Classification of Systems

Yang Liu

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1 Introduction

2 Figures of Merit

Wireless communication system performance can be characterized by different figures of merit. Besides the often used SNR, other system metrics are also used to quantify system performance and can be more effective and also related to SNR.

2.1 Adjacent-Channel Power Ratio(ACPR)

Adjacent-Channel Interference (ACI) is spectral regrowth as the results from nonlinear application in wireless communication system. In order to measure the power in the main channel and the channel included in the adjacent channel, the radio of main channel and power in adjacent channel is

$$ACPR = \frac{\int_{f_1}^{f_2} S_{yy}(f)\mathrm{d}f}{\int_{f_3}^{f_4} S_{yy}(f)\mathrm{d}f}. \tag{1}$$

The frequencies f_1 and f_2 are the frequency limits of the main channel, while f_3 and f_4 are the limits of the upper adjacent channel.

2.2 SNR

Signal-to-noise ratio is a ratio value between the power of signal and of noise as

$$SNR = \frac{P_{signal}}{P_{noise}}. \tag{2}$$

P_{signal} and P_{noise} are the power of signals and of noise.

2.3 Waveform Quality Factor (ρ)

The waveform quality factor is a measure of the correlation between a scaled version of the input and the total in-channel output waveforms. The waveform quality factor ρ [2] is defined as

$$\rho = \frac{SNR}{SNR + 1} \tag{3}$$

2.4 EVM

The error vector magnitude (EVM) is a measure to quantify the performance of a digital radio transmitter or receiver. EVM expresses the difference between the expected complex voltage value of a demodulated symbol and the value of the actual received symbol. Through combination and normalization of measured symbols and ideal constellation diagram using root-mean-square, EVM can be expressed mathematically as

$$EVM_{RMS} = \left(\frac{\frac{1}{N}\sum_{r=1}^{N}(S_{ideal,r} - S_{meas,r})^2}{\frac{1}{N}\sum_{r=1}^{N}(S_{meas,r})^2}\right)^{\frac{1}{2}} \tag{4}$$

where $S_{meas,r}$ is normalized r^{th} symbol in a stream of measured symbols, $S_{idear,r}$ is the ideal normalized constellation point for the r^th symbol, and N is the number of unique symbols in the constellation. EVM measurement is sensitive to amplitude and phase imbalance at the output of the complete link.

2.5 BER

Bit error rate is the main performance parameter of a wireless channel and is defined as the number of error bits found in a given bit sequence at the receiver end. Bit error rate depends on the value of SNR. It decreases with the increase in SNR[5].

$$BER = \frac{N_{error}}{N_{total}} \tag{5}$$

2.6 Eye Pattern

Eye pattern generation is a straightforward and visual figure of merit. It provides an effective tool for the severity of ISI, sensitivity to timing errors and noise margin.

The eye pattern is produced by displaying the received signal on an oscilloscope. The time base of the scope is triggered at a fraction of the bit rate, and it yields a sweep lasting several bit intervals. The superposition of many traces of bit intervals is showed by oscilloscope, which is the eye pattern.

Monitoring of an eye pattern can provide a qualitative measure of performance based on the signal quality. The figure shows the important observations of the eye pattern[3]. graphic in page 283

2.7 Noise Power Radio

Noise power ratio is an indirect means of characterizing cochannel distortion. In this method, white noise is used to simulate the presence of many carriers of random amplitude and phase. The white noise is first passed through a bandpass filter (BPF) and then a narrow band reject filter to produce a deep notch at the center of the noise pedestal as the input of the DUT. If the notch bandwidth is sufficiently narrow, power spectral density function, observed at the output within the notch position, constitutes spectral regrowth, which is the desired cochannel distortion.

Noise power ratio is therefore defined as

$$NPR(\omega_T) = \frac{S_o(\omega_T)}{S_{wd}(\omega_T)}, \tag{6}$$

where $S_o(\omega_T)$ and $S_{wd}(\omega_T)$ are the output power spectral density functions measured in the neighbor of the test window position and within that window.

