

# COMS 4771 Spring 2017 Homework 2

Jin Tack Lim, jl4312

Problem 2

(a) Posteriori:

$$\begin{aligned} p(\alpha|X) &\propto p(\alpha)p(X|\alpha) \\ &= \frac{1}{\sqrt{2\pi\sigma_p^2}} e^{-\frac{(\mu-\mu_p)^2}{2\sigma_p^2}} \prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \end{aligned}$$

(Remove/Add constant terms to make the following term)

$$\propto \exp \left\{ -\frac{\mu - \frac{\mu_p\sigma^2 + \sum_{i=1}^N \sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}}{2\frac{\sigma_p^2\sigma^2}{\sigma^2 + N\sigma_p^2}} \right\}$$

Once we define

$$\begin{aligned} \sigma_N &= \frac{\sigma_p^2\sigma^2}{\sigma^2 + N\sigma_p^2} \\ \mu_N &= \frac{\mu_p\sigma^2 + \sum_{i=1}^N \sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2} \end{aligned}$$

then,

$$p(\alpha|X) \propto \exp \frac{-(\mu - \mu_N)^2}{2\sigma_N^2}$$

We can apply the Gaussian integral, and get the posteriori.

$$p(\alpha|X) = \frac{1}{\sqrt{2\pi\sigma_N^2}} e^{-\frac{(\mu-\mu_N)^2}{2\sigma_N^2}}$$

(b) the maximum-a-posteriori

We can simply take the log of the posteriori, and take the derivative and set it zero to get the maximum-a-posteriori.

$$\mu_{MAP} = \mu_N = \frac{\mu_p\sigma^2 + \sum_{i=1}^N \sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}$$