Networked Dynamical Systems and Control
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Distributed control of a network of single integrators with limited angular fields of view

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***Abstract*—In the single integrator multi-agent systems with angular field of view (FOV) constraints for their sensing regions, the consensus and containment problems shall be analyzed in this paper. Firstly, we shall assume that all FOVs are half-planes (almost like humans) and then we shall develop an impulsive switching strategy such that the underlying sensing graph of the network remains uniformly quasi-strongly connected (UQSC) throughout the system evolution. The control laws for such frameworks with switched interconnected systems are designed in such a way that the objectives of consensus and containment are achieved throughout network. Then, we shall extend the problem in order to address a network of single-integrator agents with limited heterogeneous angular FOVs, on the assumption that the FOVs of all sensing devices rotate with sufficiently large angular velocities, which can be controlled independently along with the translational motion of all agents. The velocity vector and the lower bound on the magnitude of the angular velocity of the FOVs are designed so that the agents shall converge to an arbitrarily small ball, and achieve consensus. The convergence of the moving followers to the convex hull of static leaders is addressed for the containment problem as well. We shall simulate the results in order to verify the effectiveness of our proposed control strategies.**

***Index Terms*—consensus problem, containment problem, coordination and control, graph theory, vehicle angular limitations, stability analysis**

I. Introduction

In this project, we shall be dealing with the consensus and containment problem in a single integrated multi-agent system having constraints in their sensing capabilities, namely their angular field of view (FOV). By a single integrated system, we mean that the quantity of interests which in our case are the position ($q_i(t) = [x_i(t), y_i(t)]^T$) and velocity ($u_i(t)$) vectors of the i'th agent, are dependent on first derivatives as :

$$\dot{q}_i = u_i(t) \tag{1}$$

In the consensus problem, we desire the agent's certain state dependent quantity to reach a common value and in the containment problem, it is desired that the follower robots converge to the convex of the leader robots via mutual coordination. Both these aspects of coordination control techniques have vast real life applications such as formation flight of UAVs, carrying out search operations in extreme places where it is not possible for humans to carry out the operation manually, and many more.

Connectivity of the information flow graph plays a crucial role in achieving the global objective(s) of the multi-agent network. Fagnani Zampieri, 2009; Munz, Papachristodoulou, Allgöwer, 2011 discussed about variations in interaction topologies over time. Uniform connectivity on a fixed or switching directed topology is proposed in Cao, Ren, and Egerstedt (2012). Ganguli, Cortés, Bullo, 2009 talks about constraint on the field of view (FOV) of the sensing/communication devices used in multi-agent systems. . The angular FOV limitation appears in multi-agent networks equipped with certain sensing devices, this was addressed in Gerkey, Thrun, Gordon, 2006; Lee Chong, 2011; Ma Liu, 2007.

The different works done for the cooperative control of multi-agent systems with limited FOVs in prior to this however often made strong assumptions like assuming a fixed topology for the information graph, assuming that each agent is aware of the global knowledge of the network and connectivity in the network, assuming the existence of communication links apart from the sensors for information exchange, and assuming that different agents are distinguishable. In this project, no such assumptions shall be made and thus a more realistic scenario shall be considered. We will consider agents wherein the information flow is represented via a dynamic directed graph

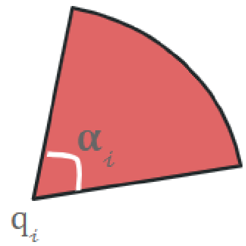

Fig. 1: 2D Conic area

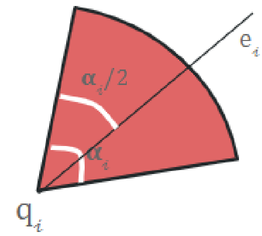

Fig. 2: Neighbour Determination

which could be connected as well as disconnected at any point of time. Also, no external communication links are needed apart from the sensors and all the agents are identical, hence no labeling is required. We will be focusing mainly on two things - Developing a control strategy such that the agents with half-plane FOVs reach a common location while preserving a quasi strong connectivity(QSC). This is done by a continuous rotation of the FOVs with sufficient angular velocity. The strategy developed in the previous part will then be generalized to solve the consensus and containment problems for agents with limited angular FOVs which are not all necessarily the same.

