Directed Acyclic Graphs

Sociol 114

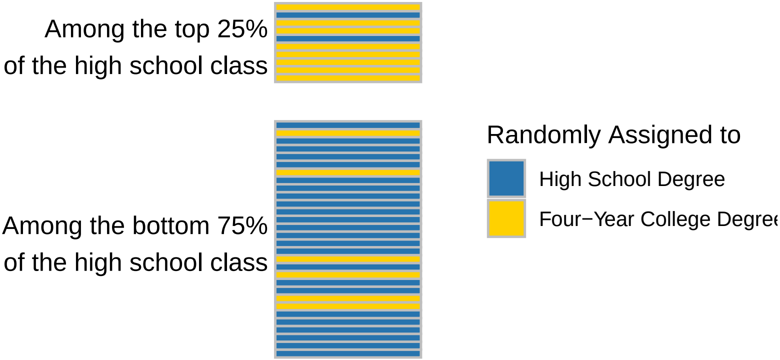
6 Feb 2025

Learning goals for today

At the end of class, you will be able to:

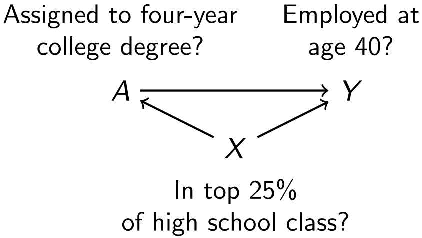
1. Read a Directed Acyclic Graph
2. Recognize causal paths
3. Understand two key structures
   - Fork structures ($\bullet \leftarrow \bullet \rightarrow \bullet$)
   - Collider structures ($\bullet \rightarrow \bullet \leftarrow \bullet$)
4. List all paths in a DAG
5. Determine which paths are blocked under a particular adjustment set
6. Select a sufficient adjustment set to isolate causal paths

A hypothetical experiment: Conditional randomization



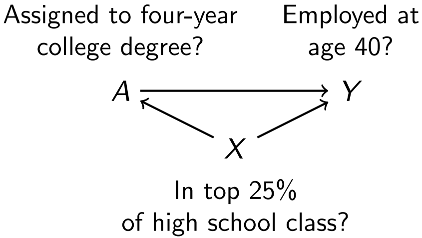
Outcome: Employed at age 40

Elements of a Directed Acyclic Graph (DAG)



- **Nodes** (X, A, Y) are random variables
- **Edges** $(\rightarrow)$ are causal relationships.
  - X has a causal effect on A
  - X has a causal effect on Y
  - A has a causal effect on Y

Elements of a Directed Acyclic Graph (DAG)



A **path** is a sequence of edges connecting two nodes.

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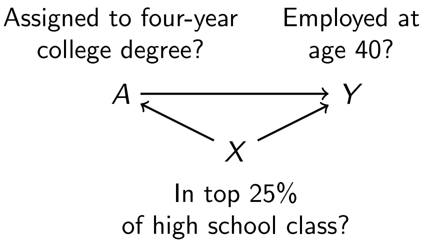

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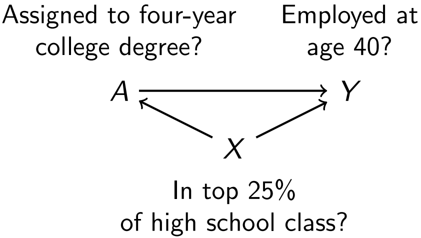
Between A and Y, what are the two paths?

Elements of a Directed Acyclic Graph (DAG)



A **path** is a sequence of edges connecting two nodes.

Between A and Y, what are the two paths?

