

Administrative

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Every Monday, 4pm-6pm, @ Room 5, Institute of Computing

Lectures are non-exhaustive.

Read references for completeness.

Final Project



Artificial Neural Networks

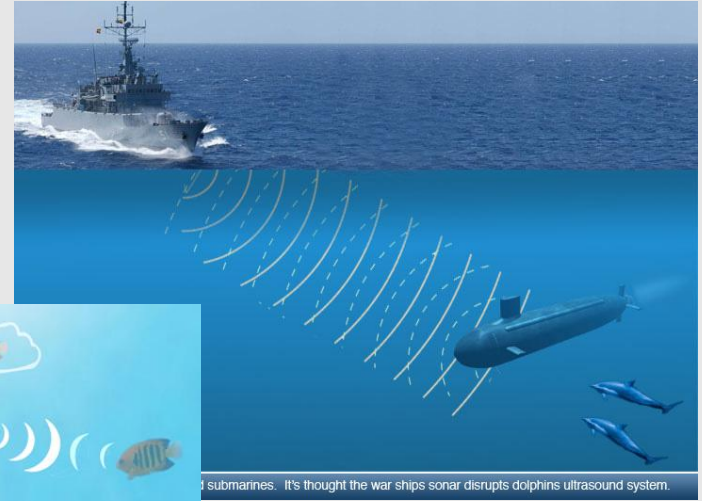
Machine Learning and Pattern Recognition

Prof. Sandra Avila

Institute of Computing (IC/Unicamp)

MC886/MO444, September 12, 2017

Many inventions were inspired by Nature ...



submarines. It's thought the war ships sonar disrupts dolphins ultrasound system.



It seems logical to look at the
brain's architecture for inspiration on
how to build an intelligent machine.

From Biological to Artificial Neurons

From Biological to Artificial Neurons

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- **1943:** Artificial Neural Networks (ANNs) were first introduced by the neurophysiologist Warren McCulloch and the mathematician Walter Pitts.

“A Logical Calculus of Ideas Immanent in Nervous Activity”,
Warren McCulloch and Walter Pitts. The bulletin of
mathematical biophysics (1943).

From Biological to Artificial Neurons

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- **Until the 1960s:** The early successes of ANNs led to the widespread belief that we would soon be conversing with truly intelligent machines.
- When it became clear that this promise would go unfulfilled funding flew elsewhere and ANNs entered a long dark era.

From Biological to Artificial Neurons

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- **1980s:** There was a revival of interest in ANNs as new network architectures were invented and **better training techniques** were developed.

“Learning representations by backpropagating errors”.

David E. Rumelhart, Geoffrey E. Hinton & Ronald J. Williams.
Nature (1986).

From Biological to Artificial Neurons

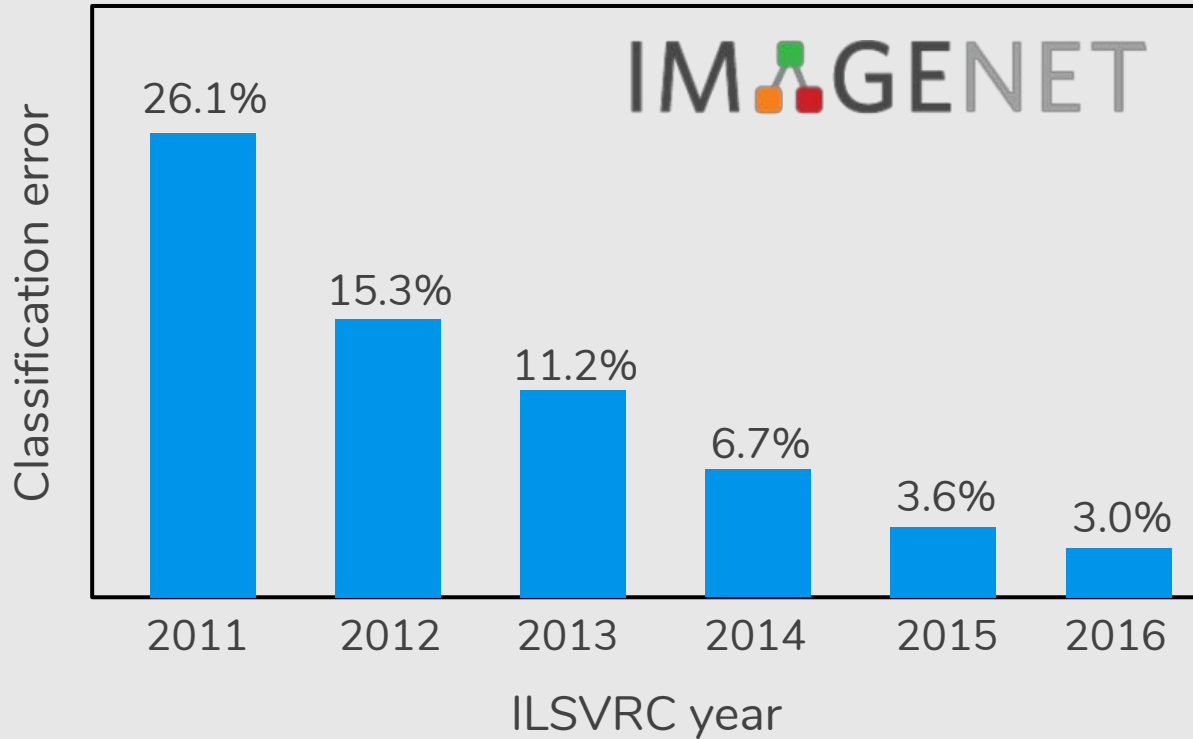
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- **1990s: Powerful alternative Machine Learning techniques** such as Support Vector Machines were favored by most researchers, as they seemed to offer better results and stronger theoretical foundations.

From Biological to Artificial Neurons

— — —

- **2010s:** We are now witnessing yet another wave of interest in ANNs.



“ImageNet classification with deep convolutional neural networks”. Alex Krizhevsky, Ilya Sutskever, Geoffrey Hinton. In: NIPS, 2012.

From Biological to Artificial Neurons

— — —

- **2010s:** We are now witnessing yet another wave of interest in ANNs.

Will this wave die out like the previous ones did?

From Biological to Artificial Neurons

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1. There is now a **huge quantity of data** available to train neural networks.



www.image-net.org

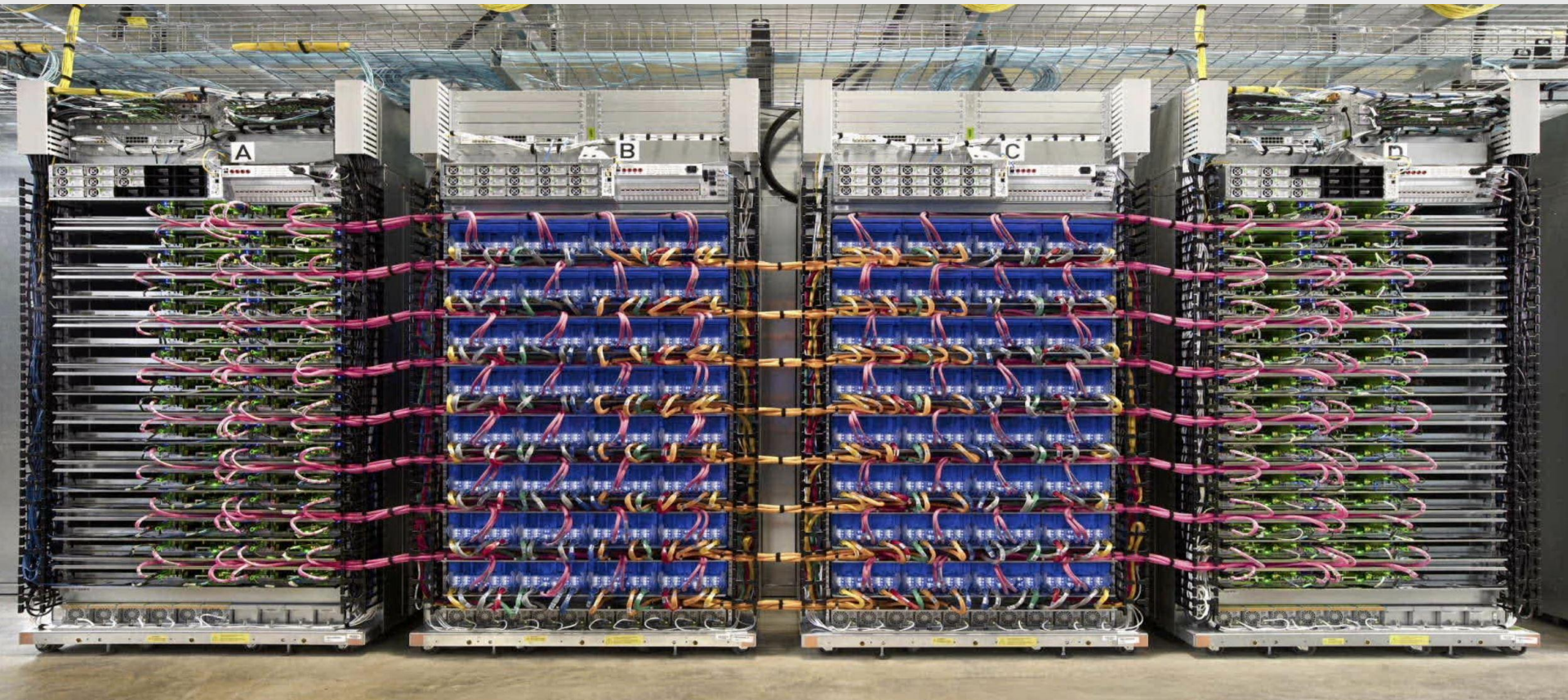
22K categories and **14M** images

- Animals
 - Bird
 - Fish
 - Mammal
 - Invertebrate
- Plants
 - Tree
 - Flower
- Food
- Materials
- Structures
 - Artifact
 - Tools
 - Appliances
 - Structures
- Person
 - Scenes
 - Indoor
 - Geological Formations
 - Sport Activities

From Biological to Artificial Neurons

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1. There is now a **huge quantity of data** available to train neural networks.
2. **Computing power** now makes it possible to train large neural networks in a reasonable amount of time.

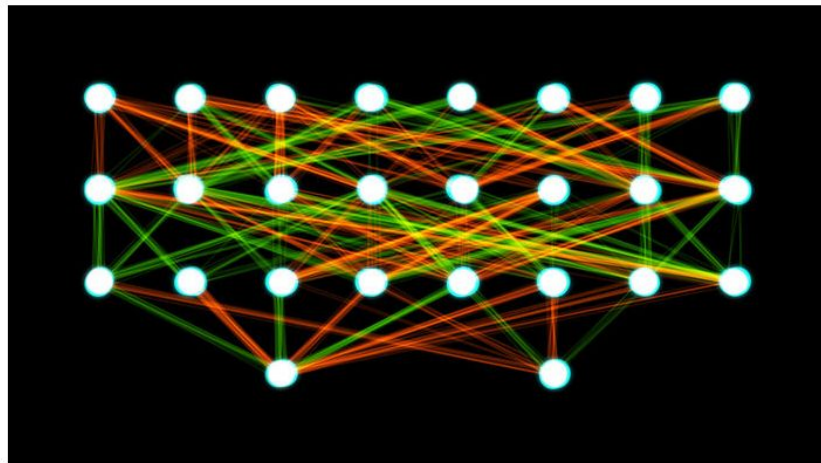


From Biological to Artificial Neurons

— — —

1. There is now a **huge quantity of data** available to train neural networks.
2. **Computing power** now makes it possible to train large neural networks in a reasonable amount of time.
3. The **training algorithms** have been **improved**.

SHARE



A representation of a neural network.

Akritasa/Wikimedia Commons

Brainlike computers are a black box. Scientists are finally peering inside

By [Jackie Snow](#) | Mar. 7, 2017, 3:15 PM

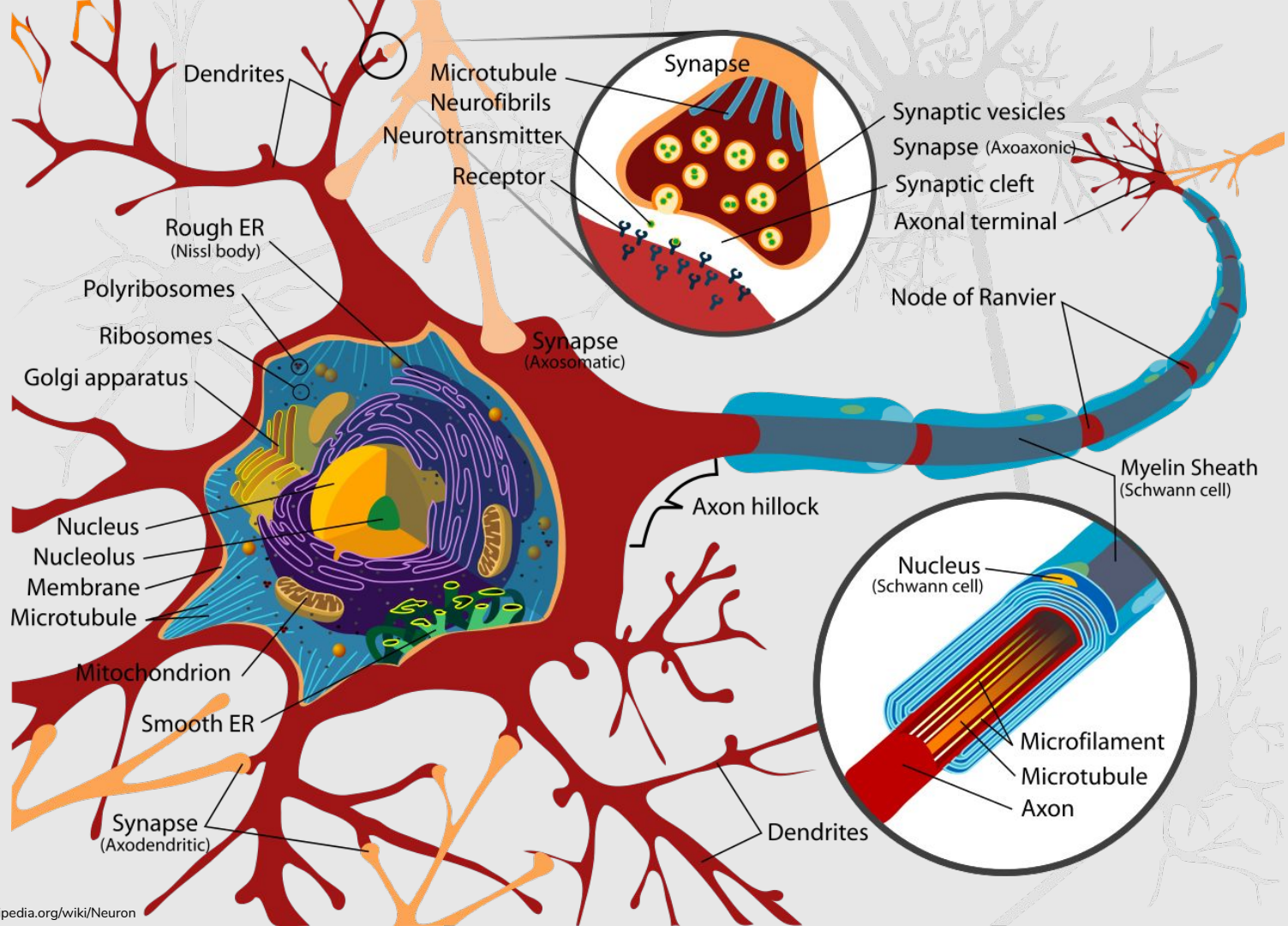
Last month, Facebook announced software that could simply look at a photo and tell, for example, whether it was a picture of a cat or a dog. A related program identifies cancerous

