Model-based causal estimation: From outcome models to treatment weighting

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Learning goals for today

At the end of class, you will be able to

- estimate average causal effects with a parametric model
  - for the outcome $\mathrm{E}(Y \mid A, \vec{L})$
  - for the treatment $\mathrm{P}(A \mid \vec{L})$

Optional reading:

- Hernán and Robins 2020 Chapter 12.1–12.5, 13, 15.1

Nonparametric estimation

Causal assumptions

$$\begin{tikzcd} \vec{L} \arrow[r] \arrow[rr, bend left] & A \arrow[r] & Y \end{tikzcd}$$

Nonparametric estimator

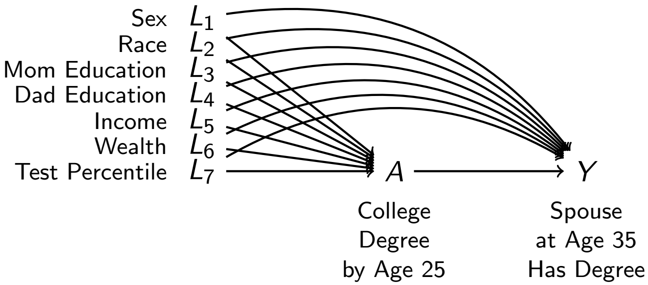
$$\hat{\mathrm{E}}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} \hat{\mathrm{E}}(Y \mid \vec{L} = \vec{\ell}_i, A = a)$$

For every unit i,

- find units who look like them on confounders $\vec{L}$
- who actually got treatment $A = a$
- take the average among those units

Then average over all units

Nonparametric estimation breaks down



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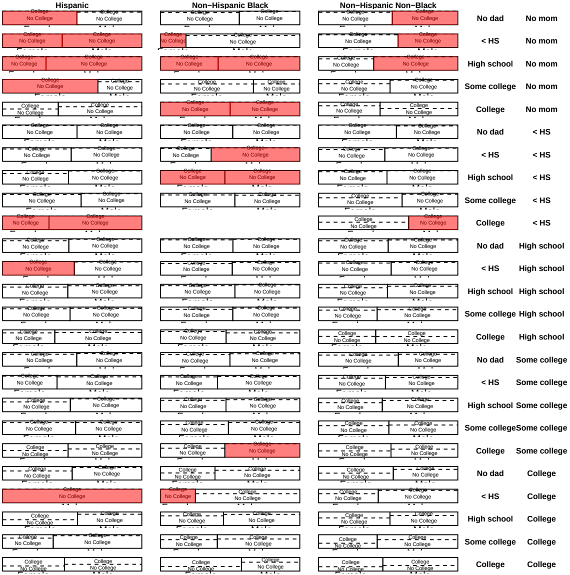

Parametric estimation: Outcome model

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Where $\hat{E}$ is a model-based prediction

$$\hat{\mathrm{E}}(Y \mid \vec{L}, A) = \hat{\alpha} + \vec{L}'\hat{\vec{\gamma}} + A\hat{\beta}$$

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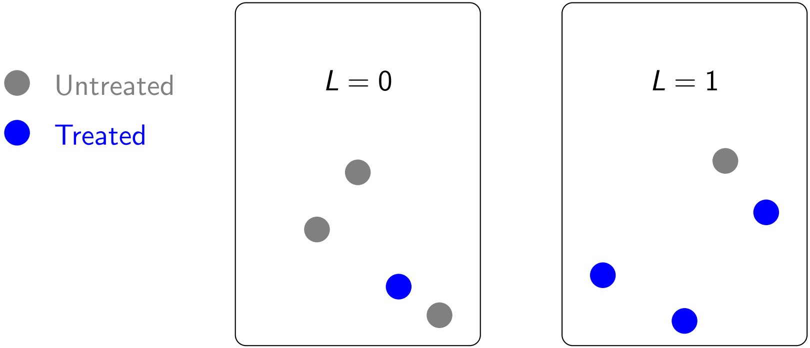
Inverse probability of treatment weighting



