Social Media & Text Analysis

lecture 8 - Paraphrase Identification and Logistic Regression

CSE 5539-0010 Ohio State University

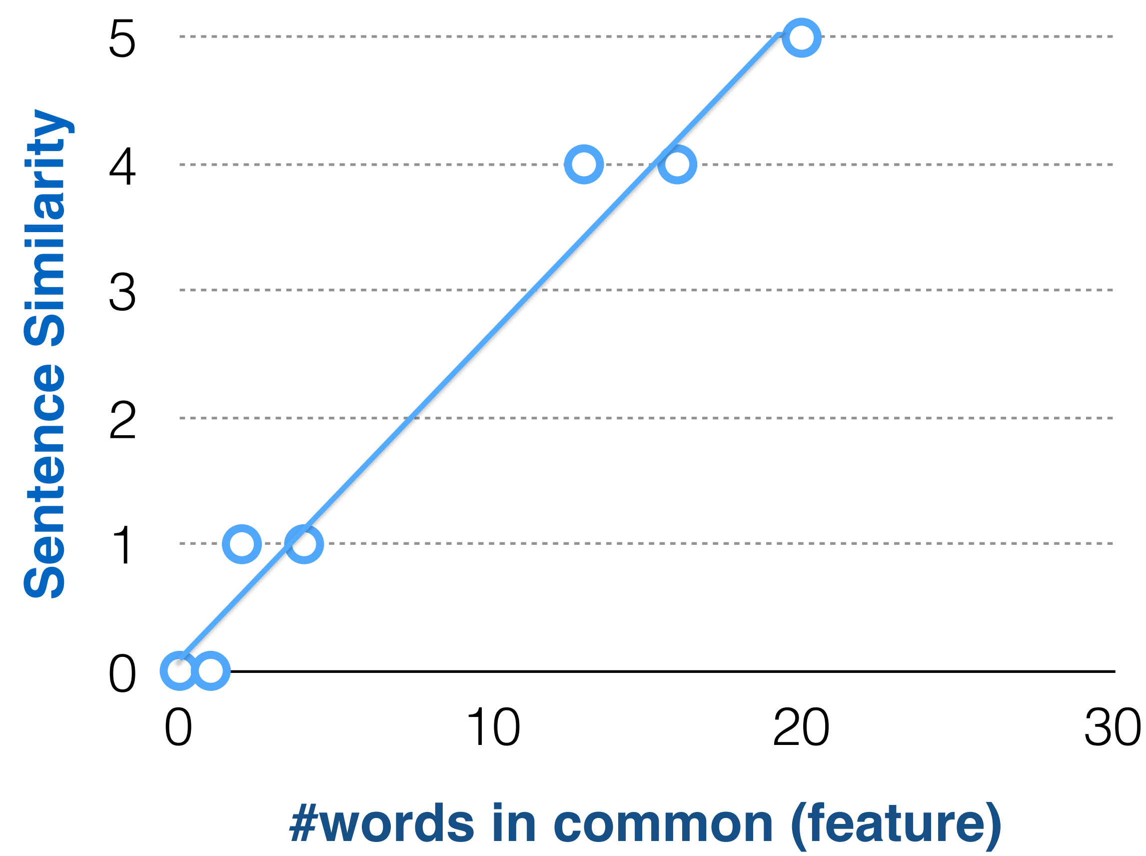
Instructor: Wei Xu

Website: socialmedia-class.org

Many slides adapted from Andrew Ng

(Recap)

Linear Regression



- also supervised learning (learn from annotated data)
- but for **Regression**: predict **real-valued** output
  (Classification: predict discrete-valued output)

(Recap)

Linear Regression

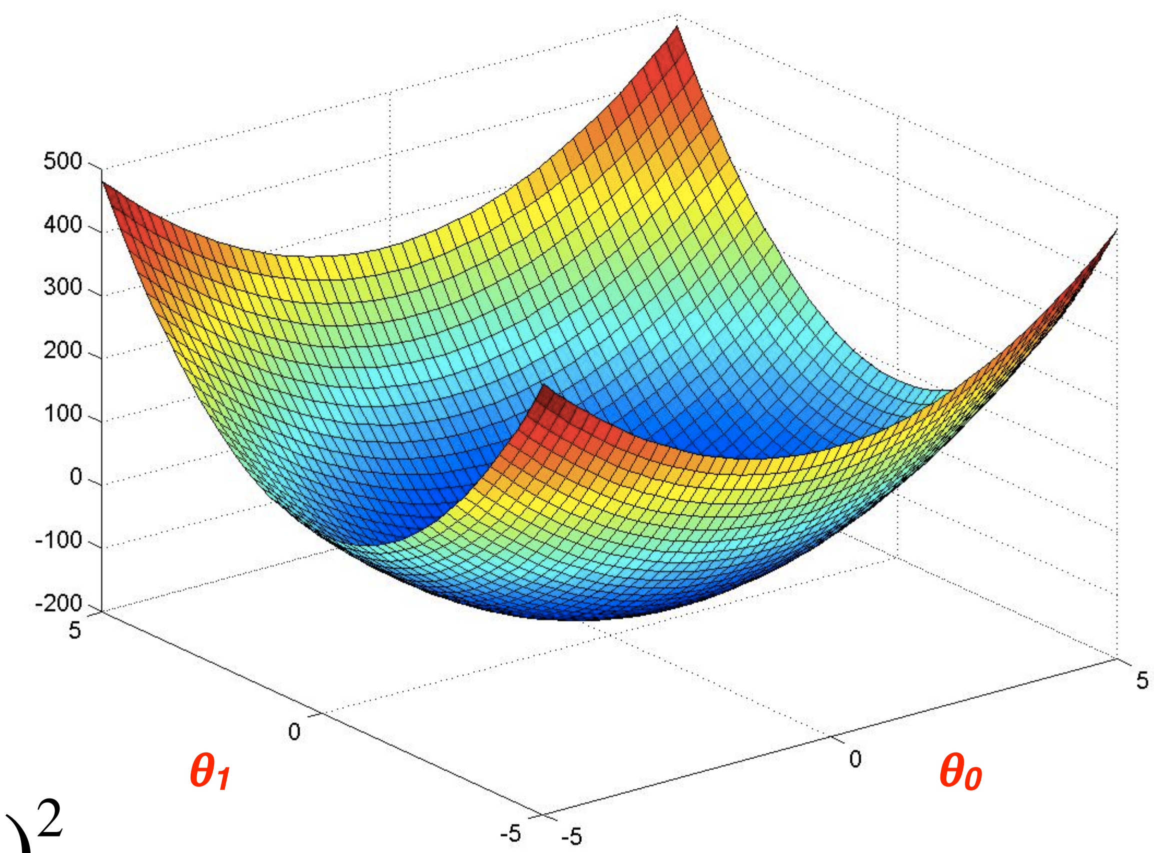
- **Hypothesis**:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- **Parameters:**

θ_0*, *θ_1

$J(\theta_1, \theta_2)$



- **Cost Function:**

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

- **Goal:** $\underset{\theta_0, \theta_1}{\text{minimize}} \, J(\theta_0, \theta_1)$

(Recap) Linear Regression w/ one variable:

Model Representation

#words in common (*x*)	Sentence Similarity (*y*)
1	0
4	1
13	4
18	5
…	…

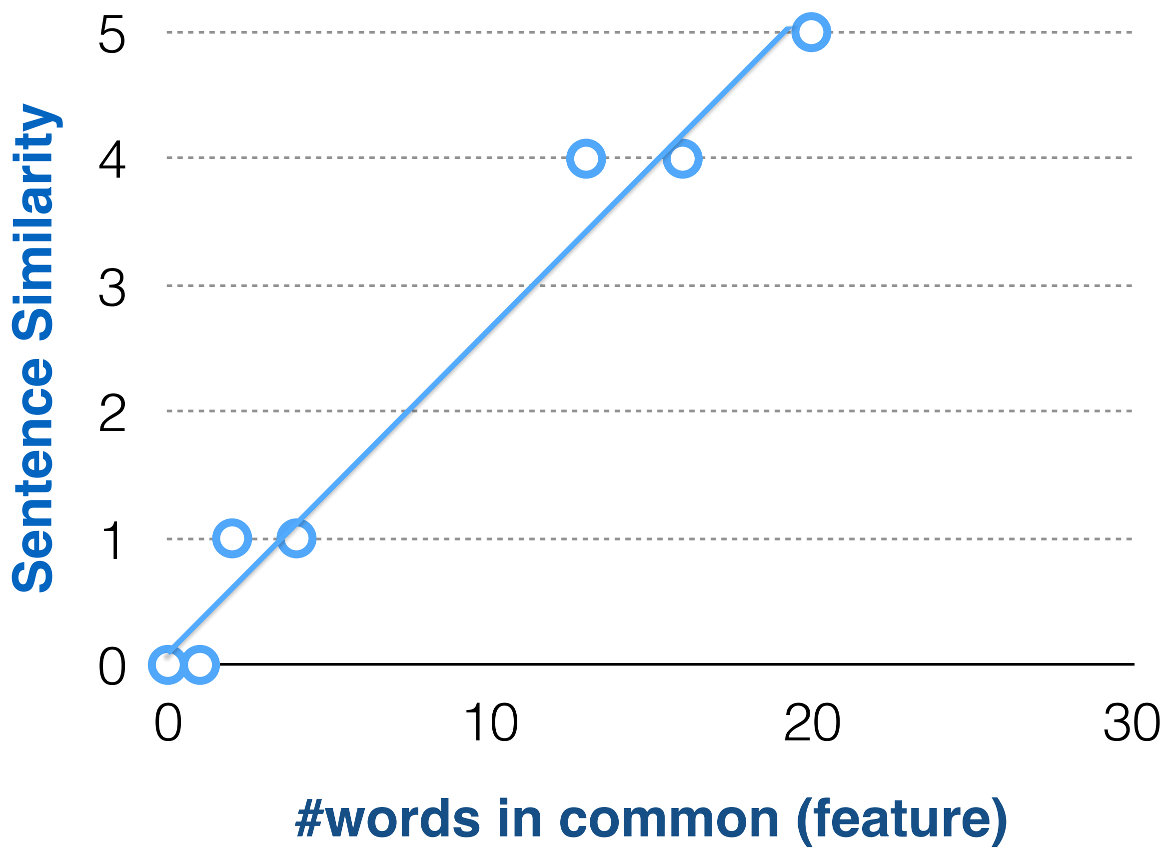
- ***m*** hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$

  θ's: parameters

Source: many following slides are adapted from Andrew Ng

(Recap) Linear Regression w/ one variable:

Cost Function



squared error function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

- **Idea:** choose ***θ_0***, ***θ_1*** so that ***$h_\theta(x)$*** is close to ***y*** for training examples ***(x, y)***

$$\underset{\theta_0, \theta_1}{\text{minimize}}\, J(\theta_0, \theta_1)$$

(Recap)

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

simultaneous update for j=0 and j=1

}

learning rate

Linear Regression w/ one variable:

Gradient Descent

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Cost Function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = ? \qquad \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = ?$$

(Recap) Linear Regression w/ one variable:

Gradient Descent

repeat until convergence {

$$\theta_0 \coloneqq \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)$$

$$\theta_1 \coloneqq \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i) \cdot x_i$$

}

simultaneous update ***θ_0***, ***θ_1***

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

$$h_\theta(x) = \theta^T x$$

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta^T x$$

- Cost function:

 # training examples

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

(Recap)

Paraphrase Identification

obtain sentential paraphrases automatically

<u>Mancini</u> has been sacked by Manchester City
<u>Mancini</u> gets the boot from Man City

Yes!

