Assignment 4.8

Problem Statement 1:

In one state, 52% of the voters are Republicans, and 48% are Democrats. In a second
state, 47% of the voters are Republicans, and 53% are Democrats. Suppose a simple
random sample of 100 voters are surveyed from each state.
What is the probability that the survey will show a greater percentage of Republican
voters in the second state than in the first state?

Solution

Step 1. Make sure the sample size is big enough to model differences with a normal population.
Because n1 P1 = 100 * 0.52 = 52,

n1(1 - P1) = 100 * 0.48 = 48,

n2 P2 = 100 * 0.47 = 47,

and n2(1 - P2) = 100 * 0.53 = 53

are each greater than 10, the sample size is large enough.

Step 2. Find the mean of the difference in sample proportions:

E(p1 - p2) = P1 - P2 = 0.52 - 0.47 = 0.05.

Step 3. Find the standard deviation of the difference.

σd = sqrt{ [P1(1 - P1) / n1] + [P2(1 - P2) / n2] }

σd = sqrt{ [(0.52)(0.48) / 100] + [(0.47)(0.53) / 100] }

σd = sqrt (0.002496 + 0.002491) = sqrt(0.004987) = 0.0706

Step 4. Find the probability.

This problem requires us to find the probability that p1 is less than p2. This is equivalent to finding the probability that p1 - p2 is less than zero. To find this probability, we need to transform the random variable (p1 - p2) into a z-score. That transformation appears below.

z p1 - p2 = (x - μ p1 - p2) / σd = = (0 - 0.05)/0.0706 = -0.7082

Using Stat Trek's Normal Distribution Calculator, we find that the probability of a z-score being -0.7082 or less is 0.24.

Therefore, the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state is 0.24.

Answer is 0.24