Differentiable Economics Literature Review

Matching is an important topic of research in economic theory that comes in many different forms. One-sided matching comes most often in the form of an auction, where a single agent or a set of individual agents submit bids for one or multiple items. Auctions are prevalent mechanisms for allocating resources to individual bidders with private valuations of items in the global economy. They are used in many applications including the fine art and non-fungible token (NFT) market [16], online auction websites like Ebay and Taobao [17], the sale of radio spectrum frequencies by the FCC [18], allocation of online advertising space for targeting consumers [19], and access to pollution permits to name a few [20]. The company Google for instance elected to IPO through a Dutch auction rather than underwriting through an investment bank in 2004, collecting a market capitalization of $23 Billion via auction [21].

Many types of auctions exist, with the four main categories of single-item auctions being ascending-bid (English) auctions, where the seller gradually raises the price of the item until bidders drop out, awarding the auction to the final bid; descending-bid (Dutch) auctions, where the seller gradually decreases the price and awards the auction to the first bidder; first-price sealed-bid auctions where bidders simultaneously submit bids to the seller and are unaware of other agents' bids, awarding the auction to the highest bid; and second-price sealed-bid (Vickrey) auctions which are similar to the previously mentioned auction instead having the auction awarded to the highest bidder at the second highest bid price (minimum price needed to maintain victory) [22]. Although these are most common, many other auction designs exist, largely devised in an effort to incentivize auction participants to behave in a certain desirable way. The 2020 nobel prize in economics for instance was awarded to Paul Milgrom and Robert B. Wilson, who created a new auction format called a simultaneous multiple round auction (SMRA) which has led to major improvements in the allocation of radio frequencies to telecommunication companies in the US.

Auction theory is an area of economic research that deals with the way that bidders act in various types of auctions as well as how the auction's rules incentivize bidder behavior. Because the actual value that each agent associates with an item in an auction is private, oftentimes, bidders can manipulate the auction by misrepresenting their item valuations with their bids and producing a Bayes-Nash equilibrium that results in unpredictable and suboptimal item allocations and pricing. For instance, telecommunication companies may misrepresent the value they place on radio broadbands which may cause the government to falsely allocate more frequencies to a company that may not produce as much social welfare as another company. In other situations, an agent misrepresenting their true item valuation with their bids can lead to lower revenues for the auction seller which oftentimes only benefits strategic agents with the resources to determine the optimal strategy thus presenting an economic burden on less sophisticated and resource-capable bidders in the auction.

As such, it is often important to design auctions in such a way that the rules ensure strategy-proofness by making sure they are **dominant-strategy incentive compatible (DSIC)**, where truthfully reporting the value assigned to an item is the dominant bidding strategy regardless of whether or not other bidders report truthfully. As such, these auctions result in predictable and efficient outcomes that allow the auction designer to optimize for desired metrics such as social welfare, revenue, allocative fairness, etc. While the field of Game Theory is primarily concerned with determining dominant strategies for individuals participating in pre-designed mechanisms like auctions, creating DSIC auctions that optimize for the metrics previously mentioned is a major goal of auction design, a subfield of the broader area of mechanism design. Where DSIC auctions may be difficult to learn, it has been shown useful in some cases to create Bayesian Incentive Compatibility (BIC) auctions where truthful reporting is the dominant strategy only if all other bidders are also reporting truthfully.

Many tools from game theory and economics have allowed researchers to analyze generic matching mechanisms, dominant strategies, and equilibria for auctions. For instance, in 1996 William Vickrey was awarded the nobel prize in economics for the Vickrey-Clarke-Groves second-price sealed-bid auction, which has been shown to be a dominant-strategy incentive compatible auction for the single-item auction. This auction not only guarantees strategy-proofness but also maximizes the welfare of all participants [23]. In 2007, Roger Myerson was awarded the nobel prize in economics for his contributions in the case where an auctioneer wants to maximize revenue. The Myerson Auction is known to guarantee strategy-proofness when selling a single item to a single agent [24]. However, beyond several specific cases, there has been little progress in the analytical discovery of strategy-proof auction designs for multiple agents and multiple items. Recent works supporting the view of 'Differentiable Economics', particularly the proposed RegretNet model [5], makes use of Neural Network architectures in Machine Learning to design auctions for situations where optimal strategy-proof auction designs are not known. Using machine learning, researchers are able to find optimal or near-optimal allocation and pricing mechanisms that optimize for metrics of interest while obeying economic constraints.

There are many considerations to account for when representing auction designs. For instance, auctioneers may wish to optimize for different goals depending on the context of the auction. While companies and sellers on platforms like Ebay for instance may wish to maximize their revenues, governments may wish to maximize allocative efficiency while minimizing the economic burden (price of payment). Bidders may not always be utility maximizers across auction contexts. For instance, in online advertising auctions, bidders instead either seek to minimize cost per click or have even more complex, multivariable constrained bidding strategies where private budget constraints may lead to an inability to place bids that accurately represent the bidder's true value associated with an object, or their strategy optimizes for metrics like pay-per-click and pay-per-acquisition rather than maximizing utility, defined as $v_i(g_i(b)) - p_i(b)$, the difference between the value of the amount of the item allocated (as a function of the agent's bid for the item where g is the allocation function and v is the value function) and the price paid for the item (as a function of the agent's bid for the item where p is the pricing function).

Additionally, there are many different bidding languages (utility functions) made possible by agents in an auction that can affect the way in which the auction design problem may be formulated. For instance, bidders may have unit-demand, where their valuation of a bundle of items is equal to the valuation of the highest value item in the bundle, a unique situation that can thus be represented as a bipartite matching problem between bidders and items where bidder preferences represented as edge weights are maximized. Additive valuation refers to the case where bidders associate a value v_j to every item j and the value of a bundle of items is equal to the sum of the individual valuations of the items in the bundle. In this case, when multiple identical items are being auctioned, the value of a bundle depends entirely on the size of the bundle, with larger bundles being valued higher than smaller ones. Finally, there may also be combinatorial or hierarchical bundles where bidders have higher or lower preferences to certain combinations of items than the individual worth of the items in the bundle, or items can be arranged in a hierarchical tree structure such that items that are near in the tree have higher valuations. It is important to note that while individual valuations are private, auctions are usually formulated such that the valuations are drawn i.i.d from known, fixed probability distributions. In most cases it is also assumed that bidders are all risk-neutral and possess symmetric information.