- $A \rightarrow Y$
- $A \leftarrow X \rightarrow Y$

Causal path

$\bullet \rightarrow \bullet \rightarrow \bullet$

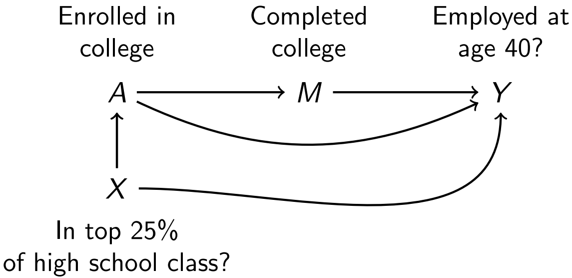
A path in which arrows point in the same direction

Causal path

$\bullet \rightarrow \bullet \rightarrow \bullet$

A path in which arrows point in the same direction

What three paths between A and Y?
Which two are causal paths?



Causal path

$\bullet \to \bullet \to \bullet$

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What three paths between A and Y?
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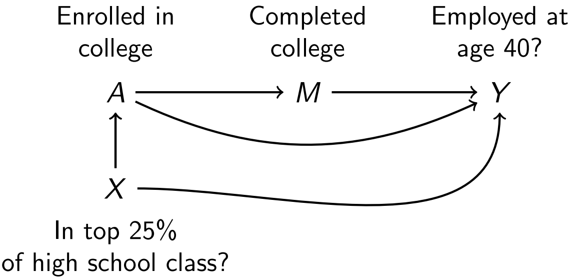

$A \to Y$
$A \to M \to Y$
$A \leftarrow X \to Y$

Causal path

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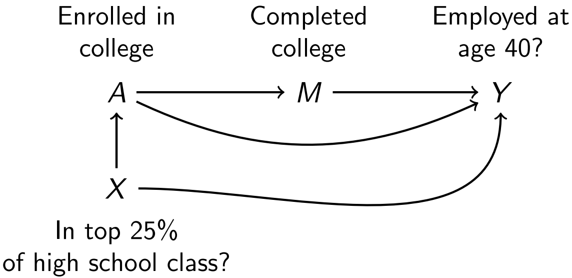

$A \to Y$ causal path

$A \to M \to Y$

$A \leftarrow X \to Y$

Causal path

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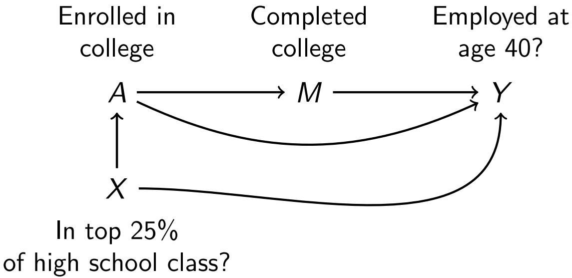

$A \to Y$ causal path
$A \to M \to Y$ causal path
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Causal path

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What three paths between A and Y?
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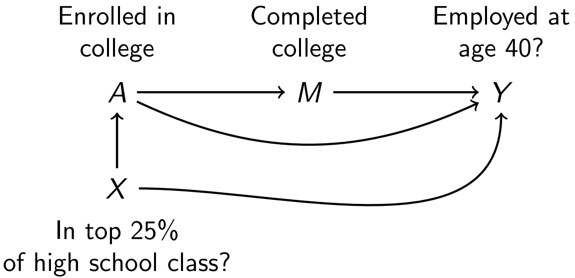

$A \to Y$	causal path
$A \to M \to Y$	causal path
$A \leftarrow X \to Y$	not a causal path

Causal path: Marginal dependence

$\bullet \to \bullet \to \bullet$

A causal path $A \to \cdots \to B$ will make the variables A and B statistically dependent

Example:

(visits grocery store) $\to$ (buys ice cream) $\to$ (eats ice cream)

Causal path: Marginal dependence

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A causal path $A \to \cdots \to B$ will make the variables A and B statistically dependent

Example:

$$\text{(visits grocery store)} \to \text{(buys ice cream)} \to \text{(eats ice cream)}$$

What if we condition:
filter to those with (buys ice cream $=$ `FALSE`)?

