

# Universal intuitions of spatial relations in elementary geometry

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## ABSTRACT

Spatial relations are central to geometrical thinking. With respect to the classical elementary geometry of Euclid's *Elements*, a distinction between *co-exact*, or qualitative, and *exact*, or metric, spatial relations has recently been advanced as fundamental. We tested the universality of intuitions of these relations in a group of Senegalese and Dutch participants. Participants performed an odd-one-out task with stimuli that in all but one case display a particular spatial relation between geometric objects. As the exact/co-exact distinction is closely related to Kosslyn's *categorical/coordinate* distinction, a set of stimuli for testing all four types was used. Results suggest that intuitions of all spatial relations tested are universal. Yet, culture has an important effect on performance: Dutch participants outperformed Senegalese participants and stimulus layouts affect the categorical and coordinate processing in different ways for the two groups. Differences in level of education within the Senegalese participants did not affect performance.

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## 1. Introduction

Euclid's theory of geometry presented in the *Elements* is a highly influential achievement of western culture. What are the intuitions at the origins of this theory? Are these intuitions universal—that is, present in all human beings independently of their culture, education, and environment? It has recently been argued that such questions are amenable to psychological investigations (Izard, Pica, Dehaene, et al., 2011). The universality of geometric intuitions has been empirically investigated in a series of studies involving the Mundurucu (Dehaene, Izard, Pica, & Spelke, 2006; Izard, Pica, Spelke, & Dehaene, 2011), an Amazonian indigene group with no schooling—and therefore no formal education of geometry, with no use of geometrical tools such as rulers, compasses or maps, and with a language possessing few words referring to spatial or geometric concepts. Dehaene and colleagues (2006) have shown that the Mundurucu possess intuitions of a wide range of geometric concepts, and are able to use geometrical information in maps to locate objects. Furthermore, Izard and colleagues (2011)

have provided evidence that the Mundurucu possess intuitions of geometric concepts beyond perceptual experience, such as infinite lines.

The premise of these empirical investigations has been criticised by Wulff in the discussion of Wulff et al. (2006) in his commentary to Dehaene et al. (2006). Wulff argues that the central feature of Euclidean geometry is its demonstrative character and its logical structure, rather than what can be seen in graphical pictures of geometric objects. Wulff thereby follows the modern view on geometric reasoning according to which pictures do not, and should not, play any role in geometric demonstrations (Hilbert, 1971). From this point of view, an empirical investigation of Euclidean geometry, which only involves pictures of geometric objects, necessarily misses the central feature of Euclidean geometry as a mathematical theory. Wulff concludes that the study of Dehaene and colleagues (2006) does not address Euclidean geometry but rather the cognitive ability of pattern recognition.

What is required to settle the dispute between these two points of view is a precise account of

the role of *pictures* or *diagrams* in the mathematical theory of Euclidean geometry. This is a well-known issue that has been extensively discussed in philosophy and mathematics. The modern view on Euclidean geometry to which Wulff subscribes denies any role for pictures or diagrams in geometric proofs. From this point of view, the picture-based approach of Dehaene and colleagues (2006) lacks any connection with Euclidean geometry as a mathematical theory. This point of view has been seriously challenged, however, by the seminal analysis of Euclid's *Elements* proposed by Manders (2008). Manders has shown that pictures or diagrams do have specific and principled role when it comes to the deductive proofs of Euclid's *Elements*. This suggests that an investigation of intuitions at the origins of the theory in the manner of Dehaene and colleagues is indeed possible and well founded, provided that one restricts one's attention to the intuitions that underlie the use of diagrams in Euclid's *Elements*.

This is precisely the standpoint of this study. Our approach is based directly on Manders' account (2008) of the role of diagrams in Euclid's geometric proofs. In this analysis, Manders notices that one can distinguish between two types of spatial relations depicted by Euclidean diagrams: *exact* relations, with metric equality of magnitudes as the primary example, and *co-exact* relations, which consist of qualitative relations such as intersection and containment. He then observes that Euclid only infers co-exact spatial relations from the diagrams in his proofs.