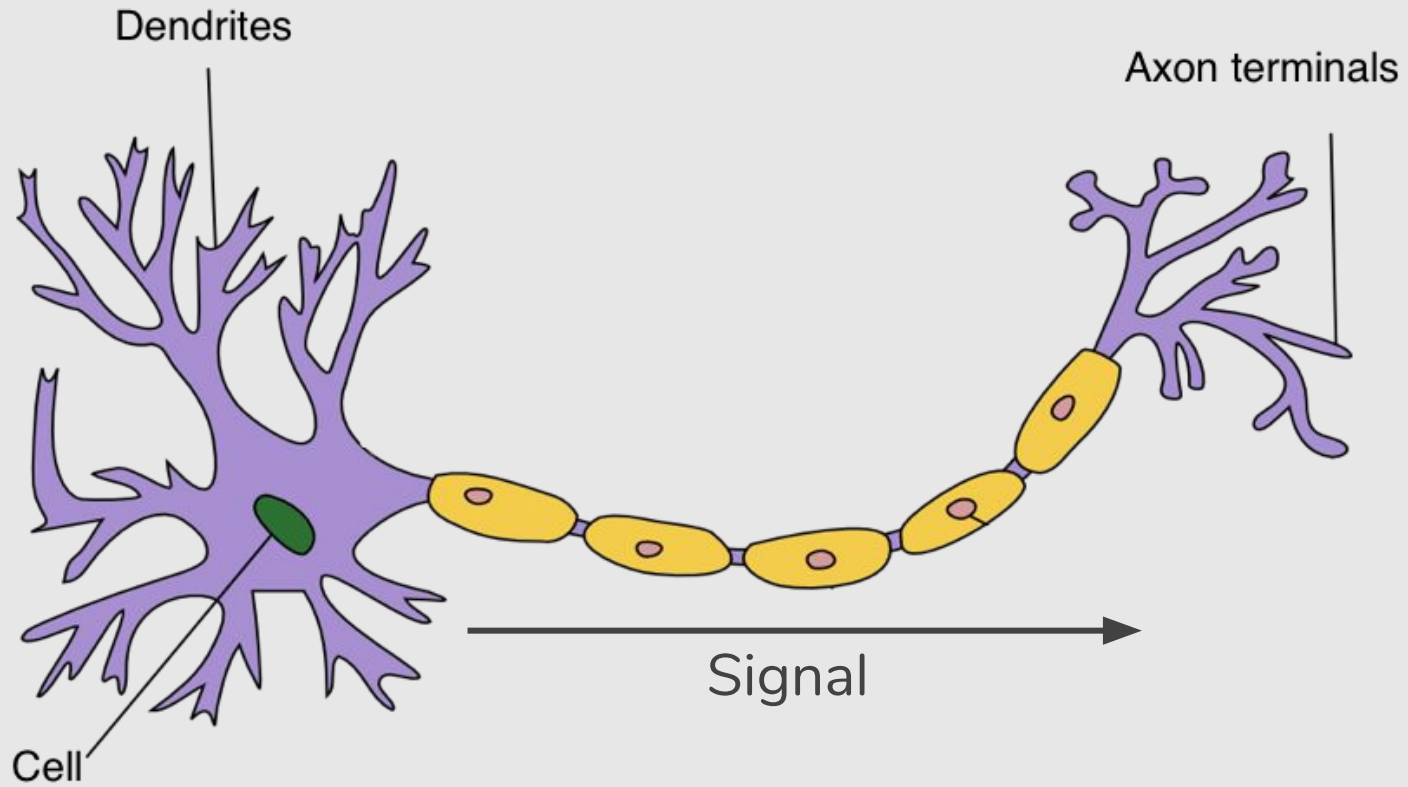
From Biological to Artificial Neurons

— — —

1. There is now a **huge quantity of data** available to train neural networks.
2. **Computing power** now makes it possible to train large neural networks in a reasonable amount of time.
3. The **training algorithms** have been **improved**.
4. ANNs seem to have entered a virtuous circle of funding and progress.

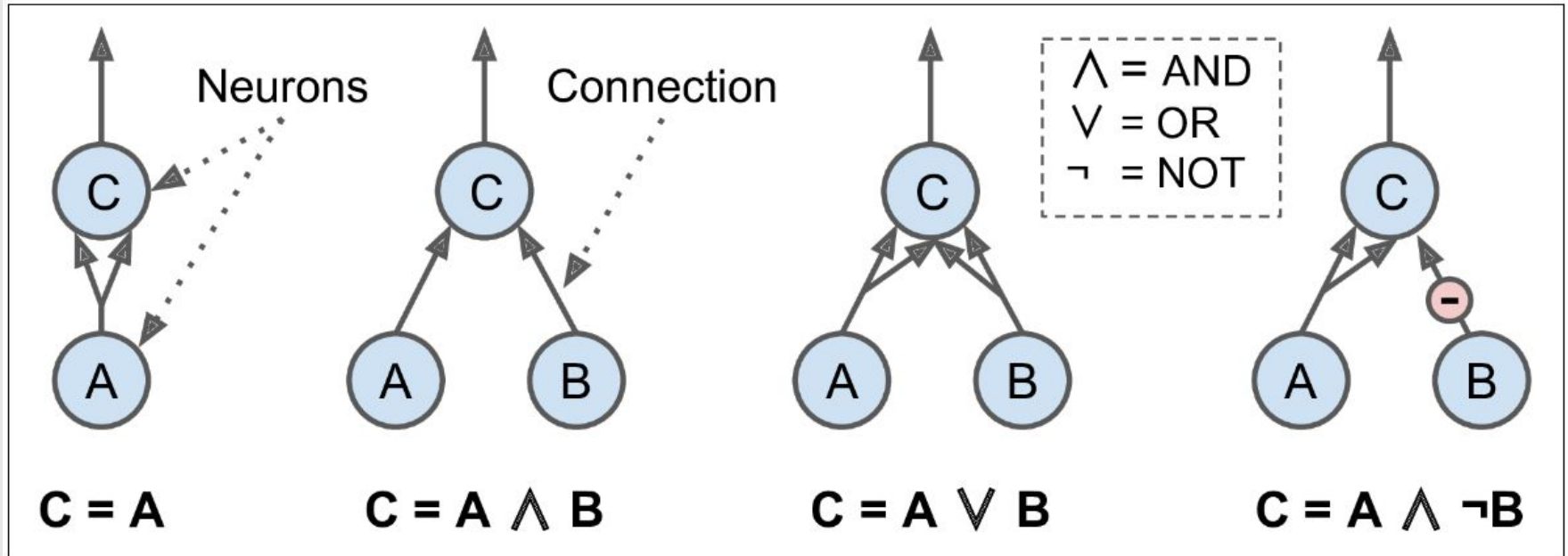
Biological Neurons





Logical Computations with Neurons

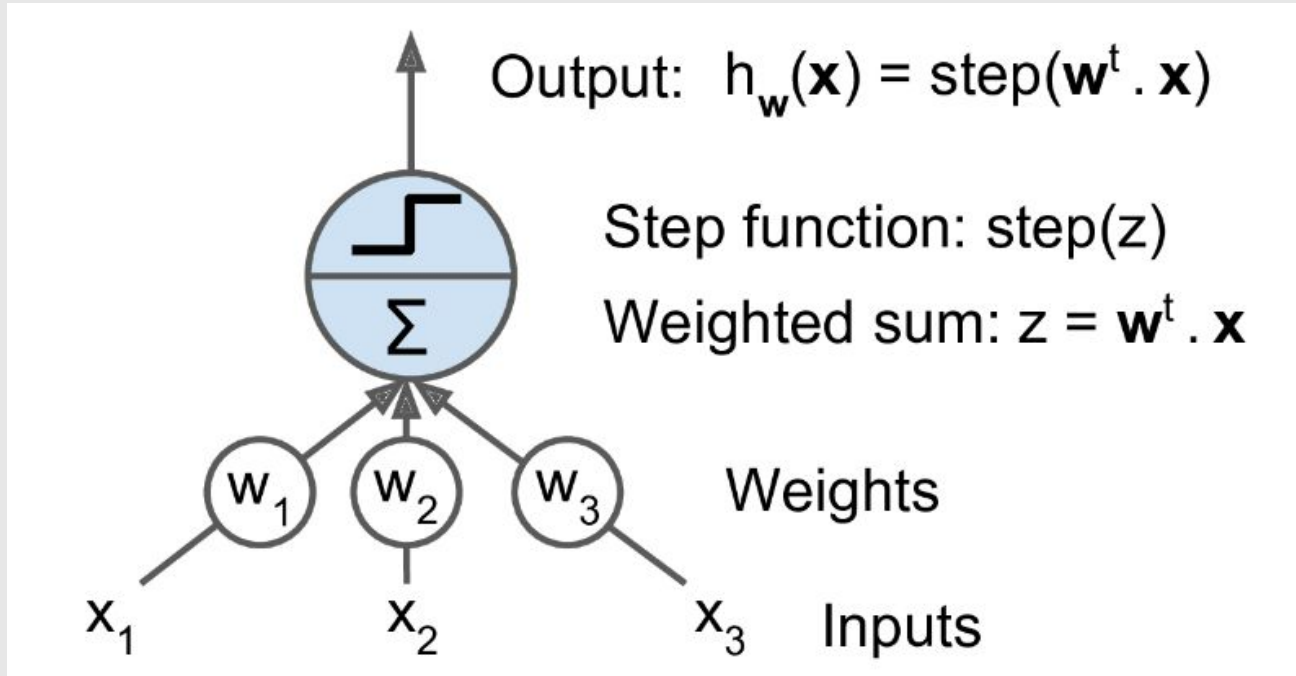
McCulloch and Pitts (1943)



Artificial Neural Networks performing simple logical computations

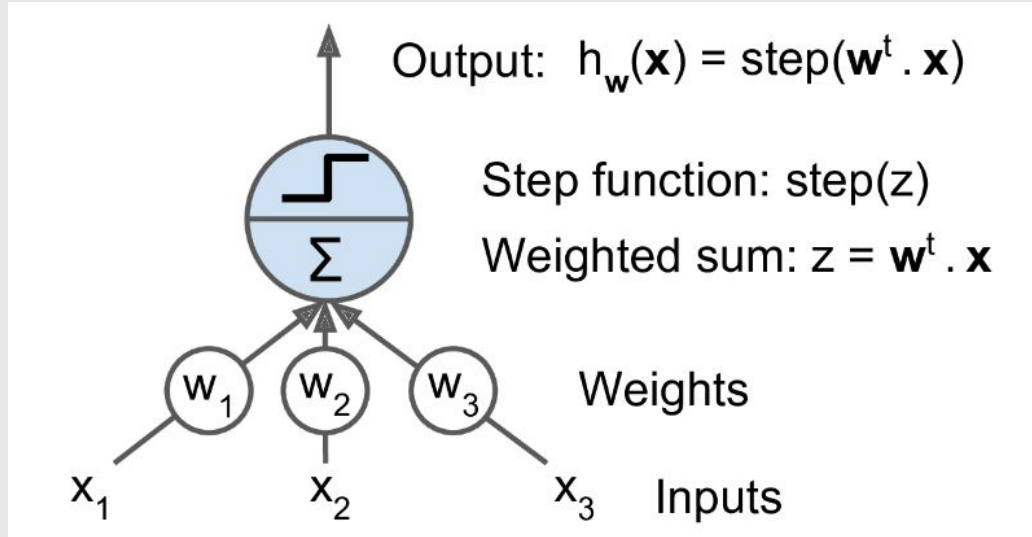
The Perceptron

The Perceptron



Linear Threshold Unit, Frank Rosenblatt (1957)

The Perceptron

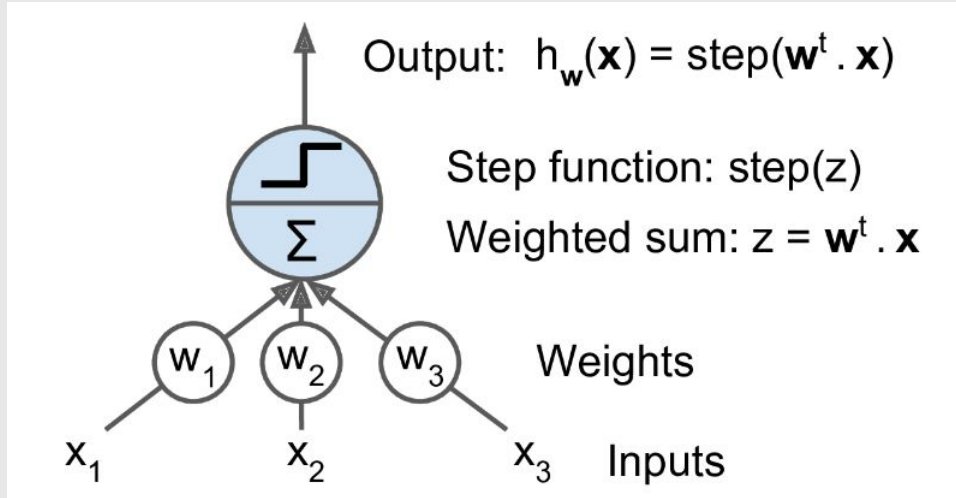


Linear Threshold Unit

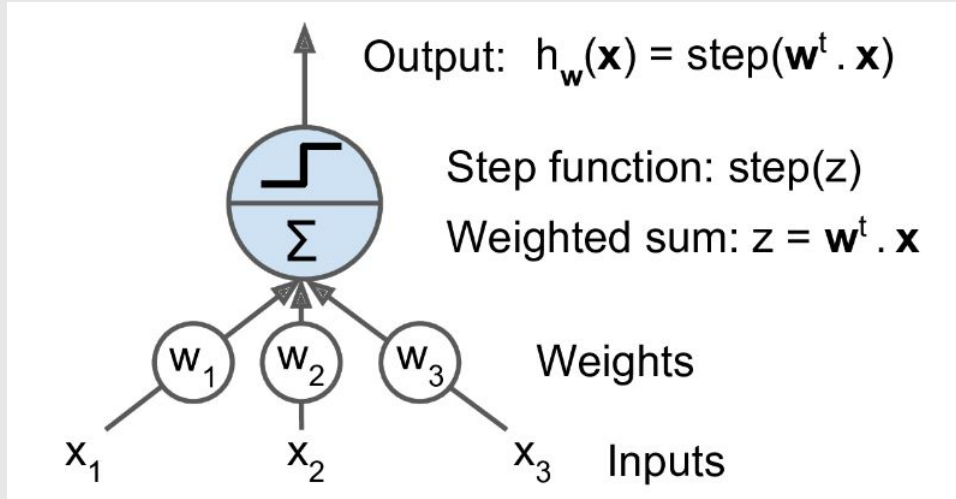
$$\text{heaviside}(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

$$\text{sign}(z) = \begin{cases} -1 & \text{if } z < 0 \\ 0 & \text{if } z = 0 \\ +1 & \text{if } z > 0 \end{cases}$$

