Minimizing Networked Epidemic Processes Using a Staggered Lockdown Strategy

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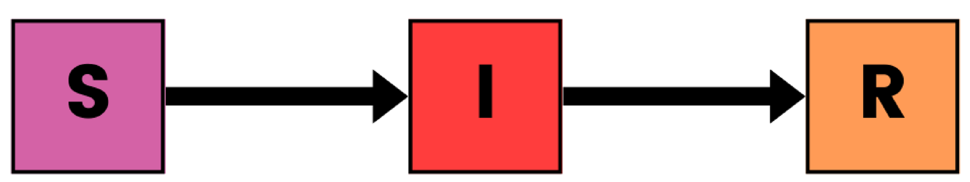
Objective

Our goal is to learn the behavior of the staggered lockdown strategy for an epidemic in a two-population network.

Background

SIR Model for a single population:



Susceptible, Infectious, Recovered/Removed

SIR Dynamics:

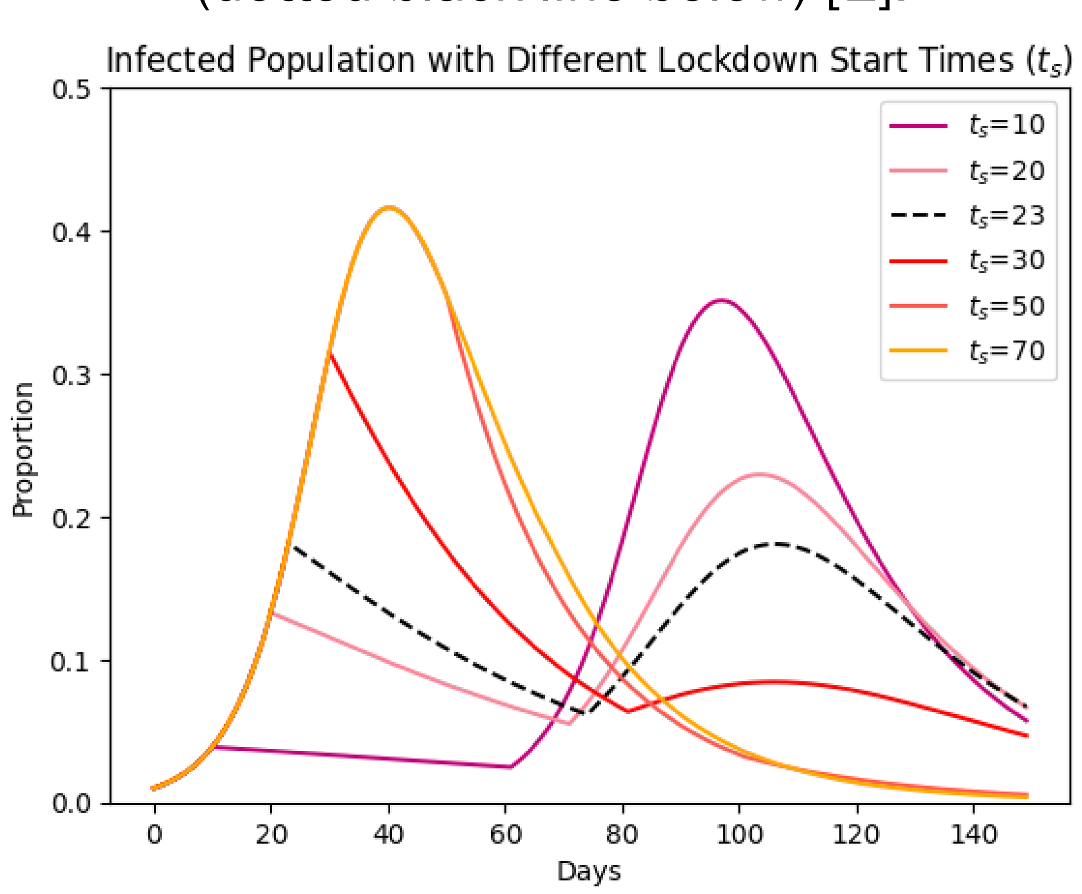
$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

