Confidence Intervals
UCLA Soc 114

Concepts for today

Statistical concepts

▶ Sampling distribution
▶ Standard error
▶ Confidence interval
▶ Bootstrap

Coding concepts

▶ Writing a custom function
▶ Writing a for loop

Example: Mean salary of MLB players

Load data:

```
baseball <- read_csv("https://soc114.github.io/data/baseba
  # Keep only a few variables for simplicity
  select(player, team, salary)
```

```
# A tibble: 944 x 3
  player            team      salary
  <chr>             <chr>      <dbl>
1 Bumgarner, Madison Arizona 21882892
2 Marte, Ketel      Arizona 11600000
3 Ahmed, Nick       Arizona 10375000
# i 941 more rows
```

Example: Mean salary of MLB players

True mean in population of all players

Example: Mean salary of MLB players

True mean in population of all players

```
baseball |> summarize(population_mean = mean(salary))


# A tibble: 1 x 1
  population_mean
            <dbl>
1       4965481.
```

Estimate from a sample

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Draw a sample of 10 players.

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```
sampled_players <- baseball |>
  slice_sample(n = 10) |>
  print(n = 3)
```

```
# A tibble: 10 x 3
  player             team         salary
  <chr>              <chr>         <dbl>
1 Montgomery, Jordan St. Louis   10000000
2 Barnes, Matt       Miami        7500000
3 Eflin, Zach        Tampa Bay   11000000
# i 7 more rows
```

Estimate from a sample

Take the mean among sampled players.
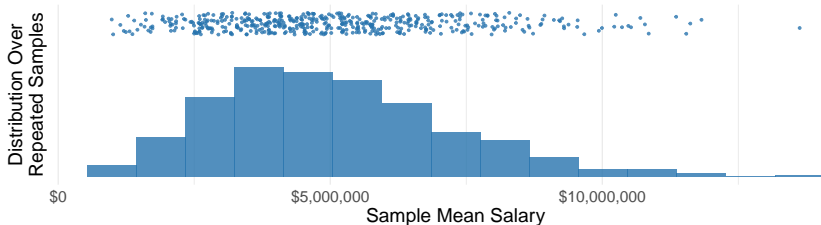
Estimate from a sample

Take the mean among sampled players.

```
sampled_players <- sampled_players |>
  summarize(sample_estimate = mean(salary)) |>
  print()


# A tibble: 1 x 1
  sample_estimate
            <dbl>
1         3435960
```
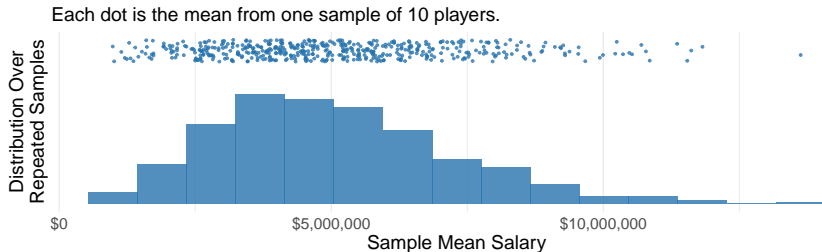
Many times

Each dot is the mean from one sample of 10 players.

If you are following, these are in many_samples.csv.

```
many_samples <- read_csv("https://soc114.github.io/data/man
```
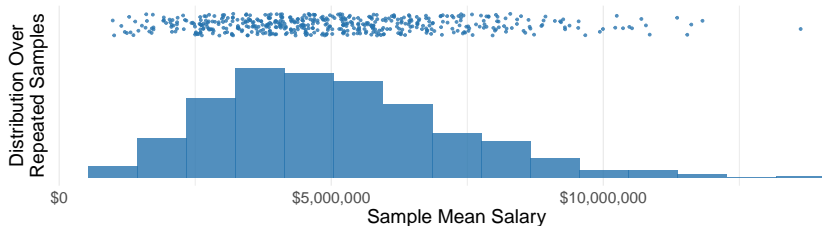
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Because each sample produces a different estimate, there is a **distribution** of different estimates across repeated samples.

Many times

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If you are following, these are in many_samples.csv.