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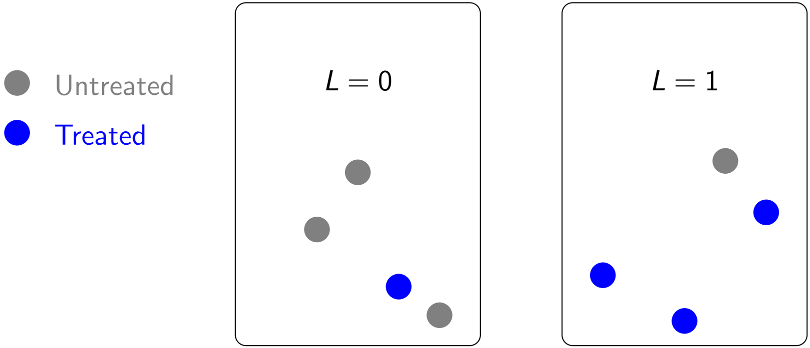

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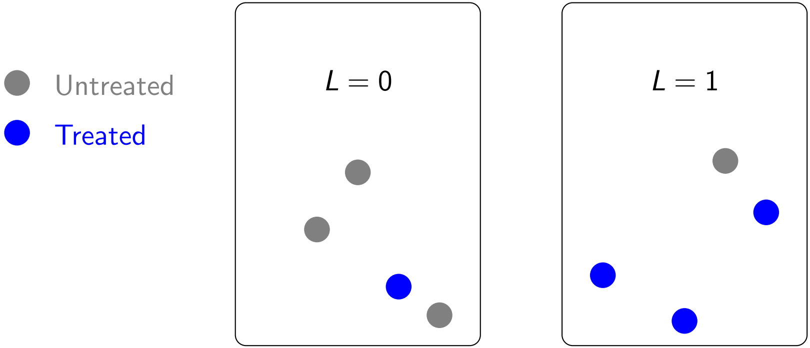

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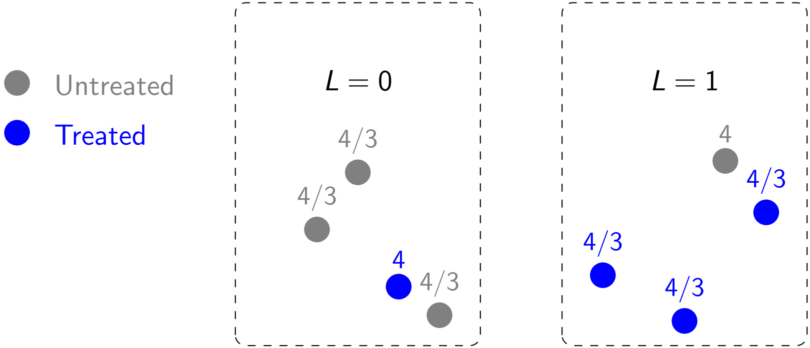

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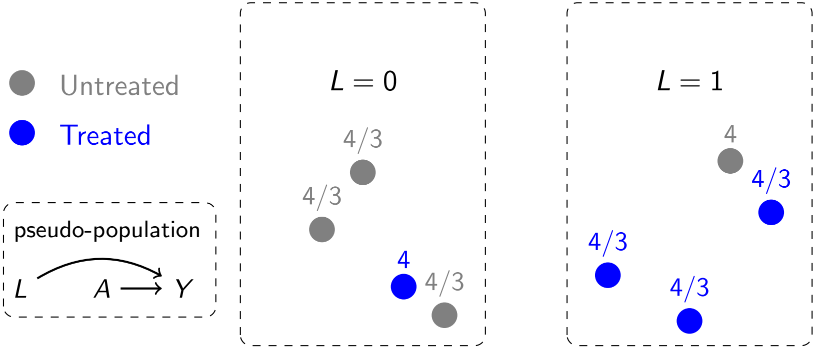

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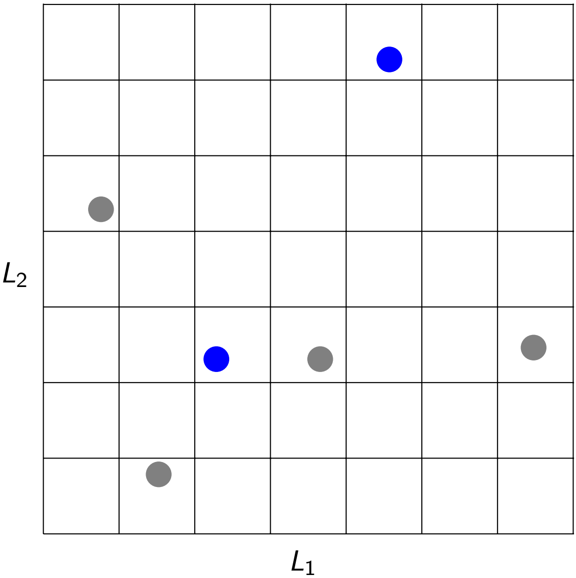
L_2
L_1

