Confidence Intervals

UCLA Soc 114

Concepts for today

Statistical concepts

- Sampling distribution
- Standard error
- Confidence interval
- Bootstrap

Coding concepts

- Writing a custom function
- Writing a for loop

Example: Mean salary of MLB players

Load data:

```
baseball <- read_csv("https://soc114.github.io/data/baseba
  # Keep only a few variables for simplicity
  select(player, team, salary)
```

```
# A tibble: 944 x 3
  player              team       salary
  <chr>               <chr>       <dbl>
1 Bumgarner, Madison Arizona 21882892
2 Marte, Ketel       Arizona 11600000
3 Ahmed, Nick        Arizona 10375000
# i 941 more rows
```

Example: Mean salary of MLB players

True mean in population of all players

Example: Mean salary of MLB players

True mean in population of all players

```
baseball |> summarize(population_mean = mean(salary))
```

```
# A tibble: 1 x 1
  population_mean
            <dbl>
1         4965481.
```

Estimate from a sample

Estimate from a sample

Draw a sample of 10 players.

Estimate from a sample

Draw a sample of 10 players.

```
sampled_players <- baseball |>
  slice_sample(n = 10) |>
  print(n = 3)
```

```
# A tibble: 10 x 3
  player              team         salary
  <chr>               <chr>         <dbl>
1 Montgomery, Jordan St. Louis  10000000
2 Barnes, Matt        Miami      7500000
3 Eflin, Zach         Tampa Bay 11000000
# i 7 more rows
```

Estimate from a sample

Take the mean among sampled players.

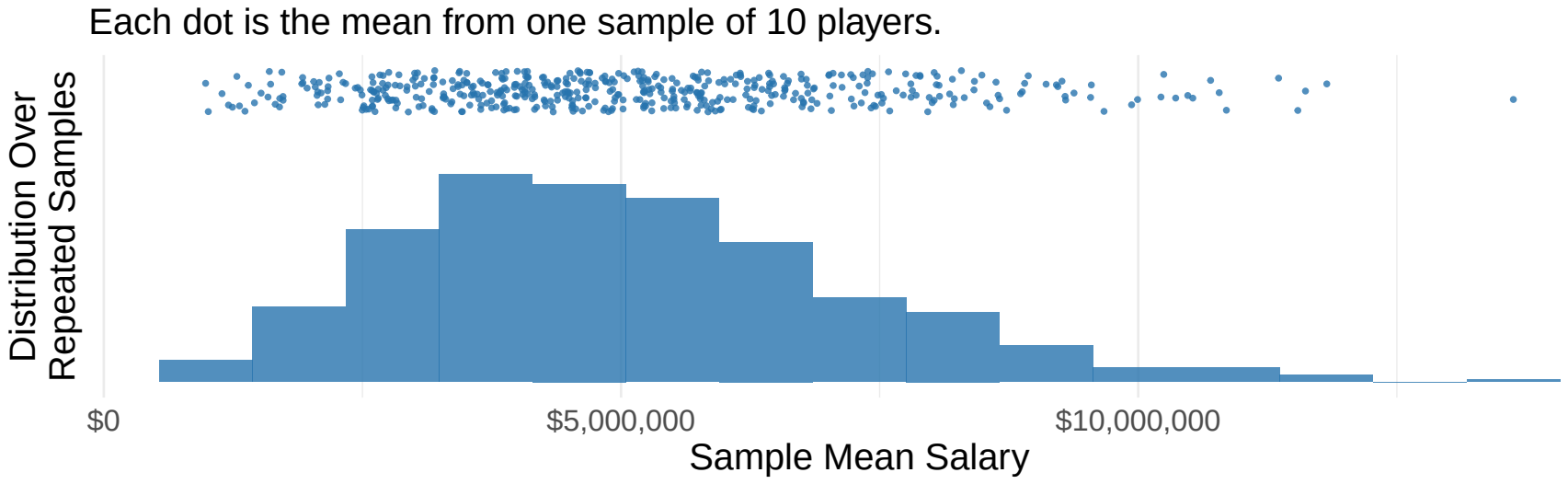
Estimate from a sample

Take the mean among sampled players.

```
sampled_players <- sampled_players |>
  summarize(sample_estimate = mean(salary)) |>
  print()
```

```
# A tibble: 1 x 1
  sample_estimate
            <dbl>
1         3435960
```