<u>WORLD OF JENKS</u> IS ON AT 11
<u>World of Jenks</u> is my favorite show on tv

No!

Wei Xu, Alan Ritter, Chris Callison-Burch, Bill Dolan, Yangfeng Ji. "Extracting Lexically Divergent Paraphrases from Twitter" In TACL (2014)

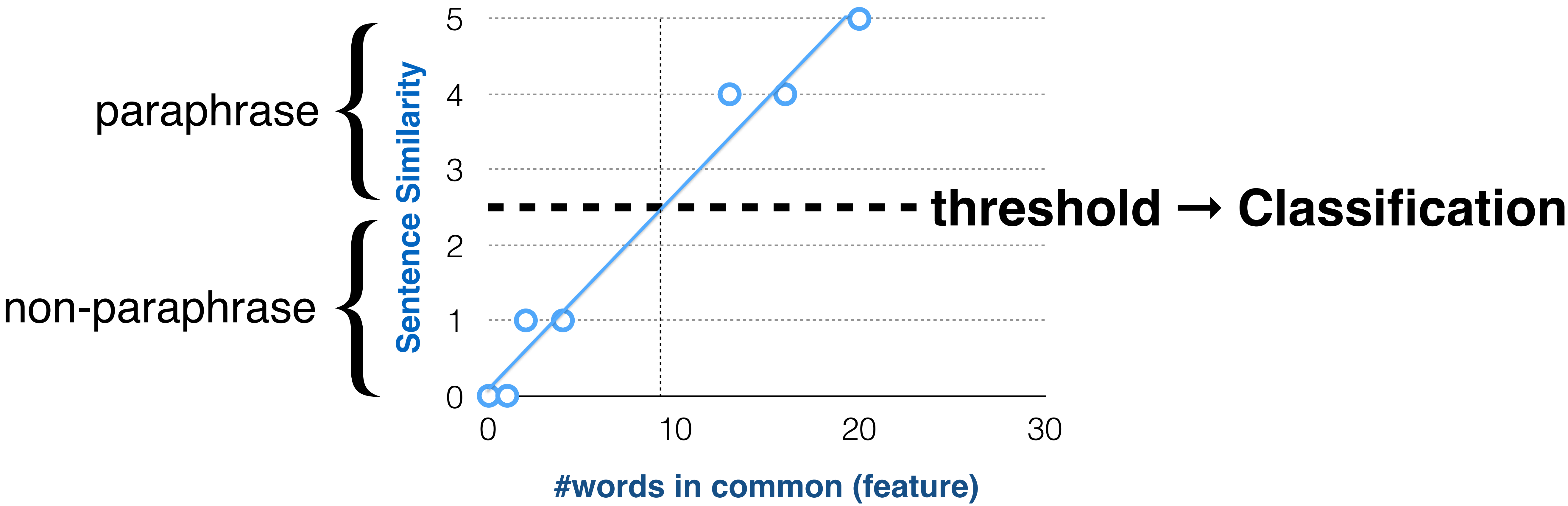
(Recap) Classification Method:

Supervised Machine Learning

- Input:
  - a sentence pair x **(represented by features)**
  - a fixed set of binary classes $Y = \{0, 1\}$
  - a training set of m hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$
- Output:
  - a learned classifier $\gamma: x \rightarrow y \in Y$ ($y = 0$ or $y = 1$)

(Recap)

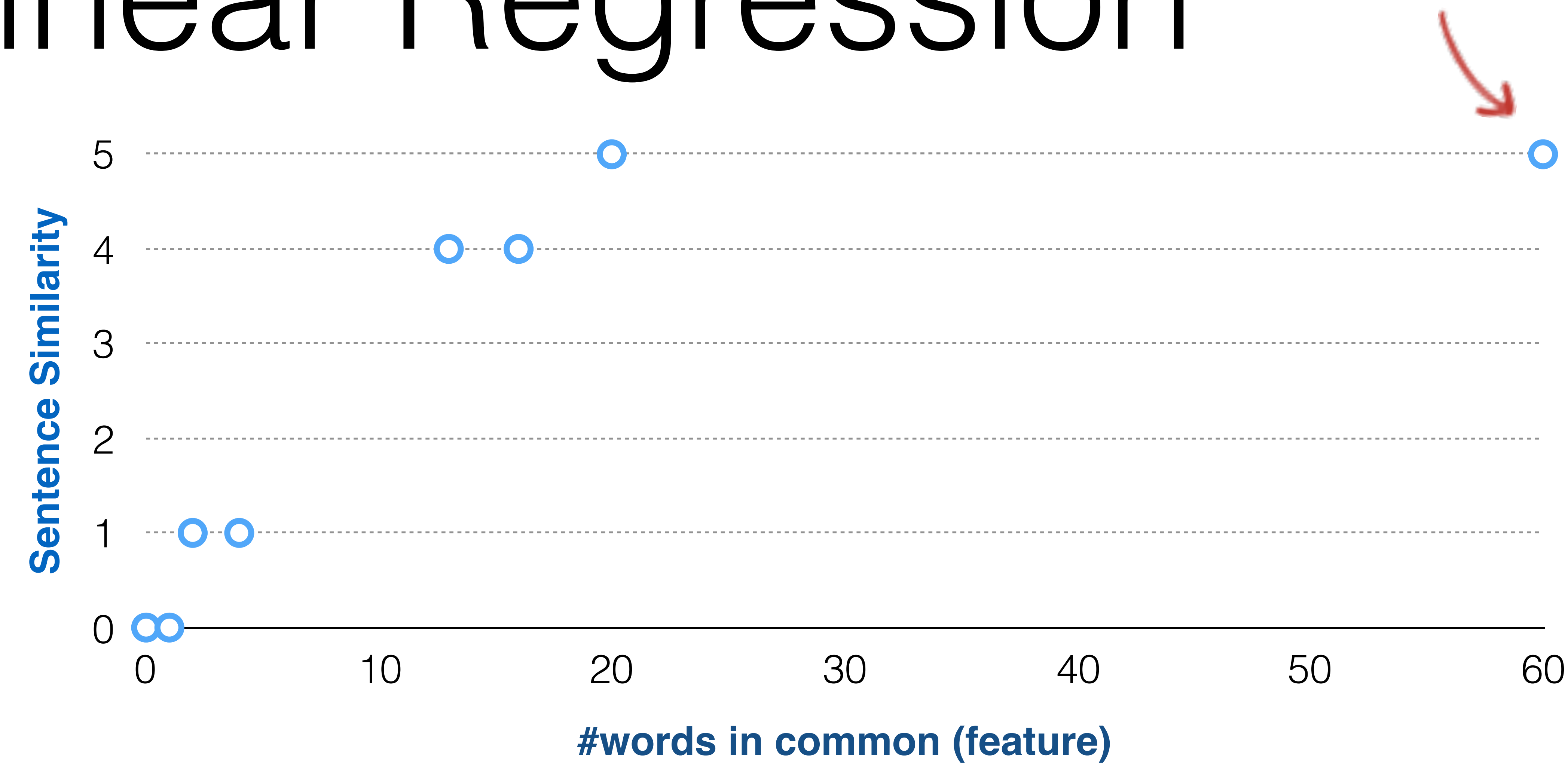
Linear Regression



- also supervised learning (learn from annotated data)
- but for **Regression**: predict **real-valued** output
  (Classification: predict discrete-valued output)