Causal path: Conditional independence

$\bullet \to \bullet \to \bullet$

A causal path $A \to \cdots \to B$ will not make the variables A and B statistically dependent if we condition on a variable along the path

Example:

$$\text{(visits grocery store)} \to \boxed{\text{(buys ice cream)}} \to \text{(eats ice cream)}$$

Causal path: Conditional independence

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A causal path $A \to \cdots \to B$ will not make the variables A and B statistically dependent if we condition on a variable along the path

Example:

$$\text{(visits grocery store)} \to \boxed{\text{(buys ice cream)}} \to \text{(eats ice cream)}$$

Among people who didn't buy ice cream today,
those who went to the store and didn't
are equally likely to be eating ice cream.

Causal path: Conditional independence

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Example:

$$\text{(visits grocery store)} \rightarrow \boxed{\text{(buys ice cream)}} \rightarrow \text{(eats ice cream)}$$

Among people who didn't buy ice cream today,
those who went to the store and didn't
are equally likely to be eating ice cream.

Conditioning on (buys ice cream $=$ `FALSE`) **blocks** this path.

Fork structure

$\bullet \leftarrow \bullet \rightarrow \bullet$

A sequence of edges within a path in which two variables are both caused by a third variable: $A \leftarrow C \rightarrow B$

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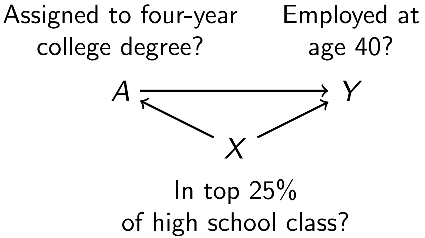
In our initial graph, what path contains a fork structure?



Recall that there are two paths:

1. $A \rightarrow Y$
2. $A \leftarrow X \rightarrow Y$

Fork structure

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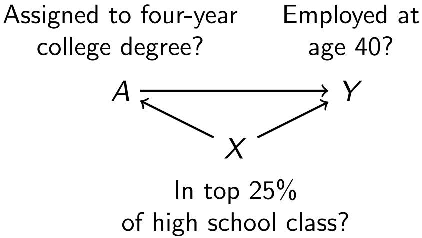

Recall that there are two paths:

1. $A \rightarrow Y$
2. $A \leftarrow X \rightarrow Y$ (this path contains a fork structure)

Fork structure: Marginal dependence

$\bullet \leftarrow \bullet \rightarrow \bullet$

A fork structure $A \leftarrow C \rightarrow B$ will make A and B statistically dependent (because C causes both).

Example:

(completed college) $\leftarrow$ (top 25% of high school) $\rightarrow$ (employed at 40)

Fork structure: Marginal dependence

$\bullet \leftarrow \bullet \rightarrow \bullet$

A fork structure $A \leftarrow C \rightarrow B$ will make A and B statistically dependent (because C causes both).

Example:

(lifeguard rescues) $\leftarrow$ (temperature) $\rightarrow$ (ice cream sales)

Fork structure: Marginal dependence

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A fork structure $A \leftarrow C \rightarrow B$ will make A and B statistically dependent (because C causes both).

Example:

$$\text{(lifeguard rescues)} \leftarrow \text{(temperature)} \rightarrow \text{(ice cream sales)}$$

On days with many lifeguard rescues,
there are also many ice cream sales.
Warm temperature causes both.

Fork structure: Marginal dependence

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Example:

(lifeguard rescues) $\leftarrow$ (temperature) $\rightarrow$ (ice cream sales)

On days with many lifeguard rescues,
there are also many ice cream sales.
Warm temperature causes both.

What if we look only at days with a given temperature?

Fork structure: Conditional independence

$\bullet \leftarrow \bullet \rightarrow \bullet$

A fork structure $A \leftarrow \boxed{C} \rightarrow B$ does not make A and B statistically dependent if we condition on C.

