# COMS 4771 Spring 2017 Homework 2

Jin Tack Lim, jl4312

Problem 2

(a) Posteriori:

$$p(\alpha|X) \propto p(\alpha)p(X|\alpha)$$

$$= \frac{1}{\sqrt{2\pi\sigma_p^2}} e^{\frac{-(\mu-\mu_p)^2}{2\sigma_p^2}} \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i-\mu)^2}{2\sigma^2}}$$

(Remove/Add constant terms to make the following term)

$$\propto exp\left\{ - \frac{\mu - \frac{\mu_p\sigma^2 + \sum_{i=1}^{N}\sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}}{2\frac{\sigma_p^2\sigma^2}{\sigma^2+N\sigma_p^2}} \right\}$$

Once we define

$$\sigma_N = \frac{\sigma_p^2\sigma^2}{\sigma^2 + N\sigma_p^2}$$

$$\mu_N = \frac{\mu_p\sigma^2 + \sum_{i=1}^{N}\sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}$$

then,

$$p(\alpha|X) \propto exp\frac{-(\mu - \mu_N)^2}{2\sigma_N^2}$$

We can apply the Gaussian integral, and get the posteriori.

$$p(\alpha|X) = \frac{1}{\sqrt{2\pi\sigma_N^2}} e^{\frac{-(\mu-\mu_N)^2}{2\sigma_N^2}}$$

(b) the maximum-a-posteriori

We can simply take the log of the posteriori, and take the derivative and set it zero to get the maximum-a-posteriori.

$$\mu_{MAP} = \mu_N = \frac{\mu_p\sigma^2 + \sum_{i=1}^{N}\sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}$$

(c) expected-a-posteriori value

Given that the posteriori is the Gaussian distribution, we can easily tell that the expected value is the mean.

$$E[posteriori] = \mu_N = \frac{\mu_p \sigma^2 + \sum_{i=1}^{N} \sigma_p^2 x_i}{\sigma^2 + N\sigma_p^2}$$

References

[1] https://www.probabilitycourse.com/chapter9/9_1_1_prior_and_posterior.php

[2] Bishop 2.3