3. MODEL CONSTRUCTION

In this chapter, the model of probabilistic traveling salesman problem with time window (PTSPTW) will be constructed and the double-time horizon criterion will be incorporated into. First the problem statement will be presented in Section 3.1. Based on the problem statement, the PTSPTW model is then formulated in Section 3.2.

3.1 Problem Statement

We consider an overnight courier mail or packages system as shown in Fig. 3.1. It can be divided into "local area" and "wide area" components [13]. The Wide Area Network (WAN) transports shipments from metropolitan regions to others, and the Local Area Network (LAN) performs pickups and deliveries within metropolitan regions. The carrier collects the mails and packages which need to be delivered and sends them to the regional or national distribution center. After the operations of sorting and classifying, the items will be transported to other metropolitan regions via air or trains. And then the contrary process is taken for deliveries. The LAN part is what we focus on. In the overnight courier mail problem, the goods are either to be picked up at a customer and then transported back to the depot for further processing or to be transported from the depot to the delivery location. In other words, a customer in the courier mail problem could either be a delivery customer or a pickup customer. The vehicle will service both kinds of customers when it is *en* route and is not permitted to return to the depot (regional distribution center) before all customers be serviced. Because the on-time delivery on next day is the common goal that all overnight carriers pursue, the vehicle must go back to the depot before a deadline for follow-up operations.