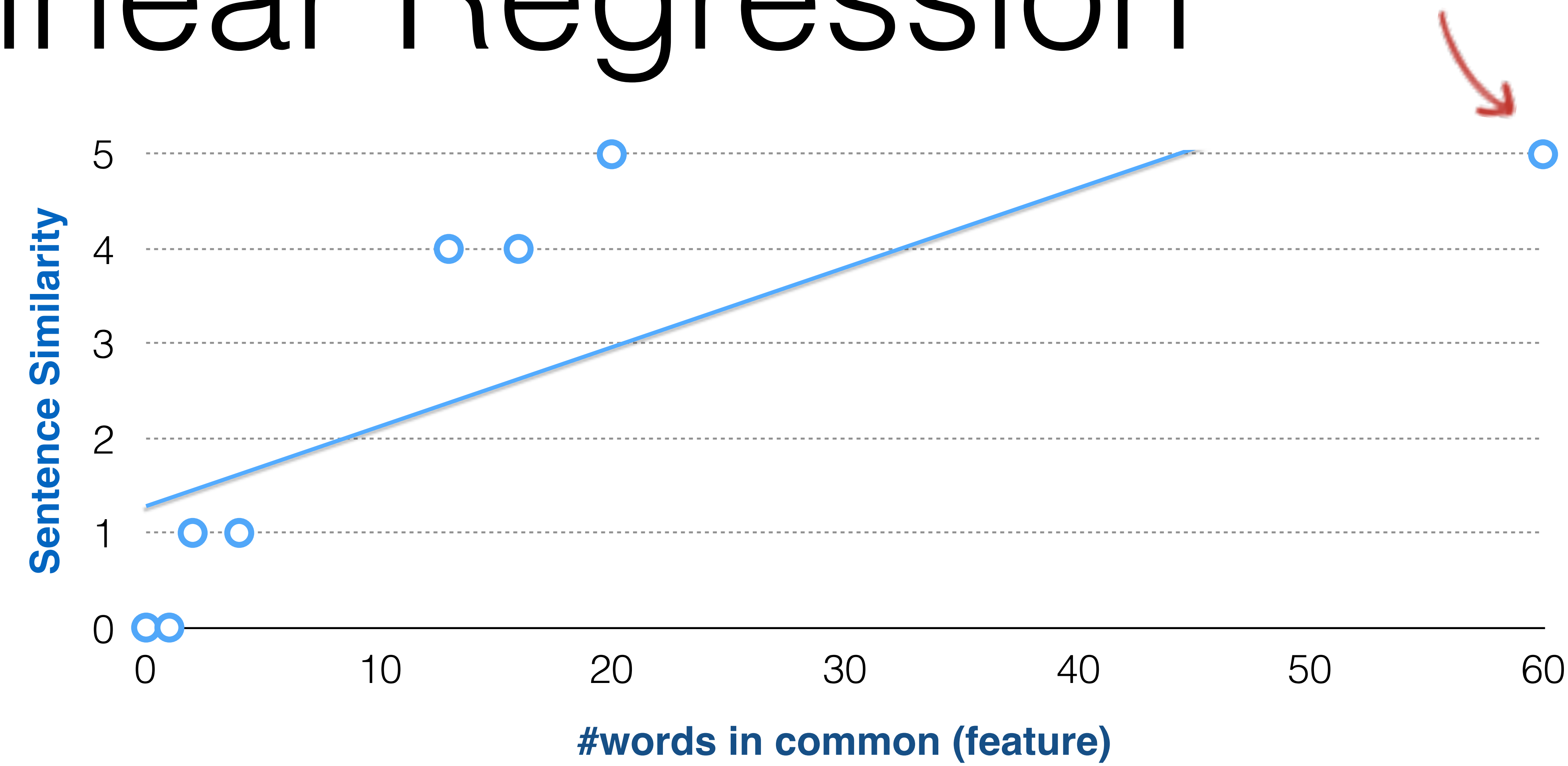
A problem in classification

Linear Regression



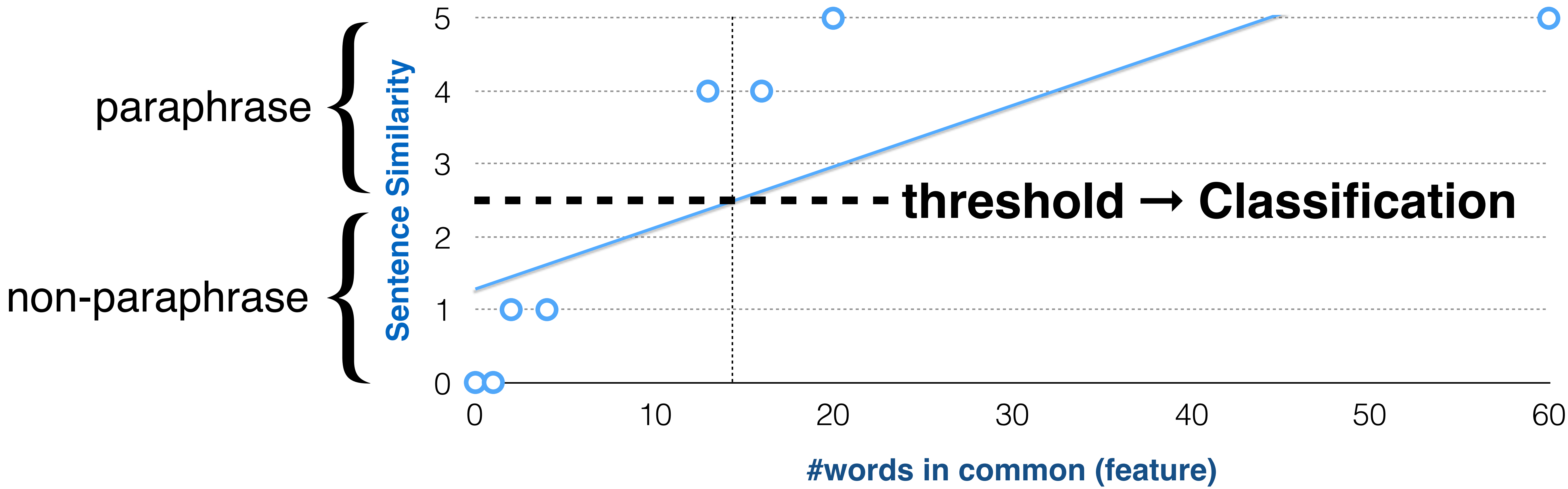
A problem in classification

Linear Regression



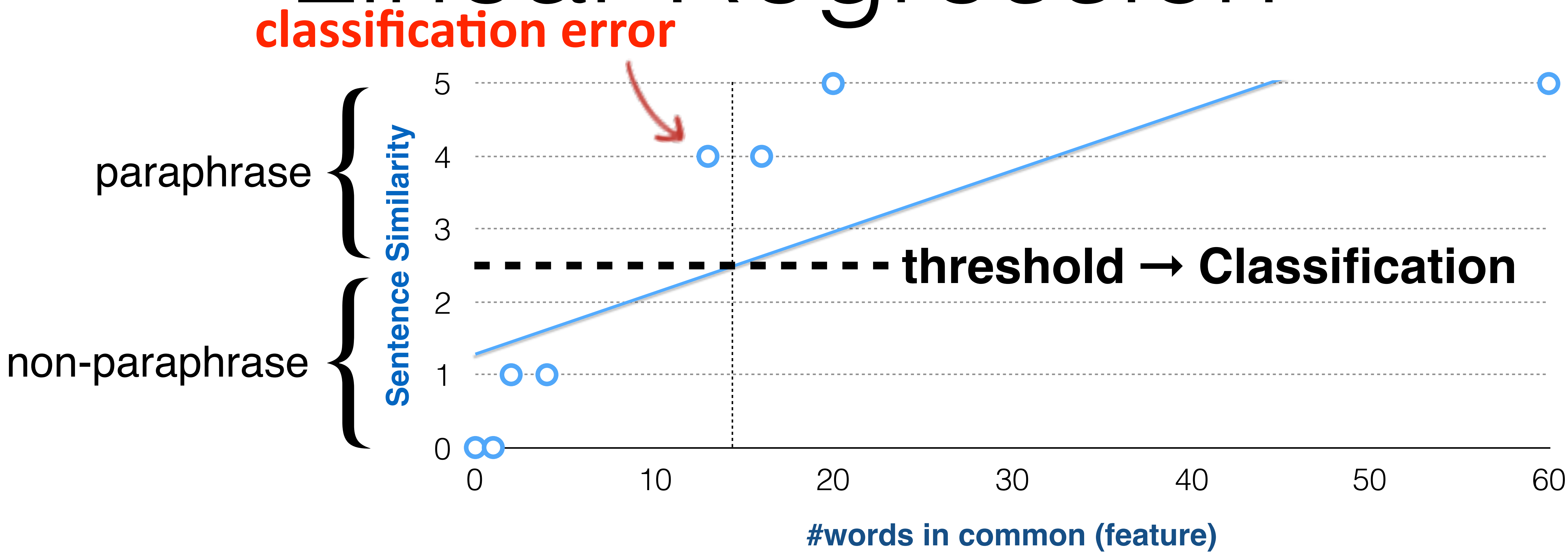
A problem in classification

Linear Regression



A problem in classification

Linear Regression



In practice, do not use linear regression for classification.

(Recap)

Logistic Regression

- One of the most useful **supervised machine learning algorithm** for classification!
- Generally high performance for a lot of problems.
- Much more robust than Naïve Bayes
(better performance on various datasets).

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0
- Logistic Regression: want $0 \le h_\theta(x) \le 1$

a classification (not regression) algorithm

Hypothesis:

Linear → Logistic Regression

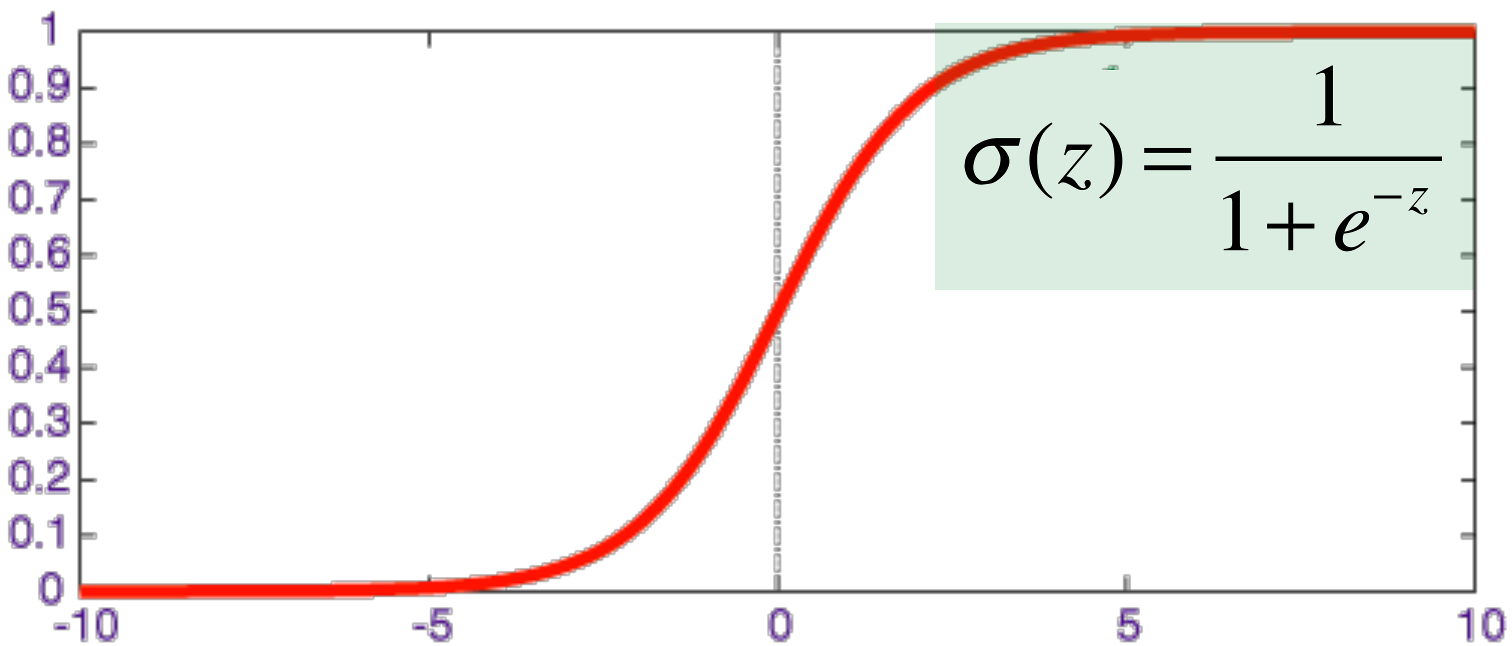
- Linear Regression: $h_\theta(x) = \theta^T x$
- Logistic Regression: want $0 \leq h_\theta(x) \leq 1$

Hypothesis:

Linear → Logistic Regression

- Linear Regression: $h_\theta(x) = \theta^T x$
- Logistic Regression: want $0 \leq h_\theta(x) \leq 1$

sigmoid (logistic) function



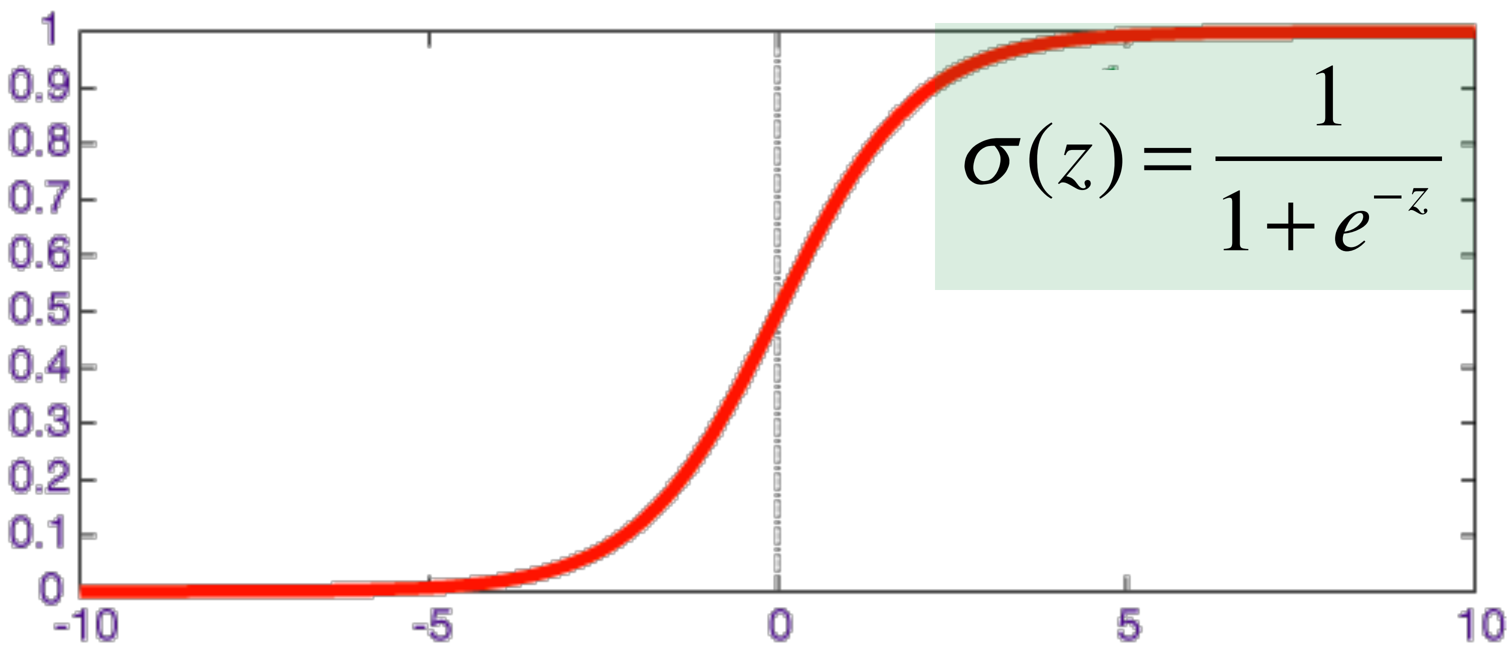
Hypothesis:

Linear → Logistic Regression

- Linear Regression: $h_\theta(x) = \theta^T x$
- Logistic Regression: want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

sigmoid (logistic) function



(Recap) Classification Method:

Supervised Machine Learning

- Input:
  - a sentence pair ***x*** **(represented by features)**
  - a fixed set of binary classes $Y = \{0, 1\}$
  - a training set of ***m*** hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots , (x^{(m)}, y^{(m)})$
- Output:
  - a learned classifier $\gamma: x \rightarrow y \in Y$ ($y = 0$ or $y = 1$)

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***y = 1*** on input

$$If \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{\#words\_in\_common} \end{bmatrix}, \; h_\theta(x) = 0.7$$

70% chance of the sentence pair being paraphrases

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$If \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{\#words\_in\_common} \end{bmatrix}, \quad h_\theta(x) = 0.7$$

70% chance of the sentence pair being paraphrases

$$h_\theta(x) = P(y = 1 \mid x; \theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

Logistic Regression:

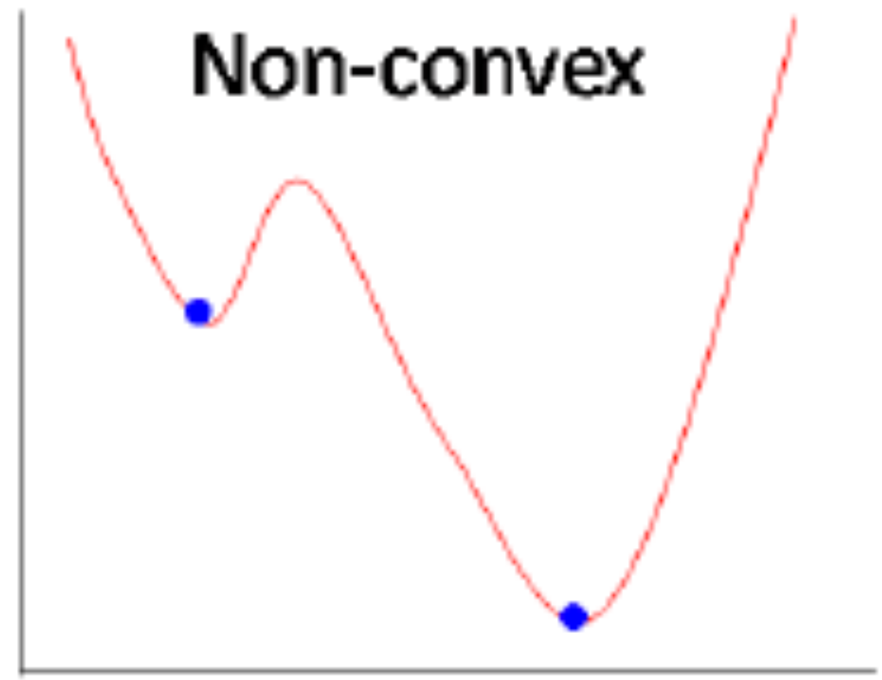
Interpretation of Hypothesis

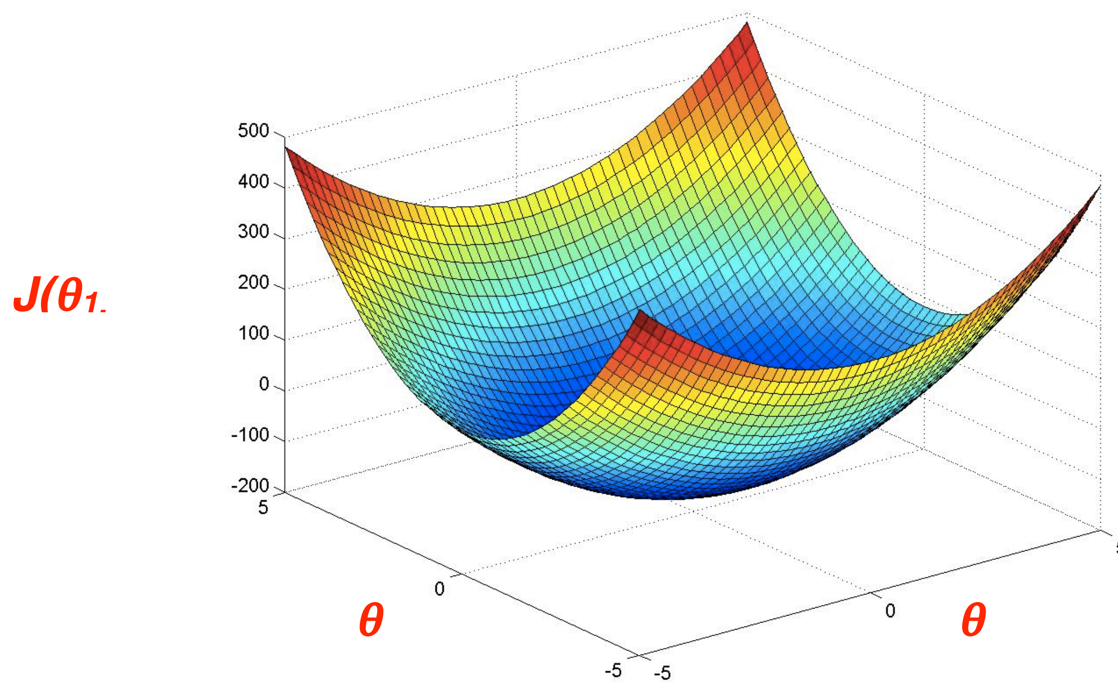
- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

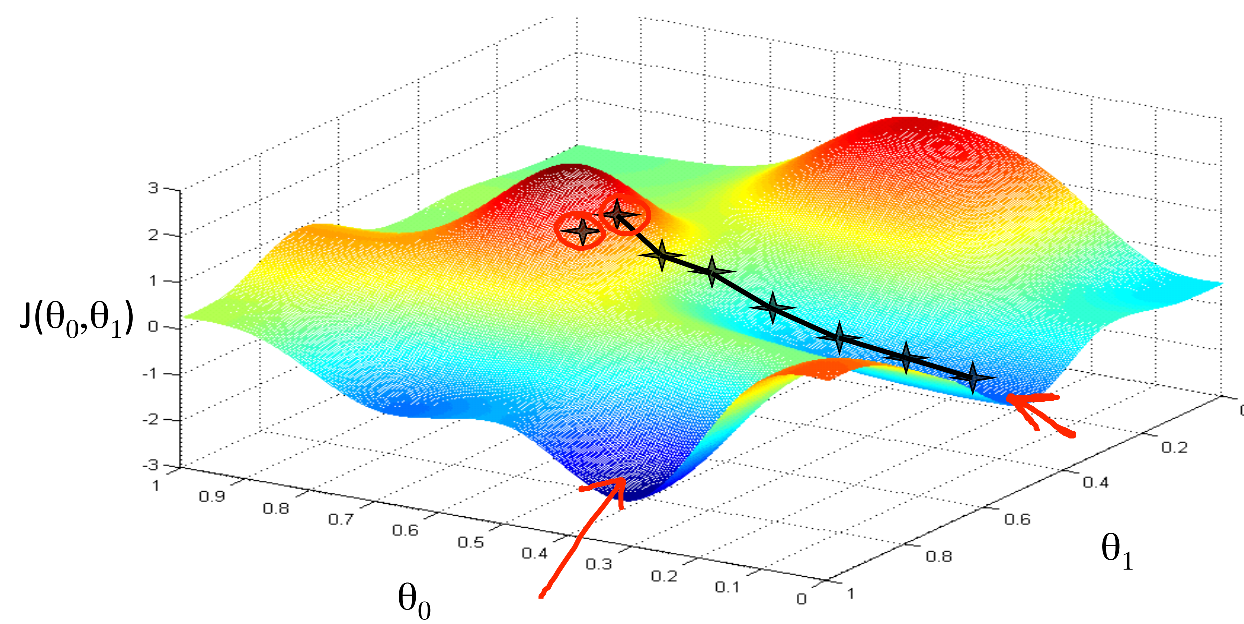
$$h_\theta(x) = P(y = 1 \mid x; \theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

we want convex! easy gradient descent!







Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$P(y=1 \mid x;\theta) + P(y=0 \mid x;\theta) = 1$$

$$h_\theta(x) = P(y=1 \mid x;\theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

Logistic Regression:

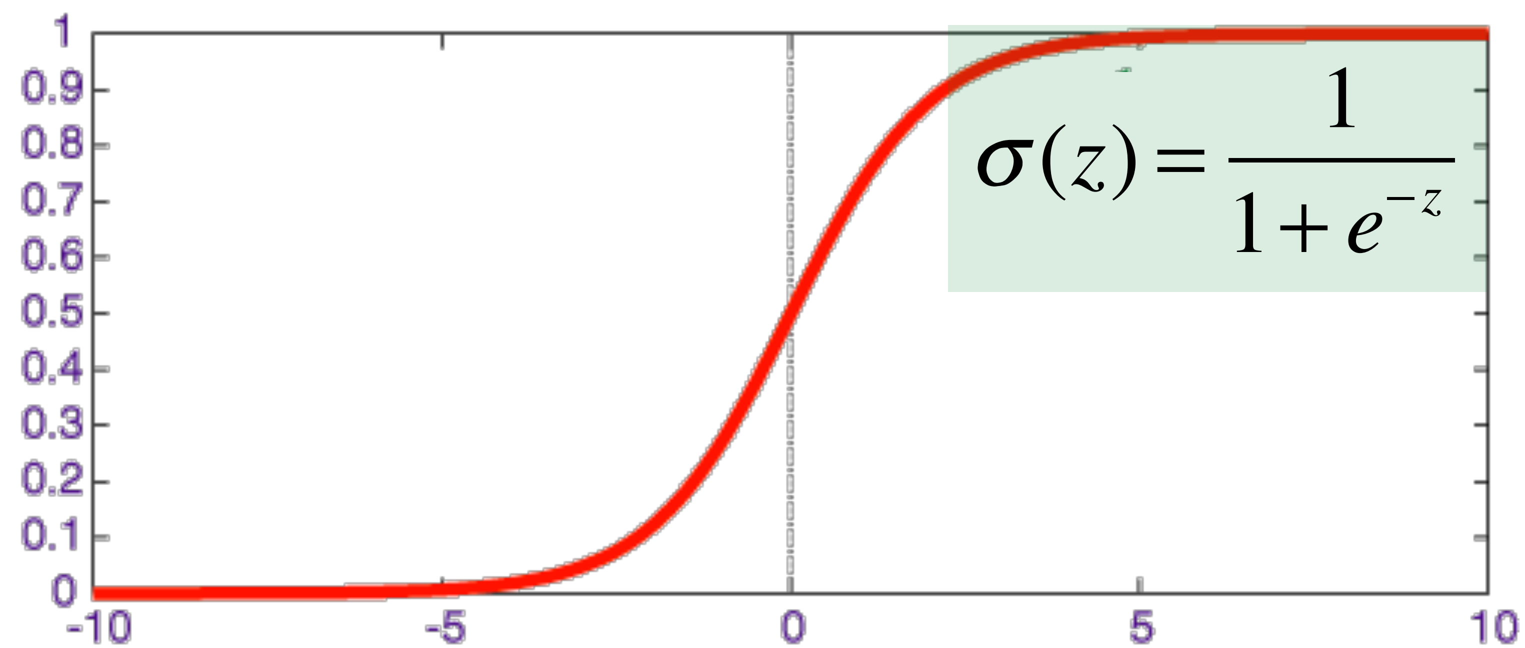
Decision Boundary

- Logistic Regression:

sigmoid (logistic) function

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$



Logistic Regression:

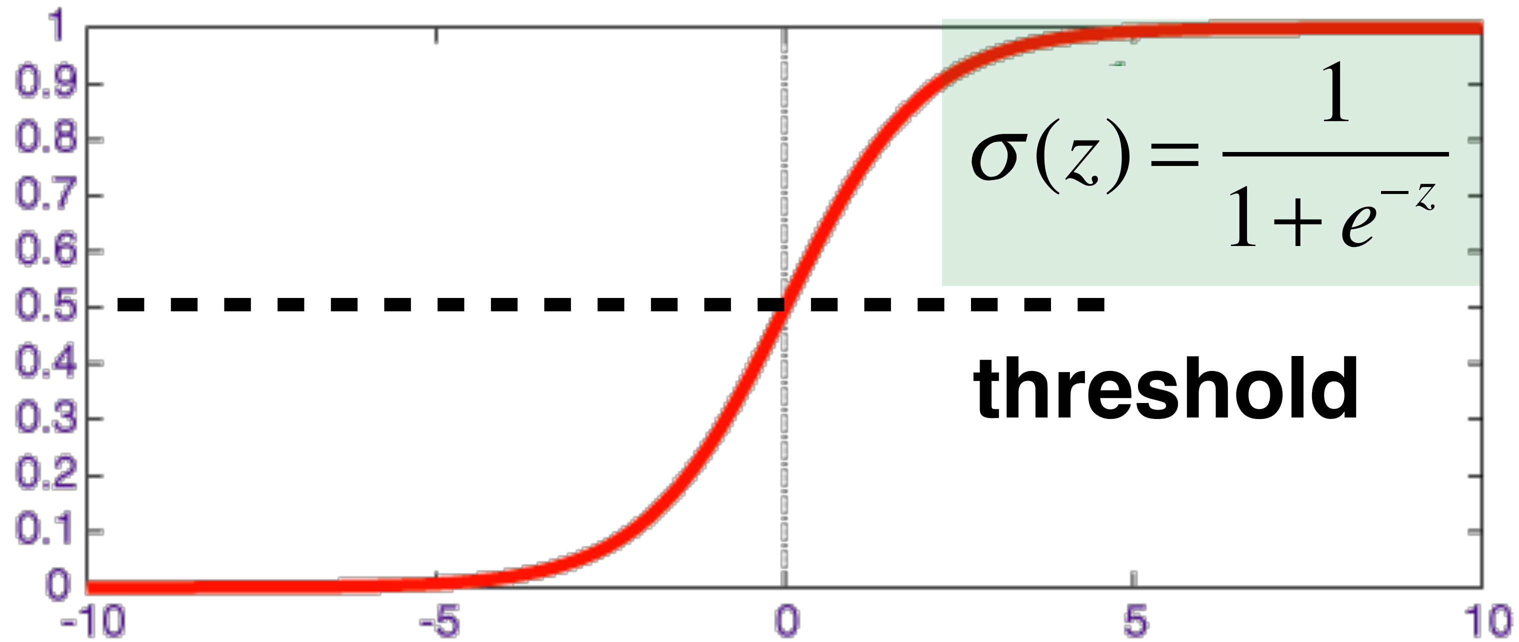
Decision Boundary

- Logistic Regression:

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

sigmoid (logistic) function



predict ***y = 1*** if ***$h_\theta(x) \geq 0.5$***

predict ***y = 0*** if ***$h_\theta(x) < 0.5$***

Logistic Regression:

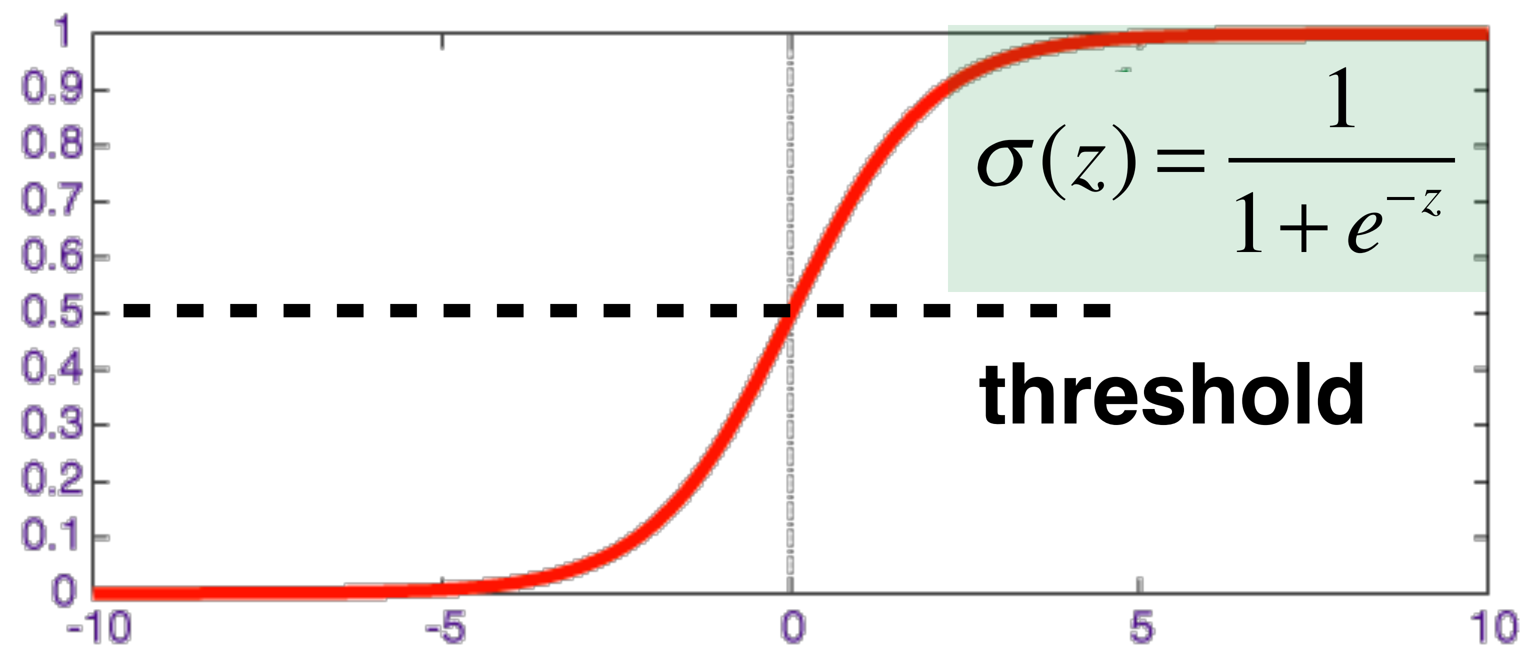
Decision Boundary

- Logistic Regression:

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

sigmoid (logistic) function

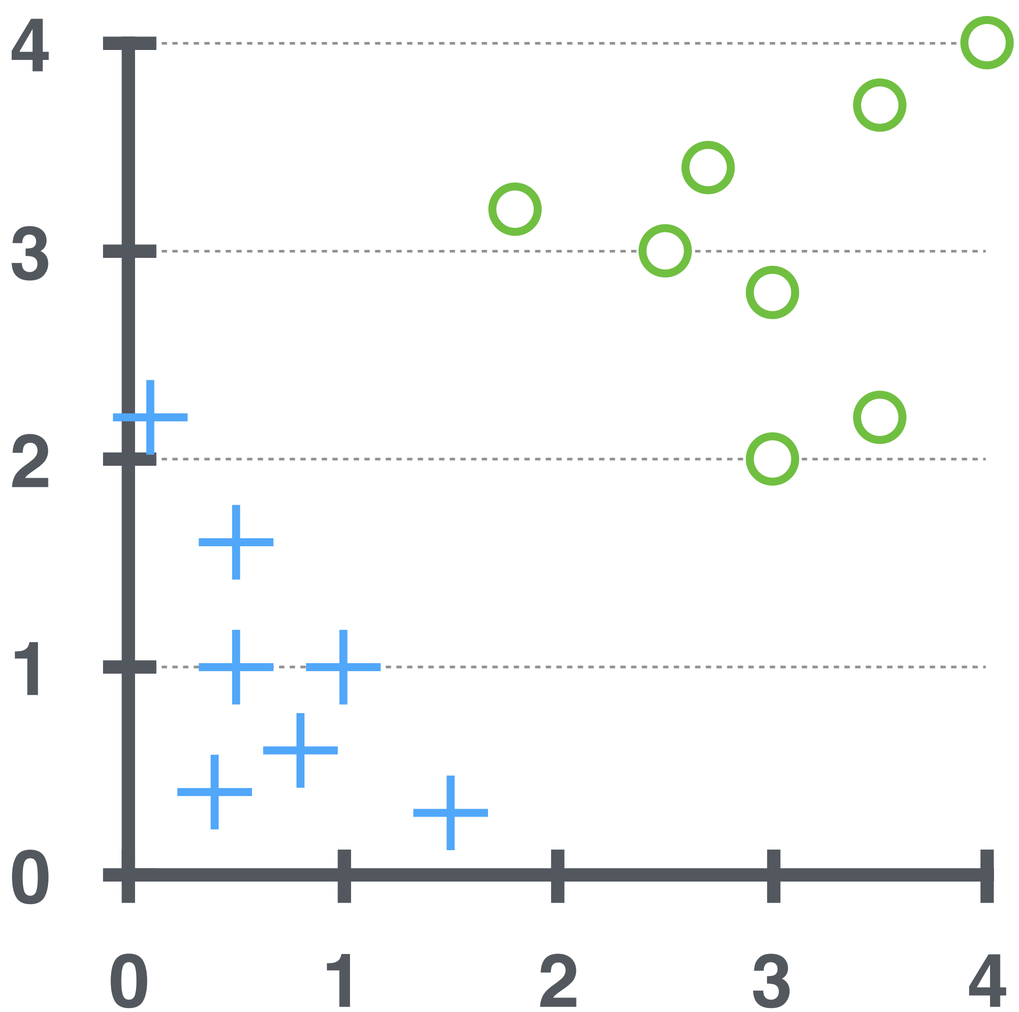


predict ***y* = 1** if $h_\theta(x) \geq 0.5$ $\longleftarrow$ when $\theta^T x \geq 0$

predict ***y* = 0** if $h_\theta(x) < 0.5$ $\longleftarrow$ when $\theta^T x < 0$

