Social Media & Text Analysis

lecture 6 - Paraphrase Identification and Logistic Regression (cont')

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CSE 5539-0010 Ohio State University

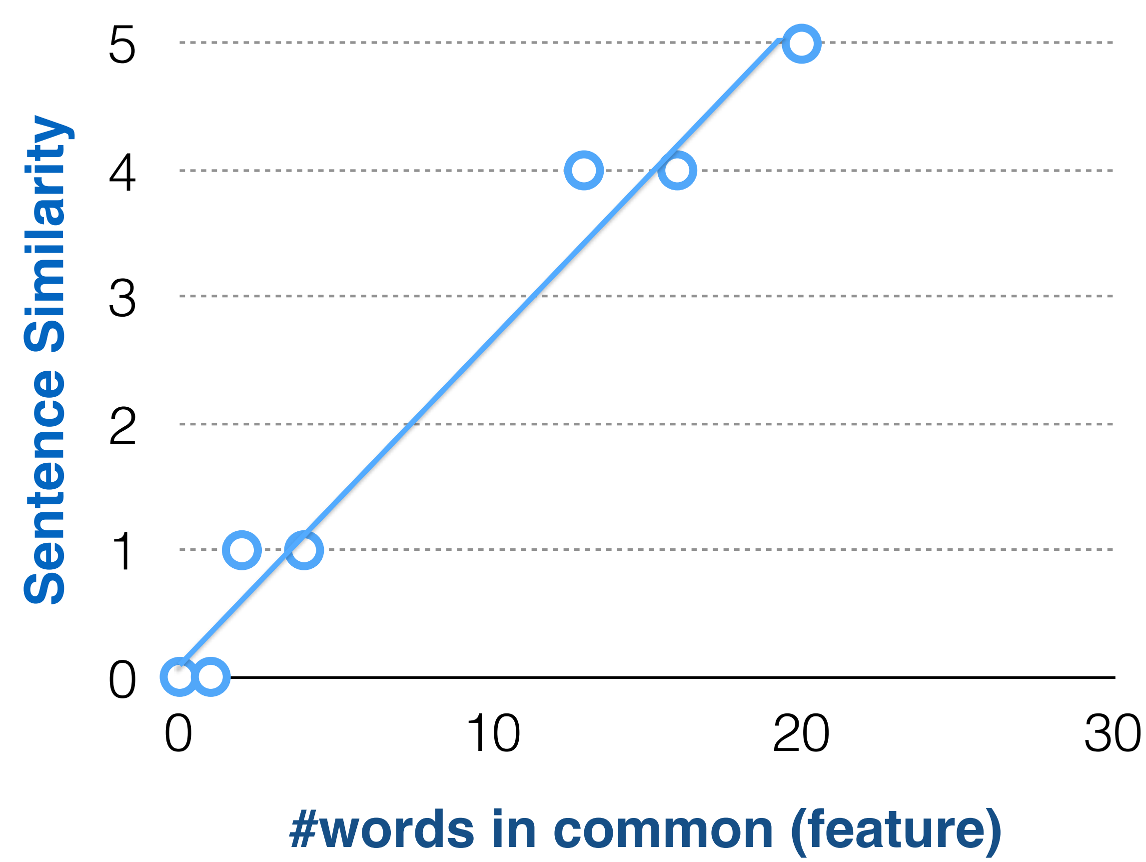
Instructor: Wei Xu

Website: socialmedia-class.org

with slides adapted from Andrew Ng

(Recap)

Linear Regression



- also supervised learning (learn from annotated data)
- but for **Regression**: predict **real-valued** output
  (Classification: predict discrete-valued output)

(Recap)

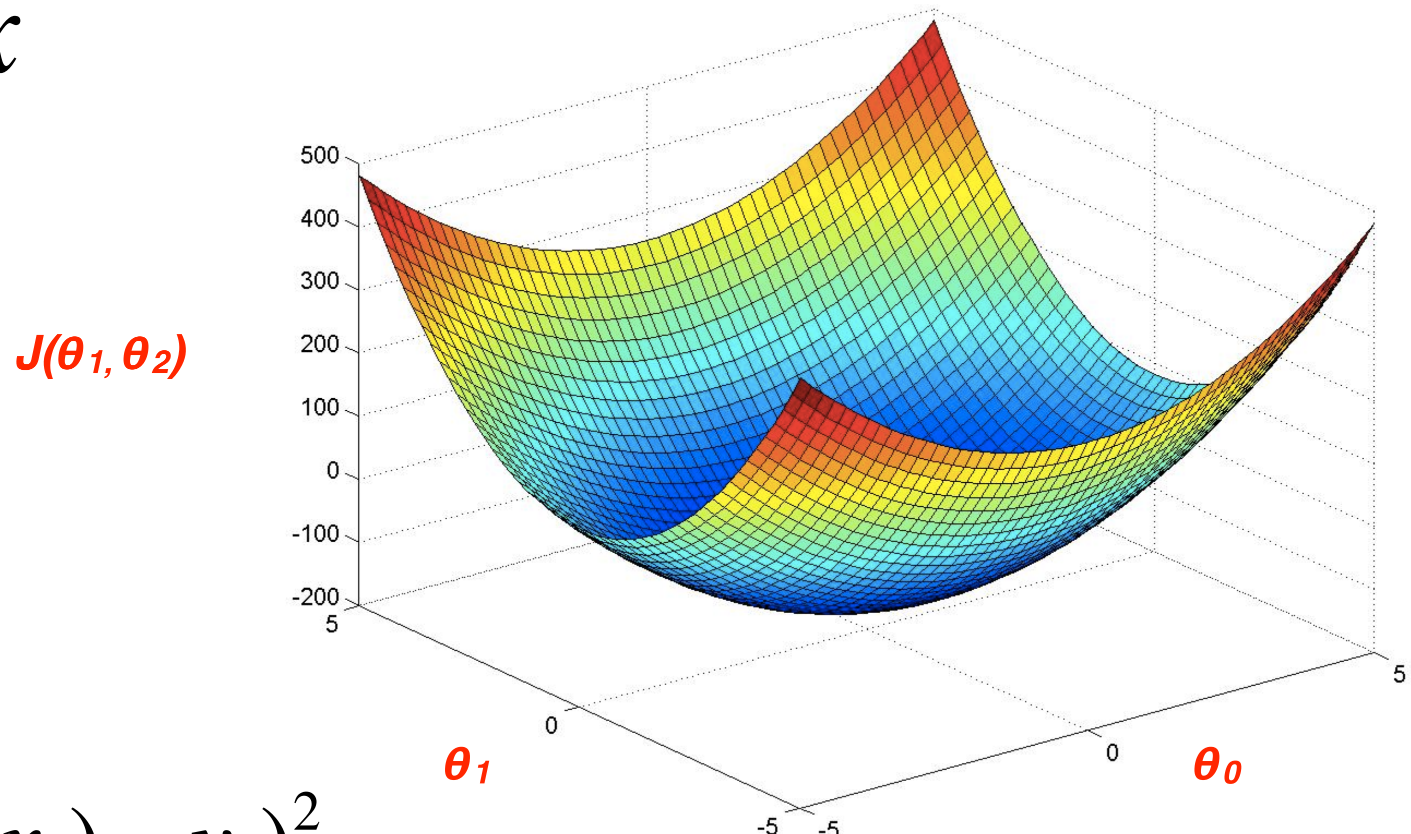
Linear Regression

- **Hypothesis**:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- **Parameters:**

θ₀*, *θ₁



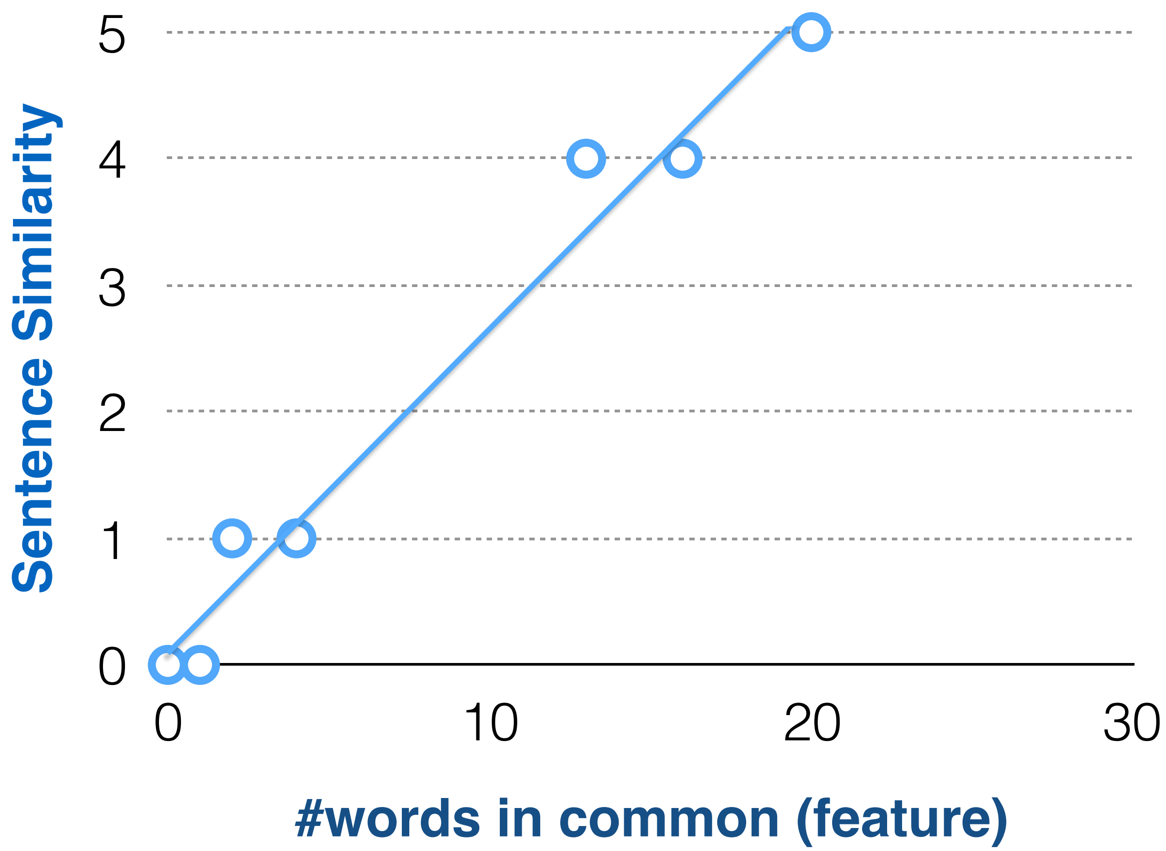
- **Cost Function:**

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$$

- **Goal:** $\underset{\theta_0,\,\theta_1}{\text{minimize}}\, J(\theta_0,\theta_1)$

(Recap) Linear Regression w/ one variable:

Cost Function



squared error function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

- **Idea:** choose ***θ_0, θ_1*** so that ***$h_\theta(x)$*** is close to ***y*** for training examples ***(x, y)***

$$\underset{\theta_0,\, \theta_1}{\text{minimize}} \, J(\theta_0, \theta_1)$$

(Recap)

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

}

simultaneous update
for j=0 and j=1

learning rate

(Recap) Linear Regression w/ one variable:

Gradient Descent

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i) \cdot x_i$$

}

simultaneous update $\boldsymbol{\theta_0}$, $\boldsymbol{\theta_1}$

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Linear Regression w/ multiple variables (features):

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$$h_\theta(x) = \theta^T x$$

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta^T x$$

- Cost function:

training examples

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

(Recap)

Paraphrase Identification

obtain sentential paraphrases automatically

<u>Mancini</u> has been sacked by Manchester City
<u>Mancini</u> gets the boot from Man City

Yes!