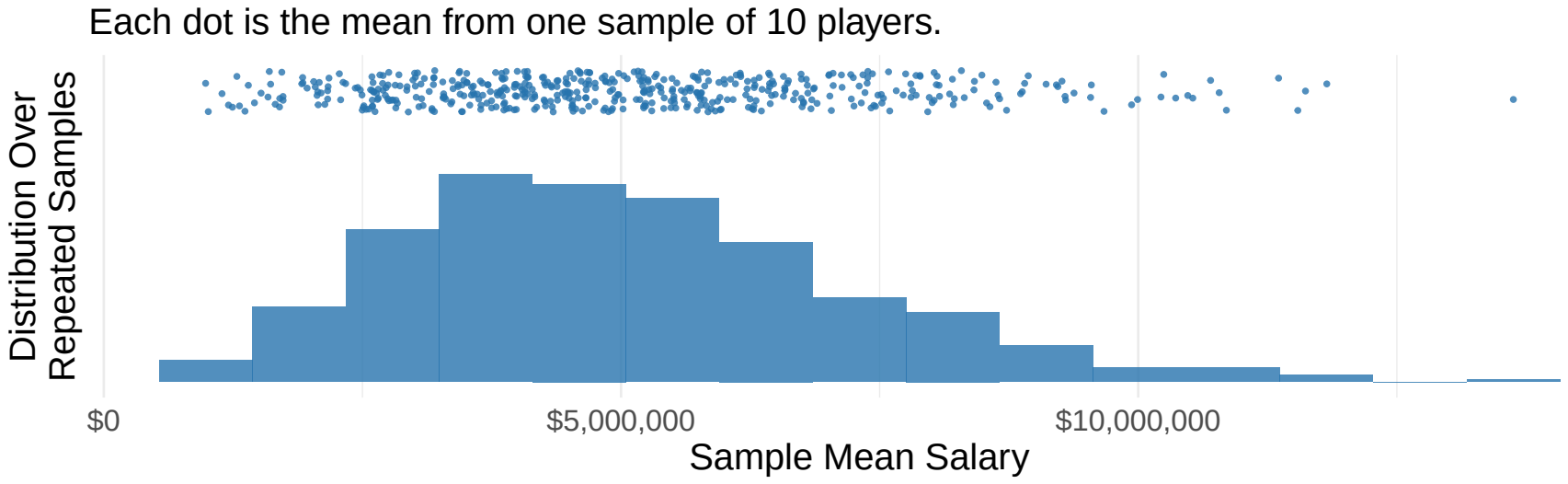
Many times



If you are following, these are in many_samples.csv.

```
many_samples <- read_csv("https://soc114.github.io/data/mar
```

Many times



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Because each sample produces a different estimate, there is a **distribution** of different estimates across repeated samples.

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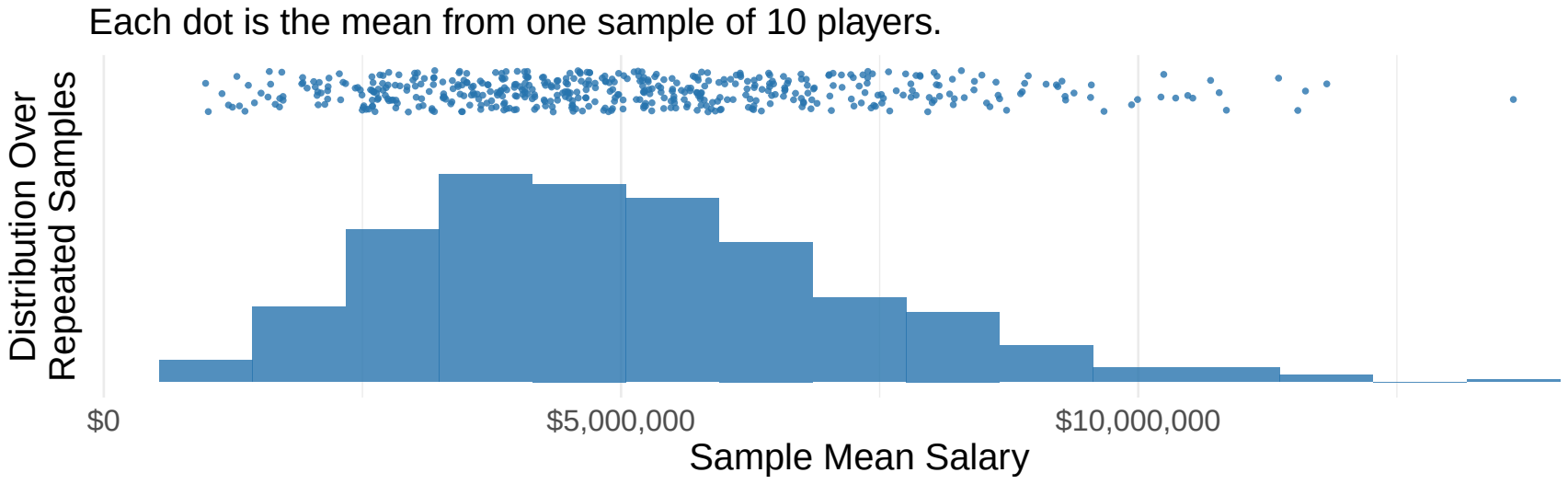