Logistic Regression:

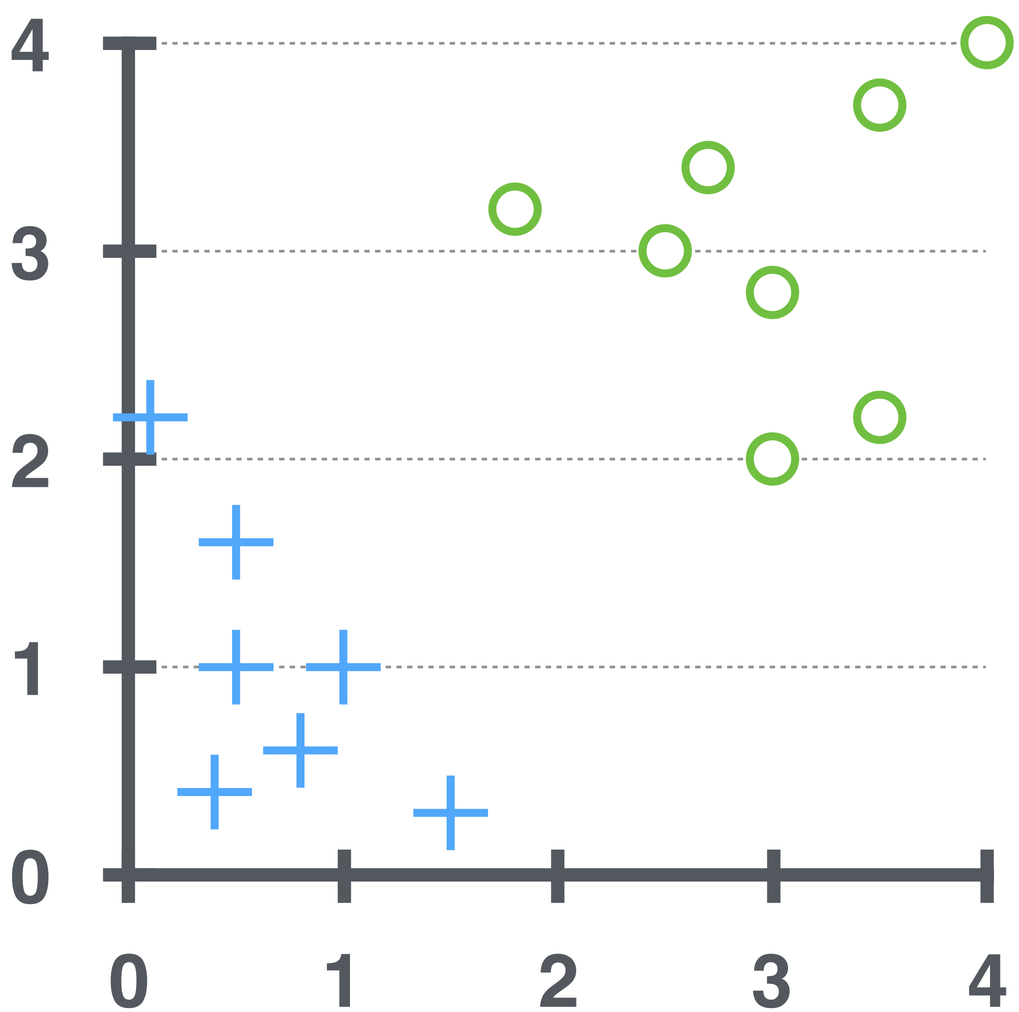
Decision Boundary



Logistic Regression:

Decision Boundary

$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$

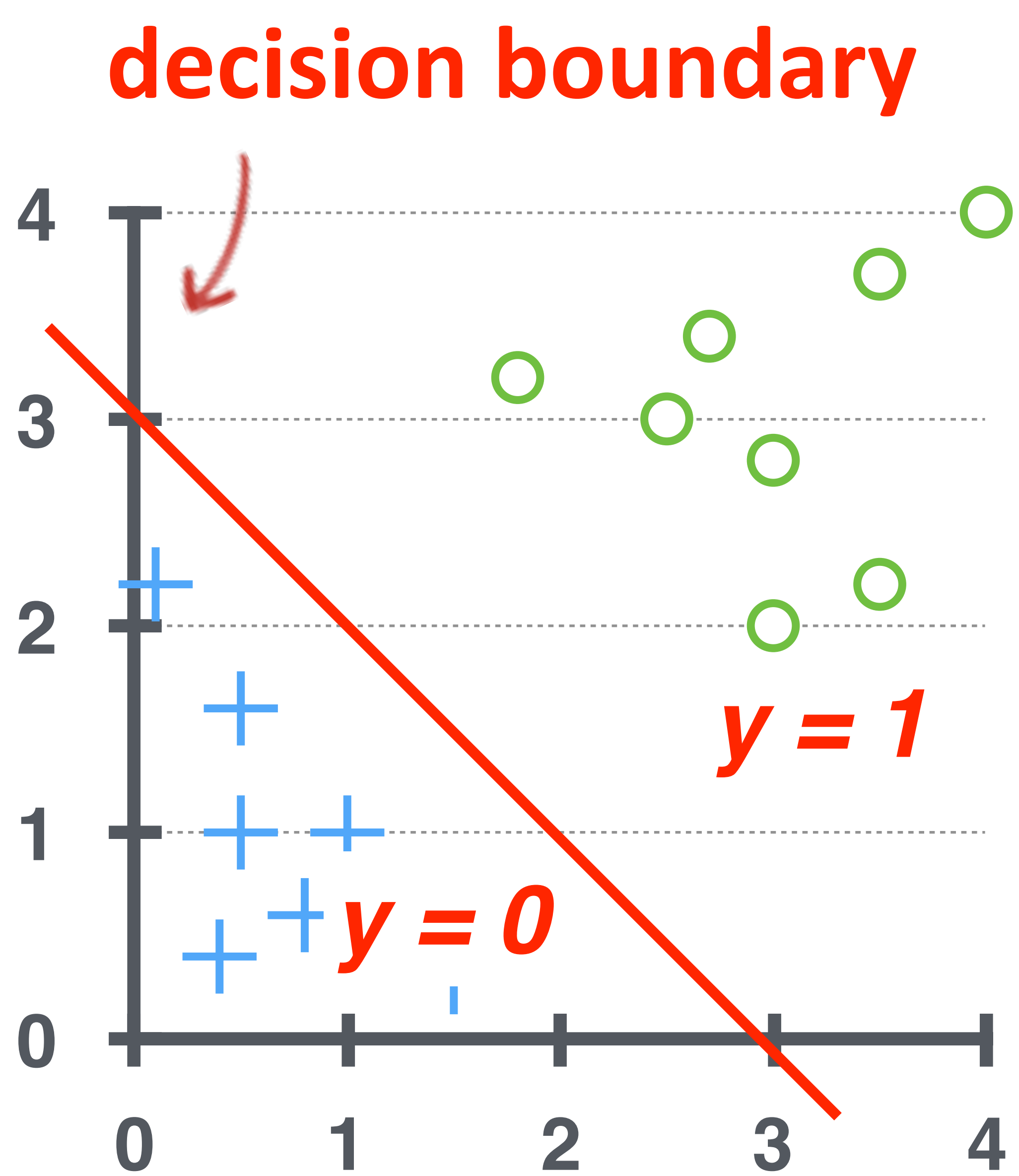


What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict ***y = 1*** if ***$\theta^T x \geq 0$***

Logistic Regression:

Decision Boundary



$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$

What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict ***y = 1*** if $\boldsymbol{\theta^T x \geq 0}$

Logistic Regression

- a training set of ***m*** hand-labeled sentence pairs
  $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$ $\quad (y \in \{0, 1\})$
-

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Cost function:

Linear → Logistic Regression

- Linear Regression: $$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{2} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

squared error function

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(\boxed{h_\theta(x^{(i)})}-y^{(i)}\right)^2$

squared error function

- Logistic Regression: $h_\theta(x)=\frac{1}{1+e^{-\theta^T x}}$

this cost function is non-convex for logistic regression

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(h_\theta(x^{(i)})-y^{(i)}\right)^2$

$$||$$

$$\mathrm{Cost}\left(h_\theta(x),y\right)$$

- Logistic Regression: $h_\theta(x)=\frac{1}{1+e^{-\theta^T x}}$

this cost function is non-convex for logistic regression

Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } \; y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \; y = 0 \end{cases}$$