II. Some Key Terminologies

2D Conic area. It is the union of a set of half-lines that start at a common apex point q and extend to infinity. The apex angle $\alpha \in (0, 2\pi]$ is defined as the angle between the two half-lines which form the boundaries of the conic area.

FOV of an agent. FOV for i^{th} agent is characterized by its apex angle α_i and the apex point q_i, representing the position of agent i. Let $\hat{e}_i$ be a unit vector which passes through q_i and is directed toward the interior of a conic area such that it bisects the angle α_i. Denoted by Ω_i, mathematically it can be formulated as

$$\Omega_i = \left\{ q \in R^2 \mid \frac{\langle \hat{e}_i, (q-q_i) \rangle}{\|q-q_i\|} \geq \cos\frac{\alpha_i}{2} \right. \tag{2}$$

Convex hull. It is the smallest convex polygon that contains all the given points inside it. Mathematically, given a set of points $\tau = q_1, ..., q_n$ in R^m the convex hull of D is defined as

$$(D) = \left\{ \sum_{i=1}^{n} \lambda_i q_i \mid q_i \in \tau, \lambda_i \in R_{\geq 0}, \sum_{i=1}^{n} \lambda_i = 1 \right. \tag{3}$$

Compliment of a digraph G = (V,E) denoted by G^c = (V,E^c), is a digraph whose vertex set is the same as G, and (i, j) $\in E^c$ if and only if (i, j) $\notin$ E for every pair of distinct vertices i, j $\in V$.

Converse of a digraph G = (V,E) denoted by G^* = (V,E^*), is a digraph whose vertex set V is the same as G, and (i,j) $\in E^*$ iff (j,i) $\in E$ for every pair of distinct vertices i, j $\in$ V.

Mirror of a digraph G = (V,E) is an undirected graph $\widetilde{G} = (V,\widetilde{E})$ where the vertex set is the same as G and $\widetilde{E} = E \cup E^*$.

Diameter of an undirected graph G = (V,E) is the maximum number of edges in the shortest path between two distinct vertices i and j in the graph, that is, diam(G) = $max_{i,j \in V} d_G(i,j)$ where $d_G(i,j)$ denotes the number of edges in the shortest path connecting two distinct vertices i,j $\in$ V.

Quasi Strongly Connected. A digraph is said to be QSC if there exists a vertex from which every two distinct vertices i and j in the digraph are reachable.

Uniformly Quasi Strongly Connected. A dynamic interaction digraph $G_{\sigma(t)}$ is uniformly quasi-strongly connected (UQSC) if there exists $T > 0$ such that the union digraph $\cup_{\tau \in [t,t+T]} G(\tau)$ for all t and $\sigma(t) \in \sum_{dwell} \tau_D$

III. Agents with Half plane FOVs

Instead of addressing the original version of the problem directly, Let's first understand how we solve the problem for a smaller version and later try to extend it to the original problem.
Let's Assume: Sensing Angle of all sensors = 180 degrees or in other terms,

$$\alpha_i = \pi \text{ , where } i \in \{1, 2, 3......., N\}$$

System model : We shall consider our usual single integrator model i.e.

$$\dot{q_i} = u_i(t) \tag{4}$$

Note: In this case we can see that, the change in vehicle heading angle is equal to the change in sensor pointing angle as the angle between the vehicle heading and the sensor heading angle is a fixed quantity.

In this case we shall consider the control input as per the usual relative distance based model in which we shall express the control input for each vehicle as the summation of relative positions of the vehicle with its neighbours, which converges to zero only when the vehicles achieve consensus.

