

Social Media & Text Analysis

lecture 6 - Paraphrase Identification
and Logistic Regression

CSE 5539-0010 Ohio State University

Instructor: Alan Ritter

Website: socialmedia-class.org

with slides adapted from Andrew Ng

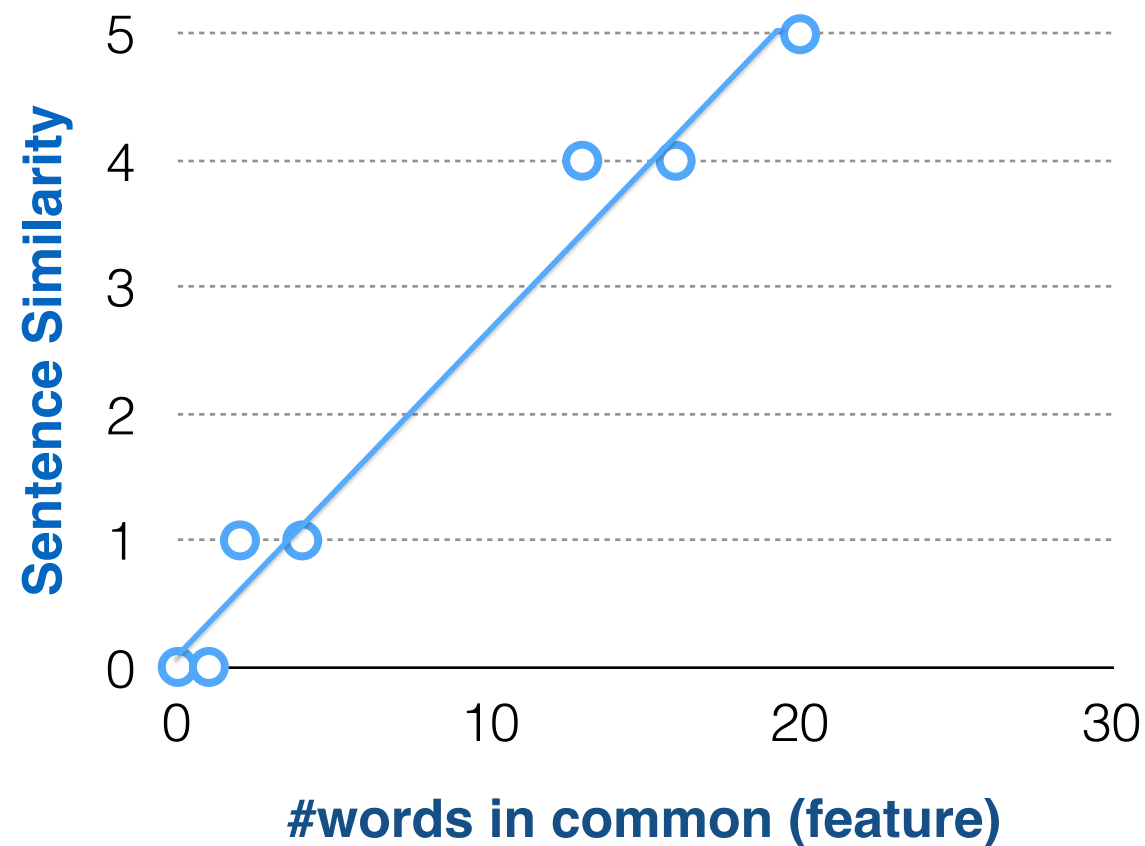
(Recap) Classification Method:

Supervised Machine Learning

- Input:
 - a sentence pair **x** (represented by features)
 - a fixed set of binary classes **$Y = \{0, 1\}$**
 - a training set of **m** hand-labeled sentence pairs **$(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})$**
- Output:
 - a learned classifier **$\gamma: x \rightarrow y \in Y$** (**$y = 0$** or **$y = 1$**)

(Recap)

Linear Regression



- also supervised learning (learn from annotated data)
- but for **Regression**: predict **real-valued** output
(Classification: predict discrete-valued output)

(Recap) Linear Regression w/ one variable:

Model Representation

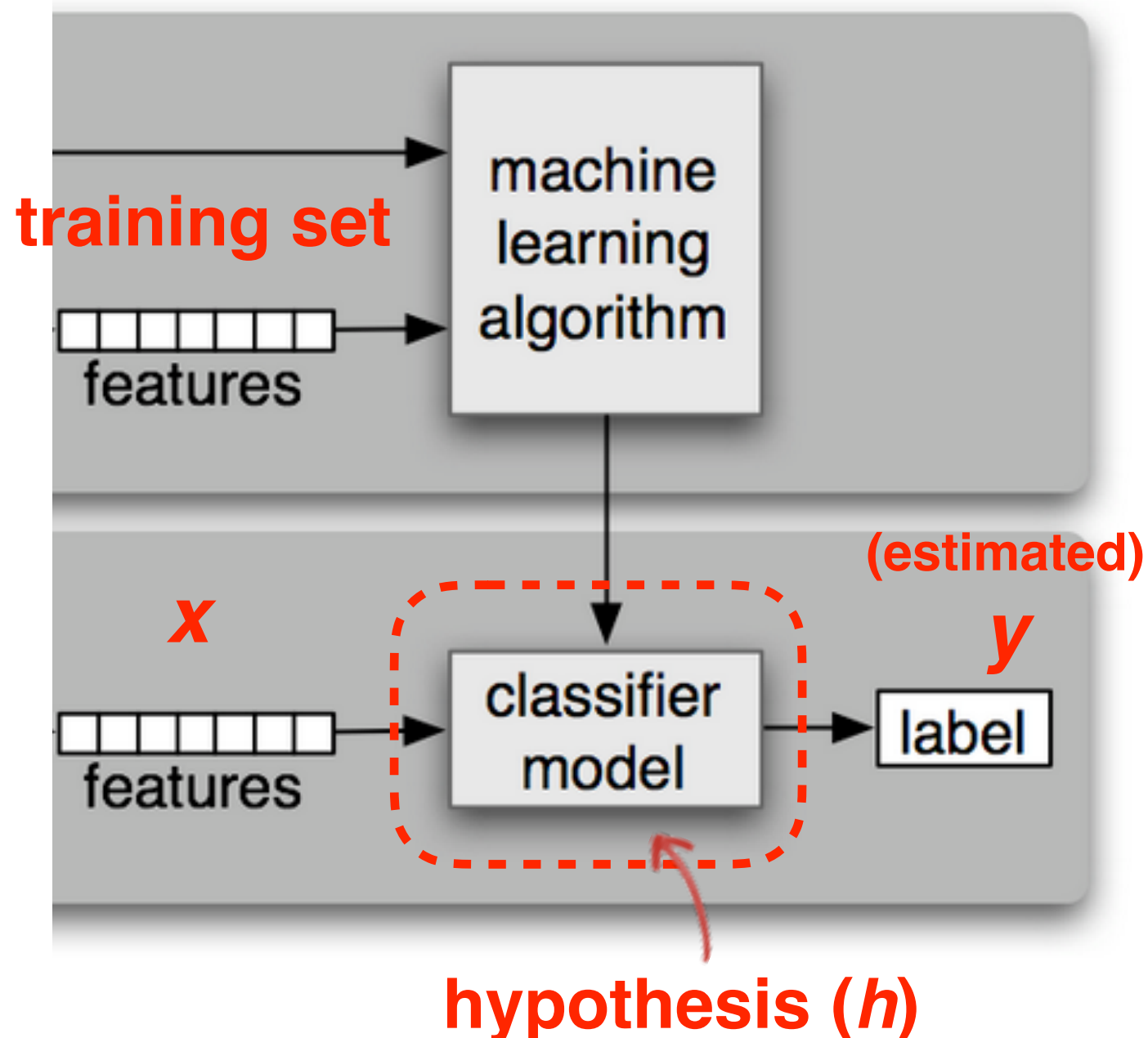
#words in common (x)	Sentence Similarity (y)
1	0
4	1
13	4
18	5
...	...

- m hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})$
 θ 's: parameters

(Recap) Linear Regression:

Model Representation

- How to represent **h** ?



$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Linear Regression
w/ one variable

(Recap)

Linear Regression

- **Hypothesis:**

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

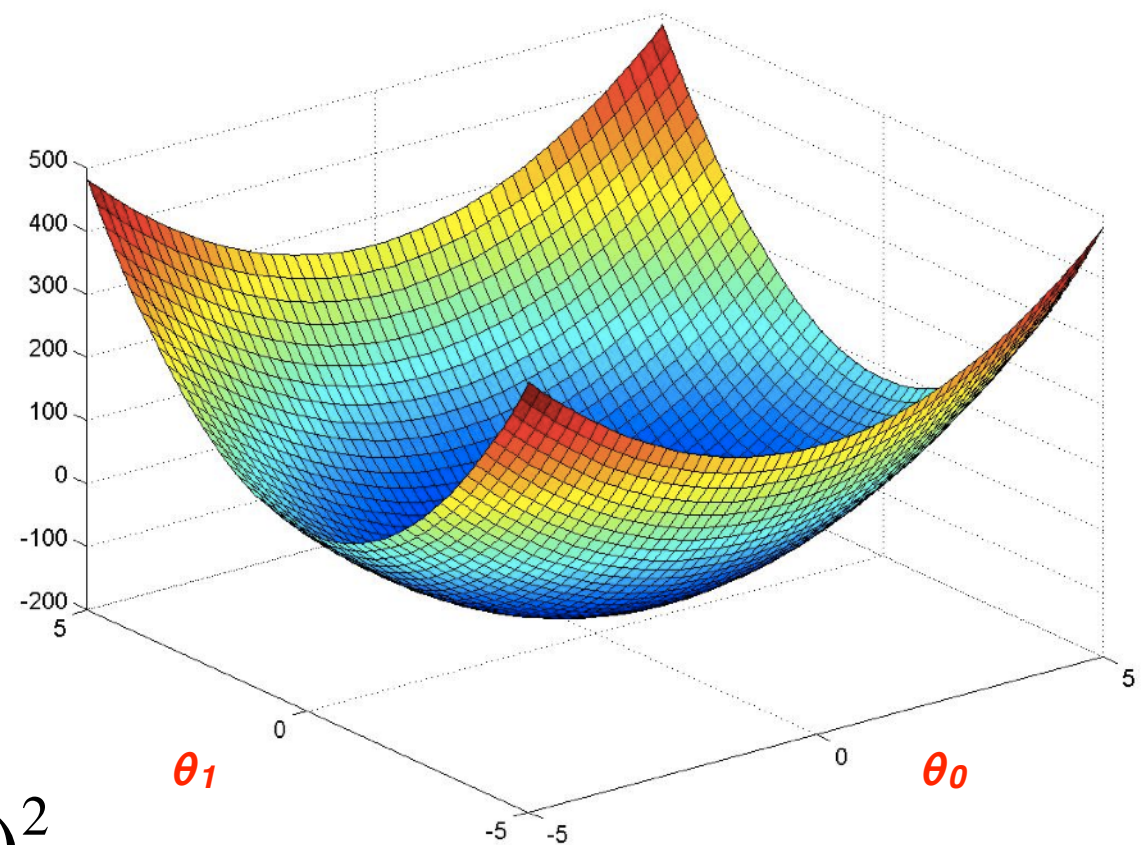
- **Parameters:**

$$\theta_0, \theta_1$$

- **Cost Function:**

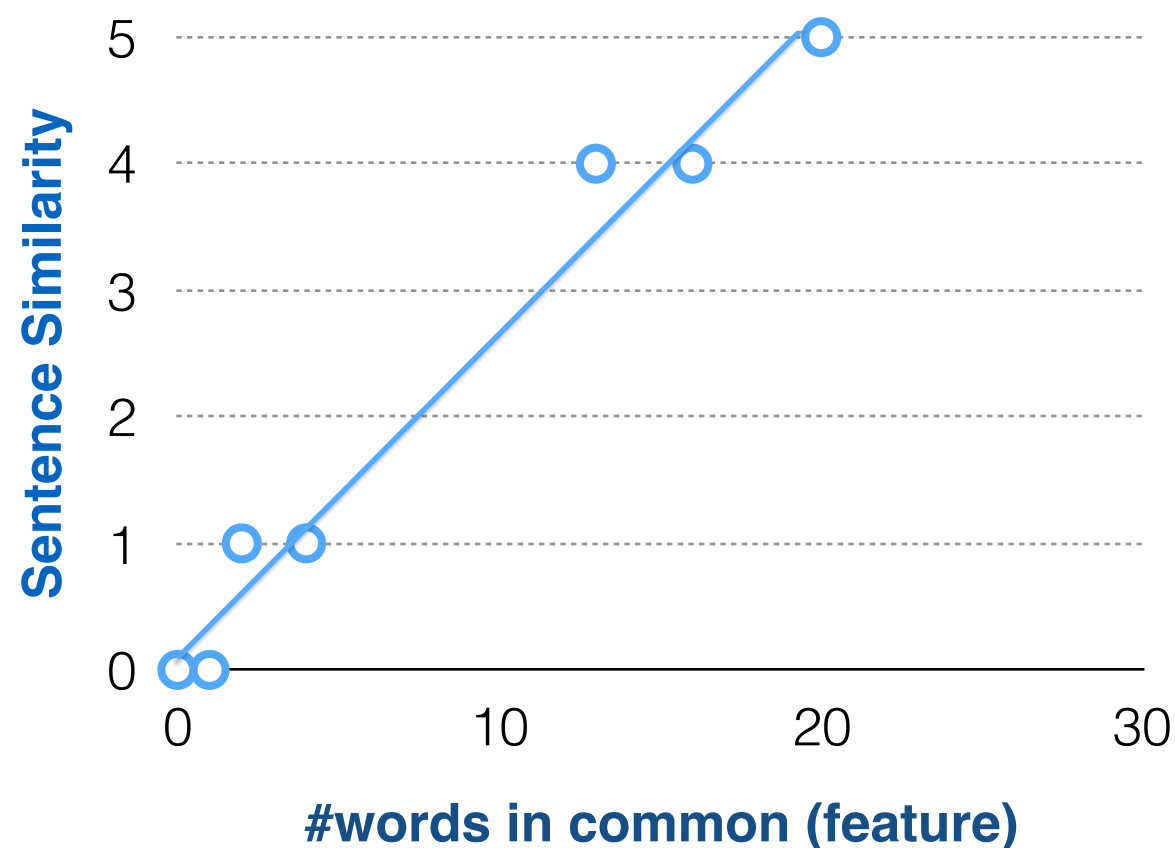
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2$$

- **Goal:** minimize $J(\theta_0, \theta_1)$
 θ_0, θ_1



(Recap) Linear Regression w/ one variable:

Cost Function



squared error function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2$$

- **Idea:** choose θ_0, θ_1 so that $h_{\theta}(x)$ is close to y for training examples (x, y) **minimize $J(\theta_0, \theta_1)$**
 θ_0, θ_1

(Recap)

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

simultaneous update
for $j=0$ and $j=1$

}


learning rate

(Recap) Linear Regression w/ one variable:

Gradient Descent

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)$$

simultaneous
update θ_0, θ_1

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) \cdot x_i$$

}

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

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$$h_{\theta}(x) = \theta^T x$$


Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_{\theta}(x) = \theta^T x$$

- Cost function: **# training examples**


$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

(Recap)

Paraphrase Identification

obtain sentential paraphrases automatically

Mancini has been sacked by Manchester City

Mancini gets the boot from Man City

Yes!



