2. LITERATURE REVIEW

The TSP and VRP have been studied for a long time. In the past 50 years, many variants of these two problems and their solution procedures are also proposed and developed. The stochastic version, for example, the probabilistic TSP (PTSP) and stochastic as well as dynamic VRP (SDVRP) attract a lot of attention in the past twenty years because of their uncertain characteristics. These models are more close to the real world and more practical in some industries in which time is an important factor, e.g., express and emergency service.

Here, we review some past important works about PTSP and DVRP in the rest of this chapter.

2.1 Probabilistic Traveling Salesman Problem (PTSP)

2.1.1 Problem Definition and Related Models

The TSP is perhaps the most extensively investigated one of all combinatorial optimization problems [16]. It is one of the core components of other routing and even scheduling problems so that a lot of efforts concentrate on it. In a deterministic context, all information of customers, including their location or size of demands is known in advance. However, when these problems are specified in a probabilistic context, i.e. some parameters are random, they are more applicable in practice.

The PTSP was first introduced by Jaillet [14] in 1985. Consider the following situation: if a carrier wants to design a tour through a set of customers and desires to minimize only the routing cost, it is then legitimate to solve the corresponding TSP as if all customers must actually be visited every day. However, concerning the regularity, to design a tour for a given prolonged period of time (more than one day) is a more practical way of looking at this problem. For this time horizon, the set of customers to be visited are different every day. Moreover, reoptimization is not desired and permitted because it is too expensive [15]. The vehicle has to only follow the predesigned route everyday and simply skip the customers who do not really request for service, as Fig. 2.1. Jaillet [15] claimed that the optimal TSP tour through all potential points does not guarantee the minimum expected length in such a probabilistic case. Jaillet [16] further suggested the formulation and analysis of the following problem. Consider a routing problem with a set of *n* customers. On any given instance of the problem, only a subsets consisting of *k* customers (who requests for service) must be visited, with the number *k* determined according to a known probability distribution. The goal of the research is to find an *a priori* tour through *n* points. These *k* points will be visited in the same order as the sequence they appear in the *a priori* tour on any given instance of the problem.

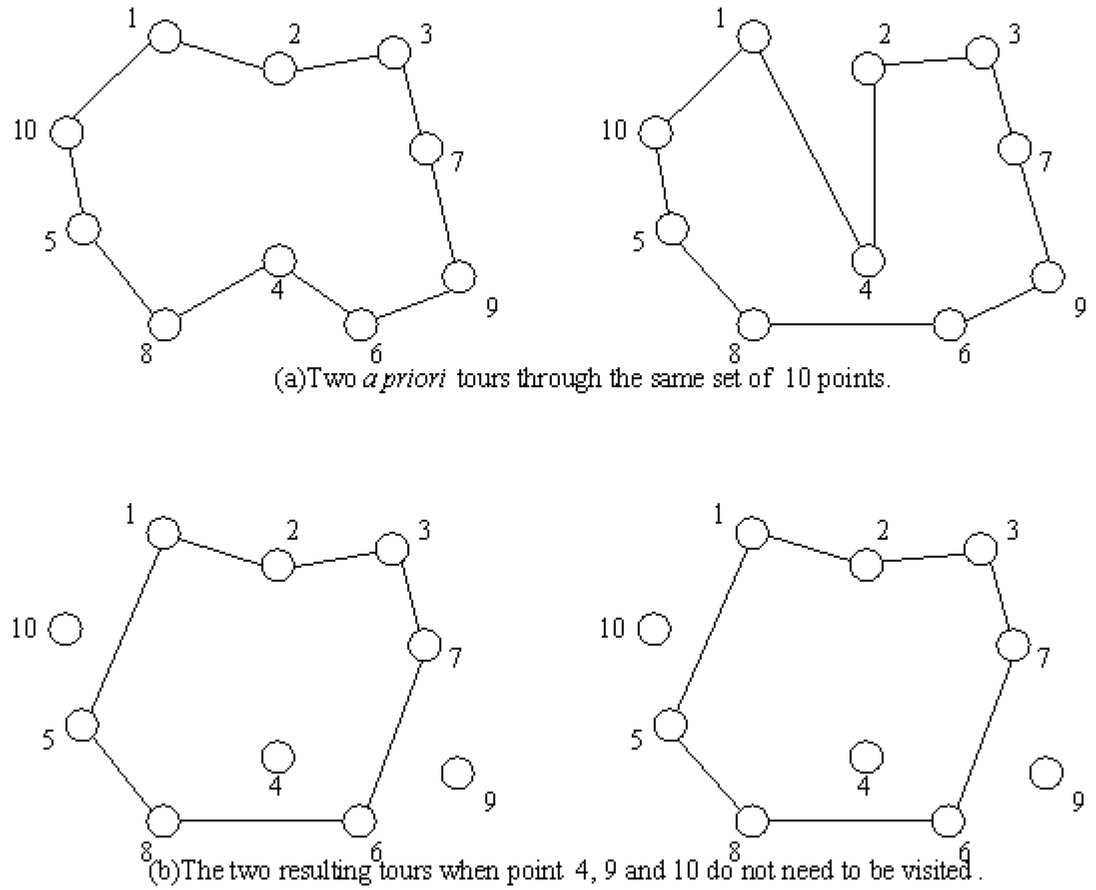


Fig. 2.1 Simple graphical example of a PTSP [3]