If you are following, these are in many_samples.csv.

```
many_samples <- read_csv("https://soc114.github.io/data/mar
```

Because each sample produces a different estimate, there is a **distribution** of different estimates across repeated samples.

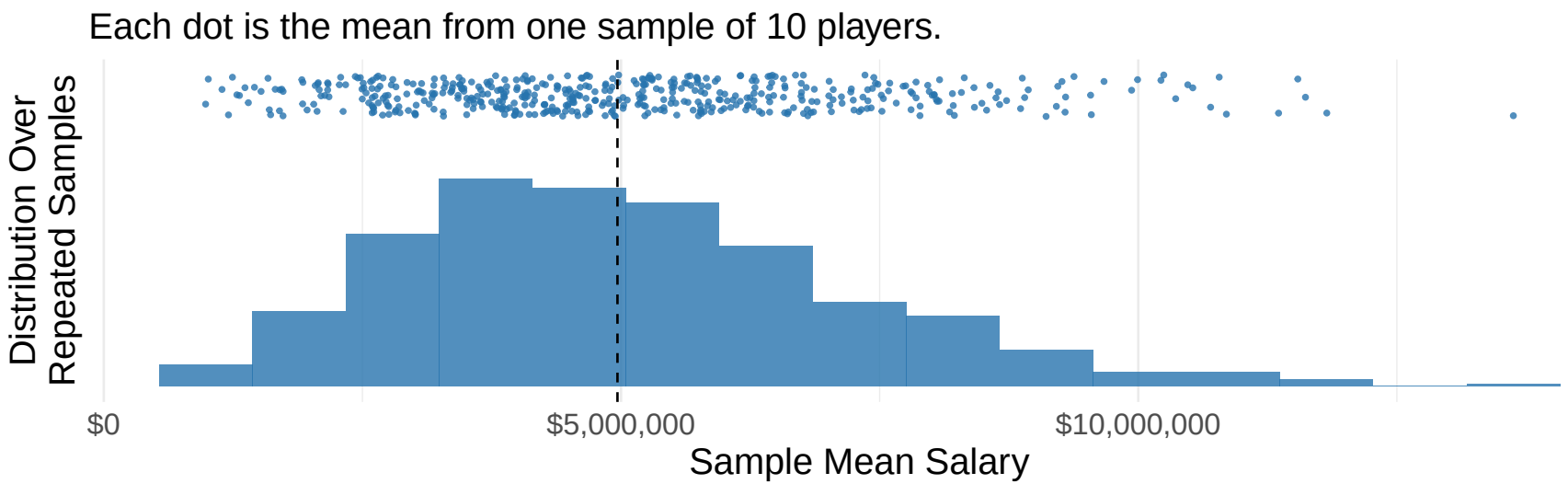
Can you propose a summary statistic for this distribution?

Mean of the distribution

Also called the **expected value**.

```
many_samples |>
  summarize(estimator_mean = mean(sample_estimate))
```

```
# A tibble: 1 x 1
  estimator_mean
           <dbl>
1       5077653.
```



