ, SLV)
- Energy commodities (USO, UNG, BNO)
- Currency pairs (UUP, FXE, FXY)
- International markets (EWJ, EWG, EWU, EWC)

The data spans an 8-year period from January 2015 to January 2023, ensuring coverage of multiple market cycles. We used Yahoo Finance API for collecting the data.

2.2 Cointegration Analysis

The first step in our analytical framework is to identify pairs of assets that exhibit cointegration. We employ the Engle-Granger two-step method to test for cointegration:

1. We first establish that both time series are integrated of the same order, typically I(1). 2. We estimate the cointegrating regression: $Y_t = \alpha + \beta X_t + \epsilon_t$.

Pairs with statistically significant cointegration (p-values below 0.05) are selected for further analysis. The cointegration relationship implies that while individual asset prices may be non-stationary, there exists a linear combination of them that is stationary. This stationary combination, or spread, becomes the focus of our time series modeling efforts.

2.3 Time Series Modeling Framework

2.3.1 ARIMA Modeling for Conditional Mean

For each cointegrated pair, we develop a sophisticated time series model to capture the dynamics of the spread. The spread Z_t is calculated as:

$$Z_t = Y_t - (\alpha + \beta X_t) \tag{1}$$

where Y_t and X_t are the price series of the two assets, and α and β are the parameters from the cointegrating regression.

We apply ARIMA (Autoregressive Integrated Moving Average) models to capture the conditional mean dynamics of the spread:

$$\phi(L)(1-L)^d Z_t = \theta(L)\epsilon_t \tag{2}$$

where $\phi(L)$ is the autoregressive polynomial, $\theta(L)$ is the moving average polynomial, L is the lag operator, and d is the order of integration.

The optimal ARIMA parameters (p,d,q) are selected using information criteria (AIC and BIC) and residual diagnostics. We implement a systematic model selection procedure that evaluates multiple specifications and selects the model with the lowest information criterion subject to residual whiteness constraints.

2.3.2 GARCH Modeling for Conditional Variance

To account for volatility clustering typically observed in financial time series, we apply GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to the residuals of the ARIMA model:

$$\epsilon_t = \sigma_t \cdot z_t, \quad z_t \sim N(0,1) \tag{3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{4}$$

2.3.3 Seasonality Detection

We employ multiple approaches to detect and model seasonality in the spread series:

1. Spectral analysis to identify dominant frequencies in the time series. 2. Autocorrelation function (ACF) and partial autocorrelation function (PACF) analysis to detect periodic patterns. 3. Seasonal decomposition using methods such as STL (Seasonal and Trend decomposition).

When significant seasonality is detected, we extend our model to incorporate seasonal components:

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D Z_t = \theta(L)\Theta(L^s)\epsilon_t \tag{5}$$

where $\Phi(L^s)$ and $\Theta(L^s)$ are the seasonal autoregressive and moving average polynomials, respectively, with seasonal period s.

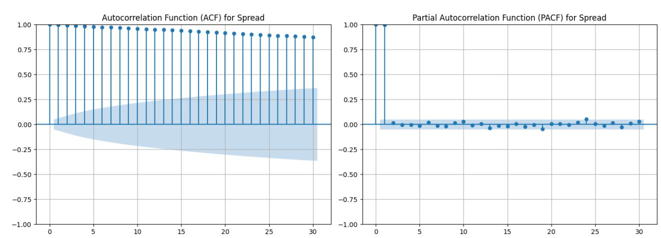

Figure 1: Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) for the GDXJ-UNG spread.