A. Consensus

Criteria for Consensus: We shall be able to achieve consensus if the connectivity digraph G is Quasi Strongly

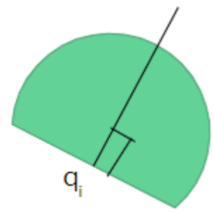

Fig. 3: Half Plane FOVs

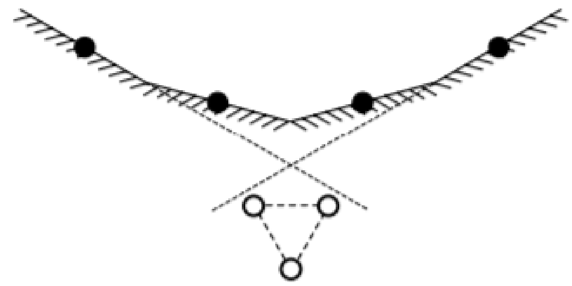

Fig. 4: Containment worst-case scenario for $n_L \geq n_f$ proof

Connected (QSC).

Statement 1: If $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$ then digraph is QSC. (Note that this is only a sufficiency condition and is not the complete set of solution for digraph being QSC).

Proof: Can be proved by contradiction. Let's say $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$ and digraph is not QSC. If digraph is not QSC then for some two nodes (u,v) : Reachability(u) $\cap$ Reachability(v) = ϕ. Which implies that - Reachability(u) + Reachability(v) $\leq$ (n-2) for digraph to be non-QSC. But, As we have $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$, therefore Reachability(u) $\geq$ d' and Reachability(v) $\geq$ d'. Hence we get, Reachability(u) + Reachability(v) $\geq$ (n-1) $>$ (n-2). Which Contradicts our initial assumption. Hence our assumption is false.

So, In addition to our regular control we make a slight addition of an impulsive switching strategy in which if for any vehicle at any point $d_i^{in} < \lceil \frac{n-1}{2} \rceil = d'$ then we shall impulsively rotate the field of view of the vehicle by π radians that will switch the field of view to cover the other half of the plane hence will automatically make sure that criteria for QSC i.e. $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$ is satisfied. Thereby ensuring that consensus is achieved.

So, the control law can be expressed as:

$$u_i(t) = \begin{cases} r_i(t) & d_i^{in} \geq d' \\ r_i'(t) & d_i^{in} < d' \end{cases} \tag{5}$$

where, r$_i(t)$ $=\Sigma_{j \in N_i}(q_j - q_i)$ and $r_i'(t) = r_i(t)$ after impulsive rotation of the vehicle by π radians, i.e., $r_i'(t) = r_i(t^+)$.

In practical cases we have constraints on number of impulsive switches happening in a particular time interval and hence we would like to limit the number of impulsive rotations to 1 in every δ seconds, where δ is a small positive constant time parameter. So, we shall start checking the QSC condition only after δ seconds have passed after an impulsive switch had ended. Therefore, the updated control law for practical scenarios is:

$$u_i(t) = \begin{cases} r_i(t) & d_i^{in} \geq d' \vee t < \varrho_i(t) + \delta \\ r_i'(t) & d_i^{in} < d' \wedge t \geq \varrho_i(t) + \delta \end{cases} \tag{6}$$

where, $\varrho_i(t)$ = time of last digraph switch for i'th vehicle

B. *Containment*

We shall apply the same control strategy as applied in the consensus subsection. But for containment we have an extra criteria that needs to be satisfied apart from the QSC condition for the consensus. The condition being: Each digraph of every follower must always contain info of at least 1 leader so as to have the presence of at least 1 leader in the followers control law at any time instant, in order to converge into the convex hull formed by the leaders. Applying this extra condition we can further prune the possible values of solutions that shall always guarantee to gives us the solution to the containment problem.

Scenario: Let's Consider that there are n_L stationary leaders and n_f moving followers. Hence we have $n = n_L + n_f$.

Statement 2: Due to the requirement of at least 1 leader in every followers digraph at every instant in order to achieve containment. This condition leads to the point that in order to guarantee containment the criteria: $n_L \geq n_f$ is to be satisfied along with the QSC condition of $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$ that we have discussed in the consensus subsection.

Proof: If we consider the worst possible case for d' in $d_i^{in} \geq \lceil \frac{n-1}{2} \rceil = d'$ in which all of d' shall be covered only by followers. So if we set d' as n_f then there will be at least 1 leader in every digraph as even if all followers are exhausted there will be 1 in-degree remaining to satisfy the QSC condition. Hence when we set d'= $n_f = \lceil \frac{n_L + n_f - 1}{2} \rceil$. We can obtain the criteria $n_L \geq n_f$ for containment.

