Distributed control of a network of single integrators with limited angular fields of view

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Based on the literature titled ‘Distributed control of a network of single integrators with limited angular fields of view’ in Automatica (Journal of IFAC), Vol. 63, No. C, January 2016

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Introduction

- Multi-agent systems are a prime focus of research in the present world. In these type of systems, it is desired to design a local control law for each agent such that a global objective is achieved over the entire network with limited information exchange between agents.
- We shall be dealing with two such global objectives, namely consensus and containment.

Literature Review

- n Cao, Ren, and Egerstedt (2012) - Switching directed topology

- Ganguli, Cortés, & Bullo, 2009 - Radial FOV limitation

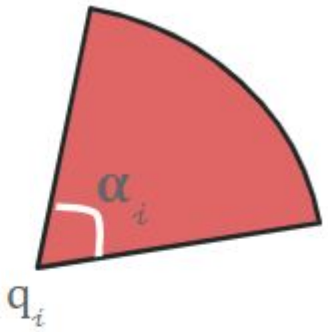
- Gerkey, Thrun, & Gordon, 2006; Lee & Chong, 2011; Ma & Liu, 2007 - Angular FOV limitation

Some Key Terminologies

- Converse of digraph : $G^* = (V, E^*)$ is the converse of the digraph $G = (V, E)$ if the vertex set V is same for both and the $(i, j) \in G^*$ iff $(j, i) \in G$.
- Mirror of digraph : Mirror of the digraph $G=(V, E)$ is an undirected graph is $G^{\sim} = (V, E^{\sim})$ with $E^{\sim} = E \cup E^*$.
- Diameter of digraph : It is the maximum number of edges in the shortest path between two distinct vertices i and j in the graph.
- Quasi Strongly Connected : A digraph is said to be QSC if there exists a vertex from which every two distinct vertices i and j are reachable.
- Convex hull : It is the smallest convex polygon that contains all the points inside it.
- A dynamic interaction digraph $G_{\sigma(t)}$ is uniformly quasi-strongly connected (UQSC) if there exists $T > 0$ such that the union digraph $\cup (\tau \in [t, t+T\,]) G(\tau\,)$ is QSC for all t and $\sigma(t) \in \Sigma dwell(\tau_D)$.

Problem Statement

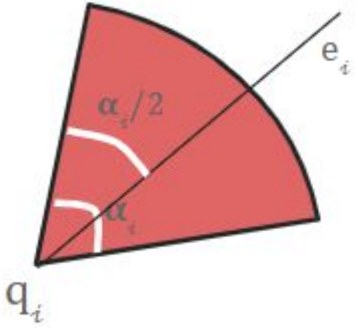
We are given N vehicles, all with built-in or mounted sensors but there is a constraint on the sensing region of the sensors that the angle of sensing is limited by an angular value - $\boldsymbol{\alpha}_i$ where $i \in \{1, 2, 3 \ldots\ldots, N\}$ and $\boldsymbol{\alpha}_i \in [0, 2\pi)$. We have to analyze the given situation for the consensus and containment problems.



Determining Neighbours

The 2D sector for each vehicle:

$$\Omega_i = \left\{ q \in R^2 \;\middle|\; \frac{\left| < e_i, (q - q_i) > \right|}{\| q - q_i \|} \geq \cos\left(\frac{\alpha_i}{2}\right) \right\}$$



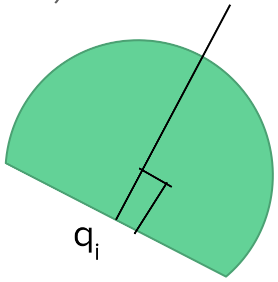
where, $q = [x\ y]'$, $q_i = [x_i\ y_i]'$ and e_i is the unit vector along heading angle of the sensor (note that it is not necessarily that of the vehicle) or in other terms, e_i is the unit vector along bisector of the sensor field of view.

Sensors with Half Plane Field of View

Instead of addressing the original version of the problem directly, Let's first understand how we solve the problem for a smaller version and later try to extend it to the original problem.