```
many_samples <- read_csv("https://soc114.github.io/data/man
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Because each sample produces a different estimate, there is a **distribution** of different estimates across repeated samples.

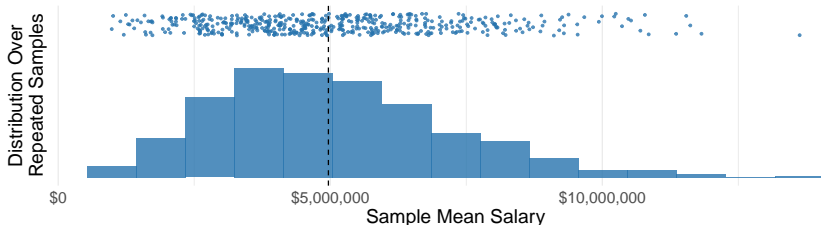Can you propose a summary statistic for this distribution?

Mean of the distribution

Also called the **expected value**.

```
many_samples |>
  summarize(estimator_mean = mean(sample_estimate))
```

```
# A tibble: 1 x 1
  estimator_mean
           <dbl>
1       5077653.
```

Each dot is the mean from one sample of 10 players.

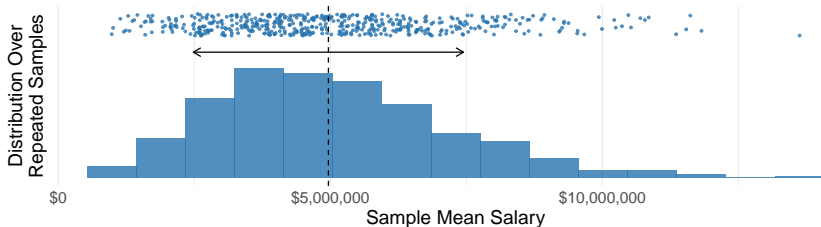(In practice, the mean of the distribution is unknown)

Standard Error

A measure of dispersion for the distribution of sample mean estimates.
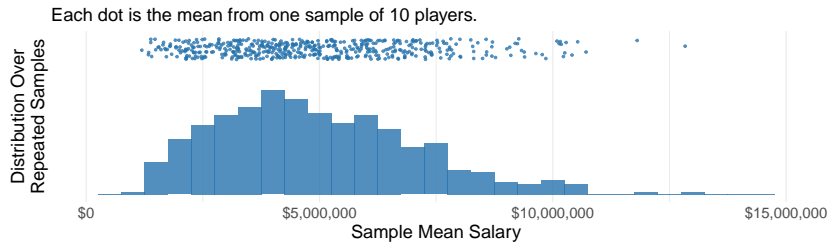
```
many_samples |>
  summarize(standard_error = sd(sample_estimate))
```

```
# A tibble: 1 x 1
  standard_error
           <dbl>
1       2158282.
```

Each dot is the mean from one sample of 10 players.

As the sample size grows

Each dot is the mean from one sample of 10 players.

Distribution Over Repeated Samples

$0 $5,000,000 $10,000,000 $15,000,000

Sample Mean Salary

As the sample size grows

Each dot is the mean from one sample of 25 players.

As the sample size grows

Each dot is the mean from one sample of 100 players.

Distribution Over Repeated Samples

$0 $5,000,000 $10,000,000 $15,000,000

Sample Mean Salary

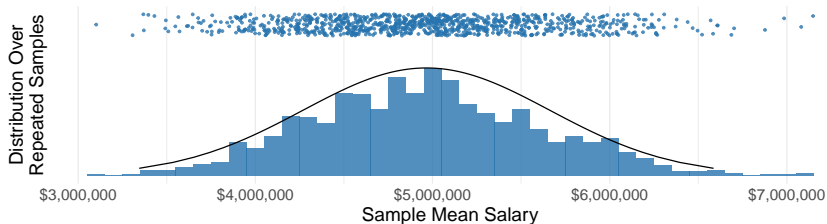# Asymptotic Normality

▶ As the sample size gets large (asymptotic)
▶ This becomes a Normal distribution

Each dot is the mean from one sample of 100 players.