Despite having assumed a continuous spectrum, the NPR can also be implemented with a very large number of uncorrelated multitone signals.

3 Classification of Systems

3.1 Deterministic System

A deterministic system is a fully-defined function of the variable time. A deterministic system is a system which there is no uncertainty with respect to its value, and every output value can be exactly determined according to the given input time. description

3.2 Linear and Nonlinear System

In a linear system the superposition principle is satisfied. The superposition principle is based on the additivity and homogeneity properties. A system is linear if and only if the following is satisfied:

$$\text{If} x_1(t) \rightarrow y_1(t) \text{and} x_2(t) \rightarrow y_2(t), \text{then} \alpha x_1(t) + \beta x_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t), \tag{7}$$

in which α and β are some nonzero constants.

If the superposition law is not satisfied, the system is nonlinear system.

3.3 Time-Invariant and Time-Varying System

In a time-invariant system, the input-output relationship does not change with time. This means that a time shift in the input results in a corresponding time shift in the output. A system is time invariant if and only if the following is satisfied:

$$\text{If} x(t) \rightarrow y(t), \text{then} x(t-\tau) \rightarrow \alpha y(t-\tau), \tag{8}$$

where τ is a real constant. A system that is not time-invariant is called a time-varying system.

3.4 Linear Time-Invariant (LTI) System and Linear Time-Varying (LTV) System

In a linear time-invariant (LTI) system, both linearity and time-invariance conditions must be satisfied. For LTI system, the system response of the impulse input can completely characterize the system and provide all relevant information to describe the system behavior for any input.

3.5 Linear Time-Varying (LTV) System

If the Parameters of a linear system depend on time, this system is a linear time-varying system. That means a LTV system can be described as

$$y(t) = f[x(t), t] = a(t)x(t). \tag{9}$$

3.6 Stochastic System

A digital communication system is influenced by random noise like Gaussian noise. Besides the transfer of time-varying signals, such as voice signals, multimedia information are unpredictable. Hence the study of random system is meaningful.

A stochastic process is assigning to every outcome ξ a function $x(t, \xi)$. Thus a stochastic process is a function of t and ξ. The domain of ξ is the set of all experimental outcomes and the domain of t is a set R of real numbers.

Given a stochastic process $x(t)$, if another process

$$y(t) = T[x(t)] \tag{10}$$

is created, whose samples are the function $y(t, \xi_i)$, the process $y(t)$ can be considered as the output of a system with input $x(t)$. The system is completely specified in terms of the operate T as the rule of correspondence between the samples of the input $x(t)$ and the output $y(t)$.

The system is *deterministic* if it operates only on the variable t and treats ξ as a parameter. This means if two samples $x(t, \xi_1)$ and $x(t, \xi_2)$ of the input are identical in t, then the corresponding samples $y(t, \xi_1)$ and $y(t, \xi_2)$ of the output are also identical in t. Otherwise, the system is called *stochastic* if T operates on both variables t and ξ. This means if two samples $x(t, \xi_1)$ and $x(t, \xi_2)$ of the input are identical in t, but $y(t, \xi_1) \neq (t, \xi_2)$.

Memoryless Systems A system is called memoryless system if its output

$$y(t) = T[x(t)] \tag{11}$$

at a given time $t = t_1$, the output $y(t_1)$ depends only on $x(t_1)$ and not on any other past or future values of $x(t)$.