Problem Formulation:

$$\min_{t_s}\{I_{max}(t_s)\}$$

where t_s is the start time of lockdown

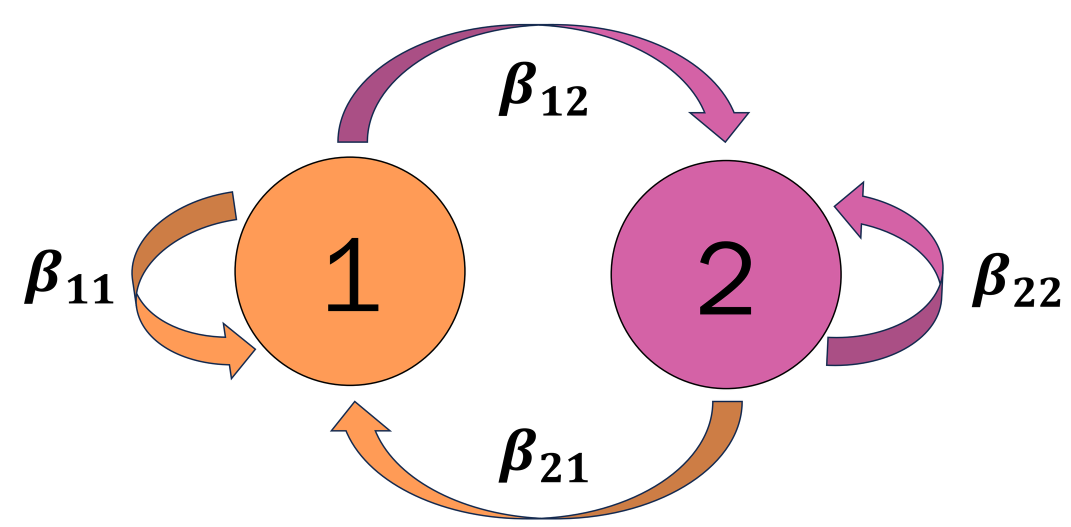
The optimal lockdown flattens the infection curve to have two equal peaks (dotted black line below) [1]:



Methodology

The two-population network case:

- Transmission occurs within and between nodes
- SIR dynamics evolve in both populations



Networked Dynamics:

$$\text{Pop. 1: } \frac{dI_1}{dt} = (1 - I_1)(I_1\beta_{11} + I_2\beta_{12}) - \gamma_1 I_1$$
$$\text{Pop. 2: } \frac{dI_2}{dt} = (1 - I_2)(I_1\beta_{21} + I_2\beta_{22}) - \gamma_2 I_2$$

Networked Problem Formulation:

$$\min_{t_{s_1},t_{s_2}} \{I_{1,max}(t_{s_1}, t_{s_2}), I_{2,max}(t_{s_1}, t_{s_2})\}$$

A staggered lockdown strategy requires:

- A single fixed period of reduced transmission
- Populations start lockdowns at different times