Example:

$$\text{(lifeguard rescues)} \leftarrow \boxed{\text{(temperature)}} \rightarrow \text{(ice cream sales)}$$

Among days with a given temperature,
lifeguard rescues and ice cream sales are unrelated.

Conditioning on (temperature) blocks this path.

Collider structure

$\bullet \rightarrow \bullet \leftarrow \bullet$

A sequence of edges within a path in which two variables both cause a third variable: $A \rightarrow C \leftarrow B$

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Example:

- sprinklers on a timer
- rain on random days
- either one can make the grass wet

$$\text{(sprinklers on)} \rightarrow \text{(grass wet)} \leftarrow \text{(raining)}$$

Collider structure

$\bullet \rightarrow \bullet \leftarrow \bullet$

A sequence of edges within a path in which two variables both cause a third variable: $A \rightarrow C \leftarrow B$

Example:

- sprinklers on a timer
- rain on random days
- either one can make the grass wet

$$\text{(sprinklers on)} \rightarrow \text{(grass wet)} \leftarrow \text{(raining)}$$

Are (sprinklers on) and (raining) statistically related?

Collider structure: Marginal independence

$\bullet \rightarrow \bullet \leftarrow \bullet$

In a collider structure $A \rightarrow C \leftarrow B$,
A and B are marginally independent.

$$\text{(sprinklers on)} \rightarrow \text{(grass wet)} \leftarrow \text{(raining)}$$

Knowing (sprinklers on $=$ `TRUE`) tells me nothing about whether (raining $=$ `TRUE`)

Collider structure: Marginal independence

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In a collider structure $A \rightarrow C \leftarrow B$,
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$$\text{(sprinklers on)} \rightarrow \text{(grass wet)} \leftarrow \text{(raining)}$$

Knowing (sprinklers on $=$ `TRUE`) tells me nothing about whether (raining $=$ `TRUE`)

What if I condition: look only at days when the grass is wet?

Collider structure: Conditional dependence

$\bullet \rightarrow \bullet \leftarrow \bullet$

$$\text{(sprinklers on)} \rightarrow \boxed{\text{(grass wet)}} \leftarrow \text{(raining)}$$

Collider structure: Conditional dependence

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$$\text{(sprinklers on)} \rightarrow \boxed{\text{(grass wet)}} \leftarrow \text{(raining)}$$

Among days when (grass wet = `TRUE`),
if (sprinklers on = `FALSE`)
then it must be (raining = `TRUE`)

(grass had to get wet somehow!)

Collider structure: Conditional dependence

$\bullet \to \bullet \leftarrow \bullet$

$$\text{(sprinklers on)} \to \boxed{\text{(grass wet)}} \leftarrow \text{(raining)}$$

Among days when (grass wet $=$ TRUE),
if (sprinklers on $=$ FALSE)
then it must be (raining $=$ TRUE)

(grass had to get wet somehow!)

In a collider structure $A \to \boxed{C} \leftarrow B$,
A and B are conditionally dependent.

Three structures

Name	Structure	A and B marginally dependent?	A and B conditionally dependent given C?
Causal path	$A \rightarrow C \rightarrow B$	Yes	No
Fork	$A \leftarrow C \rightarrow B$	Yes	No
Collider	$A \rightarrow C \leftarrow B$	No	Yes

A path can involve forks, colliders, and causal paths

(timer displays clock) $\leftarrow$ (timer works) $\rightarrow$ (sprinklers on) $\rightarrow$ (grass wet) $\leftarrow$ (raining)

A path can involve forks, colliders, and causal paths

(timer displays clock) $\leftarrow$ (timer works) $\rightarrow$ (sprinklers on) $\rightarrow$ (grass wet) $\leftarrow$ (raining)

(timer displays clock) is statistically related to which variables?

timer works
sprinklers on
grass wet
raining

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timer works	yes
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grass wet	
raining	

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timer works	yes
sprinklers on	yes
grass wet	
raining	