WORLD OF JENKS IS ON AT 11

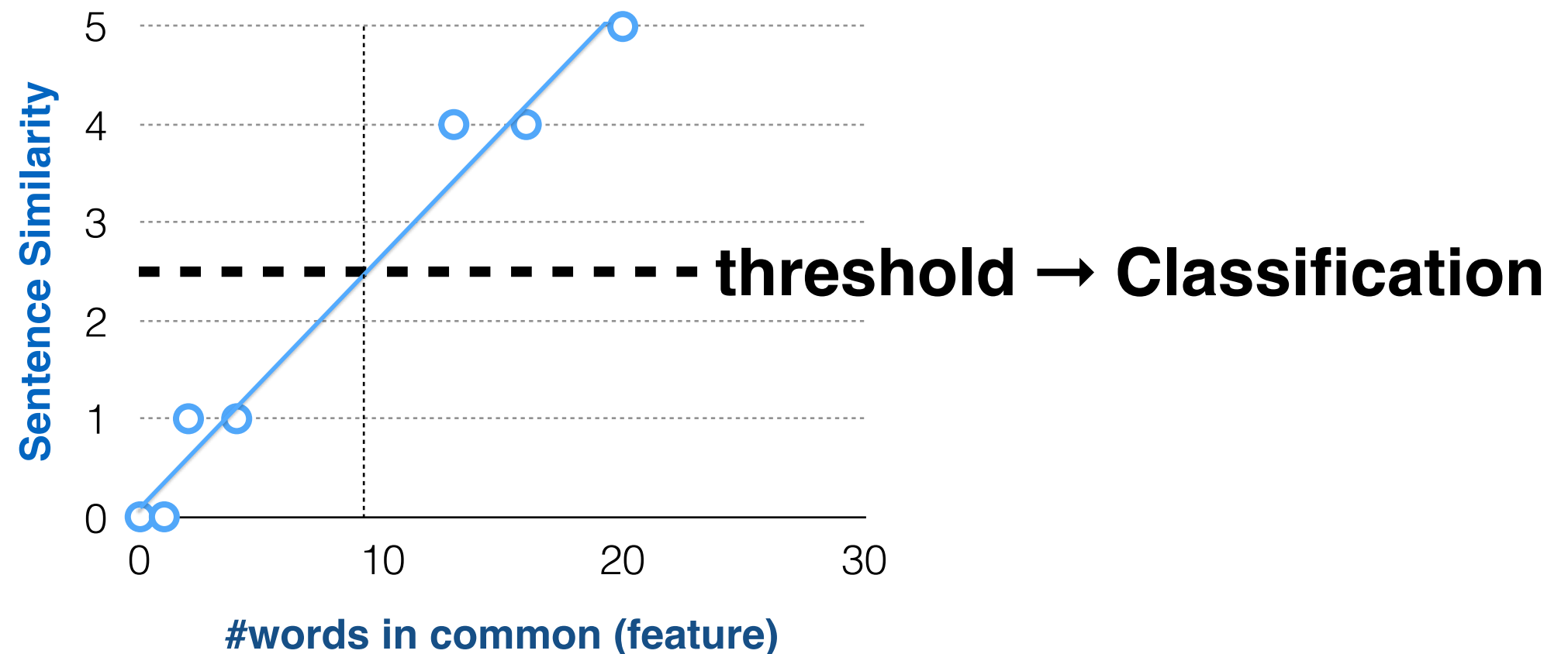
World of Jenks is my favorite show on tv

No!



(Recap)

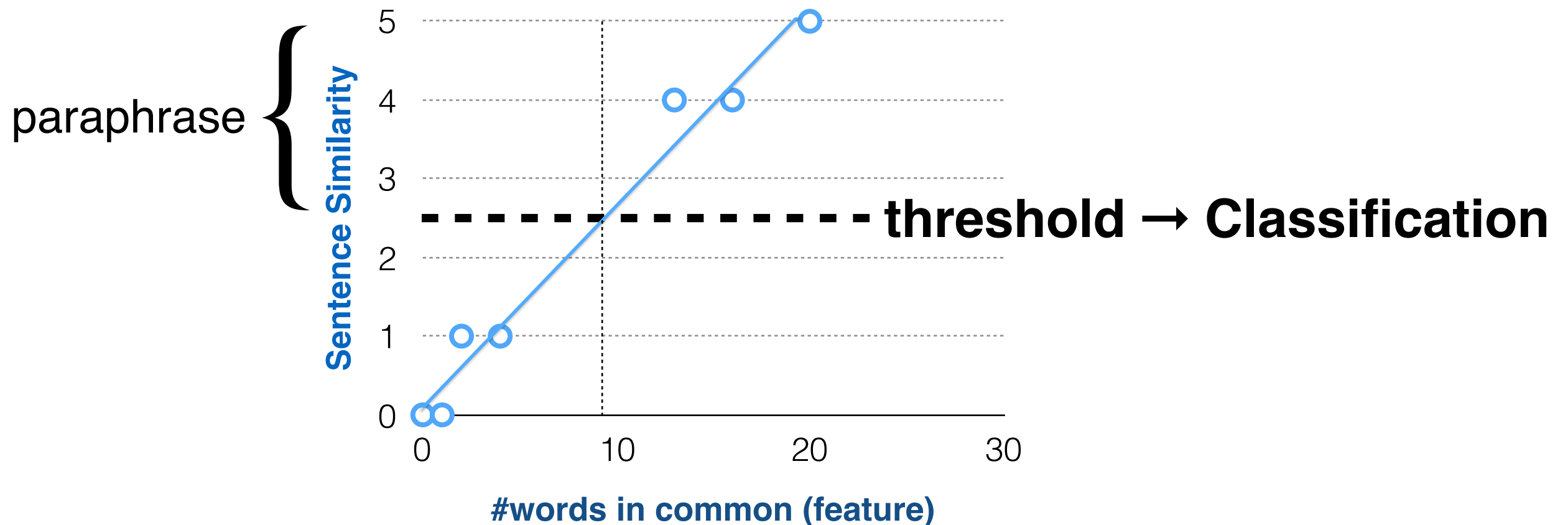
Linear Regression



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(Classification: predict discrete-valued output)

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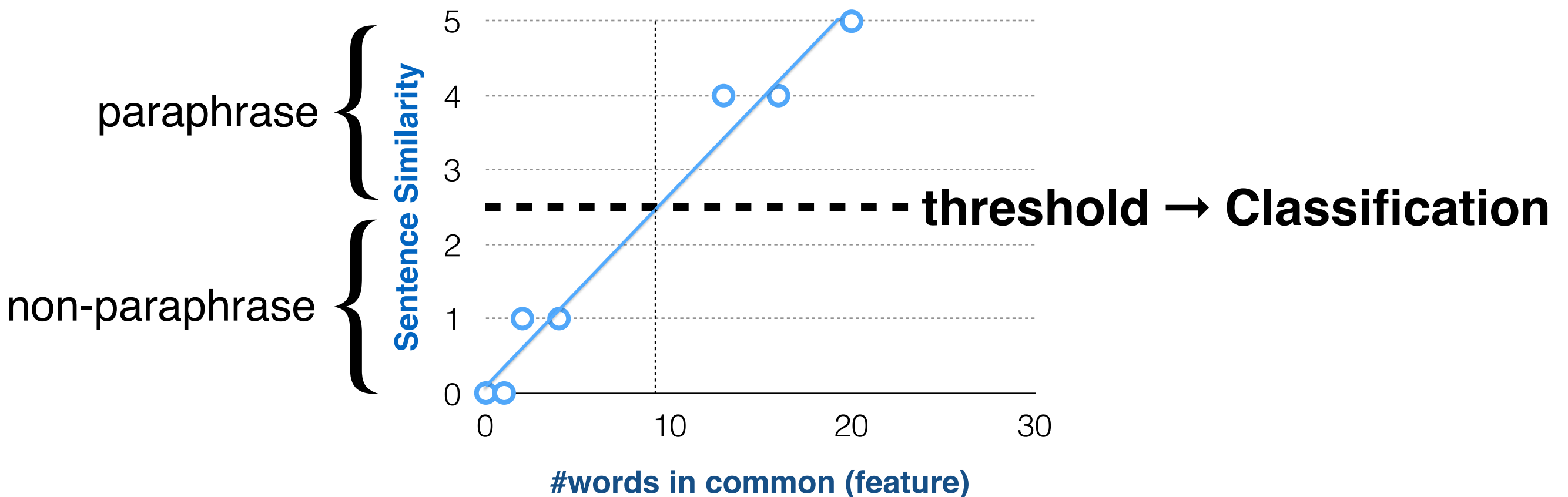
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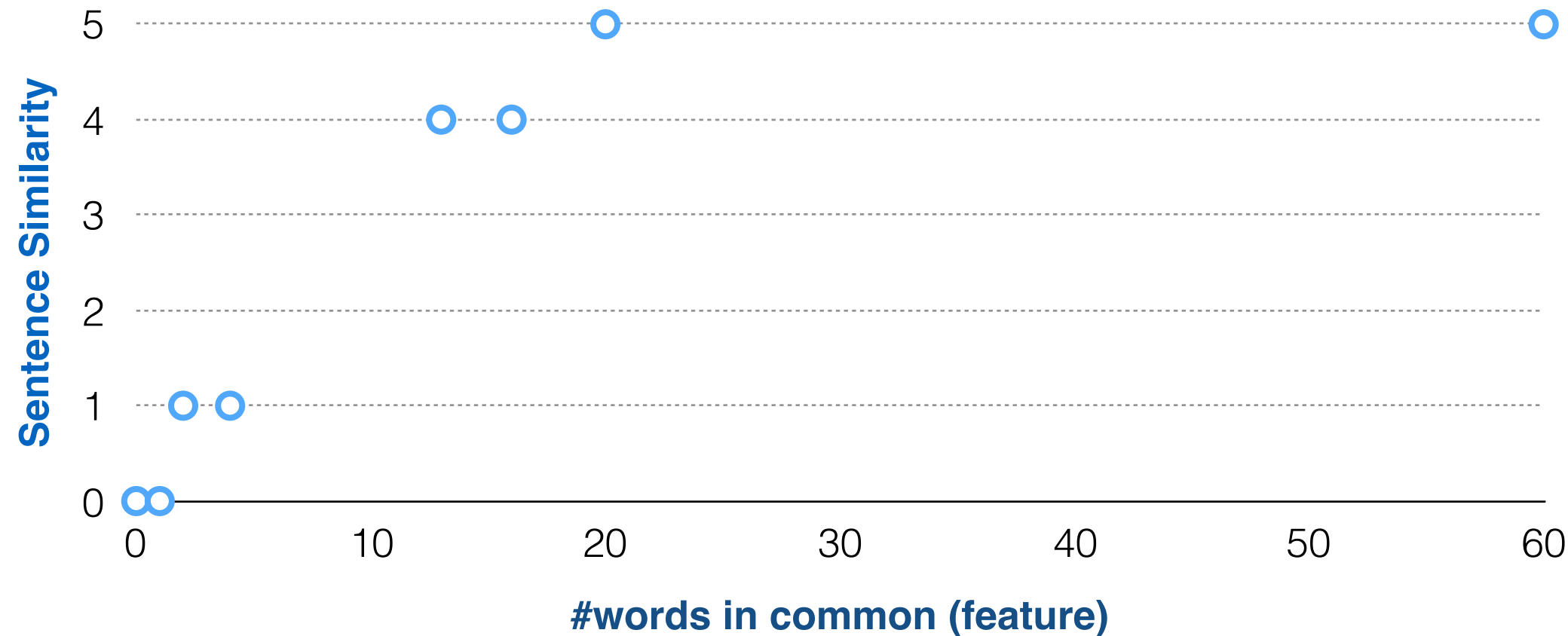
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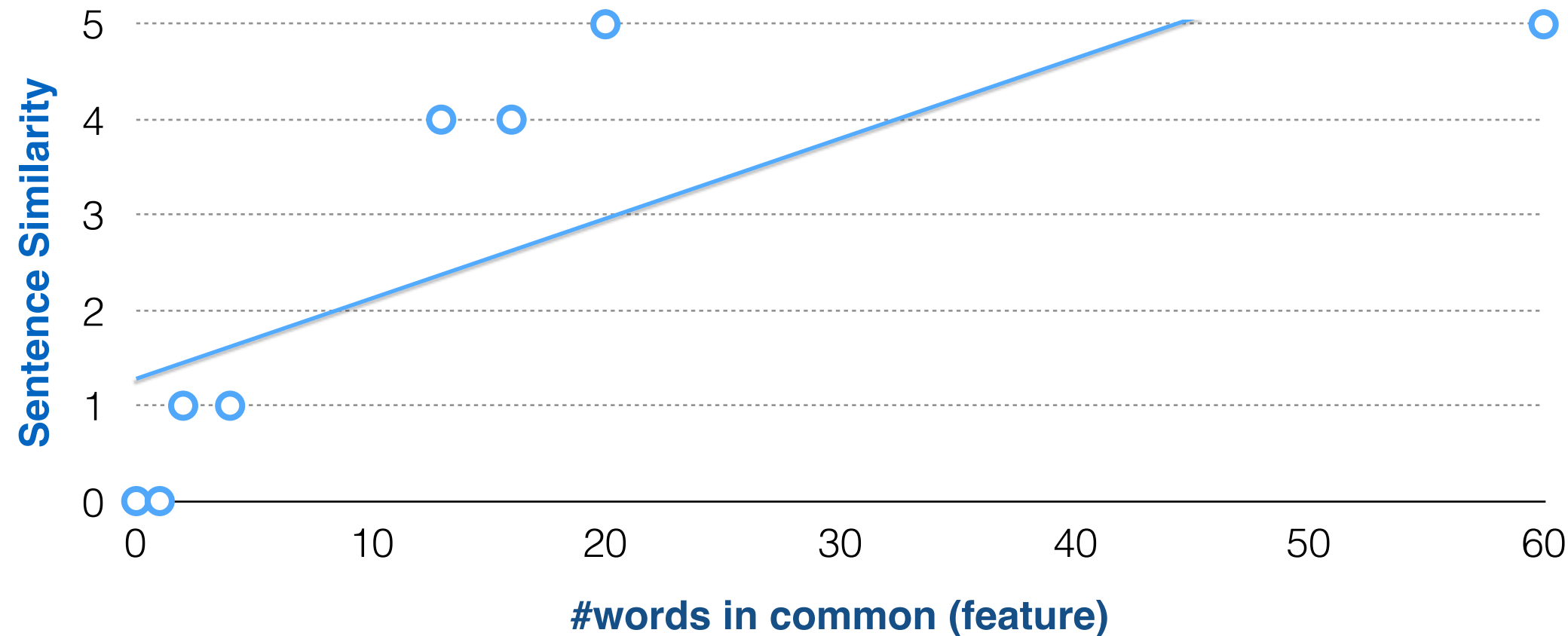
A problem in classification

