Ranking Decision Making Units by Means of Soft Computing DEA Models.

Article *in* International Journal of Uncertainty Fuzziness and Knowledge-Based Systems · February 2011

DOI: 10.1142/S0218488511006915 · Source: DBLP

International Journal of Uncertainty,
Fuzziness and Knowledge-Based Systems
Vol. 19, No. 1 (2011) 115–134

DOI: 10.1142/S0218488511006915

RANKING DECISION MAKING UNITS BY MEANS OF SOFT COMPUTING DEA MODELS

JOSÉ E. BOSCÁ
Departament d'Anàlisi Econòmica, Universidad de Valencia, Spain
jose.e.bosca@uv.es

VICENTE LIERN* and RAMÓN SALA†
Departament de Matemàtiques per a l'Economia i l'Empresa,
Universidad de Valencia, Spain
**vicente.liern@uv.es*
†*ramon.sala@uv.es*

AURELIO MARTÍNEZ
Departament d'Estructura Econòmica, Universidad de Valencia, Spain
aurelio.martinez@uv.es

Received 18 December 2008
Revised 9 November 2010

This paper presents a method for ranking a set of decision making units according to their level of efficiency and which takes into account uncertainty in the data. Efficiency is analysed using fuzzy DEA techniques and the ranking is based on the statistical analysis of cases that include representative situations. The method enables the removal of the sometimes unrealistic hypothesis of a perfect trade-off between increased inputs and outputs. This model is compared with other DEA models that work with imprecise or fuzzy data. As an illustration, we apply our ranking method to the evaluation of a group of Spanish seaports, as well as teams playing in the Spanish football league. We compare the results with other methods and we show that our method enables a total ranking of the seaports, and that the ranking of football teams is found to be more consistent with final league positions.

Keywords: DEA; fuzzy optimization; imprecise data; ranking units.

1. Introduction

Analysing the efficiency of a set of decision making units (hereafter referred to as DMUs) has at least two additional problems apart from the analysis itself: how to handle imprecise data and how to use each efficiency index. As regards imprecision, there is an abundant literature proposing methods for tackling this situation.[1–6]

*Corresponding author. Avda. dels Tarongers, s/n, E-46022 Valencia, Spain.

However, it is seldom taken into account that in order to make decisions, it may not be enough to merely know which DMUs use resources most efficiently. In reality, DMUs need to be ranked in terms of efficiency. This paper proposes a method for ranking DMUs according to their input and output values when these are uncertain.

The paper is divided into six sections. Section 2 briefly presents the classical version of data envelopment analysis (DEA) models with imprecise data. Section 3 analyses the ranking of DMUs based on fuzzy DEA models. Section 4 presents the method proposed, that is, a ranking based on statistical case analysis. Some ranking methods are compared in Sec. 5 in a simple example published by León *et al.*[3] and the quantitative results are presented. Section 6 presents two practical applications with data from a group of Spanish seaports and the Spanish First Division Football League in the 2005/6 season. The conclusions are presented in Sec. 7, and bibliographical references follow.

2. DEA Methodology

The DEA technique, developed by Charnes, Cooper, and Rhodes[7] is an extremal method rather than a non-parametric method for estimating production frontiers and assessing the efficiency of a sample of DMUs. DEA has proven to be a powerful tool for analysing the efficiency of a series of DMUs (in terms of multiple inputs and outputs), and calculates the relative efficiency of each unit with respect to the rest.

As a frontier method, DEA assesses decisions with regard to the maximum level of output attainable with a given combination of inputs, or with a minimum level of necessary inputs in the production of a given level of outputs. Furthermore, as this method is non-parametric, efficiency analysis does not require any hypotheses regarding the production frontier (the efficiency of a unit being defined by comparing it to those seen to perform the best). This analysis focuses on identifying the best performance instead of the average performance as is the case, for example, with regression analysis.

Non-parametric methodology offers great flexibility and an absence of specification errors because it is not necessary to choose any particular functional form. However, it suffers from the disadvantage of being technically deterministic and so atypical observations may bias the efficiency results and attribute any random shocks to inefficiency.[8]

Another mistake that researchers make is to use the results to rank units. If a unit has a lower efficiency ratio, this does not guarantee that one unit should have priority over another. Hence, these results cannot be used to rank units.

2.1. *DEA models*

Given n DMUs of which we know m inputs and s outputs, the efficiency of a target unit j_0 can be obtained by solving the efficiency maximization problem of a unit j_0, restricted to the efficiency of all units.

The variables in this problem are weightings and the solution produces the most favourable weightings for unit j_0 and a measure of efficiency. The mathematical programming model would be as follows:

$$\begin{aligned} \text{Max } E_o &= \sum_{r=1}^{s} u_{ro} y_{ro} \Big/ \sum_{i=1}^{m} v_{io} x_{io} \\ \text{s.t.} &\sum_{r=1}^{s} u_{ro} y_{rj} \Big/ \sum_{i=1}^{m} v_{io} x_{ij} \leq 1, \quad 1 \leq j \leq n \\ &u_{ro} \geq \epsilon > 0, \quad 1 \leq r \leq s \\ &v_{io} \geq \epsilon > 0, \quad 1 \leq i \leq m \end{aligned} \tag{1}$$

Variables u and v in the problem are required to be greater than or equal to ϵ, in order to avoid any inputs or outputs being ignored when calculating efficiency, although this requirement also serves prevent the function denominator and restrictions from taking a value of zero. The restriction coefficient has an upper limit of 1 in order to act as a reference framework for the different scales. The solution of the model provides a value for E_0, the efficiency of unit j_0, and the weightings that such efficiency produces.

From a mathematical viewpoint, as mentioned previously, we are dealing with fractional or hyperbolic programming, which can be quickly converted into a linear problem by merely changing the variables.[9,10] Furthermore, instead of working with the model itself (1), a dual model is normally used.[10]

In the models described above, constant returns to scale (CRS) have been assumed, but model (1) and its dual model can be easily modified to include variable returns to scale (VRS) by adding the convexity restriction. In the dual model (1) this restriction is

$$\sum_{j=1}^{n} \lambda_j = 1,$$

which, once included in the model, will provide us with the following programming model:

$$\begin{aligned} &\textbf{Min } \theta \\ &\textbf{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} \geq \theta x_{io}, \quad 1 \leq i \leq m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro}, \quad 1 \leq r \leq s \\ &\sum_{j=1}^{n} \lambda_j = 1, \\ &\lambda_j \geq 0, \quad 1 \leq j \leq n. \end{aligned} \tag{2}$$

As mentioned previously, data imprecision is one of the problems of practical DEA application. In order to overcome this problem, various options have been presented, ranging from bootstrapping and extreme case analysis[1] to sensitivity analysis, etc.[11] However, this latter technique must be extended to make it capable of dealing with imprecise data, as can be seen below.