When constructing auction design models there are several important constraints to keep in mind when training. For instance, we require that the auction is **Individually Rational (IR)**, such that agents can not obtain negative utility from the auction. In other words, we make sure that bidders are not harmed from simply participating in the auction and would not place a bid that exceeds the true value they place for any item. Additionally, it may be important to ensure that the auction is **Weakly Budget Balanced (WBB)** such that the sum of the payments is non-negative in order to make sure that the auction does not require a subsidy to run. Constraints often also include the requirements that items are fully allocated by the auction, each bidder may be allocated at least one item, and each bidder may only have one value assigned per item in order to avoid combinatorial explosion. Additionally, the allocation and payment mechanisms should be invariant to the order of the items or bidders.

We consider auctions with m items and n bidders such that each bidder i privately associates a value v_{ij} with each item j where each v_{ij} is from a fixed distribution known previously to the auction seller. Each bidder i then submits bids b_{ij} for each item j to the auction, such that bids do not have to accurately represent the bidder's associated value for a given item. The auction mechanism then produces an allocation $g(b_{ij})$ and price $p(b_{ij})$ for each bid by each bidder. Each bidder i then receives a utility $u(v_{ij}, b_{ij}) = v(g(b_{ij})) - p(b_{ij})$, the difference between the value associated with the allocation of the item to the bidder and the price that they paid for the item. We use a neural network architecture as a function approximator for the allocation and pricing mechanisms, such that each individual bidder acts in a way that maximizes their utility. In order to require a DSIC auction, where the dominant strategy for bidders in maximizing their utility is by truthfully reporting their item valuations, we minimize a value called regret defined as the utility a bidder could gain by misrepresenting their bids. For an auction to be DSIC, regret must equal 0 for all bidders i given any combination of bids b_{-i} from the other bidders.

Recent work has been done for neural network characterizations of single-bidder auctions that are able to learn perfectly strategy-proof auctions in cases where an optimal auction is known and in cases where the optimal auction is not known, namely the MenuNet, RochetNet, and RegretNet architectures. MenuNet proposed in [12] attempts to address the issue of previously proposed methods that suffer from an inability to find both optimal and truthful auctions, especially in multi-dimensional cases. The authors make use of a two-network architecture that produces an allocation and the buyer's action given the allocation such that individual rationality and utility maximization is held. Doing so, the authors are able to not only replicate known optimal auctions but also produce previously unknown optimal auctions across auction settings without the need for a domain-specific architecture. Additionally, they find that their approach achieves significantly faster run-time than earlier proposed linear programming approaches. Similarly, the RochetNet proposed in [5] also produces optimal and truthful auction designs in known cases for the single-bidder auction by providing a menu of possible randomized allocations and prices and maximizing the bidder's utility from the available menu of options. Proposed models such as MenuNet and RochetNet, which leverage the DSIC nature of non-decreasing convex utility functions, are however incapable of being applied on auctions with multiple bidders.

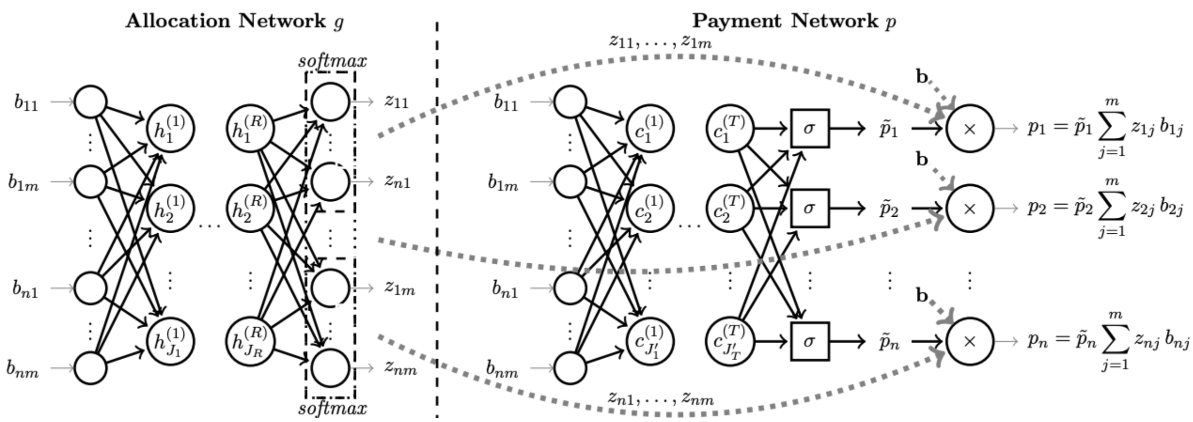


Figure 1. RegretNet Architecture includes an allocation network g and a payment network p where individual bids are used as inputs to feed-forward neural networks that compute item allocation probabilities and payments. Approximate-DSIC criteria is obtained by minimizing regret in the loss function.

