

Artificial Neural Networks Machine Learning and Pattern Recognition

Prof. Sandra Avila

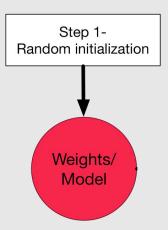
Institute of Computing (IC/Unicamp)

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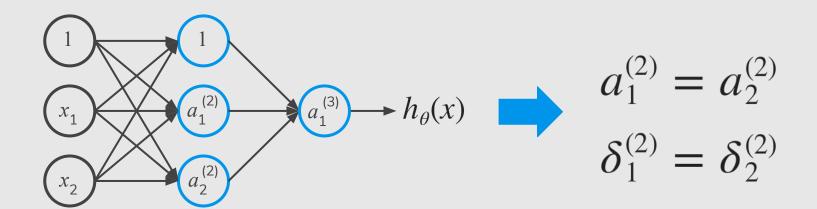
- The first thing we need to do is to select an architecture.
- **Input units:** dimensionality of the problem (features *x*)
- Output units: Number of classes
- Hidden units (per layer)

- Hidden units (per layer):
 - Usually, the more the better
 - Good start: a number close to the number of input
 - Default: 1 hidden layer. If you have >1 hidden layer, then it is interesting that you have the same number of units in every hidden layer.



Zero Initialization

Symmetric Weights

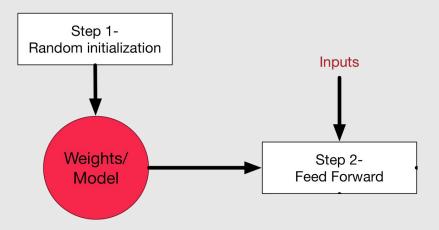


After each update, parameters corresponding to inputs going into each of two hidden units are identical.

Symmetric Breaking

- We must initialize Θ to a random value in $[-\varepsilon, \varepsilon]$ (i.e. $[-\varepsilon \le \Theta \le \varepsilon]$)
- If the dimensions of Theta1 is 3x4, Theta2 is 3x4 and Theta3 is 1x4.

```
Theta1 = random(3,4) * (2 * EPSILON) - EPSILON;
Theta2 = random(3,4) * (2 * EPSILON) - EPSILON;
Theta3 = random(1,4) * (2 * EPSILON) - EPSILON;
```



Forward Propagation

$$a^{(1)} = x$$

$$z^{(2)} = \Theta^{(1)}a^{(1)}$$

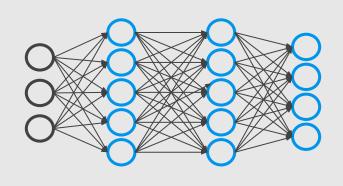
$$a^{(2)} = g(z^{(2)}) \quad (\text{add } a_0^{(2)})$$