2.2. *DEA models with imprecise data*

The most commonly used fuzzy numbers are those known as LR-fuzzy numbers.[12] These are fuzzy numbers $\tilde{M}$ expressed as

$$\tilde{M} = (m^L, m^R, \delta^L, \delta^R)_{L,R},$$

with a membership function as shown below:

$$\mu_{\tilde{M}}(r) = \begin{cases} L\left(\dfrac{m^L - r}{\delta^L}\right) & r \leq m^L \\ 1 & m^L \leq r \leq m^R \\ R\left(\dfrac{r - m^R}{\delta^R}\right) & r \geq m^R \end{cases}$$

where L and R are reference functions, i.e. $L, R\colon [0, +\infty[\to [0,1]$ are decreasing in $supp(\tilde{M}) = \{r : \mu_{\tilde{M}}(r) > 0\}$ and are upper semi-continous so that $L(0) = R(0) = 1$.

If the support of $\mu_{\tilde{M}}$ is bounded, the functions L and R are defined on $[0,1]$ and verify $L(1) = R(1) = 0$. Moreover, if L and R are linear functions, the fuzzy number is called *trapezoidal*. In the particular case that $m^L = m^R := \overline{m}$, the fuzzy number is called *triangular* and we express it as $\tilde{M} = (\overline{m}, \delta^L, \delta^R)$.

If we assume that inputs and outputs are imprecise and this uncertainty can be expressed by means of fuzzy numbers, the model (2) will be written in the following way:

$$\begin{aligned} &\textbf{Min } \theta \\ &\textbf{s.t.} \quad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \gtrsim \theta \tilde{x}_{io}, \quad 1 \leq i \leq m \\ &\qquad \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \lesssim \tilde{y}_{ro}, \quad 1 \leq r \leq s \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1 \\ &\qquad \lambda_j \geq 0, \quad 1 \leq j \leq n \end{aligned} \tag{3}$$

The problem is that in this case the efficiency of each DMU is provided by a fuzzy number with a membership function $\mu_{\tilde{E}_j}$, $1 \leq j \leq n$. Several methods appear in the literature for calculating the function $\mu_{\tilde{E}_j}$.[2–6] Essentially, these papers propose

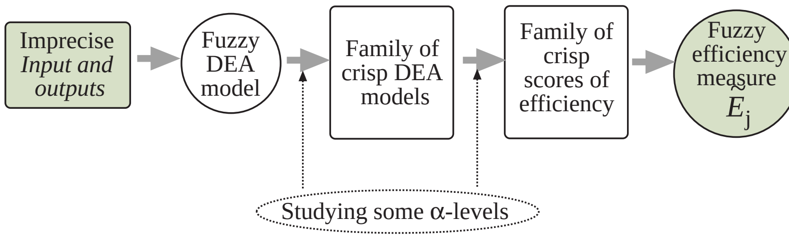

Fig. 1. Diagram of fuzzy DEA models.

fuzzy DEA models and solve them through defuzzification for some α-levels. The diagram in Fig. 1 illustrates the framework.

In some contexts this approach may be inappropriate. These include contexts in which the difficulties come from the future use of the efficiency scores, and the contexts derived from an assumed hypothesis. In the first case, we will discuss when the scores will be used to rank the DMUs, or to make predictions. Regarding the assumptions in the model, in some works inputs and outputs are incorrectly expressed as fuzzy numbers when they are actually interval-valued quantities. Furthermore, the computation of the membership function $\mu_{\tilde{E}_j}$ is usually based on α-cuts and a perfect trade-off is normally assumed between increases (or decreases) in the inputs and outputs of all the DMUs.[2,3]

This paper intends to avoid these drawbacks by using DEA models to make a statistical analysis. In Fig. 2 we have added (in a discontinuous line) our approach to the diagram in Fig. 1.

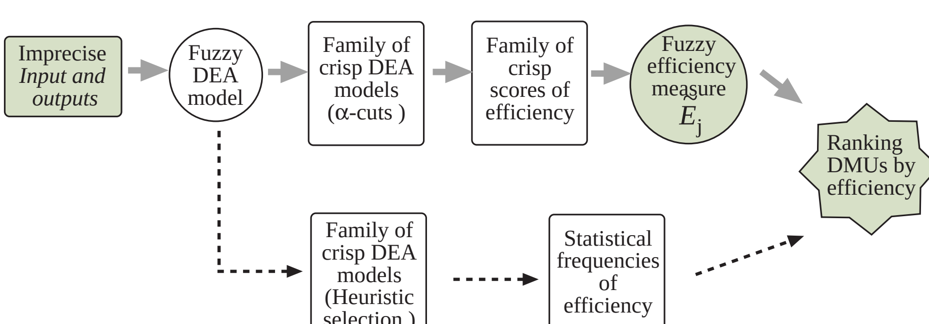

Fig. 2. Diagram of fuzzy DEA models including our proposed method.

3. Ranking DMUs by Means of Fuzzy DEA Models

We assume that the inputs and outputs can be ascertained approximately by using fuzzy numbers[2–4] that will be represented, respectively, by

$$\tilde{X}_{ij} = \{x_{ij}, \mu_{\tilde{X}_{ij}}(x_{ij})\}, \quad 1 \le i \le m, \quad 1 \le j \le n,$$
$$\tilde{Y}_{rj} = \{y_{rj}, \mu_{\tilde{Y}_{rj}}(y_{rj})\}, \quad 1 \le r \le s, \quad 1 \le j \le n.$$

The α-cuts in $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$ can be written as follows:

$$x_{ij}(\alpha) = \{x_{ij} \in \tilde{X}_{ij} \, : \, \mu_{\tilde{x}_{ij}}(x_{ij}) \ge \alpha\}, \quad 1 \le i \le m, \quad 1 \le j \le n,$$
$$y_{rj}(\alpha) = \{y_{rj} \in \tilde{Y}_{rj} \, : \, \mu_{\tilde{y}_{rj}}(y_{rj}) \ge \alpha\}, \quad 1 \le r \le s, \quad 1 \le j \le n.$$

We accept that the α-cuts can be expressed by means of intervals in the following way:[2]

$$\begin{aligned} \tilde{x}_{ij}(\alpha) &= [\min_{x_{ij}}\{x_{ij} \in \tilde{X}_{ij} : \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha\}, \ \max_{x_{ij}}\{x_{ij} \in \tilde{X}_{ij} : \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha\}] \\ &= [x_{ij}(\alpha)^L, x_{ij}(\alpha)^U], \quad 1 \le i \le m, \quad 1 \le j \le n, \end{aligned} \tag{4}$$

$$\begin{aligned} \tilde{y}_{rj}(\alpha) &= [\min_{y_{rj}}\{y_{rj} \in \tilde{Y}_{rj} : \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha\}, \ \max_{y_{rj}}\{y_{rj} \in \tilde{Y}_{rj} : \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha\}] \\ &= [y_{rj}(\alpha)^L, y_{rj}(\alpha)^U], \quad 1 \le r \le s, \quad 1 \le j \le n. \end{aligned} \tag{5}$$

Based on the Zadeh extension principle,[13] the membership function for the efficiency of DMU k can be defined as:

$$\mu_{\tilde{E}_k}(z) = \sup_{x_{ij}, y_{rj}} \min \left\{ \mu_{\tilde{X}_{ij}}(x_{ij}),\, \mu_{\tilde{Y}_{rj}}(y_{rj}),\, \forall i, j, r \mid z = E_k(x_{ij}, y_{rj}) \right\},$$

where E_k is calculated in accordance with (1).