RegretNet is proposed as an architecture for designing multi-bidder auction mechanisms. Notably, it learns the allocation mechanism and the payment mechanism by using two separate feed-forward neural networks. Bids from each agent are fed into the allocation network which outputs labels z_{ij}, the probability of item i being allocated to bidder j, with the use of two hidden layers with 128 nodes using tanh activation. To ensure that each item is only allocated to one bidder, we compute the softmax of each row of the output (corresponding to each item). For unit-demand auctions, we also ensure that each bidder is allocated at most 1 item by computing the softmax of each column of the output (corresponding to each bidder). The payment mechanism also takes bids as inputs and further ensures the IR constraint by computing a sigmoid over the output of the payment to restrict it between 0 and 1. Revenue and regret are then computed using the allocation and payment outputs. RegretNet approximates DSIC auction mechanisms by incorporating a penalty for regret in the loss function in addition to maximizing revenue (minimizing negated revenue) and thus minimizes regret through stochastic gradient descent via augmented lagrangian multipliers. To train the model, we alternate between updating model parameters and the lagrange multipliers for the regret penalty. Additionally, at each iteration in training we compute the maximum regret for a bidder i given bids b_{-i} from all other bidders through gradient ascent. Since this function is non-convex, it is not guaranteed to find an optimal solution; however, the authors show that this does not affect results drastically in experiments. While RegretNet suffers from merely approximating a DSIC auction mechanism in unknown optimal auctions, the authors show that it is able to reproduce all known optimal DSIC auctions in the single-bidder case and is able to identify some previously unknown optimal solutions in the multi-bidder case.

Most work published in auction design is tested using theoretical experimental designs where optimal analytical solutions are either known or unknown. As such auction design architectures are compared by determining whether or not they are able to replicate known solutions in theoretical cases as a benchmark and additionally if they are able to produce better allocations than existing auction designs in theoretical settings where an optimal solution is not known. Testing auction designs on real-world data is difficult and costly as access to such data is limited and high computational costs are faced when dealing with many bidders or many items as is often the case with real-world applications. Multiple public repositories exist implemented in Tensorflow or PyTorch that allow researchers to run experiments using different configurations of the neural network architectures mentioned above.

Since its publication, the original RegretNet architecture has been expanded on in multiple ways. Namely, there has been work on adjusting the loss function to allocate items based on fairness and include personalized budget constraints, improving model interpretability and training time by reducing the number of learnable parameters and adjusting or expanding on the model architecture, learning optimal and guaranteed strategy-proof auctions, and extending the architecture for two-sided matching.

Initial work done by [8] extends the RegretNet architecture with the ProportionNet to account for allocative fairness (total variation fairness) in addition to the original optimization for revenue maximization and regret minimization by including the sum of unfairness over all bidders as a penalty term in the loss function. This is incorporated using augmented lagrangian multipliers similarly to how regret is modeled as a penalty in the loss function. Unfairness is further defined as the L1 distance between the allocation of two users minus the L1 distance between the bids of two users. This attempts to mimic the idea that bidders who are similar should have similar allocations and neither individual should have an unfair advantage. This work is further expanded by [2] with the PreferenceNet which encodes fairness constraints by using labeled examples of desirable allocations. The authors experimentally test fairness allocations for metrics such as total variation fairness, used by the previous ProportionNet, as well as other metrics including entropy which prefers uniformly distributed allocation for a given agent and quota which ensures that the smallest allocation to any bidder is above a threshold. The authors also compare defined fairness metrics with fairness preferences empirically collected through human research surveys, noting that humans often have noisy definitions of fairness when it comes to allocation.

RegretNets have been adapted for various different auction scenarios. The authors in [7] construct a way to add a personalized budget constraint that would prevent bidders from placing bids that match their truthful valuations. This is done by setting utility to -inf when the budget constraint is not met. RegretNet has also been modified in [10] for auctions where efficient allocation is coupled with minimal economic burden by minimizing revenue. Additionally, they are able to again, adjust the loss function to approximate the weakly budget balanced constraint such that auctions do not have to be subsidized (total payments are non-negative). In [11], authors propose a new architecture called Deep Neural Auction (DNA) that encodes candidate ads (auction items) into context-aware rank scores for each target user. It then uses a DeepSet model architecture as part of a differentiable sorting engine to produce ad allocations and payments. The architecture improves upon RegretNet by allowing the model to optimize for multiple performance metrics (not just utility) dynamically over time.

The authors in [13] address RegretNet's limitations in recovering symmetric auctions for auctions where the known optimal solution is symmetric. A symmetric auction is one where the auction is invariant to relabeling of bidders or items, i.e. items are indistinguishable apriori and bidders are anonymous. Although RegretNet finds near optimal auctions in these cases, it rarely finds a symmetric auction design and does so by requiring a large number of valuation samples to learn the auction. Additionally, the authors show how RegretNet does poorly at generalizing to different auction settings once trained. Due to the fixed number of items and bidders in training, RegretNet is unable to generalize to an auction with a different number of bidders or items once trained. As such, the authors propose a new EquivariantNet architecture that recovers known optimal symmetric auctions without the large number of valuation samples. This is significant because their model architecture is not only able to better generalize to m bidders and n items, but it also produces more fair allocations in symmetric auctions as it addresses the row-number bias of RegretNet when faced with two bidders with similar bids. The authors in [14] propose two changes to the RegretNet architecture, partially motivated by similar shortcomings outlined in [13]. The first change proposed involves altering the neural network architecture to include a self-attention mechanism. The new architecture, named RegretFormer, also produces an allocation and payment output from a single neural network, where the input is a bid matrix with no restriction on the number of bidders and the number of items instead of a bid vector. This allows the model architecture to not only produce better allocations than RegretNet in almost all experimental settings explored by the authors but also enables the model to perform better out-of-setting as it is not fixed to a certain setting size, and it is able to perform better in symmetric auctions due to the removal of item and bidder permutation bias. The authors further improve on the model by proposing a new loss function that instead of optimizing for both revenue and regret, maximizes revenue given a maximum regret budget constraint that is pre-defined by the user. This not only reduces the number of hyperparameters estimated and thus reduces the sensitivity of the model to the initial setting of the hyperparameters in training but also produces a more interpretable hyperparameter that better defines the tradeoff between revenue and regret. The authors of [15] propose yet another model architecture called CITransNet inn an attempt to address RegretNet's poor out-of-setting generalization performance and provide an architecture that can better learn symmetric and generic auction designs as well as use context information about bidders and items in order to better learn context-integrated auctions. This is done by making clever use of transformer-based architecture and convolutional layers in the model architecture. Analytical results show that the model performs better than RegretNet or EquivariantNet in some known auction settings.