The intuitions that underlie the use of diagrams in Euclid's proofs are therefore the ability to extract co-exact spatial relations between geometric objects depicted in Euclidean diagrams. From a psychological perspective, the question arises then whether this ability is *universal* in the sense of Dehaene et al. (2006). The main aim of this study is to address this issue systematically by combining the philosophical insights of Manders (2008) with the experimental methodology of Dehaene et al. (2006). The studies on the Mundurucu do not highlight the exact/co-exact distinction, but rather focus on geometrical concepts in general. So its data are not sufficient to provide an answer to this issue. However, due to the finite and visual nature of Euclidean diagrams, this methodology appears as particularly suitable for the current purpose. With the odd-one-out paradigm, the universality of co-exact spatial relations can be studied by providing participants with six images, in which five images share the

same co-exact spatial relation between certain geometric objects while one does not. If the findings concerning universality of geometric intuitions by Dehaene et al. (2006) are correct, given Manders' analysis of Euclid's *Elements*, then universality should also be found for images reflecting exact and co-exact relations in particular.

The distinction between exact and co-exact spatial relations raises another important psychological issue. As noticed by Hamami and Mumma (2013), this distinction is closely related to an important distinction in the field of spatial relation processing, namely Kosslyn's (1987) distinction between *coordinate* and *categorical* spatial relations. This distinction dissociates abstract, propositional relations (categorical), like "left of" and "above" from precise, metric relations (coordinate) such as "2 m away" or "further apart". Many experimental studies have substantiated this by confirming that these two types of relations are processed in a distinct manner by the brain: the left parietal cortex is mainly involved in processing categorical information, whereas the right parietal cortex is mainly responsible for processing coordinate information (see e.g. Jager & Postma, 2003; Laeng, Chabris, & Kosslyn, 2003). In this study, we investigate the universality of the processing of exact versus co-exact and coordinate versus categorical spatial relations by adapting a set of stimuli traditionally used in the categorical/coordinate literature to the odd-one-out paradigm. The incorporation of typical exact and co-exact stimuli, as well as typical categorical and coordinate stimuli, will allow for the first experimental comparison of the two typologies of spatial relations. Given the high level of overlap between these definitions, we expected a very similar pattern of performance for co-exact and categorical relations, and exact and coordinate relations.

Important aspects in studying universality are specific cultural features, like education. If differences between two cultures are found, this could be the result of a difference in education levels. It can also be the result of other cultural features like language and living environment. We conducted the current experiment in adult Senegalese and Dutch participants. The culture of the Senegalese participants is very similar to that of the Mundurucu, in terms of the very limited or absent use of spatial assistive devices like maps or rulers. Moreover, the particular composition of the group of Senegalese participants allowed an analysis of the impact of education. We composed two subgroups: those

participants with at least several years of education and those with no formal education at all. As the sample was otherwise homogeneous, this allowed us to address the effect of education directly. The educated participants received at least basic mathematical training. The Dutch participants were homogeneous in terms of their education and cultural background. They were very familiar with tools like maps and rulers. These samples of participants allowed us to compare two highly different cultures and isolate the contribution of education within one of the two cultures. It should be noted that it cannot be excluded that a potential difference between the Senegalese and Dutch participants is in part attributable to a difference in education level between the two groups, due to different education laws in these two countries. If the use of the different types of spatial relations is truly universal, education should not have a significant effect on performance.

## 2. Methods

### 2.1. Participants

51 Senegalese and 30 Dutch participants performed the experiment. In Table 1, the characteristics of both groups are provided, groups were matched by mean age.

Participants were asked which languages they spoke fluently. All Senegalese participants reported Wolof to be their first language. Seven indicated to have Sereer as their second language, three of whom indicated that French was their third language. Six reported French as their second language. Linguistic fluency was difficult to explain to the Senegalese participants, as cultural characteristics may cause them to indicate that their command of a language is better than it in fact is. Therefore they were asked how proficient they were in their second and third language. The experimenter and interpreter decided whether the response given indicated fluent language skills or not. All Dutch participants indicated that Dutch was their first language, English was the second

language for all participants, five reported to speak a third language fluently; French (1), Spanish (2), Japanese (1), and German (1).

The Senegalese participants all lived in or around Dakar, the capital of Senegal (population around 2.5 million). Therefore, they had all been exposed to the same, relatively urban living environment. All Dutch participants lived in urban areas in the Netherlands: the cities of Utrecht (population 335,000) and Leiden (population 122,000), both urban environments.

Testing took place in accordance with the declaration of Helsinki, and approval of the local ethical committee was obtained.