Kao and Liu[2] proposed obtaining the membership function by means of intervals $E_k(\alpha) = [E_k^L(\alpha), E_k^U(\alpha)]$ that can be obtained using the following mathematical programming models[a]

$$\begin{aligned} E_{j_0}^L(\alpha) = \ & \mathbf{Min}\,\theta \\ & \text{s.t.} \sum_{j=1, j \ne j_0}^{n} \lambda_j x_{ij}^L(\alpha) + \lambda_{j_0} x_{ij_0}^U(\alpha) \ge \theta x_{io}, \quad 1 \le i \le m, \\ & \sum_{j=1, j \ne j_0}^{n} \lambda_j y_{rj}^U(\alpha) + \lambda_{j_0} y_{rj_0}^L(\alpha) \le y_{ro}, \quad 1 \le r \le s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \ge 0, \quad 1 \le j \le n. \end{aligned} \tag{6}$$

[a] In reality, Kao and Liu[2] employ the dual programmes such as those presented in this research.

$$
\begin{aligned}
E_{j_0}^U(\alpha) = \mathbf{Min}\,\theta & \\
\text{s.t.} \sum_{j=1, j\neq j_0}^{n} \lambda_j x_{ij}^U(\alpha) + \lambda_{j_0} x_{ij_0}^L(\alpha) \geq \theta x_{io}, & \quad 1 \leq i \leq m, \\
\sum_{j=1, j\neq j_0}^{n} \lambda_j y_{rj}^L(\alpha) + \lambda_{j_0} y_{rj_0}^U(\alpha) \leq y_{ro}, & \quad 1 \leq r \leq s, \\
\sum_{j=1}^{n} \lambda_j = 1, & \\
\lambda_j \geq 0, \quad 1 \leq j \leq n. &
\end{aligned}
\tag{7}
$$

In this work we are interested in ranking the set of scores,

$$\left\{\tilde{E}_j = [E_j^L(\alpha), E_j^U(\alpha)], \quad \alpha \in [0,1]\right\}_{j=1}^{n},$$

in order to obtain a ranking of the DMUs according to their efficiency. There are many proposals for comparing fuzzy numbers, and Wang and Kerre[14,15] have collected and classified many of these methods. Most proposals require the membership functions of the fuzzy numbers to be ranked. However, given the way scores have been obtained, to accomplish our objective methods based on the α-cuts would be the most useful. Moreover, when DEA models are applied to real cases, it is common to work with many DMUs, inputs, and outputs; and this makes it inadvisable to use ranking methods based on fuzzy relations (those that Wang and Kerre[15] classify as a "third class of ordering approach") because they need $\frac{n(n-1)}{2}$ comparisons (sometimes for each α-cut). For these reasons, methods based on index numbers are used - including those based on α-cuts. We briefly summarise some of these procedures below.

- *Yager's approach.* R. R. Yager[16] proposed four indexes to rank fuzzy quantities in $[0,1]$, of which the most commonly used and intuitive is the following:

$$Y(\tilde{E}_j) = \int_0^{\alpha_{\max}} \frac{1}{2}\left(E_j^L(\alpha) + E_j^U(\alpha)\right) d\alpha. \tag{8}$$

- *Campos and Muñoz's approach.*[17] Given $\lambda \in [0,1]$ an indicator of the optimism of the decision maker, Campos and Muñoz define an index family,[14] from which the most commonly used index is

$$CM_\lambda(\tilde{E}_j) = \lambda \int_0^1 E_j^U(\alpha) + (1-\lambda) \int_0^1 E_j^L(\alpha) d\alpha. \tag{9}$$

  This index is also propsed by Herrera and Lamata[18] for ranking graded numbers.
- *Choobinech and Li's approach.*[19] Let a and b be two numbers satisfying $a \leq \inf\left\{x : x \in \cup_{j=1}^n supp(\tilde{E}_j)\right\}$ and $d \geq \sup\left\{x : x \in \cup_{j=1}^n supp(\tilde{E}_j)\right\}$

$$CL(\tilde{E}_j) = \frac{1}{2}\left(\alpha_{\max} - \frac{1}{d-a}\left(\int_0^{\alpha_{\max}} (d - E_j^U(\alpha))d\alpha - \int_0^{\alpha_{\max}} (E_j^L(\alpha) - a)d\alpha\right)\right). \tag{10}$$

We use the models (6), (7) to ascertain the efficiency for different values of α, for instance $\alpha_\ell = \ell/N, \quad 0 \le \ell \le N$,

$$\left\{ \left[E_j(\alpha_\ell)^L, E_j(\alpha_\ell)^U \right] \right\}_{\ell=0}^{N}, \quad 1 \le j \le n. \tag{11}$$

In this case, the Lebesgue integral of expressions (8), (9), and (10) is transformed in sums and the probability distribution of α_ℓ is uniform.

For fuzzy DEA models, Kao and Liu[2] propose using the index suggested by Chen and Klein,[20] because it provides an efficient and accurate fuzzy ranking method that requires only three of four α -cuts. This increases efficiency as well as being applicable to a wider range of fuzzy numbers.

- *Chen and Klein's approach.*[20] These authors consider the index

$$CK(\tilde{E}_j) = \frac{\sum_{\ell=0}^{N} (E_j^U(\alpha_\ell) - c)}{\sum_{\ell=0}^{N} (E_j^U(\alpha_\ell) - c) - \sum_{\ell=0}^{N} (E_j^L(\alpha_\ell) - d)}, \tag{12}$$

where $c = \min_{i,j}\{E_{ij}^L(\alpha_\ell)\}$ and $d = \max_{i,j}\{E_{ij}^U(\alpha_\ell)\}$.

Having previous indexes in mind, we can establish a total ranking in the set of scores.