Hence, The control law is the same as the one defined in Equation (6) for consensus subsection but with the additional constraint of: $n_L \geq n_f$ for guaranteed containment said above.

IV. Agents with Heterogeneous FOVs

Now we shall extend our previously developed solutions to a more general case where the robots do not necessarily have a half plane FOV, that is, the FOVs of agents need not be a half-plane or be the same for all agents. Due to non-half plane FOVs, switching is no more sufficient to cover the entire plane of motion and so now there is a new state variable in the system. It is assumed that FOVs of all the robots are rotating with a constant angular velocity.

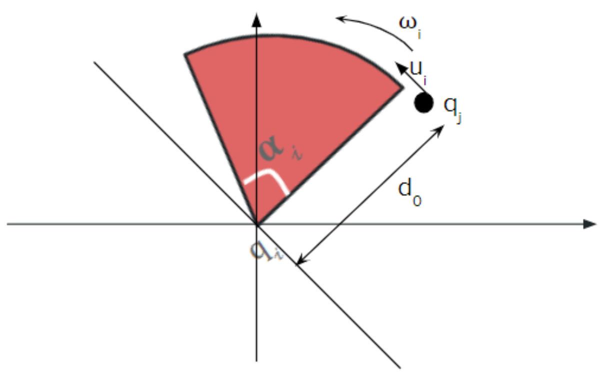

Fig. 5: Heterogenous FOVs

System Model : Now the system equations for the i^{th} agent is given by

$$\dot{q_i} = u_i(t), \tag{7}$$

$$\dot{\theta}_i = \omega_i(t) \tag{8}$$

A. *Consensus*

Criteria for Consensus: We shall be able to achieve consensus if the connectivity digraph G is Uniformly Quasi Strongly Connected (UQSC).

Statement 3: If the angular velocities ω_i is lower bounded by some constant ω_o, and all agent velocities $\|u_k(t)\|$ is upper bounded by some constant u_0 for $k \in \{1, 2, 3 \ldots\ldots, N\}$ then if we have, $\|q_i(t) - q_j(t)\| > \frac{3\pi u_o}{\omega_o}$ at time t, then $\exists\ t^{'}$ in $[t, t+T]$ such that $q_j \in \Omega_i(t')$ for all $T > \frac{3\pi}{\omega_o}$. In other words, the interaction interaction digraph is UQSC, which means that the j^{th} agent can be detected by the FOV of agent i at some time in the interval $[t, t+T]$.

Proof: Let us consider two agents, agent i denoted by q_i and agent j denoted by q_j (fig.5). d_o is the distance between the two robots at time t, that is, $d_o = \|q_i(t) - q_j(t)\|$. This is also the shortest path that agent j can take to leave the half plane (the plane formed by the line perpendicular to this shortest path). It is assumed that the forward velocities of all the agents is upper bounded by a constant value, that is, $\|u_i(t)\| < u_o$ for all t, therefore the minimum time required for agent j to leave the half-plane and reach an arbitrary point on the other half-plane is $T_{min} = \frac{d_o}{u_o}$. This also means that within $[t, t+T_{min}]$, it remains in the current half-plane. If we look at agent i, it's rotating FOV will cover this entire half plane if it rotates at least 3π radians, regardless of the apex angle α_i. This means that it will take at most, $T_{max} = \frac{3\pi}{\omega_o}$ to do this. Therefore, it is now very straight forward to notice that agent i will be able to detect agent j at some time instant $t^{'} \in [t, t+T]$ if it holds that $T_{min} > T_{max}$ for all $T \geq T_{max}$. This inequality gives a lower bound on the distance between agent i and j such that agent j is detectable and hence serves as the basis to determine the neighbours of agent i,

$$\|q_i(t) - q_j(t)\| > \frac{3\pi u_o}{\omega_o} \tag{9}$$

There is a need for a slight adjustment in statement 3 to be relevant in our current problem as in statement 3 we have considered the vehicle q_i to be having only rotational motion and no translational motion, then we derived the criteria of ω_o based on it for neighbour detection but in our case the vehicle of consideration q_i also moves translationally.