In this perspective, the traditional TSP is a special case where the request probabilities of all customers equal to 1. The objective function of PTSP is to minimize the expected length of the *a priori* route. The expectation is computed over all possible instances of the problem, i.e., over all subsets of the vertex set $V = \{1, 2, ..., n\}$. Bertsimas *et al.* [4] proposed the following way of computing expectation. Given an *a priori* tour τ, if the problem instance $S \subseteq V$ will occur with probability $p(S)$ and will require converting a total distance $L_\tau(S)$ to visit the subset S of customers, then the problem instance will receive a weight of $p(S)L_\tau(S)$ in the computation of the expected length. If we denote the length of τ by L_τ (a random variable), the problem is to find an *a priori* tour τ_p through all n potential customers, which minimize the quantity

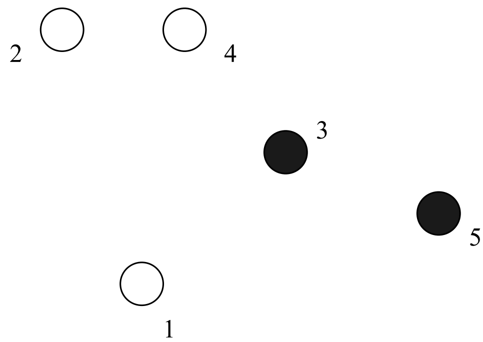
$$E[L_\tau] = \sum_{S \subseteq V} p(S)L(S)$$

with the summation over all subsets of V [4].

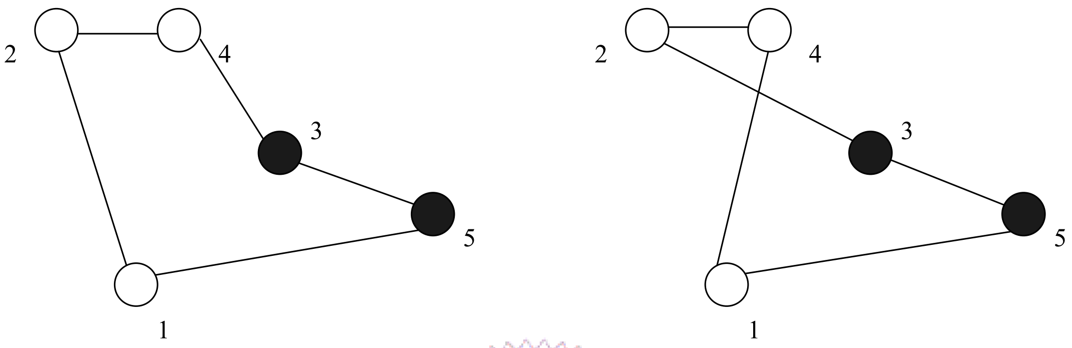
Because of the probabilistic context, the results obtained from PTSP may have radical difference with TSP [4]. In TSP, the selected arcs in the optimal route should not intersect with each other. The order in which the points on convex hull appear in an optimal TSP tour must be the same as the order in which these points appear on the convex hull. But this property is not guaranteed in PTSP as shown in Fig.2.2.

2.1.2 Methodology and Complexities

Bertsimas *et al.* [4] observed that *a priori* and reoptimizing strategies have very close asymptotic performance. The PTSP requires the computation of only one solution and is easily updated, while reoptimazing strategies require the computation of an optimal solution for every problem instance. However, it is not easy to find an



(a) The set of five points



(b) The optimal PTSP tour when $p > 0.75$ (c) The optimal PTSP tour when $p < 0.25$

Note: Black nodes denote the customers whose request probabilities equal to one

Fig. 2.2 Intersection of the optimal PTSP tour

optimal solution of PTSP in the complexity perspective [1]. Although we can efficiently compute the expected length of any given PTSP route, it is still NP-hard to find an optimal *a priori* solution [7]. Jaillet [14] proposed a branch-and-bound scheme and several heuristic methods, and derived an $O(n^3)$ formula for the expected cost of the *a priori* solution in his further study [16]. Laporte *et al.* [17] proposed an exact algorithm for determining an *a priori* tour of least expected cost. Beraldi *et al.* [2] developed an efficient heuristic called Neighborhood Evaluation Procedure which can be carried out in $O(n^3)$ steps in recently works.