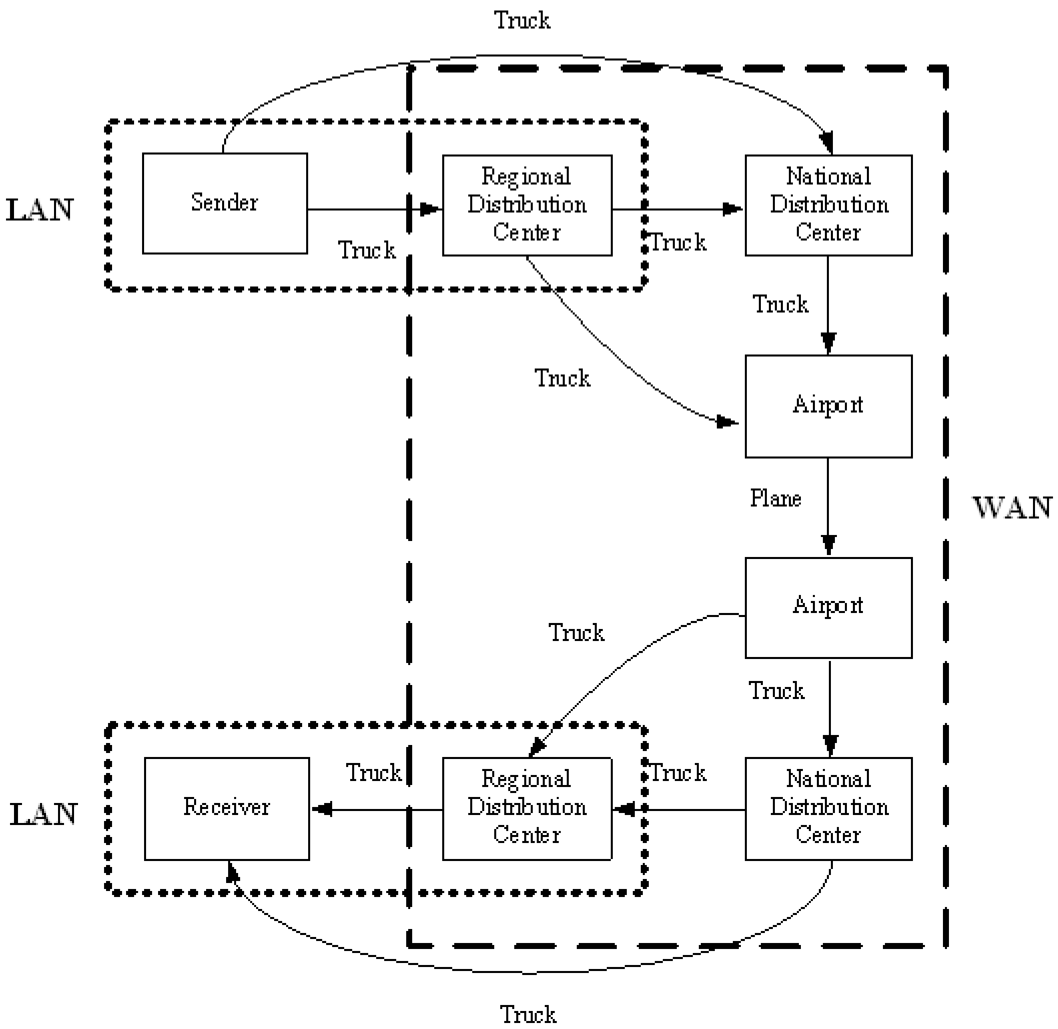

Fig. 3.1 Simplified framework of common carrier network [18]

Each customer may assign a time interval in which it is good for the pickup or delivery service. It is not permitted to begin service before the early time window of the customer because the mails or packages may not be available. Beginning service after the late time window is allowed, however some additional cost will occur due to dissatisfaction of the customer.

Pickup and delivery customers have some difference in information of customers. All information of delivery customers, including their locations and time windows is known before the route start. But information of some pickup customers cannot be achieved in advance because most overnight carriers provide the real-time call-ins service. To deal with the pickup customers easily, we divide them into three categories:

(1) Daily pickup customers: Some customers scheduled a pickup service every day. The information of these customers, both locations and time windows are available before the route begins.

(2) Regular pickup customers: Some customers request for pickup service very often but not everyday. The carrier has the information about the locations of customers in his database, but he dose not assure that whether a specific customer requests for pickup in certain day.

(3) Immediate customers: These customers demand for pickups when the vehicle is *en route*. The carrier does not have any information about them.

In practice of courier service, the area covered by a regional distribution center is partitioned into several subregions and each subregion is served by a fixed vehicle with a fixed driver. Therefore, we only have to consider the activities in one of these subregions. In addition, because of small volume of a single item, the capacity constraints are not considered in this problem. As mentioned above, this problem can be viewed as a probabilistic and dynamic traveling salesman problem with time window constraints.

The objective of the carrier is to collect all mail and packages safely within the given time restrictions while keeping the operating costs, including traveling cost and dissatisfaction of customers, as low as possible.

3.2 Model Framework

To deal with the uncertain requests from regular pickup and immediate customers, we must construct a stochastic and dynamic model. In chapter 2, we have reviewed some literatures about PTSP and DVRP. However, we have also mentioned that the problem has characteristics of both PTSP and DVRP were rarely studied. We will propose an *a priori* model to cope with the regular pickup customers firstly, and insert immediate requests into the route dynamically when they occur. The restriction of vehicle capacity is not considered in this problem. If the termination time of one route does not exceed the collecting deadline of distribution center, T, and the service of each customer does not begin before his early time window, e_i, this route is a feasible one.

3.2.1 A multi-objective *a priori* PTSP model

For routing problems, the most important objective is to reduce every kind of cost due to customer demands. Both traveling time and the cost of customer dissatisfaction due to delayed pickups are addressed in this study. However, the immediate requests arising in the future make this problem more complicated. According to the phenomenon proposed by Mitrović-Minić *et al.* [21], accumulating more slack time in the distant future is helpful to satisfy immediate requests more efficiently in *a priori* routing. Hence, we divide the total routing time-horizon into two: short-term and long-term and set different goals in these two horizons. The short-term goal is to reduce traveling time and customer dissatisfaction, while the long-term goal is to maintain the route in a state that will enable them to easily respond to future requests. We set two objectives in the short-term horizon: to reduce lateness and traveling time; and set three long-term objectives: to reduce lateness and traveling time, and accumulate slack time, as shown in Fig. 3.2.