### 2.2. Materials and design

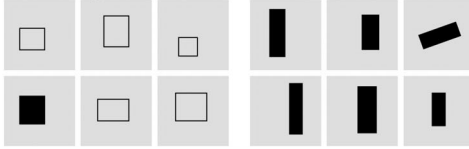
The experimental design was based on Dehaene et al. (2006). A single trial consisted of a page with six different pictures displaying the same combination of geometric objects. To test a particular spatial relation, five pictures displayed situations in which the spatial relation holds between the objects, while in the last one the relation did not hold (for recent experimental work using the odd-one-out paradigm for geometric stimuli see Giofrè, Mammarella, Ronconi, & Cornoldi, 2013; Giofrè, Mammarella, & Cornoldi, 2014; Izard & Spelke, 2009; for a computational analysis see Lovett & Forbus, 2011). In total, two training trials and 26 actual trials were used. All 28 stimuli are depicted in Figure 1. The actual trials could be assigned to four different conditions. The *co-exact* condition consisted of different types of co-exact spatial relations, based on Hamami and Mumma (2013). The *exact* condition consisted of the five trials taken from the original Dehaene stimuli that represent “exact” spatial relations. The remaining two conditions, *categorical* and *coordinate*, made use of four conventional stimulus layouts to test categorical and coordinate spatial relation processing: dot bar, large cross, small cross, and diagonal cross (see e.g. Hellige & Michimata, 1989; Van der Ham, Raemaekers, Van Wezel, Oleksiak, & Postma, 2009; Van der Ham, van Wezel, Oleksiak, & Postma, 2007). Four trials with abstract figures, that is, the dot bar and cross dot stimuli, were used. In the categorical trials, the spatial category (e.g. above versus below, left versus right) was the same in five pictures and different in one picture. In the coordinate trials, the same types of stimuli layouts were used as in the categorical trials. Instead of categories, now the distance between the two elements in the figure was

**Table 1.** Descriptive statistics of all participants.

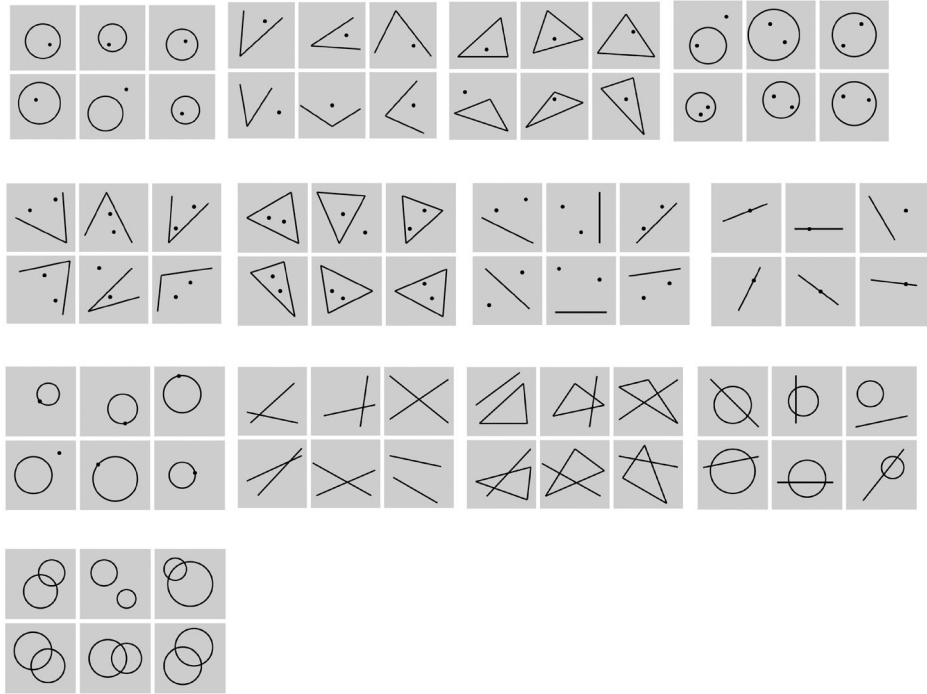
Group	N (F/M)	Mean age (SD)	Mean years of education (SD)
Senegalese	51 (12/39)	23.8 (5.6)	3.5 (4.3)
No education	25 (4/21)	23.2 (6.0)	0.0 (0.0)
Education	26 (8/18)	24.5 (5.3)	6.9 (3.6)
Dutch	30 (23/7)	24.6 (4.9)	16.2 (2.9)

Note: M, male; F, female; SD, standard deviation.