Linear Regression



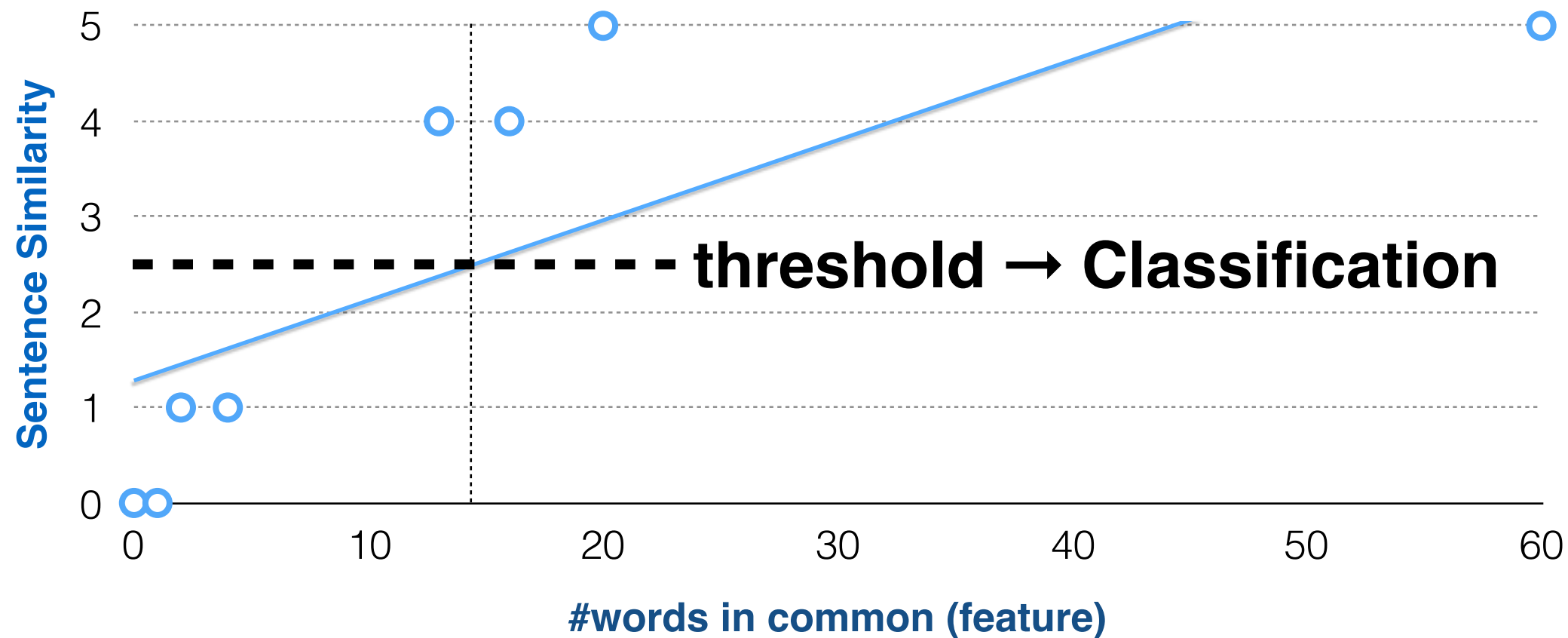
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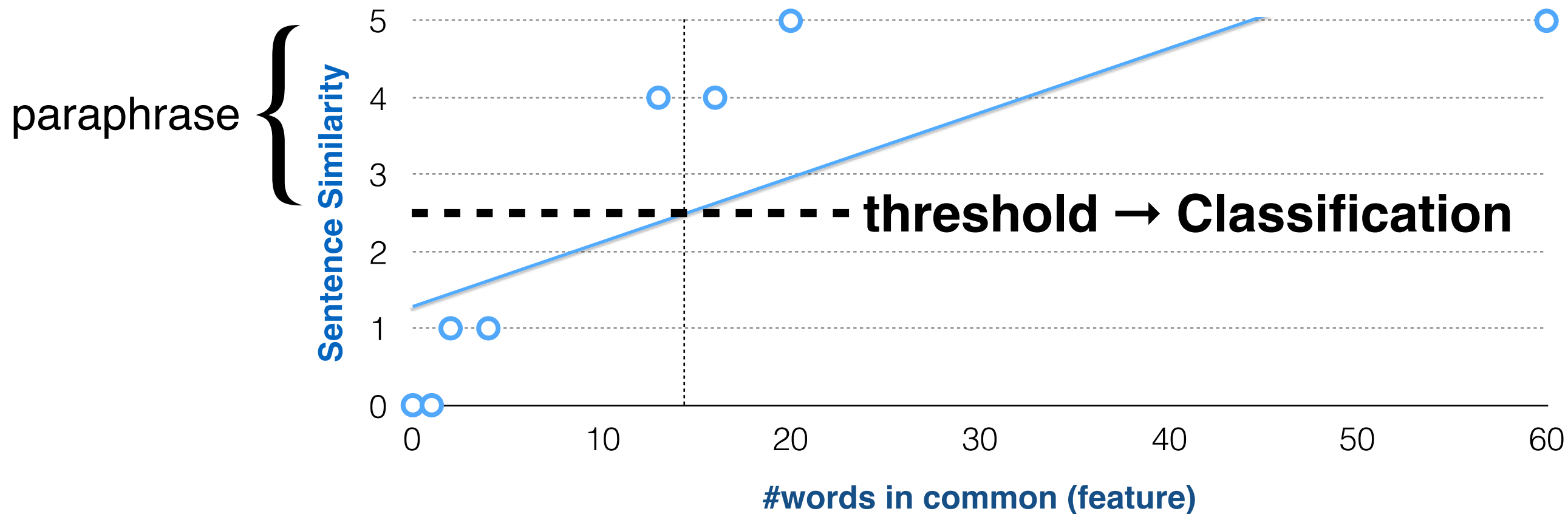
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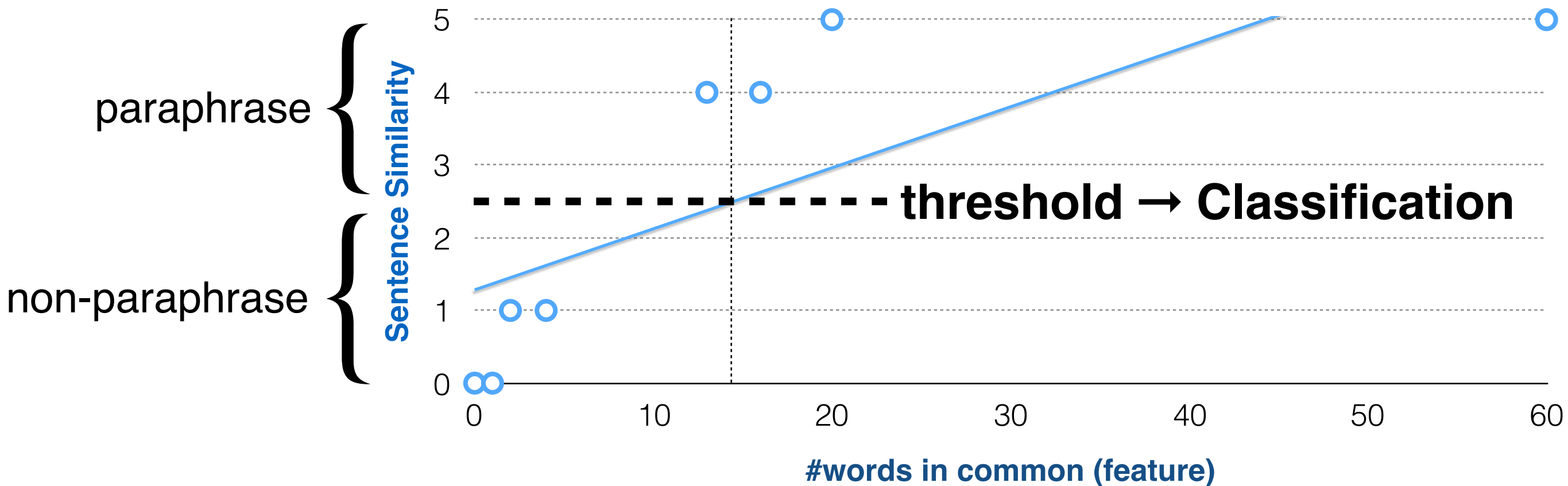
A problem in classification

Linear Regression



A problem in classification

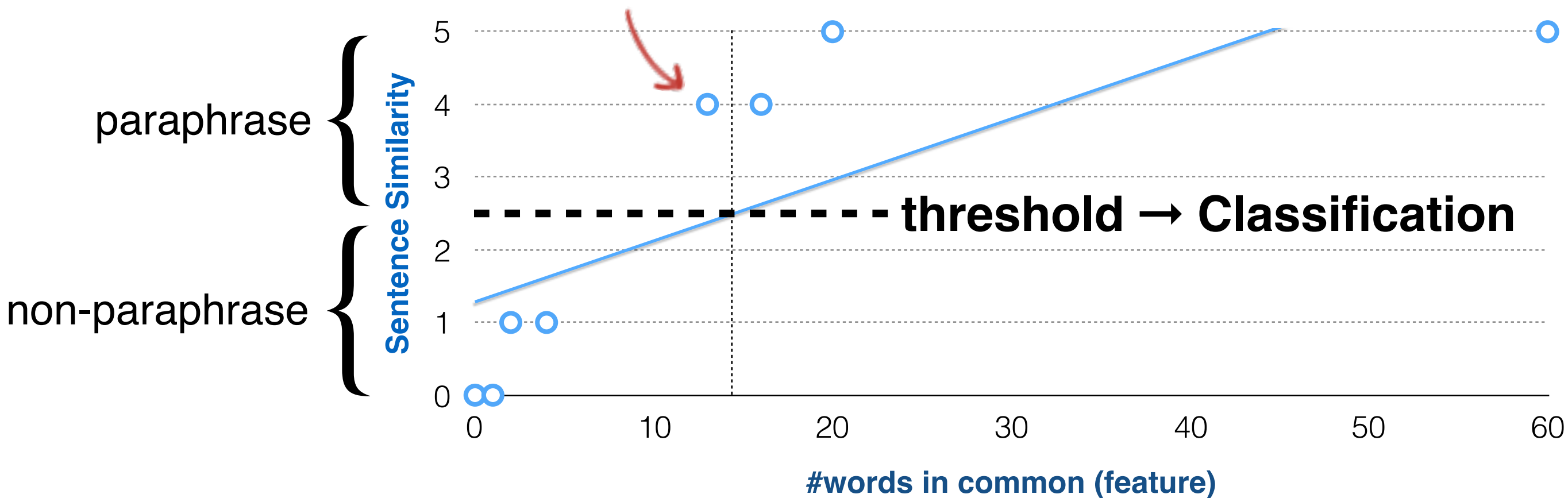
Linear Regression



A problem in classification

Linear Regression

classification error

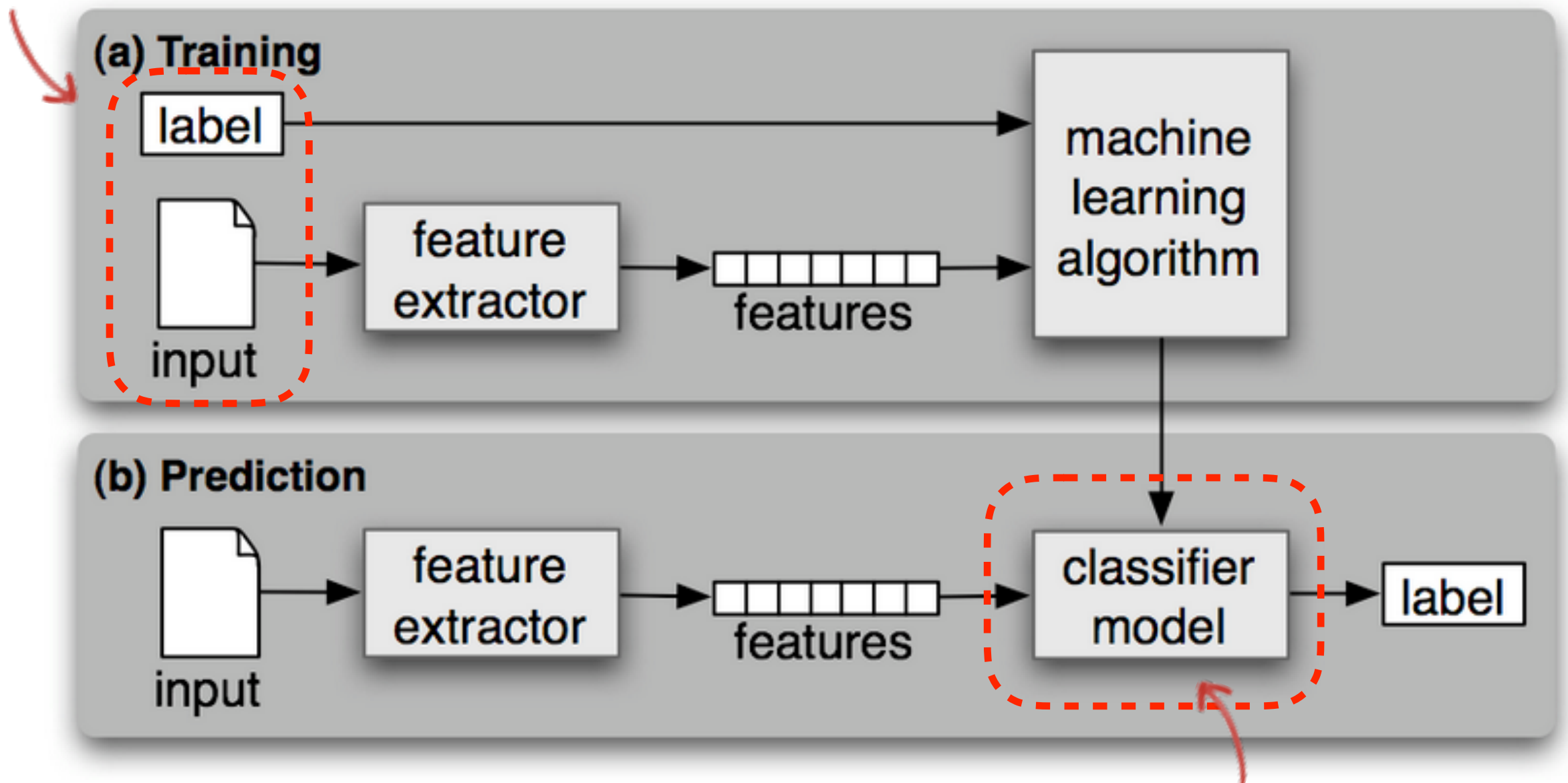


In practice, do not use linear regression for classification.

(Recap) Classification:

Supervised Machine Learning

training set



(also called) hypothesis

(Recap)

Logistic Regression

- One of the most useful **supervised machine learning algorithm** for classification!
- Generally high performance for a lot of problems.
- Much more robust than Naïve Bayes (better performance on various datasets).

Hypothesis:

Linear \rightarrow Logistic Regression

Classification: $y = 0$ or $y = 1$

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 **a classification (not regression) algorithm**

Hypothesis:

Linear \rightarrow Logistic Regression

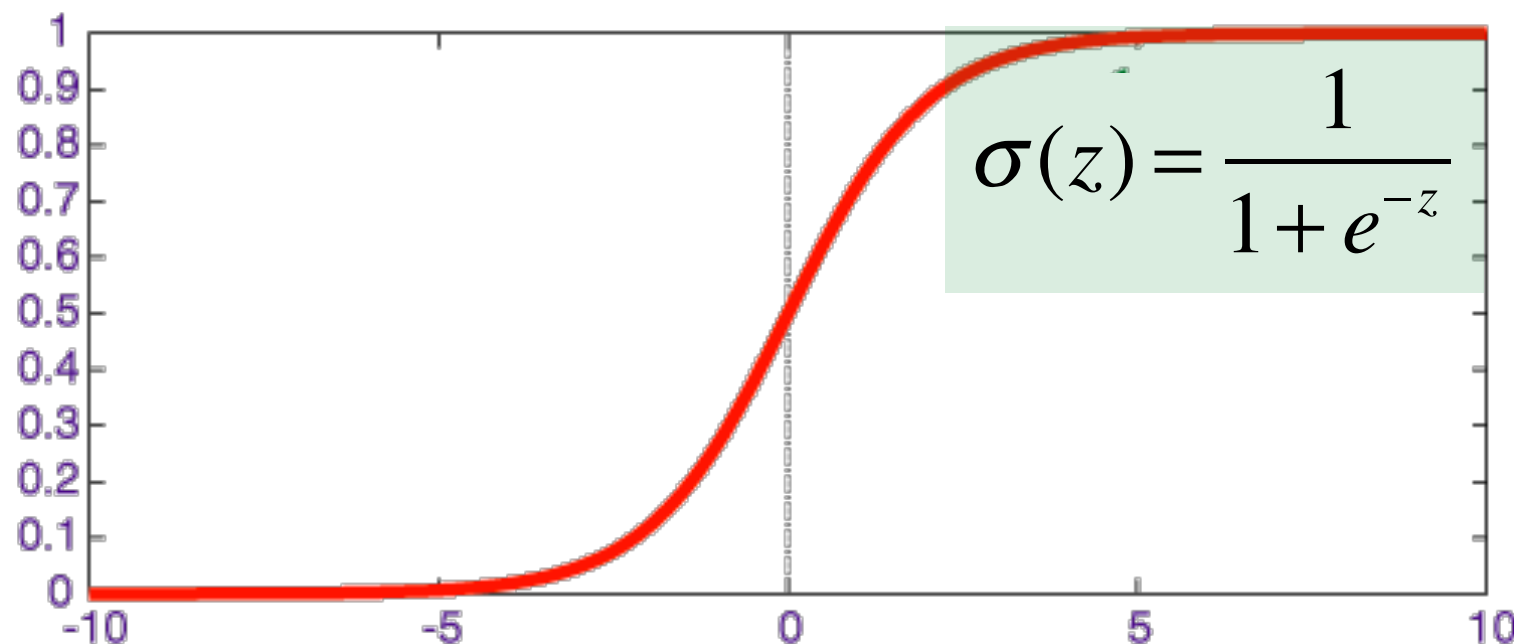
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sigmoid (logistic) function

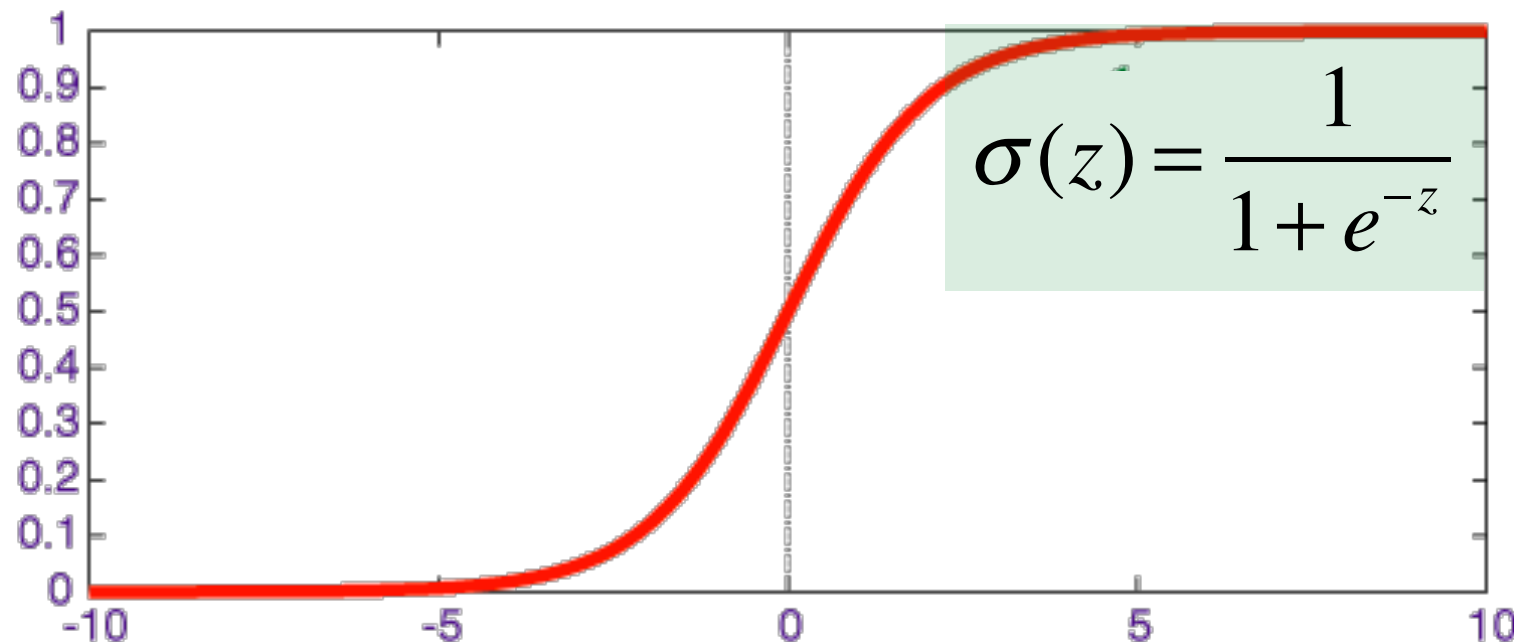


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Logistic Regression:

Interpretation of Hypothesis

- $h_{\theta}(x)$ = estimated probability that $y = 1$ on input

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$$\text{If } x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{\#words_in_common} \end{bmatrix}, h_{\theta}(x) = 0.7$$

70% chance of the sentence pair being paraphrases

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Interpretation of Hypothesis

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70% chance of the sentence pair being paraphrases

$$h_{\theta}(x) = P(y = 1 \mid x; \theta)$$



probability that $y = 1$, given x , parameterized by θ

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Logistic Regression:

Interpretation of Hypothesis

- $h_{\theta}(x)$ = estimated probability that $y = 1$ on input

$$P(y = 1 \mid x; \theta) + P(y = 0 \mid x; \theta) = 1$$

$$h_{\theta}(x) = P(y = 1 \mid x; \theta)$$



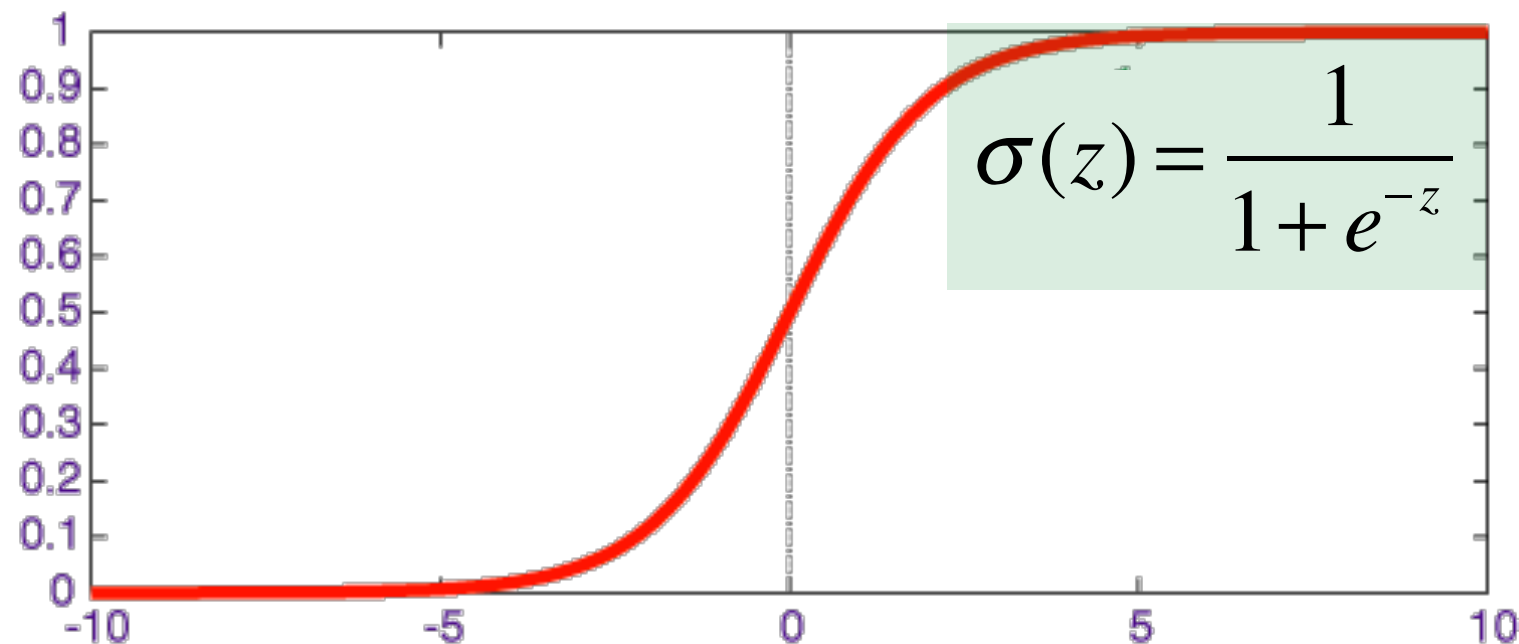
probability that $y = 1$, given x , parameterized by θ

Logistic Regression:

Decision Boundary

- Logistic Regression: **sigmoid (logistic) function**

$$h_{\theta}(x) = \sigma(\theta^T x)$$
$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$

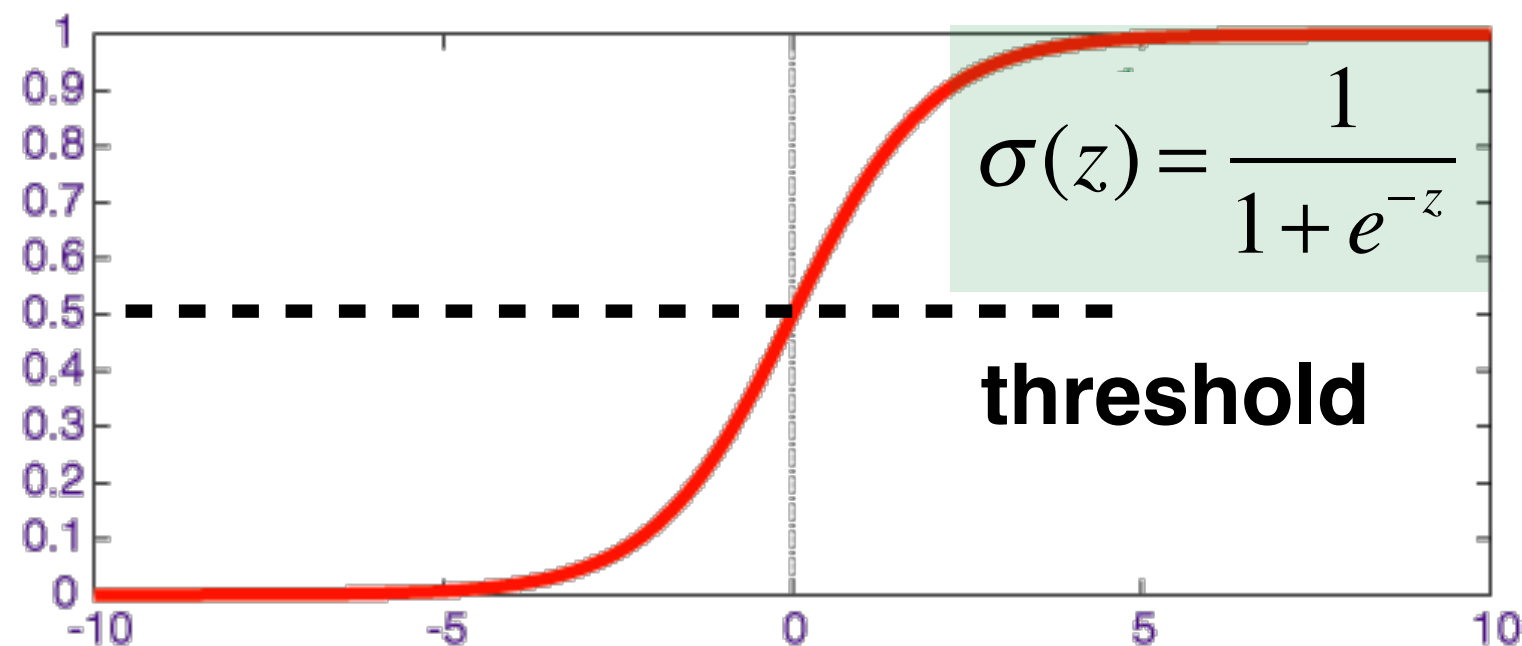


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predict **$y = 1$** if **$h_{\theta}(x) \geq 0.5$**

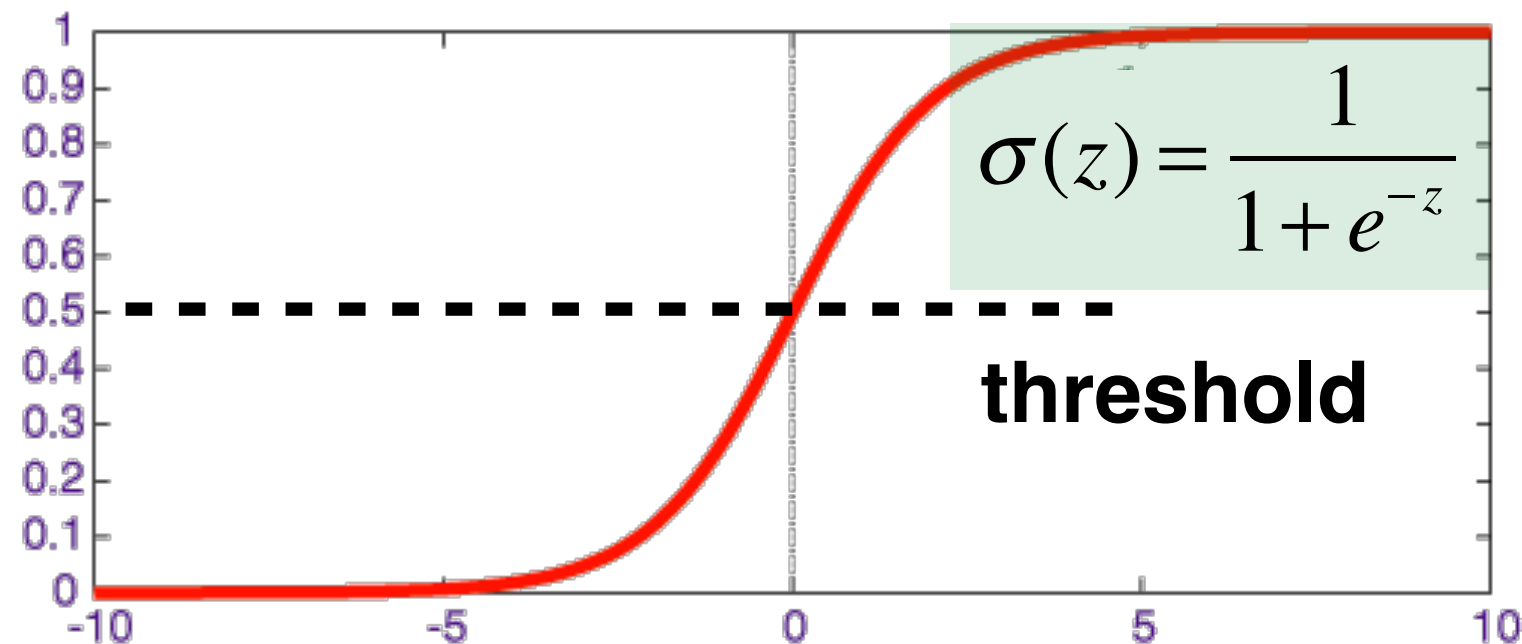
predict **$y = 0$** if **$h_{\theta}(x) < 0.5$**

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Decision Boundary

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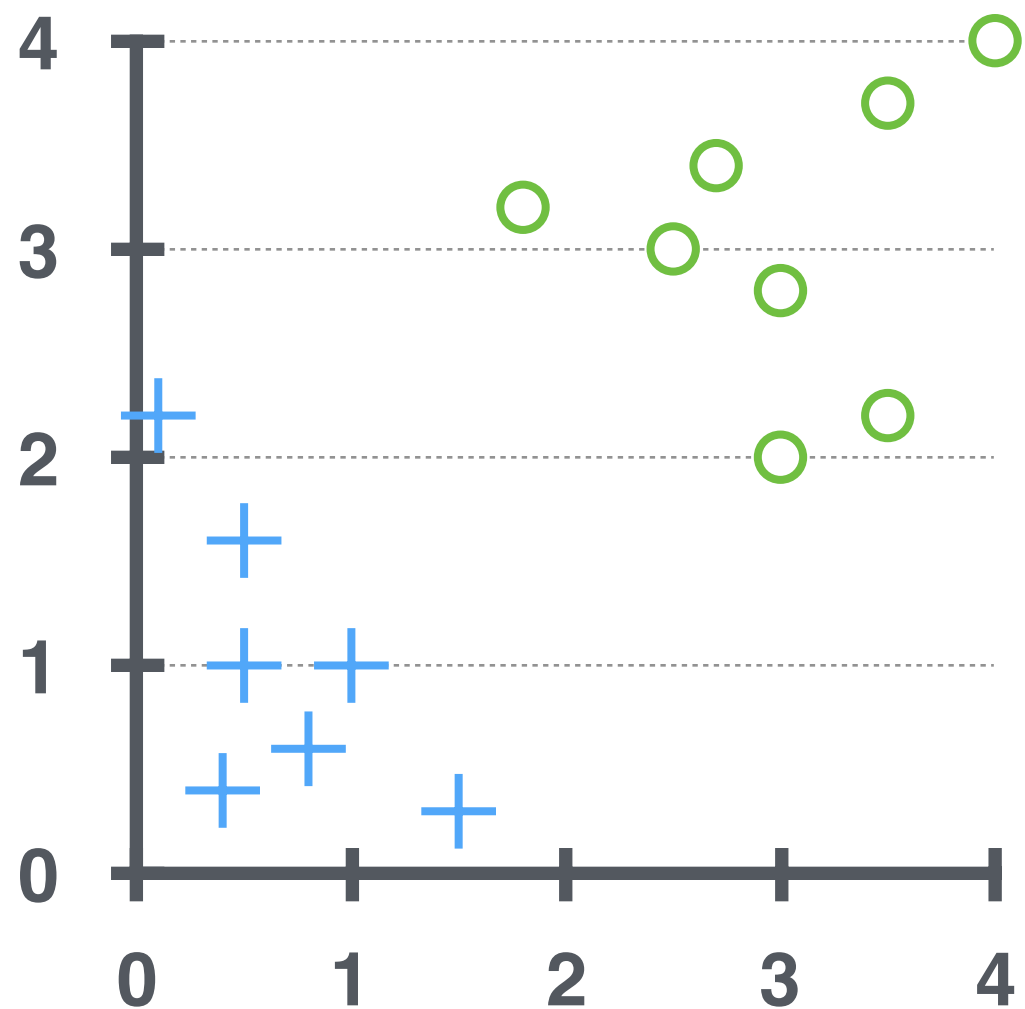


predict **$y = 1$** if **$h_{\theta}(x) \geq 0.5$** ← when **$\theta^T x \geq 0$**

predict **$y = 0$** if **$h_{\theta}(x) < 0.5$** ← when **$\theta^T x < 0$**

Logistic Regression:

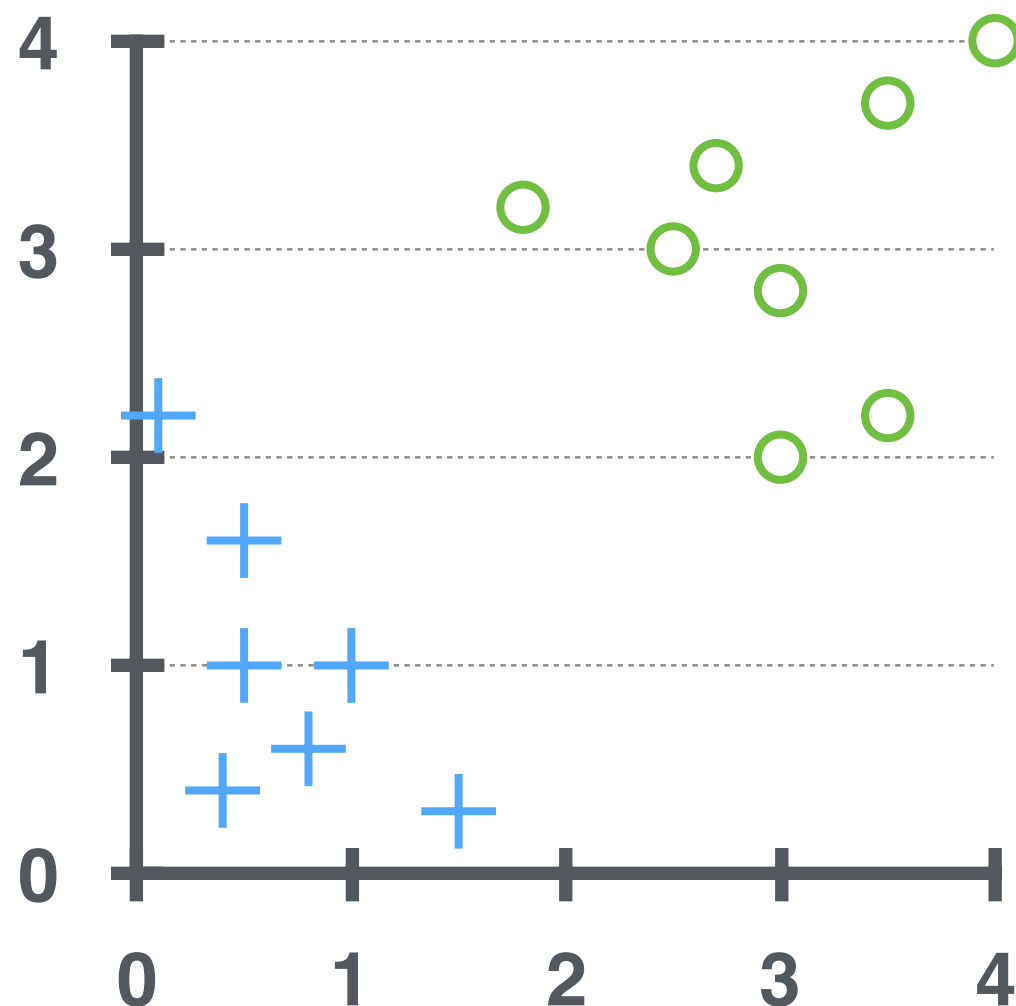
Decision Boundary



Logistic Regression:

Decision Boundary

$$h_{\theta}(x) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$



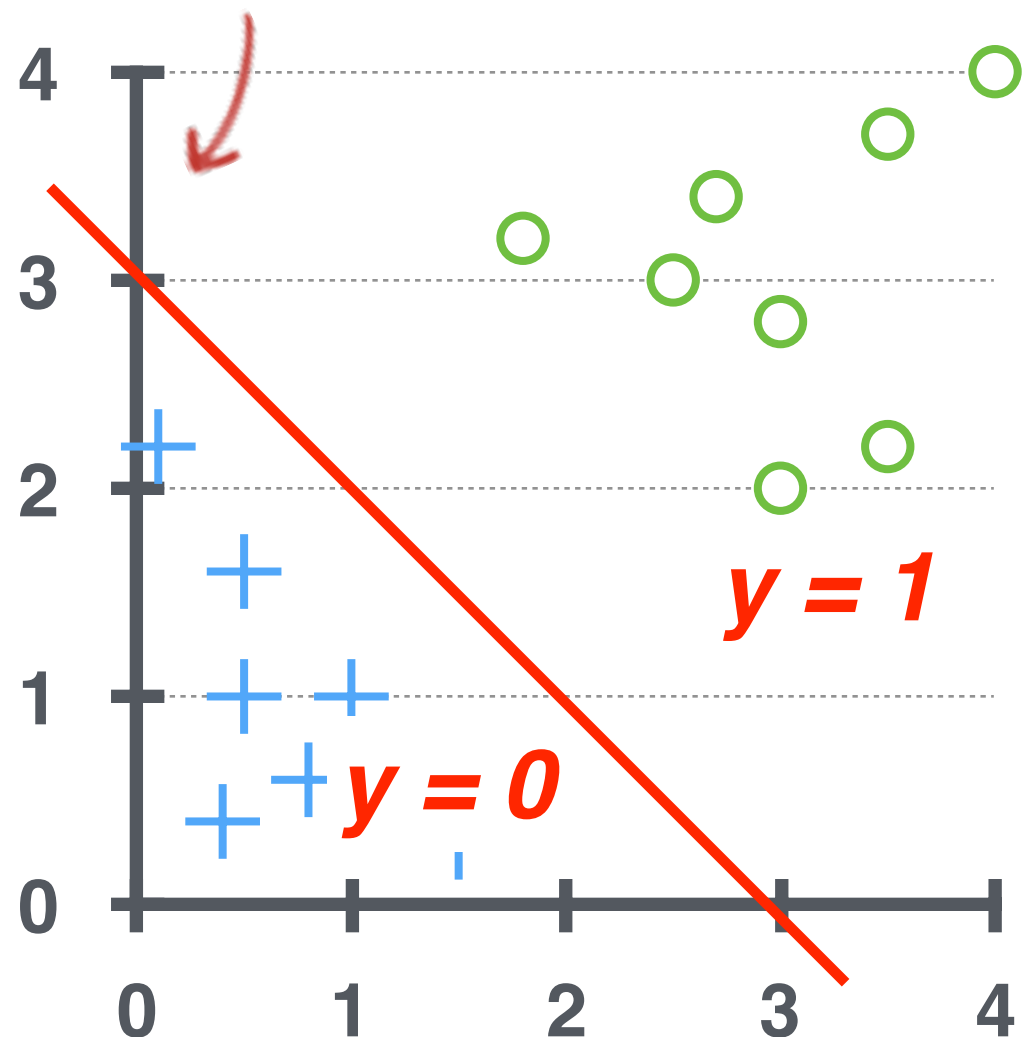
What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict **$y = 1$** if **$\theta^T x \geq 0$**

Logistic Regression:

Decision Boundary

decision boundary



$$h_{\theta}(x) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

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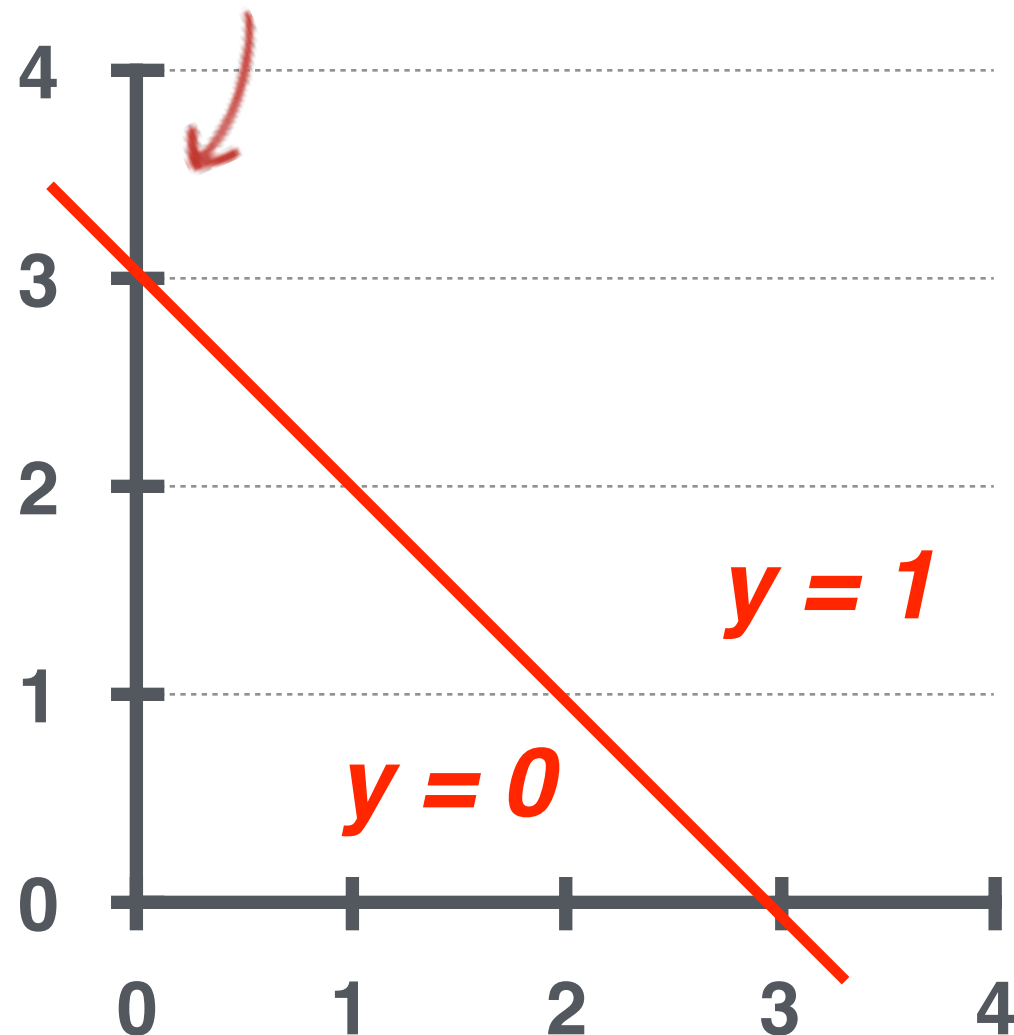
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Logistic Regression:

Decision Boundary

decision boundary

$$h_{\theta}(x) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$



a property of the hypothesis
a property of the parameters
~~a property of the dataset~~

Logistic Regression

- a training set of m hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})$ ($y \in \{0, 1\}$)

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta) = \frac{1}{m} \sum_{i=1}^m \frac{1}{2} \left(h_{\theta}(x^{(i)}) - y^{(i)} \right)^2$

squared error function

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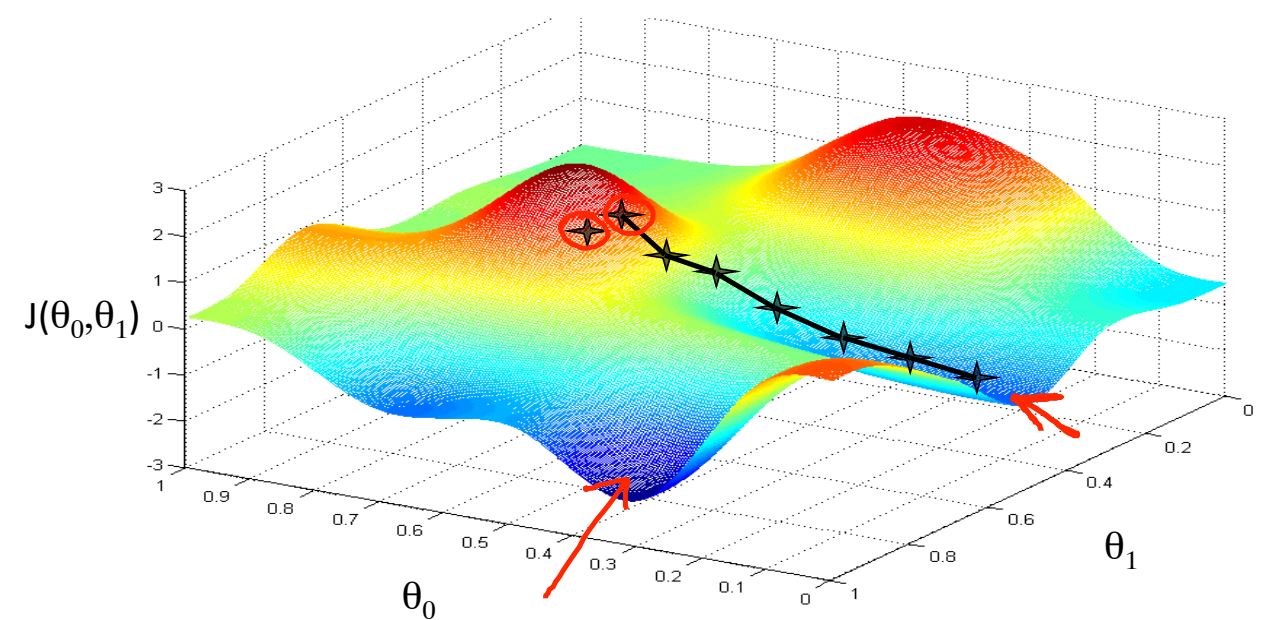
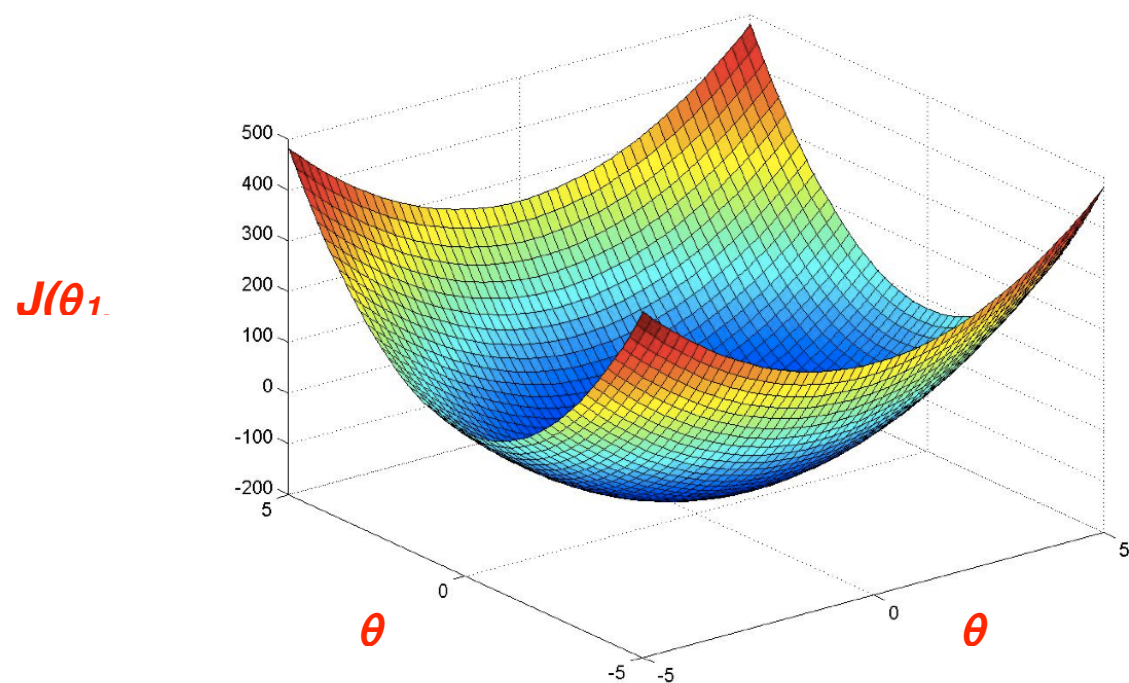
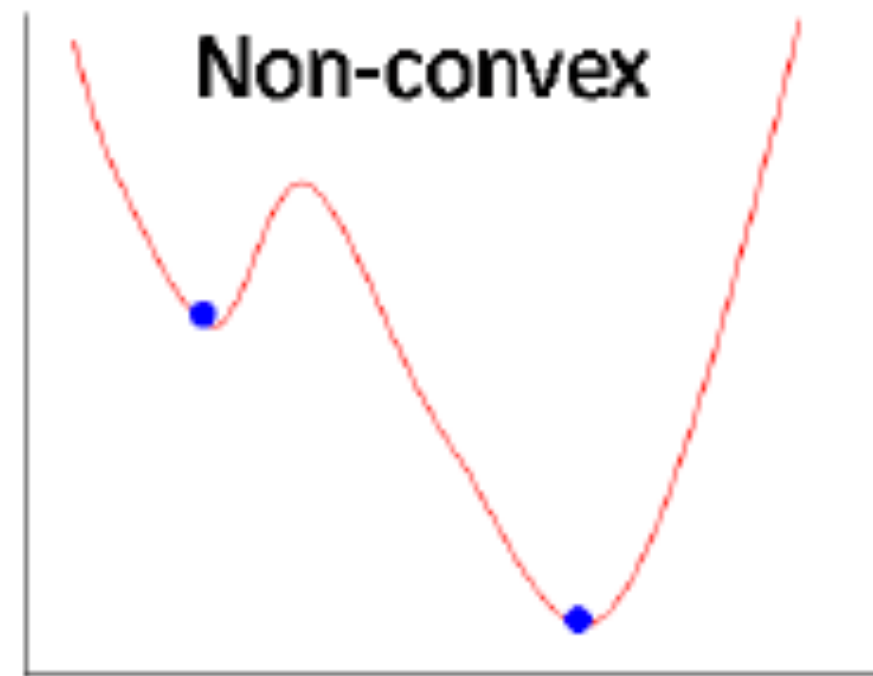
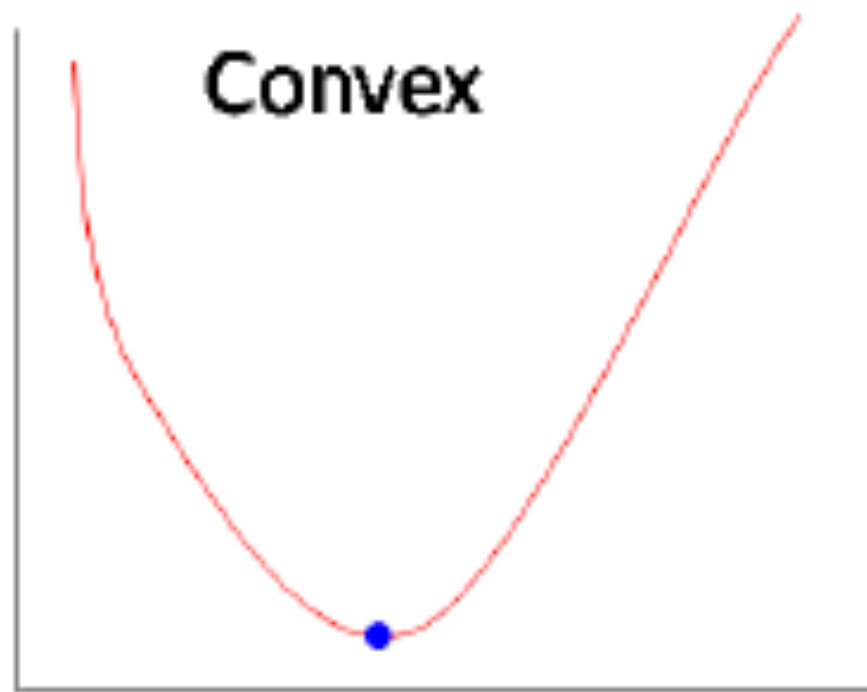
this cost function is non-convex for logistic regression