Neuron Model: Logistic Unit



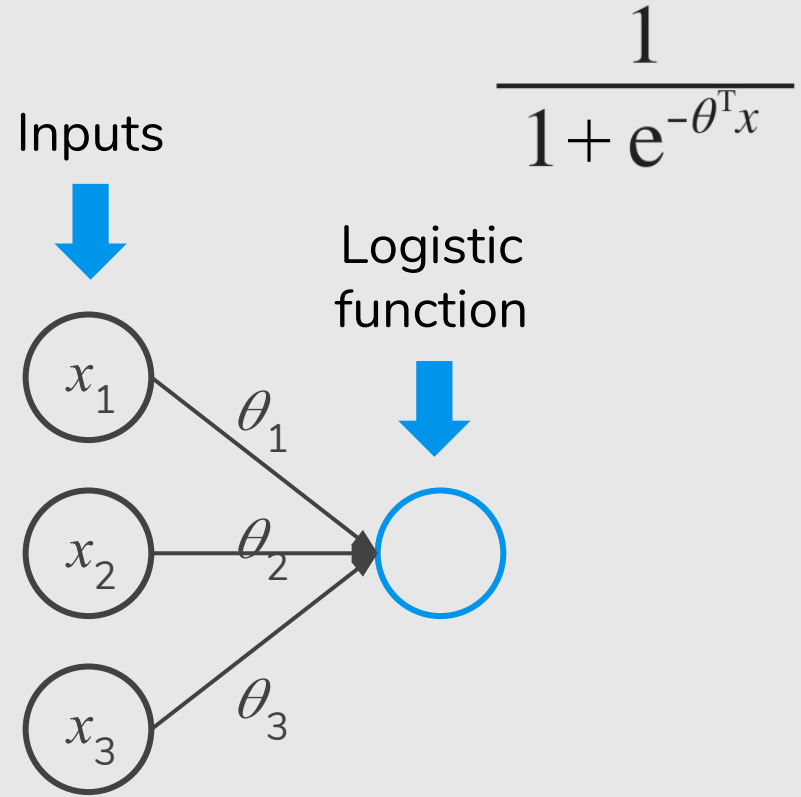
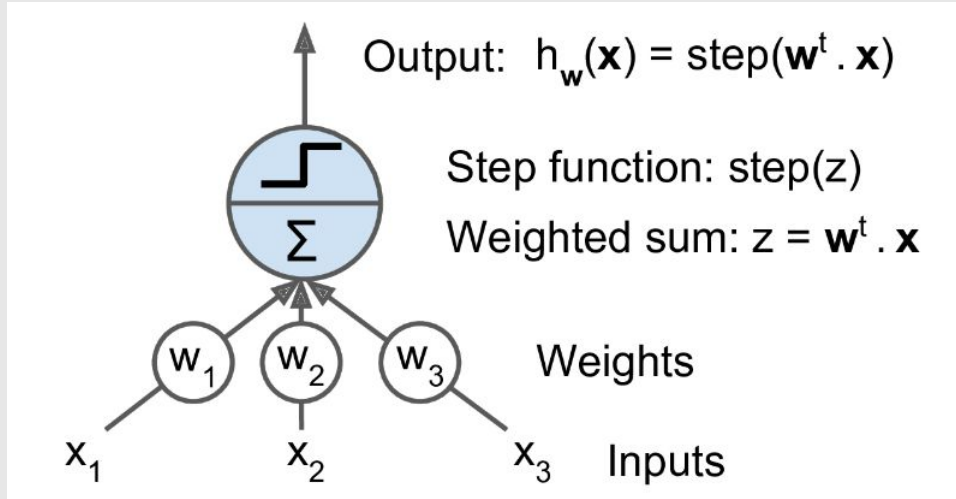
Neuron Model: Logistic Unit



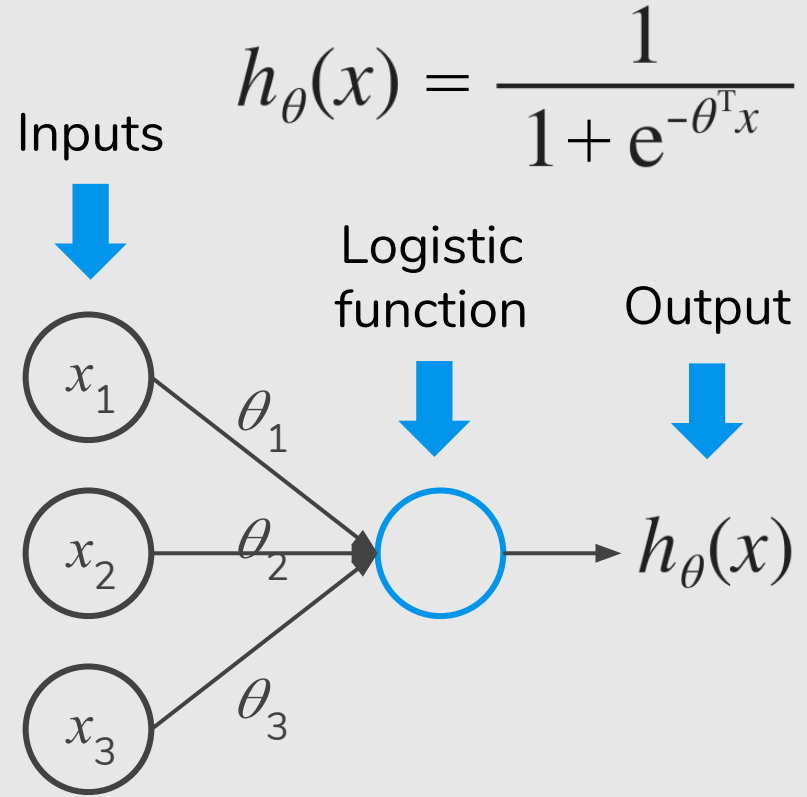
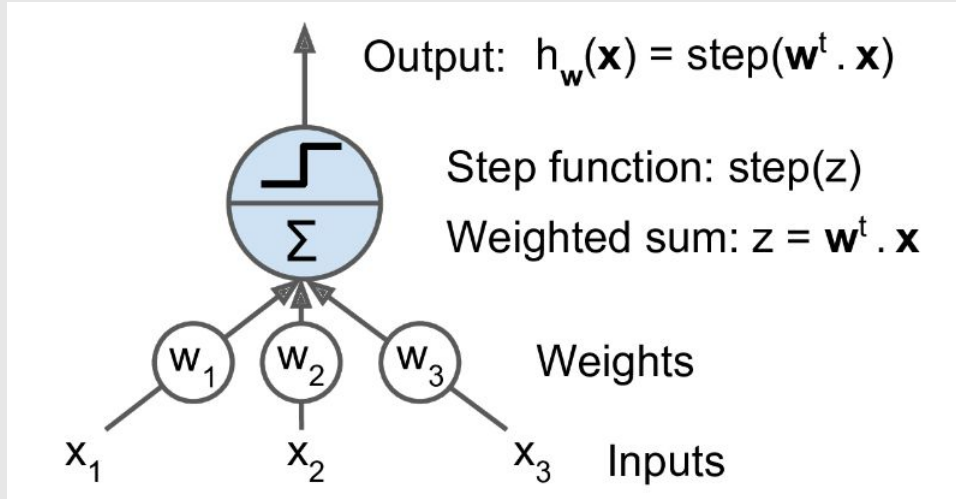
Inputs



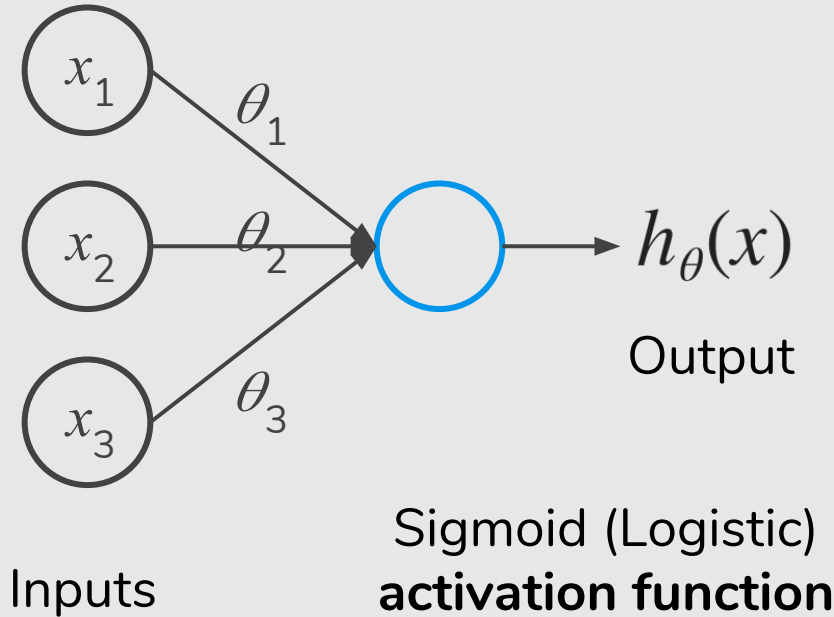
Neuron Model: Logistic Unit



Neuron Model: Logistic Unit



Neuron Model: Logistic Unit

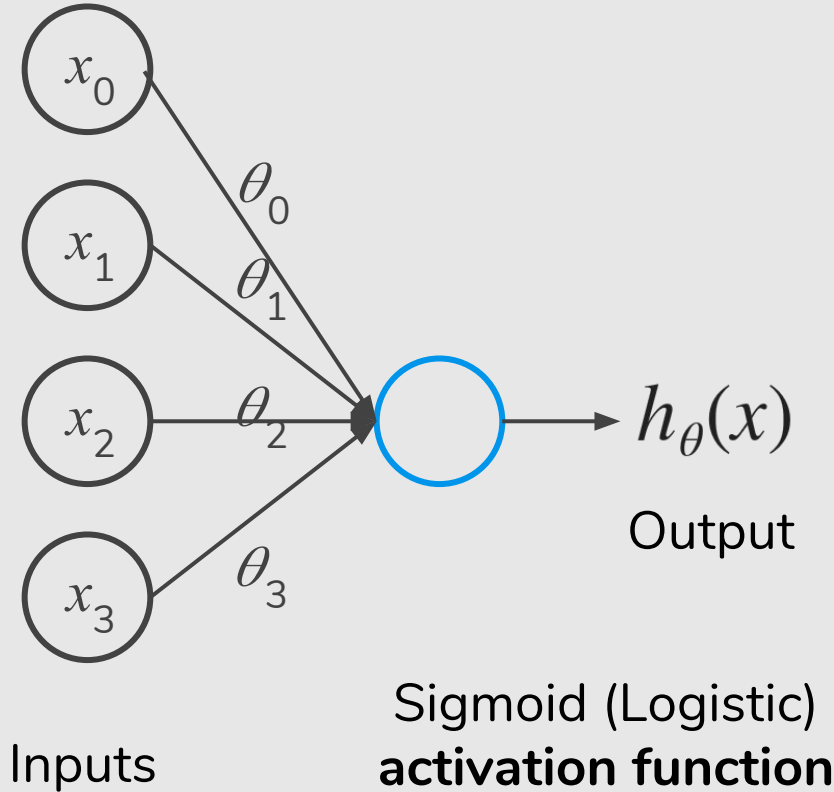


weights
↓

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Neuron Model: Logistic Unit



weights

↓

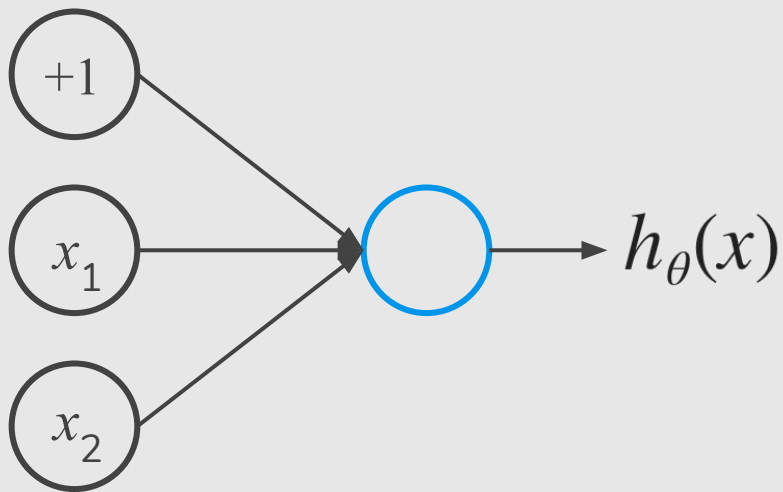
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$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Examples