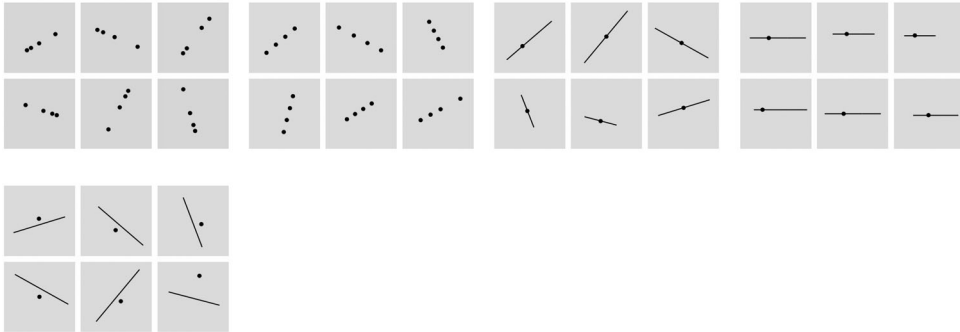
### Training



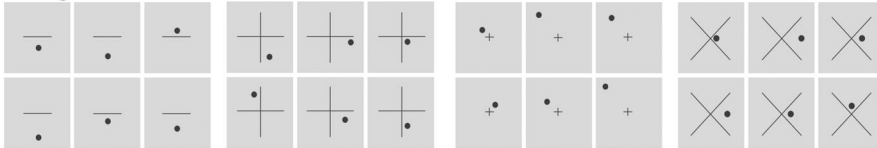
### Co-exact



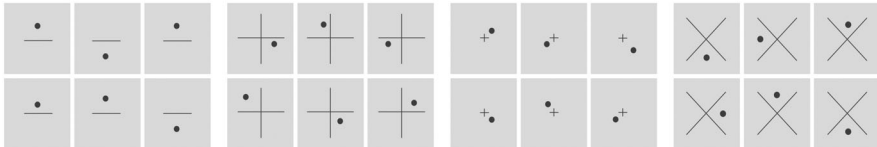
### Exact



### Categorical



### Coordinate



**Figure 1.** All trials used in the experiment.

the same in five pictures, and longer or shorter in one picture.

### 2.3. Procedure

Testing in Senegal was performed with assistance of two interpreters. Both interpreters were Senegalese by birth. They were living in the Netherlands for over 15 years, but made regular visits to Senegal. They were fluent in both Wolof and Dutch. They were uninformed about the goal of the experiment and only translated the basic task instructions and any questions participants asked or comments they made. The task was designed to require very little verbal instructions.

Senegalese participants were tested in a quiet area and shown the stimuli on paper. They responded by pointing out the picture that they thought did not belong to the other pictures on the page.

Dutch participants were also tested in a quiet area. They received verbal instructions and were shown the stimuli on paper as well. Their responses consisted of circling the picture they thought did not belong to the other pictures on the page with a pen. No further indication of what rules to apply was given. All instructions were identical for all participants, and simply entailed that they had to point out the one picture that did not belong to the other five objects because it looked “strange” or “different”. No further indication about what distinctions to look for was given.

The experiment started with two examples, identical to the training trials used by Dehaene et al. (2006). When participants understood the two training trials they continued with the 26 actual trials. No other instructions were given, aside from encouragement to continue and guess if necessary when participants indicated uncertainty about what to do. It was made clear that they could take as much time as they needed to complete the experiment. They were not allowed to go back to previous trials. The order of the trials was pseudo-randomised. Stimuli from the different conditions were distributed equally over the whole stimulus set. A stimulus was never directly followed by a stimulus from the same condition. Eight different orders were created to avoid any undesired sequence effects of particular stimuli.

### 2.4. Statistical analyses

First of all, the frequencies of responses for each trial and each group of participants were calculated. For

each condition, average accuracy was calculated. Accuracy was defined as the selection of the response that fits the geometrical rule used for that particular trial. First, the effect of culture was studied by comparing responses of the Senegalese participants to the responses of the Dutch participants, by means of a repeated-measures General Linear Model (GLM). Second, the effect of education was analysed by comparing participants with no education to those who had education within the Senegalese participants, by means of a repeated-measures GLM.

More detailed analyses were performed within the categorical and coordinate conditions. Also the observations, in particular during testing Senegalese participants, were recorded.