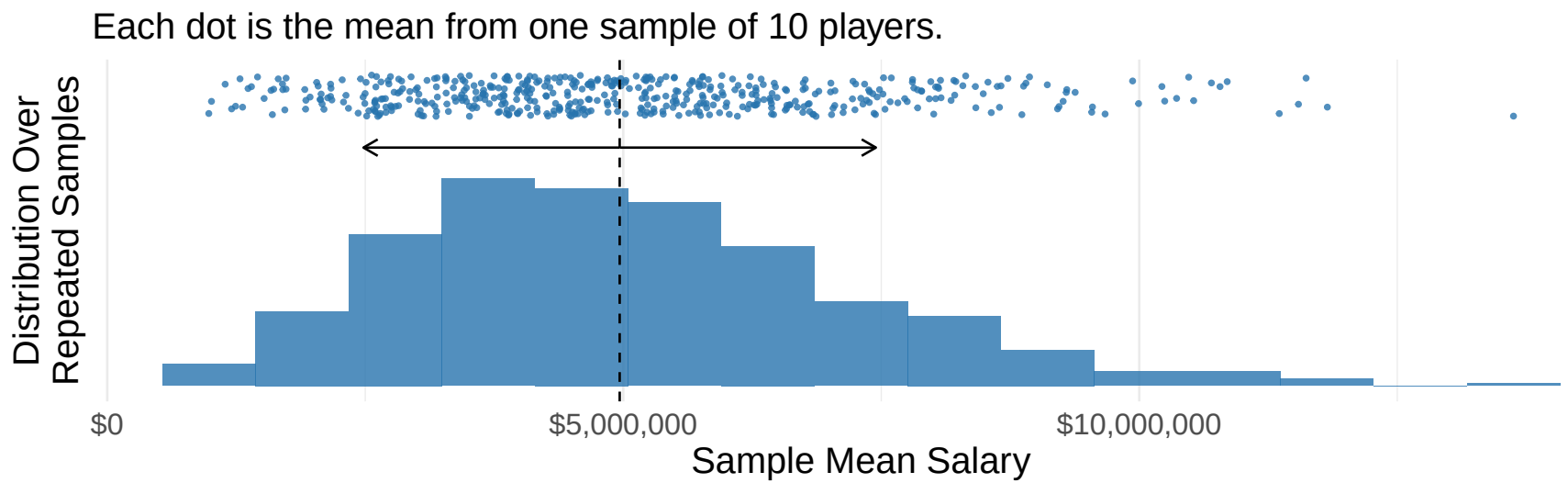
(In practice, the mean of the distribution is unknown)

Standard Error

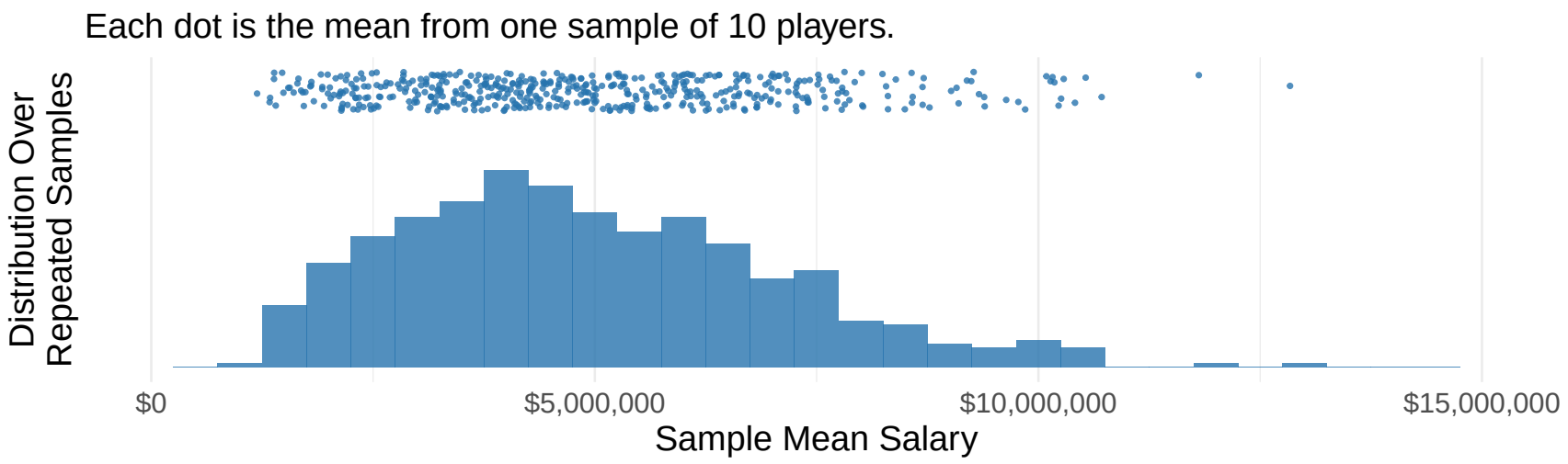
A measure of dispersion for the distribution of sample mean estimates.

```
many_samples |>
  summarize(standard_error = sd(sample_estimate))
```

```
# A tibble: 1 x 1
  standard_error
           <dbl>
1       2158282.
```