Definition 1. The set of fuzzy numbers $\{\tilde{E}_j\}_{j=1}^n$, can be ordered as follows:

$$\begin{aligned} &\tilde{E}_j \succ \tilde{E}_k \quad \text{by} \quad \Psi \quad \Leftrightarrow \quad \Psi_j > \Psi_k, \\ &\tilde{E}_j \sim \tilde{E}_k \quad \text{by} \quad \Psi \quad \Leftrightarrow \quad \Psi_j = \Psi_k, \\ &\tilde{E}_j \succsim \tilde{E}_k \quad \text{by} \quad \Psi \quad \Leftrightarrow \quad \Psi_j > \Psi_k \quad \text{or} \quad \Psi_j = \Psi_k, \end{aligned} \tag{13}$$

where Ψ represents the index Y, CM_λ, CL or CK.

4. Ranking Based on Statistical Analysis

Let n DMUs whose inputs and outputs may be expressed by the following LR-fuzzy numbers:

$$\begin{aligned} \tilde{X}_{ij} &= (x_{ij}^L, x_{ij}^U, \gamma_{ij}^L, \gamma_{ij}^U), \quad 1 \le i \le m, \quad 1 \le j \le n, \\ \tilde{Y}_{rj} &= (y_{rj}^L, y_{rj}^U, \delta_{rj}^L, \delta_{rj}^U), \quad 1 \le r \le s, \quad 1 \le j \le n. \end{aligned}$$

Solving (3) our proposal involves a heuristic procedure. We select representative crisp cases for these LR-fuzzy numbers which represent the inputs and outputs. If we solve all the possibilities with the selected crisp numbers, we can analyze for how many more scenarios a DMU will be efficient. DMU A is preferable to another, B, if A is more efficient than B. We are going to express this in an algorithmic form.

Step 1: We select the following crisp numbers:

$$\begin{aligned} x_{ij}^t &:= x_{ij}^L - \gamma_{ij}^L, \quad x_{ij}^A := \frac{x_{ij}^L + x_{ij}^U}{2}, \quad x_{ij}^T := x_{ij}^U + \gamma_{ij}^U, \\ y_{rj}^t &:= y_{rj}^L - \delta_{rj}^L, \quad y_{rj}^A := \frac{y_{rj}^L + y_{rj}^U}{2}, \quad y_{rj}^T := y_{rj}^U + \delta_{rj}^U, \end{aligned} \tag{14}$$

Remark: The choice of values may be different to those proposed here.

Step 2: To analyze the efficiency of the DMU j_0, we solve the following DEA models

$$E(x^a_{ij}, x^b_{ij_0}, y^c_{rj}, y^d_{rj_0}) = \textbf{Min} \quad \theta$$

$$\begin{aligned} \text{s.t.} \sum_{j=1}^{n} \lambda_j x^a_{ij} &\geq \theta x^b_{io}, \quad 1 \leq i \leq m \\ \sum_{j=1}^{n} \lambda_j y^c_{rj} &\leq y^d_{ro}, \quad 1 \leq r \leq s \\ \sum_{j=1}^{n} \lambda_j &= 1 \\ \lambda_j &\geq 0, \quad 1 \leq j \leq n \end{aligned} \tag{15}$$

where $a, b, c, d \in \{t, A, T\}$.

Step 3: We construct the set

$$\Gamma_{j_0} = \left\{ E(x^a_{ij}, x^b_{ij_0}, y^c_{rj}, y^d_{rj_0}) \mid a, b, c, d \in \{t, A, T\} \right\},$$

Step 4: We calculate the two following ratios for the j_0-th unit:

$$R^1_{j_0} := \frac{e_{j_0}}{\text{card}(\Gamma_{j_0})}, \quad R^2_{j_0} := \begin{cases} \dfrac{S_{j_0} - e_{j_0}}{\text{card}(\Gamma_{j_0}) - e_{j_0}} & \text{card}(\Gamma_{j_0}) \neq e_{j_0}, \\ 0 & \text{card}(\Gamma_{j_0}) = e_{j_0}, \end{cases} \tag{16}$$

where $\text{card}(\Gamma_{j_0})$ is the cardinal of Γ_{j_0}, $S_{j_0} = \sum_{a,b,c,d} E(x^a_{ij}, x^b_{ij_0}, y^c_{rj}, y^d_{rj_0})$ and e_{j_0} is the number of times that the DMU j_0 is efficient, that is, the optimal value of (15) is 1.

Remark 1. In DEA models in Step 2, where there is a set of inputs and outputs, the total number of combinations of variations in the data involves 81 alternatives, $\text{card}(\Gamma_{j_0}) = 3^4 = 81$. However, in (16) we preferred to use expression $\text{card}(\Gamma_{j_0})$ instead of 81, because there are models that analyze efficiency using inputs, outputs, and undesirable outputs, and in this case the cardinal would be $\text{card}(\Gamma_{j_0}) = 3^6 = 729$. In addition, we want to remark that the procedure has been described for (15) is a BCC model. However, what we have said remains valid if we eliminate the convexity restriction, $\sum \lambda_j = 1$, that is, if we work with CCR models.

If we repeat the above process for all DMUs, we have the following ratios:

$$\left\{ R^1_j, \ R^2_j \right\}_{i=1}^{n}, \tag{17}$$

and from (17) the DMUs can be ordered.

Definition 2. The set of fuzzy numbers $\{\tilde{E}_j\}_{j=1}^n$ can be ordered as:

$$
\begin{aligned}
\tilde{E}_j \succ \tilde{E}_k \quad &\Leftrightarrow \quad R_j^1 > R_k^1 \quad \text{or} \quad [R_j^1 = R_k^1 \quad \text{and} \quad R_j^2 > R_k^2],\\
\tilde{E}_j \sim \tilde{E}_k \quad &\Leftrightarrow \quad R_j^1 = R_k^1 \quad \text{and} \quad R_j^2 = R_k^2,\\
\tilde{E}_j \succsim \tilde{E}_k \quad &\Leftrightarrow \quad R_j^1 \geq R_k^1 \quad \text{or} \quad R_j^2 \geq R_k^2.
\end{aligned} \tag{18}
$$

It is important to mention that the method described above can also be applied when inputs and outputs are described by intervals instead of LR-fuzzy numbers.

5. Computational Results

To test the two methods described in the previous sections, we will use the data from a simple example published by León *et al.*[3] in which an efficiency analysis is made, but which does not aim to rank DMUs. The example consists of eight DMUs, each having one known input and output, both of which are expressed in symmetrical triangular fuzzy numbers presented in Table 1.

Table 1. Inputs and outputs as symmetrical triangular fuzzy numbers.

DMU	Input $(x_j, \alpha_j^L, \alpha_j^R)$	Output $(y_j, \beta_j^L, \beta_j^R)$
A	(3, 2, 2)	(3, 1, 1)
B	(4, 0.5, 0.5),	(2.5, 1, 1)
C	(4.5, 1.5, 1.5)	(6, 1, 1)
D	(6.5, 0.5, 0.5)	(4, 1.25, 1.25)
E	(7, 2, 2)	(5, 0.5, 0.5)
F	(8, 0.5, 0.5)	(3.5, 0.5, 0.5)
G	(10, 1, 1)	(6, 0.5, 0.5)
H	(6, 0.5, 0.5)	(2, 1.5, 1.5)

In the following subsections we will see the ranking obtained using the methods presented in Sec. 3.