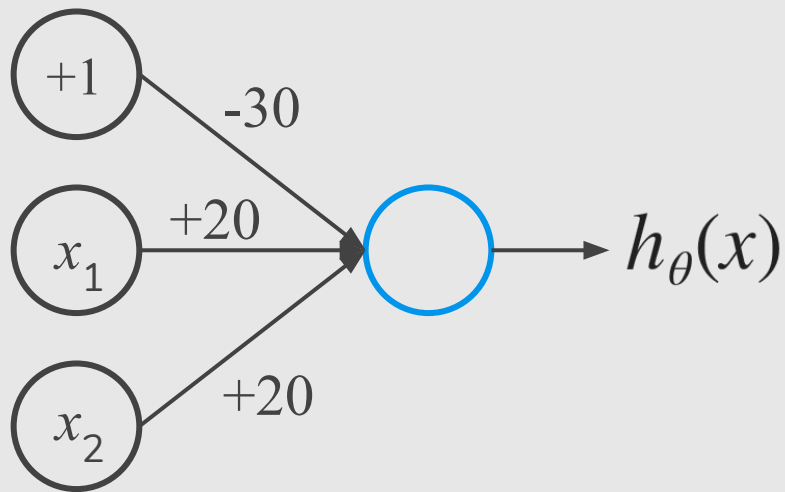
Simple Example: AND

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ AND } x_2$$



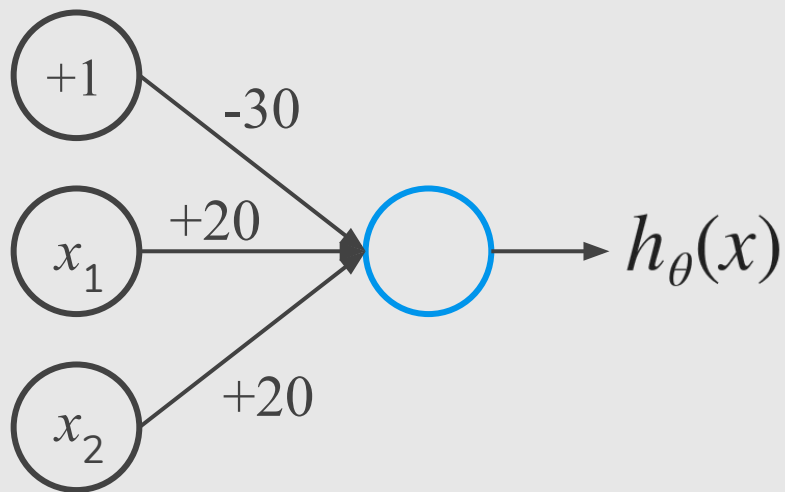
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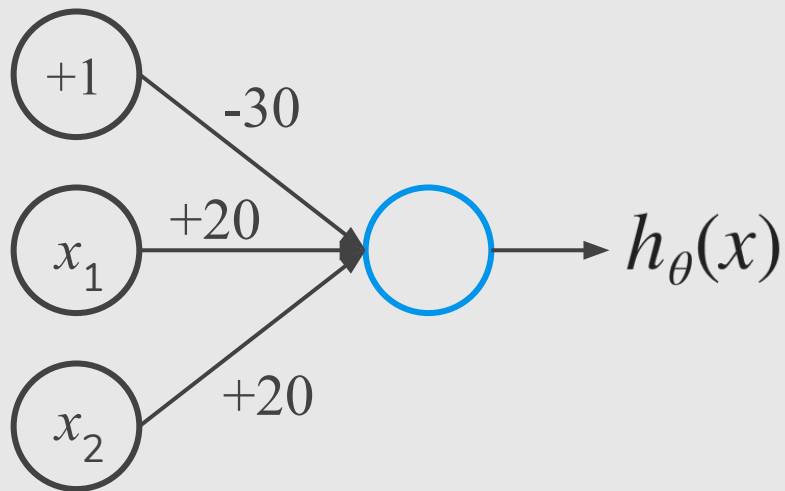
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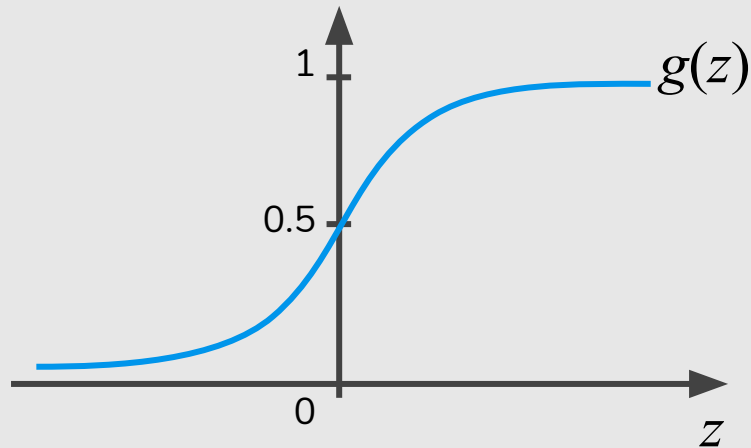
$$h_{\theta}(x) = g(-30 + 20x_1 + 20x_2)$$

Simple Example: **AND**

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ AND } x_2$$



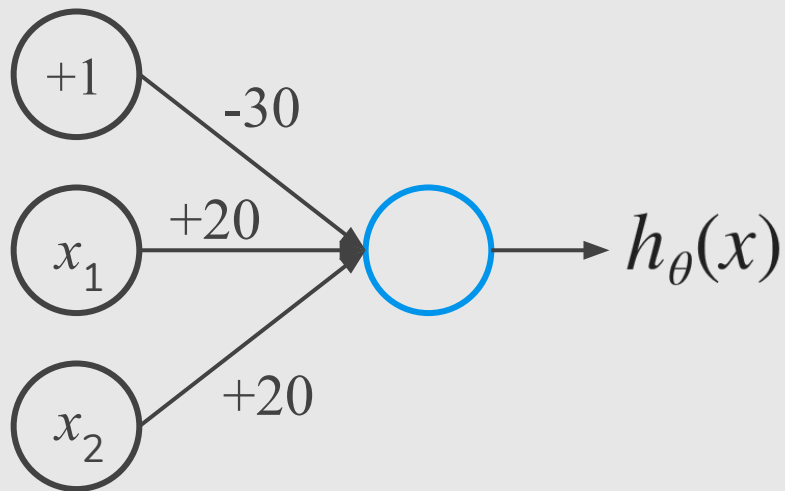
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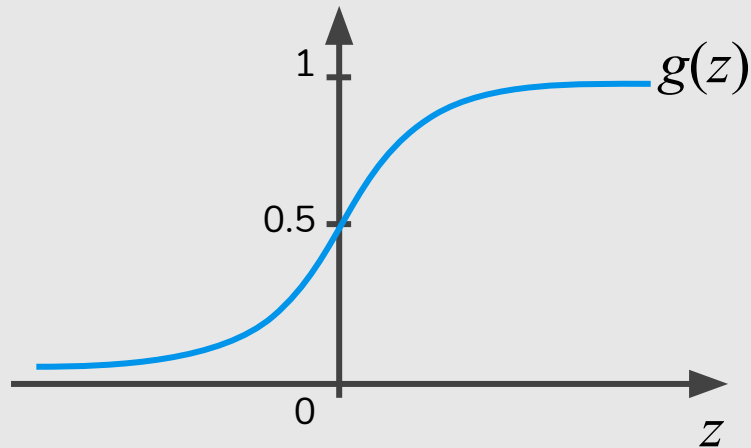
x_1	x_2	$h_\theta(x)$
0	0	
0	1	
1	0	
1	1	

Simple Example: AND

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ AND } x_2$$



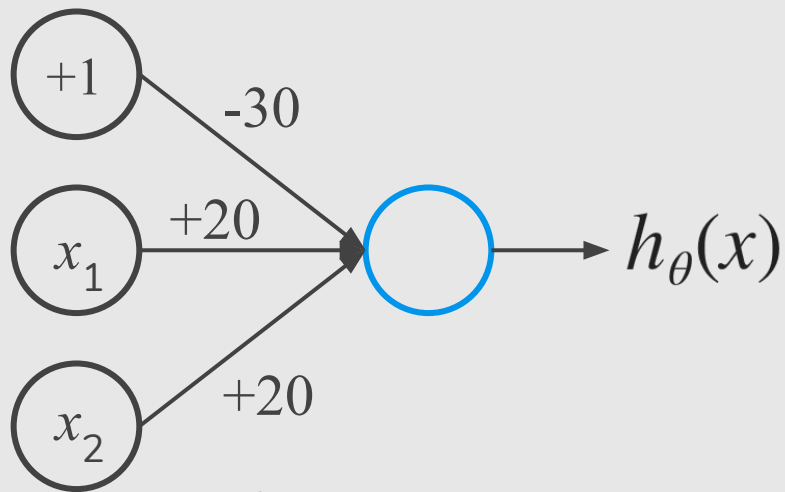
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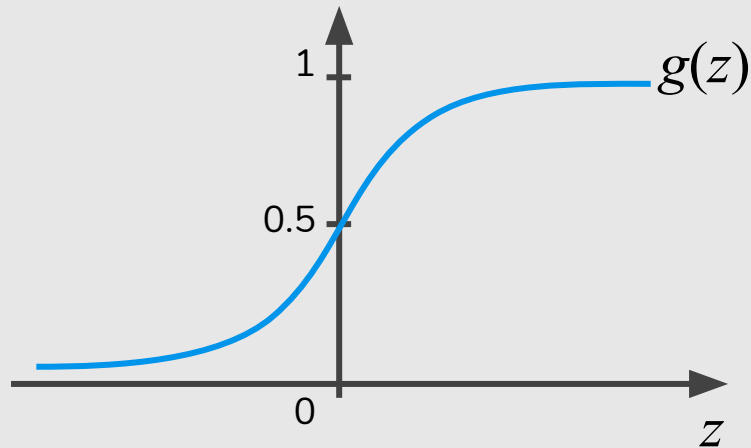
x_1	x_2	$h_\theta(x)$
0	0	$g(-30) \approx 0$
0	1	
1	0	
1	1	

Simple Example: AND

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ AND } x_2$$



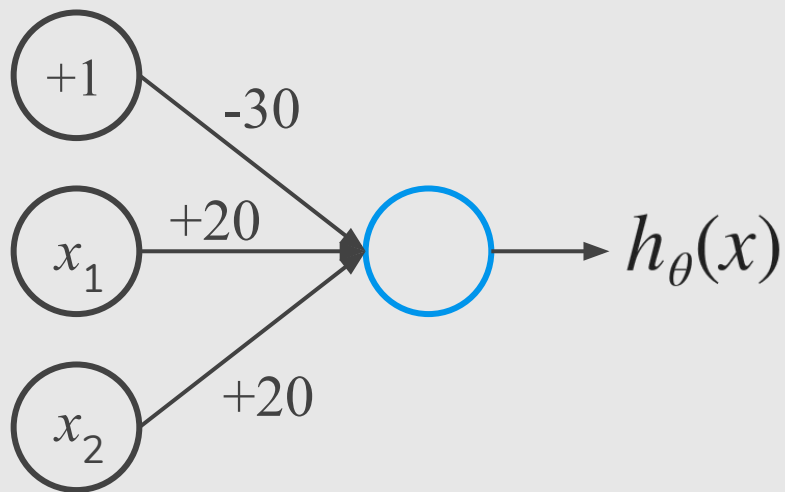
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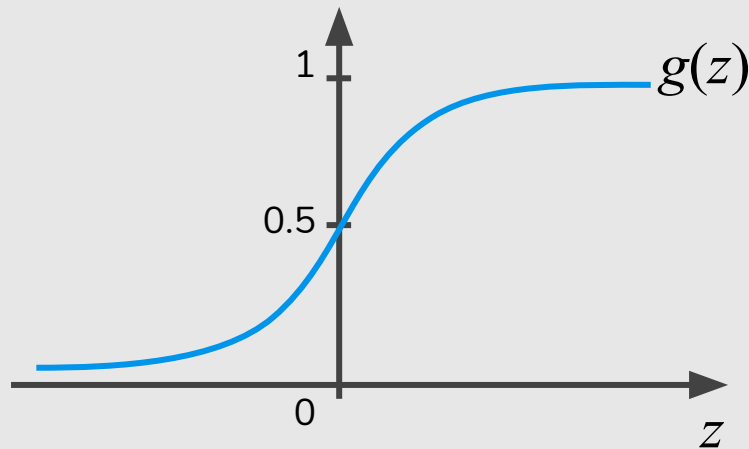
x_1	x_2	$h_{\theta}(x)$
0	0	$g(-30) \approx 0$
0	1	$g(-10) \approx 0$
1	0	
1	1	

Simple Example: AND

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ AND } x_2$$



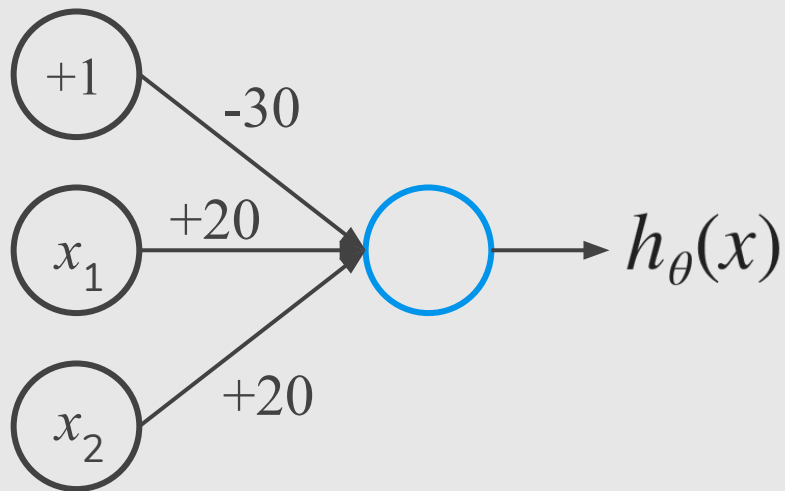
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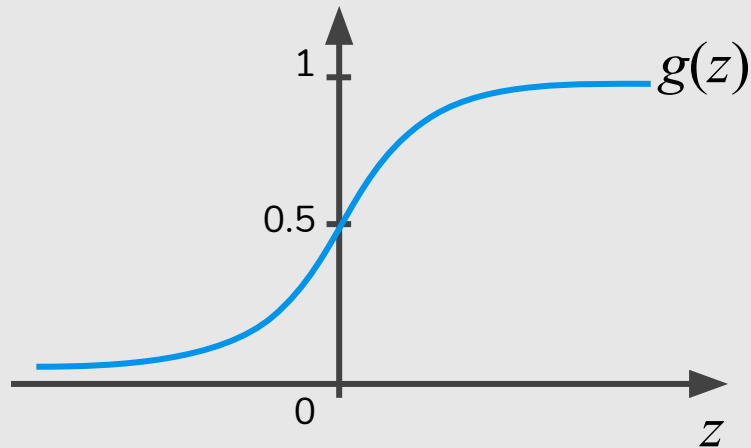
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1	1	

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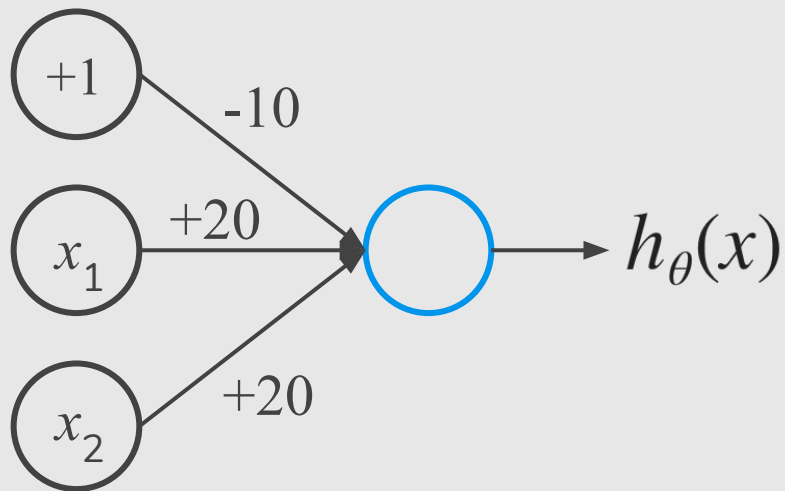
$$h_{\theta}(x) = g(-30 + 20x_1 + 20x_2)$$