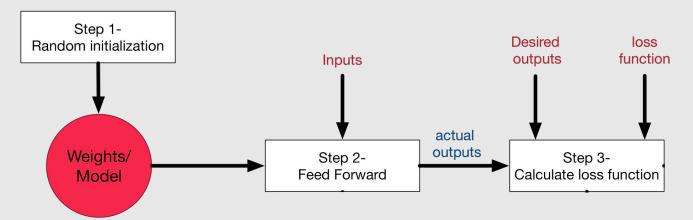
$$z^{(3)} = \Theta^{(2)}a^{(2)}$$

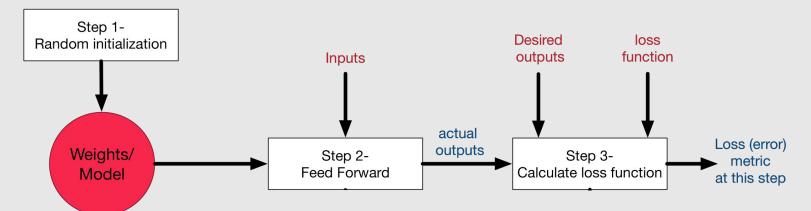
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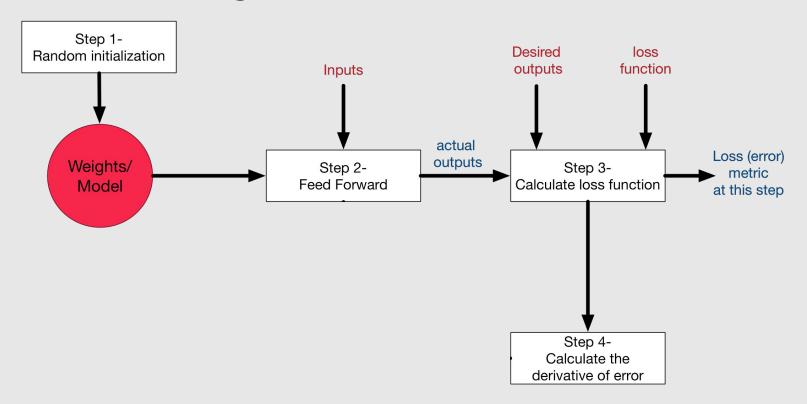
$$z^{(4)} = \Theta^{(3)}a^{(3)}$$

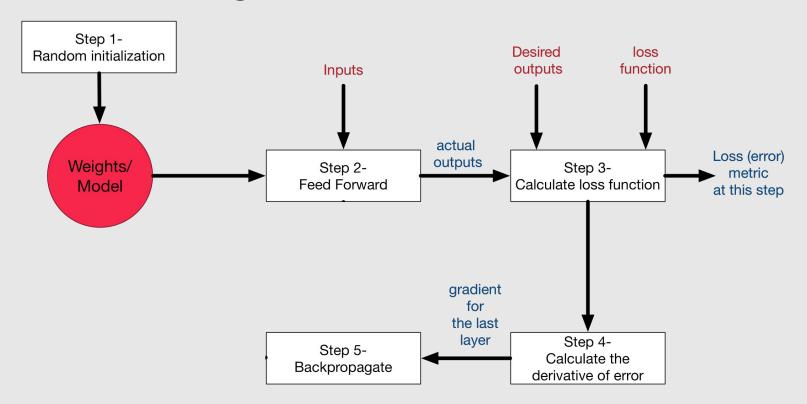
$$a^{(4)} = h_{\Theta}(x) = g(z^{(4)})$$











Training Set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$

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Set $\Delta_{ii}^{(l)} = 0$ (for all l, i, j)



will be used as accumulators for computing $\frac{\partial}{\partial \Theta_{i,i}^{(l)}} J(\Theta)$

```
Training Set: (x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})

Set \Delta_{ij}^{(l)} = 0 (for all l, i, j)

For i = 1 to m

Set a^{(1)} = x^{(i)}
```

Training Set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$

Set $\Delta_{ij}^{(l)} = 0$ (for all l, i, j)

For i = 1 to m

Set $a^{(1)} = x^{(i)}$

Performed forward propagation to compute $a^{(l)}$ for l = 2, 3, ..., L

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Using $y^{(i)}$, compute $\delta^{(L)} = a^{(L)} - y^{(i)}$

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Using $y^{(i)}$, compute $\delta^{(L)} = a^{(L)} - y^{(i)}$

Compute $\delta^{(L-1)}$, $\delta^{(L-2)}$,..., $\delta^{(2)}$

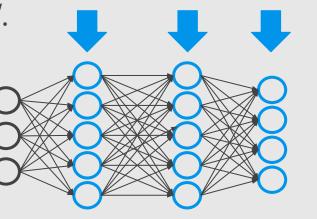
Intuition:
$$\delta_j^{(l)} =$$
 "error" of node j in layer l .