Model the treatment assignment

$$\hat{\text{P}}(A = 1 \mid \vec{L}) = \text{logit}^{-1}\left(\hat{\alpha} + \hat{\vec{\gamma}}\vec{L}\right)$$

Predict the propensity score for each unit

$$\hat{\pi}_i = \begin{cases} \text{logit}^{-1}\left(\hat{\alpha} + \hat{\vec{\gamma}}\vec{L}\right) & \text{if } A_i = 1 \\ 1 - \text{logit}^{-1}\left(\hat{\alpha} + \hat{\vec{\gamma}}\vec{L}\right) & \text{if } A_i = 0 \end{cases}$$

Estimate by inverse probability weighting

$$\hat{\text{E}}(Y^a) = \frac{1}{N}\sum_{i:A_i=a}\frac{Y_i}{\hat{\pi}_i}$$

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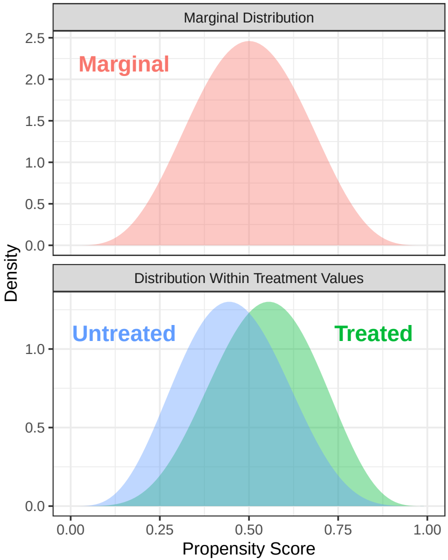
Both solutions accept bias in order to reduce variance

Accepting bias to reduce variance: Trimming



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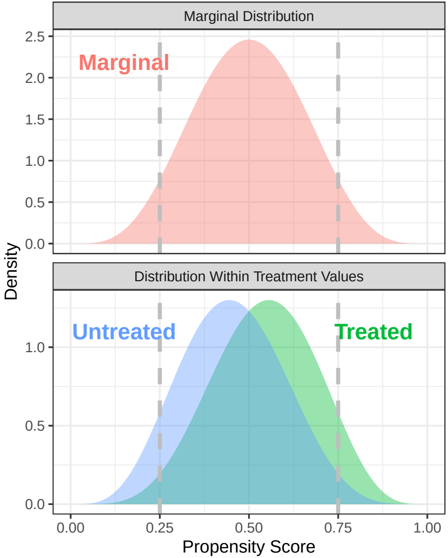

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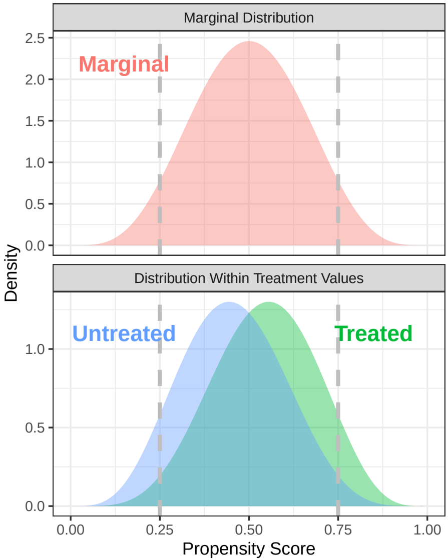

Drop units with extreme weights

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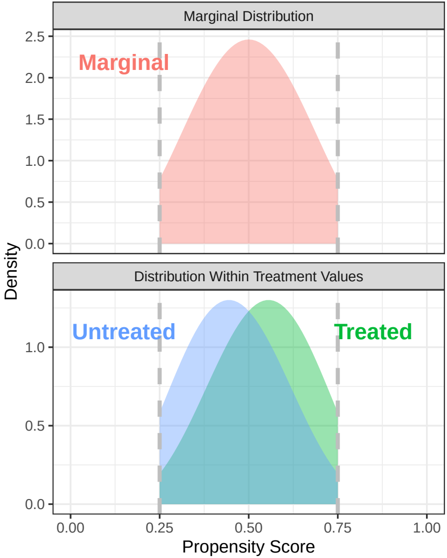