Let's Assume: Sensing Angle of all sensors = 180 degrees or in other terms,

$$\boldsymbol{\alpha}_i = \pi, \text{ where } i \in \{1, 2, 3 \ldots\ldots, N\}$$



System model :

We shall consider our usual single integrator model i.e. ,

$$\dot{q}_i(t) = u_i(t), \text{ where } i \in \{1, 2, 3 \ldots\ldots, N\}$$

Note: In this case we have our vehicle heading angle = sensor pointing angle.

Consensus In Half Plane FOV

We know that we shall be able to achieve consensus if the digraph G is Quasi Strongly Connected.

Statement: If d_{in} (for all nodes) >= ceil((n-1)/2)=d' then digraph is QSC. (Note that this is only a sufficiency condition and is not the complete set of solution for digraph being QSC).

Can be proved by contradiction. Let's say d_{in}>=d' and digraph is not QSC.

If digraph is not QSC then for some two nodes (u,v) :

$$\text{Reachability}(u) \cap \text{Reachability}(v) = \phi$$

Which implies that - Reachability(u) + Reachability(v)<=n-2 for digraph to be non-QSC.

But, As we have d_{in}>=d' , Reachability(u)>=d' and Reachability(v)>=d'

We have, Reachability(u) + Reachability(v)>=ceil(n-1)>n-2. Hence Contradicts.

Control law

1. For, Consensus problem we define,

$$u_i(t) = \begin{cases} r_i(t)\ ,\ if\ d_i^{in}(t) \geq \tilde{d} \\ r_i'(t)\ ,\ if\ d_i^{in}(t) < \tilde{d} \end{cases}$$

Where, $r_i(t) = \sum_{j \in N_i(t)} (q_j - q_i)$; $\tilde{d} = ceil\left(\frac{(n-1)}{2}\right)$;

$r'_i(t) = r_i(t)$ after rotating the vehicle by 180 degrees $= r_i(t^+)$

For theoretical analysis, we can impulsively rotate as many times as we want but keeping practical restrictions we shall introduce a positive constant, **δ** a small minimum time that we shall wait for before switching digraph.

$$u_i(t) = \begin{cases} r_i(t)\ ,\ if\ d_i^{in}(t) \geq \tilde{d} \lor t < \varrho_i(t) + \delta \\ r_i'(t)\ ,\ if\ d_i^{in}(t) < \tilde{d} \land t \geq \varrho_i(t) + \delta \end{cases}$$

Where,

$\rho_i(t) = \{$time of last digraph switch of i'th vehicle$\}$

2. Containment problem

Given N_L leaders (stationary) and N_F followers we have to design a control law such that the followers converge into the convex hull formed by the leaders.

Same control as in Consensus but shall have one excess condition of $N_L >= N_F$ in order to guarantee containment.

Proof Idea: In each digraph if we have at least one leader then we can guarantee containment occurrence so let's equate d' to the worst possible case of exhausting all followers then we will have to have 1 leader always. That will gives us the above condition.

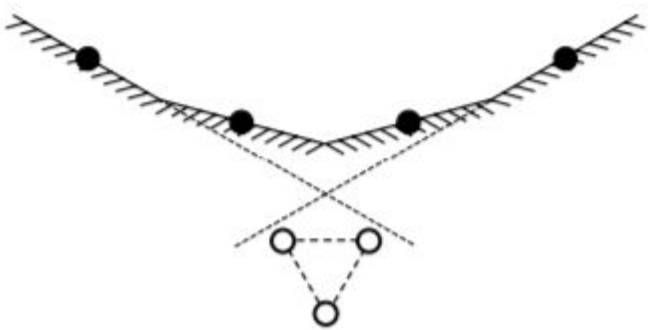


Fig adopted from 'Distributed control of a network of single integrators with limited angular fields of view' in Automatica (Journal of IFAC), Vol. 63, No. C, January 2016

Sensors with Heterogeneous Field of View

Now let us extend our solution to a more general case where the robots do not necessarily have a half plane FOV, and also it is not necessary that they all have the same FOV.