As the sample size grows



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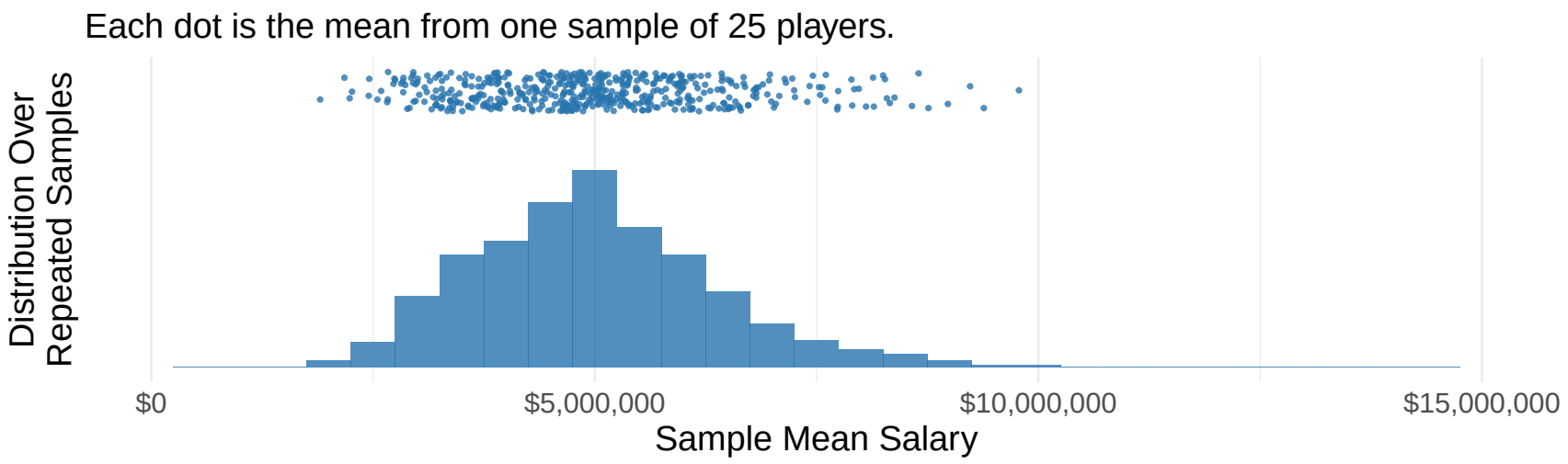