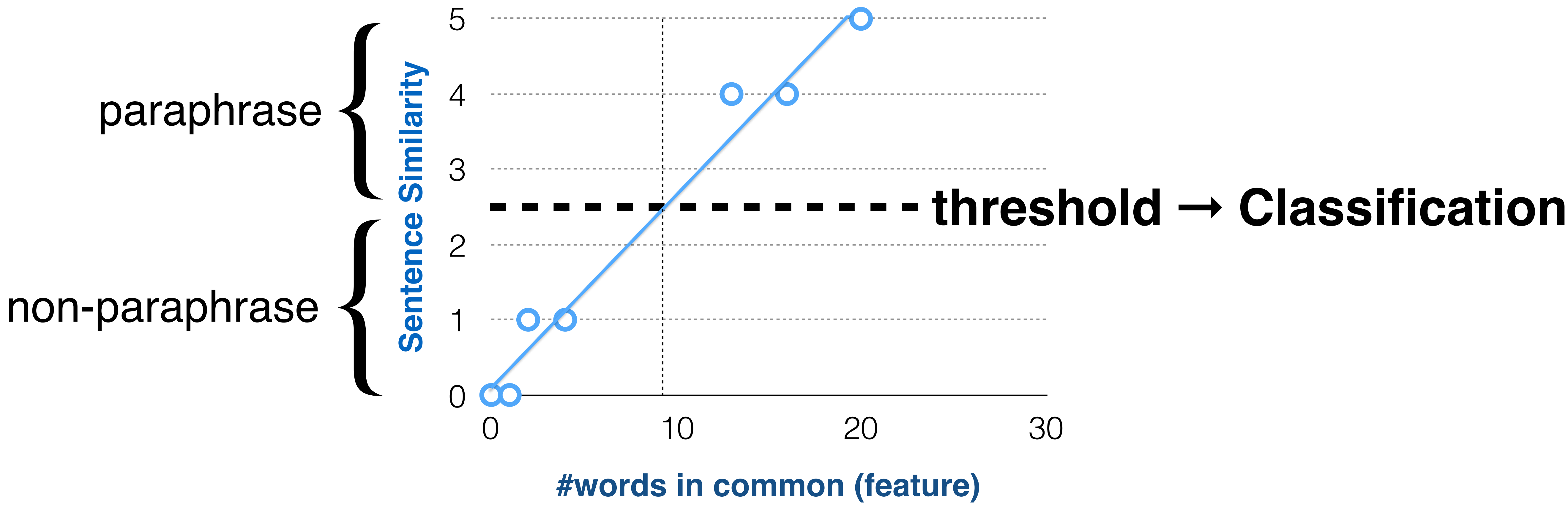
<u>WORLD OF JENKS</u> IS ON AT 11
<u>World of Jenks</u> is my favorite show on tv

No!

Wei Xu, Alan Ritter, Chris Callison-Burch, Bill Dolan, Yangfeng Ji. "Extracting Lexically Divergent Paraphrases from Twitter" In TACL (2014)

(Recap)

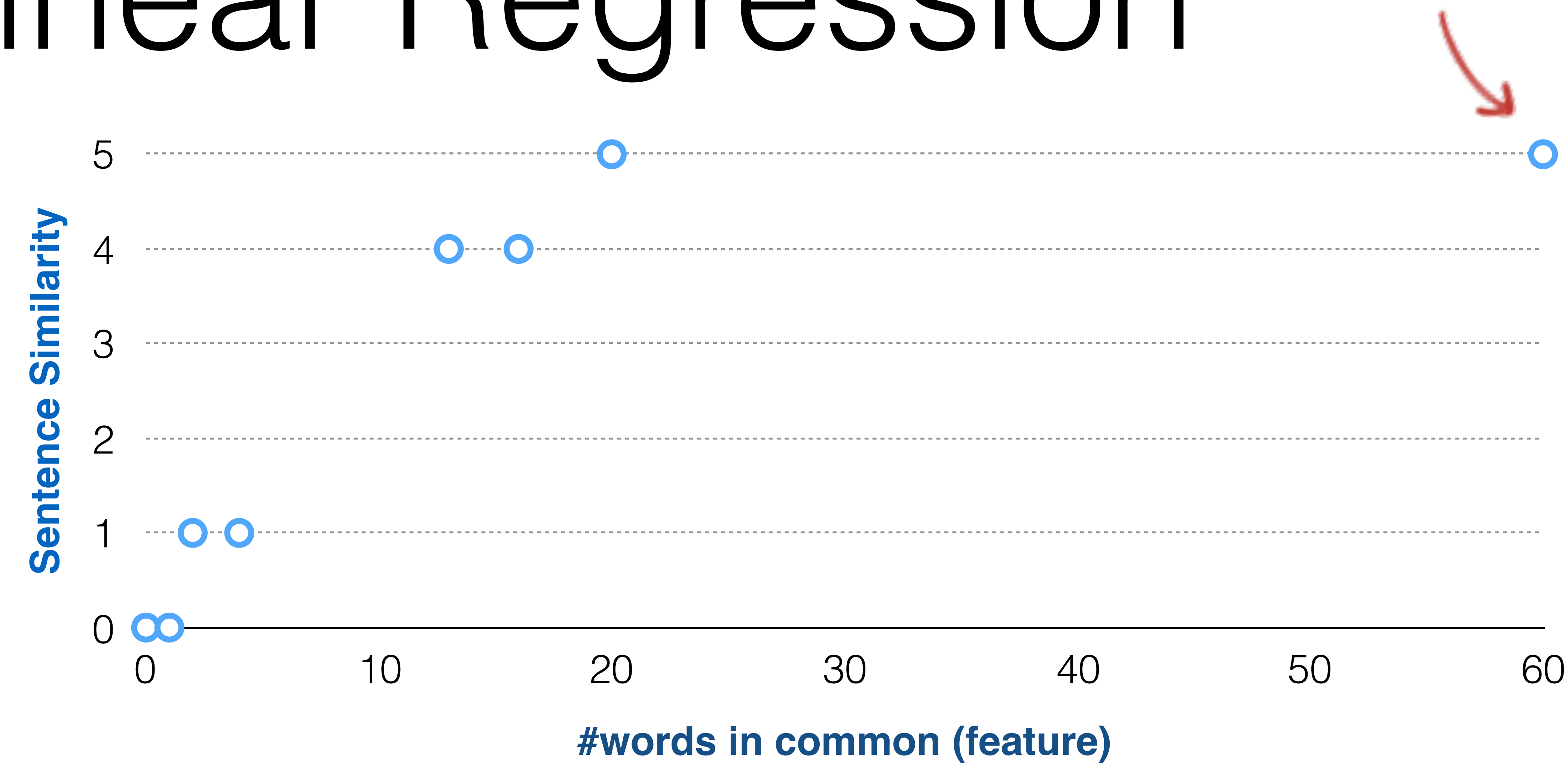
Linear Regression



- also supervised learning (learn from annotated data)
- but for **Regression**: predict **real-valued** output
  (Classification: predict discrete-valued output)