For each output unit (layer L = 4)

$$\delta_j^{(4)} = a_j^{(4)} - y_j$$

For each hidden unit

$$\delta^{(3)} = (\Theta^{(3)})^{\mathrm{T}} \delta^{(4)} \cdot *g'(z^{(3)}) \qquad a^{(3)} (1 - a^{(3)})$$
$$\delta^{(2)} = (\Theta^{(2)})^{\mathrm{T}} \delta^{(3)} \cdot *g'(z^{(2)})$$



 $\delta^{(2)} \quad \delta^{(3)} \quad \delta^{(4)}$

Training Set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$

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 $\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)}$

Training Set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$

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 $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)}$

Training Set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$

Set
$$\Delta_{ij}^{(l)} = 0$$
 (for all l, i, j)

For i = 1 to m

Set
$$a^{(1)} = x^{(i)}$$

Performed forward propagation to compute $a^{(l)}$ for l = 2, 3, ..., L

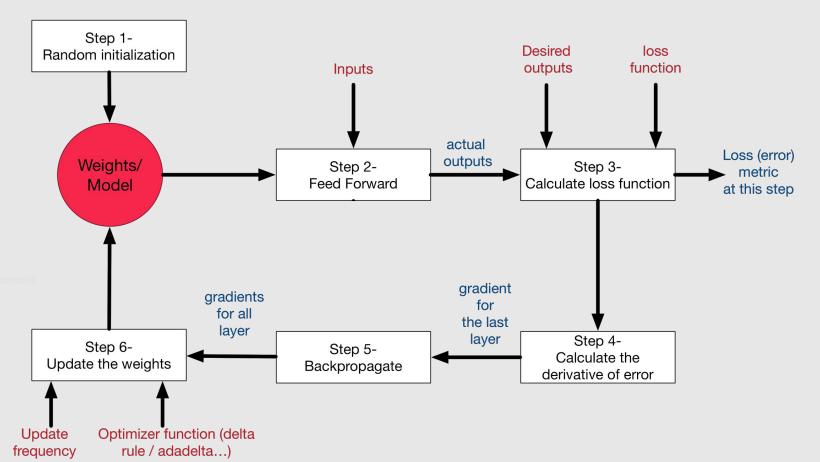
Using
$$y^{(i)}$$
, compute $\delta^{(L)} = a^{(L)} - y^{(i)}$

Compute $\delta^{(L-1)}$, $\delta^{(L-2)}$,..., $\delta^{(2)}$

$$\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)}$$

$$D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)}$$

$$\frac{\partial}{\partial \Theta_{i,j}^{(l)}} J(\Theta) = D_{ij}^{(l)}$$



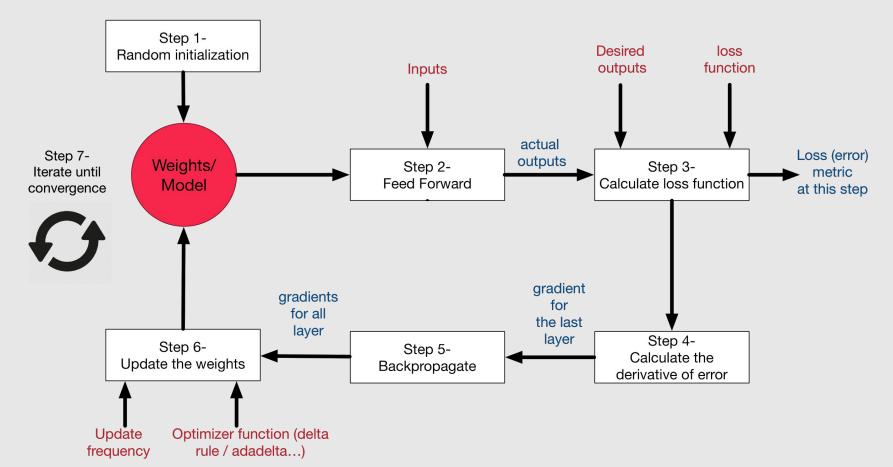
Gradient Descent

$$J(\Theta) = -\frac{1}{m} \left[\sum_{i=1}^{m} \sum_{k=1}^{K} y_k^{(i)} \log(h_{\Theta}(x^{(i)}))_k + (1 - y_k^{(i)}) \log(1 - (h_{\Theta}(x^{(i)}))_k) \right]$$

Want $\min_{\Theta} J(\Theta)$:

repeat {

$$\Theta_{ij}^{(l)} := \Theta_{ij}^{(l)} - \alpha \frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$$