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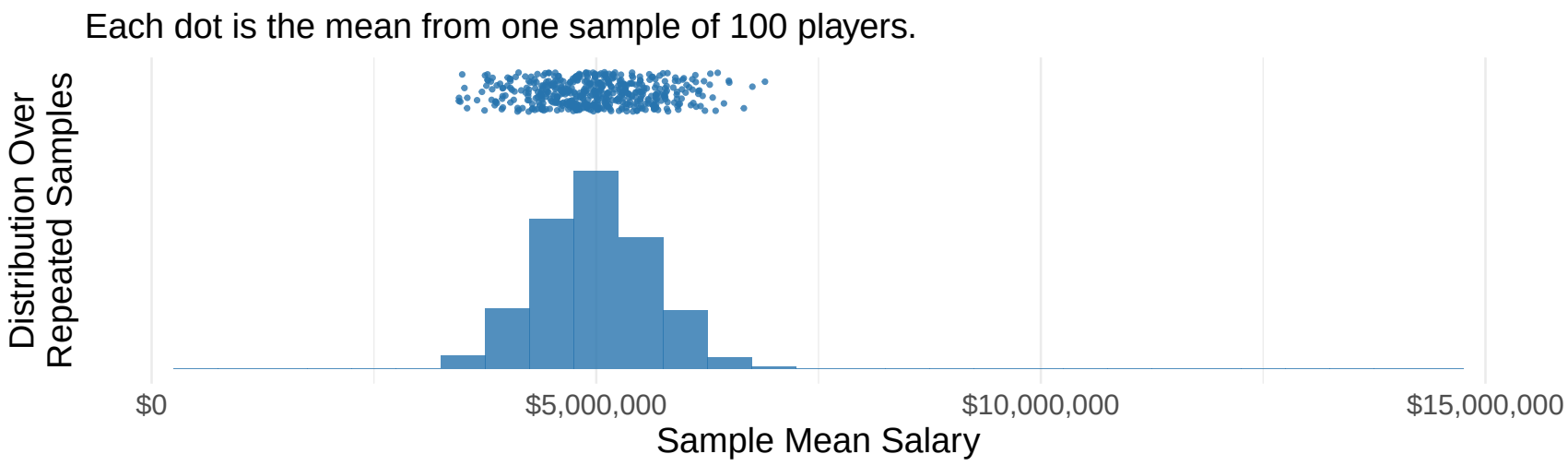
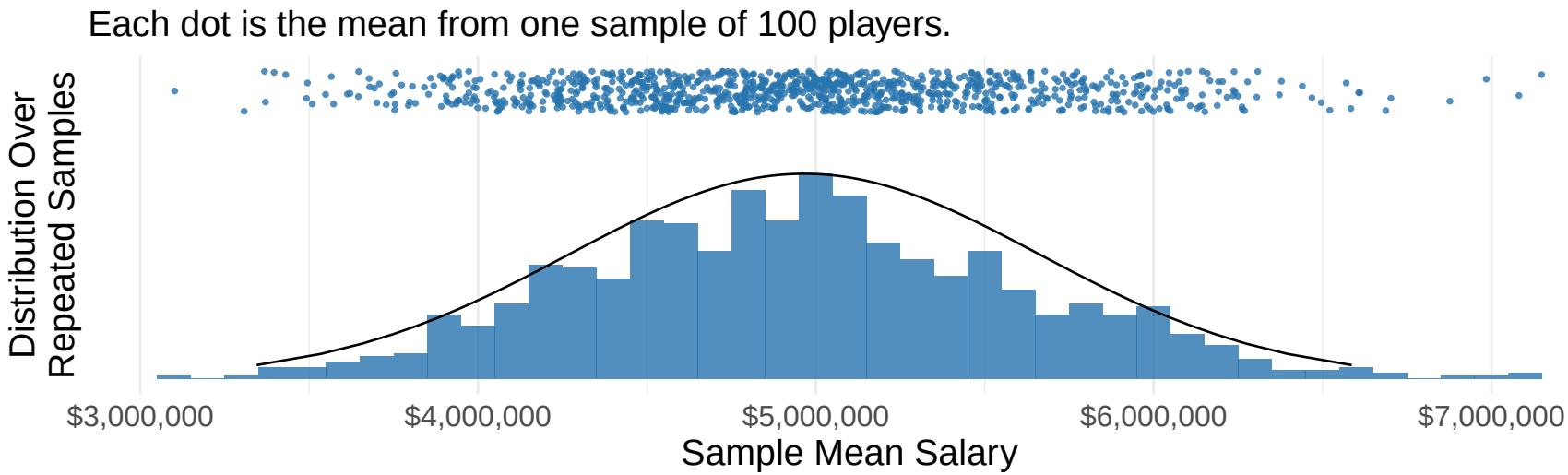