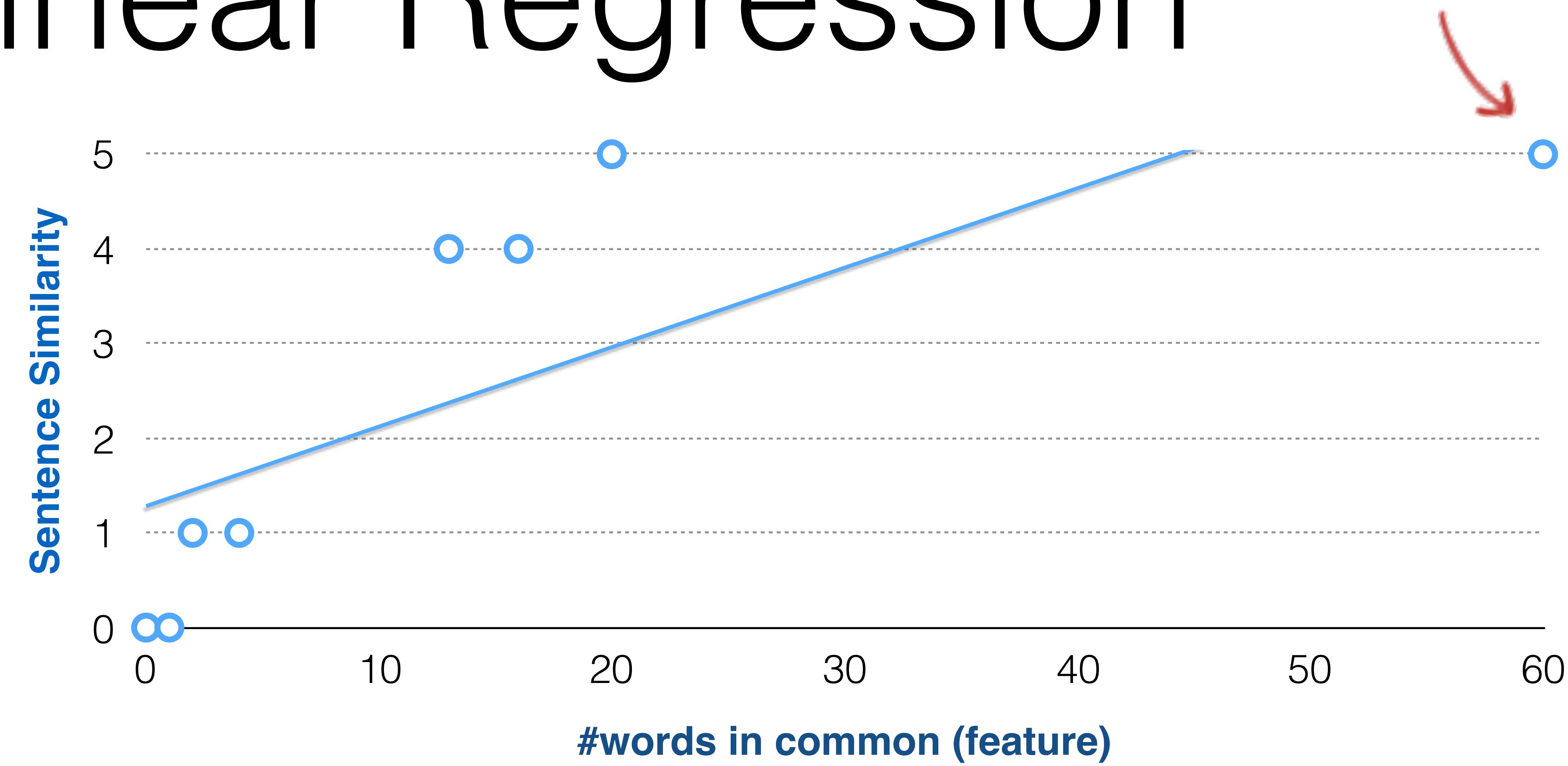
A problem in classification

Linear Regression



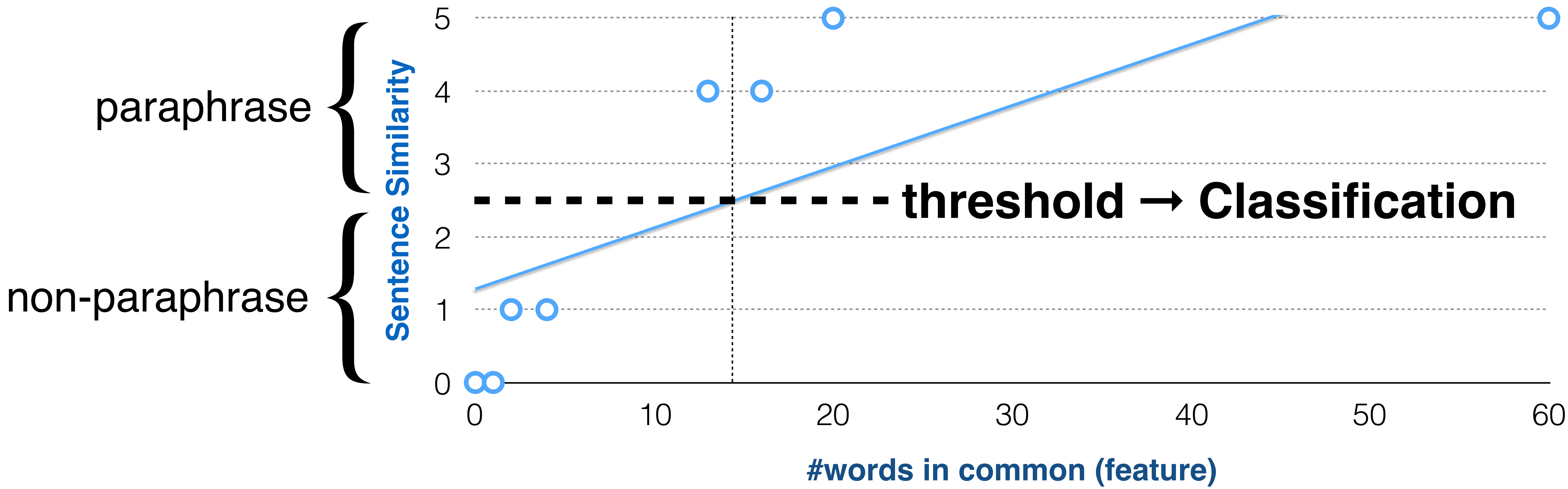
A problem in classification

Linear Regression



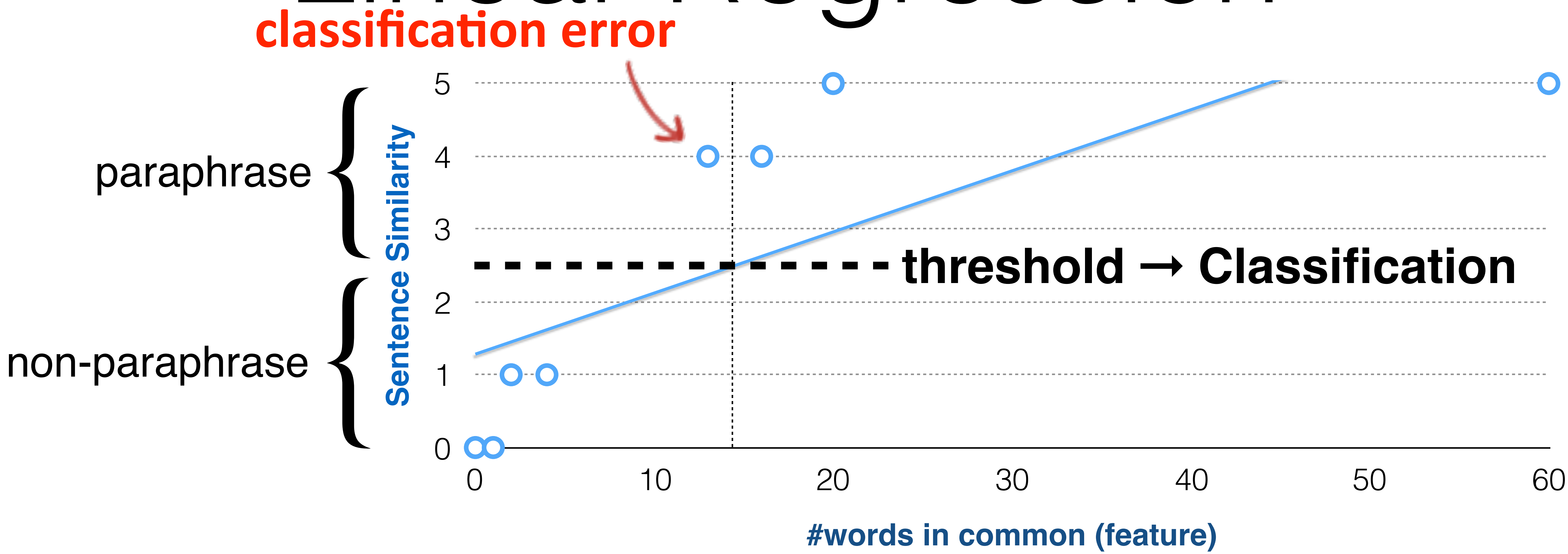
A problem in classification

Linear Regression



A problem in classification

Linear Regression

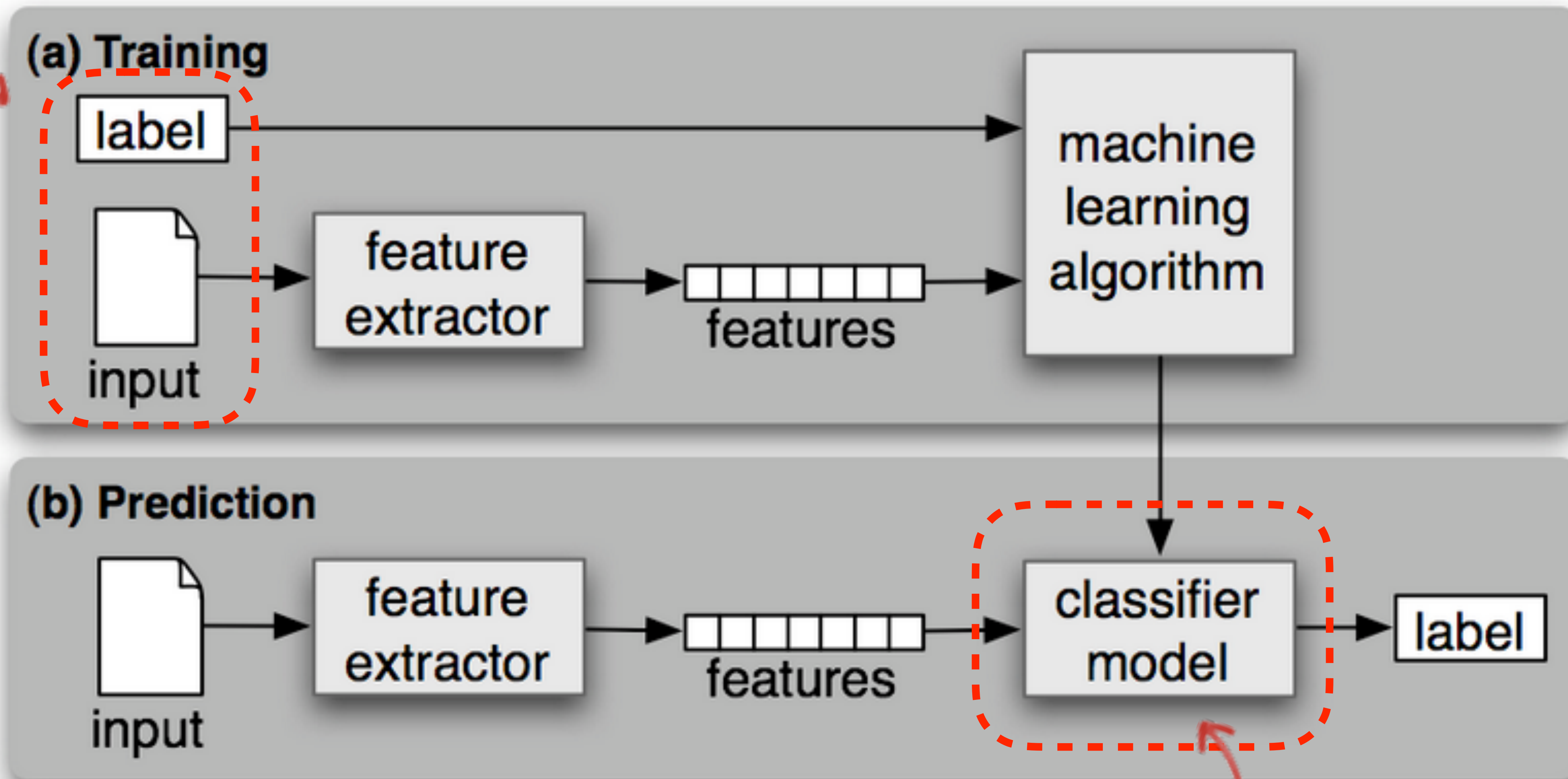


In practice, do not use linear regression for classification.