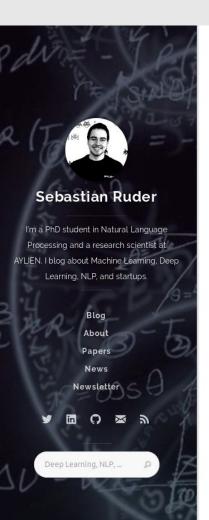


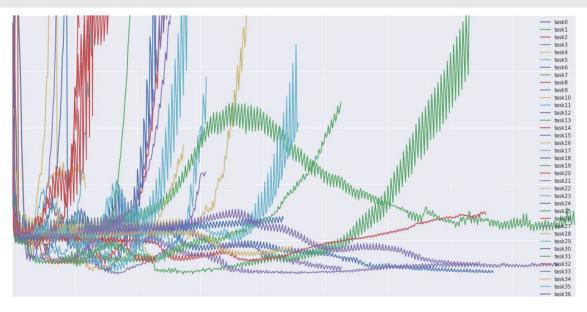
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- 3. It depends on the optimization method.

http://ruder.io/optimizing-gradient-descent/





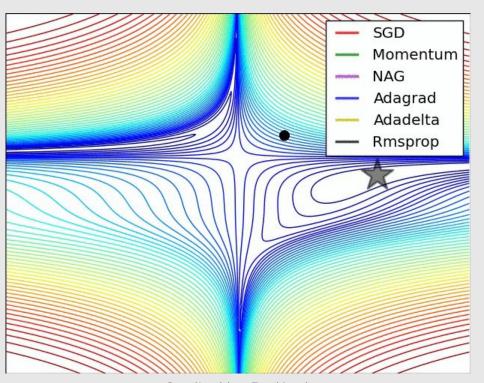
An overview of gradient descent optimization algorithms

- Momentum
 - Nesterov
- Adagrad

- Adadelta
- RMSprop
- Adam

- AdaMax
- Nadam

Batch gradient descent



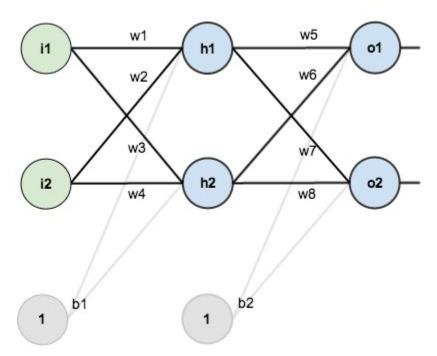
Credit: Alec Radford.

- 1. It depends on the meta-parameters of the network (how many layers, how complex the non-linear functions are).
- 2. It depends on the learning rate.
- 3. It depends on the optimization method.
- 4. It depends on the random initialization of the network.

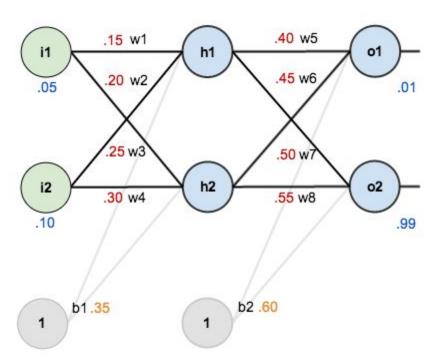
How many iterations are needed to converge?

- 1. It depends on the meta-parameters of the network (how many layers, how complex the non-linear functions are).
- 2. It depends on the learning rate.
- 3. It depends on the optimization method.
- 4. It depends on the random initialization of the network.
- 5. It depends on the quality of the training set.

A Step by Step Backpropagation Example



Given inputs 0.05 and 0.10, we want the neural network to output 0.01 and 0.99.



Initial weights, the biases, and training inputs/outputs.

The Forward Pass

Here's how we calculate the total net input for h_1 :

$$net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

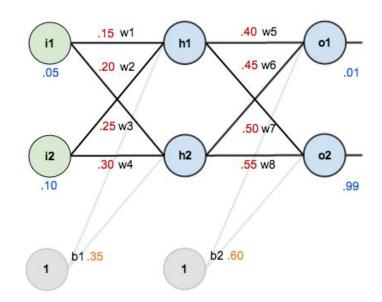
$$net_{h1} = 0.15 * 0.05 + 0.2 * 0.1 + 0.35 * 1 = 0.3775$$

We then squash it using the logistic function to get the output of h_1 :

$$out_{h1} = \frac{1}{1 + e^{-net_{h1}}} = \frac{1}{1 + e^{-0.3775}} = 0.593269992$$

Carrying out the same process for h_2 we get:

$$out_{h2} = 0.596884378$$



The Forward Pass

We repeat this process for the output layer neurons, using the output from the hidden layer neurons as inputs.