Asymptotic Normality

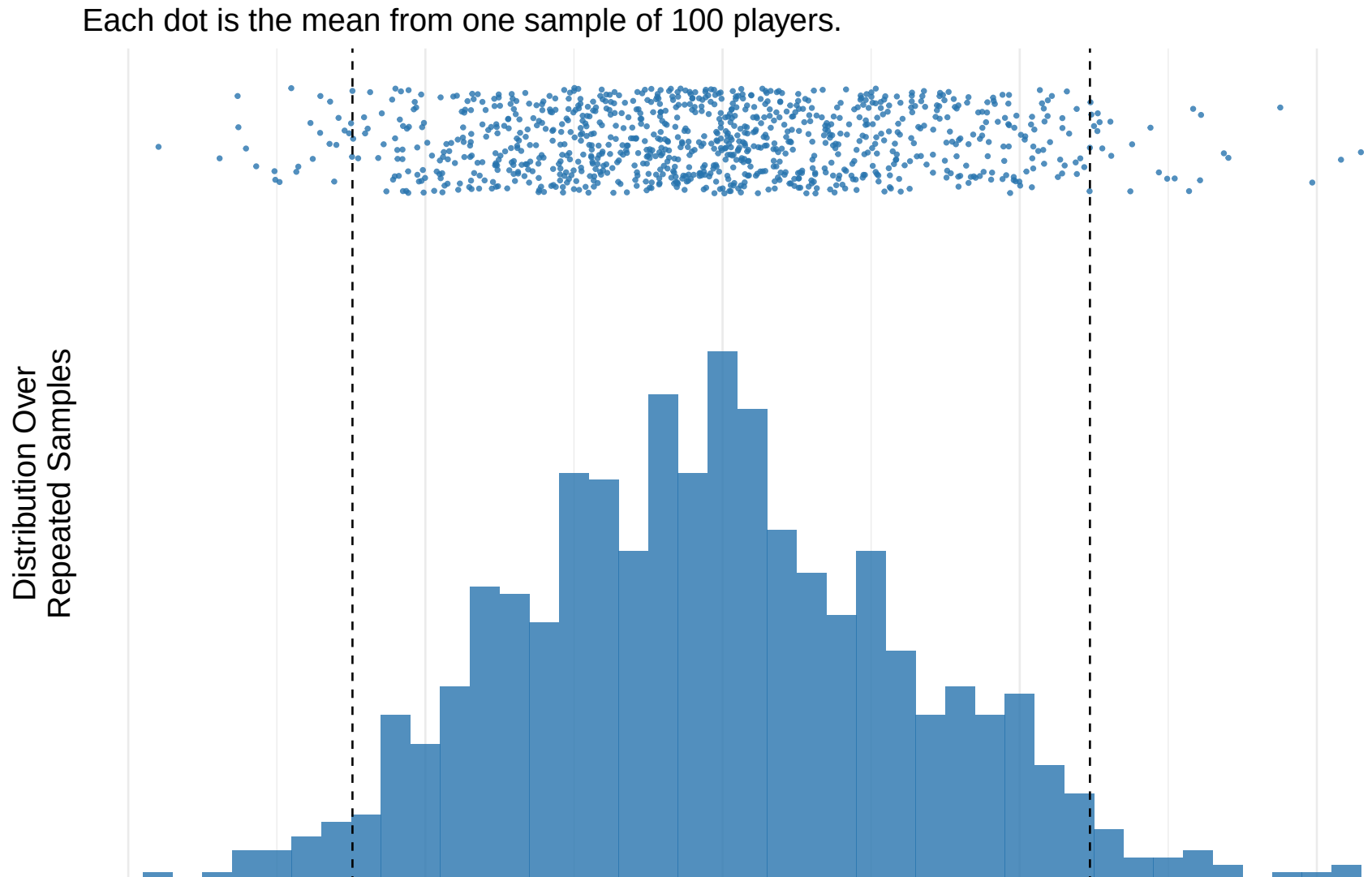
- As the sample size gets large (asymptotic)
- This becomes a Normal distribution



Middle 95% sampling interval

We might want to summarize:

- The mean of the estimator
- A range containing the middle 95% of sample estimates



Confidence interval via the bootstrap

What we want:

1. We would want many samples: `sample_1`, `sample_2`, `sample_3`,...
2. We estimate with each
3. We summarize the middle 95%

Confidence interval via the bootstrap

What we can do:

1. We get only one `sample`
   - So we simulate hypothetical `sample_sim_1`, `sample_sim_2`,...
2. We estimate with each
3. We summarize the middle 95%

How to generate bootstrap samples

Start with your one sample.

```
sampled_players <- baseball |>
  slice_sample(n = 100)
```

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```
sampled_players <- baseball |>
  slice_sample(n = 100)
```

Resample n players with replacement.

```
sampled_players_bootstrap <- sampled_players |>
  slice_sample(prop = 1, replace = TRUE)
```

How to generate bootstrap samples: Example

Here is a sample of 3 players:

```
a_small_sample <- baseball |>
  slice_sample(n = 3) |>
  print()
```

```
# A tibble: 3 x 3
  player         team          salary
  <chr>          <chr>          <dbl>
1 Bohm, Alec     Philadelphia 748000
2 Ginkel, Kevin  Arizona      746600
3 Bednar, David  Pittsburgh   745000
```

How to generate bootstrap samples: Example

Here is a bootstrap sample of those 3 players.

```
a_small_sample |>
  slice_sample(prop = 1, replace = TRUE) |>
  print()
```

```
# A tibble: 3 x 3
  player          team          salary
  <chr>           <chr>          <dbl>
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```

Coding concepts

We will analyze hundreds of bootstrap samples.

We need two coding concepts.

1. How to write an `estimator` function
2. How to write a `for` loop

How to write an `estimator` function

A function (like `mean`) takes an input and returns an output. You can write your own.

```
estimator <- function(data) {
  data |>
    summarize(estimate = mean(salary)) |>
    pull(estimate)
}
```

The function takes `data` and returns an estimate.

```
estimator(data = sampled_players)
```

```
[1] 5522424
```

How to write a `for` loop

Useful for tasks you will repeat.

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Useful for tasks you will repeat.

First, initialize a vector to hold results.

```
vector_for_results <- rep(NA, 3)
```

The `rep` function repeates the value `NA` 3 times.

How to write a `for` loop

Useful for tasks you will repeat.

First, initialize a vector to hold results.

```
vector_for_results <- rep(NA, 3)
```

The `rep` function repeates the value `NA` 3 times.