A Sinkhorn-based differentiable bipartite matching method proposed in [1] attempts to address the fact that RegretNet provides separate architectures depending on the bidder demand type. Unlike vanilla RegretNet, this method can easily represent different demand types by simply defining the marginals rather than adjusting the allocation model output. The authors in [9] propose ALGNet as an alternative network architecture to RegretNet that attempts to solve several issues with the RegretNet architecture. While it does not improve upon revenue-maximizing performance or constraints, the authors are able to construct a new architecture that uses time-independent lagrangians that drastically reduce the model's dependence on specific hyperparameter values. ALGNet also improves upon model training time by amortizing the gradient ascent loop for expected regret maximization. In [4] the authors propose a Lottery Affine Maximizer Auction (AMA) model that guarantees strategy-proof auction mechanisms for multi-bidder auctions without finding optimal solutions, while previously methods only existed that either found optimal solutions for multi-bidder auctions without the strategy-proof guarantee (RegretNet and ALGNet) or found optimal, strategy-proof solutions but only in the single-bidder case (MenuNet and RochetNet). There has yet to be a proposed model architecture capable of producing optimal, perfectly strategy-proof mechanisms in multi-bidder auctions. While RegretNet is able to empirically find strategy-proof auction mechanisms, it does not have a way to explicitly verify strategy-proofness. The authors in [6] are able to calculate bounds on the level of exploitation by strategic agents by replacing the softmax activation in RegretNet with a piecewise linear sparsemax. Additionally, they experiment with adding the individual rationality constraint as a learned penalty in the loss function as an augmented lagrangian.

While prices can often be the leading drivers for matching mechanisms, as in bidding auctions, a plethora of markets and mechanisms exist where prices are not the main drivers for matching. Two-sided matching for instance is often driven by discrete preferences that individual agents in one set have for agents in another set and vice versa. Examples of two-sided matching mechanisms include date matching on apps like Tinder and OkCupid, rider and ride-share driver matching on apps like Uber and Lyft, job hiring between employees and employers in labor markets, applicant matching between students and schools or residents and hospital systems, matching of medical patients to donors for kidneys, bone marrow, hearts, etc., matching of government research funds to research proposals, matching of food bank resources to people in need, etc [27]. Naturally, this exercise can be easily extended to multi-sided matching where there are n disjoint sets of agents where an individual agent in each set has a set of preferences for agents within other sets. An example of such a multi-sided matching problem is matching between phd students, advisors, and co-advisors [28]. Complications quickly arise for different matching scenarios. For instance, the type of matching relationship, one-to-one, one-to-many, and many-to-many is entirely dependent on the matching problem at hand. Some constraints are also application-dependent. For instance, in the case of Kidney matching, matches are constrained by the blood type of the donor and recipient, the proximity of the donor to the recipient and time in which a transplant would be possible, among other medical domain-specific factors [29]. Other applications are often also limited by the amount of information available. For instance, matching is often required in applications where individual agents do not or are unable to submit bids or preferences over the entire set of available matches on the other side of the market such that not all preferences for an individual are known or a rank of preferences is known but not the strength of each preference.

Extensive work has been done on analyzing two-sided matching problems analytically. For instance, the 2012 nobel prize in economics was awarded to Lloyd Shapley for the theoretic development of the stable matching, illustrated best by the marriage problem. The marriage problem is one where we wish to match men and women together based on their individual preferences of members of the other gender assuming only heterosexual preferences. We attempt to reach a stable matching such that no pair of men and women (M, W) would both be better off not matched. The finding of the work was that a stable matching exists for any setting of the marriage problem and can be found using the deferred acceptance algorithm defined in the work. However, it is impossible to achieve an outcome such that the matching is strategyproof for both sides and always results in a stable outcome [25]. The National Resident Matching Problem where medical students are matched with residency programs is a classic example of the two-sided matching at work, where deferred acceptance was used to allocate students to residency programs [26].

Differentiable Economics, including work done on the RegretNet architecture have been mostly applied to one-sided matching mechanisms (auctions). As of now, only one extension of this work has been applied to two-sided matching mechanisms. In [3], the authors empirically characterize the trade-off between auction stability and strategy-proofness for two-sided matching mechanisms, where it is known that achieving both is impossible. The work goes on to explain how while *Deferred Acceptance (DA)* is not a strategy-proof mechanism, it is a stable mechanism, where no pair of agents mutually prefer each other to the other agent pairs. Similarly, *Random Serial Dictatorship (RSD*) is not a stable mechanism but it is strategy-proof. By using a neural network architecture inspired by RegretNet, the authors are able to construct a model that produces a better trade-off between stability and strategy-proofness than the simple convex combination of DA and RSD. Each worker w has a preference ranking of firms f. Similarly, each firm f has a preference ranking of workers w. We create (w, f) pairs such that they prefer each other and define an individually rational mechanism such that no worker or firm finds its pair unacceptable and would rather prefer to be unmatched. Envy is created when a worker prefers a different firm to the one it is matched with as well as when a firm prefers a different worker to the one it is matched with (or if either finds each other unacceptable). Approximate-stable matching is obtained by minimizing envy while Strategy-proofness is obtained by minimizing regret (which is represented similarly to the RegretNet representation). By constructing such a model, the authors are able to empirically characterize the tradeoff between these two properties and examine the differences with increasing observed correlated preferences. The authors adapt the RegretNet architecture for discrete preferences. Assuming full preferences, the discrete ranking can be represented as evenly spaced entries from 1/|W| to 1 where W is the number of agents in the other group (women for marriage problem). Converting the problem into a continuous and differentiable one, the author calculates the expected ex post stability violation (envy) for each possible blocking pair (i, c) and uses this calculation as part of the loss.