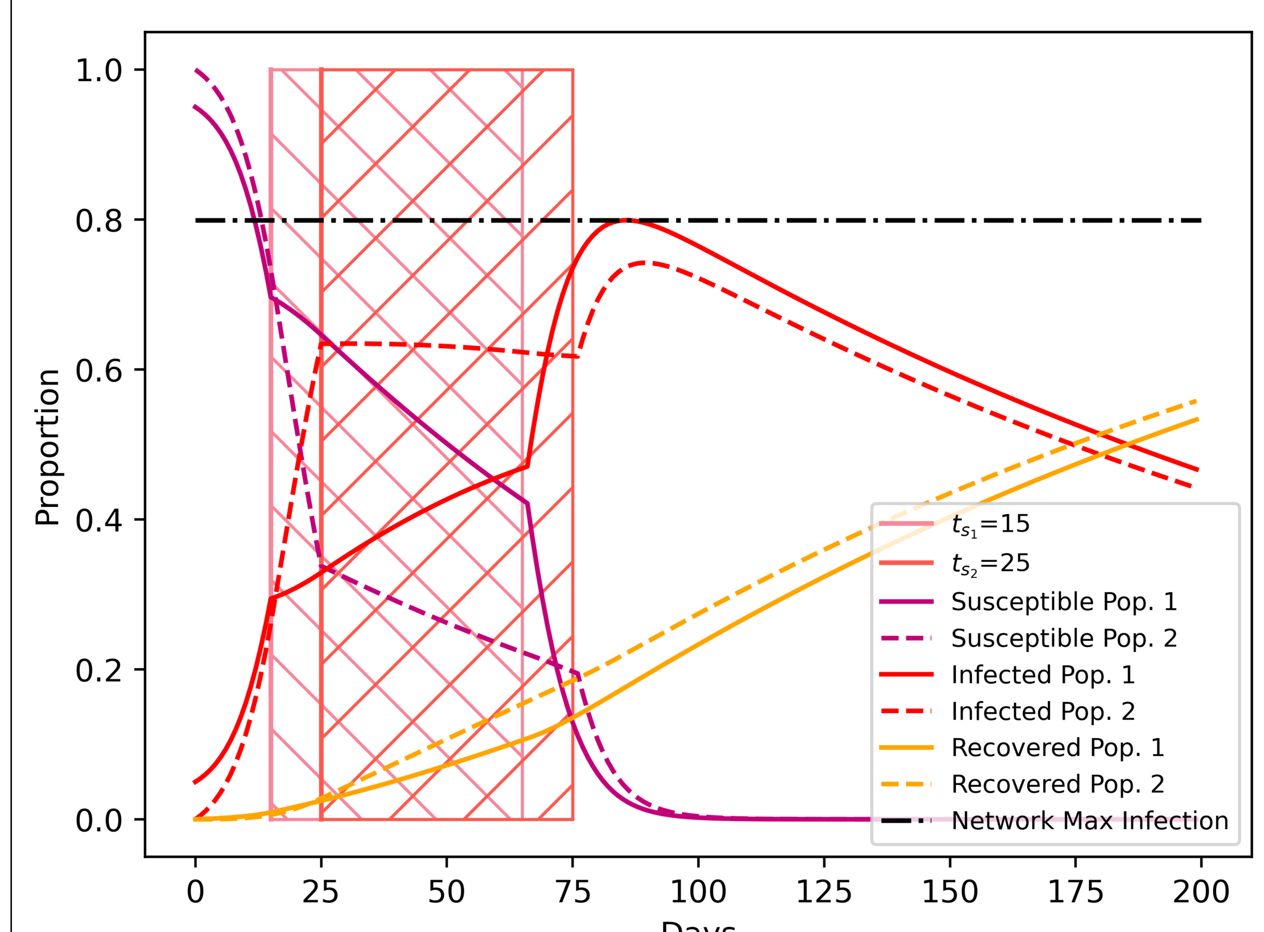


Fig. 1 SIR Model of Two Populations with Staggered Lockdown

Results

The two surfaces in Fig. 2 represent each population's maximum infection peak for that combination of lockdown start times.