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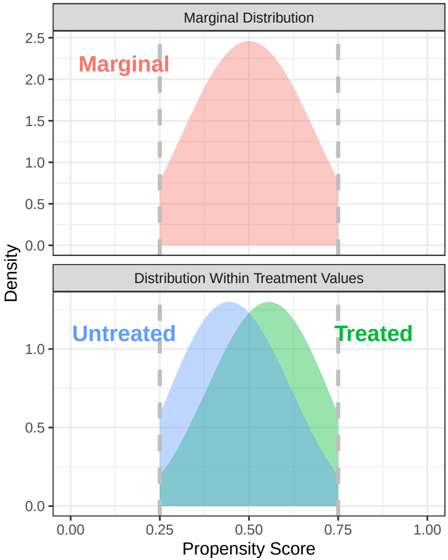
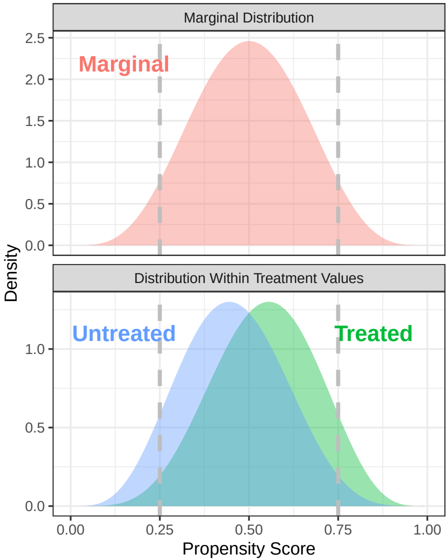


Drop units with extreme weights

Changes target population — Biased for full population

Accepting bias to reduce variance: Weight truncation



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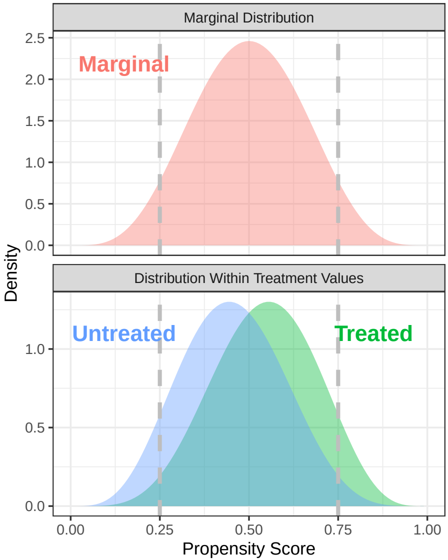

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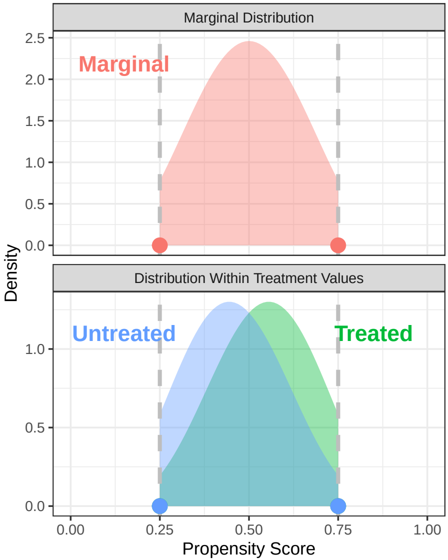
Accepting bias to reduce variance: Weight truncation



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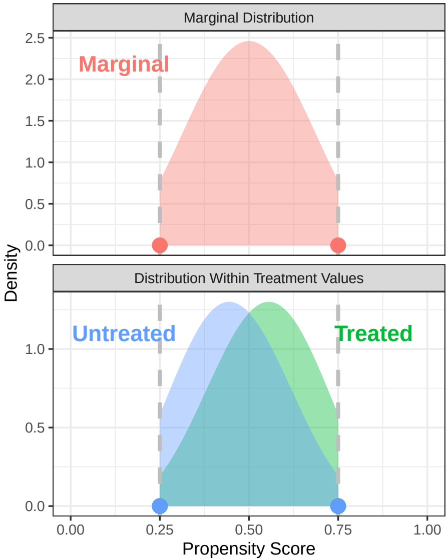

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Biased: Ignores some confounding

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