- Due to non half plane FOVs, switching is not enough to cover the entire plane.
- It is assumed that all the FOVs are rotating with a constant angular velocity

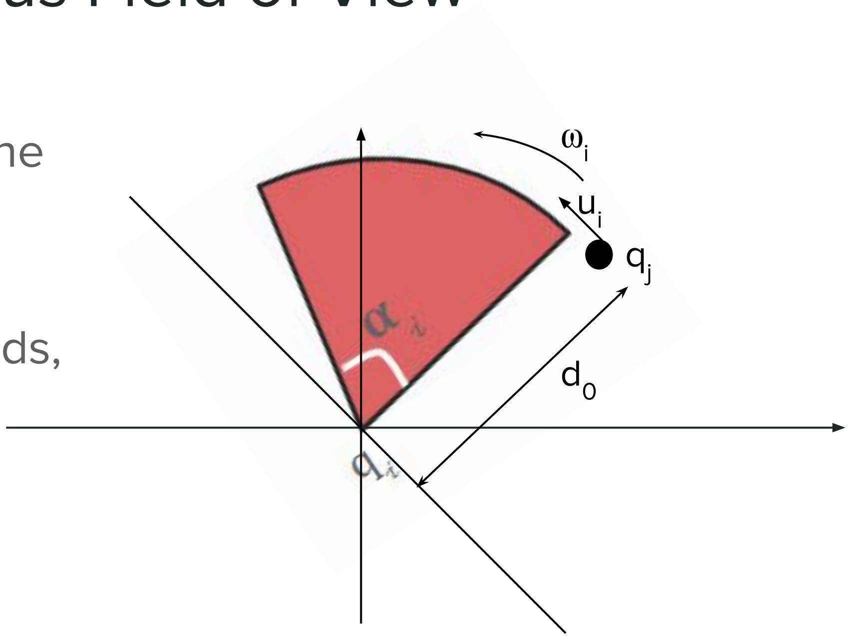
Hence our single integrator model now has one more component-

$$\dot{q}_i(t) = u_i(t), \text{ where } i \in \{1, 2, 3 \ldots\ldots, N\}$$

$$\dot{\theta}_i(t) = \omega_i(t)$$

Consensus in Heterogeneous Field of View

We know that an agent j is said to be the neighbour of agent i if there exists a directed path from j to i or in other words, agent i can detect agent j using its limited FOV.



$$\| q_i(t) - q_j(t) \| > \frac{3\pi u_o}{\omega_o}$$

$$\| u_i(t) \| < u_0$$

Control law

$$u_i(t) = \sum_{j \in N_i(t)} a_{ij}(q)\left(q_j - q_i\right)$$

, where $a_{ij}(q) = \dfrac{k}{1 + \sum_{j \in N_i(t)} \left\| q_j - q_i \right\|}$

k is a positive finite constant

$$\left\| u_i(t) \right\| < k$$

$$\omega_0 = \frac{6\pi k}{\varepsilon}$$

Containment in Heterogeneous Field of View

Now let us extend previous result for the containment problem in a leader-follower network. We have N_L stationary leaders and N_F moving followers which we want to converge into the convex hull formed by the leaders.

- First task is to ensure that there is at least one leader $j \in V_L$ for each follower $i \in V_F$
- For this, we need to make sure that a follower is able to detect at least one leader in its field of view, which is a similar case to our previous problem and so the previously obtained result can be utilized here.