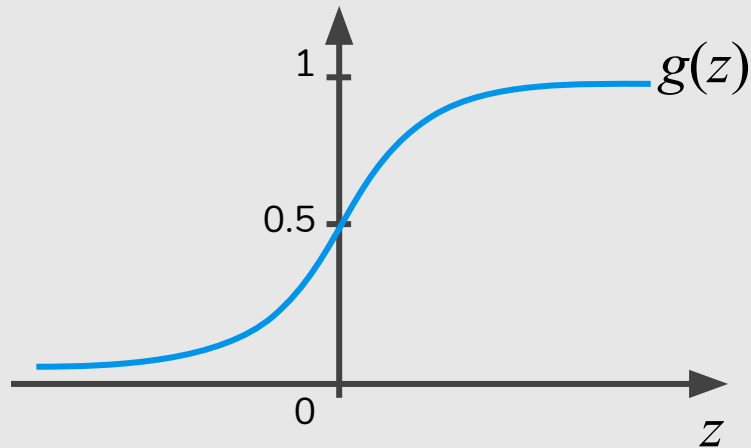
x_1	x_2	$h_{\theta}(x)$
0	0	$g(-30) \approx 0$
0	1	$g(-10) \approx 0$
1	0	$g(-10) \approx 0$
1	1	$g(10) \approx 1$

Simple Example: OR

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ OR } x_2$$



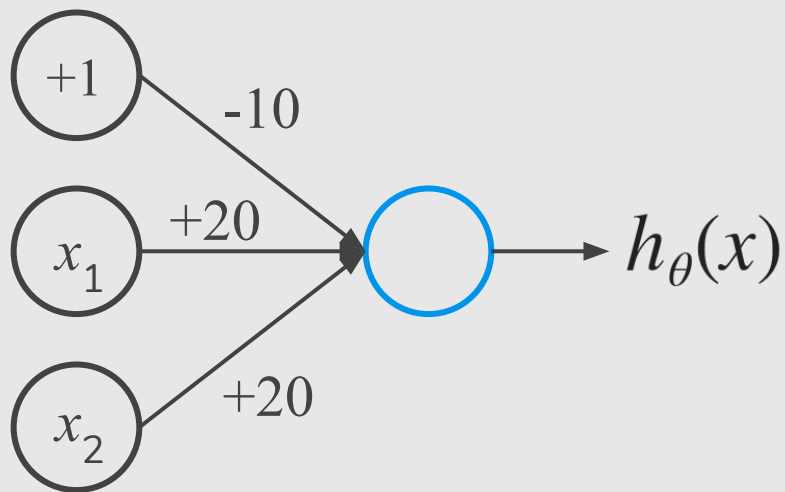
$$h_\theta(x) = g(-10 + 20x_1 + 20x_2)$$



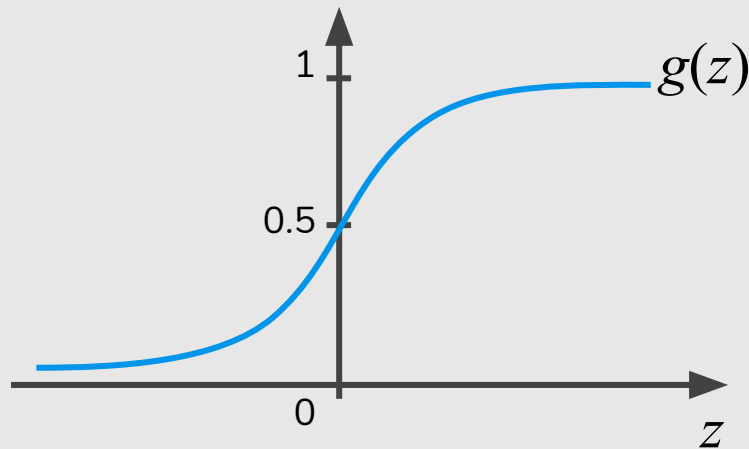
x_1	x_2	$h_\theta(x)$
0	0	
0	1	
1	0	
1	1	

Simple Example: OR

$$x_1, x_2 \in \{0,1\} \quad y = x_1 \text{ OR } x_2$$



$$h_\theta(x) = g(-10 + 20x_1 + 20x_2)$$



x_1	x_2	$h_\theta(x)$
0	0	$g(-10) \approx 0$
0	1	$g(10) \approx 1$
1	0	$g(10) \approx 1$
1	1	$g(30) \approx 1$

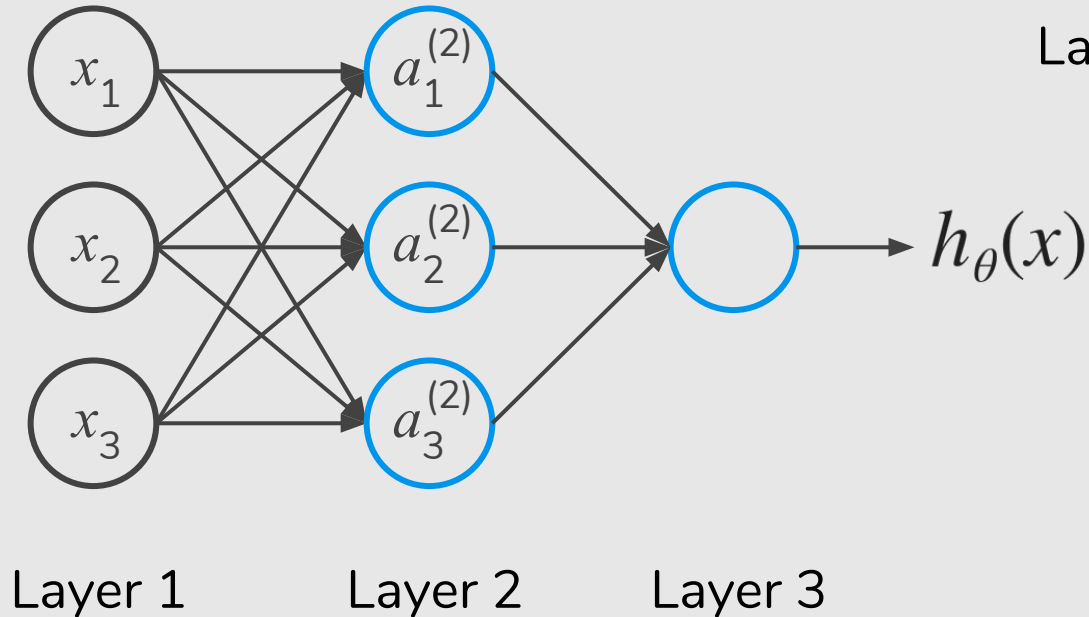
Neural Network

Neural Network

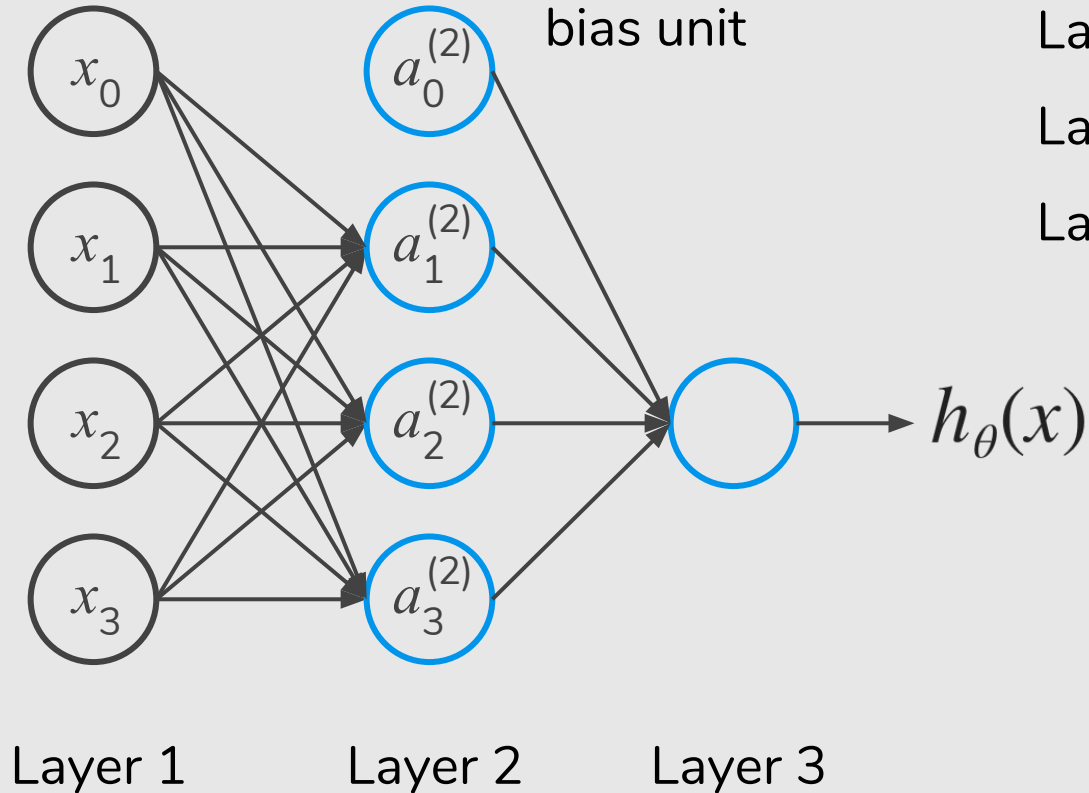
Layer 1 = Input layer

Layer 2 = Hidden layer

Layer 3 = Output layer



Neural Network

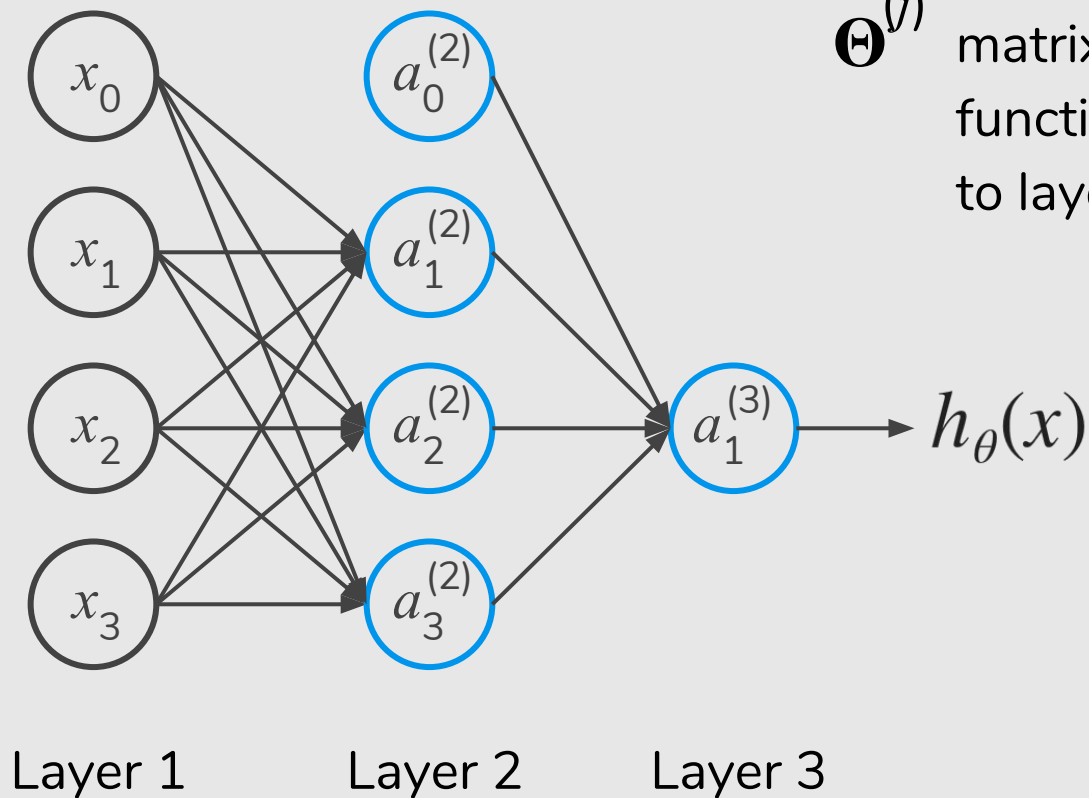


Layer 1 = Input layer

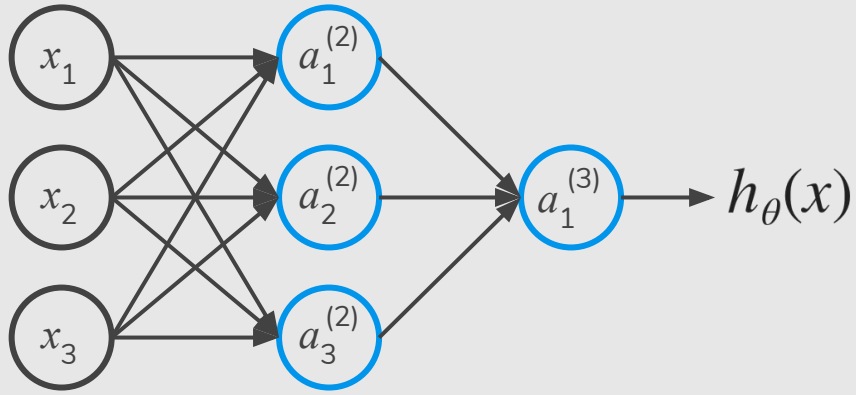
Layer 2 = Hidden layer

Layer 3 = Output layer

Neural Network

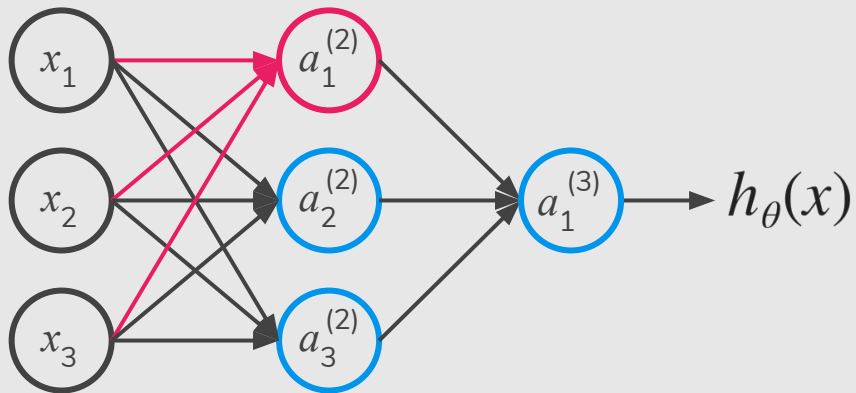


$a_i^{(j)}$ “activation” of unit i in layer j
 $\Theta^{(j)}$ matrix of weights controlling function mapping from layer j to layer $j + 1$