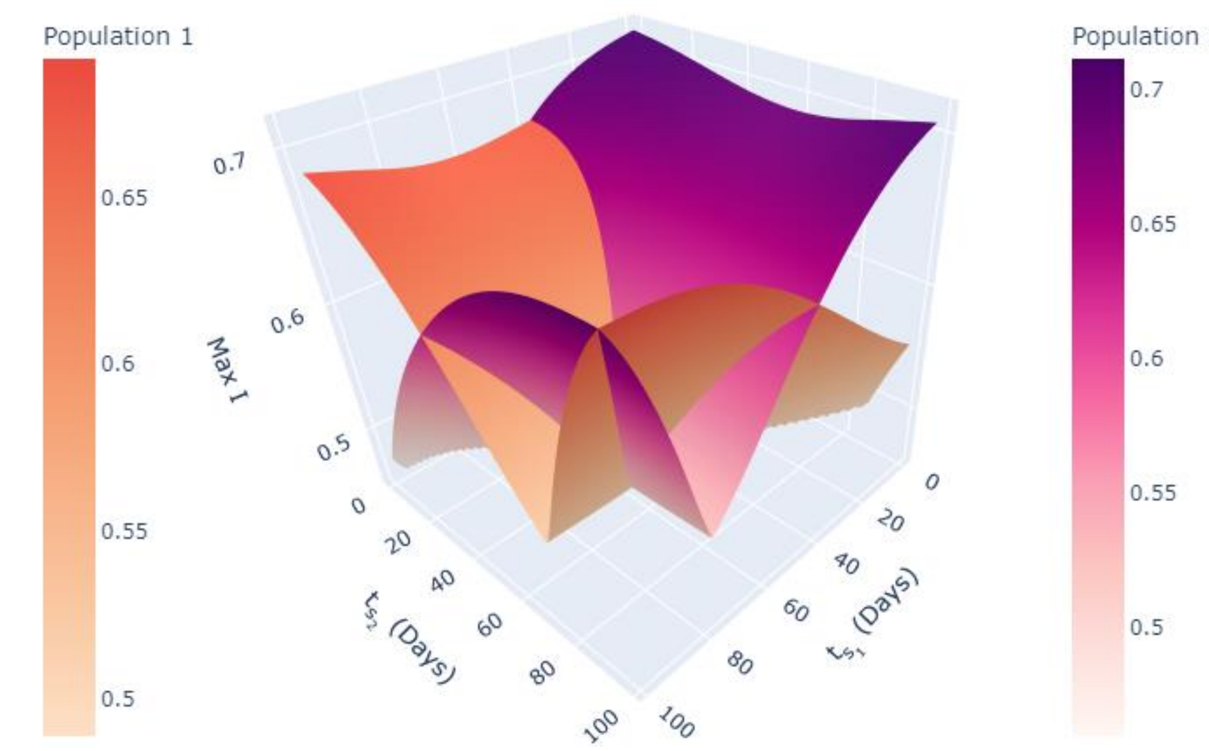


Fig. 2 Maximum Infection Peaks for Two Population with Different Lockdown Start Times (t_s)

The optimal lockdown start times occur at the minimum of the intersections between the two surfaces in Fig 2.

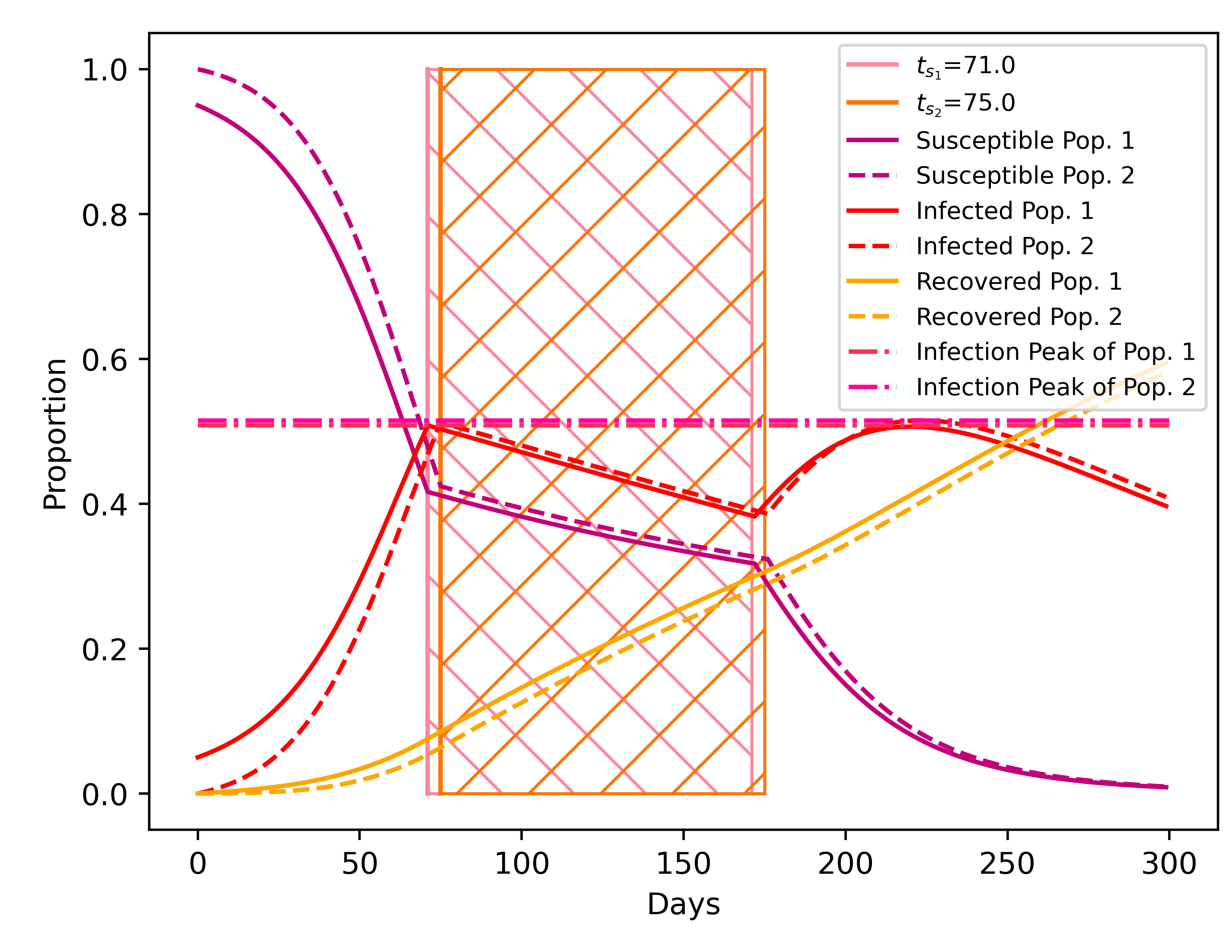


Fig. 3 SIR Model of Two Populations Using Optimal Staggered Lockdown Start Times

The optimal combination leads to the most flattened curves for the entire network.