$$\omega_0 = \frac{3\pi k'}{\varepsilon'} \qquad \|q_i - q_j\| > \varepsilon'$$

Where, k' is a positive constant

ε' is the radius of the smallest circle enclosing all the leaders

$i \in V_F$, $j \in V_L$

Control law

$$u_i(t) = \sum_{j \in N_i(t)} a_{ij}(q)\left(q_j - q_i\right)$$

, where

$$a_{ij}(q) = \frac{k'}{\left(N_L + N_F - 1\right)R}$$

k’ is a positive finite constant

R = largest distance between all pairs in the initial configuration

Input values

Followers(half plane), xy_f = [x y θ] , where θ is the heading angle

Followers(heterogeneous FOVs), xy_f = [x y θ ∝], where is the FOV

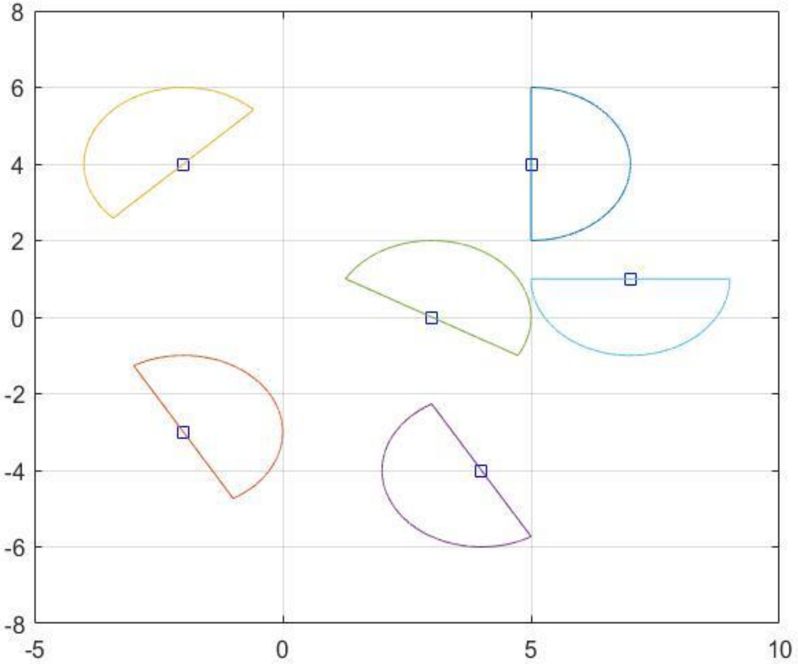
Leaders(both cases) = xy_l = [x y]

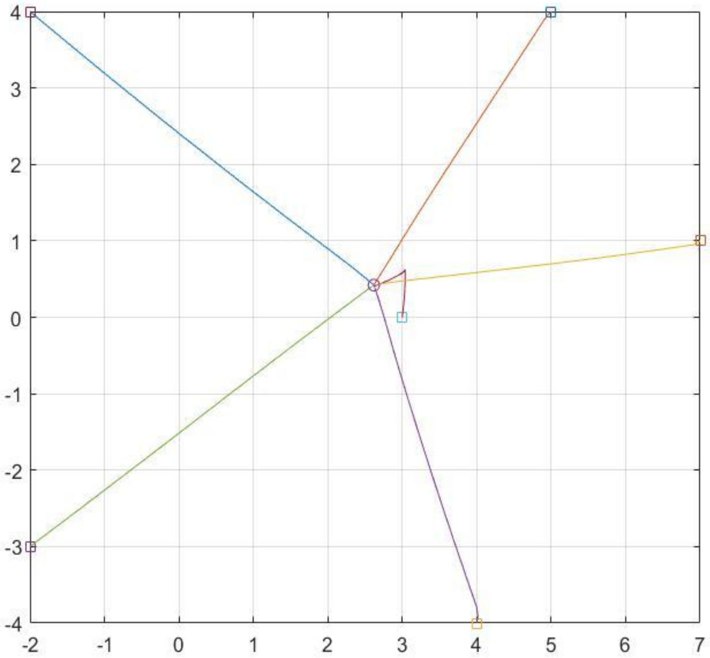
$$xy\_f = \begin{bmatrix} 5 & 4 & 0 \\ -2 & -3 & 30 \\ -2 & 4 & 135 \\ 4 & -4 & 210 \\ 3 & 0 & 60 \\ 7 & 1 & 270 \end{bmatrix}$$

$$xy\_f = \begin{bmatrix} 5 & 4 & 0 & \frac{\pi}{4} \\ -2 & -3 & 30 & \pi \\ -2 & 4 & 135 & \frac{\pi}{2} \\ 4 & -4 & 210 & \frac{2\pi}{3} \\ 3 & 0 & 60 & \frac{\pi}{3} \\ 7 & 1 & 270 & \frac{3\pi}{4} \end{bmatrix}$$