Second, loop through and fill your vector.

```
for (index in 1:3) {
  vector_for_results[index] <- index
}
```

Square brackets `[]` extract an element of a vector.

Analyze 500 bootstrap samples

Initialize a vector to hold the result.

```
bootstrap_estimates <- rep(NA, times = 500)
```

Analyze 500 bootstrap samples

Write a `for` loop that will repeat 500 times.

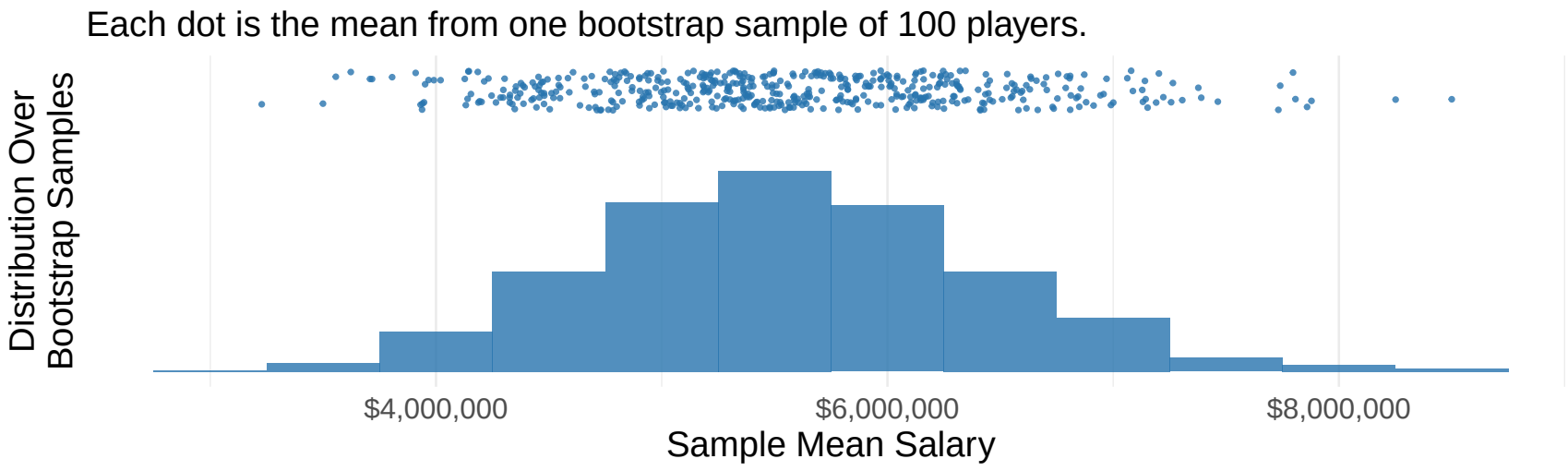
```
for (index in 1:500) {

  # Draw a bootstrap sample
  bootstrap_sample <- sampled_players |>
    slice_sample(prop = 1, replace = TRUE)

  # Construct an estimate
  estimate_this_index <- estimator(bootstrap_sample)

  # Store that estimate
  bootstrap_estimates[index] <- estimate_this_index
}
```

Bootstrap results



Bootstrap results: Summary statistics

Bootstrap estimate of the standard error.

```
sd(bootstrap_estimates)
```

```
[1] 864362.2
```

Middle 95% of bootstrap estimates

```
quantile(x = bootstrap_estimates, prob = c(.025, .975))
```

```
   2.5%   97.5%
3958991 7261108
```

Confidence interval

An interval from lower(sample) to upper(sample) with the property: across repeated samples, 95% of intervals constructed this way would contain the population parameter.

Confidence interval: Example

Middle 95% of bootstrap estimates is a confidence interval.

- The true population mean salary is $4,965,481
- Our sample mean is $5,522,424
- Our confidence interval is:

```
quantile(x = bootstrap_estimates, prob = c(.025, .975))
```

```
   2.5%   97.5%
3958991 7261108
```

Across repeated samples, 95% of intervals constructed this way will contain the population mean salary.

Recap

- Statistical concepts
- Coding concepts

Recap: Statistical concepts

Statistical concepts

- Sampling distribution
  - Cannot be directly observed. We have one sample.
- Standard error
  - Spread of the sampling distribution
- Confidence interval
  - Covers truth in 95% of samples
- Bootstrap
  - Method of constructing the CI with one sample

Recap: Coding concepts

- Writing a custom function (R4DS Ch 25)
- Writing a for loop (R4DS Ch 27.5)