This work is limited however by the fact that bidder preferences in two-sided matching mechanisms are discrete rather than continuous as in one-sided matching, which presents issues for computing utility gradients in the model. Additionally, the current model introduces a computational bottleneck for the calculation of regret which prevents empirical results for larger markets. While there have been many extensions on the RegretNet architecture for multi-bidder auctions, there are still many limitations to this work for both one-sided multi-bidder matching mechanisms and two-sided matching mechanisms with discrete agent preferences. Additionally, more work needs to be done to account for variations in auction constraints such as the introduction of risk-averse and risk-seeking bidders, correlated values between items particularly in the case for the *winner's curse* where the utility of the auction winner decreases due to the realization that other bidders' estimated value is less than them [30], the presence of a linkage principle that presents a dependence between bidder values, as well a the presence of royalties and incentive payments established by the auction seller.

In conclusion, despite breakthroughs in economic research in mechanism design, particularly auction theory, very little is known beyond a few simple settings of the auction design problem. Recently, work aiming at leveraging the power of Neural Networks as universal function approximators has led to exciting new developments in deep learning for mechanism design in one- and two-sided markets. Despite new breakthroughs leading to optimal mechanisms for previously unknown auctions, many limitations and weaknesses remain to be addressed.

Personal Note:

For CMSC499A this semester, I dove deep into the literature behind the topic of mechanism design. I conducted a literature review of work relevant to deep learning for auction design in one-sided matching and for mechanism design in two-sided matching. Having no prior exposure to this field of study, I learned a lot about both the economics and computer science of auctions and other matching mechanisms as well as got a stronger foundation in the concepts of game theory that drive research in this field. I have learned an extensive amount about the current state of research in the field including the motivation, applications, relevant experimental methods and analysis tools, neural network architectures (as well as the mathematics behind constrained optimization which I had no prior exposure to), and public code repositories. I feel confident in my understanding of the research material and prior work as well as the process used by researchers to train networks, test them, and evaluate and compare their performance. Additionally, I am relatively aware of current limitations in the research and have developed a deep interest in some of these limiting areas including adjusting RegretNet to address time-evolving auctions, testing auctions with imperfect preference information that may require prediction of bidder preferences for items where bidders were unable to give preference to, testing auctions with bidder coordination, creating a continuous approximation of discrete preferences for two-sided matching, adjusting RegretNet for the three-sided matching problem, testing auctions with other bidder parameters like risk-seeking, correlated bids, and other constraints like allocation caps and item delivery. As such, next semester, I hope to run experiments on specific auction settings under various new constraints and parameters, and address some of these current limitations of the deep-learning approach for matching mechanisms.

Annotated Paper Notes:

[1] Learning Revenue-Maximizing Auctions With Differentiable Matching

- Uses the Sinkhorn algorithm to perform a differentiable bipartite matching
- Strategyproof revenue-maximizing mechanisms in settings not learnable by the previous RegretNet architecture (no free disposal, at least one item)

Auctions:

- Used in broadcasting, advertising, electricity markets, etc.
- Auctioneer solicits bids from participants and awards the item to the winner
- Participants can lie about their worth of the items to manipulate the auction
- Must design strategy-proof auction so that participants must be honest
- Welfare of all participants:
    - Vickrey-Clarke-Groves Auction
- Revenue for auctioneer:
    - Single item sale: Myerson Mechanism
    - Approximation: RegretNet
        - Enforces Strategyproofness in loss function
        - Cannot enforce constraints well (all participants receive k items)
        - Explicitly apply matching constraint using Sinkhorn Algorithm (if allocation is a linear function of valuation)
    - Approximation: Rochet Net
        - single-agent auctions

- Auction mechanism (allocation and payment): (g(b), p(b)), b_i are bids
    - G_{ij} probability of allocating item j to bidder i
    - Utility: $u_i(v_i, b) = v(g(b)) - p(b)$
    - SIMPLIFYING ASSUMPTION: each bidder has one value per item
    - Regret(v_i, b_i) = utility bidder could have gotten if they lied.
        - Regret should be 0 for all bidders with valuation v_i and any bids $b_{\_i}$.
        - Individually Rational (non-negative utilities)
- Bidder Demands:
    - Unit-demand: value of bundle is max value item in bundle
        - K-unit demand (sum of top k items in bundle)
    - Exactly one demand: each bidder can be only allocated one item
- RegretNet
    - Ensures individual rationality by using sigmoid activation (0 to 1)
    - Additive Allocation
        - Probability for each item at most 1: row-wise softmax of network outputs
    - Unit-Demand

        - Row-wise softmax and column-wise softmax to ensure only one allocation per item
    - Training:
        - Gradient Descent on augmented Lagrangian
- Sinkhorn time costly, biased towards deterministic outcomes
- Support randomized allocations, computational improvements, extensions to other mechanism design problems where the allocation decision can be expressed as a matching

Unsure: Why tanh activation? 2 hidden layers? 128 nodes?

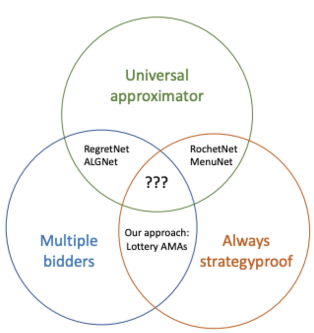
[2] PreferenceNet: Encoding Human Preferences in Auction Design with Deep Learning
- Auctions that benefit third parties (electromagnetic spectrum distribution)
- Fairness with respect to protected characteristics
- Total Variation fairness might not be good
- Trained using responses on surveys on “fairness”
- dominant-strategy incentive compatible (DSIC) = strategy-proof. Truthful bids.
- Preference Classification Accuracy (PCA) = assign label s(b) = [0,1] for each bid. Take average of s(b) over n test bids.
- Assign label based on nearest neighbor in ground truth set of allocation -> label
- Pairwise comparison to compare positive and negative labels
- Optimize binary cross-entropy loss
- Class misbalance handled by oversampling minority class

Fairness:
- Total Variational Fairness
    - Distance between allocations of two users = distance between users (minimize to 0)
- Entropy
    - Allocations uniformly distributed for a given bidder
- Quota
    - Smallest allocation of item to person j is greater than a threshold
    - Hard to enforce in additive auctions with many agents
    - Nxm auctions (n is number of agents, m number of items)
    - Probability of fairness label being unperturbed increases with distance from 0.7 decision boundary. Follows normal distribution. Humans are noisy at choosing fairness.