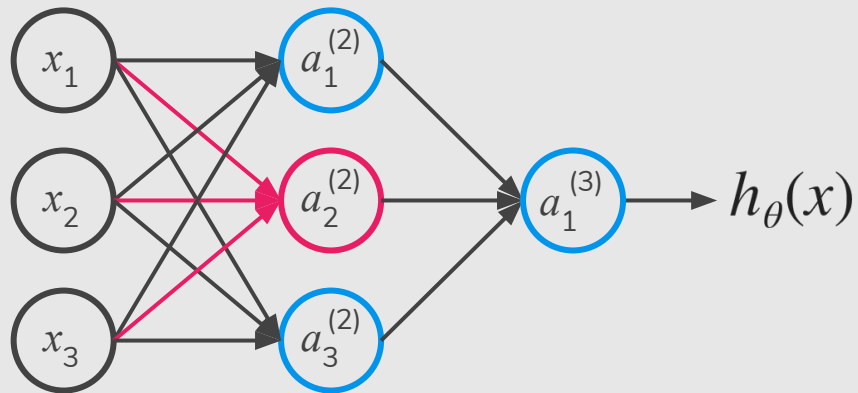
$a_i^{(j)}$ “activation” of unit i in layer j

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$a_i^{(j)}$ “activation” of unit i in layer j
 $\Theta^{(j)}$ matrix of weights controlling
 function mapping from layer j
 to layer $j + 1$

$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

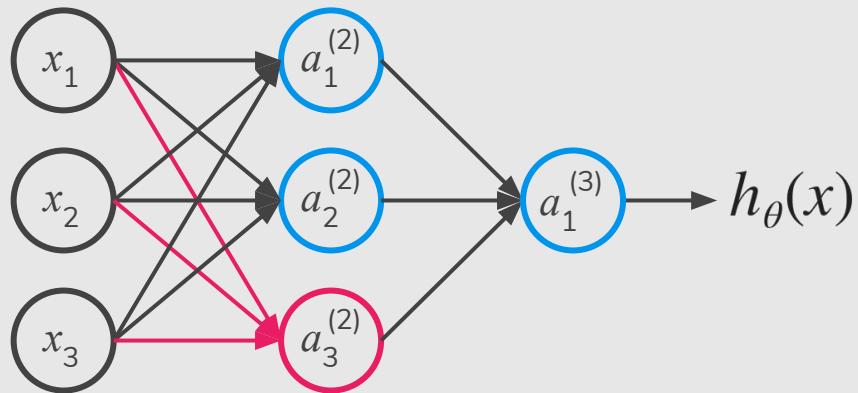


$a_i^{(j)}$ “activation” of unit i in layer j

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$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$



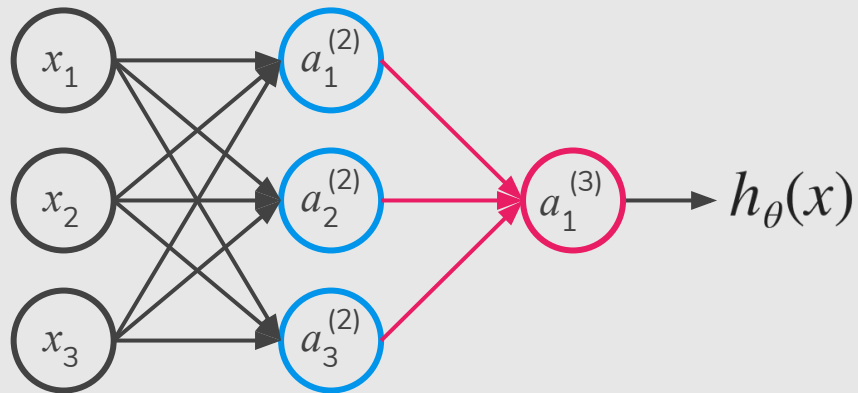
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$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$

$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$



$a_i^{(j)}$ “activation” of unit i in layer j

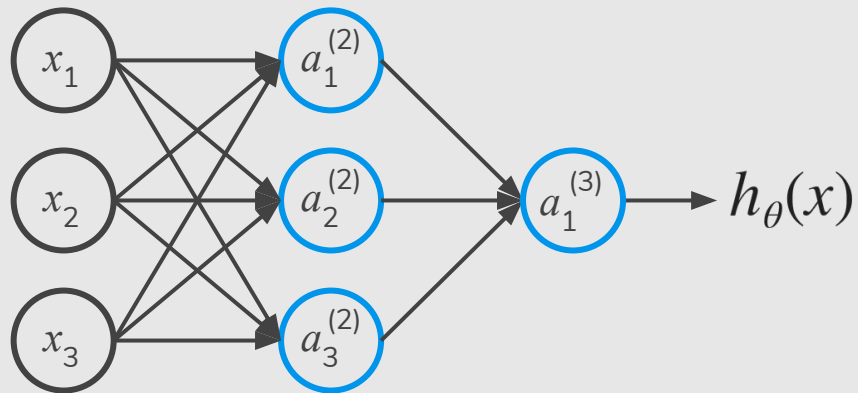
$\Theta^{(j)}$ matrix of weights controlling function mapping from layer j to layer $j + 1$

$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$

$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

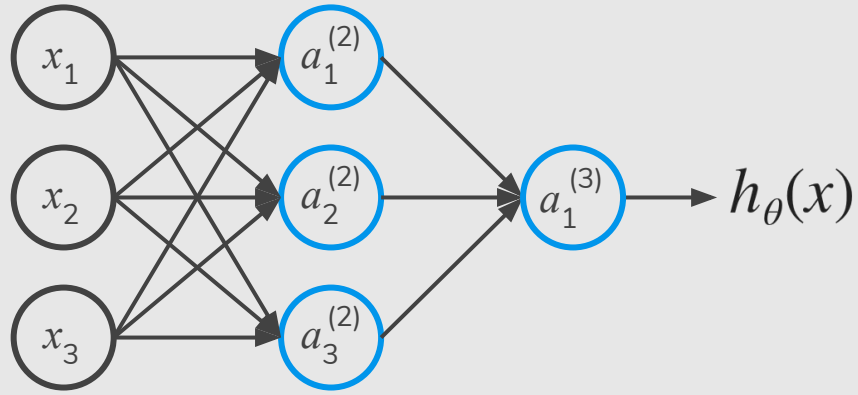
$$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$



$a_i^{(j)}$ “activation” of unit i in layer j
 $\Theta^{(j)}$ matrix of weights controlling
 function mapping from layer j
 to layer $j + 1$

Feedforward Neural Network (forward propagating)

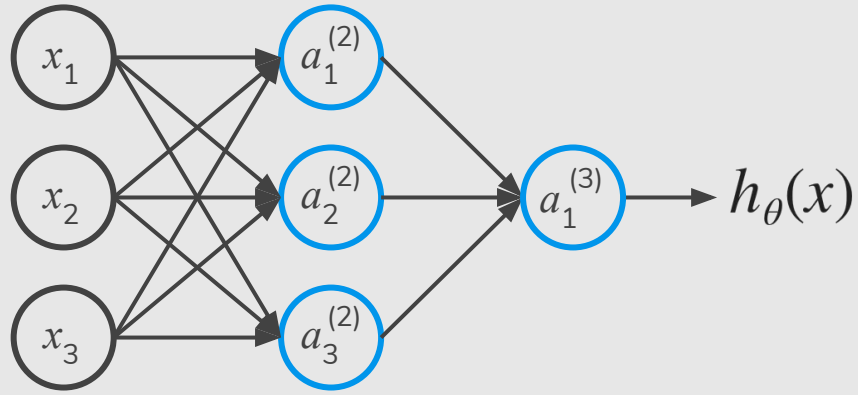
$$h_{\Theta}(x) = a_1^{(3)} = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$



$a_i^{(j)}$ “activation” of unit i in layer j

$\Theta^{(j)}$ matrix of weights controlling function mapping from layer j to layer $j + 1$

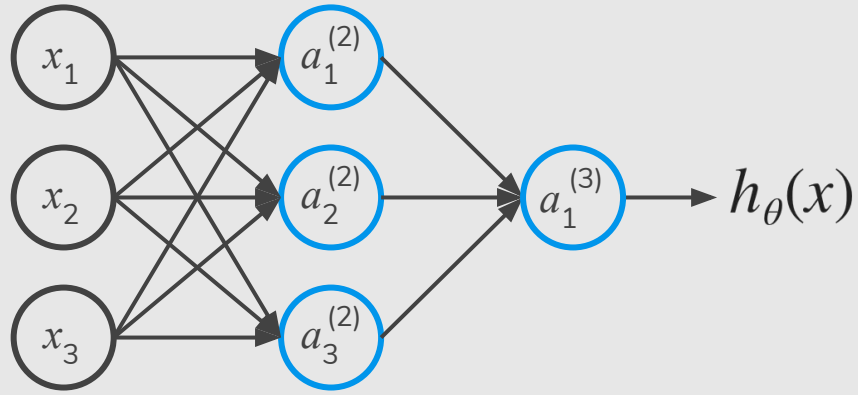
If network has s_j units in layer j , s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.



$a_i^{(j)}$ “activation” of unit i in layer j
 $\Theta^{(j)}$ matrix of weights controlling
 function mapping from layer j
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If network has s_j units in layer j , s_{j+1} units in layer $j+1$,
 then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.

E.g.: If layer 1 has 2 input nodes and layer 2 has 4 activation nodes,

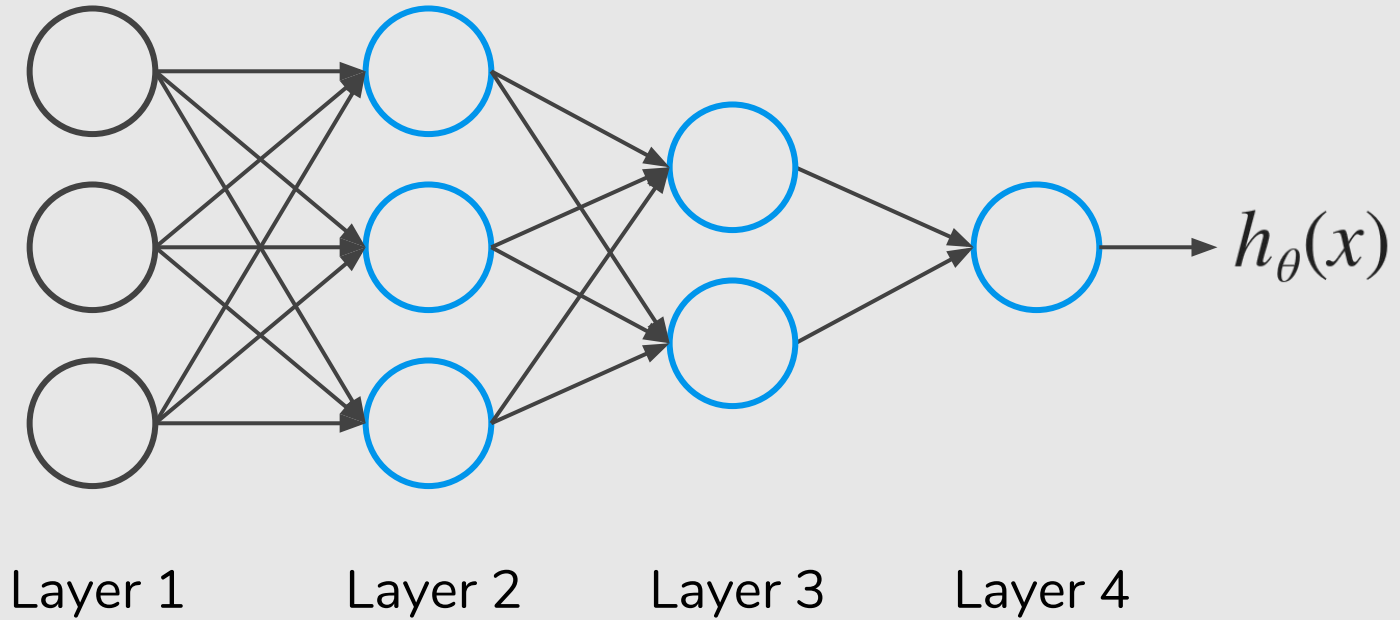


$a_i^{(j)}$ “activation” of unit i in layer j
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If network has s_j units in layer j , s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.

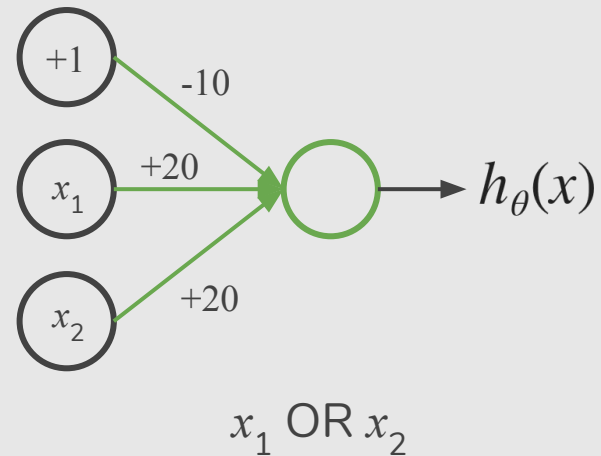
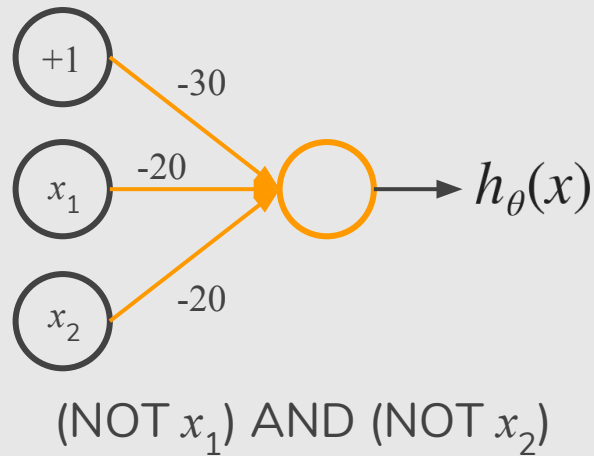
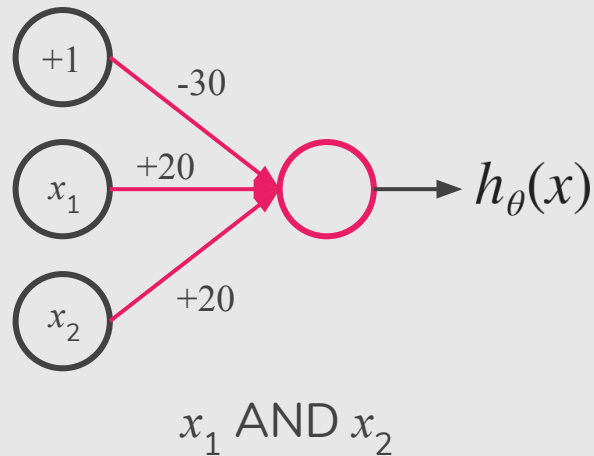
E.g.: If layer 1 has 2 input nodes and layer 2 has 4 activation nodes, dimension of $\Theta^{(1)}$ is going to be 4×3 where $s_j = 2$ and $s_{j+1} = 4$, so $s_{j+1} \times (s_j + 1) = 4 \times 3$.