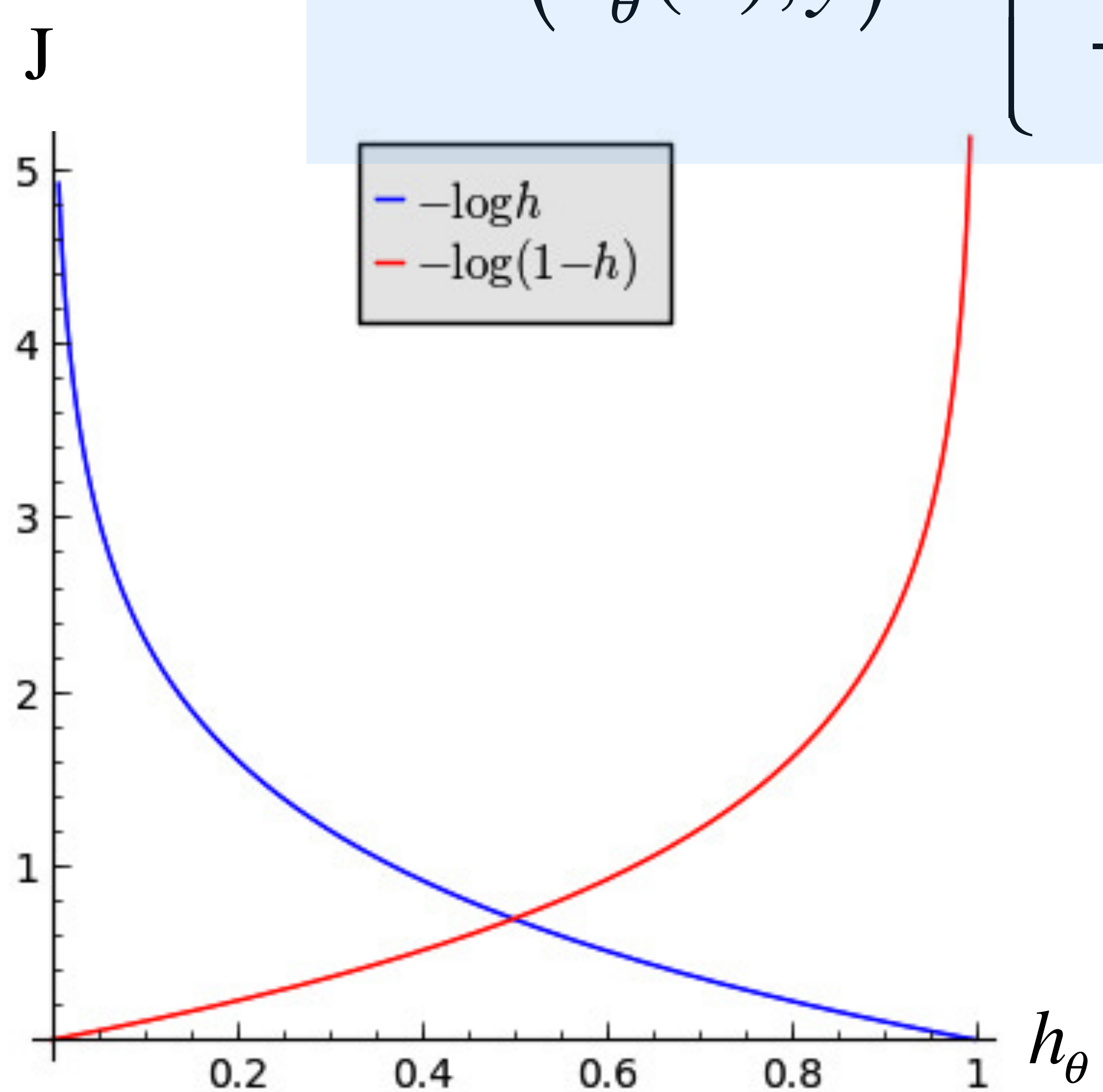
remember that

$$0 \le h_\theta(x) \le 1$$

Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } \ y=1 \\ -\log(1-h_\theta(x)) & \text{if } \ y=0 \end{cases}$$



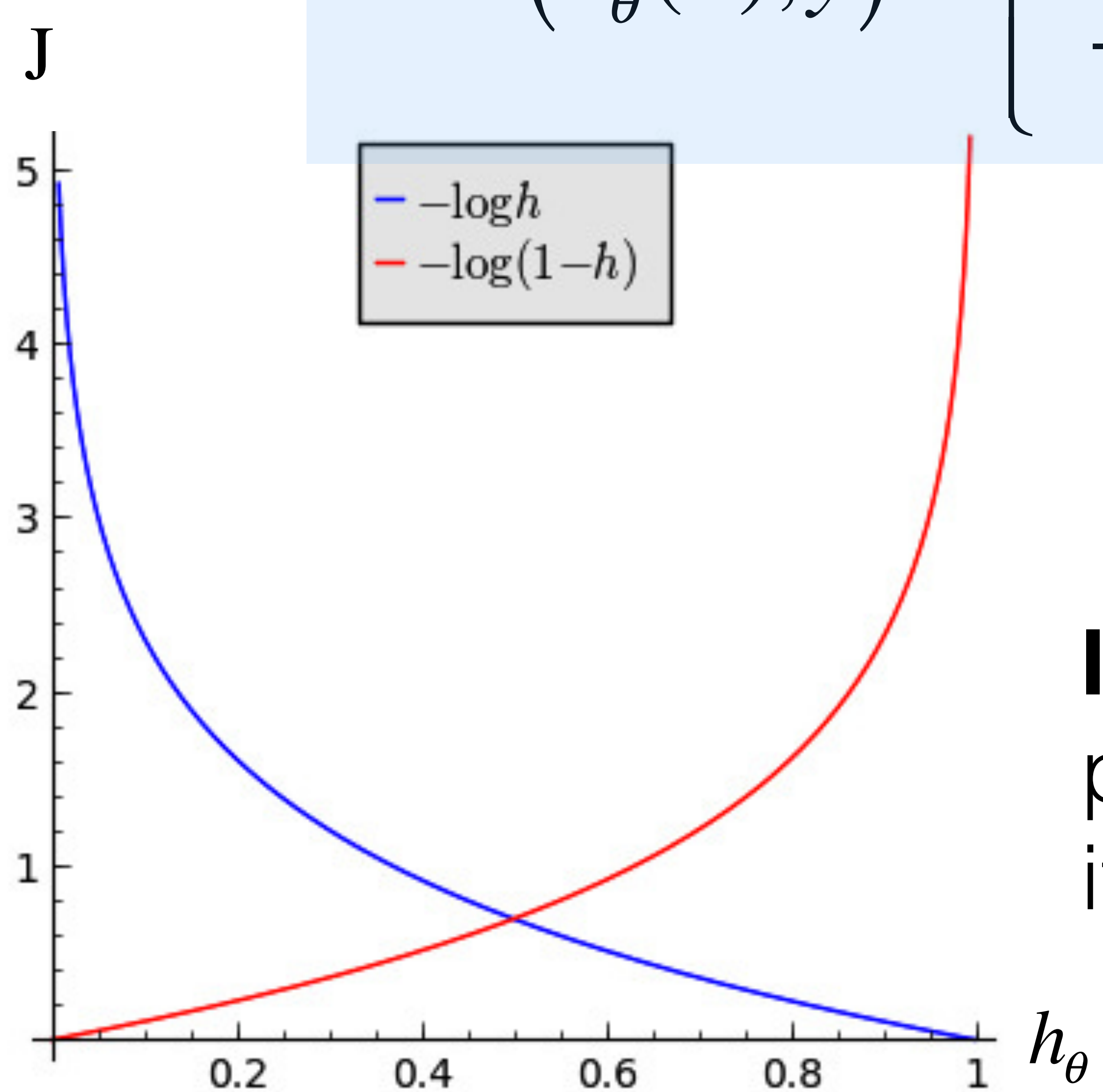
remember that

$$0 \le h_\theta(x) \ \le 1$$

Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } \; y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \; y = 0 \end{cases}$$



Cost = 0 if ***y = 1***, ***h_θ(x) = 1***

But as ***h_θ(x) → 0*** , ***Cost → ∞***

Intuition:

penalize learning algorithm

if ***h_θ(x) = 0*** (predict **P(y=1|x;θ) = 0**),

but ***y = 1***

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$\mathrm{Cost}\left(h_\theta(x), y\right) = \begin{cases} -\log(h_\theta(x)) & \text{if } \ y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \ y = 0 \end{cases}$$

remember that $y = 0$ or 1 always

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\text{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$\text{Cost}\left(h_\theta(x), y\right) = \begin{cases} -\log(h_\theta(x)) & \text{if } \; y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \; y = 0 \end{cases}$$

remember that $y = 0$ or 1 always

the same

cross entropy loss:

$$\text{Cost}\left(h_\theta(x), y\right) = -y\log(h_\theta(x)) - \left(1 - y\right)\log(1 - h_\theta(x))$$

Learn more: https://en.wikipedia.org/wiki/Cross_entropy

Logistic Regression

- **Cost Function:**

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$= -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + \left(1 - y^{(i)}\right)\log(1 - h_\theta(x^{(i)}))$$

- **Goal:**

learn parameters θ to $\underset{\theta}{\text{minimize}}\, J(\theta)$

- **Hypothesis (to make a prediction):** $h_\theta(x) = \dfrac{1}{1+e^{-\theta^T x}}$

P(y=1|x;θ)

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

simultaneous update for all θ_j

learning rate

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

simultaneous update for all θ_j

learning rate

 # training examples

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

This look the same as linear regression!!???

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

$$h_\theta(x^{(i)}) = \frac{1}{1+e^{-\theta^T x}}$$

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

using different hypothesis from linear regression

Classification Method:

Supervised Machine Learning

- Naïve Bayes
- Logistic Regression
- Support Vector Machines (SVM)
- …
- Hidden Markov Model (HMM)
- Conditional Random Fields (CRF)
- …

sequential models

Classification Method:

Sequential Supervised Learning

- Input:
  - rather than just individual examples ***(w_1 = the, c_1 = DT)***
  - a training set consists of ***m*** sequences of labeled examples ***(x_1, y_1), … , (x_m, y_m)***

