

Assignment 5.1

Problem Statement 1:

Is gender independent of education level? A random sample of 395 people were surveyed and each person was asked to report the highest education level they obtained. The data that resulted from the survey is summarized in the following table:

	High School	Bachelors	Masters	Ph.d.	Total
Female	60	54	46	41	201
Male	40	44	53	57	194
Total	100	98	99	98	395

Question: Are gender and education level dependent at 5% level of significance? In other words, given the data collected above, is there a relationship between the gender of an individual and the level of education that they have obtained?

Solution

Step 1: Define the Null and Alternate Hypotheses.

Null Hypothesis H_0 : Gender and Education level are independent

Alternate Hypothesis H_1 : Gender and Education level are dependent.

Step 2: State the significance level (α).

Significance Level : 5% (0.05).

Step 3: Calculate degree of freedom.

$$df = (\text{rows} - 1) * (\text{columns} - 1)$$

$$= (2-1)*(4-1) = 3$$

Step 4: State decision rule.

For $\alpha = 0.05$ and $df = 3$, refer the chi-square table.

The critical value of χ^2 with 3 degree of freedom is

If χ^2 is greater than 7.81473, reject H_0

Step 5: Calculate test statistics

$$\chi^2 = \sum (O - E)^2 / E$$

the table of expected counts:

	High School	Bachelors	Masters	Ph.d.	Total
Female	50.886	49.868	50.377	49.868	201
Male	49.114	48.132	48.623	48.132	194
Total	100	98	99	98	395

$$\begin{aligned} \text{So, } \chi^2 &= (60 - 50.886)^2 / 50.886 + (54 - 49.868)^2 / 49.868 + (46 - 50.377)^2 / 50.377 \\ &\quad + (41 - 49.868)^2 / 49.868 + (40 - 49.114)^2 / 49.114 + (44 - 48.132)^2 / 48.132 \\ &\quad + (53 - 48.623)^2 / 48.623 + (57 - 48.132)^2 / 48.132 \\ &= 8.006 \end{aligned}$$

Step 5: State result

$$\chi^2 = 8.006$$

Result: Reject H_0

Step 6: State conclusion

As $8.006 > 7.81473$ It means, There is some relationship between gender and level of education.

We conclude that Gender and Education level are dependent, $\chi^2 = 8.006$, $p < 0.05$