Here's the output for o_1 :

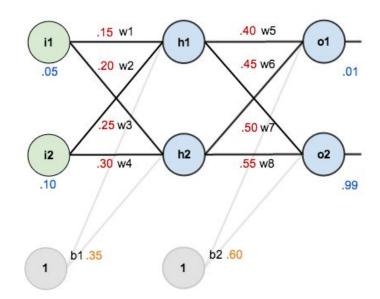
$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$net_{o1} = 0.4 * 0.593269992 + 0.45 * 0.596884378 + 0.6 * 1 = 1.105905967$$

$$out_{o1} = \frac{1}{1 + e^{-net_{o1}}} = \frac{1}{1 + e^{-1.105905967}} = 0.75136507$$

And carrying out the same process for o_2 we get:

$$out_{o2} = 0.772928465$$



The Error

We can now calculate the error for each output neuron using the <u>squared error</u> <u>function</u> and sum them to get the total error:

$$E_{total} = \sum \frac{1}{2} (target - output)^2$$

For example, the target output for o_1 is 0.01 but the neural network output 0.75136507, therefore its error is:

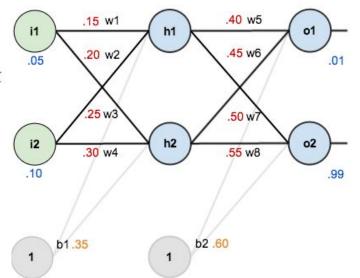
$$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2 = \frac{1}{2}(0.01 - 0.75136507)^2 = 0.274811083$$

Repeating this process for o_2 (remembering that the target is 0.99) we get:

$$E_{o2} = 0.023560026$$

The total error for the neural network is the sum of these errors:

$$E_{total} = E_{o1} + E_{o2} = 0.274811083 + 0.023560026 = 0.298371109$$



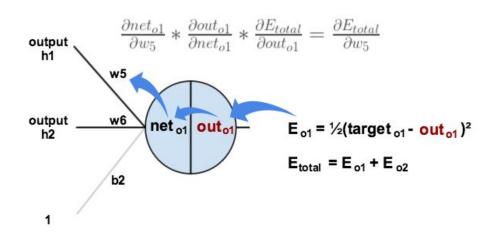
Output Layer

Consider w_5 . We want to know how much a change in w_5 affects the total error, aka $\frac{\partial E_{total}}{\partial w_5}$.

 $\frac{\partial E_{total}}{\partial w_5}$ is read as "the partial derivative of E_{total} with respect to w_5 ". You can also say "the gradient with respect to w_5 ".

By applying the <u>chain rule</u> we know that:

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$



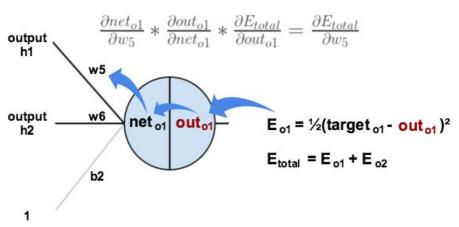
We need to figure out each piece in this equation.

First, how much does the total error change with respect to the output?

$$E_{total} = \frac{1}{2}(target_{o1} - out_{o1})^2 + \frac{1}{2}(target_{o2} - out_{o2})^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2} (target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1}) = -(0.01 - 0.75136507) = 0.74136507$$

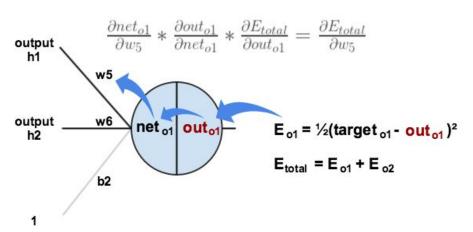


Next, how much does the output of O_1 change with respect to its total net input?