Statement 4: In order to address this issue we shall use the concept of relative velocity and make the vehicle of consideration q_i to have no translational velocity but have only angular velocity. Relatively now any vehicle j will have the velocity vector as the sum of the vectors: $u_j(t) + u_i(t)$. Now we can apply the condition of statement 3 and here we have $\|u_i(t)\| \leq u_o$, $\|u_j(t)\| \leq u_o$ so we can say $u_j(t) + u_i(t) \leq 2u_0$. So the upper bound on translation velocity is the only change from statement 3. Hence the updated inequality that gives a lower bound on the distance between agent i and j such that agent j is detectable and hence serves as the basis to determine the neighbours for a translationally moving agent i is :

$$\|q_i(t) - q_j(t)\| > \frac{6\pi u_o}{\omega_o} \tag{10}$$

So the control law in this case can be expressed as:

$$u_i(t) = \Sigma_{j \in N_i} a_{ij}(q)(q_j - q_i) \tag{11}$$

where, $a_{ij}(q) = \frac{\kappa}{1+\Sigma_{j\in N_i}\|q_j - q_i\|}$, κ is a finite positive constant. Also here we can observe that $\|u_i(t)\| \leq \kappa = u_0$.

B. *Containment*

Now let us extend previous result for the containment problem in a leader-follower network. We have N_L stationary leaders and N_F moving followers which we want to converge into the convex hull formed by the leaders. Our first task is to ensure that there is at least one leader j $\in V_L$ for each follower i $\in V_F$. For this, we need to make sure that a follower is able to detect at least one leader in its field of view, which is a similar case to our previous problem and so the previously obtained result can be utilized here.

Statement 5: Let $\epsilon^{'}$ be the radius of the smallest circle enclosing all leaders, and $\kappa^{'}$ be a positive finite constant representing an upper bound on the magnitude of the velocity vector of all followers. If the angular velocity of the FOV of all followers is lower-bounded by $\bar{\omega_o} = \frac{3\pi\kappa^{'}}{\epsilon^{'}}$ then there is at least one leader $j \in V_L$ for each follower $i \in V_F$ such that a directed path exists from j to i in the union sensing digraph $\cup_{\tau\in[t,t+\bar{T}]} G(\tau)$ for all $\bar{T} \geq \frac{\epsilon^{'}}{\kappa^{'}}$

Proof: We prove this by contradiction. Suppose $\exists$ a follower $i \in V_F$ such that its distance from all leaders is less than $\epsilon^{'}$, that is, $\|q_j - q_i\| < \epsilon^{'}$ for all $j \in V_L$. This in turn means that there is a circle centered at q_i which encompasses all the leaders and its radius is less than $\epsilon^{'}$ but this contradicts the definition of $\epsilon^{'}$. Therefore, it is now easy to say that $\exists$ at least one leader j for each follower i such that $\|q_j - q_i\| > \epsilon^{'}$. Now consider a fixed leader j and a moving follower i such that $\|q_j - q_i\| > \epsilon^{'}$ for some time t, and the forward velocity of the follower i is upper bounded by a fixed constant, that is, $\|u_i(t)\| < \kappa^{'}$ for all t, then it follows from statement 5, if the magnitude of the angular velocity ω_i is lower-bounded by $\bar{\omega_0} = \frac{3\pi\kappa^{'}}{\epsilon^{'}}$ then there exists $t^{'} \in [t, t+\bar{T}$ such that $q_j \in \Omega_i(t^{'}$ for every $\bar{T} > \frac{\epsilon^{'}}{\kappa^{'}}$, which means that there exists a directed path of length one from j to i in the union sensing digraph $\cup_{\tau\in[t,t+\bar{T}]}$. This completes the proof.

So the control law in this case can be expressed as:

$$u_i(t) = \Sigma_{j\in N_i} a_{ij}(q_j - q_i) \tag{12}$$

where, $R = max_{i,j\in V}\|q_i(t) - q_j(t)\|$, $a_{ij} = \frac{\kappa'}{(n_L+n_f-1)R}$, κ' is a finite positive constant. Also here we can observe that $\|u_i(t)\| \leq \kappa' = u_0$.