    x_1 = <the back door> and y_1= <DT JJ NN>

- Output:
  - a learned classifier to predict label sequences ***γ: x → y***

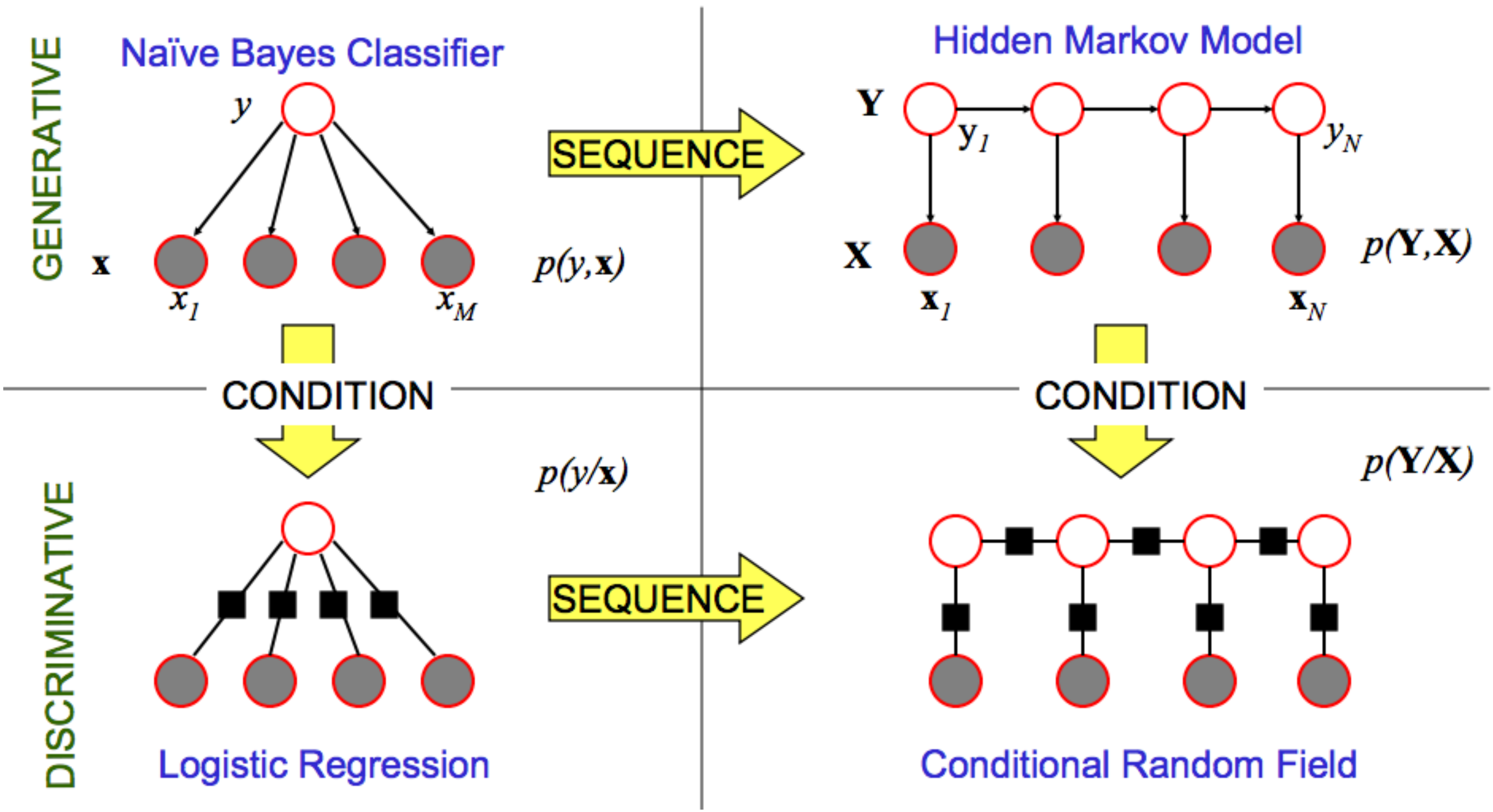
Features for Sequential Tagging

- Words:
  - current words
  - previous/next word(s) — context
- Other linguistic information:
  - word substrings
  - word shapes
  - POS tags
- Contextual Labels
  - previous (and perhaps next) labels

word shapes

Varicella-zoster	Xx-xxx
mRNA	xXXX
CPA1	XXXd

Probabilistic Graphical Models

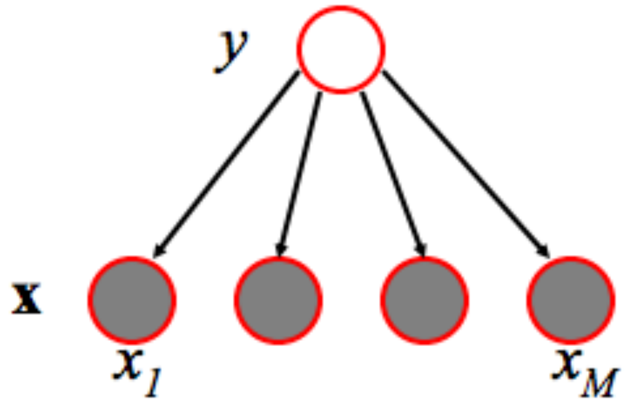


Source: Andrew McCallum, Sargur Srihari

Probabilistic Graphical Models

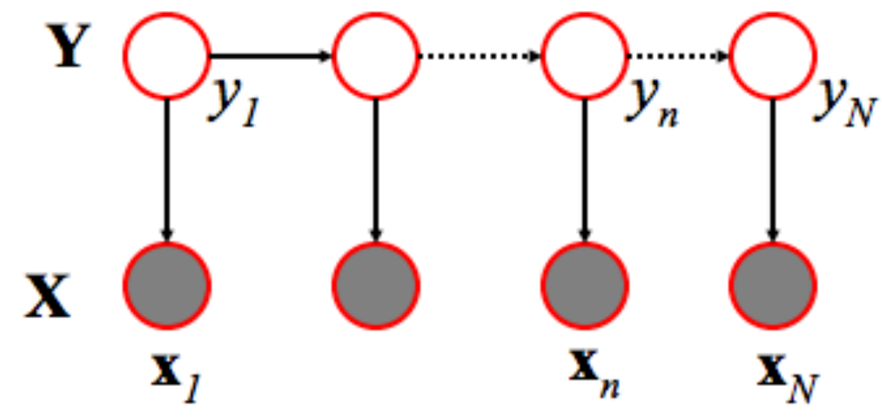
GENERATIVE

Naïve Bayes Classifier



$$p(y,\mathbf{x}) = p(y)\prod_{m=1}^{M} p(x_m \mid y)$$

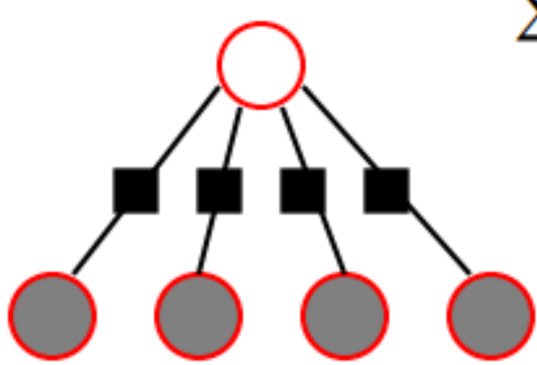
Hidden Markov Model



$$p(\mathbf{Y},\mathbf{X}) = \prod_{n=1}^{N} p(y_n \mid y_{n-1})\, p(\mathbf{x}_n \mid y_n)$$

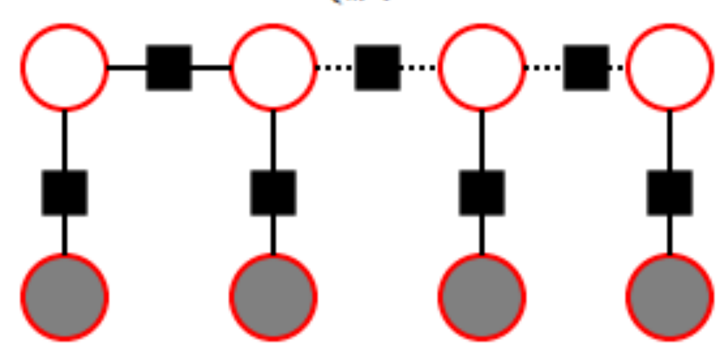
DISCRIMINATIVE

$$p(y \mid \mathbf{x}) = \frac{\exp\left\{\sum_{m=1}^{M} \lambda_m f_m(y,\mathbf{x})\right\}}{\sum_{y'} \exp\left\{\sum_{m=1}^{M} \lambda_m f_m(y',\mathbf{x})\right\}}$$



Logistic Regression

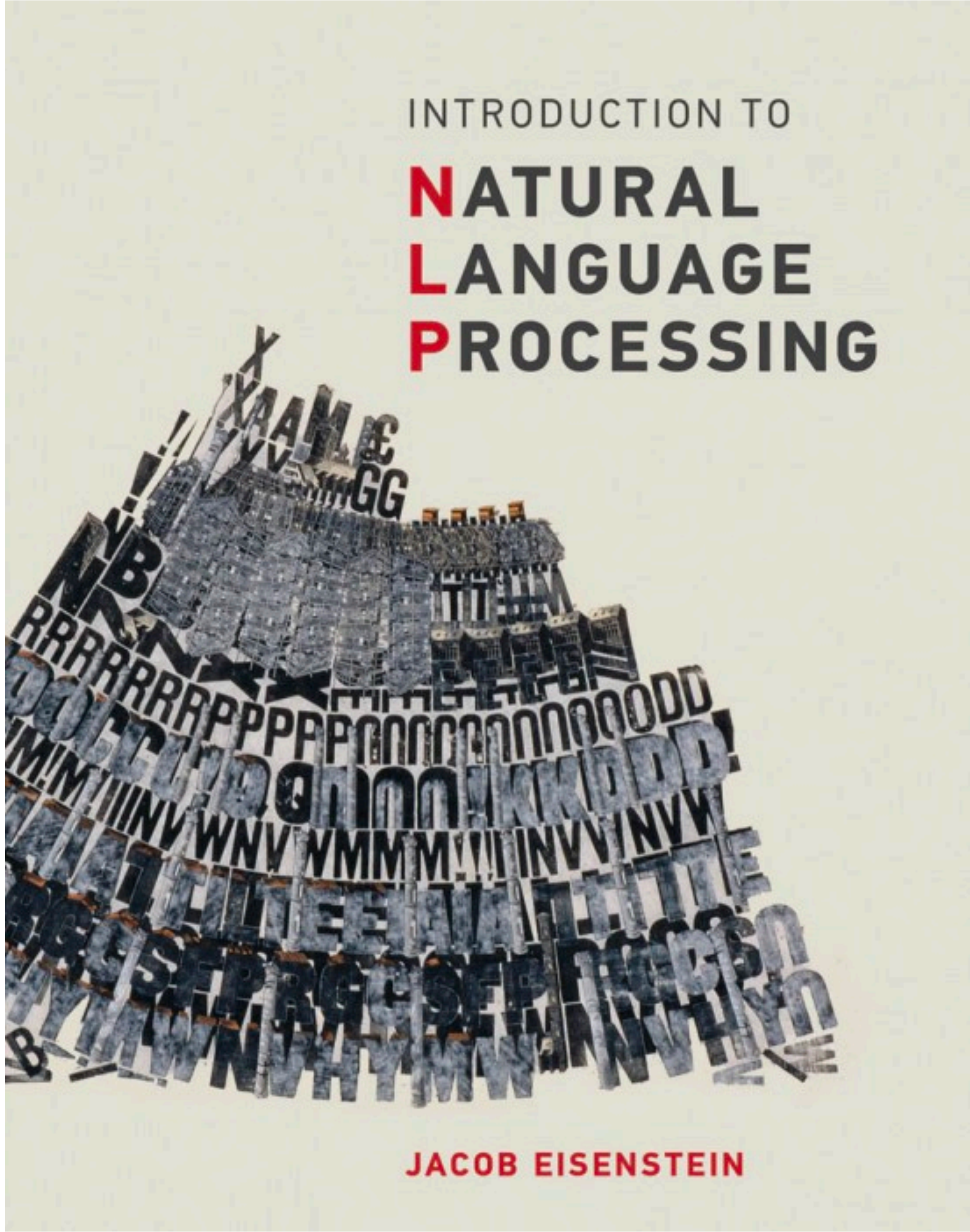
$$p(\mathbf{Y} \mid \mathbf{X}) = \frac{\exp\left\{\sum_{m=1}^{M} \lambda_m f_m(y_n, y_{n-1}, \mathbf{x}_n)\right\}}{\sum_{\mathbf{y}'} \exp\left\{\sum_{m=1}^{M} \lambda_m f_m(y_n', y_{n-1}', \mathbf{x}_n)\right\}}$$



Conditional Random Field

Source: Andrew McCallum, Sargur Srihari

New Textbook



October 2019
MIT Press
https://mitpress.mit.edu/books/introduction-natural-language-processing

Follow @cocoweixu

Instructor: Wei Xu

http://web.cse.ohio-state.edu/~weixu/

Course Website: socialmedia-class.org