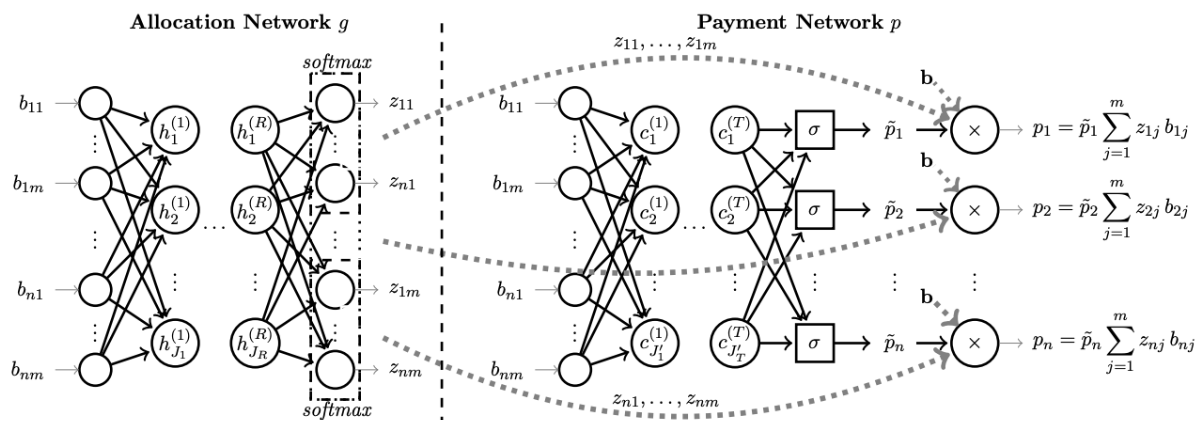
[3] Deep Learning for Two-Sided Matching
- Two-sided matching is impossible to achieve both strategy-proofness and stability
- Convex Combination of Deferred Acceptance:

    - Stable and strategy-proof for only one side of market
        - Equal preference between two agents, but can misreport preferences
        - Used in Doctor-hospital matching, school choice, and cadet matching
    - Strategy-proof but not stable (randomized serial dictatorship)
- Trying to model frontier of tradeoff between stability and strategy-proofness for correlated and uncorrelated preferences
- Increasing correlation decreases the ability for strategy
- Must figure out how to handle discrete preference inputs for computing utility gradients

[4] Differentiable Economics for Randomized Affine Maximizer Auctions

- MenuNet and RochetNet are strategy-proof for single-bidder auctions
- RegretNet for multi-bidder auctions approximates any mechanism, but approximates strategyproofness
- Affine Maximizer Auction with Lottery is perfectly strategyproof for multi-bidders
    - VCG that adds positive weight to each bidder's welfare
    - Revenue can be increased by adjusting weights and boosts
- AMD approaches:
    - Designing mechanism from scratch in tabular form
    - Conducting search over parameters of mechanism class
    - Incremental mechanism design
- Interpretable mechanisms
    - All allocations, weights and boosts much easier to interpret than parameters in NN
- Recover MenuNet & RochetNet in single-bidder case
- Lotteries can improve revenue:
    - offering more menu items can improve revenue by allowing finer price discrimination, and there are always fewer deterministic allocations than possible lotteries.
- every strategyproof mechanism must take the form of an AMA (when no restriction on utility)
- 

[5] Optimal Auctions through Deep Learning

- First, general purpose, end-to-end approach for solving the multi-item auction design problem
- Objective is to maximize revenue
- Non-decreasing convex utility functions are IC
- Minimizing expected ex-post regret via augmented lagrangian
- Can optimize over broad class of mechanisms
- NN much faster than linear programming because number of parameters is smaller
- Loss Function: minimize negated expected revenue
- Regret is equal to the maximum increase in utility considering all possible non-truthful bids
- 

- Tanh activation function in hidden nodes
- Softmax over each item to ensure item only allocated to one bidder
- Payment output is fed through sigmoid function to ensure IR
- Unit-demand: Allocation is minimum of row and column normalized allocations (softmax)
- Include quadratic penalty for regret
- Training alternates on update for model parameters and lagrange multipliers
- We solve for max regret using gradient ascent (non-convex so not guaranteed to find optimal solution)

[6] Certifying Strategyproof Auction Networks
- Propose making changes to make RegretNet Strategy-proofness verifiable
- Finding rigorous bounds on how much they can be exploited by strategic agents
- Replace softmax activation in RegretNet with piecewise linear sparsemax
- Add individual rationality as a learned penalty in the lagrangian
- Solve non-convex integer program (slow)

[7] Deep Learning for Revenue-Optimal Auctions with Budgets
- No optimal auctions are known for when bidders have budget constraints
- Utility is negative infinity if payment goes over budget

    - Refine regret to include budget violations
- Model finds higher revenues with low errors in several cases, in others it finds the known optimal solutions
- Explore both DSIC and BIC auctions
    - DSIC: truth is optimal no matter what other bidders do
    - BIC: truth is optimal if other bidders are truthful
- Changes in Type Distributions
- Explore effect of correlation between value and budget

