

CBCS SCHEMEUSN

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BMATS201

Second Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024**Mathematics – II for CSE Stream**

Time: 3 hrs.

Max. Marks: 100

- Note:* 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. VTU Formula Hand Book is permitted.
 3. M : Marks , L: Bloom's level , C: Course outcomes.

Module – 1

			M	L	C
			7	L2	CO1
Q.1	a.	Evaluate $\iiint_{-1 \leq x \leq z} (x + y + z) dy dx dz$.			
	b.	Evaluate $\iint_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates.	7	L3	CO1
	c.	Show that $\beta(m, n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$	6	L2	CO1

OR

			M	L	C
			7	L3	CO1
Q.2	a.	Evaluate $\iint_0^{\sqrt{xy}} (x^2y + xy^2) dx dy$ by changing the order of integration.			
	b.	Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$	7	L2	CO1
	c.	Using mathematical tools, write the code to find the area of an ellipse by double integration $A = 4 \int_0^a \int_0^{b/\sqrt{a^2 - x^2}} dy dx$, taking $a = 4, b = 6$.	6	L3	CO5

Module – 2

			M	L	C
			7	L2	CO2
Q.3	a.	Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along vector $2i - 3j + 6k$.			
	b.	Show that the vector $\vec{F} = \frac{xi + yi}{x^2 + y^2}$ is both solenoidal and irrotational.	7	L2	CO2
	c.	Prove that the spherical coordinate system is orthogonal.	6	L3	CO2

OR

			M	L	C
			7	L2	CO2
Q.4	a.	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z^2 + y^2 - x = 3$ at $(2, -1, 2)$.			
	b.	Express the vector $\vec{A} = zi - 2xj + yk$ in cylindrical coordinates.	7	L2	CO2
	c.	Using mathematical tools, write the code to find the curl of $\vec{F} = x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$.	6	L3	CO5

Module – 3

Q.5	a. Prove that the subset $W = \{(x, y, z) : ax + by + cz = 0; x, y, z \in \mathbb{R}\}$ of the vector space \mathbb{R}^3 is a subspace of \mathbb{R}^3 .	7	L2	CO3
	b. Determine the following vectors are linearly independent or not, $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$ and $x_3 = (1, 2, 2)$ in \mathbb{R}^3 .			
	c. Show that the function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $T(x, y) = (x + y, x - y, y)$ is a linear transformation.			

OR

Q.6	a. Determine whether the vectors $v_1 = (1, 2, 3)$, $v_2 = (3, 1, 7)$ and $v_3 = (2, 5, 8)$ are linearly dependent or linearly independent.	7	L2	CO3
	b. Verify the Rank-Nullity theorem for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.			
	c. Consider the vectors $u = (1, 2, 4)$, $v = (2, -3, 5)$, $w = (4, 2, -3)$ in \mathbb{R}^3 . Find: i) $\langle u, v \rangle$ ii) $\langle u, w \rangle$ iii) $\langle v, w \rangle$ iv) $\langle (u + v), w \rangle$			

Module – 4

Q.7	a. Find an approximate value of the root of the equation $x^3 - x^2 - 1 = 0$, using the Regula-Falsi method upto four decimal places of accuracy, where root lies between 1.4 and 1.5.	7	L2	CO4						
	b. Using Newton's divided difference formula evaluate $f(4)$ from the following:									
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr> <tr> <td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr> </table>				x	0	2	3	6	f(x)
x	0	2	3	6						
f(x)	-4	2	14	158						

c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Trapezoidal rule by taking 7 ordinates.

6 **L3** **CO4****OR**

Q.8	a. Find an approximate root of the equation $x \log_{10}x - 1.2 = 0$ corrected to five decimal places where root lies near 2.5 by Newton-Raphson method.	7	L2	CO4								
	b. The area A of a circle of diameter d is given for the following values. Calculate the area of a circle of diameter 82 by using Newton's forward interpolation formula.											
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>d</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr> <tr> <td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr> </table>				d	80	85	90	95	100	A	5026
d	80	85	90	95	100							
A	5026	5674	6362	7088	7854							

c. Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.

6 **L2** **CO4**

Module – 5

Q.9	a. Find by Taylor's series method the value of y at x = 0.1 to five places of decimals from $\frac{dy}{dx} = x^2 y - 1$ with an initial condition y(0) = 1.	7	L2	CO4
	b. Using the Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2 taking h = 0.2.	7	L2	CO4
	c. Given that $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979. Compute y at x = 1.4 by applying Milne's method.	6	L2	CO4
OR				
Q.10	a. Using modified Euler's method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ at x = 0.1 corrected to four decimal places by taking h = 0.1, with initial condition y(0) = 1.	7	L2	CO4
	b. Given that $\frac{dy}{dx} = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y(0.8) by Milne's method.	7	L2	CO4
	c. Using mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at y(2) taking h = 0.2. Given that y(1) = 2 by Runge-Kutta method of 4 th order.	6	L3	CO5