5.1. *Ranking based on the Kao and Liu approach*

To apply the ranking methods based on Definition 1, we will first express the fuzzy numbers in Table 1 as α-cuts (Table 2). We use the data in Table 3 to estimate the models $E_j^L(\alpha)$ and $E_j^U(\alpha)$ for the values of $\alpha = 0, 0.1, 0.2, \ldots, 1$ (see (6), (7)).

Taking into account the results expressed in Table 4, the DMUs are ranked in terms of efficiency as shown in Table 5, where the numbers on the rows indicate the ranking of each DMU.

5.2. *Ranking based on our approach*

As pointed out in the previous section, we have taken the input and output cases from the example that we are interested in (with the notation of (14)).

Table 2. Some α-cuts for each input and output.

DMU	$x(\alpha)$	$y(\alpha)$
A	$[3 - 2(1-\alpha),\ 3 + 2(1-\alpha)]$	$[3 - 1(1-\alpha),\ 3 + 1(1-\alpha\ )]$
B	$[4 - 0.5(1-\alpha),\ 4 + 0.5(1-\alpha)]$	$[2.5 - 1(1-\alpha),\ 2.5 + 1(1-\alpha)]$
C	$[4.5 - 1.5(1-\alpha),\ 4.5 + 1.5(1-\alpha)]$	$[6 - 1(1-\alpha),\ 6 + 1(1-\alpha)]$
D	$[6.5 - 0.5(1-\alpha),\ 6.5 + 0.5(1-\alpha)]$	$[4 - 1.25(1-\alpha),\ 4 + 1.25(1-\alpha)]$
E	$[7 - 2(1-\alpha),\ 7 + 2(1-\alpha)]$	$[5 - 0.5(1-\alpha),\ 5 + 0.5(1-\alpha)]$
F	$[8 - 0.5(1-\alpha),\ 8 + 0.5(1-\alpha)]$	$[3.5 - 0.5(1-\alpha),\ 3.5 + 0.5(1-\alpha)]$
G	$[10 - 1(1-\alpha),\ 10 + 1(1-\alpha)]$	$[6 - 0.5(1-\alpha),\ 6 + 0.5(1-\alpha)]$
H	$[6 - 0.5(1-\alpha),\ 6 + 0.5(1-\alpha)]$	$[2 - 0.5(1-\alpha),\ 2 + 0.5(1-\alpha)]$

Table 3. Efficiency intervals for some values of α.

α	$E_A(\alpha)$	$E_B(\alpha)$	$E_C(\alpha)$	$E_D(\alpha)$
0	[0.6, 1]	[0.222, 1]	[0.611, 1]	[0.143, 1]
.1	[0.656, 1]	[0.270, 1]	[0.735, 1]	[0.173, 1]
.2	[0.717, 1]	[0.318, 1]	[0.860, 1]	[0.203, 0.922]
.3	[0.784, 1]	[0.368, 1]	[0.987, 1]	[0.234, 0.876]
.4	[0.857, 1]	[0.419, 1]	[1, 1]	[0.265, 0.829]
.5	[0.938, 1]	[0.471, 1]	[1, 1]	[0.296, 0.782]
.6	[1, 1]	[0.524, 1]	[1, 1]	[0.337, 0.734]
.7	[1, 1]	[0.578, 0.947]	[1, 1]	[0.388, 0.686]
.8	[1, 1]	[0.634, 0.872]	[1, 1]	[0.438, 0.637]
.9	[1, 1]	[0.692, 0.810]	[1, 1]	[0.489, 0.588]
1	[1, 1]	[0.75, 0.75]	[1, 1]	[0.539, 0.539]
α	$E_E(\alpha)$	$E_F(\alpha)$	$E_G(\alpha)$	$E_H(\alpha)$
0	[0.148, 1]	[0.118, 0.743]	[0.182, 1]	[0.154, 0.974]
.1	[0.184, 1]	[0.142, 0.714]	[0.209, 1]	[0.186, 0.928]
.2	[0.222, 1]	[0.167, 0.687]	[0.235, 1]	[0.219, 0.885]
.3	[0.260, 1]	[0.192, 0.653]	[0.262 1]	[0.252, 0.830]
.4	[0.300, 0.924]	[0.217, 0.618]	[0.289, 1]	[0.286, 0.772]
.5	[0.341, 0.858]	[0.242, 0.583]	[0.316, 1]	[0.320, 0.714]
.6	[0.384, 0.795]	[0.268, 0.548]	[0.342, 1]	[0.355, 0.655]
.7	[0.428, 0.735]	[0.298, 0.513]	[0.369, 1]	[0.390, 0.615]
.8	[0.474, 0.678]	[0.334, 0.478]	[0.396, 1]	[0.426, 0.576]
.9	[0.522, 0.623]	[0.370, 0.442]	[0.423, 1]	[0.463, 0.538]
1	[0.571, 0.571]	[0.406, 0.406]	[0.45, 0.45]	[0.500, 0.500]

Taking into acoount the data expressed in Table 6, we solve the 81 cases described in Model (15) for each unit. Rather than displaying a statistical list of these results, a summary table with the cases that provide the largest and smallest efficiency ratios has been prepared (see Table 7). To illustrate the efficiency ratios obtained for each case in units B, C, D and G, we present a graph (see Fig. 3).

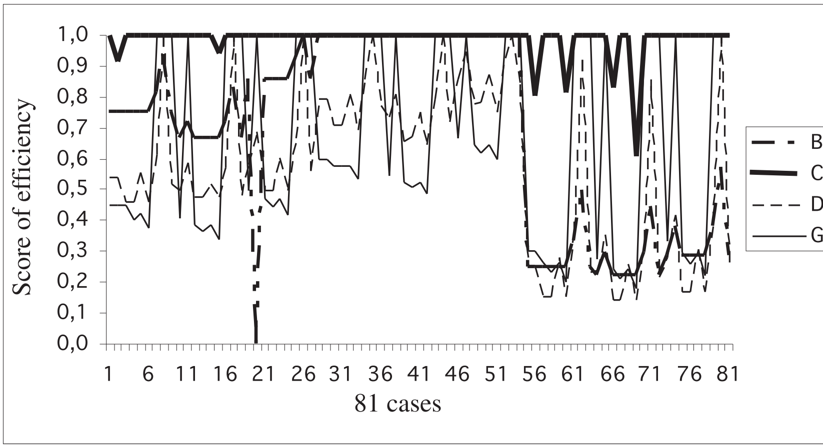

Fig. 3. Graph of the efficiency ratios for units B, C, D, and G.

