Final Research Project – EE514, 2016

(Dated: April 27, 2016)

Project Goal: Assess and compare the performance of various error avoidance/suppression/correction strategies in overcoming asymmetric decoherence.

Project Rules:

1. Up to two persons per team. Teams of two need to declare what percentage of effort each person contributed to the project, and the grade will be in proportion to that declared percentage (i.e., for percentages x and y, the maximum possible scores are $2x$ and $2y$, respectively).
2. You will be assessed foremost on creativity and originality. It is unrealistic to expect that anyone will offer a complete solution, but some degree of progress is expected.
3. At a minimum you must provide **(A) a clear problem statement** and **(B) a description of one attempted solution approach, including mathematical details**.
4. Submissions must be typed and typeset in latex, and emailed to lidar@usc.edu by the deadline, **noon, May 11, 2016**. **No extensions will be possible**.

Problem Description:

Consider the system-bath Hamiltonian

$$H_{SB} = \left(\sum_{i=1}^{N} \sigma_i^x\right) \otimes B^x + \left(\sum_{i=1}^{N} \sigma_i^y\right) \otimes B^y + \sum_{i=1}^{N} \sigma_i^z \otimes B_i^z \tag{1}$$

acting on N qubits. Note that the x and y terms are collective while the z term has every qubit coupled independently to the bath.

The bath Hamiltonian H_B may be set to zero, but in a more realistic treatment it is a general bounded operator that doesn't commute with H_{SB}.

The system Hamiltonian $H_S(t)$ may be chosen so as to assist in the implementation of any desired control operations, whether in dynamical decoupling (DD) or in quantum logic gates, etc.

The full Hamiltonian is then

$$H(t) = H_S(t) + H_B + H_{SB}. \tag{2}$$

Assume that you can prepare the N qubits in the pure state $|\psi(0)\rangle = |0\dots0\rangle$ at time $t = 0$.

The goal of this project is to develop and compare different methods for preserving the initial state of as many of the N qubits as possible. That is, to find a way to get the final (encoded) system state at time T, $\bar{\rho}(T)$ as close as possible to $|\phi(0)\rangle\langle\phi(0)|$, where $|\phi(0)\rangle = |\bar{0}\dots\bar{0}\rangle$ is the state of $N' \leq N$ (encoded) qubits.

To determine the closeness of two states use either the trace-norm distance, D, or the Uhlmann fidelity, F, or both.

The different methods must include DFS/NS, DD, and QEC. You may also consider hybrid methods, in particular DFS/NS-DD, DFS/NS-QEC, and DD-QEC. Encoding (into a DFS/NS or QEC) of course results in some sacrifice of qubits, which is why N' above can be $< N$. To make sure the comparisons you make are fair, in using DD you can assume that the pulses are instantaneous but separated by a finite pulse interval τ; likewise in QEC you can assume that all operations are instantaneous but separated by a finite interval τ. The number of pulses in a DD scheme can then be compared to the number of operations (gates, measurements) in a QEC scheme.

For each of the methods above, report at a minimum (i) the fidelity, (ii) the space resources (number of qubits, or better: the code rate), (iii) the time resources in units of τ. You may also include plots reporting simulation results.

Where appropriate use a perturbative expansion of the time-evolution operator, at least to second order in time. Then report the fidelity (or trace-norm distance) using this expansion.

There is no need to account for fault tolerance considerations, but you may do so if you wish. Likewise, you may consider higher-order DD sequences such as CDD, to improve the result.

All optional statements above ("you may") will result in a higher score if carried out correctly.