Because it is a probabilistic problem, we should measure the performance by expected value. Theses five objectives can be expressed as follows:

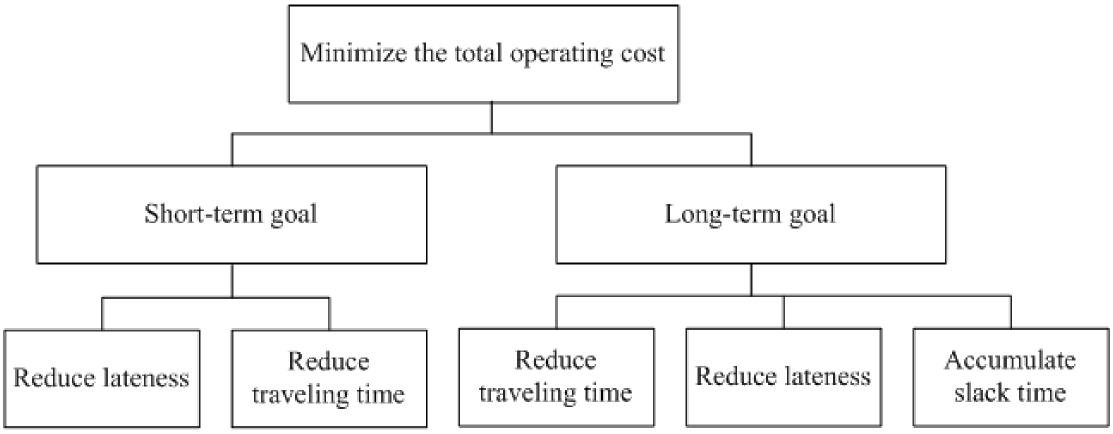

Fig. 3.2 Hierarchical objectives

(1) Minimize the expected traveling time in the short-term horizon

$$\text{Min} \quad Z_1(r_k) = E\left(q_k^S\right) \tag{1}$$

(2) Minimize the expected dissatisfaction cost due to lateness in the short-term horizon

$$\text{Min} \quad Z_2(r_k) = E\left(c_k^S\right) \tag{2}$$

(3) Maximize the expected slack time in the long-term horizon

$$\text{Max} \quad Z_3(r_k) = E\left(h_k^L\right) \tag{3}$$

(4) Minimize the expected traveling time in the long-term horizon

$$\text{Min} \quad Z_4(r_k) = E\left(q_k^L\right) \tag{4}$$

(5) Minimize the expected dissatisfaction cost due to lateness in the long-term horizon

$$\text{Min} \quad Z_5(r_k) = E\left(c_k^L\right) \tag{5}$$

where

r_k : route *k*, which goes through all regular and advance customers

F : the set of feasible routes

q_k^S : traveling time of route *k* in the short-term horizon

q_k^L : traveling time of route *k* in the long-term horizon

h_k^L : average slack time over all locations in the long-term horizon of route *k*

c_k^S : dissatisfaction cost due to lateness in the short-term horizon of route k

c_k^L : dissatisfaction cost due to lateness in the long-term horizon of route k

In practice, the dissatisfaction cost of customer can be viewed as a certain of quality loss and cannot be easily measured in terms of money or time. On the other hand, the traveling time and accumulated slack time can both be computed in time. Therefore, to deal with this multi-objective problem, we transform objective (2) and (5) to constraints by the ε-Constraints method and combine objective (1), (3) and (4) by weighting method. The mathematical model is as follows:

$$Z(r_k) = E\left[\alpha \cdot q_k^S + (1-\alpha)\left((1-\beta)q_k^L - \beta h_k^L\right)\right] \tag{6}$$

$$\min_k \; Z(r_k)$$

$$\begin{aligned} &s.t. \\ &\quad \mathrm{Z}_2(r_k) \le \varepsilon_1 \\ &\quad \mathrm{Z}_5(r_k) \le \varepsilon_2 \\ &\quad r_k \in F \end{aligned}$$

where α and β are weights and in the interval $[0,1]$. By adjusting ε_1 and ε_2, we can find the optimal *a priori* solution. For the convenience of of solving this problem, we can satisfy the lateness constraints by setting wider but hard time windows. Therefore, the problem can be transformed into Probability Traveling Salesman Problem with hard time windows.