## 3. Results

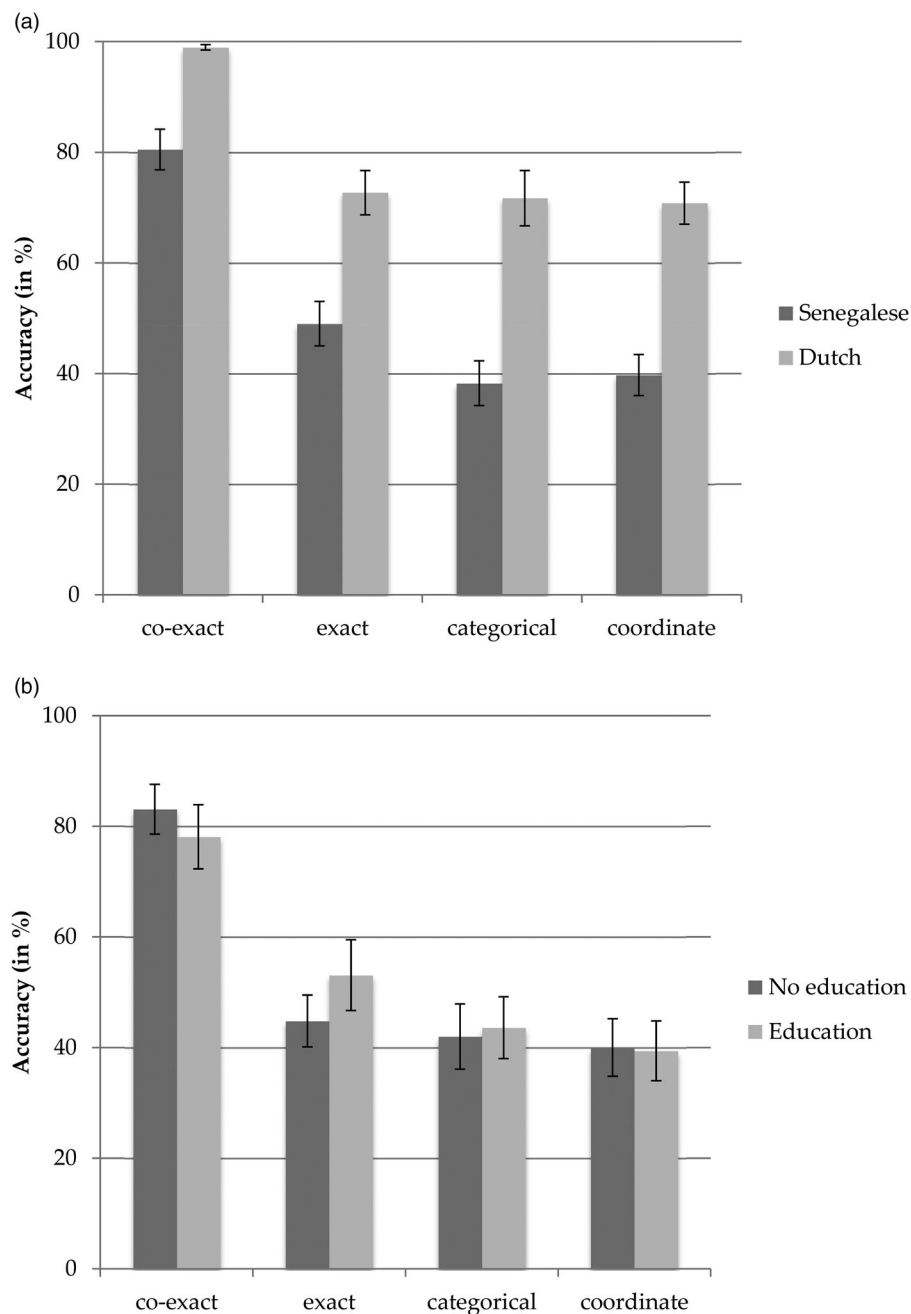
In Figure 2(a,b), the mean accuracy for all groups is provided for each condition.

### 3.1. Culture

First, we examined the effect of culture by comparing the Dutch and Senegalese participants, Figure 2(a). A repeated-measures GLM with condition as within subject factor and group as between subject factor showed a significant main effect of condition,  $F(3, 77) = 38.65$ ,  $p < .001$ , partial  $\eta^2 = .329$ , and a significant main effect of group,  $F(1, 79) = 57.43$ ,  $p < .001$ , partial  $\eta^2 = .421$ . Performance on the co-exact condition was significantly more accurate, compared to all three other conditions. No further differences were found. Overall accuracy in Senegalese participants was lower, compared to Dutch participants ( $p < .001$  in all cases).