(Recap) Classification:

Supervised Machine Learning

training set



(also called) hypothesis

(Recap)

Logistic Regression

- One of the most useful **supervised machine learning algorithm** for classification!
- Generally high performance for a lot of problems.
- Much more robust than Naïve Bayes
(better performance on various datasets).

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0

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- Logistic Regression: want $0 \le h_\theta(x) \le 1$

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0
- Logistic Regression: want $0 \leq h_\theta(x) \leq 1$

a classification (not regression) algorithm

Hypothesis:

Linear → Logistic Regression

- Linear Regression: $h_\theta(x) = \theta^T x$
- Logistic Regression: want $0 \leq h_\theta(x) \leq 1$

Hypothesis:

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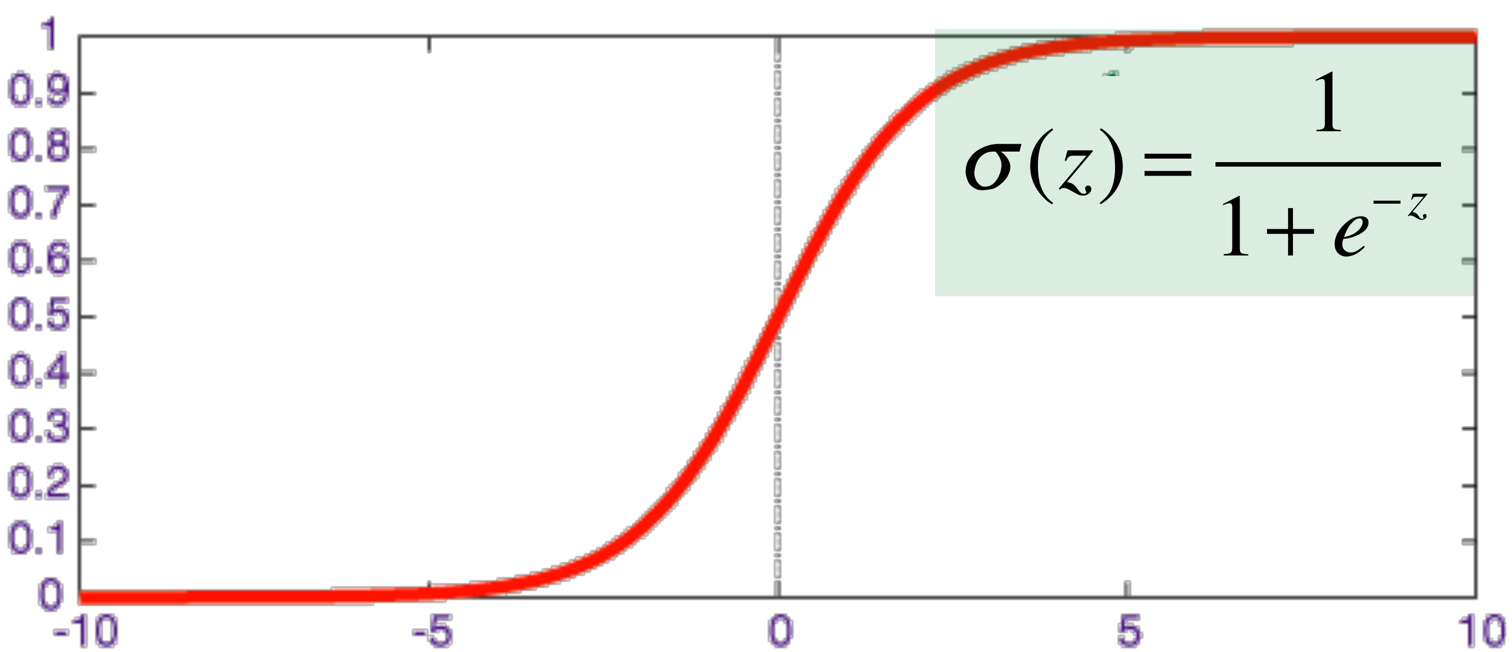
sigmoid (logistic) function



$$\sigma(z) = \frac{1}{1+e^{-z}}$$

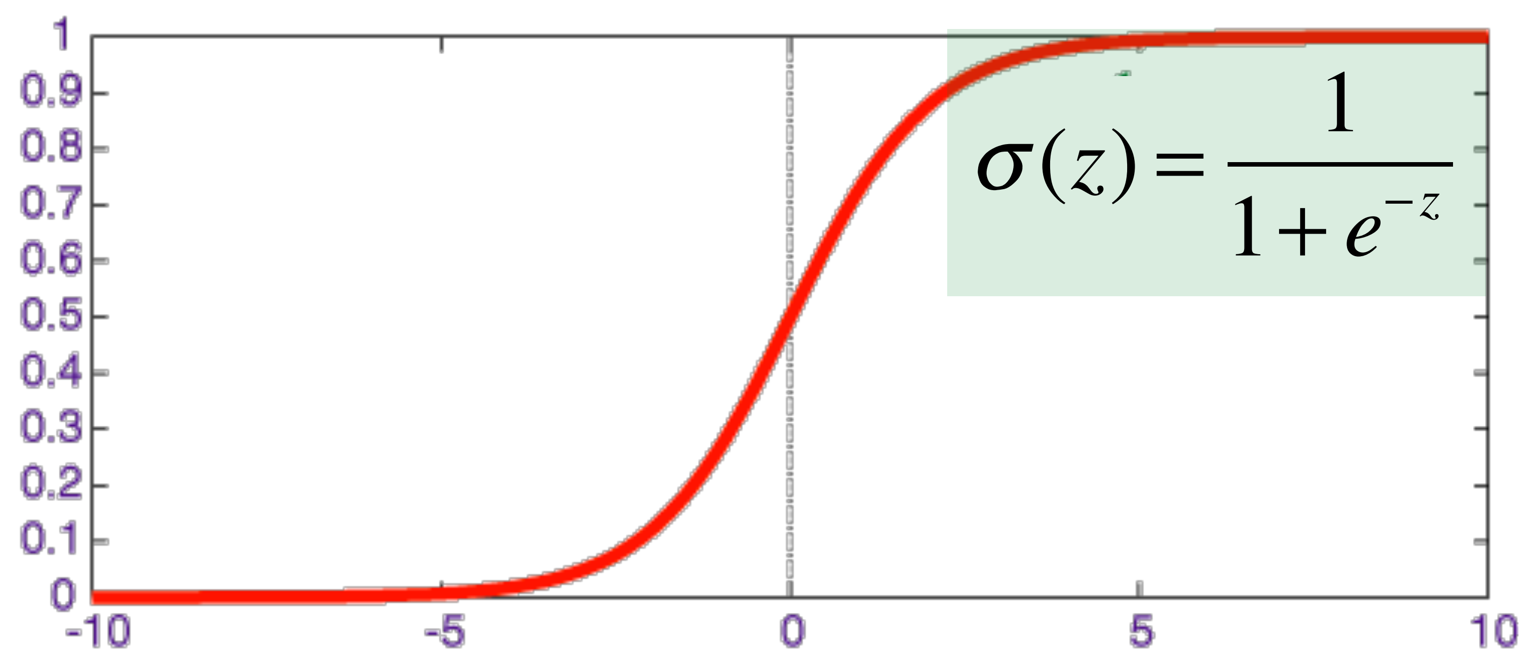
Hypothesis:

Linear → Logistic Regression

- Linear Regression: $h_\theta(x) = \theta^T x$
- Logistic Regression: want $0 \le h_\theta(x) \le 1$

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

sigmoid (logistic) function



(Recap) Classification Method:

Supervised Machine Learning

- Input:
  - a sentence pair x **(represented by features)**
  - a fixed set of binary classes $Y = \{0, 1\}$
  - a training set of m hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$
- Output:
  - a learned classifier $\gamma: x \rightarrow y \in Y$ ($y = 0$ or $y = 1$)

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***y = 1*** on input

$$If \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \#words\_in\_common \end{bmatrix}, \; h_\theta(x) = 0.7$$

tell that 70% change of sentence pairs being paraphrases

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$If \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \#words\_in\_common \end{bmatrix}, \; h_\theta(x) = 0.7$$

tell that 70% change of sentence pairs being paraphrases

$$h_\theta(x) = P(y = 1 \mid x; \theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$h_\theta(x) = P(y = 1 \mid x; \theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$P(y=1 \mid x;\theta) + P(y=0 \mid x;\theta) = 1$$

$$h_\theta(x) = P(y=1 \mid x;\theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