Other Network Architectures

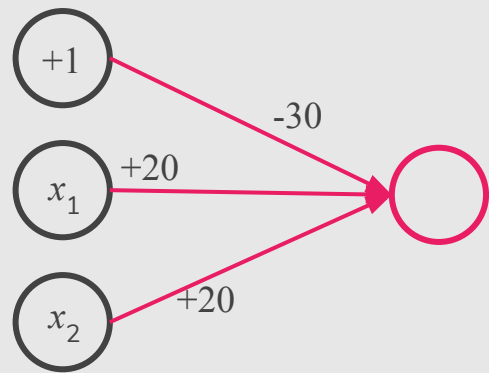
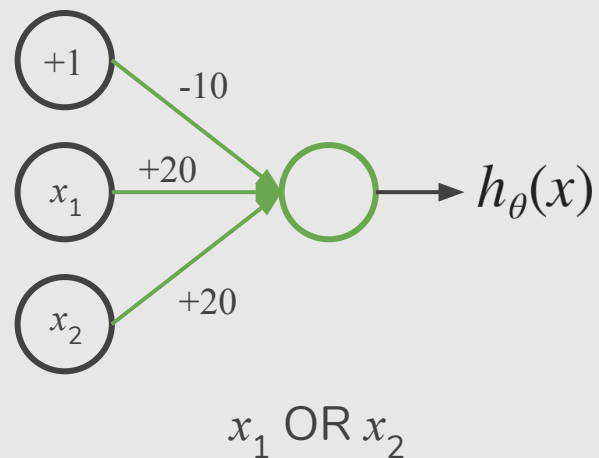
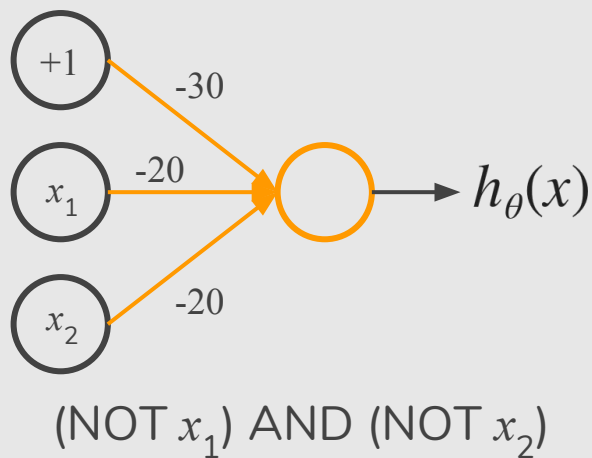
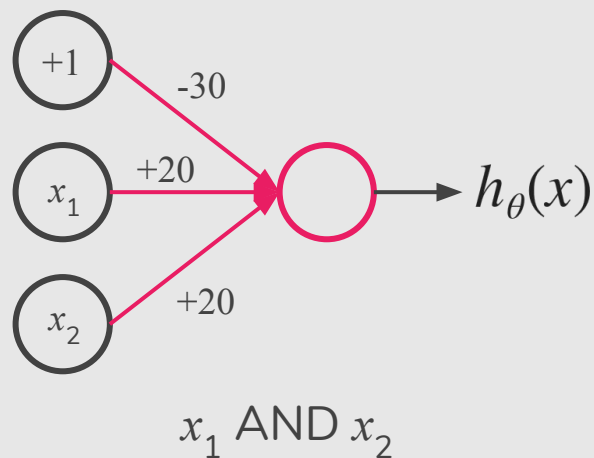


Example: **XNOR** $x_1, x_2 \in \{0,1\}$ $y = x_1 \text{ XNOR } x_2$

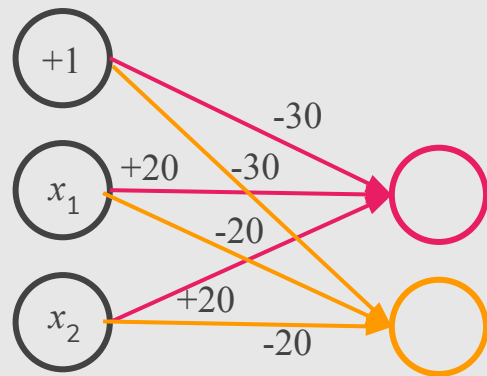
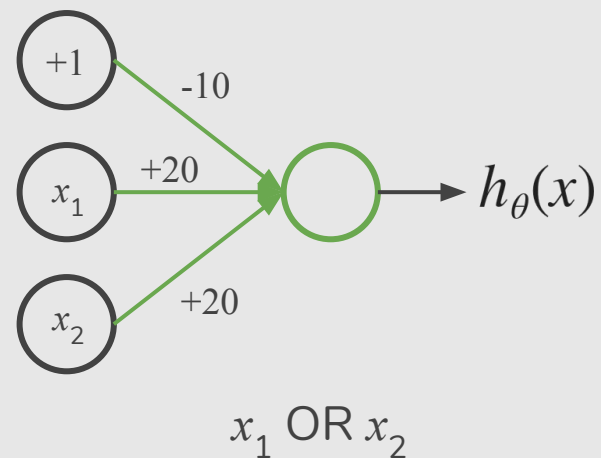
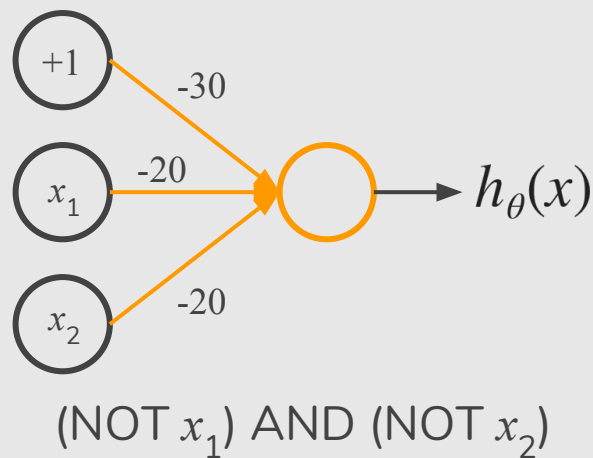
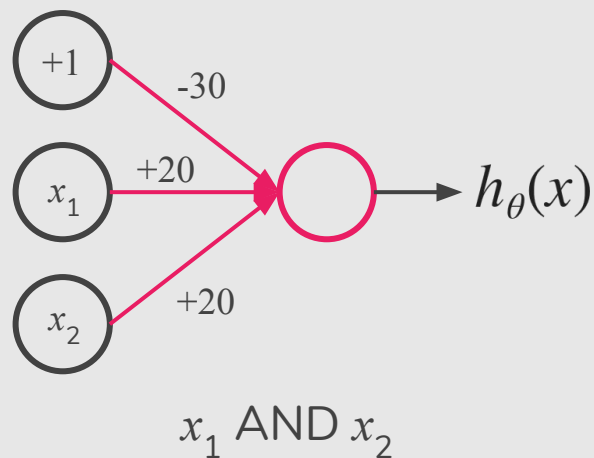
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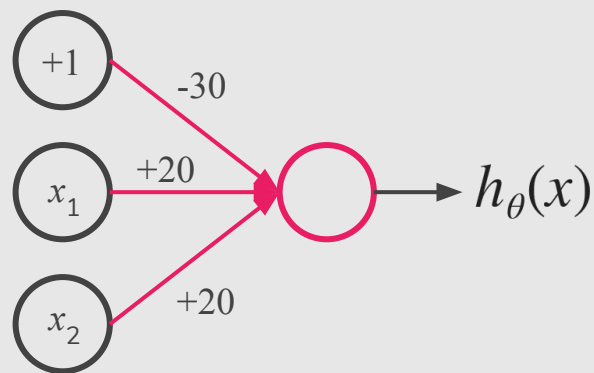
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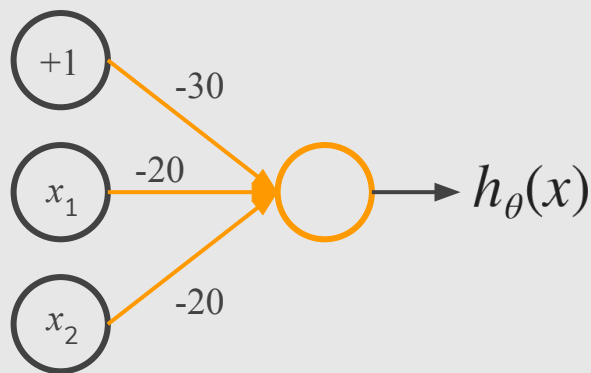
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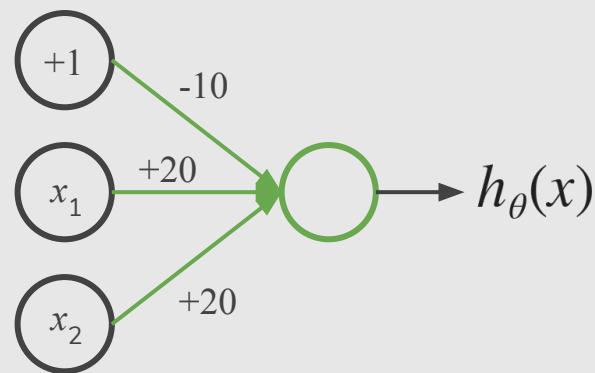
Example: **XNOR** $x_1, x_2 \in \{0,1\}$ $y = x_1 \text{ XNOR } x_2$