Table 4. Index values for each DMU.

DMU	Y	$CM_{0.5}$	$CM_{0.4}$	CL	CK
A	0.9342	0.9342	0.9079	0.9254	0.8702
B	0.7102	0.7102	0.6169	0.6715	0.6122
C	0.9633	0.9633	0.9486	0.9584	0.9232
D	0.5499	0.5499	0.4574	0.4897	0.4933
E	0.5921	0.5921	0.4947	0.5376	0.5240
F	0.4154	0.4154	0.3494	0.3372	0.3817
G	0.6333	0.6333	0.5066	0.5842	0.5488
H	0.5369	0.5369	0.4612	0.4749	0.4732

Table 5. Ordering with the analyzed methods.

Ranking method	A	B	C	D	E	F	G	H
Yager	2	3	1	6	5	8	4	7
Campos-Muñoz ($\lambda = 0.5$)	2	3	1	6	5	8	4	7
Campos-Muñoz ($\lambda = 0.4$)	2	3	1	7	5	8	4	6
Choobinech -Li	2	3	1	6	5	8	4	7
Chen-Klein	2	3	1	6	5	8	4	7
Extensive analysis	2	4	1	6	5	8	3	7

The ratios R_j^1 and R_j^2 (see Expression (16)), are expressed in Table 7. Therefore, according to Definition 2 , the following ranking is obtained:

$$\tilde{E}_C \succ \tilde{E}_A \succ \tilde{E}_G \succ \tilde{E}_B \succ \tilde{E}_E \succ E_D \succ \tilde{E}_H \succ \tilde{E}_F \qquad (19)$$

Table 6. Values $\{x^t_{ij}, x^A_{ij}, x^T_{ij}\}$ and $\{y^t_{rj}, y^A_{rj}, y^T_{rj}\}$ for the data in Table 1.

DMU	Input	Output
A	{1, 3, 5}	{2, 3, 4}
B	{3.5, 4, 4.5}	{1.5, 2.5, 3.5}
C	{3, 4.5, 6}	{5, 6, 7}
D	{6, 6.5, 7}	{2.75, 4, 5.25}
E	{5, 7, 9}	{4.5, 5, 5.5}
F	{7.5, 8, 8.5}	{3, 3.5, 4}
G	{9, 10, 11}	{5.5, 6, 6.5}
H	{5.5, 6, 6.5}	{0.5, 2, 3.5}

Table 7. Value of efficiency scores, R^1_j and R^2_j for each DMUs.

DMU	Best	Original	Worst	R^1_j	R^2_j
A	1	1	0.6000	0.7037	0.7637
B	1	0.7500	0.2222	0.3580	0.5259
C	1	1	0.6111	0.9259	0.8209
D	1	0.5385	0.1429	0.0864	0.5288
E	1	0.5714	0.1482	0.1975	0.5272
F	0.7429	0.4065	0.1176	0	0.4056
G	1	0,4500	0.1818	0.4074	0.4187
H	0.9740	0,5000	0.1538	0	0.4961

5.3. *Comparing the methods*

In the example previously under study, the rankings obtained are expressed in Table 5 (the numbers on the rows indicate the ranking of each DMU).

Notice that we have used crisp numbers as inputs and outputs (that is, $(x_j, 0, 0)$ and $(y_j, 0, 0)$ with values x_j and y_j given in Table 1) and have ranked the DMUs by the value of their scores,

$$E_A = E_C > E_B > E_E > E_D > E_H > E_G > E_F \tag{20}$$

which is certainly not consistent with either of the rankings presented above. Therefore, we believe it is worth insisting that the values of the respective indexes are not useful when it comes to ranking units according to their efficiency. The real meaning of these values are those of magnitude coefficients for establishing preferences among units.

As can be seen in Table 5, the rankings are practically identical except for the permutation of units B and G which are inverted (the same occurs for the permutation of units D and H with the Campos and Muñoz method for $\lambda = 0.4$). For this reason, we consider the statistical analysis method to be a tool that enables different units to be ranked when data is imprecise.

6. Applications

In this section we present two real applications of our method and make a comparison with other methods used in previous sections. The first application deals with the ranking of a group of Spanish seaports and the second with the ranking of football teams in the Spanish professional football league. In this second application, given that we know the final ranking of teams at the end of the season, we can compare the different methods with the real ranking.

Unless otherwise stated, all methods based on α-cuts have been performed with eleven equally spaced values of α, i. e. $\alpha_\ell = \ell/10, \quad 0 \leq \ell \leq 10$.

6.1. *Efficiency ranking of Spanish seaports*

From the 28 seaports that belong to the Spanish port system, we have selected 6 that can be termed efficient, given that they have obtained a score 1 for at least one year between 1994 and 2008. This fact would mean that some ranking methods for DEA models would consider these units as tied.[21,22] However, this difficulty does not arise for the procedures presented in this paper. In addition, these seaports present a homogeneous structure where general commodity and container traffic predominates, in comparison with other ports that have a more heterogeneous traffic structure.[1]

For the DEA analysis we consider one input, equipment (I1), and three outputs, liquid bulk (O1), solid bulk (O2), and general commodities (O3).

An efficiency analysis of seaports would usually use data collected over several years. Sometimes this data is estimated and unreal, given that data is subject to measurement error and time variation.[23] For this reason, in Bonilla *et al.*[1] uncertainty affecting data is accounted for by using tolerances. More specifically, both the input and the outputs can be expressed through the following triangular numbers:

$$\begin{aligned} \tilde{X}_j &= (x_j, 0.049x_j, 0.1x_j), && 1 \leq j \leq 6, \\ \tilde{Y}_{rj} &= (y_{rj}, 0.052y_{rj}, 0.106y_{rj}), && 1 \leq r \leq 3, \quad 1 \leq j \leq 6, \end{aligned} \tag{21}$$

where x_j and y_{rj} are the values expressed in Table 8.

Table 8. Inputs and outputs for selected seaports in 2008.

Port (DMU)	I1	O1	O2	O3
Baleares	338087.32	2074	2131	9018
Valencia	1185683.25	5968	5137	48224
Las Palmas	752384.41	4739	1159	17842
Vigo	221339.26	58	458	4102
Barcelona	1671092.03	12105	3506	34935
Melilla	127557.69	71	34	641

Table 9. Values[23] of inputs and outputs for selected seaports in 2008.