Middle 95% sampling interval
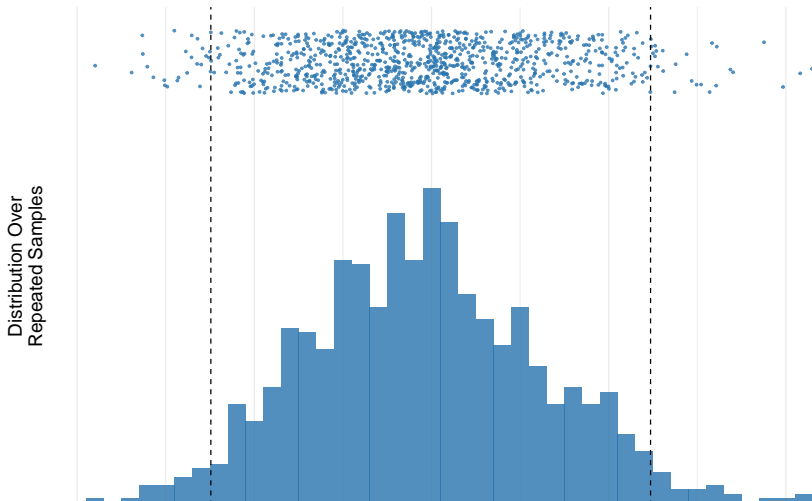
We might want to summarize:

▶ The mean of the estimator
▶ A range containing the middle 95% of sample estimates

Each dot is the mean from one sample of 100 players.

Distribution Over
Repeated Samples

Confidence interval via the bootstrap

What we want:

1. We would want many samples: `sample_1`, `sample_2`, `sample_3`,…
2. We estimate with each
3. We summarize the middle 95%

Confidence interval via the bootstrap

What we can do:

1. We get only one `sample`
   - So we simulate hypothetical `sample_sim_1`, `sample_sim_2`,…
2. We estimate with each
3. We summarize the middle 95%

How to generate bootstrap samples

Start with your one sample.

```
sampled_players <- baseball |>
  slice_sample(n = 100)
```

How to generate bootstrap samples

Start with your one sample.

```r
sampled_players <- baseball |>
  slice_sample(n = 100)
```

Resample n players with replacement.

```r
sampled_players_bootstrap <- sampled_players |>
  slice_sample(prop = 1, replace = TRUE)
```

How to generate bootstrap samples: Example

Here is a sample of 3 players:

```
a_small_sample <- baseball |>
  slice_sample(n = 3) |>
  print()
```

```
# A tibble: 3 x 3
  player         team          salary
  <chr>          <chr>          <dbl>
1 Bohm, Alec     Philadelphia  748000
2 Ginkel, Kevin  Arizona       746600
3 Bednar, David  Pittsburgh    745000
```

How to generate bootstrap samples: Example

Here is a bootstrap sample of those 3 players.

```
a_small_sample |>
  slice_sample(prop = 1, replace = TRUE) |>
  print()
```

```
# A tibble: 3 x 3
  player        team         salary
  <chr>         <chr>         <dbl>
1 Bednar, David Pittsburgh    745000
2 Bohm, Alec    Philadelphia  748000
3 Bednar, David Pittsburgh    745000
```

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  player        team       salary
  <chr>         <chr>       <dbl>
1 Bednar, David Pittsburgh 745000
2 Bednar, David Pittsburgh 745000
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```

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Here is a bootstrap sample of those 3 players.

```
a_small_sample |>
  slice_sample(prop = 1, replace = TRUE) |>
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```
# A tibble: 3 x 3
  player         team         salary
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1 Bohm, Alec     Philadelphia 748000
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3 Ginkel, Kevin  Arizona      746600
```

Coding concepts

We will analyze hundreds of bootstrap samples.