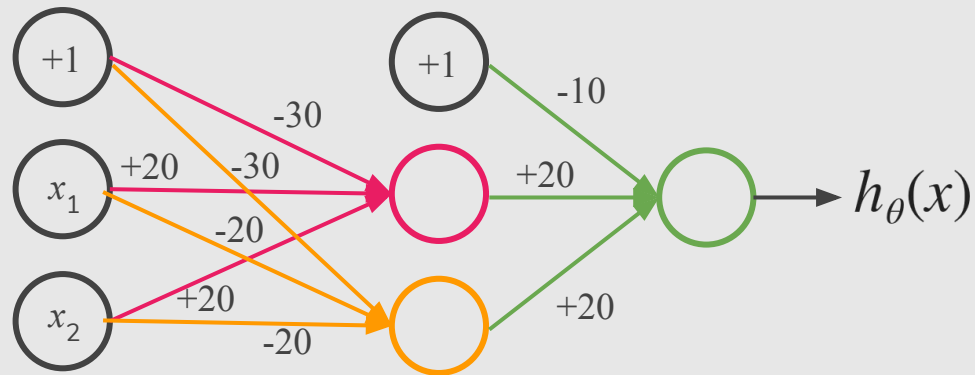
$x_1 \text{ AND } x_2$



$(\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$

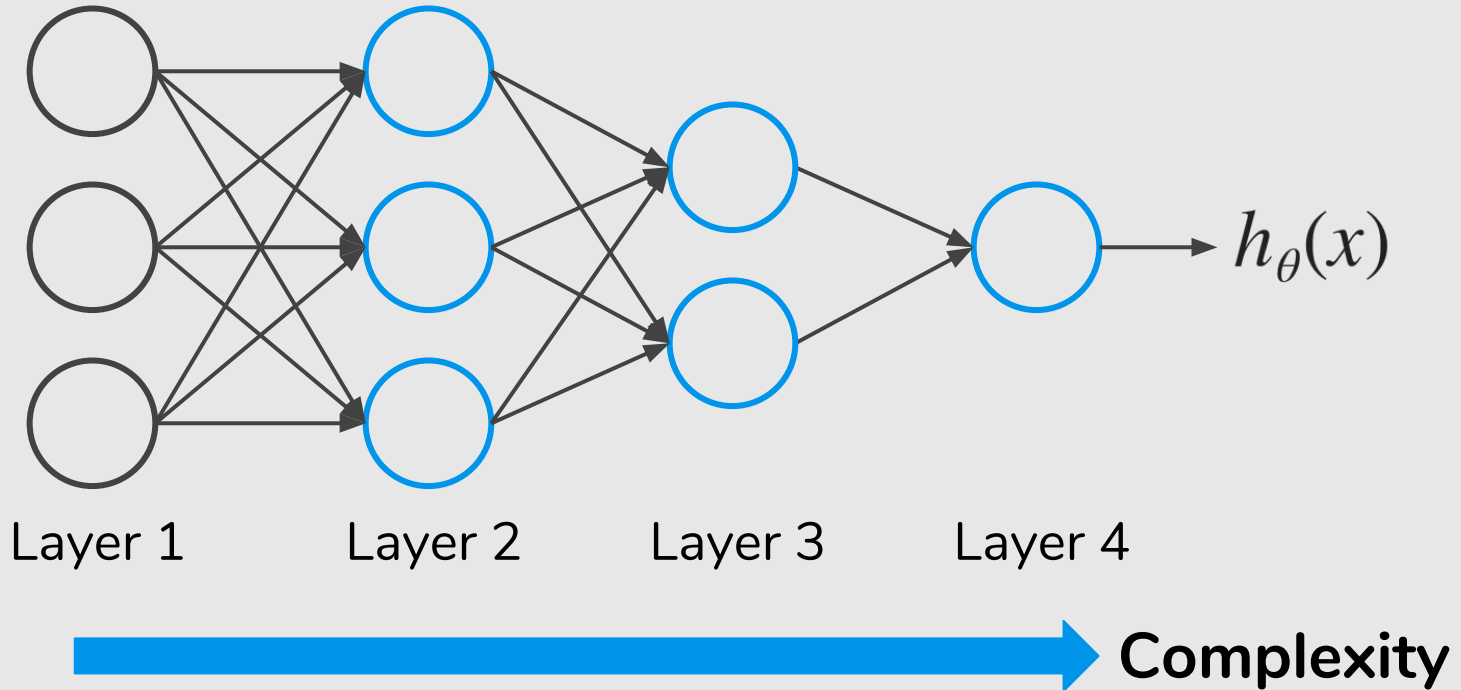


$x_1 \text{ OR } x_2$



x_1	x_2	$h_\theta(x)$
0	0	1
0	1	0
1	0	0
1	1	1

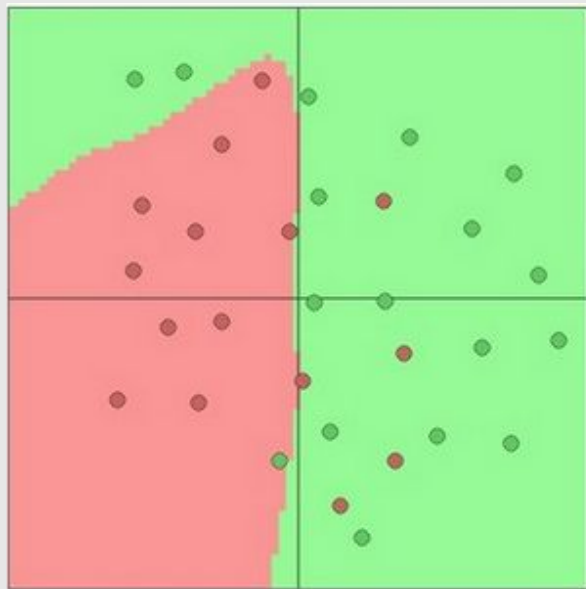
Neural Network Intuition



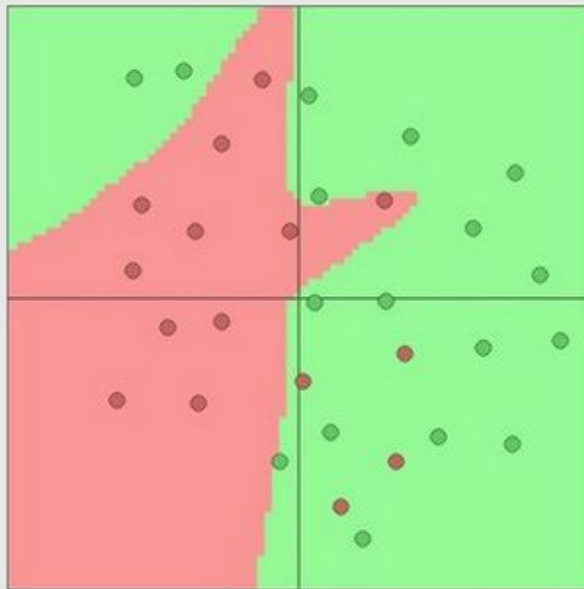
Neural Network Intuition

Toy 2d classification with 2-layer neural network

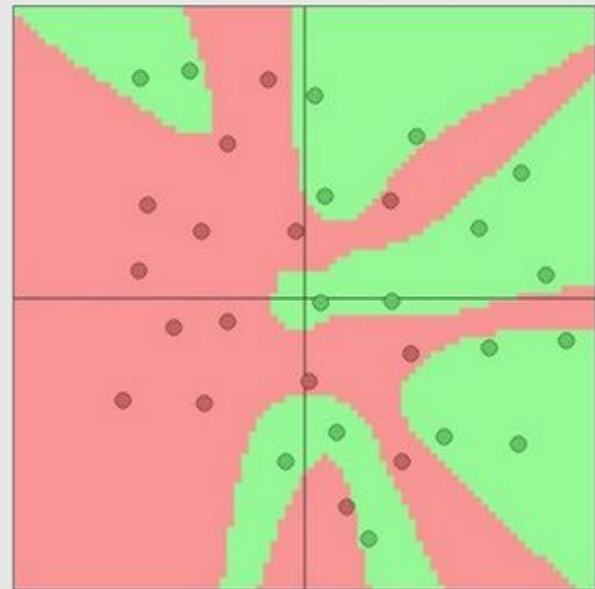
<http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html>



3 hidden neurons

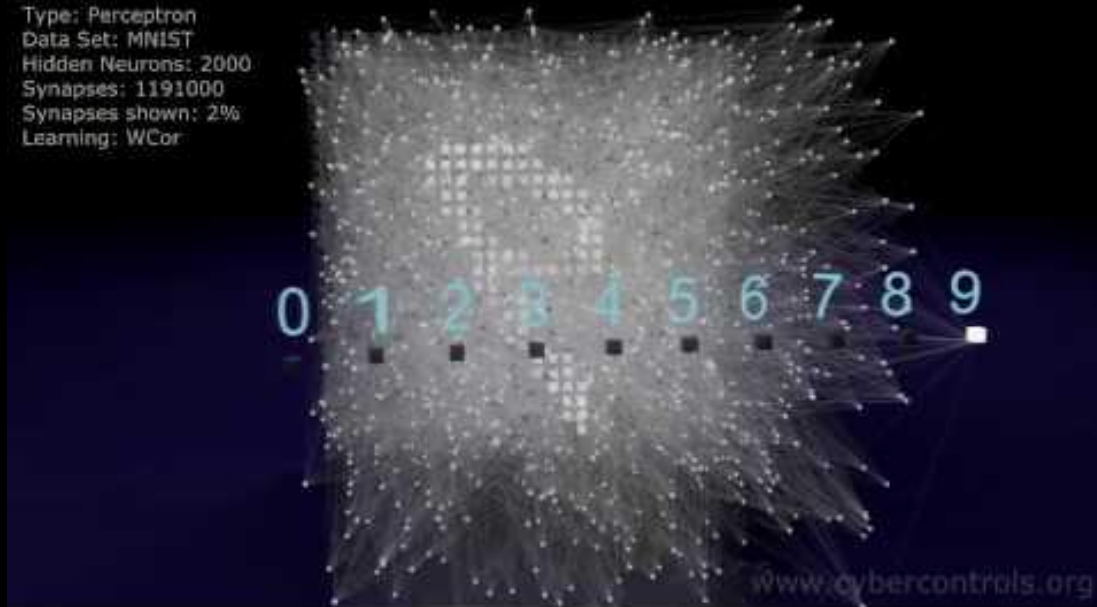


6 hidden neurons



20 hidden neurons

Type: Perceptron
Data Set: MNIST
Hidden Neurons: 2000
Synapses: 1191000
Synapses shown: 2%
Learning: WCor



<https://youtu.be/3JQ3hYko51Y>

Multi-class Classification



Cat



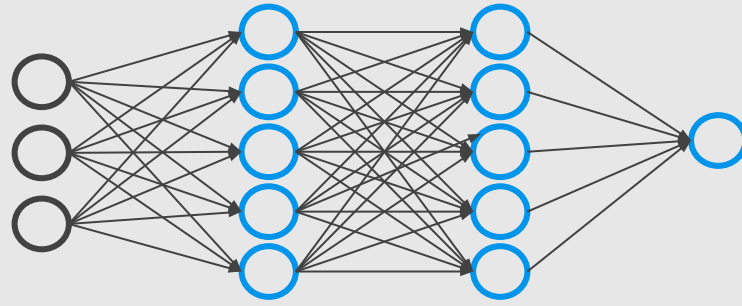
Dog



Frog



Car





Cat



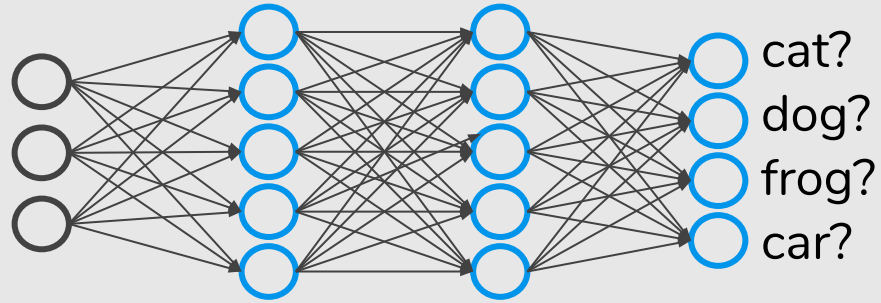
Dog



Frog



Car





Cat



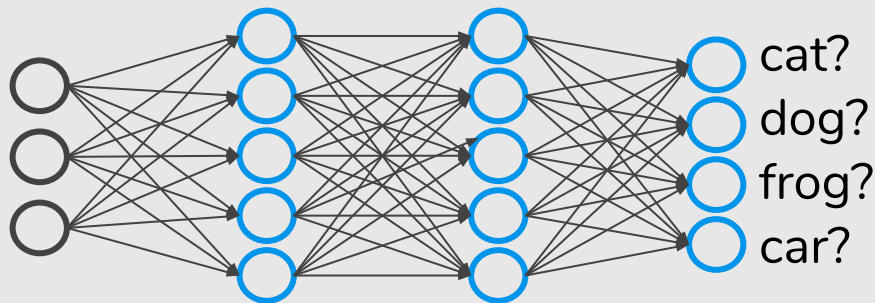
Dog



Frog



Car



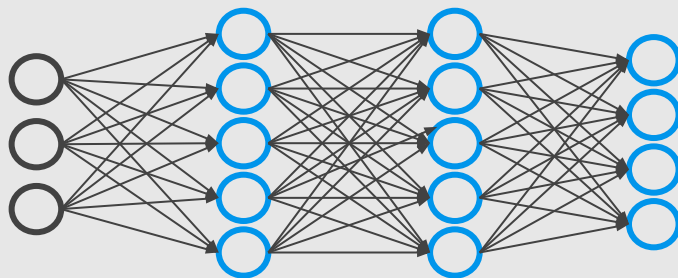
Want $h_{\Theta}(x) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, when cat

$h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, when dog

$h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, when frog

$h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ when car

Multiple Output Units



$$\begin{array}{cccc} \text{Want } h_{\Theta}(x) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, & h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, & h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, & h_{\Theta}(x) \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\ \text{when cat} & \text{when dog} & \text{when frog} & \text{when car} \end{array}$$

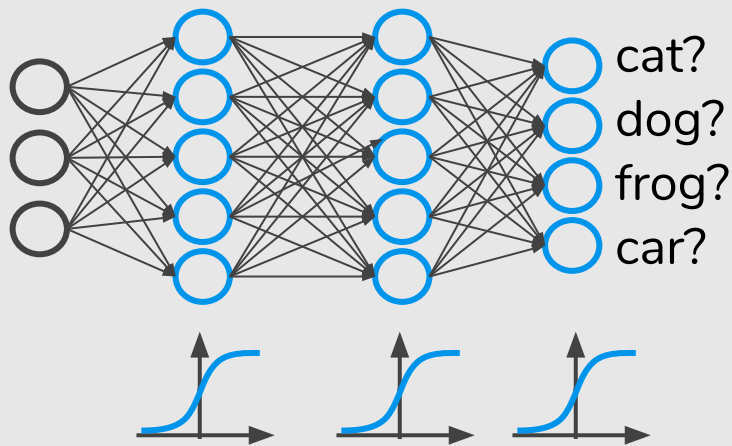
Training Set: $(x^{(1)}, y^{(1)})$, $(x^{(2)}, y^{(2)})$, ..., $(x^{(m)}, y^{(m)})$

$$y^{(i)} \text{ one of } \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Softmax Classification

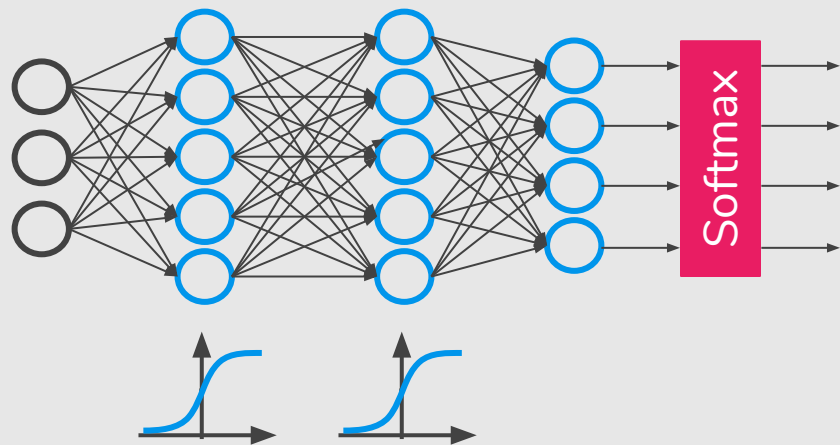
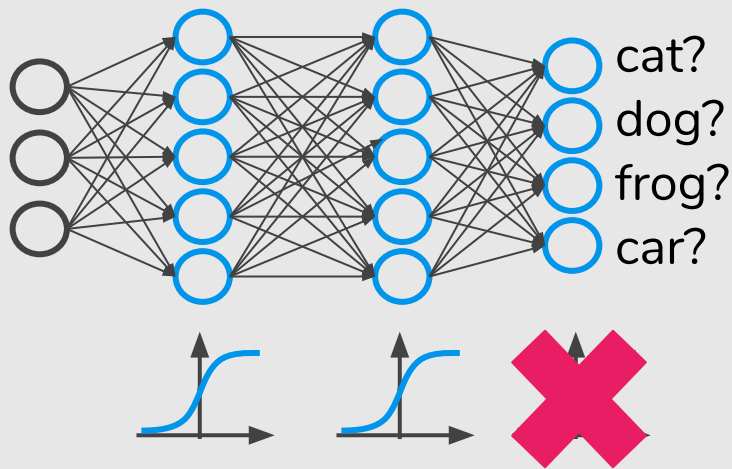
Softmax Classification

The **output layer** is typically modified by replacing the individual activation functions by a **shared softmax** function.



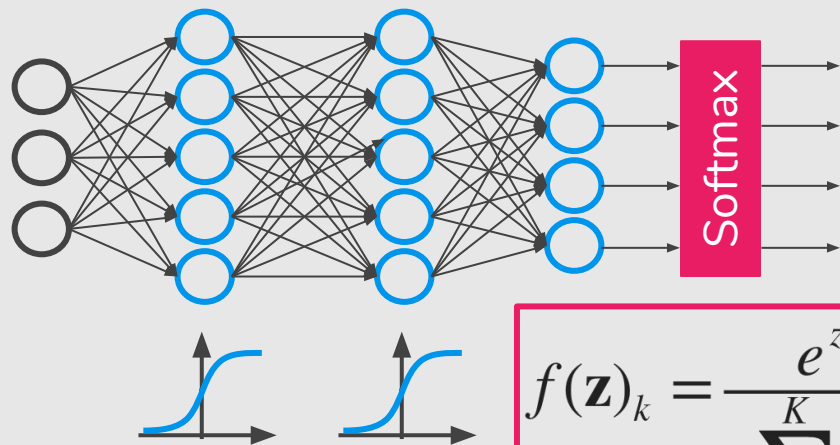
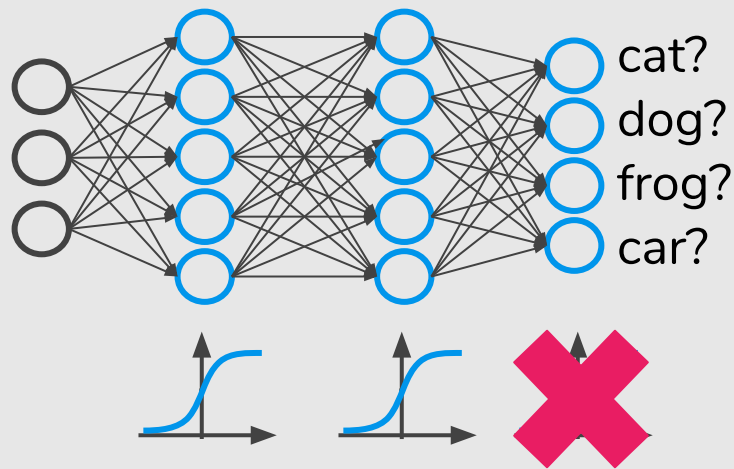
Softmax Classification

The **output layer** is typically modified by replacing the individual activation functions by a **shared softmax** function.



Softmax Classification

The **output layer** is typically modified by replacing the individual activation functions by a **shared softmax** function.



$$f(\mathbf{z})_k = \frac{e^{z_k}}{\sum_{j=1}^K e^{z_j}}$$

To be continued ...

References

— — —

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 10
- Pattern Recognition and Machine Learning, Chap. 5
- Pattern Classification, Chap. 6
- Free online book: <http://neuralnetworksanddeeplearning.com>

Machine Learning Courses

- <https://www.coursera.org/learn/machine-learning>, Week 4 & 5
- <https://www.coursera.org/learn/neural-networks>