Cost function:

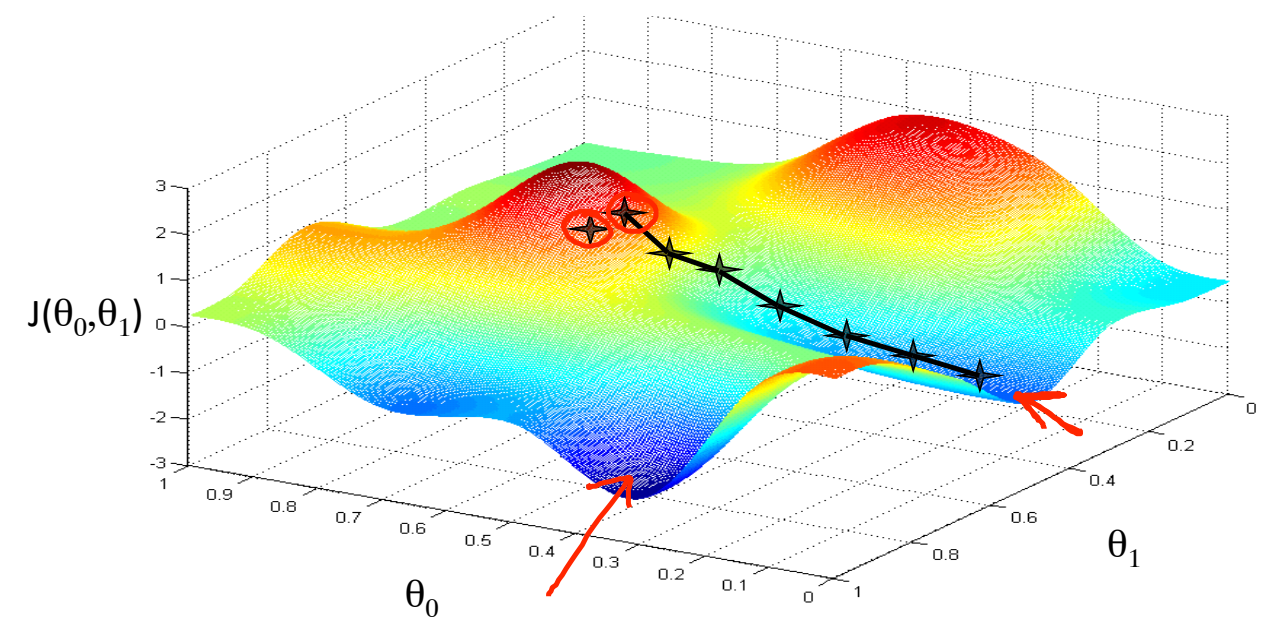
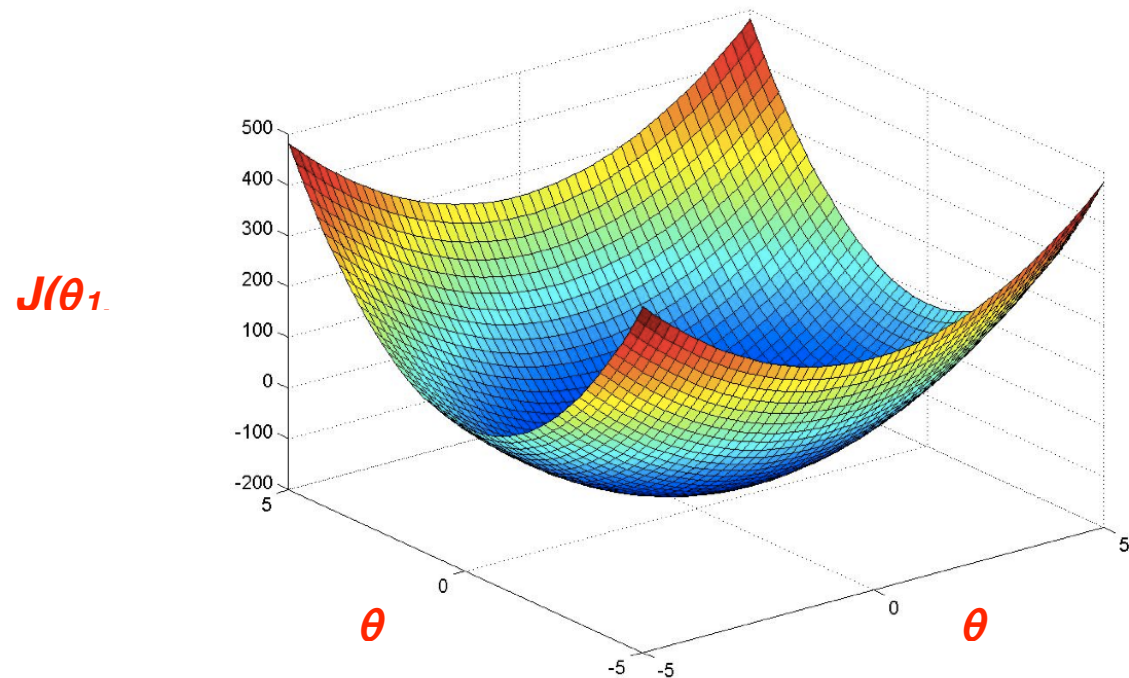
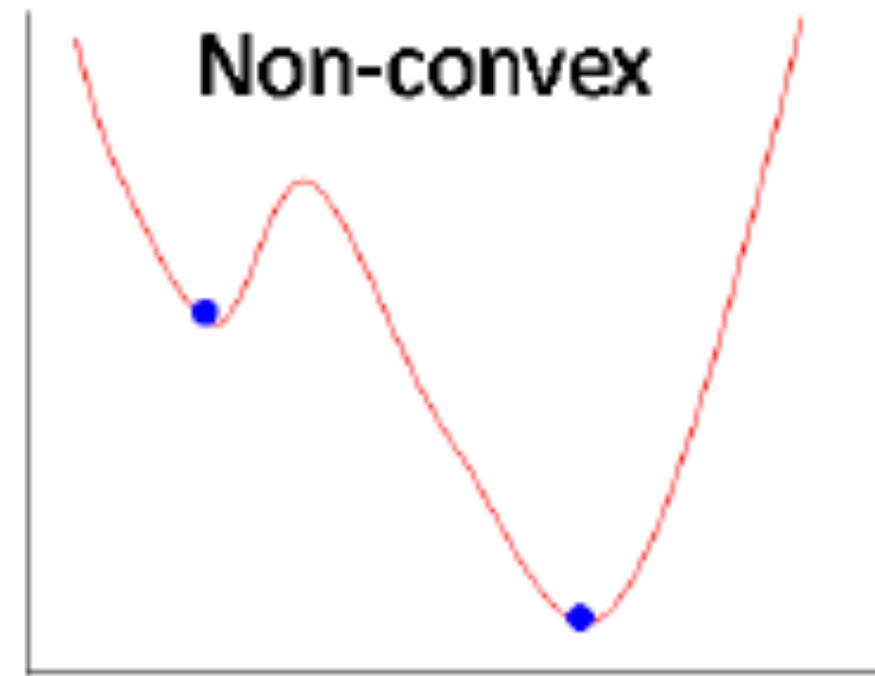
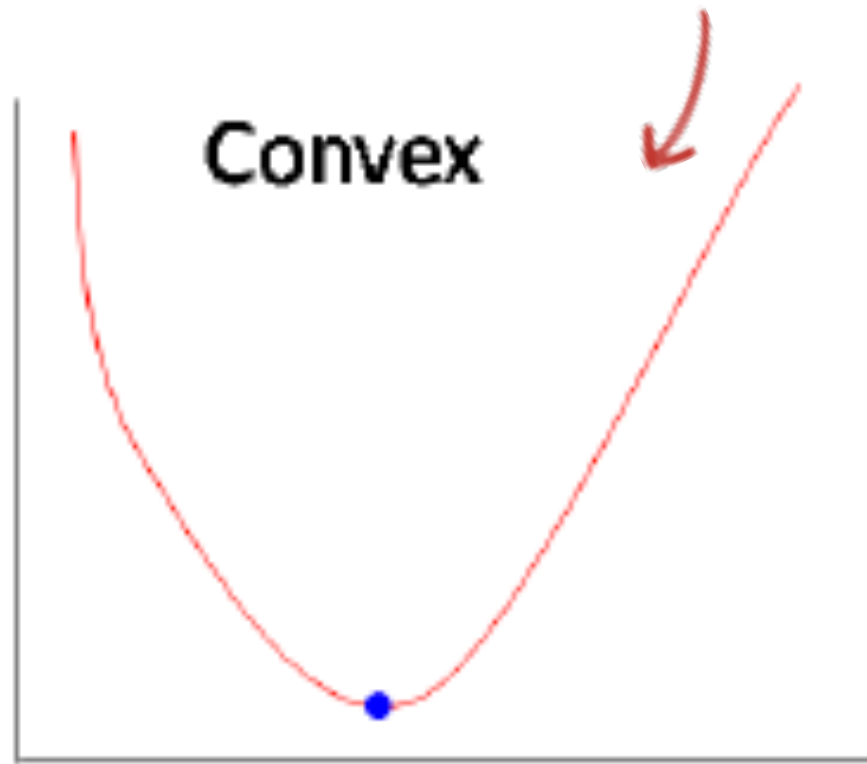
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||
 $\text{Cost}(h_{\theta}(x), y)$
- Logistic Regression: $h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$

this cost function is non-convex for logistic regression



we want convex! easy gradient descent!



Logistic Regression:

Cost Function

$$\text{Cost}(h_{\theta}(x), y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