2.2 Dynamic Traveling Salesman Problem/ Dynamic Vehicle Routing Problem (DTSP/DVRP)

2.2.1 Introduction

Real-time decision problems in transportation are playing an increasingly important role in the economy due to advances in communication and information technologies that allow real-time information to be quickly obtained and processed [5]. Séguin *et al.* [24] proposed what is likely distinguish most distribution problems today from equivalent problems in the past is that the information that needed to come up with a set of good vehicle routes and schedules is dynamically revealed to the decision maker. Such real-time availability of information was rare or non-existent in the past [23]. In the DTSP or DVRP, a set of vehicles is routed over a particular time-horizon (typically, a day) while new service requests are occurring in real-time. With each new request, the current solution may be reconfigured to better service the new request, as well as those already assigned to a route [9].

Gendreau and Potvin [9] divided the DVRP along the characteristics:

- many-to-many where each request includes both pickup and delivery service versus one-to-many where each request includes either pickup or delivery service.
- capacitated where each vehicle has a fixed capacity versus uncapacitated.

Table 2.1 shows specific application example for each category of dynamic local area vehicle routing, and the category of courier mail delivery is what we concentrate in this thesis. On the side, traditional VRP including time window constraints of customers (VRPTW) is often discussed. In the dynamic context of VRPTW, these time windows are often soft and can be violated to some extent to account for early or late arrivals. In addition, Psarafits [24], Gendreau and Potvin [10] and Ghiani *et al.*

[13] provided literature reviews about the topic of DTSP/DVRP.

Table 2.1 Dynamic local area vehicle routing and dispatching [9]

	many-to-many	one-to-many
capacitated	dial-a-ride	feeder system
uncapacitated	express mail delivery	**courier mail or repair service**

2.2.2 Courier mail services

A courier service refers to the local portion of international express mail service (e.g., Federal Express) [9, 23]. Larsen [18] proposed the following problem. A mini-van goes around a city district collecting parcels and express packages. The deliveries of mail or packages form a static routing problem because all information of the delivery customers are known before the routing starts. However, the pickup to be handled during the deliveries has the effect that the problem become dynamic in the sense that the driver and the decision maker (dispatcher) do not have all information about where and when the immediate requests are going to take place. Requests for service arrive to the courier’s central dispatching office in real time and are automatically relayed either by phone or by other device to the mini-van. The intelligent routing and dispatching device in the dispatching center or installed on the van processes each immediate requests, by appropriately inserting it into the sequence of pickups. Because of small volume of mail and packages, the capacitated restrictions sometimes are ignored [19]. Besides, Gendreau *et al.* [10] exploited the power of new computing technologies, in particular parallel processing. The solution procedure is based on an adaptive memory tabu search heuristic, initially designed for the static version of the problem where all requests are known in advance.

2.2.3. Double-horizon (DH) based heuristic

If the future is divided to the near and the distant ones, events which arise in the distant future are usually more difficult to be predicted and dealt with. Mitrović-Minić *et al.* [21] observed that the unknown requests in the DVRP have the similar characteristic. It is likely that most requests belonging to the time interval in the near future are already known. In contrast, when the time interval is in the distant future, many requests belonging to this portion are unknown in the meantime of decision making. They believe that better managing slack time in the distant future may help reduce routing cost. For example, if we want to minimize the total traveled distance, it may be preferable to accumulate slack time in the distant future rather than concentrating on distance minimization, since larger slack times make future requests insertions easier. Therefore, they proposed a heuristic based on double-horizon idea. The total time horizon is divided into two portions and both of them are set different goals, as shown in Fig. 2.2. The short-term goal is to reduce travel distance, while the long-term goal is to maintain the route in a state that will enable them to easily respond to future requests. The resulting model is a multi-objective optimization problem. They also proposed an objective function as follows:

$$z = \frac{l}{s}\beta q_S + (1-\beta)\big((1-\alpha)q_L + \alpha h_L\big)$$

where q_S is total length of the route portions belonging to the short-term time horizon, q_L is the total length of the remaining parts of the routes, h_L is the average slack time over all locations in the long-term time horizon, l is the length of the long-term time horizon, and s is the length of the short-term time horizon. By setting $0 < \alpha < 1$ and $0 < \beta < 1$, different values of the objective function over two time horizons are obtained. This objective function is used in the tabu search procedure. To well distribute the slack time over the time horizon can be view as a scheduling subproblem in the DVRP. This is an important component in the dynamic routing problems, for instance, Psaraftis [22] and Mitrović-Minić and Laporte [20] did some related studies in this topic.

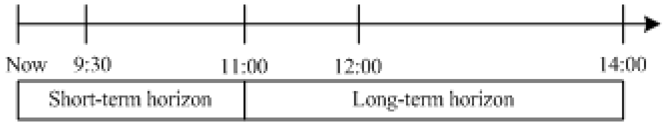


Fig. 2.3 The two time horizons