V. Simulation Results

In both Half-plane FOV's as well as Heterogeneous FOV's simulations we have considered the following values: $N_f = 6$, $N_L = 0$ or in other terms $N = 6$ for consensus and $N_f = 6$, $N_L = 5$ for containment.

Also, The position values for leaders is taken common for Half-plane FOV's as well as Heterogeneous FOV's containment problem and are as follows:

$$[x, y]_{leaders} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ -1 & 2 \\ 1 & 3 \\ 3 & 2 \end{bmatrix}$$

The example simulated for Half-plane FOVs is:

$$[x, y, \theta]_{followers} = \begin{bmatrix} 5 & 4 & 0 \\ -2 & -3 & 30 \\ -2 & 4 & 135 \\ 4 & -4 & 210 \\ 3 & 0 & 60 \\ 7 & 1 & 270 \end{bmatrix}$$

The example simulated for Heterogeneous FOVs is:

$$[x, y, \theta, \alpha]_{followers} = \begin{bmatrix} 5 & 4 & 0 & \frac{\pi}{4} \\ -2 & -3 & 30 & \pi \\ -2 & 4 & 135 & \frac{\pi}{2} \\ 4 & -4 & 210 & \frac{2\pi}{3} \\ 3 & 0 & 60 & \frac{\pi}{3} \\ 7 & 1 & 270 & \frac{3\pi}{4} \end{bmatrix}$$