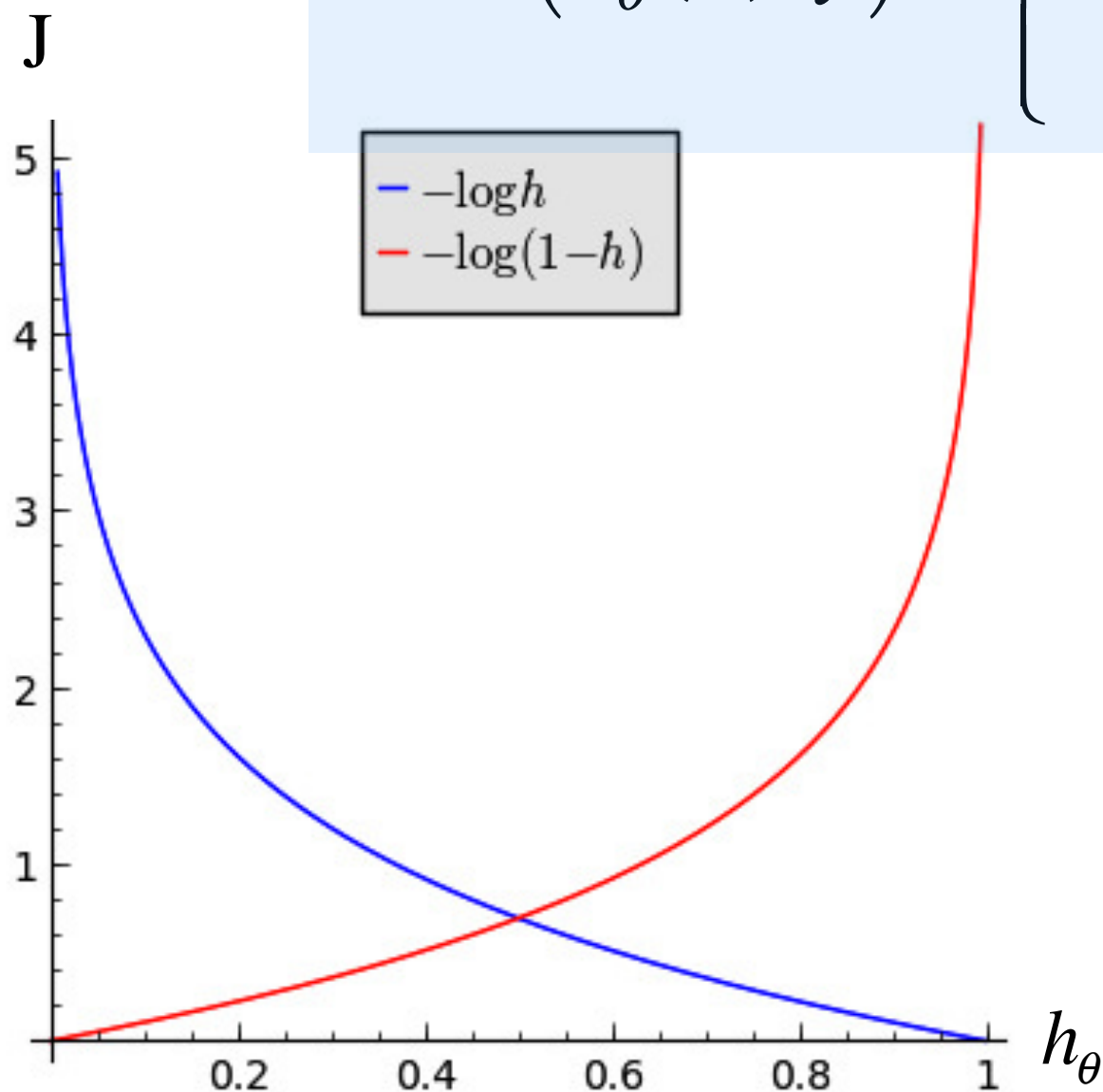
remember that

$$0 \leq h_{\theta}(x) \leq 1$$

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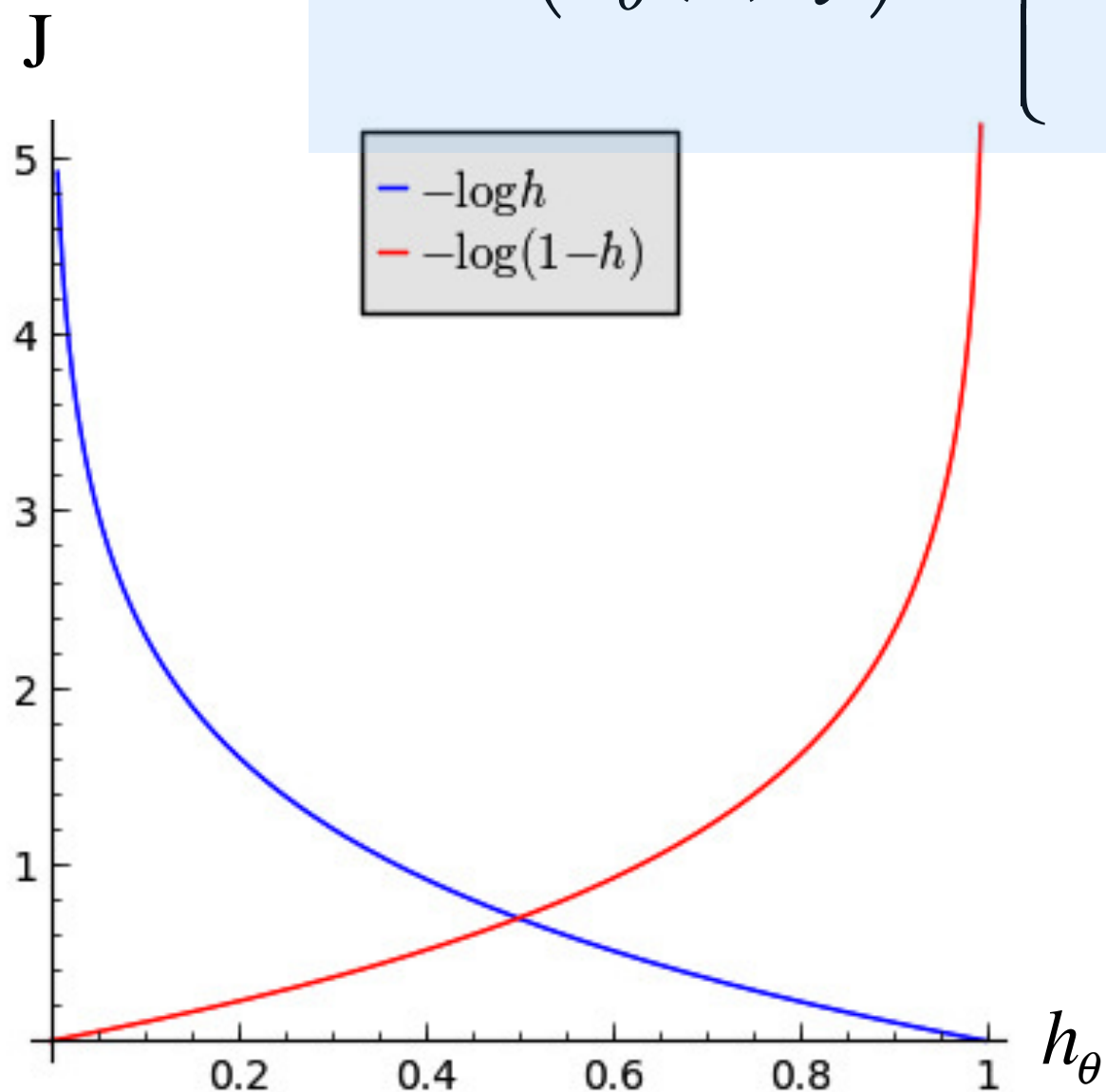
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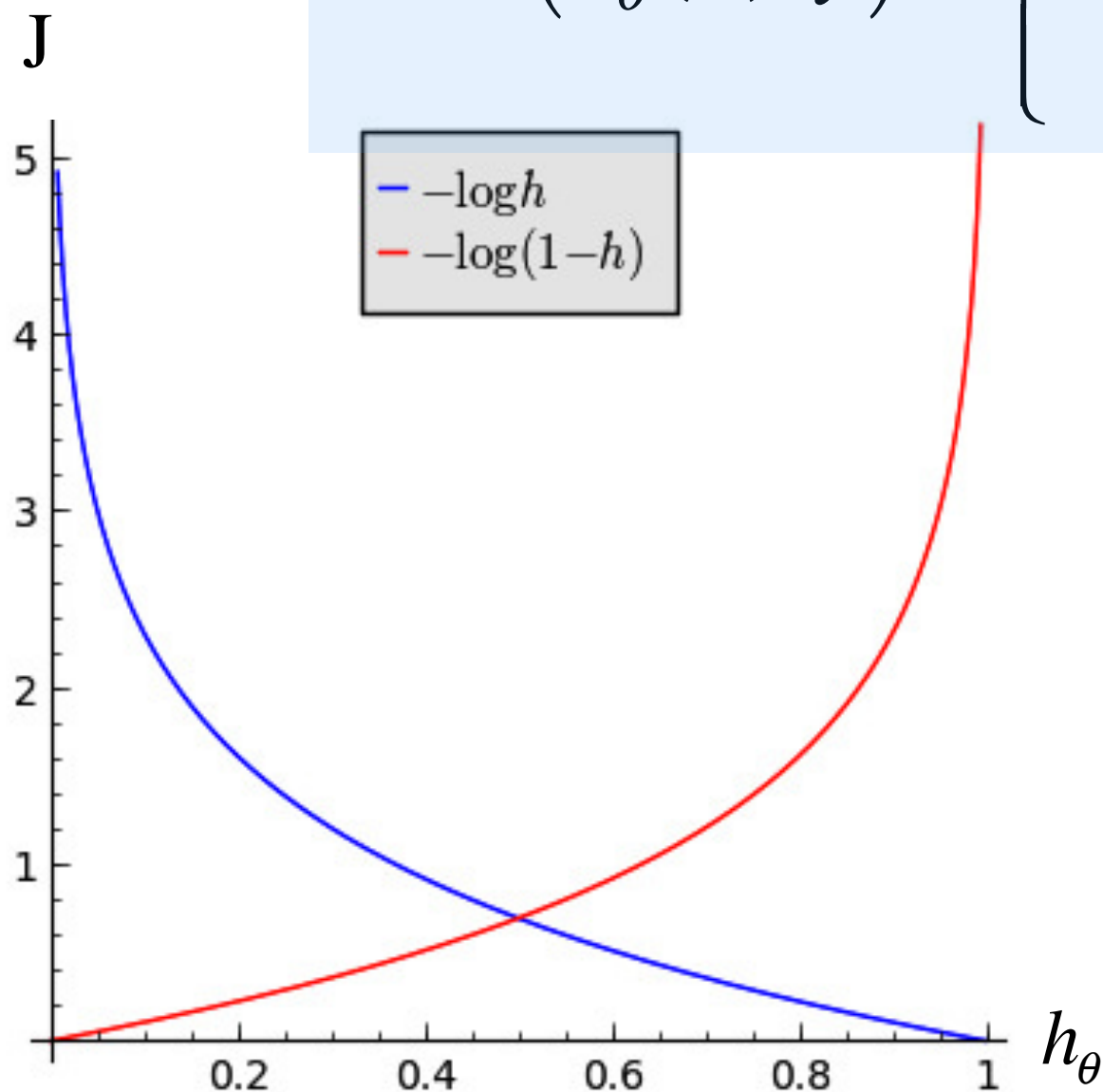
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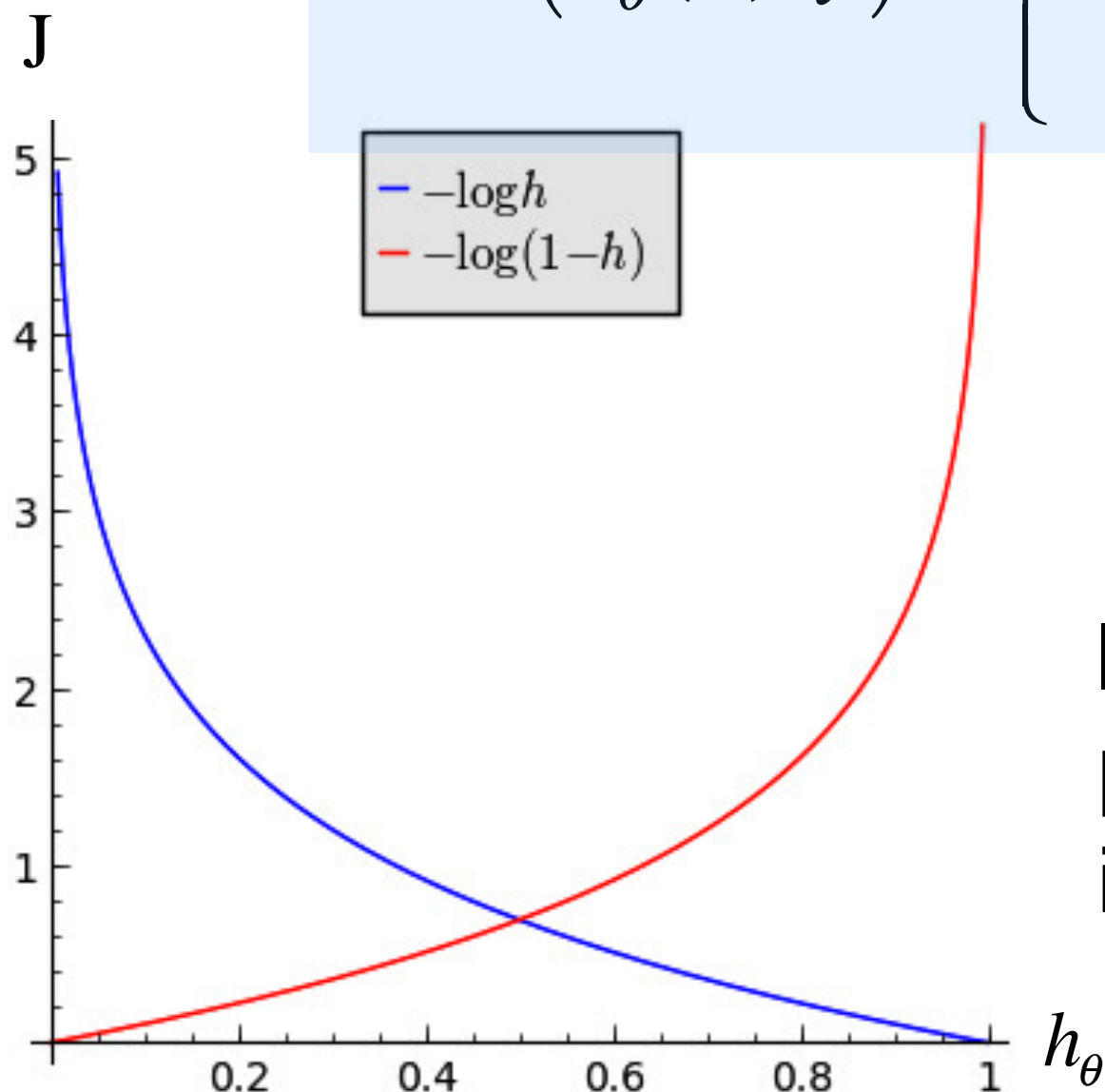
Cost = 0 if **$y = 1$** , **$h_{\theta}(x) = 1$**

But as **$h_{\theta}(x) \rightarrow 0$** , **Cost $\rightarrow \infty$**

Logistic Regression:

Cost Function

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Cost = 0 if **$y = 1$** , **$h_{\theta}(x) = 1$**

But as **$h_{\theta}(x) \rightarrow 0$** , **Cost $\rightarrow \infty$**

Intuition:

penalize learning algorithm

if **$h_{\theta}(x) = 0$** (predict **$P(y=1|x;\theta) = 0$**),

but **$y = 1$**

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\text{Cost}(h_{\theta}(x), y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

remember that $y = 0$ or 1 always

Logistic Regression:

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remember that $y = 0$ or 1 always

the same

cross entropy loss:

$$\text{Cost}(h_{\theta}(x), y) = -y \log(h_{\theta}(x)) - (1 - y) \log(1 - h_{\theta}(x))$$

Logistic Regression


- **Cost Function:**

$$\begin{aligned} J(\theta) &= \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}) - y^{(i)}) \\ &= -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \end{aligned}$$

- **Goal:**

learn parameters θ to minimize $J(\theta)$

- **Hypothesis (to make a prediction):** $h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$

$P(y=1|x;\theta)$ 

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

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learning rate



simultaneous update
for all θ_j

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learning rate

training examples

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^m \left(h_{\theta}(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

Logistic Regression:

Gradient Descent

repeat until convergence { simultaneous update
for all θ_j

$$\theta_j := \theta_j - \alpha \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

}

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Logistic Regression:

Gradient Descent

repeat until convergence { simultaneous update
for all θ_j

$$\theta_j := \theta_j - \alpha \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

}

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

This look the same as linear regression!!???

Logistic Regression:

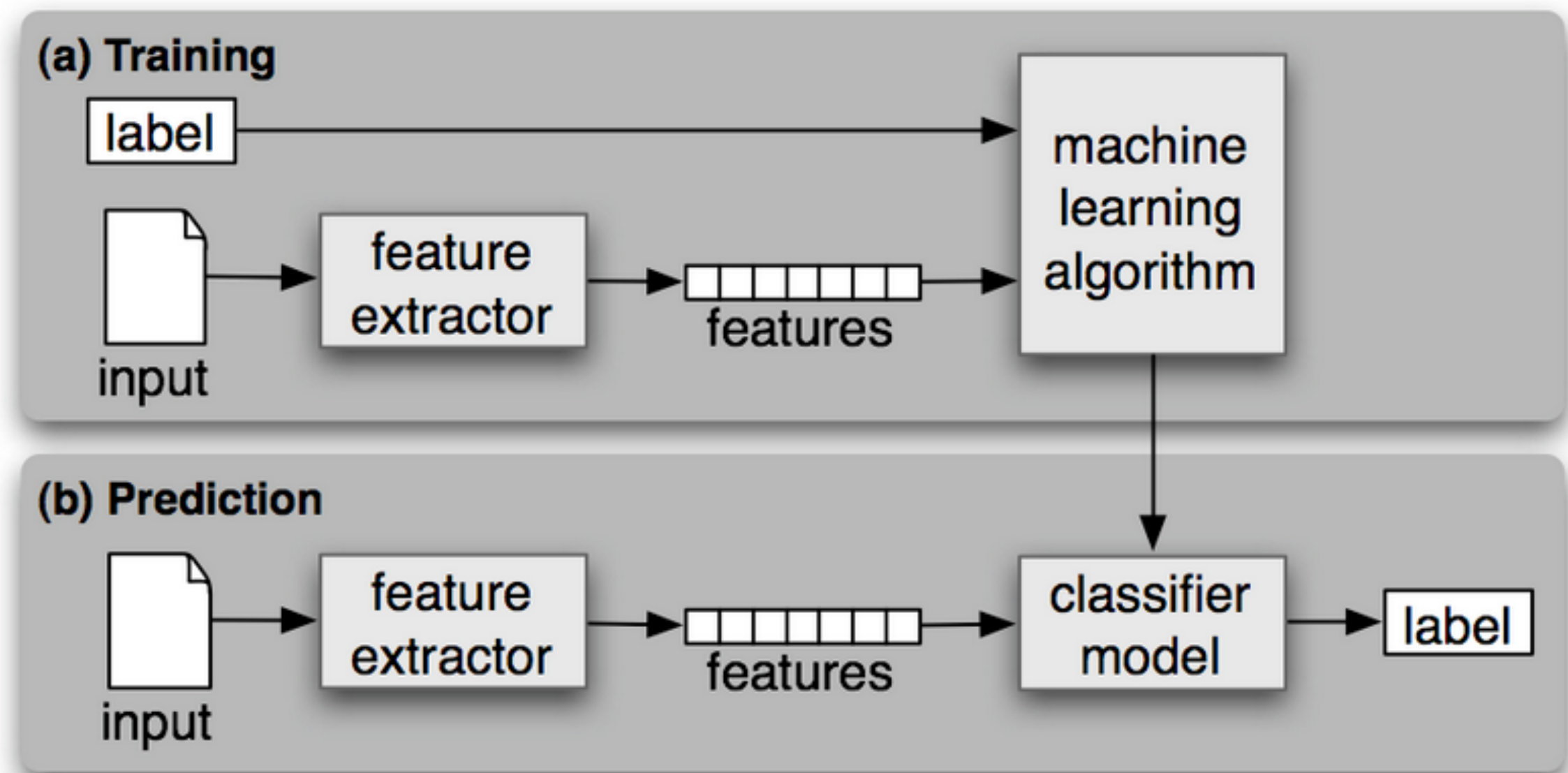
Gradient Descent

repeat until convergence { simultaneous update
for all θ_j

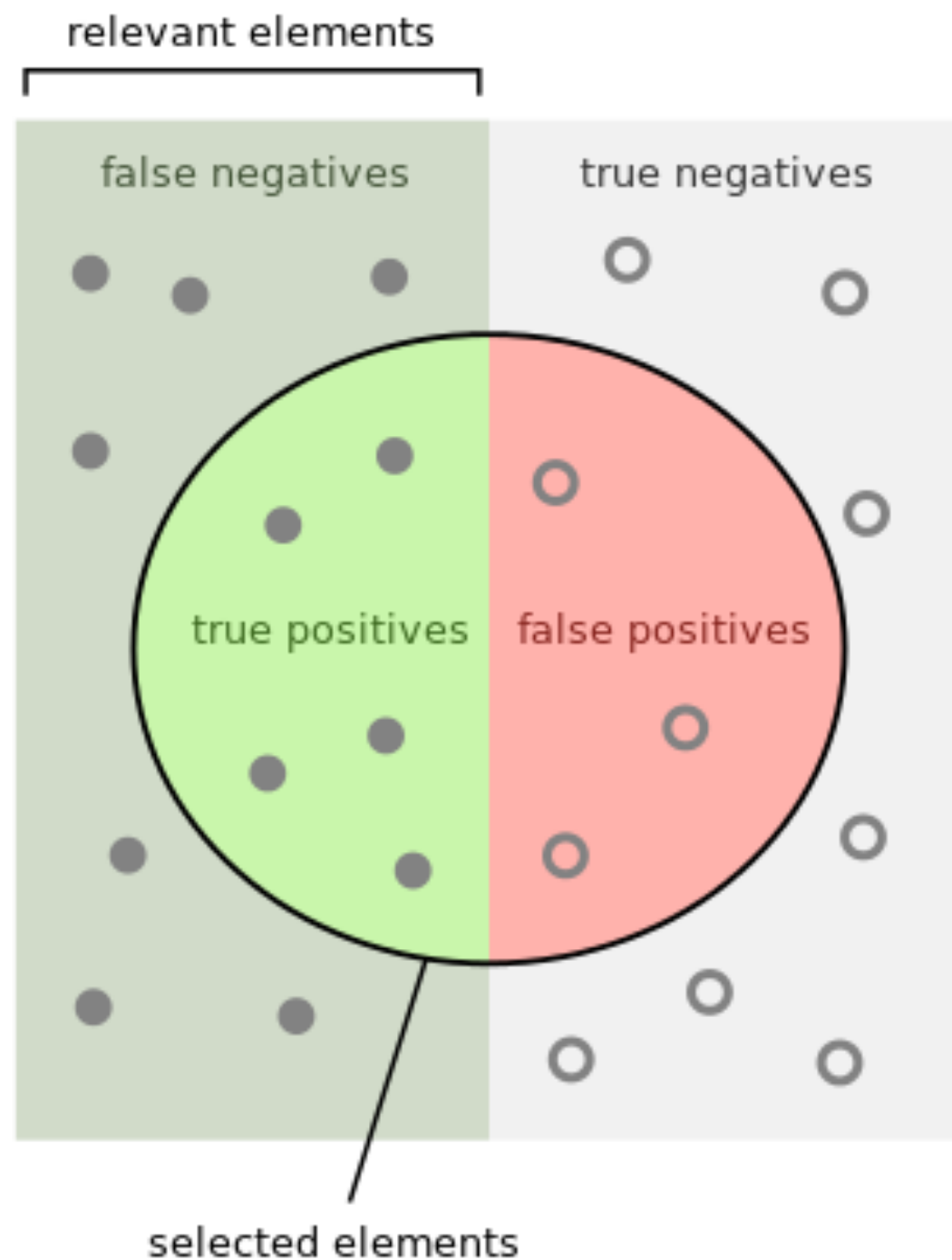
$$\theta_j := \theta_j - \alpha \sum_{i=1}^m \left(\underbrace{h_{\theta}(x^{(i)})}_{\substack{= \\ \frac{1}{1 + e^{-\theta^T x}}}} - y^{(i)} \right) x_j^{(i)}$$
$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

using different hypothesis from linear regression

[Recap] Classification Method: Supervised Machine Learning



Classification Evaluation



How many selected items are relevant?