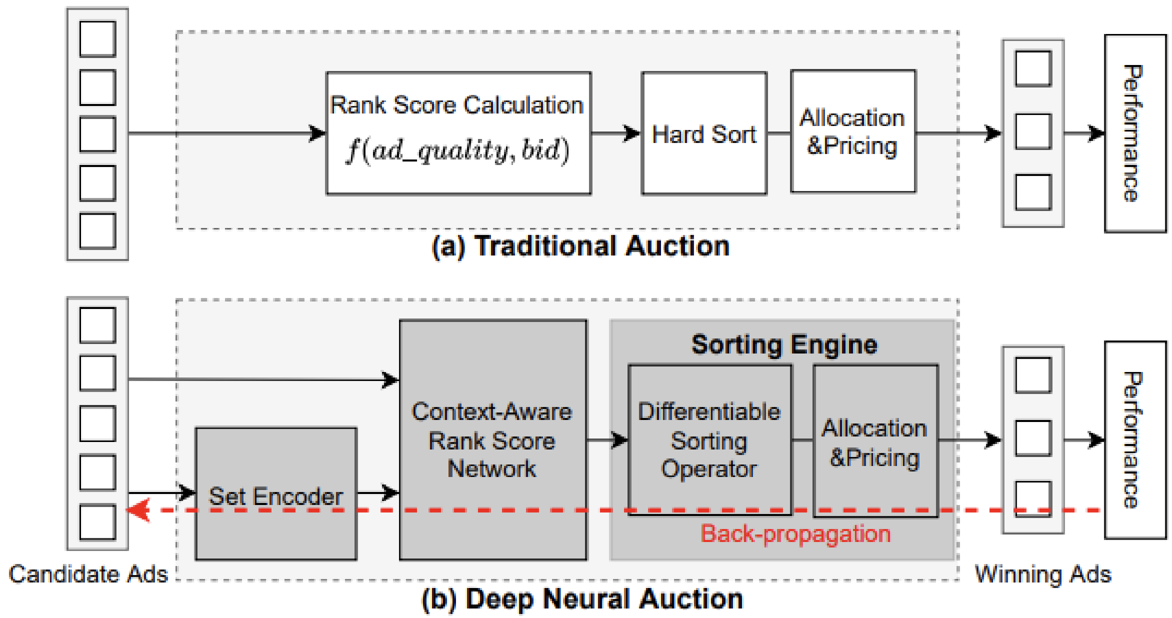
[8] ProportionNet: Balancing Fairness and Revenue for Auction Design with Deep Learning
- Fairness in terms of protected characteristics:
    - Race, gender, national-origin
- Deals with additive and unit-demand but not combinatorial valuations
- Total Variation Fairness:
    - L1 distance between allocation for two users is at most the distance between the two users
- Sum of unfairness over all agents in game added to loss function
- Unfairness is a big problem in auctions (examples given)

[9] Auction learning as a two-player game
- Introduce time-independent Lagrangians
- Ammortize gradient ascent for computing optimal misreports with another neural network
- Problems with RegretNet:
    - Hard to train because objective is non-stationary (time-dependent lagrangians)
    - No good metric to compare auctions with different revenues and truthfulness
    - Slow inner-loop for regret calculation
- Algnet:
    - Easier tuning, less hyperparameters
    - Metric for comparing auctions
    - Good for Online auctions: bids/valuation varies over time
- Finding good parameters for learning requires a large search for good hyperparameters
- Formulate it as a two-player game between the auctioneer and the agent
    - Auctioneer Module
    - Misreporter's Module
- Algnet finds revenue/regret faster than RegretNet with fewer parameters

[10] A neural architecture for designing truthful and efficient auctions
- Assume that participants' valuations for the items for sale are independently sampled from an unknown but fixed distribution

- producing truthful and efficient auctions that minimize the economic burden on participants
- Mechanism design is a field in economics that deals with setting incentives and interaction rules among self-interested agents so as to achieve desired objectives for the group as a whole. It is sometimes referred to as "inverse game theory": in game theory we set the rules of a game, and study the behaviors that emerge, while in mechanism design we have a target behavior we wish to encourage, and we set the rules of the game so that agents acting in their own self-interest will gravitate towards that desired outcome
- Sum of payments is non-negative (design mechanism that does not require a subsidy)
- VCG auctions are truthful and efficient but they do not minimize economic burden.
- Bidding Languages:
    - Multi-unit auctions with decreasing marginal utilities
        - Preference depends on size of bundle of items
    - Heterogeneous objects with unit demand
        - Value of bundle is max value of item in bundle
        - Allocation found by solving the maximum-weighted bipartite matching between bidders and items
    - Hierarchical bundles
        - Binary tree representation of items (builders and lots)
- Design truthful auction that is rational, weakly budget balanced, allocatively-efficient while minimizing the sum of payments.
    - Each player payment uses same function
    - Payment is player order invariant
    - Robust to changing number of participants
- Input is a tensor of all player valuations except i, along with valuation of each item and utility. Fed into 2-layer CNN, multilayer-perceptron for each player (2 layers, 64 units), then sum pooled and decoded to a single value
- Loss Function is sum of payment mechanism (with non-deficit constraint)
- Baselines: VCG auctions, Guo and Conitzer (only multi-unit auctions), Manisha et al. (unit-demand, no hierarchical), 2 layer, 128-unit MLP trained on flat data.
- While method does not guarantee weakly budgeted or rational auctions (next to zero examples are found)
- Data representation is key contribution

[11] Neural Auction: End-to-End Learning of Auction Mechanisms for E-Commerce Advertising
- Propose Deep Neural Auction (DNA) that relaxes discrete sorting operation.
- GSP fixed allocation rule limits its capability to optimize multiple performance metrics in dynamic environments