The partial <u>derivative of the logistic function</u> is the output multiplied by 1 minus the output:

$$out_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1}) = 0.75136507(1 - 0.75136507) = 0.186815602$$



Finally, how much does the total net input of o1 change with respect to w_5 ?

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

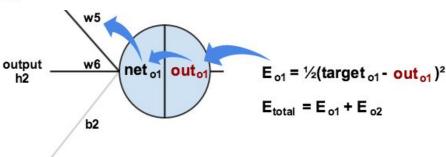
$$\frac{\partial net_{o1}}{\partial w_5} = 1 * out_{h1} * w_5^{(1-1)} + 0 + 0 = out_{h1} = 0.593269992$$

Putting it all together:

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.74136507 * 0.186815602 * 0.593269992 = 0.082167041$$

$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



You'll often see this calculation combined in the form of the delta rule:

$$\frac{\partial E_{total}}{\partial w_5} = -(target_{o1} - out_{o1}) * out_{o1}(1 - out_{o1}) * out_{h1}$$

Alternatively, we have $\frac{\partial E_{total}}{\partial out_{o1}}$ and $\frac{\partial out_{o1}}{\partial net_{o1}}$ which can be written as $\frac{\partial E_{total}}{\partial net_{o1}}$, aka δ_{o1} (the Greek letter delta) aka the *node delta*. We can use this to rewrite the calculation above:

$$\delta_{o1} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} = \frac{\partial E_{total}}{\partial net_{o1}}$$

$$\delta_{o1} = -(target_{o1} - out_{o1}) * out_{o1}(1 - out_{o1})$$

Therefore:

$$\frac{\partial E_{total}}{\partial w_5} = \delta_{o1} out_{h1}$$

To decrease the error, we then subtract this value from the current weight (optionally multiplied by some learning rate, eta, which we'll set to 0.5):

$$w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 * 0.082167041 = 0.35891648$$

Some sources use α (alpha) to represent the learning rate, others use η (eta), and others even use ϵ (epsilon).

We can repeat this process to get the new weights w_6 , w_7 , and w_8 :

$$w_6^+ = 0.408666186$$

$$w_7^+ = 0.511301270$$

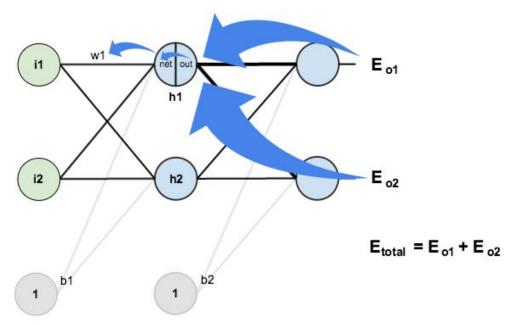
$$w_8^+ = 0.561370121$$

Hidden Layer

Next, we'll continue the backwards pass by calculating new values for w_1 , w_2 , w_3 , and w_4 .

Big picture, here's what we need to figure out:

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$



We're going to use a similar process as we did for the output layer, but slightly different to account for the fact that the output of each hidden layer neuron contributes to the output (and therefore error) of multiple output neurons. We know that out_{h1} affects both out_{o1} and out_{o2} therefore the $\frac{\partial E_{total}}{\partial out_{h1}}$ needs to take into consideration its effect on the both output neurons:

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

Starting with $\frac{\partial E_{o1}}{\partial out_{h1}}$:

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}}$$

We can calculate $\frac{\partial E_{o1}}{\partial net_{o1}}$ using values we calculated earlier:

$$\frac{\partial E_{o1}}{\partial net_{o1}} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} = 0.74136507 * 0.186815602 = 0.138498562$$

And $\frac{\partial net_{o1}}{\partial out_{h1}}$ is equal to w_5 :

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$

Plugging them in:

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}} = 0.138498562 * 0.40 = 0.055399425$$

Following the same process for $\frac{\partial E_{o2}}{\partial out_{b1}}$, we get:

$$\frac{\partial E_{o2}}{\partial out_{b1}} = -0.019049119$$

Therefore:

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}} = 0.055399425 + -0.019049119 = 0.036350306$$

Now that we have $\frac{\partial E_{total}}{\partial out_{h1}}$, we need to figure out $\frac{\partial out_{h1}}{\partial net_{h1}}$ and then $\frac{\partial net_{h1}}{\partial w}$ for each weight:

$$out_{h1} = \frac{1}{1 + e^{-net_{h1}}}$$

$$\frac{\partial out_{h1}}{\partial net_{h1}} = out_{h1}(1 - out_{h1}) = 0.59326999(1 - 0.59326999) = 0.241300709$$

We calculate the partial derivative of the total net input to h_1 with respect to w_1 the same as we did for the output neuron:

$$net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

$$\frac{\partial net_{h1}}{\partial w_1} = i_1 = 0.05$$

Putting it all together:

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.036350306 * 0.241300709 * 0.05 = 0.000438568$$

We can now update w_1 :

$$w_1^+ = w_1 - \eta * \frac{\partial E_{total}}{\partial w_1} = 0.15 - 0.5 * 0.000438568 = 0.149780716$$

Repeating this for w_2 , w_3 , and w_4

$$w_2^+ = 0.19956143$$

$$w_3^+ = 0.24975114$$

$$w_4^+ = 0.29950229$$

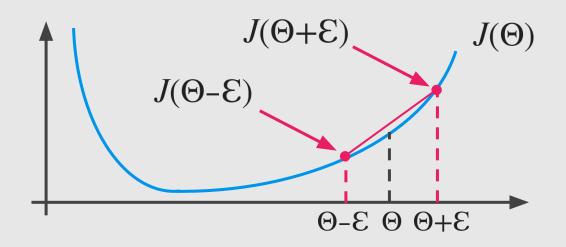
Finally, we've updated all of our weights! When we fed forward the 0.05 and 0.1 inputs originally, the error on the network was 0.298371109. After this first round of backpropagation, the total error is now down to 0.291027924. It might not seem like much, but after repeating this process 10,000 times, for example, the error plummets to 0.0000351085. At this point, when we feed forward 0.05 and 0.1, the two outputs neurons generate 0.015912196 (vs 0.01 target) and 0.984065734 (vs 0.99 target).

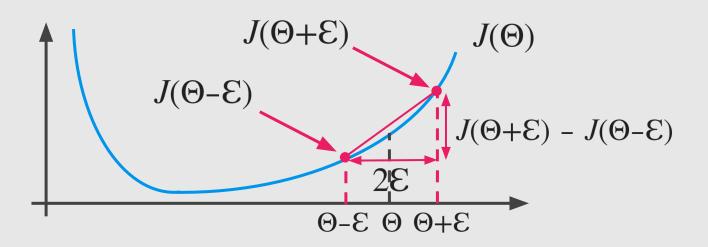
Gradient Checking

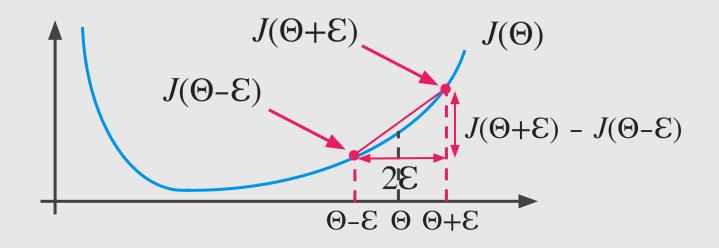
How can we check the Backpropagation algorithm

along with its optimization function for finding

parameters (e.g. gradient descent) are actually working?