We need two coding concepts.

1. How to write an `estimator` function
2. How to write a `for` loop

How to write an `estimator` function

A function (like `mean`) takes an input and returns an output. You can write your own.

```
estimator <- function(data) {
  data |>
    summarize(estimate = mean(salary)) |>
    pull(estimate)
}
```

The function takes `data` and returns an estimate.

```
estimator(data = sampled_players)
```

```
[1] 5522424
```

How to write a `for` loop

Useful for tasks you will repeat.

How to write a `for` loop

Useful for tasks you will repeat.

First, initialize a vector to hold results.

```
vector_for_results <- rep(NA, 3)
```

The `rep` function repeates the value `NA` 3 times.

How to write a `for` loop

Useful for tasks you will repeat.

First, initialize a vector to hold results.

```
vector_for_results <- rep(NA, 3)
```

The `rep` function repeats the value `NA` 3 times.

Second, loop through and fill your vector.

```
for (index in 1:3) {
  vector_for_results[index] <- index
}
```

Square brackets [] extract an element of a vector.

Analyze 500 bootstrap samples

Initialize a vector to hold the result.

```r
bootstrap_estimates <- rep(NA, times = 500)
```
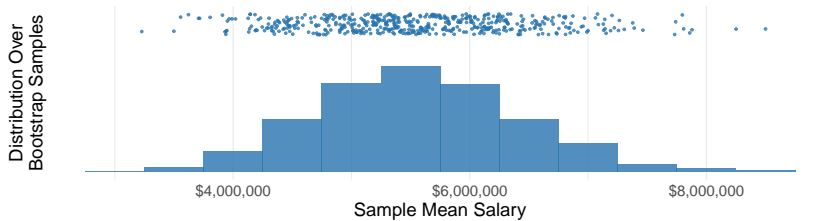
Analyze 500 bootstrap samples

Write a `for` loop that will repeat 500 times.

```r
for (index in 1:500) {

  # Draw a bootstrap sample
  bootstrap_sample <- sampled_players |>
    slice_sample(prop = 1, replace = TRUE)

  # Construct an estimate
  estimate_this_index <- estimator(bootstrap_sample)

  # Store that estimate
  bootstrap_estimates[index] <- estimate_this_index
}
```

Bootstrap results

Each dot is the mean from one bootstrap sample of 100 players.

Distribution Over Bootstrap Samples

$4,000,000 $6,000,000 $8,000,000

Sample Mean Salary

Bootstrap results: Summary statistics

Bootstrap estimate of the standard error.

```
sd(bootstrap_estimates)
```

```
[1] 864362.2
```

Middle 95% of bootstrap estimates

```
quantile(x = bootstrap_estimates, prob = c(.025, .975))
```

```
    2.5%    97.5%
3958991 7261108
```

Confidence interval

An interval from lower(sample) to upper(sample) with the property: across repeated samples, 95% of intervals constructed this way would contain the population parameter.

Confidence interval: Example

Middle 95% of bootstrap estimates is a confidence interval.

- ▶ The true population mean salary is $4,965,481
- ▶ Our sample mean is $5,522,424
- ▶ Our confidence interval is:

```
quantile(x = bootstrap_estimates, prob = c(.025, .975))
```

```
   2.5%    97.5%
3958991 7261108
```

Across repeated samples, 95% of intervals constructed this way will contain the population mean salary.

Recap

▶ Statistical concepts
▶ Coding concepts

Recap: Statistical concepts

Statistical concepts

▶ Sampling distribution
  ▶ Cannot be directly observed. We have one sample.
▶ Standard error
  ▶ Spread of the sampling distribution
▶ Confidence interval
  ▶ Covers truth in 95% of samples
▶ Bootstrap
  ▶ Method of constructing the CI with one sample

Recap: Coding concepts

▶ Writing a custom function (R4DS Ch 25)
▶ Writing a for loop (R4DS Ch 27.5)