

How to estimate in a conditionally randomized experiment

More formally: Nonparametric estimation by subgroup means under conditional exchangeability

Soc 114

Nonparametric estimation

(a fancy words for model-free estimation, such as with `group_by` and `summarize()`)

Simulated setting: In math

- ▶ Treatment a takes values 0 and 1
- ▶ Conditionally randomized within x , taking values 0, 1 and 2
 - ▶ $P(A = 1 | X = 0) = .3$
 - ▶ $P(A = 1 | X = 1) = .5$
 - ▶ $P(A = 1 | X = 2) = .8$
- ▶ Outcome Y is numeric

Simulated setting: In data

```
# A tibble: 1,000 x 5
  id      x     a     y sampling_weight
  <int> <int> <int> <dbl>             <dbl>
1 1       1     1     1   0.814            1
2 2       2     0     0   0.750            1
3 3       3     0     0   -0.699           1
4 4       4     1     0   0.507            1
5 5       5     1     0   1.65             1
# i 995 more rows
```

1) Estimate within subgroups

Estimate each mean potential outcome in each subgroup $E(Y^a | X = x)$ by the subgroup sample weighted mean among those with $X = x$ and $A = a$.

$$\begin{aligned}\hat{E}(Y^a | X = x) &= \hat{E}(Y | X = x, A = a) \\ &= \frac{\sum_{i:X_i=x, A_i=a} w_i Y_i}{\sum_{i:X_i=x, A_i=a} w_i}\end{aligned}$$

1) Estimate within subgroups

```
average_potential_outcomes <- simulated |>
  group_by(x,a) |>
  summarize(mean_y = weighted.mean(y, w = sampling_weight))
```

```
# A tibble: 6 x 3
# Groups:   x [3]
  x     a   mean_y
  <int> <int>   <dbl>
1 0     0 -0.0206
2 0     1 -0.0369
3 1     0 -0.0849
4 1     1  0.907 
5 2     0 -0.241 
6 2     1  1.91
```

2) Pivot across treatment A to take differences

```
cate <- average_potential_outcomes |>
  # Pivot wider and difference over A to estimate CATE
  pivot_wider(
    names_from = a,
    names_prefix = "mean_y",
    values_from = "mean_y"
  ) |>
  mutate(cate = mean_y1 - mean_y0) |>
  print()
```

```
# A tibble: 3 x 4
# Groups:   x [3]
      x  mean_y0  mean_y1     cate
  <int>   <dbl>   <dbl>   <dbl>
1     0 -0.0206 -0.0369 -0.0163
2     1 -0.0849  0.907   0.992 
3     2 -0.241   1.91    2.15
```

3) Re-aggregate to the Average Treatment Effect

The average treatment effect (ATE) is the weighted average of the conditional average treatment effects (CATEs), weighted by size.

$$\underbrace{\hat{E}(Y^1 - Y^0)}_{\text{estimated overall average effect}} = \sum_x \underbrace{\hat{P}(X = x)}_{\substack{\text{size of subgroup} \\ \text{sum over subgroups}}} \underbrace{\hat{E}(Y^1 - Y^0 \mid X = x)}_{\text{average effect within subgroup}}$$

3) Re-aggregate to the Average Treatment Effect

First, determine the size of the subgroups:

```
stratum_sizes <- simulated |>
  # Count sum of sampling weight in each stratum
  count(x, wt = sampling_weight) |>
  # Convert count to a proportion of the population
  mutate(stratum_size = n / sum(n)) |>
  select(-n)
```

```
# A tibble: 3 x 2
  x stratum_size
  <int>      <dbl>
1     0        0.255
2     1        0.492
3     2        0.253
```