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sprinklers on	yes
grass wet	yes
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timer works	yes
sprinklers on	yes
grass wet	yes
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(timer displays clock) $\leftarrow$ (timer works) $\rightarrow$ (sprinklers on) $\rightarrow$ (grass wet) $\leftarrow$ (raining)

(timer displays clock) is statistically related to which variables?

timer works	yes
sprinklers on	yes
grass wet	yes
raining	no

We just learned: One collider can block an entire path

A path can involve forks, colliders, and causal paths

(timer displays clock) $\leftarrow$ (timer works) $\rightarrow$ $\boxed{\text{(sprinklers on)}}$ $\rightarrow$ (grass wet) $\leftarrow$ (raining)

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timer works
grass wet
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timer works	yes
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timer works	yes
grass wet	no
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A path can involve forks, colliders, and causal paths

(timer displays clock) $\leftarrow$ (timer works) $\rightarrow$ $\boxed{\text{(sprinklers on)}}$ $\rightarrow$ (grass wet) $\leftarrow$ (raining)

(timer displays clock) is statistically related to which variables?

timer works	yes
grass wet	no
raining	no

We just learned: One conditioned non-collider can block an entire path

Rules for whether paths are open or blocked

- If a path contains an unconditioned collider, it is blocked
- If a path contains a conditioned non-collider, it is blocked
- Otherwise, the path is open

Open paths create statistical dependence.
Blocked paths do not.

Determining statistical dependence: A procedure

How do you know if two nodes are dependent?

1. List all paths between the two nodes
2. Cross out any blocked paths that are blocked
3. If any paths remain, the two nodes are dependent

Determining statistical dependence: A procedure

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Practice: Are A and C statistically independent or dependent?

$$\begin{tikzcd}
 & B & & \\
A \arrow[ur] & & C \arrow[ul] \arrow[r] & D \\
X \arrow[u] \arrow[urrr, bend right] & & &
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- $A \to B \leftarrow C$
- $A \leftarrow X \to D \leftarrow C$

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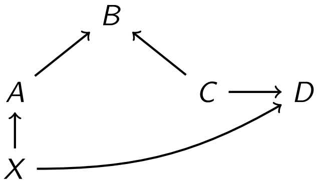

- $\cancel{A \rightarrow B \leftarrow C}$
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No unblocked paths. A and C are independent!

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- $\cancel{A \to B \leftarrow C \to D}$
- $A \leftarrow X \to D$

One remains. A and D are dependent!

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Practice: Are A and statistically independent or dependent?

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Practice: Are A and statistically independent or dependent?

$$\begin{tikzcd}
 & B & & \\
A \arrow[ur] & & C \arrow[ul] \arrow[r] & D \\
X \arrow[u] \arrow[urrr, bend right] & & &
\end{tikzcd}$$

- $A \to B \leftarrow C \to D$
- $A \leftarrow X \to D$

Determining statistical dependence: A procedure

How do you know if two nodes are dependent?

1. List all paths between the two nodes
2. Cross out any blocked paths that are blocked
3. If any paths remain, the two nodes are dependent

Practice: Are A and statistically independent or dependent?

$$
\begin{tikzcd}
 & B & & \\
A \arrow[ur] & & C \arrow[ul] \arrow[r] & D \\
X \arrow[u] \arrow[urrr, bend right] & & &
\end{tikzcd}
$$

- $A \rightarrow B \leftarrow C \rightarrow D$
- $A \leftarrow X \rightarrow D$

Learning goals for today

At the end of class, you will be able to:

1. Read a Directed Acyclic Graph
2. Recognize causal paths
3. Understand two key structures
   - Fork structures ($\bullet \leftarrow \bullet \rightarrow \bullet$)
   - Collider structures ($\bullet \rightarrow \bullet \leftarrow \bullet$)
4. List all paths in a DAG
5. Determine which paths are blocked under a particular adjustment set
6. Select a sufficient adjustment set to isolate causal paths