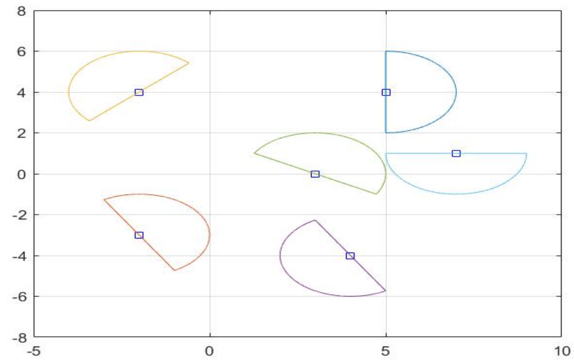

Fig. 6: Input Plot for Half Plane Consensus

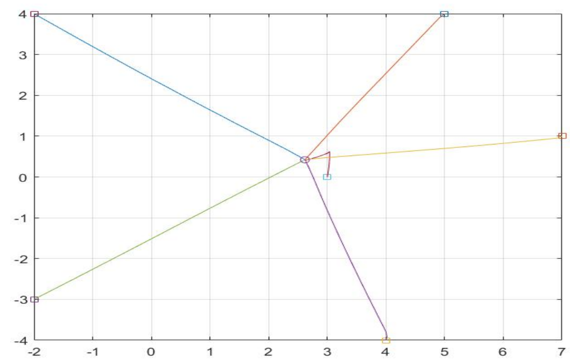

Fig. 7: Output Plot for Half Plane Consensus

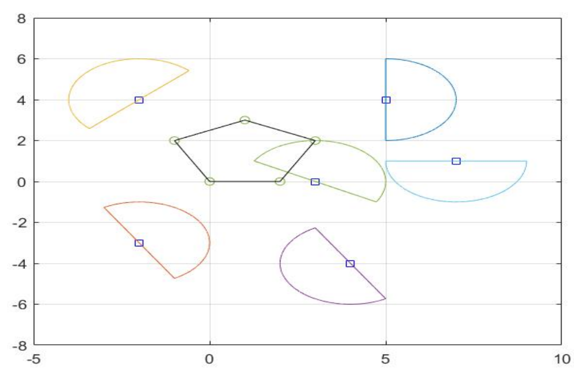

Fig. 8: Input Plot for Half Plane Containment

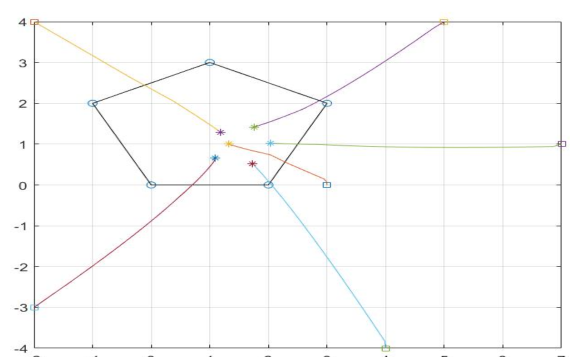

Fig. 9: Output Plot for Half Plane Containment

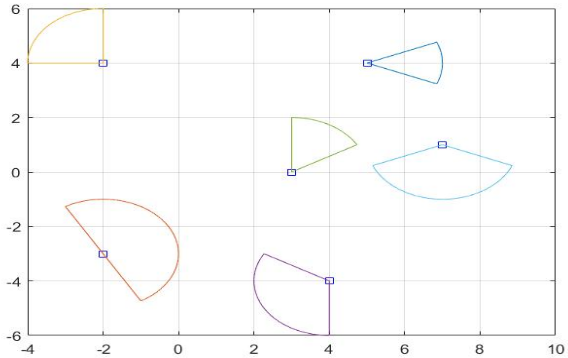

Fig. 10: Input Plot for Heterogeneous Consensus

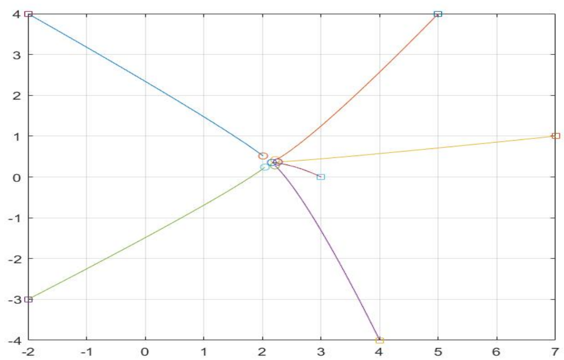

Fig. 11: Output Plot for Heterogeneous Consensus

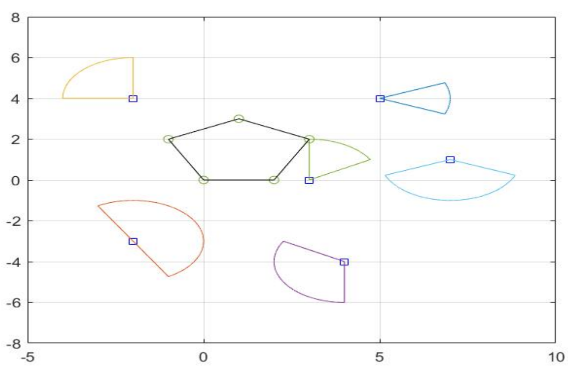

Fig. 12: Input Plot for Heterogeneous Containment

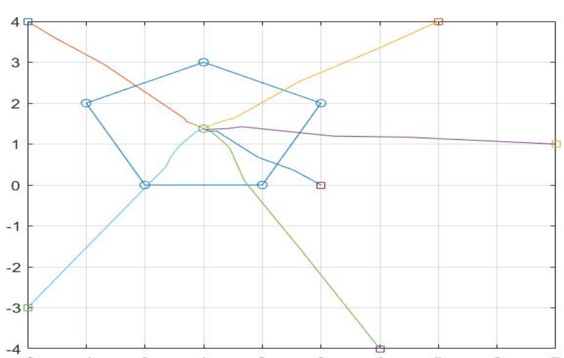

Fig. 13: Output Plot for Heterogeneous Containment

VI. Conclusion

- Cooperative control strategy is developed for a network of single integrator agents with sensing limitations in terms of their FOVs.
- For the special case of half plane FOVs, an impulsive switching control was developed to maintain QSC of the sensing graph.
- A consensus strategy is developed which is then modified for containment problem for a leader-follower network.
- These results are then extended to a general case of heterogeneous FOVs.
- Appropriate lower bound on the angular FOVs were proposed to achieve consensus and then modified appropriately for containment problem.

VII. Future Work

- One can explore these problems by combining radial sensing limitation and angular FOV limitations.
- Considering more complex situations like time delay in sensing, moving leaders, etc.
- Automatic path adjustment based on opaque objects sensed in path.

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- ode45 function which is used to solve the first order differential equation for the given problems.