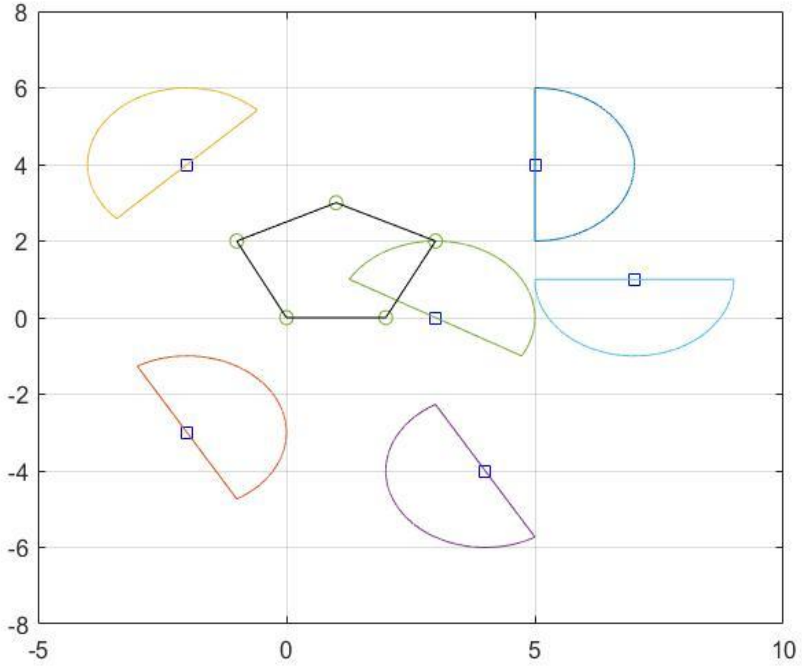
$$xy\_l = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ -1 & 2 \\ 1 & 3 \\ 3 & 2 \end{bmatrix}$$