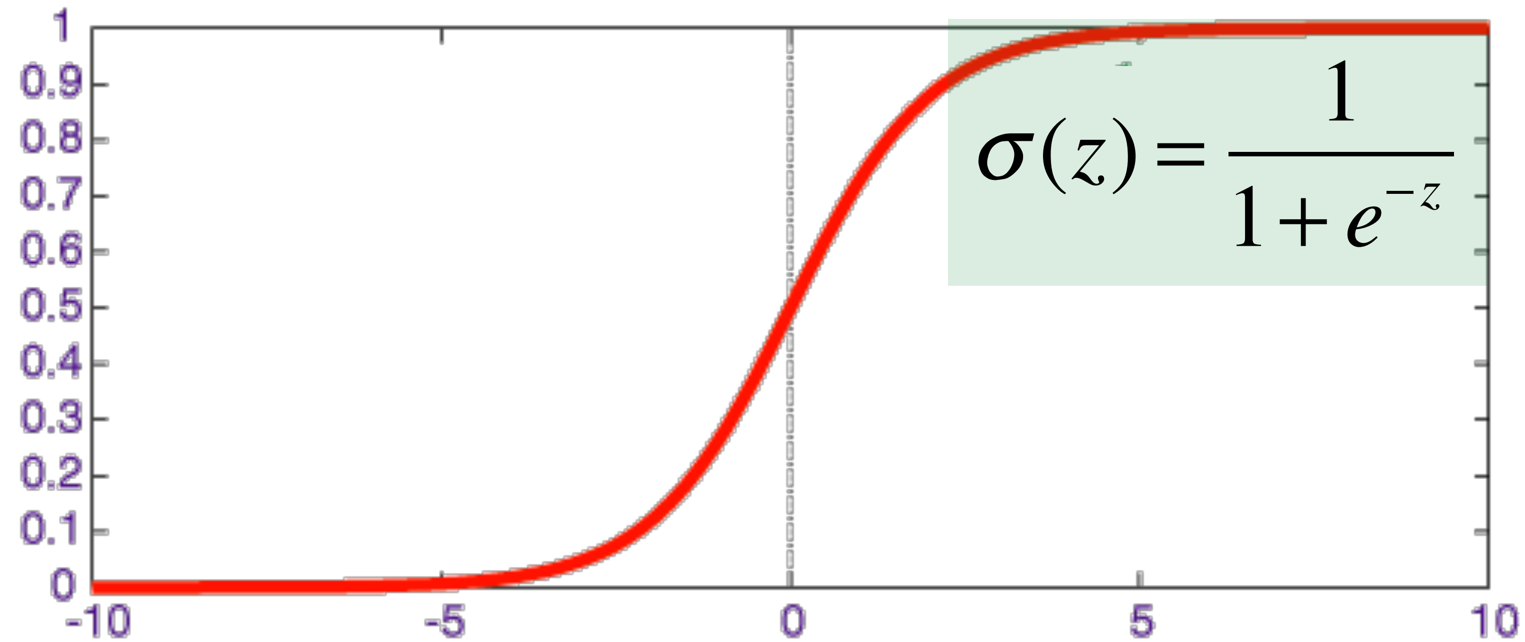
Decision Boundary

- Logistic Regression:

sigmoid (logistic) function

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$



Logistic Regression:

Decision Boundary

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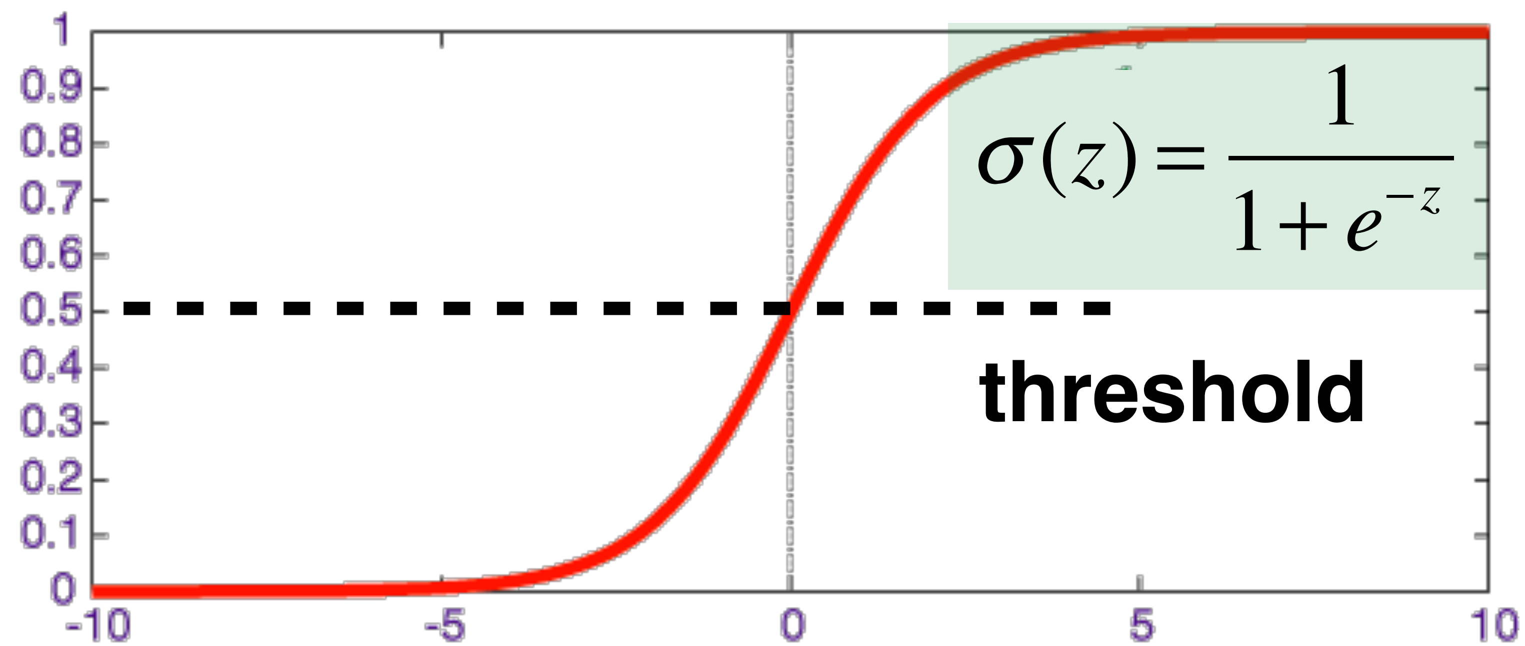
sigmoid (logistic) function



predict ***y = 1*** if ***$h_\theta(x) \geq 0.5$***

predict ***y = 0*** if ***$h_\theta(x) < 0.5$***

Logistic Regression:

Decision Boundary

- Logistic Regression:

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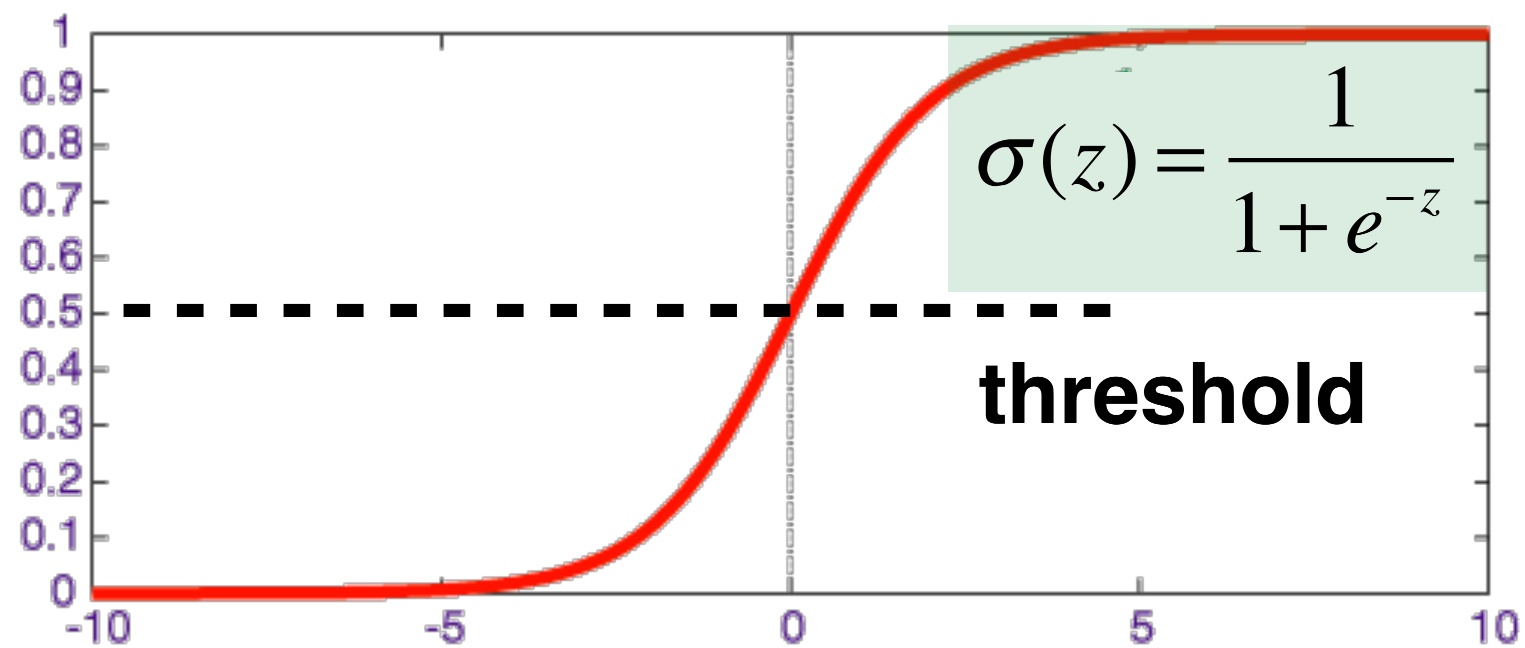
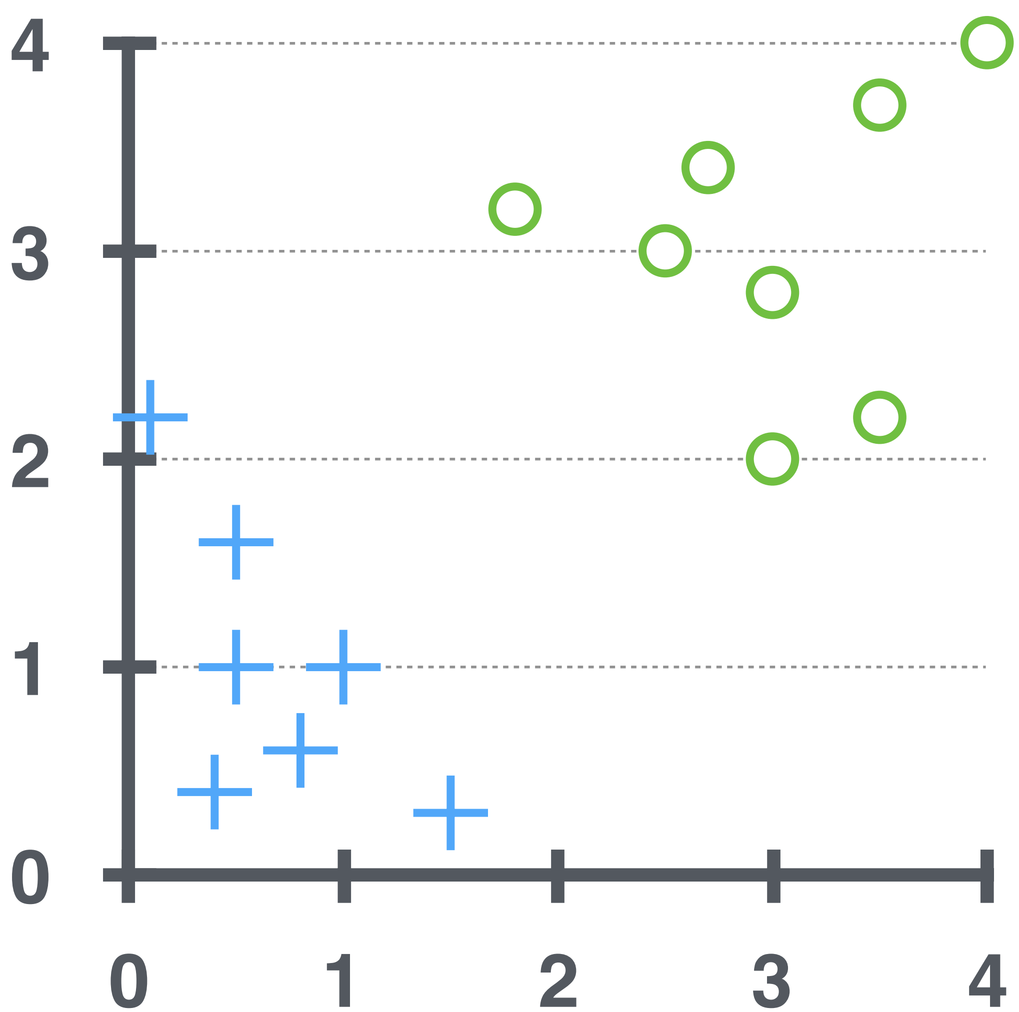