Port (DMU)	$Y = CM_{0.5}$	$CM_{0.4}$	CL	CK	R^1	R^2
Baleares	1	1	1	1	1	0
Valencia	1	1	1	1	1	0
Las Palmas	0.9201	0.9087	0.9128	0.8698	0.2839	0.8525
Vigo	0.4696	0.4601	0.4159	0.4228	0	0.4597
Barcelona	0.9882	0.9859	0. 9871	0.9776	0.8272	0.8191
Melilla	0.1234	0.1207	0.0346	0.0474	0	0.1221

Using the data in (21), in Table 9 we show the values for the indexes analyzed in Secs. 3 and 4. We can see that in all cases the ranking is as follows:

$$\tilde{E}_{\text{Baleares}} \sim \tilde{E}_{\text{Valencia}} \succ \tilde{E}_{\text{Barcelona}} \succ \tilde{E}_{\text{Las Palmas}} \succ \tilde{E}_{\text{Vigo}} \succ \tilde{E}_{\text{Melilla}}.$$

Therefore, this application shows that our procedure is as effective as the other rankings used.

6.2. *Team rankings in the Spanish professional football league*

The application presented below is interesting because it meets all the conditions that have been outlined in previous sections: the data is imprecise and efficiency analysis is used to rank the teams and make future decisions. For club trainers it may be useful when designing future strategies to see a ranking based on technical efficiency. In addition, the example has the added value that we have at our disposal the ranking of teams at the end of the season. This fact makes it possible to compare some of the proposed ranking methods with the final league rankings.

Although at first glance it does not seem to be the case, the DEA models show uncertainty for several reasons: the data is imprecise because it was collected by non-professional evaluators, plays are sometimes credited to the wrong teams, (for example, in Table 10 we collect data about own goals), referee decisions, etc.

Table 10. Percentage of own goals.

Season	Own goals	Total goals	Percentage
2000/2001	27	1041	2.6%
2001/2002	24	961	2.5%
2002/2003	25	1016	2.5%
2003/2004	38	1019	3.7%
2004/2005	26	980	2.7%
2005/2006	25	936	2.7%

Four efficiency variants can be analysed (attacking, defending, home, and away), using a series of inputs (proxies of player skills) to achieve outputs (goals).[9] In this application, we used the most representative measure: attacking at home. The measure has been selected to enable external measurement to be made of the final position in the league — referred to by the two rankings discussed in the previous section. The output chosen for this analysis is goals scored (O1) by each of the teams during home matches. The inputs are: balls kicked into the centre area of the opposing team (I1), attacking plays made by the team (I2), minutes of possession (I3), and shots-on-goal (I4). More detailed information about the suitability of these inputs and outputs, and about cross-correlations among them can be found in Bosca *et al.*[9] A description of the data used in our example can be seen in Table 11.

Table 11. Description of data.

RANKS	O1	I1	I2	I3	I4
Max	45	650	111	574	385
Min	16	474	38	405	222
Average	25.95	556.80	63.45	454.85	268.95
Deviation	7.65	53.85	20.67	42.61	38.95

Using these four inputs and the output (goals scored), a DEA efficiency analysis was made on the 20 teams in the Spanish first division during the 2005/06 season. On attempting to include uncertainty in the analysis, we have encountered non-uniform situations in all teams that are illustrated by different variation percentages. A cluster analysis of the last 10 football seasons has been carried out and clubs have been grouped under two criteria: annual income and sporting history in the Spanish league since they were founded. This did not make it possible to group the teams into five categories, so that variation percentages were homogeneous within each group, but little difference is appreciated among the groups themselves.

Note that if the same variation percentage were applied to all the teams, we would not be able to compare the two indexes. Table 12 includes the tolerances (in percentage) applied to each team and the group to which it belongs. Once tolerances have been established, inputs and output are expressed through the following triangular numbers:

$$\begin{aligned} \tilde{X}_{ij} &= \left(x_{ij}, x_{ij}\frac{p_{ij}}{100}, x_{ij}\frac{p_{ij}}{100}\right), \quad 1 \le i \le 4, \quad 1 \le j \le 20, \\ \tilde{Y}_j &= \left(y_j, \frac{q_j}{100}, \frac{q_j}{100}\right) \qquad\qquad 1 \le j \le 20, \end{aligned} \tag{22}$$

where p_{ij} and q_j are the values expressed in Table 12.

Table 13 shows the ranking of the three methods applied together with the final league classification. In order to measure the differences between index CK and our index, we decided to compare both with the final league classification. The indexes Y and CK display, respectively, a correlation of 0.625 and 0.634 with the final league ranking (column 4), while the correlation between our index and the final ranking is 0.741.

Table 12. Tolerance applied for each group.

	GROUP	O1 q_{rj}	I1 p_{1j}	I2 p_{2j}	I3 p_{3j}	I4 p_{4j}
Ath. Bilbao	4	10	4	13	3	3
Atl. Madrid	3	12	4	13	6	4
Cádiz	5	10	5	12	5	4
Dep. Coruña	2	8	5	13	5	4
Dep. Alavés	5	10	5	12	5	4
Espanyol	4	10	4	13	3	3
F.C. Barcelona	1	6	3	12	4	3
Getafe	5	10	5	12	5	4
Málaga C.F.	4	10	4	13	3	3
Osasuna	4	10	4	13	3	3
R.C. Celta Vigo	3	12	4	13	6	4
R.C.D. Mallorca	5	10	5	12	5	4
Rac. Santander	5	10	5	12	5	4
Real Betis	3	12	4	13	6	4
Real Madrid	1	6	3	12	4	3
R. Sociedad	4	10	4	13	3	3
R. Zaragoza	3	12	4	13	6	4
Sevilla F.C.	2	8	5	13	5	4
Valencia C.F.	2	8	5	13	5	4
Villarreal	3	12	4	13	6	4

Table 13. Ranking for teams.

Team	R1	R2	Rank league	Y	CK	Our method
Ath. Bilbao	0	0.6274	12	15	16	15
Atl. de Madrid	0	0.6337	10	16	15	14
Cádiz	0,0370	0,7915	19	8	8	8
Dep. A Coruña	0	0.5953	8	19	18	17
Dep. Alavés	0	0.0370	18	13	13	19
Espanyol	0.0123	0.8126	15	4	4	9
F.C. Barcelona	0.9877	0.9735	1	1	1	1
Getafe	0.2963	0.8680	9	5	5	4
Málaga C.F.	0	0.5570	20	18	19	18
Osasuna	0.1728	0.8470	4	6	6	5
R.C. Celta de Vigo	0.0370	0.7917	6	9	9	7
R.C.D. Mallorca	0.0123	0.7380	13	12	12	12
Racing de Santander	0	0.6458	16	14	14	13
Real Betis	0	0.6091	14	17	17	16
Real Madrid	0.6173	0.9150	2	2	2	2
Real Sociedad	0	0.0123	17	20	20	20
Real Zaragoza	0.0123	0.7417	11	10	11	11
Sevilla F.C.	0.0123	0.7769	5	11	10	10
Valencia C.F.	0.4815	0.9070	3	3	3	3
Villarreal	0.0864	0.8293	7	7	7	6

The ranking given by the index Y coincides with the ranking collected in the table for the index CK when the number of α-cuts is greater or equal to 18. As can be appreciated, the indexes display high coefficients and therefore largely explain how the teams performed in the Spanish football league. However, our method achieved slightly better results than the Chen-Klein and Yager methods.