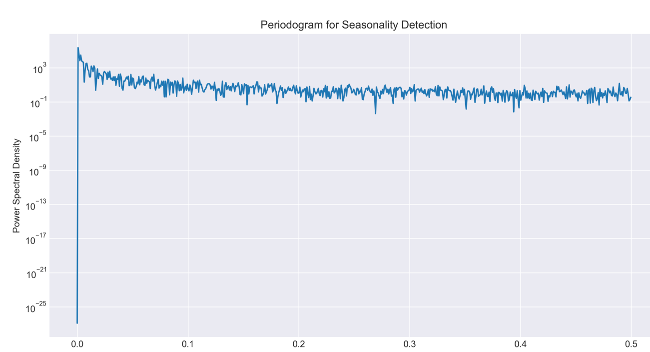

Figure 2: Seasonality Detection for GDXJ-UNG Spread. The plot visualizes the periodogram and spectral density of the spread, revealing cyclical patterns at specific frequencies.

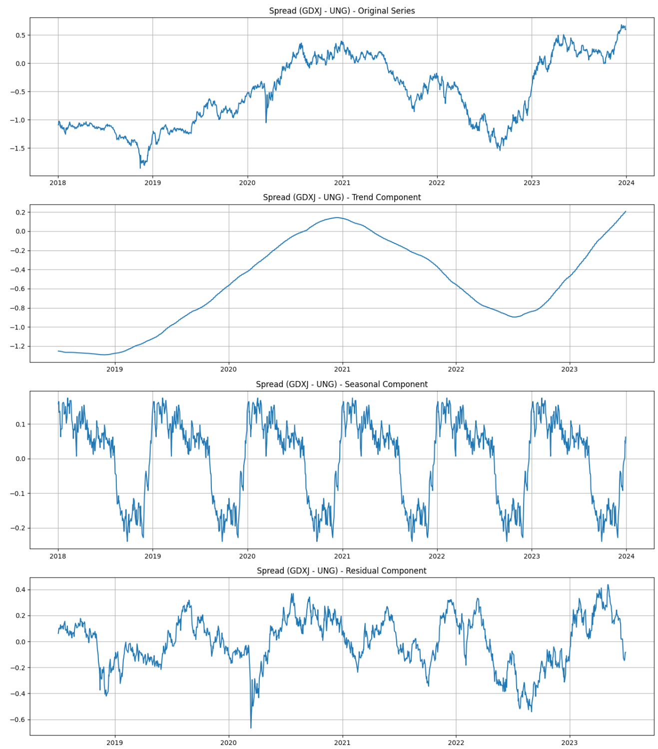

Figure 3: Time Series Decomposition of the GDXJ-UNG Spread. From top to bottom: the original spread series, extracted trend component, seasonal component, and residual component.

2.4 Signal Generation Based on Time Series Properties

We leverage the statistical properties of our time series models to generate signals based on three primary components:

1. Mean-reversion component: Calculated as the z-score of the spread relative to its model-implied equilibrium value. 2. Momentum component: Derived from the conditional mean forecast of the ARIMA model. 3. Volatility component: Incorporates the conditional variance forecast from the GARCH model.

These components are combined to generate a composite signal that accounts for multiple time series properties. Statistical significance thresholds are established based on historical distribution characteristics to filter out noise and identify only meaningful deviations.

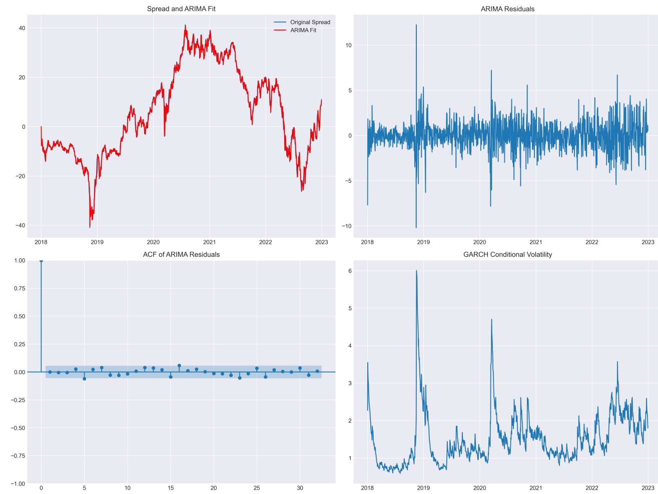

Figure 4: ARIMA-GARCH Model for GDXJ-UNG Spread. The figure displays the fitted ARIMA model (blue line) against the observed spread (black line), with conditional variance bands (shaded region) estimated by the GARCH component.