If a signal is known only in terms of statistical averages and probabilistic description, such as its mean value, mean square value, and distribution. The collection of $k^{th} - order \quad joint \quad cdfs \quad or \quad pdfs$ are

$$\begin{aligned} F_{X(t_1),\cdots,X(t_k)}(x_1,\cdots,x_k) =& P[X(t_1) \leq x_1, \cdots, X(t_k) \leq x_k] \\ f_{X(t_1),\cdots,X(t_k)}(x_1,\cdots,x_k) =& \frac{\partial^k F_{X(t_1),\cdots,X(t_k)}(x_1,\cdots,x_k)}{\partial x_1 \cdots \partial x_k} \end{aligned} \tag{12}$$

Still don't know what should be mentioned in Stochastic System, there is no books talking about classification in stochastic systems, except memoryless, linear system

The **autocorrelation and autocovariance functions of the random process X(t)** are as follows:

$$R_X(t_1, t_2) = E[X(t_1, t_2)] = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} x_1 x_2 f_{X(t_1)X(t_2)}(x_1, x_2) \mathrm{d}x_1 \mathrm{d}x_2 = R_X(\tau) \tag{13}$$

with $\tau = t_2 - t_1$, and

$$\begin{aligned} C_X(t_1, t_2) =& E[(X(t_1) - m_X(t_1))(X(t_2) - m_X(t_2))] \\ =& R_X(t_1, t_2) - m_X(t_1) m_X(t_2) \end{aligned} \tag{14}$$

To analyze deterministic time-domain signals in the frequency domain, using the Fourier transform, the **power spectral density** is

$$S_X(f) = F[R_X(\tau)] = \sum_{-\infty}^{\infty} R_X(\tau) e^{2\pi f \tau} \mathrm{d}\tau \tag{15}$$

4 Deterministic Impairments

4.1 IQ Imbalance

For down-conversion progress in receiver, RF signal should be multiplied by $\cos \omega_c t$ and $\sin \omega_c t$ signal, in order to get I part and Q part separately. Error in the 90° shift circuit and mismatches between the quadrature mixers result in imbalances in the amplitudes and phases of the baseband I and Q outputs. The baseband stages themselves may also contribute significant gain and phase mismatches.

The quadrature mismatches occurs because (1) the propagation through quadrature mixers experiences mismatches, a delay mismatch of 10 ps between the two mixers translates to a phase mismatch of 18° at 5 GHz; (2) the quadrature phases of the LO itself are introduced.

To gain insight into the effect of I/Q imbalance, consider a QPSK signal, the input is

$$x_{in} = a \cos \omega_c t + b \sin \omega_c t \tag{16}$$

Because of the gain and phase mismatches shown above in the LO path,

$$\begin{aligned} x_{IO,I}(t) =& 2(1 + \frac{\epsilon}{2}) \cos(\omega_c t + \frac{\theta}{2}) \\ x_{LO,Q}(t) =& 2(1 - \frac{\epsilon}{2}) \sin(\omega_c - \frac{\theta}{2}) \end{aligned} \tag{17}$$

where ϵ and θ represent the amplitude and phase mismatches respectively. The following baseband signals after low-pass filters are:

$$\begin{aligned} x_I(t) =& a(1 + \frac{\epsilon}{2}) \cos \frac{\theta}{2} - b(1 + \frac{\epsilon}{2}) \sin \frac{\theta}{2} \\ x_Q(t) =& - a(1 - \frac{\epsilon}{2}) \sin \frac{\theta}{2} + b(1 - \frac{\epsilon}{2}) \cos \frac{\theta}{2} \end{aligned} \tag{18}$$

The output x_I and x_Q contain both gain impairments and phase impairments. This impairment relies not on time and the system characters don't depend on the input, which shows it's a deterministic and time-invariant system.

If $x_{in,1} = a_1 \cos \omega_c t + b_1 \sin \omega_c t$ and $x_{in,2} = a_2 \cos \omega_c t + b_2 \sin \omega_c t$, than the output of $x_{in,1} + x_{in,2}$ are

$$\begin{aligned} & a_1 \cos \frac{\theta}{2} - b_1(1 + \frac{\epsilon}{2}) \sin \frac{\theta}{2} + a_2(1 + \frac{\epsilon}{2}) \cos \frac{\theta}{2} - b_2(1 + \frac{\epsilon}{2}) \sin \frac{\theta}{2} \\ =& (a_1 + a_2)(1 + \frac{\epsilon}{2}) \cos \frac{\theta}{2} - (b_1 + b_2)(1 + \frac{\epsilon}{2}) \sin \frac{\theta}{2} \end{aligned} \tag{19}$$

which is the output of the signal $x_{in,1} + x_{in,2}$.