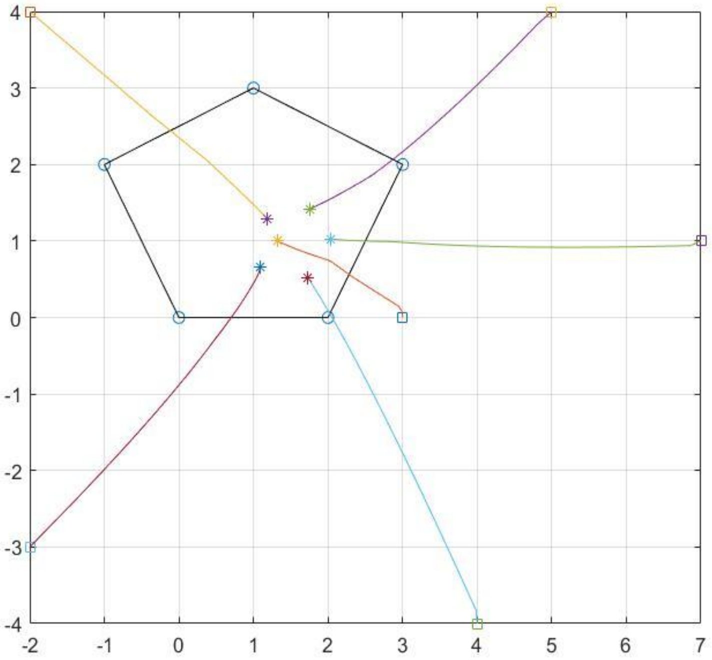
Simulation results



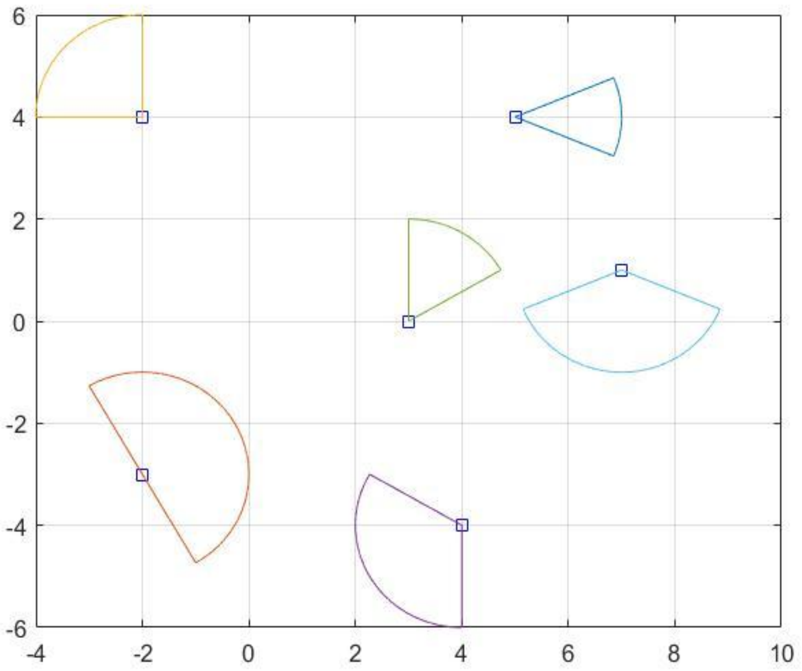


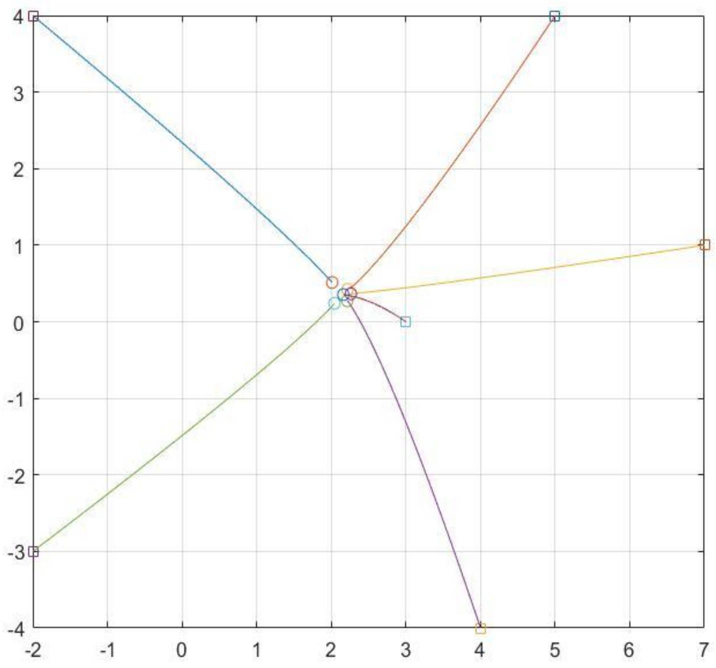
Containment with Half plane FOVs



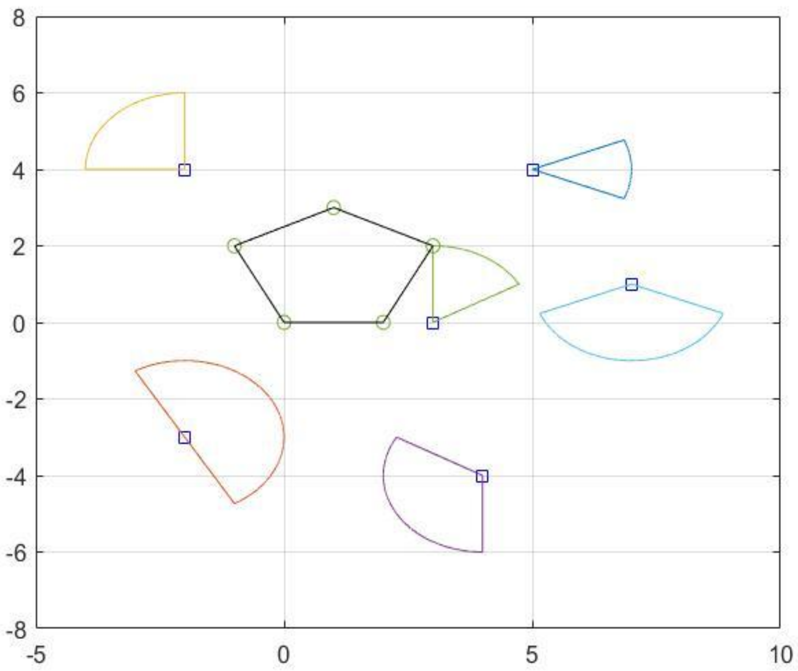


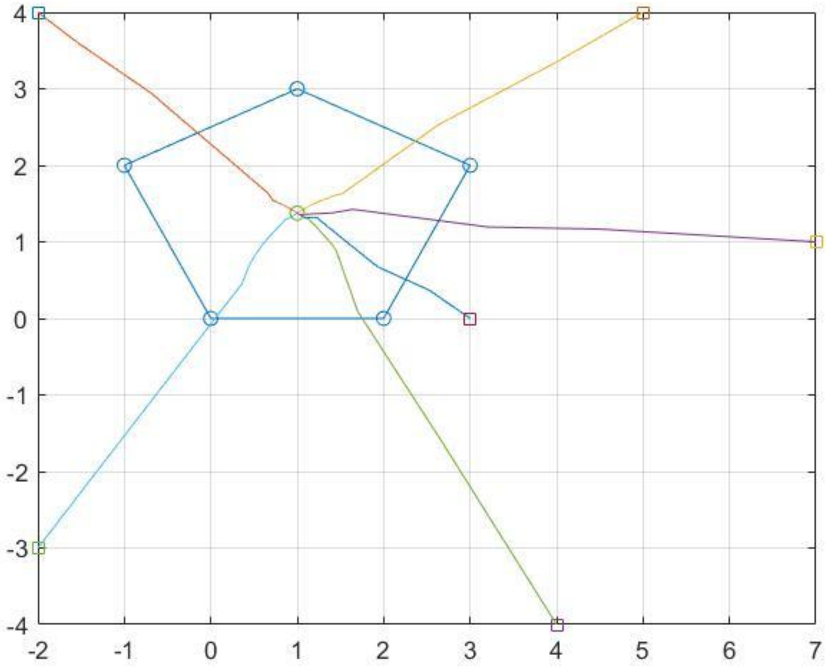
Consensus with Heterogeneous FOVs





Containment with Heterogeneous FOVs





Conclusion

- Cooperative control strategy is developed for a network of single integrator agents with sensing limitations in terms of their FOVs.
- For the special case of half plane FOVs, an impulsive switching control was developed to maintain QSC of the sensing graph.
- A consensus strategy is developed which is then modified for containment problem for a leader-follower network.
- These results are then extended to a general case of heterogeneous FOVs.
- Appropriate lower bound on the angular FOVs was proposed to achieve consensus and then modified appropriately for containment problem.

Future work

- One can explore these problems by combining radial sensing limitation and angular FOV limitations.
- Considering more complex situations like time delay in sensing, moving leaders, etc.
- Automatic path adjustment based on opaque objects sensed in path.

References

- Mohammad Mehdi Asadi, Amir Ajorlou, Amir G. Aghdam (2016), Distributed control of a network of single integrators with limited angular fields of view. In Automatica 63 (2016) 187–197
- ode45 function which is used to solve the first order differential equation for the given problems

Thank you