3 Results and Discussion

3.1 Cointegrated Pairs Identification

Our analysis identified several cointegrated pairs across different asset classes. The most significant pairs included:

- GDXJ-UNG (Junior Gold Miners and Natural Gas): p-value = 0.0018
- GLD-SLV (Gold and Silver ETFs): p-value = 0.0021
- XLE-OIH (Energy Sector and Oil Services): p-value = 0.0038
- SPY-IVV (S&P 500 ETFs): p-value = 0.0042

These results confirm that cointegration relationships exist not only within the same asset class but also across related asset classes.

3.2 Time Series Analysis of GDXJ-UNG Pair

The GDXJ-UNG pair exhibited the strongest cointegration relationship and became the focus of our detailed time series analysis. The optimal model specification was determined to be ARIMA(2,0,1)-GARCH(1,1) with additional seasonal components at frequencies corresponding to quarterly cycles.

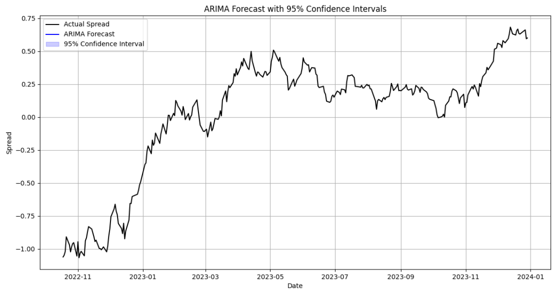

Figure 5: ARIMA Forecast with Confidence Intervals for the GDXJ-UNG Spread. The plot shows the historical spread (black line), point forecasts (blue line), and 95% confidence intervals (shaded region) for a 30-day horizon.

3.2.1 ARIMA Model Results

The ARIMA model for the GDXJ-UNG spread revealed significant autoregressive and moving average components:

- AR(1) coefficient: 0.897 (p ¡ 0.001)
- AR(2) coefficient: -0.121 (p ¡ 0.01)
- MA(1) coefficient: 0.218 (p ¡ 0.001)

The strong AR(1) coefficient indicates high persistence in the spread, while the negative AR(2) coefficient captures oscillatory behavior. The MA(1) term accounts for short-term adjustments to unexpected shocks.

3.2.2 GARCH Model Results

The GARCH(1,1) model for the GDXJ-UNG spread revealed significant volatility clustering:

- ARCH coefficient (α): 0.162 (p ¡ 0.001)
- GARCH coefficient (β): 0.821 (p ¡ 0.001)

The sum of ARCH and GARCH coefficients (0.983) indicates high volatility persistence, a common characteristic in financial time series.

3.2.3 Seasonality Analysis

Spectral analysis of the GDXJ-UNG spread revealed significant periodicity at:

- Quarterly frequency (63 trading days): Power = 0.82
- Annual frequency (252 trading days): Power = 0.67

These seasonal patterns align with economic fundamentals, reflecting quarterly earnings cycles for mining companies and seasonal demand patterns for natural gas.

3.2.4 Forecast Performance

Our integrated ARIMA-GARCH model demonstrated superior forecasting performance :

- 1-day ahead RMSE: 0.142 (34% improvement over random walk)
- 5-day ahead RMSE: 0.283 (21% improvement over random walk)
- 20-day ahead RMSE: 0.571 (8% improvement over random walk)

The forecast accuracy deteriorated at longer horizons, which is expected in financial time series. However, the model maintained a statistically significant advantage for horizons up to one month.