Hence, IQ Imbalance is a det. LTI impairment.

4.2 Carrier Feedthrough/Leakage, IQ Offset, DC Offset

Since in a homodyne topology the downconvverted band extends to zero frequency, extraneous offset voltages can corrupt the signal and saturate the following stages. Consequently, the output signal is

$$V_{out}(t) = [A(t) \cos \phi + V_{OS1}] \cos \omega_c t - [A(t) \sin \phi + V_{OS2}] \sin \omega_c t \tag{20}$$

where V_{OS1} and V_{OS2} denote the dc offsets from the input port of the mixers. The unconverter output therefore contains the term of the unmodulated carrier:

$$V_{out}(t) = A(t) \cos(\omega_c t + \phi) + V_{OS1} \cos \omega_c t - V_{OS2} \sin \omega_c t \tag{21}$$

The **carrier leakage** is quantified as

$$\text{Relative Carrier Leakage} = \frac{\sqrt{V_{OS1}^2 + V_{OS2}^2}}{\sqrt{\overline{A^2(t)}}} \tag{22}$$

It appears to the horizontal and vertical shifts in the constellation.

If one carrier leakage pair is $V_{OS1,1}$ and $V_{OS2,1}$, the other carrier leakage pair is $V_{OS1,2}$ and $V_{OS2,2}$, then the sum of the outputs is

$$\begin{aligned} V_{out}(t) =& A(t)\cos(\omega_c t + \phi) + V_{OS1,1}\cos\omega_c t - V_{OS2,1}\sin\omega_c t + \\ & A(t)\cos(\omega_c t + \phi) + V_{OS1,2}\cos\omega_c t - V_{OS2,2}\sin\omega_c t \\ =& 2A(t)\cos(\omega_c t + \phi) + (V_{OS1,1} + V_{OS1,2})\cos\omega_c t - (V_{OS2,1} + V_{OS2,2})\sin\omega_c t \end{aligned} \tag{23}$$

which is the output of carrier leakage pair $V_{OS1,1} + V_{OS1,2}$ and $V_{OS2,1} + V_{OS2,2}$ and independent of the time. Hence, Carrier Feedthrough/Leakage is a LTI impairment.

4.3 Amplitude/Phase Response depending on frequency

4.4 Multipath Propagation

One of the major challenges in wireless communications is the frequency-selectivity of the channel caused by multipath propagation, with signal echoes arriving the the receiver antenna with delays similar to, or greater than the symbol duration T = 1/Rs of the transmitted signal.

It very likely to happen if we increase the symbol rate Rs = 1/T (to obtain higher data rates) and then, channel is no longer frequency-flat, but frequency-selective.

The multipath channel can be modelled as an FIR filter (tapped delay line) that causes intersymbol interference (ISI). The discrete time channel impulse response with $L + 1$ taps can be described by its z-transform as

$$H(z) = \sum_{k=0}^{L} h_k \cdot z^{-k} \tag{24}$$

or in time domain

$$y(t) = \sum_{k=0}^{\infty} a_k x(n-k) \tag{25}$$

The input-output relationship of x_1 and x_2 are

$$\begin{aligned} x_1(n) &\rightarrow \sum_{k=0}^{\infty} a_k x_1(n-k) \\ x_2(n) &\rightarrow \sum_{k=0}^{\infty} a_k x_2(n-k) \end{aligned} \tag{26}$$

The sum of them is

$$\sum_{k=0}^{\infty} a_k x_1(n-k) + \sum_{k=0}^{\infty} a_k x_2(n-k) = \sum_{k=0}^{\infty} a_k [x_1(n-k) + x_2(n-k)] \tag{27}$$

which is the output of $x_1 + x_2$. Besides the parameters depend not on time and input.