For both groups, we examined whether mean accuracy deviated significantly from chance level (16.67%) and from perfect performance (100%). In all cases, performance was above chance level ( $p < .001$  in all cases) and below perfect performance in all but one instance ( $p < .05$ ). For Dutch participants, co-exact performance differed from perfect performance only at trend level,  $t(24) = 1.81$ ,  $p = .083$ .

As the differences between the two groups were clearly significant, we also looked into the difficulty profile of the two groups. For each trial, the mean performance for each group was calculated. Next, the performance of the Senegalese and Dutch participants were correlated,  $r^2 = 68.5\%$ ,  $p < .001$ . This



**Figure 2.** (a) The mean accuracy for the Senegalese and Dutch participants, for each of the four main conditions. Error bars represent standard error of the mean (SEM). (b) The mean accuracy for the Senegalese participants with and without education, for each of the four main conditions. Error bars represent SEM.

indicated that there was a very similar profile of difficulty for these two groups.

### 3.2. Education

The participants with and without education in the Senegalese sample were compared, in Figure 2(b). A repeated-measures GLM with condition as within subject factor and education level as between

subject factor showed a significant main effect of condition,  $F(3, 47) = 31.77$ ,  $p < .001$ , partial  $\eta^2 = .393$ . Performance on the co-exact condition was more accurate than performance on the other three conditions ( $p < .001$  in all cases). There was no significant main effect of group,  $F(1, 49) = 0.057$ ,  $p = .812$ , partial  $\eta^2 = .001$ . Again, for both Senegalese groups, mean accuracy was compared to chance level (16.67%) and perfect performance



(100%). In all cases, performance was well above chance level ( $p < .01$  in all cases). Performances on all conditions were significantly different from perfect performance ( $p < .05$ ).

There was considerable variation in years of education in the Senegalese subgroup with education. Although basic mathematical training occurs very early on in educational programs, it could be that this variation has substantial effect on geometrical skills. Therefore, a correlation was calculated for the Senegalese sample to identify potential significant correlations between number of years of education and the conditions. None of these correlations were significant (all  $p$ 's  $> .10$ ).

As some of the Senegalese participants indicated to be bi- or trilingual, this was also taken into account. Bilingualism (including trilingualism) was added as a between participant factor and did not interact significantly with condition,  $F(3, 47) = 0.13$ ,  $p = .923$ , partial  $\eta^2 = .003$ , or show a significant main effect,  $F(1, 49) = 0.33$ ,  $p = .571$ , partial  $\eta^2 = .007$ .

Again, the difficulty profile was examined by correlating performance between the two groups. This revealed a high level of correlation,  $r^2 = 89.2\%$ ,  $p < .001$ .

As the education level between the Dutch and Senegalese groups of participants is clearly different, one further analysis was performed regarding education level. The same GLM as described under "3.1 Culture" section was performed, but now with education level as a between subjects variable. This analysis provides an indication of the extent of the effect of education level on performance of the two groups. The results of this analysis show that education level has no significant effect on performance,  $F(1, 78) = 0.07$ ,  $p = .788$ , partial  $\eta^2 = .001$ , or in interaction with condition,  $F(3, 76) = 1.13$ ,  $p = .332$ , partial  $\eta^2 = .014$ .

### 3.3. Categorical and coordinate conditions

The categorical and coordinate conditions made use of four different commonly used stimulus layouts. The reference cross was either large or small and the cross was either positioned vertically or

diagonally. This allowed for a follow-up analysis on these stimulus features. As every participant performed each of these trials once, a binomial logistic regression analysis was selected. The regression analysis was performed to ascertain the effects of nationality (Dutch and Senegalese), condition (categorical and coordinate), and layout (dot bar, large cross, small cross, and diagonal cross) on the likelihood that participants give a correct answer. The logistic regression model was statistically significant,  $\chi^2(3) = 66.77$ ,  $p < .0001$ . The model explained 13.1% (Nagelkerke  $R^2$ ) of the variance in accuracy and correctly classified 64.8% of cases. Senegalese participants were 3.85 times less likely to give a correct answer than Dutch participants. Condition and layout did not significantly add to the model. Means and standard deviations are provided in Table 2.