For the indexes described in Sec.3, once the intervals $E_j(\alpha)$ are obtained for each α-cut, a possibly large number of arithmetic operations must be performed, while the number of operations is considerably lower with our method.

To calculate the scores of the 81 cases, a file was created in GAMS[24] with loops that are capable of solving all 81 problems for each DMU. CPLEX 7.0 was used to solve the linear problems and the 1,620 problems in this exercise took 138 seconds to be resolved on a Pentium IV personal computer.

7. Conclusions

One of the main disadvantages of using the DEA technique, as with all deterministic models, is model data consistency. Many procedures have been used to overcome this problem. This research presents another method.

A decision-maker is often unable to allocate probabilities or possibilities for the realisation of inputs and outputs and simply knows extreme values for the model. The use of the statistical analysis model is fully justified in these cases and this is also when it is most useful.

One of the advantages for many end-users of DEA is that there are commercial packages (DEA-Solver, DEAP, etc.) that make it possible to obtain the results of an analysis quickly and without having to use any type of mathematical instrument. This *ad hoc* use of DEA models prevents the exploration of possibilities of a broader analysis.

From an end-user point of view, the presented model suffers from the disadvantage of not being usable "mechanically" because applying the model requires the creation of a GAMS file with loops to calculate the 81 cases for each of the units together. However, the time required is minimal given that the problem with 20 DMUs and their corresponding cases took a Pentium IV personal computer 138 seconds to compute the results, or 0.08 seconds per linear problem. However, from a computational viewpoint, we must bear in mind that the data does not require time-consuming preparation, unlike other techniques.

Statistical data analysis makes it easy to rank units in such a way that the units that are most often efficient, according to traditional DEA criteria, can be identified immediately. Furthermore, should two units be tied in terms of inefficiency, we have a second criterion that also enables a ranking to be made.

The results of the computational tests provide comparative information with respect to other criteria in which there are only small variations in the ranking. The tests have the advantage of being usable in non-fuzzy environments and are easier to calculate than making individual α-cuts for each problem.

Finally, we must highlight the fact that our model may be immediately applied to real decision making problems where it is necessary to rank units involved in a process. In this sense, in the Spanish football application (in which team efficiency is very clearly reflected by the final league classification), the ranking provided by our method resemble the final league classification for the 2005/06 season more closely than the results of the Chen-Klein method for the same choice of tolerances.

Acknowledgments

We wish to thank Maria Teresa Lamata for her valuable help. We would like to thank the referees for their useful comments and suggestions. In addition, we would like to express our thanks to GECA Sport for generously supplying the data used in this study. The authors also wish to acknowledge partial financial support through projects TIN2008-06872-C04-02, NOVEDAR (Ref. CSD2007-00055), ECO2009-09569, and ECO2008-04669 from the Spanish government.

References

1. M. Bonilla, T. Casasús, A. Medal and R. Sala, An efficiency analysis with tolerances of the Spanish Ports System, *Int. J. Transport Economics* **31** (2004) 379–400.
2. C. Kao and S. T. Liu, Fuzzy efficiency measures in data envelopment analysis, *Fuzzy Sets and Systems* **119** (2000) 149–160.
3. T. León, V. Liern, J. Ruiz and I. Sirvent, A possibilistic programming approach to the assessment of efficiency with DEA models, *Fuzzy Sets and Systems*, **139** (2003) 407–419.
4. J. K. Sengupta, A fuzzy systems approach in data envelopment analysis, *Computers and Mathematics with Applications*, **24** (1992) 259–266.
5. I. Sirvent, Algunos aspectos del problema de la evaluación de la eficiencia con modelos DEA, Ph. D. thesis, Universidad Miguel Hernández, 2001.
6. K. Triantis and O. Girod, A mathematical programming approach for measuring technical efficiency in a fuzzy environment, *Journal of Productivity Analysis* **10** (1998) 85–102.
7. A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research*, **2** (1978) 429–444.
8. M. Bonilla, T. Casasús, A. Medal and R. Sala, Traffic in Spanish ports: an efficiency analysis, *Int. J. Transport Economics* **29** (2002) 215–230.
9. J. E. Boscá, V. Liern, A. Martínez and R. Sala, Increasing offensive or defensive efficiency? An analysis of Italian and Spanish football, *Omega* **37** (2009) 63–78.
10. J. E. Boscá, V. Liern, A. Martínez and R. Sala, Analysis and evolution of efficiency in Spanish Soccer League (200/01–2007/08), *Journal of Quantitative Analysis in Sports* **5** (2009) 1–22.
11. A. Charnes, W. W. Cooper, A. Y. Lewin, R. C. Morey and J. Rousseau, Sensitivity and stability in DEA, *Annals of Operations Research* **2** (1985) 139–156.
12. J. K. Slowiński, *Fuzzy Sets in Decicision Analysis, Operations Research and Statistics* (Kluwer, Boston, 1998).
13. L. A. Zadeh, Fuzzy Sets, *Information and Control* **8** (1965) 338–353.
14. X. Wang and E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), *Fuzzy Sets and Systems* **118** (2001) 375–385.

15. X. Wang and E. Kerre, Reasonable properties for the ordering of fuzzy quantities (II), *Fuzzy Sets and Systems* **118** (2001) 387–405.
16. R. R. Yager, A procedure for ordering fuzzy subsets of the unit interval, *Information Sciences* **24** (1981) 143–161.
17. L. Campos and A. Munoz, A subjective approach for ranking fuzzy numbers, *Fuzzy Sets and Systems* **29** (1989) 145–153.
18. J. A. Herrera and M. T. Lamata, A total order for the graded numbers used in decision problems, *Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems* **7** (1999) 267–276.
19. F. Choobineh and H. Li, An index for ordering fuzzy numbers, *Fuzzy Sets and Systems* **54** (1993) 287–294.
20. C. B. Chen and C. M. Klein, An efficient approach to solving fuzzy MADM problems, *Fuzzy Sets and Systems* **88** (1997).
21. J. Jablonsky, P. Fiala, Y. Smirlis and D. K. Despotis DEA with interval data: An illustration using the evaluation of branches of a Czech bank, *Central European Journal of Operations Research* **12** (2004) 323–337.
22. D. K. Despotis and Y. G. Smirlis, Data envelopment analysis with imprecise data, *European Journal of Operational Research* **140** (2002) 24–36.
23. A. Medal, Análisis de la eficiencia del Sistema Portuario Español: estructura, evolución y perspectivas, Ph. D. thesis, Universitat de València, 2010.
24. O. B. Olesen and N. C. Petersen, A presentation of GAMS for DEA, *Computers and Operations Research*, **23** (1996) 323–339.