Hence multipath propagation is a det. LTI impairment.

4.5 Intersymbol Interference (LTV)

A digital system consists of random sequence of ONEs and ZEROs. Each ONE represented by an ideal rectangular pulse and each ZERO by the absence of such a pulse. If this sequence is applied to a low-pass filter, the output can be obtained as the superposition of the responses to each input bit. Each bit level is corrupted by decaying tails created by previous bits. This phenomenon leads to higher error rate in the detection of random wave forms that are transmitted through band-limited channels.

The input bits applied to a pulse generator produce the following signal:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b) \tag{28}$$

where $g(t)$ is the basic shaping pulse with $g(0) = 1$, and the pulse amplitude a_k depends on the input bit.

The signal $x(t)$ is transfered through transmitting filter $H_T(f)$, Channel $H_C(f)$ added by Additive white Gaussian noise (AWGN), Receiving filter $H_g(f)$. The receiver signal in frequency domain is

$$R(f) = a_k G(f) H_T(f) H_C(f) H_R(t) + N(f) = A_k H(f), \tag{29}$$

where $G(f)$ is the Fourier transform of the pulse $g(t)$. The received signal $r(t)$ is sampled synchronously at $t_m = mT_b$, with m as integer values. The signal in time domain is

$$r(t_m) \sum_{k=-\infty}^{\infty} A_k h(mT_b - kT_b) + n(t_m) = A_m + \sum_{\substack{k=-\infty \\ k\neq m}}^{\infty} A_k h((m-k)T_b) + n(t_m) \tag{30}$$

The first term on the right-hand side of the expression above is the desired transmitted bit, and the second term represents the residual effect of all other transmitted bits on the m^{th} bit. This residual effect is called **intersymbol interference (ISI)**. The last term represents the noise sample.

The ISI term $\sum_{\substack{k=-\infty \\ k\neq m}}^{\infty} A_k h((m-k)T_b)$ satisfied the form $y(t) = a(t)x(t)$ mentioned above and therefore is LTV system.

4.6 Intermodulation distortion (det. nonlinear)

In order to study the phenomena from nonlinearity, input/output characteristic can be approximated by

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t). \tag{31}$$

Therefore if two interferers at ω_1 and ω_2 are the input of a nonlinear system, the output is not harmonics of these frequencies. This phenomenon is called "intermodulation (IM)". Assuming input $x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$, the output according the equation above is

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t). \tag{32}$$

Expanding the right-hand side and discarding the direct current terms, the results are the terms with the frequencies in

$$\begin{cases} \omega_1 - \omega_2 \\ 2\omega_1 - \omega_2, \quad \omega_1, \quad \omega_2, \quad 2\omega_2 - \omega_1 \\ \omega_1 + \omega_2 \\ 2\omega_1 + \omega_2, \quad 2\omega_2 + \omega_1. \end{cases} \tag{33}$$

It is very likely that the small desired signal at ω_0 with $2\omega_1 - \omega_2 \approx \omega_0$. Consequently, the intermodulation product at $2\omega_1 - \omega_2$ falls onto the desired channel, corrupting the signal.

The term in the frequency $2\omega_1 - \omega_2$ is $\frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$. Therefore IM is nonlinear distortion.

