Numerical modelling and dynamic analysis of rocking rigid blocks

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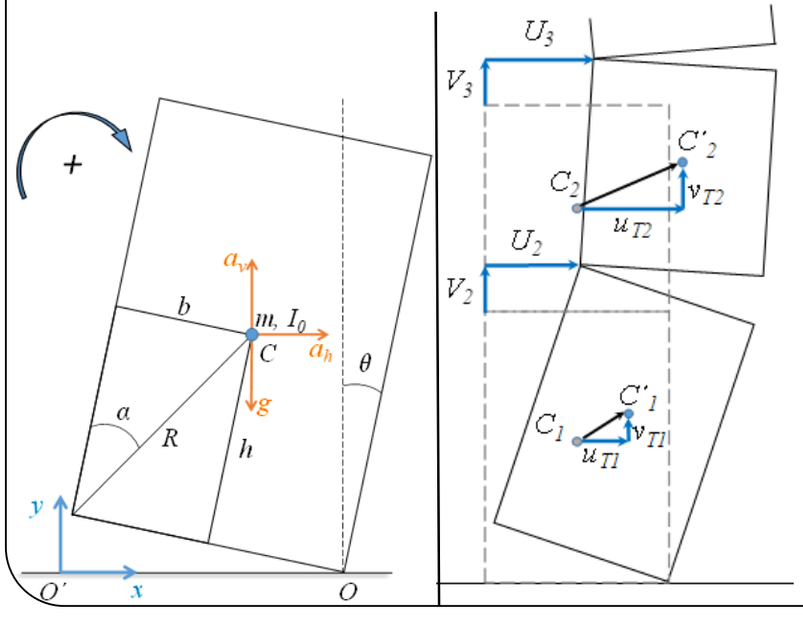
Introduction

The rocking block problem: Rotation of one or more rigid bodies on top of each other. Earthquake response of columns and modular structures.

Aims of the project:

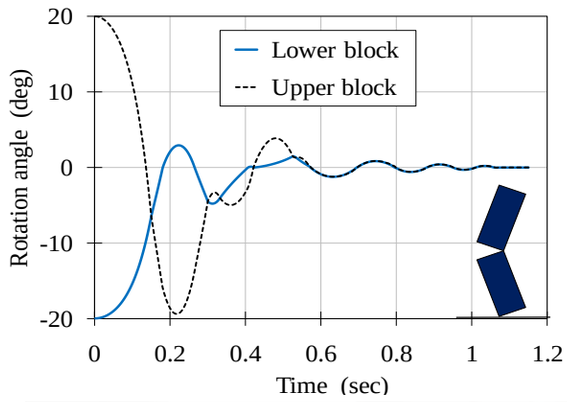
- Derive a compact mathematical model describing the equations of motion of one & two rocking blocks.
- Develop a computer software for numerical simulation of free or forced rocking of a block system.
- Set up a test platform in the lab and measure corresponding free and forced motion of rocking blocks.

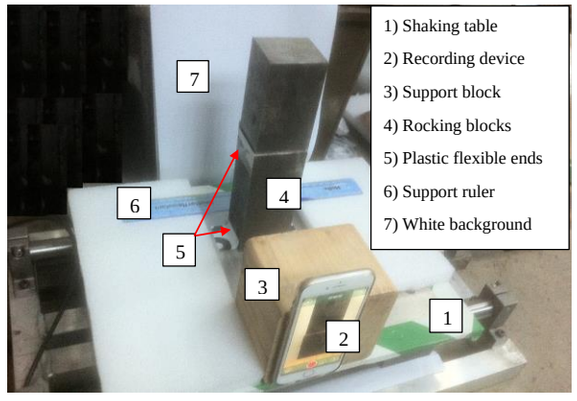


$$\ddot{\theta}_1 = \begin{bmatrix} -m_1\left(u_h\frac{du_{T1}}{d\theta_1} + (g+a_h)\frac{dv_{T1}}{d\theta_1}\right) - m_2\left(u_h\frac{du_{T2}}{d\theta_1} + (g+a_h)\frac{dv_{T2}}{d_1}\right) \\ -2m_2R_2h_1\left(\dot{\theta}_2^2\cdot\sin(\gamma)\cdot C_{Z2} + \ddot{\theta}_2\cos(\gamma)\right) \end{bmatrix} \cdot \frac{1}{I_{O1}+m_2.c}$$

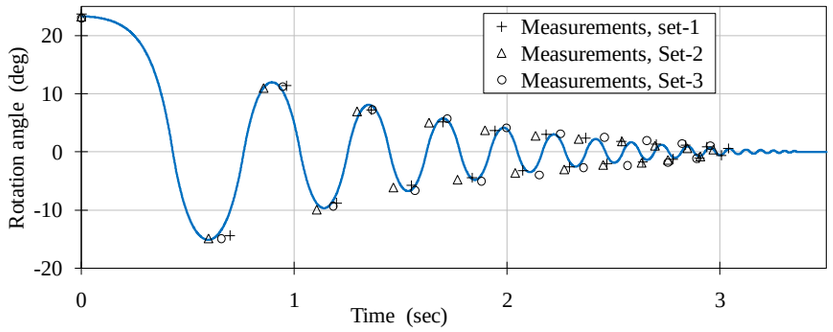
$$\ddot{\theta}_2 = \begin{bmatrix} -m_1\left(u_h\frac{du_{T1}}{d\theta_2} + (g+a_h)\frac{dv_{T1}}{d\theta_2}\right) - m_2\left(u_h\frac{du_{T2}}{d\theta_2} + (g+a_h)\frac{dv_{T2}}{d\theta_2}\right) \\ -2m_2R_2h_1\left(\ddot{\theta}_1\cdot\cos(\gamma) - \dot{\theta}_1^2\cdot\sin(\gamma)\cdot C_{Z2}\right) \end{bmatrix} \cdot \frac{1}{I_{O2}}$$

Numerical integration of the Euler-Lagrange dynamic equation, and block impacts modelling using conservation of angular momentum.

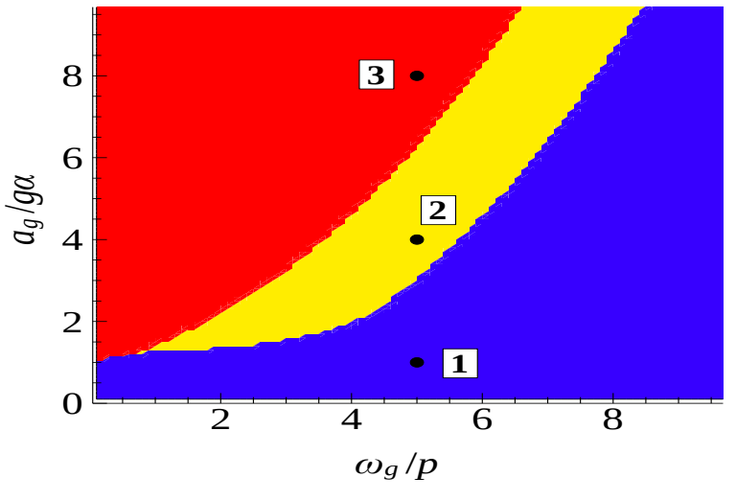




Recording of free and forced rocking motion using an iPhone 5S. Sinusoidal earthquake input using an academic shaking table.



Successful self-validation of code and high numerical results accuracy. Good repeatability of measurements. Satisfactory agreement.



Model application: Overturning map of a block subjected to single pulse excitation.

Conclusions

Reliable numerical modelling of the rocking block problem is feasible, but requires experimental data to make the analytic model results applicable for real materials and actual block geometry.