### 3.4. Comparison to the Mundurucu

As the five trials in the exact condition were taken directly from the stimulus set Dehaene et al. (2006) used, we could directly compare the performance on each of these trials. In Table 3, the mean accuracy is given for all three samples. It should be noted here that this comparison is based on a limited number of trials, but it is nonetheless informative to allow a general comparison between three different populations. As the Dehaene sample consisted of indigene Amazonian participants, the comparison to the Senegalese participants is most meaningful. Performance of the American participants in the Dehaene study was not reported for single stimuli. It shows large similarity for equidistance, increasing distance, and middle of segment. However, Senegalese participants showed clearly lower accuracy for the distance and fixed proportion trials. Moreover, the Dutch participants also showed much lower accuracy for the fixed proportion trial. A closer look at the data indicates that the Dutch and Senegalese participants preferred the third option in the fixed proportion trial, in which the line was the shortest (63% and 51%, respectively).

**Table 2.** Means for both groups on each of the categorical and coordinate trials.

Group	Categorical condition				Coordinate condition			
	Dot bar	Large cross	Small cross	Diagonal cross	Dot bar	Large cross	Small cross	Diagonal cross
Senegalese	45.1 (50.3)	33.3 (47.6)	27.5 (45.1)	47.1 (50.4)	27.5 (45.1)	54.9 (50.3)	62.8 (48.8)	13.7 (34.8)
Dutch	80.0 (40.7)	43.3 (50.4)	90.0 (30.5)	73.3 (45.0)	83.3 (37.9)	66.7 (47.5)	93.3 (25.3)	40.0 (45.0)

Note: Standard deviation in parentheses.

**Table 3.** Comparison of performance on distance trials with Dehaene's dataset (2006).

Triallabel	Dehaene sample	Dutch sample	Senegalese sample
Distance	93%	100%	69%
Equidistance	55%	90%	65%
Increasing distance	45%	80%	49%
Middle of segment	36%	63%	25%
Fixed proportion	86%	30%	37%

Note: Percentages reflect the proportion of participants selecting the stimulus that differs from the other five, based on the respective geometric properties.

### 3.5. Observations

With both groups, participants were observed while performing the task. All of them were serious and concentrated while performing the task, as observed by the experimenter and interpreters. The interpreters made sure all participants felt at ease when doing the experiment. It was also clear that they were unfamiliar with the images used and the type of decisions they had to make. On the occasions where participants picked a seemingly random answer, most participants reported specific spatial rules to solve the task when asked about their strategies; however those rules appeared to differ between Senegalese and Dutch participants. The Senegalese participants would for instance focus on the image that was the largest or the smallest out of the six, or they would report to choose based on overall shape characteristics, such as the "most round" image. It should be noted that these observations were not obtained in a structured manner and serve as anecdotal evidence.

Furthermore, the interpreters were asked about spatial aspects of living in Senegal. The use of rulers or maps is highly uncommon. Spatial descriptions are usually given in very elaborate verbal narratives. Despite the elaborate descriptions, spatial details are commonly lacking; especially coordinate, or metric information is rarely used. An anecdotal example is how directions to a specific person are given:

walk on this road until you reach the end of the village, watch out for the tall tree and make a turn in this (speaker points) direction when you reach the tree, you should find the person you look for in that area.

## 4. Discussion

Spatial relations are central to geometrical thought and are at the heart of the appeal to intuition in the diagrammatic reasoning of Euclid's *Elements*.

Spatial relations in Euclidean diagrams can be classified into two categories—exact and co-exact—and only co-exact relations between geometric objects from Euclidean diagrams are used in the deductive reasoning of the *Elements* (Manders, 2008). With a sample of Senegalese and Dutch participants, we assessed the universality of co-exact and exact spatial relations, as well as the universality of the processing of categorical and coordinate spatial relations. We investigated the specific effects of culture and education to further characterise these processes of spatial cognition, and we directly compared the Senegalese and Mundurucu performances on the exact condition.