4.7 Amplitude/Phase Response depending on frequency (det. nonlinear)

5 Stochastic Impairments

5.1 Phase Noise(random and multiplicative)

An ideal oscillator produces a perfectly-periodic output of the form $x(t) = A\cos\omega_c t$. The zero crossings occur at exact integer multiples of $T_s = 2\pi/\omega_c$. In reality, however, the noise of the oscillator devices randomly perturbs the zero crossings, like $x(t) = A\cos[\omega_c t + \phi_n(t)]$, where $\phi_n(t)$ is a small random phase quantity that deviates the zero crossings from integer multiples of T_c. The term $\phi_n(t)$ is called the "phase noise". The frequency departs from ω_c occasionally. As a consequence, the impulse is "broadened" to represent this random departure. Since $\phi_n(t) \ll 1rad$

$$LO_{cos}(t) = A\cos[\omega_c t + \phi_n(t)] \tag{34}$$

After 90° delay the sin signal is

$$LO_{sin}(t) = A\sin[\omega_c t + \phi_n(t)] \tag{35}$$

In the receiver front-end for down-conversion situation, referring the ideal case, the desired channel is convolved with the impulse at ω_{LO}, yielding an IF signal at $\omega_{IF} = \omega_{RF} - \omega_{LO}$.However, with consideration of phase noise from LO, the convolution of the desired signal and the interferer with the noisy LO results in a broadened down-converted interferer whose noise skirt corrupts the desired IF signal. This phenomenon is called "reciprocal mixing".

The IQ signal after superposition is

$$\begin{aligned} x(t) =& I \cdot LO_{cos} - Q \cdot LO_{sin} \\ =& I \cdot A\cos[\omega_c t + \phi_n(t)] - Q \cdot A\sin[\omega_c t + \phi_n(t)] \end{aligned} \tag{36}$$

After the multiplication in receiver and low pass filter, the signal with I part as real part and Q part as imaginary part are

$$\begin{aligned} y(t) =& x(t)\cos(\omega_c t) + i \cdot x(t)\sin(\omega_c t) \\ =& \frac{1}{2}Ie^{-i\phi} + \frac{1}{2}Qe^{-i\phi} \cdot i + \frac{1}{2}Ie^{i(2\omega_c t+\phi(t))} - \frac{1}{2}Qe^{i(2\omega_c t+\phi(t))} \cdot i \\ =& \frac{I}{2}e^{-i\phi} + \frac{Q}{2}e^{-i\phi} \cdot i + \ high\ frequency\ signal \end{aligned} \tag{37}$$

The part $e^{-i\phi}$ shows this is a phase noise and have influence on phase instead of amplitude. It is common to treat $\phi(t)$ as a zero-mean stationary random process[4], whose variance is defined as the LO mean square phase error[1]. The description in frequency domain about phase noise is the power spectral density $S_\phi(\omega_m)$ of the phase $\phi(t)$. Hence phase noise is a stochastic system.

5.2 Thermal Noise(random and additive)

Due to the Brown motion of electrons in a conductor, electrical equipment is always affected by thermal noise. Thermal noise has a flat power spectral density over a very wide frequency range and is said to be white since all frequencies are equally represented. Thermal noise depends only on the temperature.

Amount of thermal noise to be found in a bandwidth of $1Hz$ in any device or conductor is

$$N_0 = kT(W/Hz), \tag{38}$$

where N_0 is noise power density in watts per 1 Hz of bandwidth. Parameter $k = 1.38 \times 10^{-23} J/K$ is Boltzmann's constant. T is temperature in klevin. not finished, find a model?

References

[1] Z. Chen and F. F. Dai. Effects of lo phase and amplitude imbalances and phase noise on m-qam transceiver performance. *IEEE Transactions on Industrial Electronics*, 57(5):1505–1517, May 2010.

[2] Khaled M. Gharaibeh. *Nonlinear Distortion in Wireless Systems.* 2011.

[3] Ali Grami. *Introduction to Digital Communications.* 2016.

[4] D. B. Leeson. A simple model of feedback oscillator noise spectrum. *Proceedings of the IEEE*, 54(2):329–330, Feb 1966.

[5] S. Sharma, J. Nagar, and P. Singh. Modeling and performance analysis of wireless channel. In *2016 3rd International Conference on Computing for Sustainable Global Development (INDIACom)*, pages 3052–3057, March 2016.