predict ***y = 1*** if $h_\theta(x) \geq 0.5$ $\longleftarrow$ when $\theta^T x \geq 0$

predict ***y = 0*** if $h_\theta(x) < 0.5$ $\longleftarrow$ when $\theta^T x < 0$

Logistic Regression:

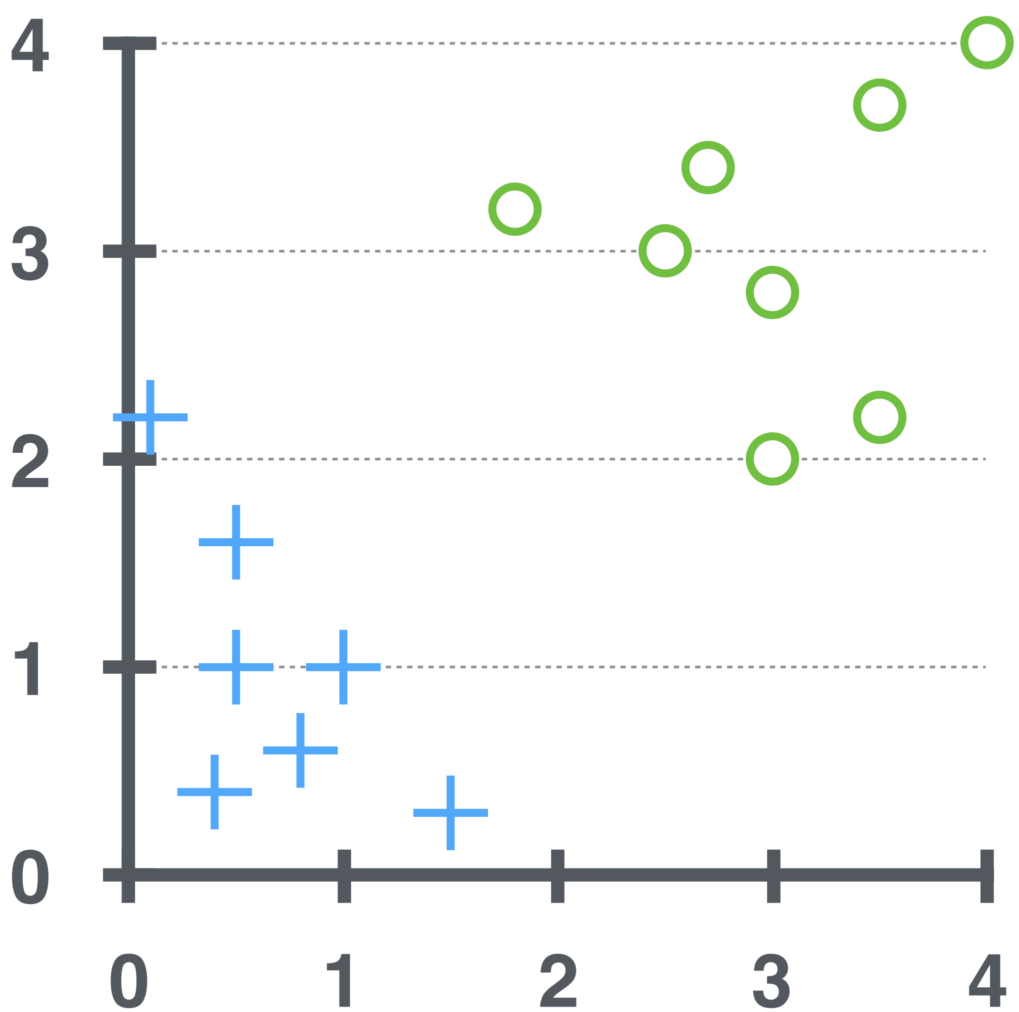
Decision Boundary



Logistic Regression:

Decision Boundary

$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$

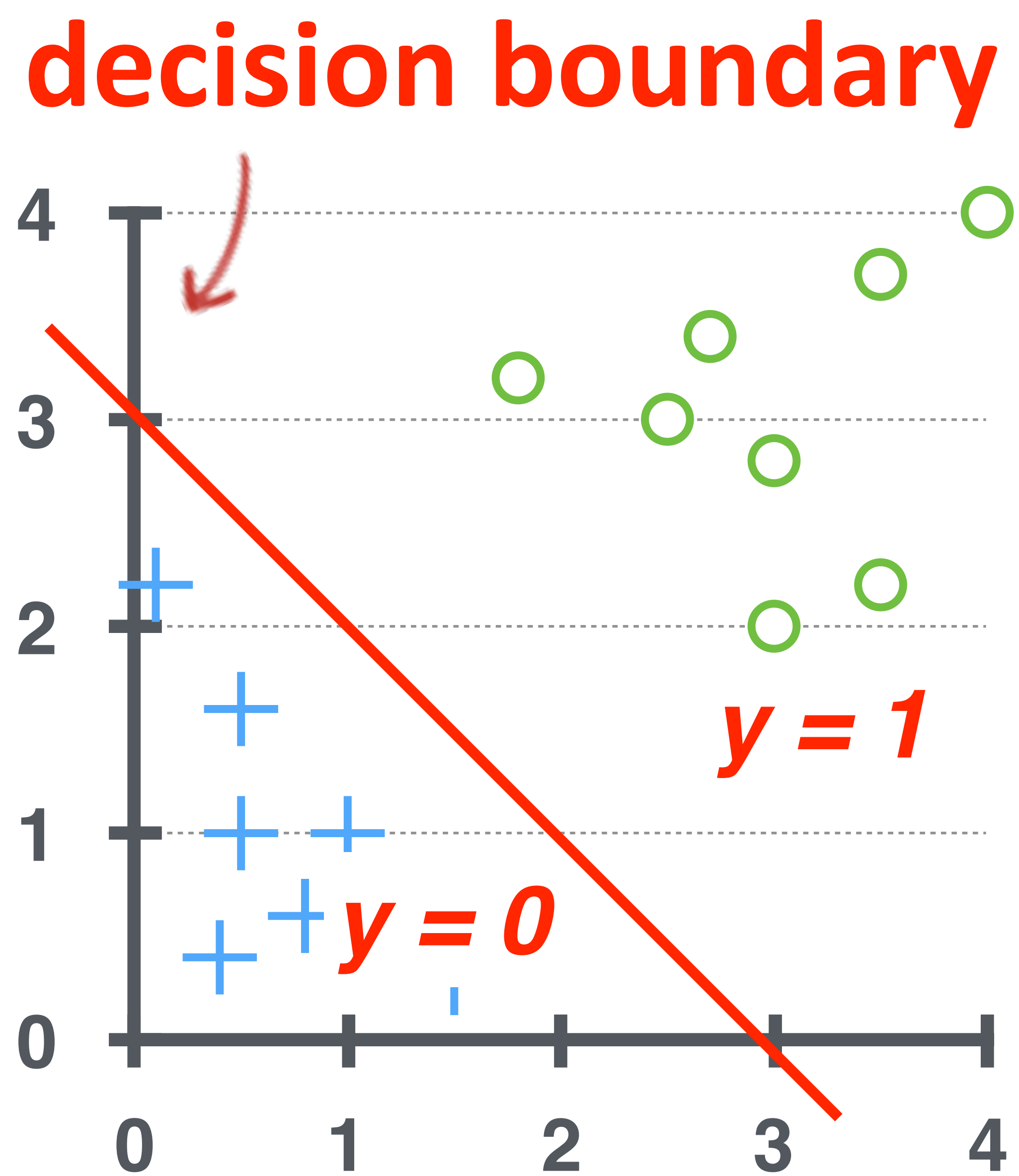


What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict ***y = 1*** if ***$\theta^T x \geq 0$***

Logistic Regression:

Decision Boundary



$$h_\theta(x) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

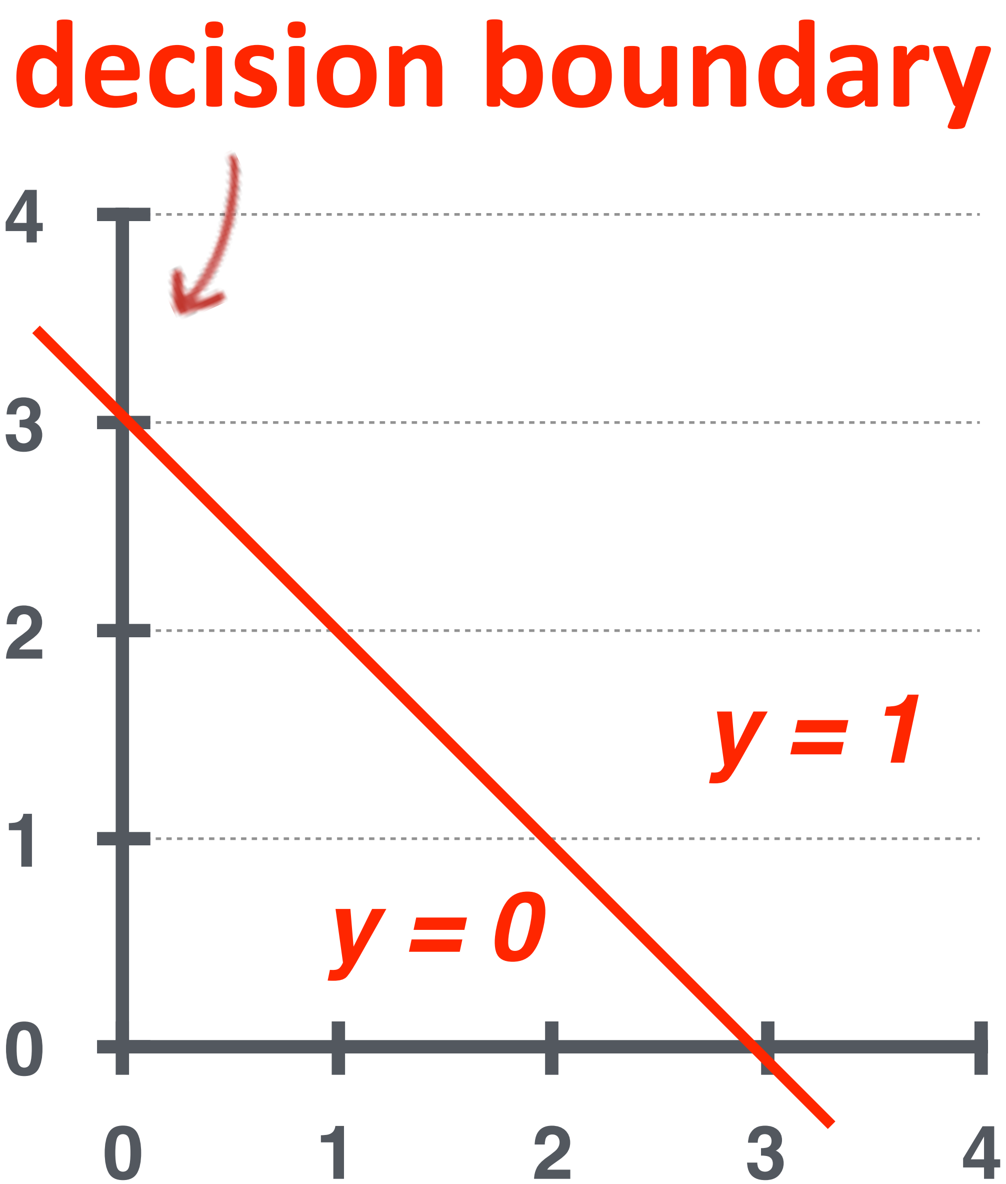
What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict ***y = 1*** if $\boldsymbol{\theta^T x \geq 0}$

Logistic Regression:

Decision Boundary

$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$



a property of the hypothesis

a property of the parameters

~~a property of the dataset~~

Logistic Regression

- a training set of ***m*** hand-labeled sentence pairs
  $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$ ($y \in \{0, 1\}$)

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

squared error function

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(\boxed{h_\theta(x^{(i)})} - y^{(i)}\right)^2$

squared error function

- Logistic Regression: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

this cost function is non-convex for logistic regression

Cost function:

Linear → Logistic Regression

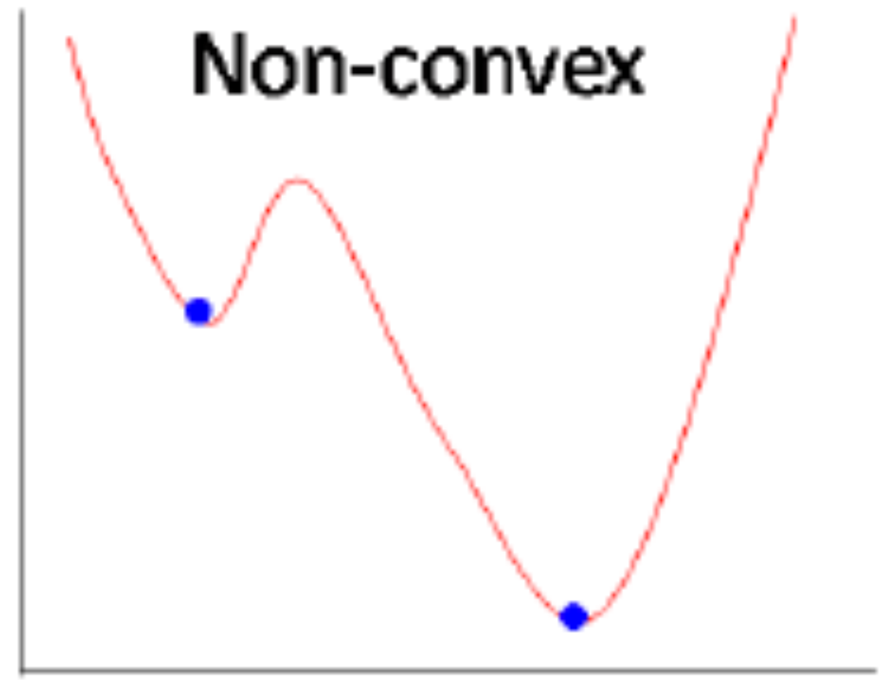
- Linear Regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

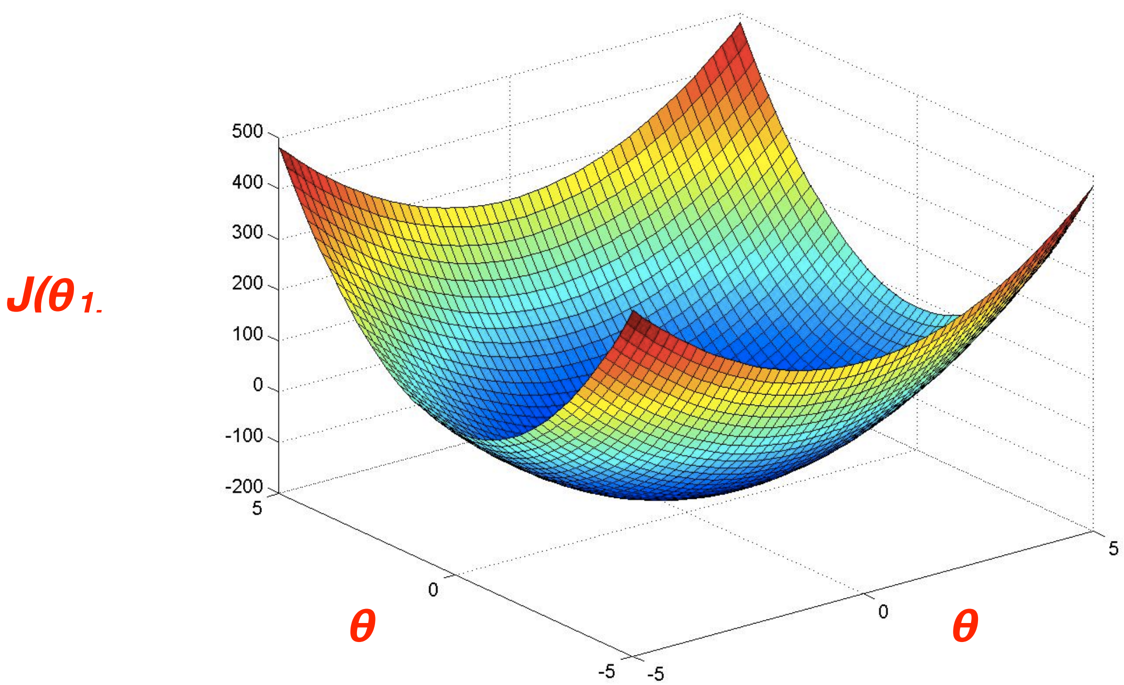
$$\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2 = \mathrm{Cost}\left(h_\theta(x), y\right)$$

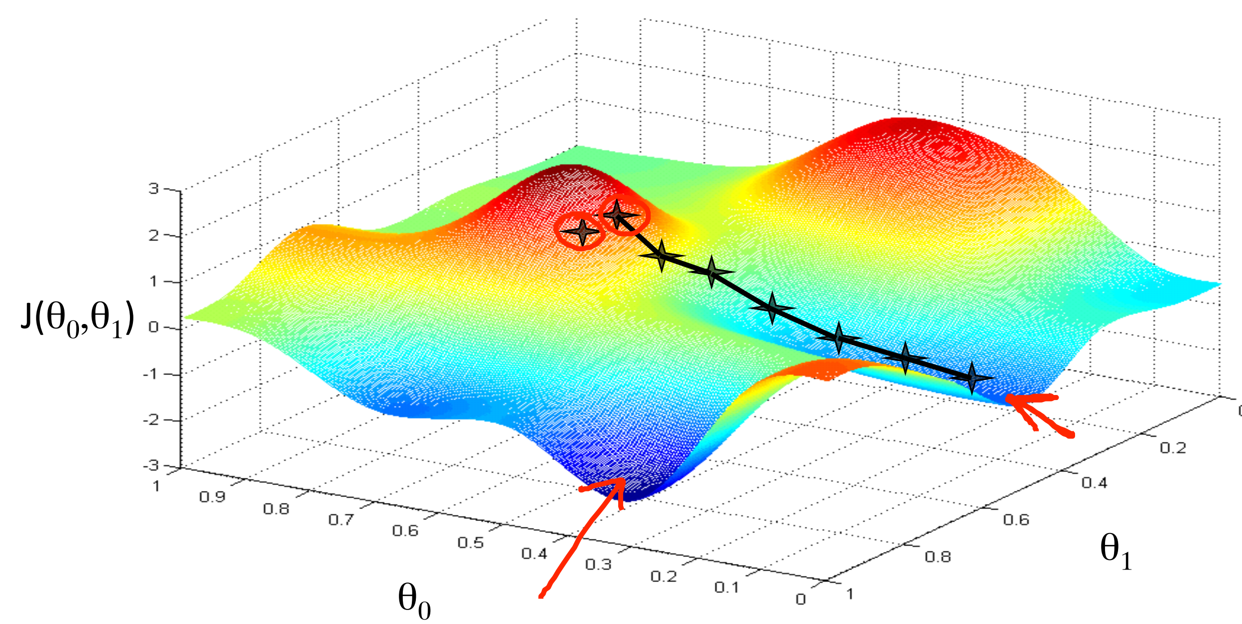
- Logistic Regression: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

this cost function is non-convex for logistic regression

we want convex! easy gradient descent!







Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

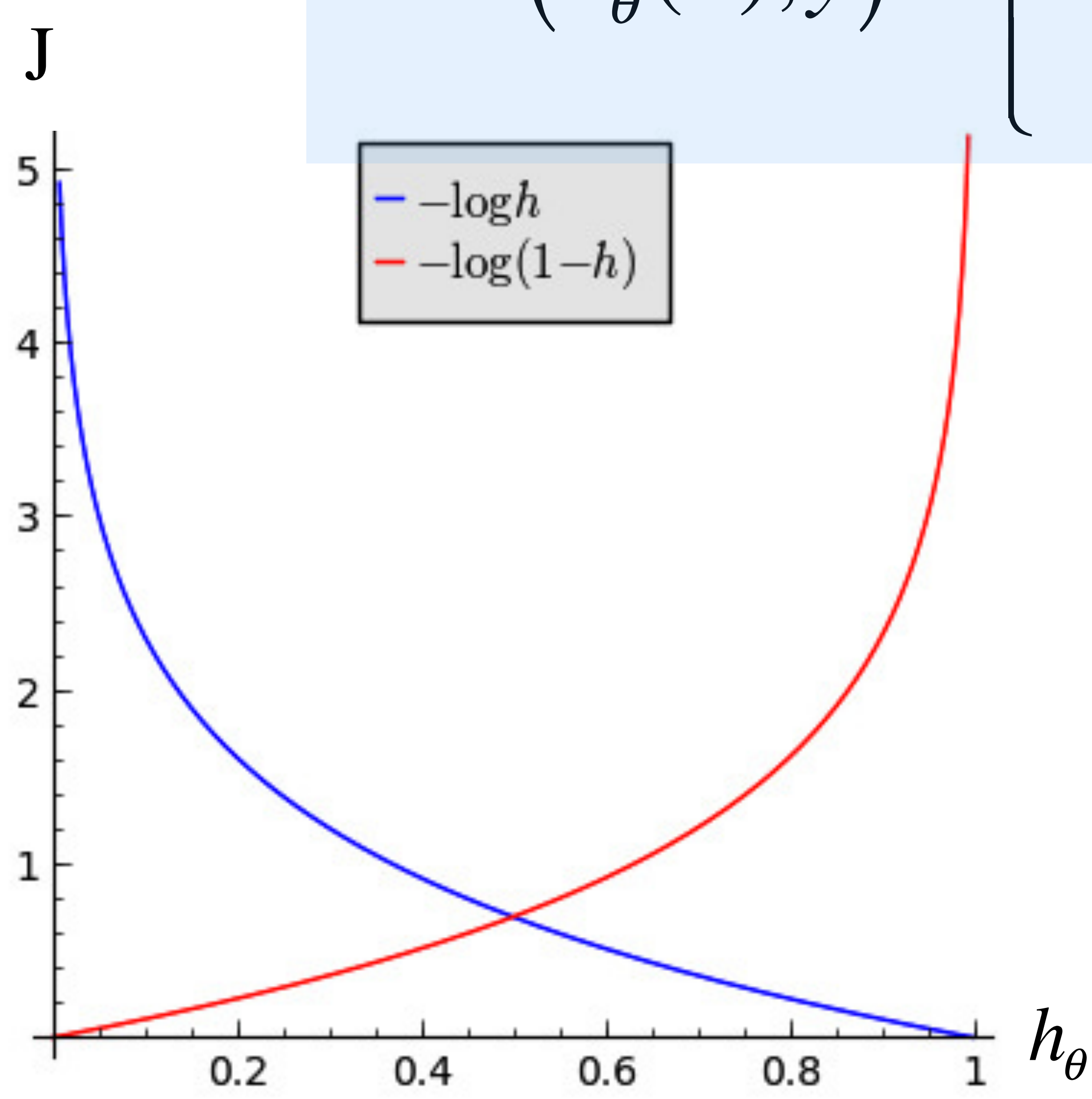
remember that

$$0 \le h_\theta(x) \le 1$$

Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } \ y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \ y = 0 \end{cases}$$



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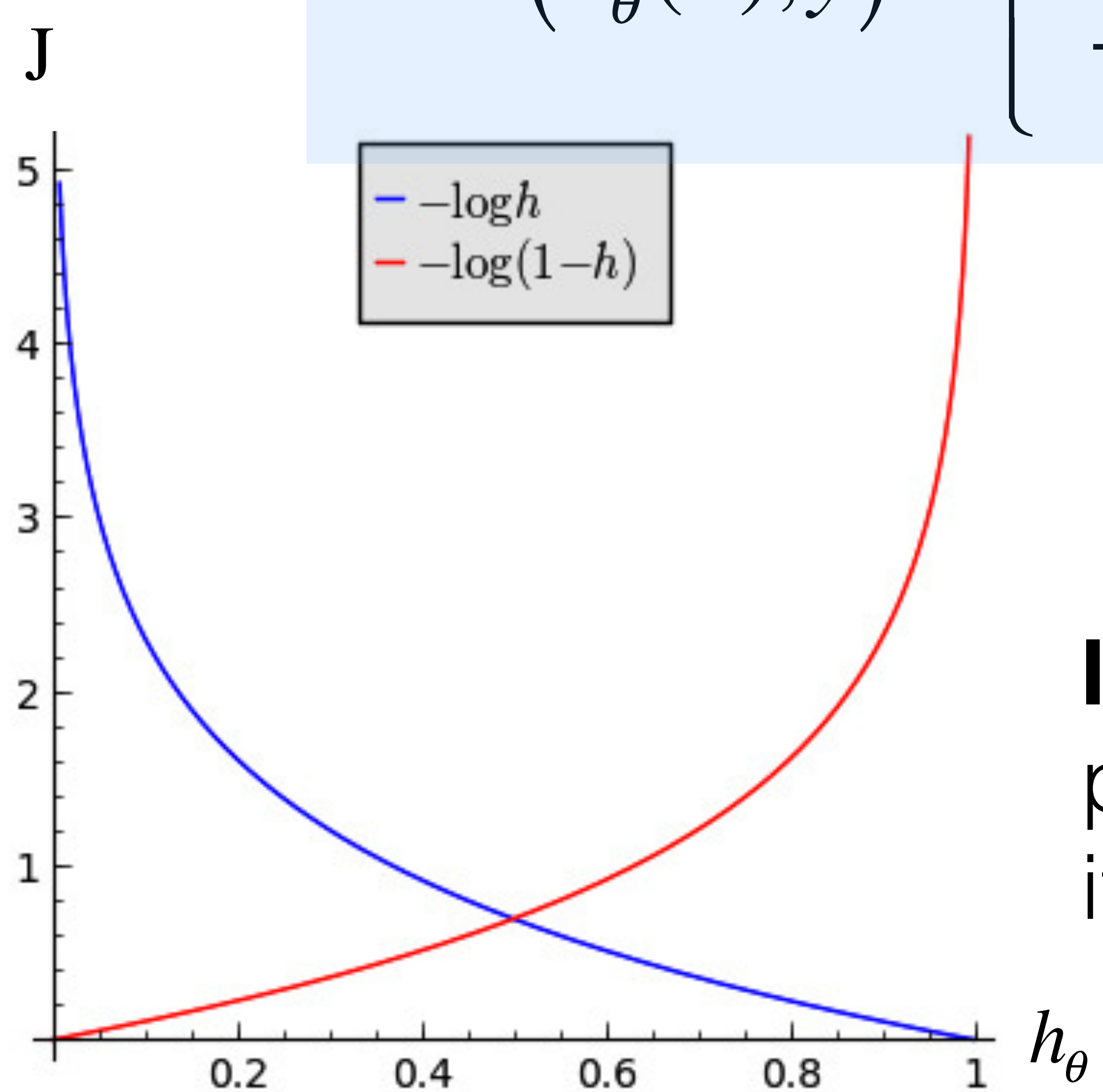

Cost = 0 if ***y = 1***, ***h_θ(x) = 1***

But as ***h_θ(x) → 0***, ***Cost → ∞***

Intuition:
penalize learning algorithm
if ***h_θ(x) = 0*** (predict **P(y=1|x;*θ*) = 0**),
but ***y = 1***

Logistic Regression:

Cost Function

$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)})-y^{(i)}\right)$$

$$\mathrm{Cost}\left(h_\theta(x),y\right)=\begin{cases}-\log(h_\theta(x)) & \text{if } y=1\\ -\log(1-h_\theta(x)) & \text{if } y=0\end{cases}$$

remember that $y=0$ or 1 always

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$\mathrm{Cost}\left(h_\theta(x), y\right) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

remember that $y = 0$ or 1 always

the same

$$\mathrm{Cost}\left(h_\theta(x), y\right) = -y\log(h_\theta(x)) - \left(1 - y\right)\log(1 - h_\theta(x))$$

Logistic Regression

- **Cost Function:**

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$= -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + \left(1 - y^{(i)}\right)\log(1 - h_\theta(x^{(i)}))$$

- **Goal:**

learn parameters θ to $\underset{\theta}{\mathrm{minimize}}\, J(\theta)$

- **Hypothesis (to make a prediction):** $h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$

P(y=1|x;θ)

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

simultaneous update for all θ_j

learning rate

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

simultaneous update for all θ_j

learning rate

 # training examples

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

This look the same as linear regression!!???

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(\boxed{h_\theta(x^{(i)})} - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$= \boxed{\frac{1}{1+e^{-\theta^T x}}}$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

using different hypothesis from linear regression

Next Class:

- Paraphrases in Twitter
- more about Homework #2
- Evan Kozliner & Evan Jaffe
- Wei Sun & Wuwei Lan

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