

# Directed Acyclic Graphs

Sociol 114

6 Feb 2025

# Learning goals for today

At the end of class, you will be able to:

1. Read a Directed Acyclic Graph
2. Recognize causal paths
3. Understand two key structures
  - ▶ Fork structures ( $\bullet \leftarrow \bullet \rightarrow \bullet$ )
  - ▶ Collider structures ( $\bullet \rightarrow \bullet \leftarrow \bullet$ )
4. List all paths in a DAG
5. Determine which paths are blocked under a particular adjustment set
6. Select a sufficient adjustment set to isolate causal paths

## A hypothetical experiment: Conditional randomization

Among the top 25%  
of the high school class



Among the bottom 75%  
of the high school class



Randomly Assigned to

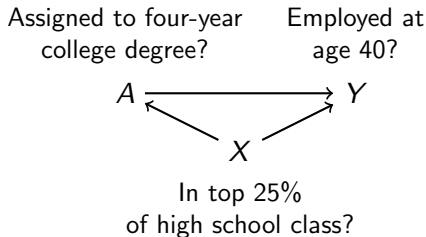


High School Degree

Four-Year College Degree

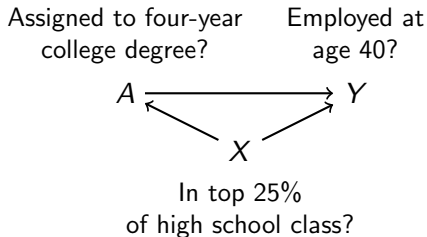
Outcome: Employed at age 40

# Elements of a Directed Acyclic Graph (DAG)



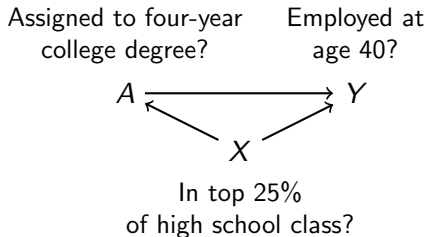
- ▶ **Nodes** ( $X, A, Y$ ) are random variables
- ▶ **Edges** ( $\rightarrow$ ) are causal relationships.
  - ▶  $X$  has a causal effect on  $A$
  - ▶  $X$  has a causal effect on  $Y$
  - ▶  $A$  has a causal effect on  $Y$

# Elements of a Directed Acyclic Graph (DAG)



A **path** is a sequence of edges connecting two nodes.

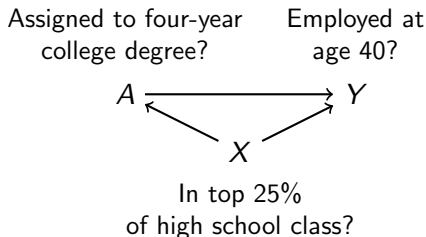
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Between *A* and *Y*, what are the two paths?

# Elements of a Directed Acyclic Graph (DAG)



A **path** is a sequence of edges connecting two nodes.

Between A and Y, what are the two paths?

- ▶  $A \rightarrow Y$
- ▶  $A \leftarrow X \rightarrow Y$

# Causal paths

In a **causal path**, all arrows point in the same direction.

(● → ● → ●)



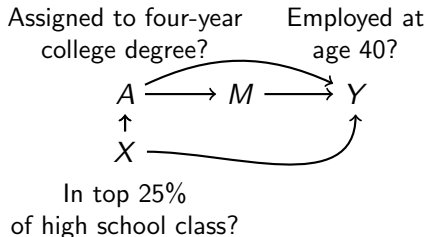
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( $\bullet \rightarrow \bullet \rightarrow \bullet$ )

What three paths between  $A$  and  $Y$ ?

Which two are causal paths?