3.2.2 Expected values in PTSP

To show the probabilistic context in this model, we use expected value to evaluate the performance on each attribute. Assume a set of known customers, $D = \{d_1, d_2, ..., d_N\}$, where daily and regular pickup customers are included. The request probability of customer i is denoted as p_i. For daily pickup customers, $p_i = 1$, and $p_i < 1$ for regular ones. There are numbers of instances, which represent the true request condition of each day, in a certain route r_k. For each instance l, we have the following function:

$$I_{kl} = \prod_{d_i \in D_P} \prod_{d_j \in D_U} p_i \left(1 - p_j\right) \left(\sum_{D_p} t_{m,m+1} \right) \tag{7}$$

where

D_P : the customer set which request for service in this instance

D_U : the customer set which do not request for service in this instance

$t_{m,m+1}$: the traveling time needed between two consecutive nodes which belongs to D_p

I_{kl} : the total traveling time multiplied by the occurrence probability of instance l of route k

Therefore, the expected total traveling time of route k is expressed as follows:

$$E\left(\text{travel time of route } k\right) = \sum_l I_{kl} \tag{8}$$

For example, consider a customer set with four elements, as shown in Fig. 3.3. Customers 1 and 3 are advance customers, i.e. $p_1 = p_3 = 1$, and Customer 2 and 4 are regular customers. The route $r_1 : 1 \to 2 \to 3 \to 4 \to 1$ has 4 instances:

(1) $1 \to 2 \to 3 \to 4 \to 1$

(2) $1 \to 2 \to 3 \to 1$

(3) $1 \to 3 \to 4 \to 1$

(4) $1 \to 3 \to 1$

For each instance l, I_{1l} can be easily calculated:

$$I_{11} = p_2 p_4 \left(t_{12} + t_{23} + t_{34} + t_{41}\right)$$

$$I_{12} = p_2 \left(1 - p_4\right)\left(t_{12} + t_{23} + t_{31}\right)$$

$$I_{13} = \left(1 - p_2\right) p_4 \left(t_{13} + t_{34} + t_{41}\right)$$

$$I_{14} = (1 - p_2)(1 - p_4)(t_{13} + t_{31})$$

Therefore,

$$E(\text{travel time of } r_1) = I_{11} + I_{12} + I_{13} + I_{14}$$

Except for the traveling time, the expected accumulated slack time can be calculated in the similar procedure.

Note: Customer 2 and 4 are regular customers.

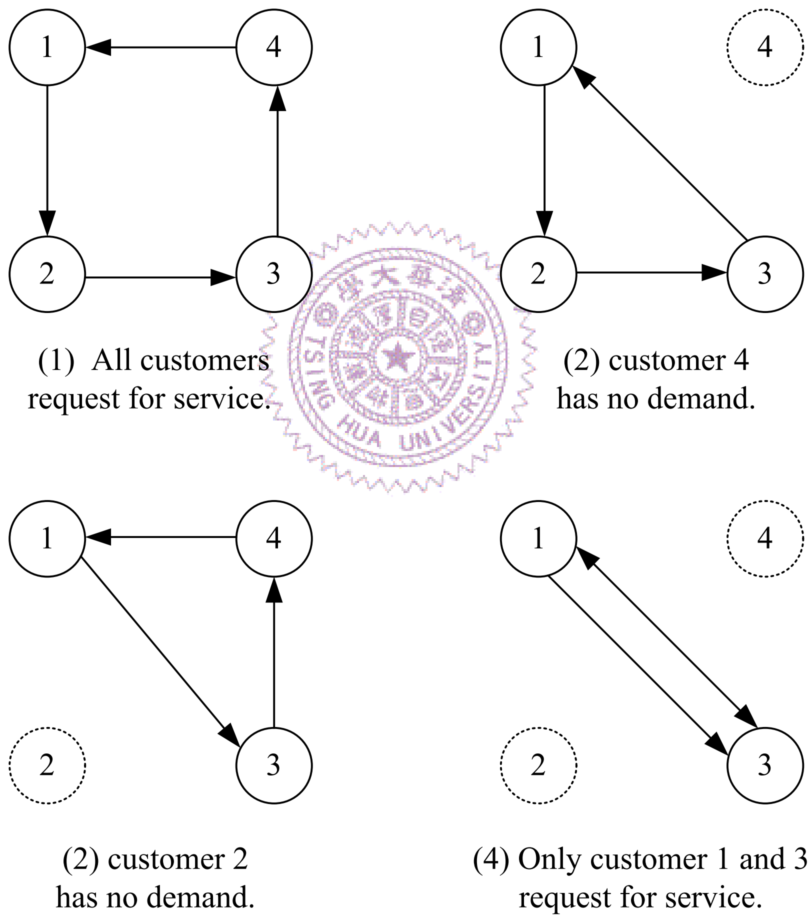

Fig. 3.3 Four instances of a four-nodes example