3) Re-aggregate to the Average Treatment Effect

Then, re-aggregate across subgroups:

```
cate_with_stratum_size <- cate |>
  left_join(stratum_sizes, by = join_by(x)) |>
  ungroup()
```

```
# A tibble: 3 x 5
  x mean_y0 mean_y1    cate stratum_size
  <int>   <dbl>   <dbl>   <dbl>      <dbl>
1 0 -0.0206 -0.0369 -0.0163      0.255
2 1 -0.0849  0.907   0.992       0.492
3 2 -0.241   1.91    2.15       0.253
```

```
ate <- cate_with_stratum_size |>
  summarize(ate = weighted.mean(cate, w = stratum_size))
```

```
# A tibble: 1 x 1
  ate
  <dbl>
1 1.03
```

Takeaways

- ▶ In a **conditionally randomized experiment**, units are randomly assigned to treatment within subgroups of X .
- ▶ We get **conditional exchangeability**:

$$\underbrace{\{Y^1, Y^0\}}_{\text{Potential Outcomes}} \quad \underbrace{\parallel}_{\text{Are Independent of}} \quad \underbrace{A}_{\text{Treatment}} \quad \underbrace{|}_{\text{Within Subgroups}} \quad \underbrace{X}_{\text{of } X}$$

- ▶ Under conditional exchangeability, causal effects within subgroups can be estimated by a difference in means

Bonus: Estimation by treatment weighting

Treatment weighting is an approach that begins from a different angle but ends up at a mathematically equivalent estimator.

For each unit, we observe Y^{A_i} with some probability

$$P(A = A_i | X = x_i)$$

We can estimate by the proportion within each subgroup to receive treatment $A = A_i$.

- ▶ For units with $A_i = 1$, this is the subgroup mean of $A = 1$
- ▶ For units with $A_i = 0$, this is the subgroup mean of $A = 0$

Bonus: Estimation by treatment weighting

```
data_with_probabilities <- simulated |>
  group_by(x) |>
  mutate(probability_of_a = case_when(
    # For treated units, proportion treated
    a == 1 ~ mean(a),
    # For untreated units, proportion untreated
    a == 0 ~ mean(1 - a)
  )) |>
  ungroup() |>
  print()
```

# A tibble: 1,000 x 6						
	id	x	a	y	sampling_weight	probability_of_a
	<int>	<int>	<int>	<dbl>	<dbl>	<dbl>
1	1	1	1	0.814	1	0.530
2	2	0	0	0.750	1	0.655
3	3	0	0	-0.699	1	0.655
4	4	1	0	0.507	1	0.470
5	5	1	0	1.65	1	0.470
6	6	1	1	0.641	1	0.530
7	7	1	0	-0.584	1	0.470
8	8	1	0	-0.519	1	0.470

Bonus: Estimation by treatment weighting

Sampling weights are 1 divided by the probability of inclusion in the sample. Likewise, treatment weights are 1 divided by the probability of receiving a particular treatment.

```
data_with_weights <- data_with_probabilities |>
  mutate(
    total_weight = sampling_weight * (1 / probability_of_a)
  ) |>
  print(n = 3)
```

```
# A tibble: 1,000 x 7
  id     x     a     y sampling_weight probability_of_a total_weight
  <int> <int> <int> <dbl>             <dbl>            <dbl>           <dbl>
1     1     1     1   0.814             1       0.530          1.53
2     2     0     0   0.750             1       0.655          1.55
3     3     0     0  -0.699             1       0.655          1.55
# i 997 more rows
```

Bonus: Estimation by treatment weighting

Once we have inverse probability of treatment weights, we can estimate by the sample weighted mean in each treatment group.

```
ate <- data_with_weights |>
  group_by(a) |>
  summarize(estimate = weighted.mean(y, w = total_weight)) |>
  pivot_wider(
    names_from = a,
    names_prefix = "mean_y",
    values_from = estimate
  ) |>
  mutate(ate = mean_y1 - mean_y0) |>
  print()
```

```
# A tibble: 1 x 3
  mean_y0  mean_y1    ate
  <dbl>    <dbl> <dbl>
1 -0.108    0.920  1.03
```