$$\text{Precision} = \frac{\text{true positives}}{\text{true positives} + \text{false positives}}$$

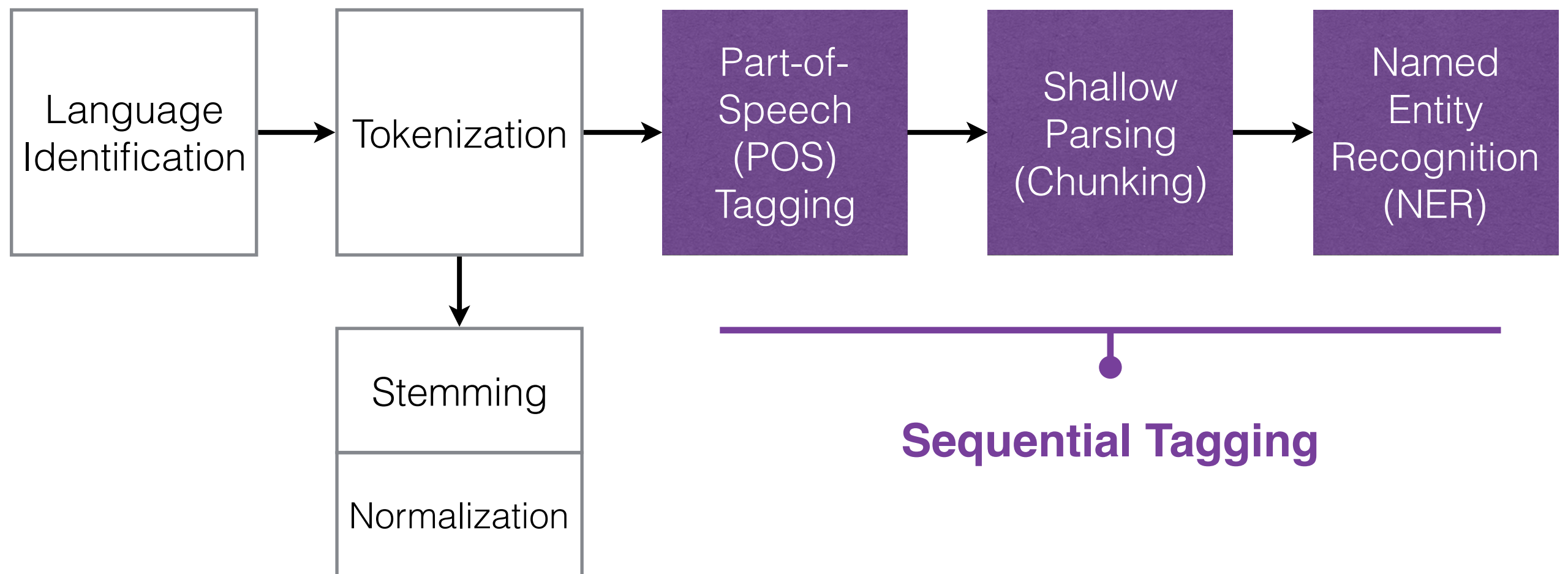
How many relevant items are selected?

$$\text{Recall} = \frac{\text{true positives}}{\text{true positives} + \text{false negatives}}$$

F-measure:

$$F_1 = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

NLP Pipeline (next)



Part-of-Speech (POS) Tagging

Cant	MD
wait	VB
for	IN
the	DT
ravens	NNP
game	NN
tomorrow	NN
...	:
go	VB
ray	NNP
rice	NNP
!!!!!!!	.



Chunking

Cant	VP
wait	
for	PP
the	NP
ravens	
game	NP
tomorrow	
...	
go	VP
ray	NP
rice	
!!!!!!!	



Named Entity Recognition(NER)

Cant	
wait	
for	
the	
ravens	ORG
game	
tomorrow	
...	
go	
ray	
rice	PER
!!!!!!!	.



ORG: organization

PER: person

LOC: location

IO tag encoding

Cant		VP	
wait	VP		VP
for	PP		PP
the			NP
ravens	NP		NP
game			NP
tomorrow	NP		NP
...			O
go	VP		VP
ray			NP
rice	NP		NP
!!!!!!!			O



IO tag encoding

Cant		VP	B-VP
wait	VP	VP	I-VP
for	PP	PP	B-PP
the		NP	B-NP
ravens	NP	NP	I-NP
game		NP	I-NP
tomorrow	NP	NP	B-NP
...		O	O
go	VP	VP	B-VP
ray		NP	B-VP
rice	NP	NP	I-VP
!!!!!!!		O	O



I: Inside

O: outside

B: Begin

BIO allows separation of adjacent chunks/entities

Classification Method:

Supervised Machine Learning

- Naïve Bayes
- Logistic Regression
- Support Vector Machines (SVM)
- ...
- Hidden Markov Model (HMM)
- Conditional Random Fields (CRF)
- ...

**sequential
models**

Classification Method:

Sequential Supervised Learning

- Input:
 - rather than just individual examples *(w₁ = the, c₁ = DT)*
 - a training set consists of *m* sequences of labeled examples *(x₁, y₁), ..., (x_m, y_m)*
x₁ = <the back door> and y₁ = <DT JJ NN>
- Output:
 - a learned classifier to predict label sequences *$\gamma: x \rightarrow y$*

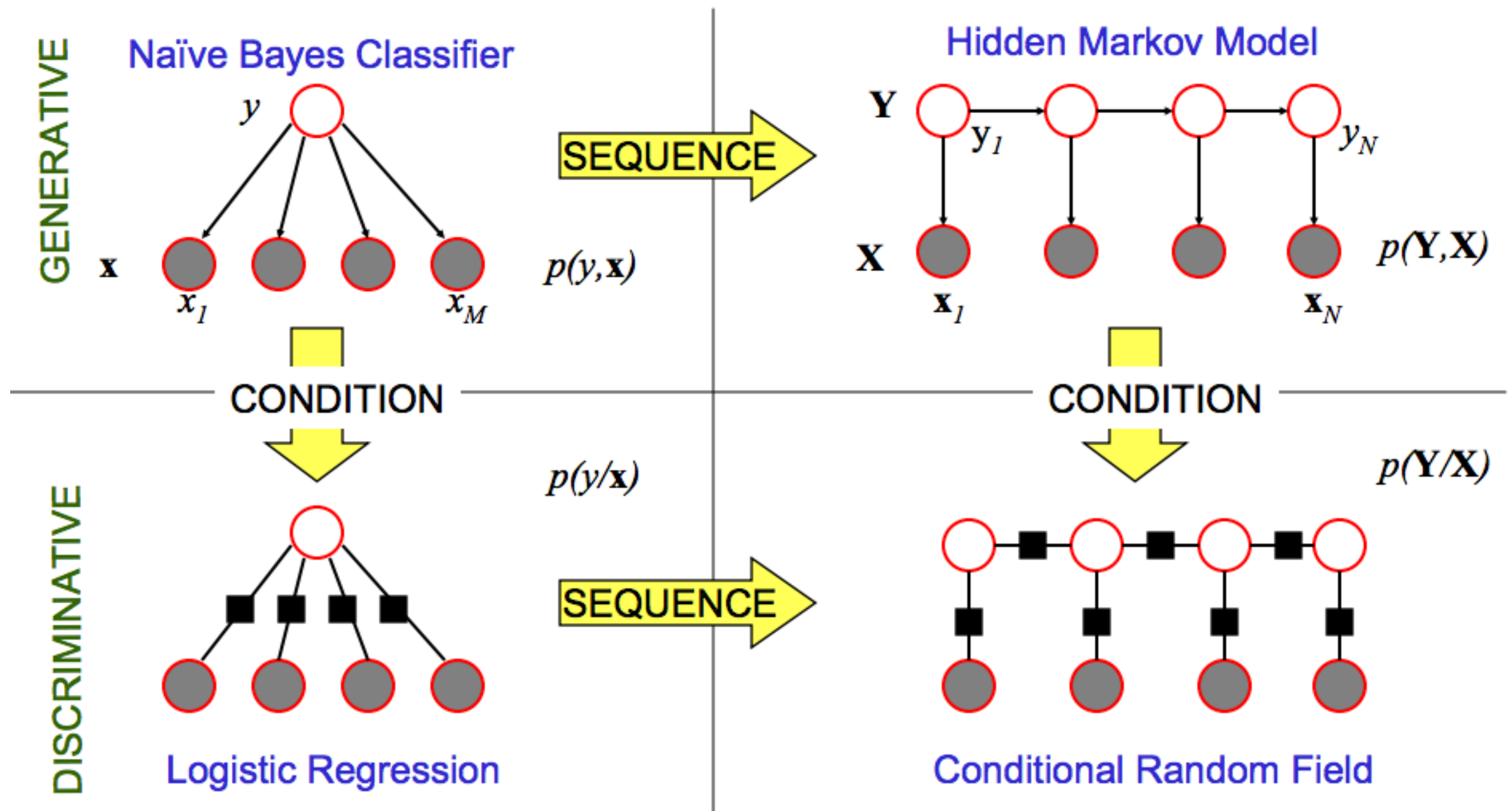
Features for Sequential Tagging

- Words:
 - current words
 - previous/next word(s) — context
- Other linguistic information:
 - word substrings
 - word shapes
 - POS tags
- Contextual Labels
 - previous (and perhaps next) labels

word shapes

Varicella-zoster	Xx-xxx
mRNA	xXXX
CPA1	XXXd

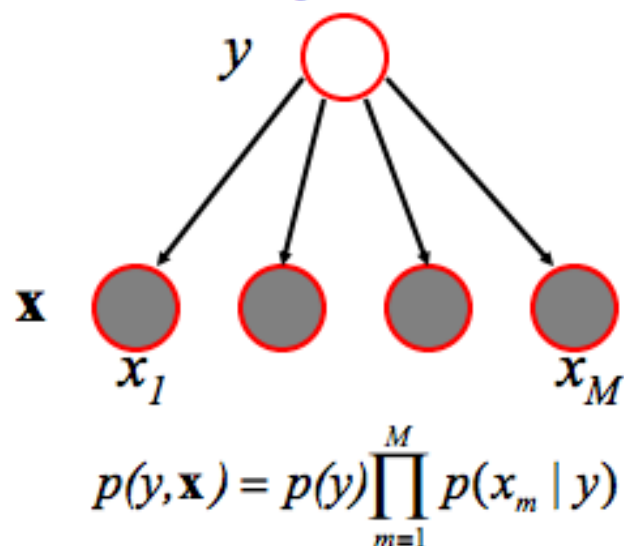
Probabilistic Graphical Models



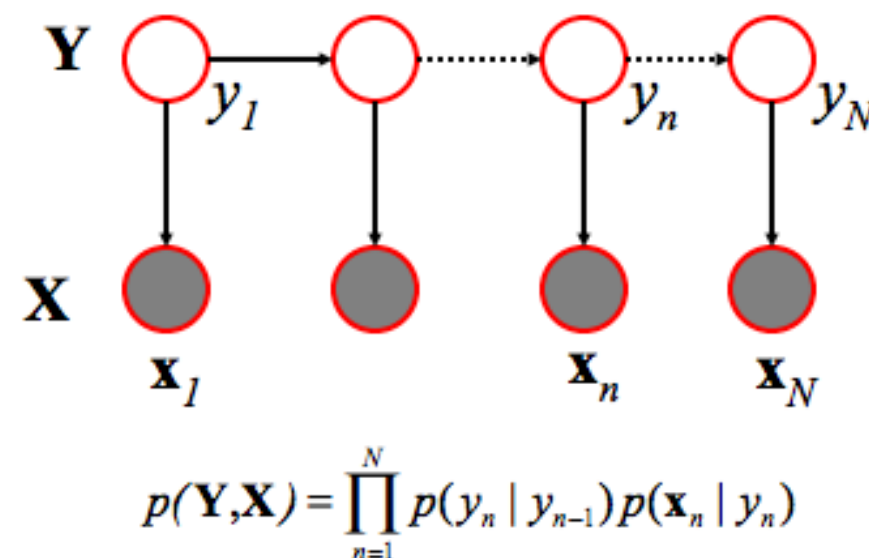
Probabilistic Graphical Models

GENERATIVE

Naïve Bayes Classifier

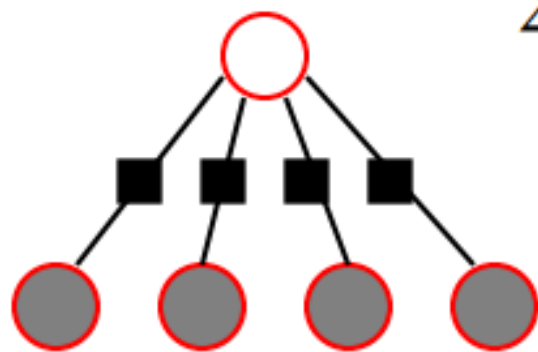


Hidden Markov Model



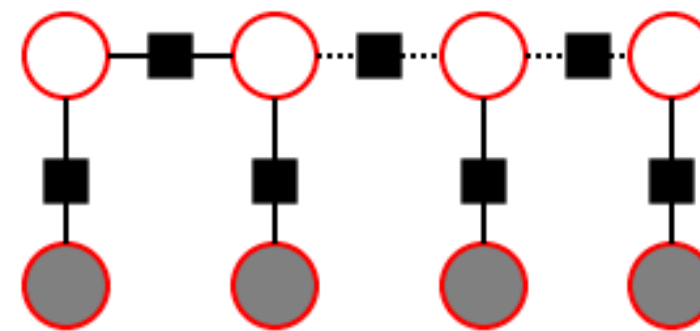
DISCRIMINATIVE

$$p(y | \mathbf{x}) = \frac{\exp \left\{ \sum_{m=1}^M \lambda_m f_m(y, \mathbf{x}) \right\}}{\sum_{y'} \exp \left\{ \sum_{m=1}^M \lambda_m f_m(y', \mathbf{x}) \right\}}$$



Logistic Regression

$$p(\mathbf{Y} | \mathbf{X}) = \frac{\exp \left\{ \sum_{m=1}^M \lambda_m f_m(y_n, y_{n-1}, \mathbf{x}_n) \right\}}{\sum_{y'} \exp \left\{ \sum_{m=1}^M \lambda_m f_m(y'_n, y'_{n-1}, \mathbf{x}_n) \right\}}$$



Conditional Random Field

Probabilistic Graphical Models