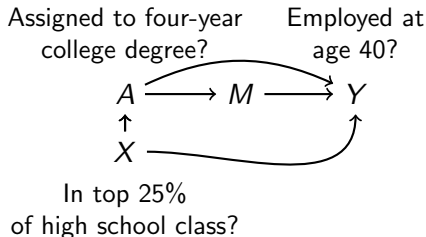
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$$A \rightarrow Y$$

$$A \rightarrow M \rightarrow Y$$

$$A \leftarrow X \rightarrow Y$$

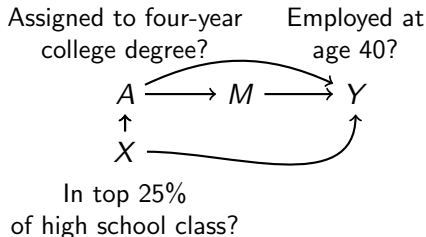
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$A \rightarrow Y$  causal path

$A \rightarrow M \rightarrow Y$

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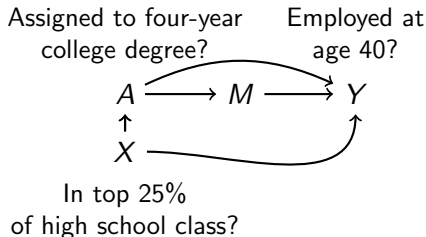
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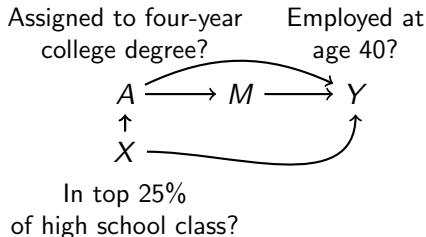
# Causal paths

In a **causal path**, all arrows point in the same direction.

( $\bullet \rightarrow \bullet \rightarrow \bullet$ )

What three paths between  $A$  and  $Y$ ?

Which two are causal paths?



- |                                 |                   |
|---------------------------------|-------------------|
| $A \rightarrow Y$               | causal path       |
| $A \rightarrow M \rightarrow Y$ | causal path       |
| $A \leftarrow X \rightarrow Y$  | not a causal path |

# Causal paths

When two variables are connected by a causal path, those variables are statistically associated (because the first variable causes the second) unless you hold constant some variable along the path

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Example: Among people with a fever,

( $A$ : friend offers Tylenol)  $\rightarrow$  ( $M$ : person takes Tylenol)

$\rightarrow$  ( $Y$ : Fever subsides quickly)

- ▶ Marginally, this causal path makes  $A$  associated with  $Y$ 
  - ▶ Fevers subsided more quickly among those whose friend offered Tylenol
- ▶ Conditional on  $M = 0$ ,  $A$  and  $Y$  are unrelated
  - ▶ Among those who didn't take Tylenol, it doesn't matter whether a friend offered them Tylenol or not. Fevers were the same.

# Fork structures

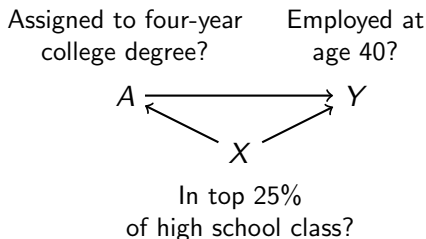
A **fork structure** is a sequence of edges within a path in which two variables are both caused by a third variable:  $A \leftarrow C \rightarrow B$



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In our initial graph, what path contains a fork structure?



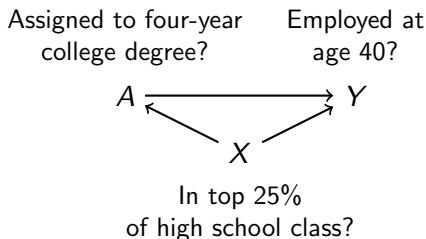
Recall that there are two paths:

1.  $A \rightarrow Y$
2.  $A \leftarrow X \rightarrow Y$

# Fork structures

A **fork structure** is a sequence of edges within a path in which two variables are both caused by a third variable:  $A \leftarrow C \rightarrow B$

In our initial graph, what path contains a fork structure?



Recall that there are two paths:

1.  $A \rightarrow Y$
2.  $A \leftarrow X \rightarrow Y$  (this path contains a fork structure)

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