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- Contradictory labels for same advertiser feature profile (one-to-many) problem for training.
- stick to the rank score-based allocation and second-price payment procedures inherited from GSP auction.
- advertisers with top-K scores would win the corresponding ad slots, with ties broken randomly.
- Payment is inverted rank score of next highest advertiser
- Two types of advertises (not just utility maximizing):
    - Optimized Cost per Click (OCPC): upper bound on bids
    - Multi-variable Constrained Bidding (MCB): constraints over budgets and average costs like pay-per-click (PPC) and pay-per-acquisition (PPA)
- Value-Maximizers:optimize value vi while keeping pi below maximum willingness to pay mi. IC if monotonic (An advertiser would win the same or a higher slot if she reports a higher bid), Critical price: The payment for the winning advertiser is the minimum bid that she needs to report to maintain the same slot.
- ARCHITECTURE: set encoder, a context-aware rank score function, and a differentiable sorting engine
- Embedding attached as feature for each ad (uses DeepSet to embed permutation-equivariant representation)
- Use Neural-sort to derive a differentiable top-K permutation matrix,
- Sort using row-stochastic permutation matrix (uses softmax approximation)
- Loss is row-wise cross-entropy between ground truth and predicted row stochastic matrix
- Performance metrics: Revenue per Mile (RPM), Click-Through Rate (CTR), Conversion Rate (CVR), GMV per mile (GPM)
- Benchmarks: GSP, utility-based GSP, Deep GSP (uses RL)

[12] Automated Mechanism Design via Neural Networks

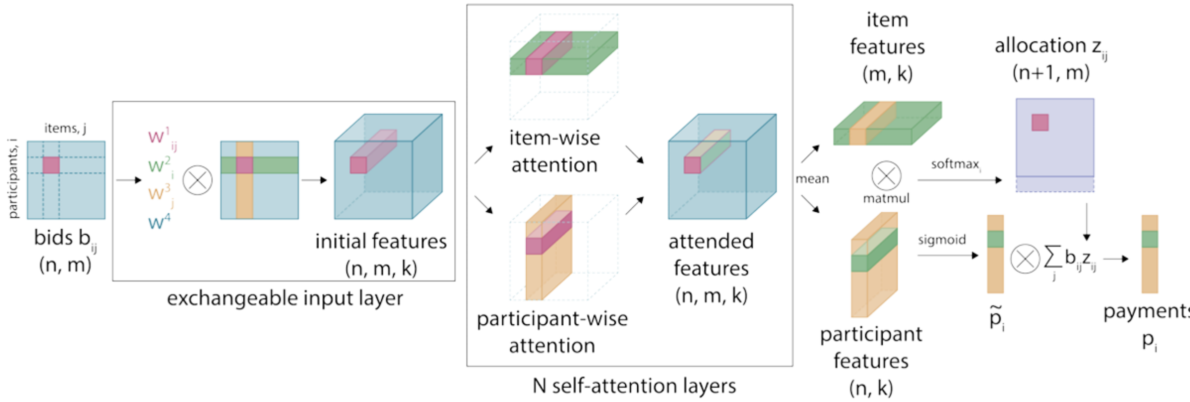
- Representation of Auction Design likely involves searching a space that does not even contain the optimal design
- Auction Design found is likely not truthful or far from optimal
- New setting is required for each domain application.
    - Authors propose MenuNet (works only for single buyer case)
- Most work has been in approximately-optimal or approximately-truthful auctions, not exact.
- MenuNet has an allocation network and a buyer behavior network that is modeled as a GAN. Produces list of valuation, outcome tuples.
- Buyer network does not require utility because it outputs the buyer's strategy which may not align with their utility.
- Performance is significantly faster than linear programs
- Able to recover several known optimal allocations and some previously unknown optimal allocations.

[13] A Permutation-Equivariant Neural Network Architecture For Auction Design

- RegretNet is not data efficient and may require a large number of valuation samples to learn an optimal auction, when not making use of inductive bias.
- Multi-bidder symmetric auctions are invariant to relabeling bidders or items. Items are indistinguishable apriori and bidders are anonymous.
- "Bundling together" auctions are symmetric, where bidders either pay a fixed price to get everything or get nothing.
- RegretNet finds a near-optimal auction for known symmetric auctions but rarely outputs a symmetric auction by design (poor in fairness because row number matters).
- RegretNet cannot generalize to a different number of bidders than it is trained on.
- Propose EquivariantNet

[14] Optimal-er Auctions Through Attention

- Propose two changes: new architecture based on attention mechanism called RegretFormer, and new loss function that is interpretable and relies on a single hyperparameter
- Authors compare performance out-of-setting



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- RegretFormer is insensitive to order of items and bidders which is good for learning symmetric auctions.
- Does not require fixed input size.
- Maximize revenue given fixed regret budget to allow for interpretable trade-off between regret and revenue
- RegretNet is highly sensitive to the hyperparameters in training. How to select hyperparameters for new setting that has not been studied is difficult to know.
- RegretFormer only uses a single neural network to predict allocation and payments. It also does not use a flat input vector, it uses a bid matrix that does not require fixed m and n.

[15] A Context-Integrated Transformer-Based Neural Network for Auction Design
- Another architecture that attempts to address RegretNet's inability to generalize well to out-of-setting auctions, attempts to generalize the model to perform well in the general case, the symmetric case, and in the case where context is provided for each bid and bidder.
- Uses transformer-based approach with convolutional layers in architecture.
- Analytically performs better than RegretNet or EquivariantNet in some known optimal auctions.

[16] Under the Skin of Foundation NFT Auctions

[17] Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet

[18] Discovering Auctions: Contributions of Paul Milgrom and Robert Wilson*

[19] Real-time bidding for online advertising: measurement and analysis

[20] Revenue and efficiency in pollution permit allocation mechanisms

[21] Going Dutch: The Google IPO

[22] Auction Theory: A Guide to the Literature

[23] COUNTERSPECULATION, AUCTIONS, AND COMPETITIVE SEALED TENDERS

[24] Optimal Auction Design

[25] College Admissions and the Stability of Marriage

[26] The Effects of the Change in the NRMP Matching Algorithm

[27] Two-Sided Matching Platforms: Characteristics, Welfare, and Design

[28] Finding Stable Matchings in PhD Markets with Consistent Preferences and Cooperative Partners

[29] Kidney Exchange

[30] Anomalies: The Winner's Curse