First of all, both Senegalese and Dutch participants perform well above chance level in each condition, which supports the hypothesis that all the geometric intuitions tested here are universal. Moreover, our analyses show a substantial difference between the two nationalities. The Dutch participants outperformed the Senegalese participants in all four conditions. Yet, the difficulty profile of both groups was very similar, indicating that the geometric intuitions tested here are highly similar in all participants, across the different conditions. These results provide empirical evidence that intuitions of spatial relations prominent in Euclid's theory of geometry are universal. Moreover, accuracy of responses with the co-exact condition for both groups was much higher than for the other conditions. This seems to connect directly to Manders' (2008) explanation for why Euclid restricts himself to only infer co-exact spatial relations from diagrams: judgements of co-exact spatial relations in Euclidean diagrams are far more reliable than exact ones. Our study provides the first empirical evidence for this central claim of Manders' analysis.

We were able to assess effects of education within the Senegalese sample, as participants with and without education were included. There was no difference in performance between these two groups for any of the conditions. Although this suggests education level does not affect performance, an alternative explanation for this could be that the education level of the educated Senegalese group is too modest to render significant the difference between the educated and non-educated Senegalese groups. Education level within the educated group showed large variety, ranging from a few years of elementary school to university level education. On the other hand, though the average level of education among the educated Senegalese



participants is lower than that among the Dutch participants, every member of the latter group received basic mathematical training directly relevant to the task. This supports the alternative explanation that the education level does not affect performance. To explore this matter further, the relation between years of education and performance at a continuous scale was also assessed. There was no such relation. Moreover, when adding education level as an additional factor to the analysis of nationality, it did not affect the outcome. This suggests that the large differences between the Senegalese and Dutch participants are likely not fully explained by education. More general cultural differences in how people use spatial information in their daily activities appear to be a more likely candidate to explain these differences, as observed during interviews. It should be noted, however, that we did not explicitly match the different education groups with regard to socio-economic status or type of employment and that the Dutch participants on average received more years of education than the education Senegalese participants, due to different national education laws. Therefore, the findings concerning education should be interpreted with some caution. One other critical point is that our approach does not control for potential difference in overall innate spatial ability. However, the lack of difference between the groups of Senegalese participants with and without education provides an indication that their general cognitive abilities are likely to be comparable (see e.g. Giofré et al., 2014). A limitation of the current study is that we do not have objective measures of general intelligence available for our participants.

In previous studies (e.g. Van der Ham et al., 2012), it has been reported that stimulus layout can affect categorical and coordinate decisions in different ways. A closer examination of the different layouts used to test categorical and coordinate relations showed that condition or specific features of the stimulus layouts did not affect performance of participants.

The current study is the first to allow a comparison between, on the one hand, exact and co-exact processing and, on the other hand, categorical and coordinate processing. Despite general differences in performance between the Senegalese and Dutch groups, the difficulty profiles are very similar and suggest a similar approach to the stimuli. In this study, co-exact processing appeared easier than categorical processing. However, this could

be due to the amount of visual information required to solve both tasks; in order to determine quadrant membership, more information needs to be processed than to determine whether something is “on” or “off” a line.

Finally, trials from the original Dehaene stimulus set formed the exact condition, and moreover allowed for a comparison between their participants and ours. The Mundurucu data should mainly be compared to the Senegalese participants. The main difference between these two groups is that the language of the Senegalese participants does include spatial prepositions. Geometric training and use of spatial assistive devices is comparable between both groups. The comparison shows that for three stimuli performance was highly comparable, but the Mundurucu participants were somewhat better in the distance stimulus and clearly better in the fixed proportion stimulus. Strikingly, also the Dutch participants were much worse in the fixed proportion stimulus. Both the Dutch and Senegalese participants paid more attention to the absolute size, instead of the proportions.

Observations made during testing indicate that the Dutch participants may have a stronger tendency to look at details, whereas the Senegalese participants anecdotally reported to perceive the stimuli more globally. They took into account overall size or general shape of the objects more often. This more global tendency is in line with the way spatial descriptions are given in their culture, directions are usually very general and environment oriented, whereas the Dutch commonly provide more specific details.

## 5. Conclusion

In conclusion, our findings suggest that intuitions of the spatial relations tested here, prominent in Euclid’s theory of geometry, could be universal. Moreover, high performances in the co-exact condition suggest an initial cognitive explanation for the reliability of judgements involving co-exact spatial relations advocated by Manders (2008) to explain why only co-exact relations are inferred from diagrams in Euclid’s deductive reasoning. However, culture greatly affects performance in two respects: Dutch performances are proportionally higher than Senegalese in the different conditions tested and culture has an effect on the way different stimulus layouts affect the processing of categorical and coordinate spatial relations. Furthermore, education

level does not appear to affect performance, in line with universality.

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## Disclosure statement

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