$$\frac{d}{d\Theta}J(\Theta) \approx \frac{J(\Theta+E) - J(\Theta-E)}{2E}$$

$$\theta=\theta_1,\!\theta_2,\!\theta_3,\!...,\!\theta_n$$
 (E.g, θ is "unrolled" version of $\Theta^{(1)},\!\Theta^{(2)},\!...,\!\Theta^{(n)}$)

$$\frac{\partial}{\partial \theta_{1}} J(\theta) \approx \frac{J(\theta_{1} + \mathcal{E}, \theta_{2}, \theta_{3}, \dots, \theta_{n}) - J(\theta_{1} - \mathcal{E}, \theta_{2}, \theta_{3}, \dots, \theta_{n})}{2\mathcal{E}}$$

$$\frac{\partial}{\partial \theta_{2}} J(\theta) \approx \frac{J(\theta_{1}, \theta_{2} + \mathcal{E}, \theta_{3}, \dots, \theta_{n}) - J(\theta_{1}, \theta_{2} - \mathcal{E}, \theta_{3}, \dots, \theta_{n})}{2\mathcal{E}}$$

$$\vdots$$

 $\frac{\partial}{\partial \theta_n} J(\theta) \approx \frac{J(\theta_1, \theta_2, \theta_3, ..., \theta_n + \mathcal{E}) - J(\theta_1, \theta_2, \theta_3, ..., \theta_n - \mathcal{E})}{2\mathcal{E}}$

$$\theta=\theta_1, \theta_2, \theta_3, ..., \theta_n$$
 (E.g, θ is "unrolled" version of $\Theta^{(1)}, \Theta^{(2)}, ..., \Theta^{(n)}$)

$$\frac{\partial}{\partial \theta_{1}} J(\theta) \approx \frac{J(\theta_{1} + \mathcal{E}, \theta_{2}, \theta_{3}, ..., \theta_{n}) - J(\theta_{1} - \mathcal{E}, \theta_{2}, \theta_{3}, ..., \theta_{n})}{2\mathcal{E}}$$

$$\frac{\partial}{\partial \theta_{2}} J(\theta) \approx \frac{J(\theta_{1}, \theta_{2} + \mathcal{E}, \theta_{3}, ..., \theta_{n}) - J(\theta_{1}, \theta_{2} - \mathcal{E}, \theta_{3}, ..., \theta_{n})}{2\mathcal{E}}$$

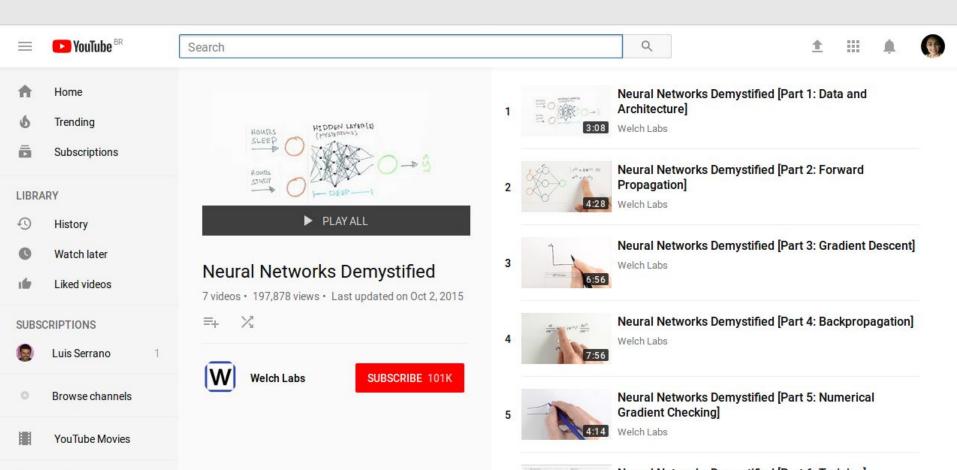
$$\vdots$$

 $\frac{\partial}{\partial \theta} J(\theta) \approx \frac{J(\theta_1, \theta_2, \theta_3, \dots | \theta_n + \mathcal{E}) - J(\theta_1, \theta_2, \theta_3, \dots | \theta_n - \mathcal{E})}{2\mathcal{E}}$

Implementation Note

 Backpropagation is very efficient compared to gradient numerical checking. We should use it just in one or few iterations to double check backpropagation results.

Neural Networks Demystified (in Python)



References

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Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 10
- Pattern Recognition and Machine Learning, Chap. 5
- Pattern Classification, Chap. 6
- Free online book: http://neuralnetworksanddeeplearning.com

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 4 & 5
- https://www.coursera.org/learn/neural-networks