Results

To see the plot from Fig. 3 more clearly, the axes are zoomed in:

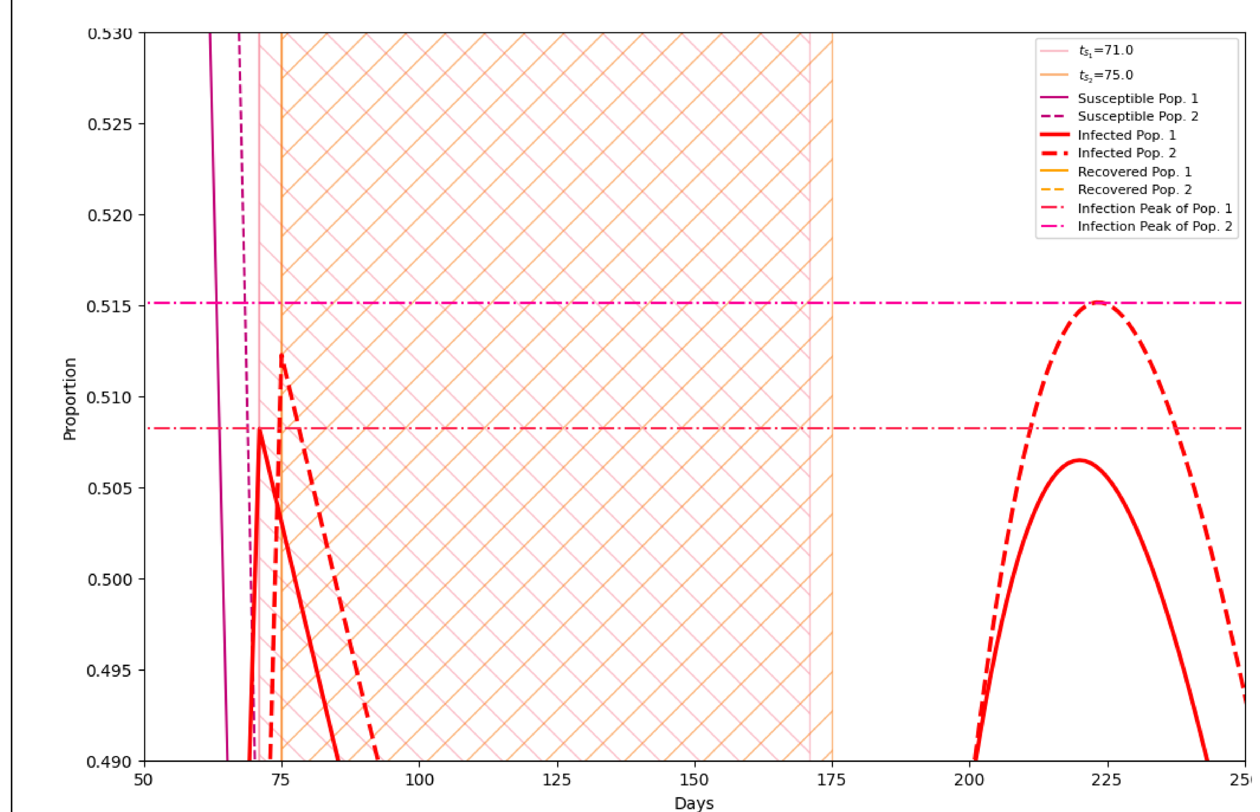


Fig. 4 Small Scale SIR Model of Two Populations Using Optimal Staggered Lockdown Start Times

Fig. 4 shows that optimal lockdown does not lead to equal peaks for individual populations in a network.

Conclusions

- Proof by example that results from [1] do not hold in a two-population network.
- Optimal lockdown strategy almost always has staggered lockdown instead of uniform ones.
- Future work could examine other lockdown strategies or epidemic models.

References

[1] J.M. Greene and E.D. Sontag (2021). Minimizing the infected peak utilizing a single lockdown: A technical result regarding equal peaks