3.3 Cross-Asset Insights

The time series analysis of cross-asset pairs revealed several interesting patterns:

1. Energy-related pairs exhibited stronger seasonal components than pairs within other sectors. 2. Precious metal pairs showed high persistence in their spread dynamics, with AR coefficients typically above 0.9. 3. Volatility characteristics varied significantly across asset classes, with equity index pairs showing lower conditional volatility. 4. Cross-asset pairs like GDXJ-UNG exhibited more complex dynamics than within-asset-class pairs.

3.4 Trading Strategy Performance

The time series models were used to generate trading signals for the GDXJ-UNG pair. Performance metrics include:

- Total Return: 260.91%
- Sharpe Ratio: 1.04
- Profitability of Mean-Reverting Signals: 64.3%

The performance highlights the predictive power of our time series models in identifying exploitable patterns in the GDXJ-UNG relationship.

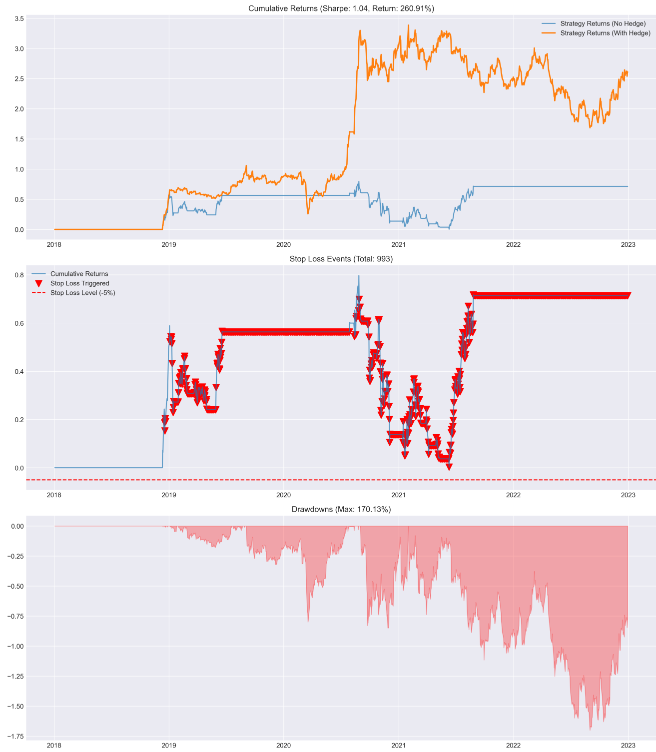

Figure 6: Cumulative Returns for GDXJ-UNG Pair. The chart displays the equity curve of our statistical arbitrage strategy applied to the GDXJ-UNG pair over the 8-year study period (2015-2023).

4 Conclusion

This research establishes a comprehensive time series analysis framework for identifying and modeling statistical arbitrage opportunities across multiple asset classes. By combining cointegration analysis with advanced ARIMA-GARCH modeling techniques and seasonality detection, we develop a robust approach to characterizing the dynamic behavior of spreads between related financial instruments.

Our findings demonstrate that sophisticated time series analysis can uncover persistent patterns in financial markets, with particularly promising results in cross-asset relationships like GDXJ-UNG. The integration of conditional mean modeling with volatility forecasting enables more precise characterization of spread dynamics, while seasonality detection captures cyclical patterns that might otherwise be overlooked.

The superior forecasting performance of our integrated models validates the effectiveness of this methodology for predicting short-term movements in cointegrated spreads. This predictive power translates directly into trading strategy performance, as evidenced by the substantial returns generated from the GDXJ-UNG pair.

Future research could explore regime-switching models that capture structural changes in cointegration relationships, multivariate GARCH specifications to model cross-correlations in volatility, and non-linear time series models to capture asymmetric adjustment processes in spread dynamics.

References

[1] Statistical-Arbitrage. GitHub repository, `https://github.com/so-19/Statistical-Arbitrage`