Social Media & Text Analysis

lecture 6 - Paraphrase Identification and Logistic Regression

CSE 5539-0010 Ohio State University
Instructor: Alan Ritter
Website: socialmedia-class.org

Many slides adapted from Andrew Ng

(Recap) Classification Method:

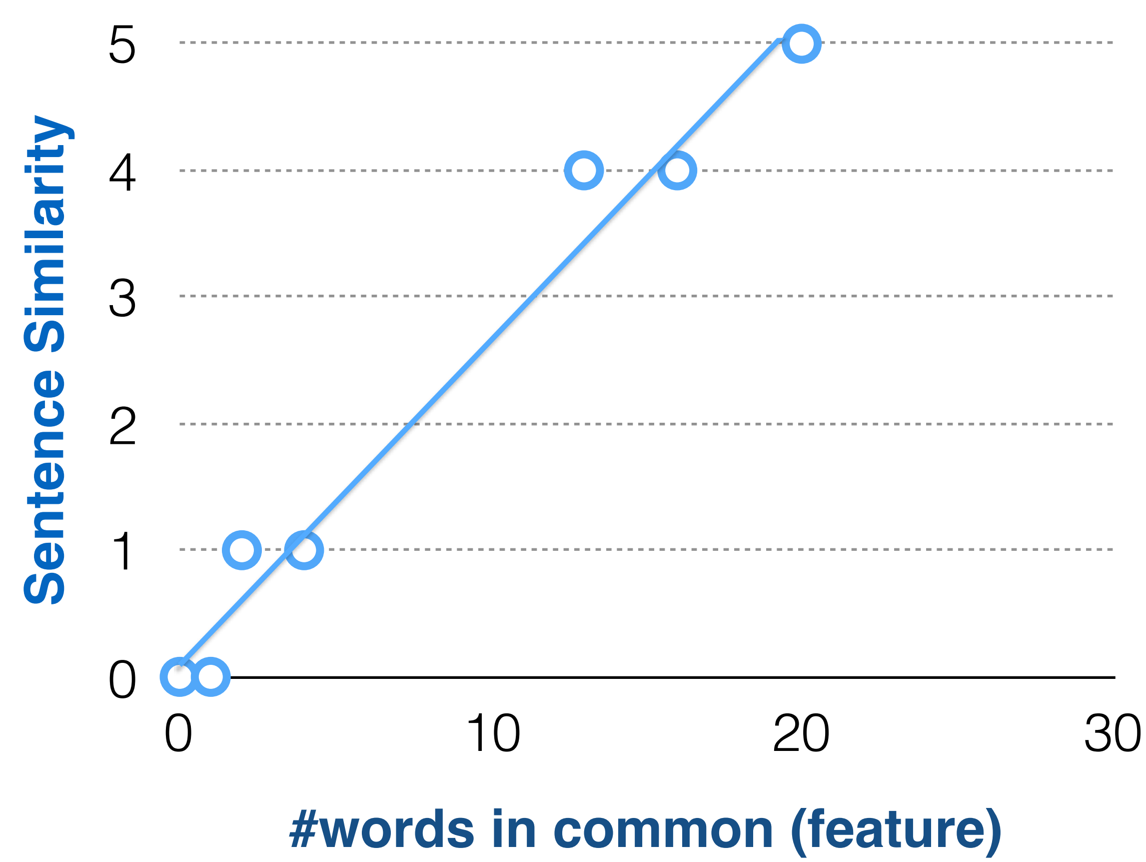
Supervised Machine Learning

- Input:
  - a sentence pair $\boldsymbol{x}$ **(represented by features)**
  - a fixed set of binary classes $\boldsymbol{Y = \{0, 1\}}$
  - a training set of $\boldsymbol{m}$ hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$

- Output:
  - a learned classifier $\gamma: x \rightarrow y \in Y$ ($y = 0$ or $y = 1$)

(Recap)

Linear Regression



- also supervised learning (learn from annotated data)

- but for **Regression**: predict **real-valued** output
  (Classification: predict discrete-valued output)

(Recap) Linear Regression w/ one variable:

Model Representation

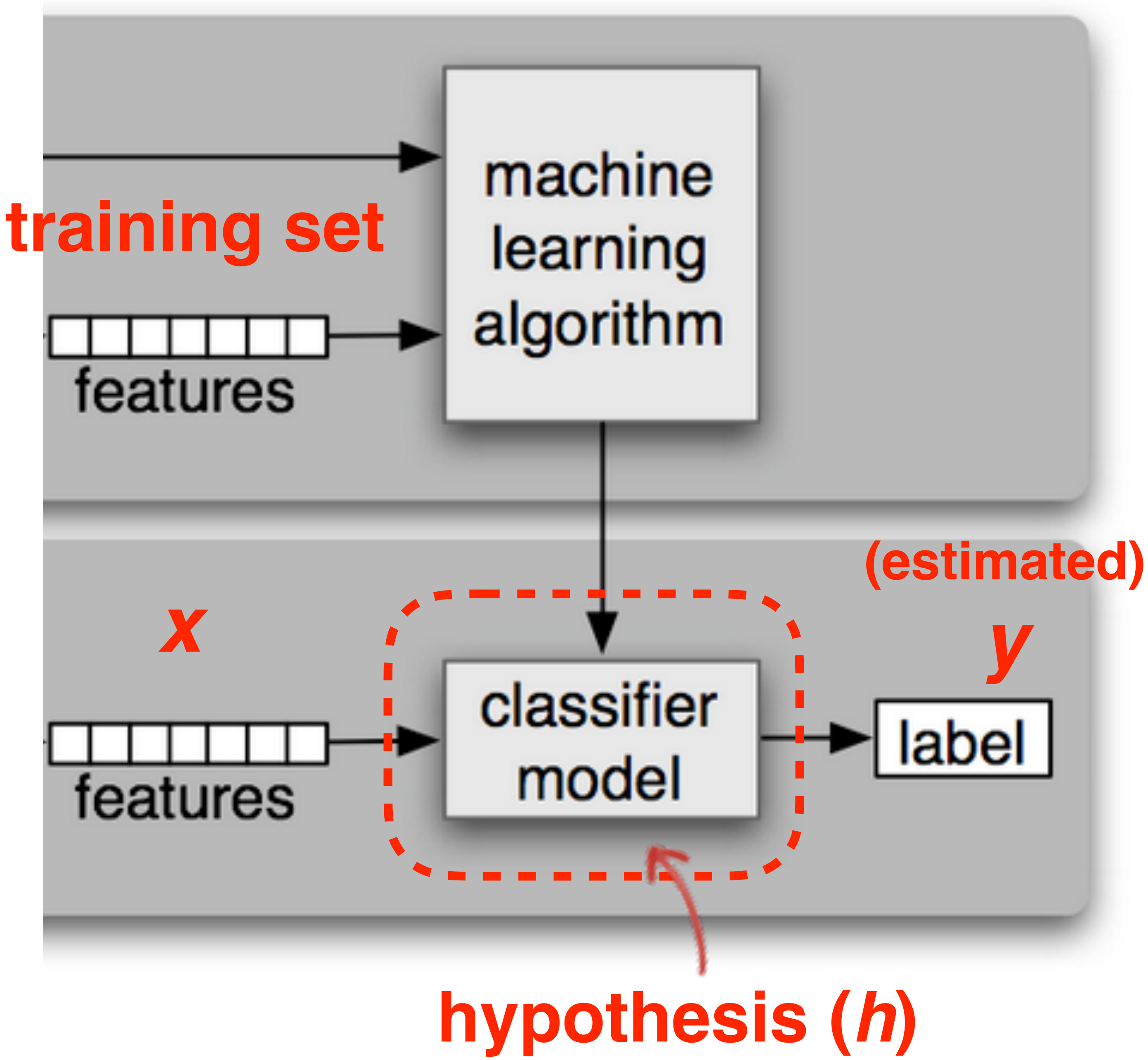
#words in common (***x***)	**Sentence Similarity** (***y***)
1	0
4	1
13	4
18	5
…	…

- ***m*** hand-labeled sentence pairs $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$
  θ's: parameters

Source: many following slides are adapted from Andrew Ng

(Recap) Linear Regression:

Model Representation



- How to represent ***h*** ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Linear Regression
w/ one variable

(Recap)

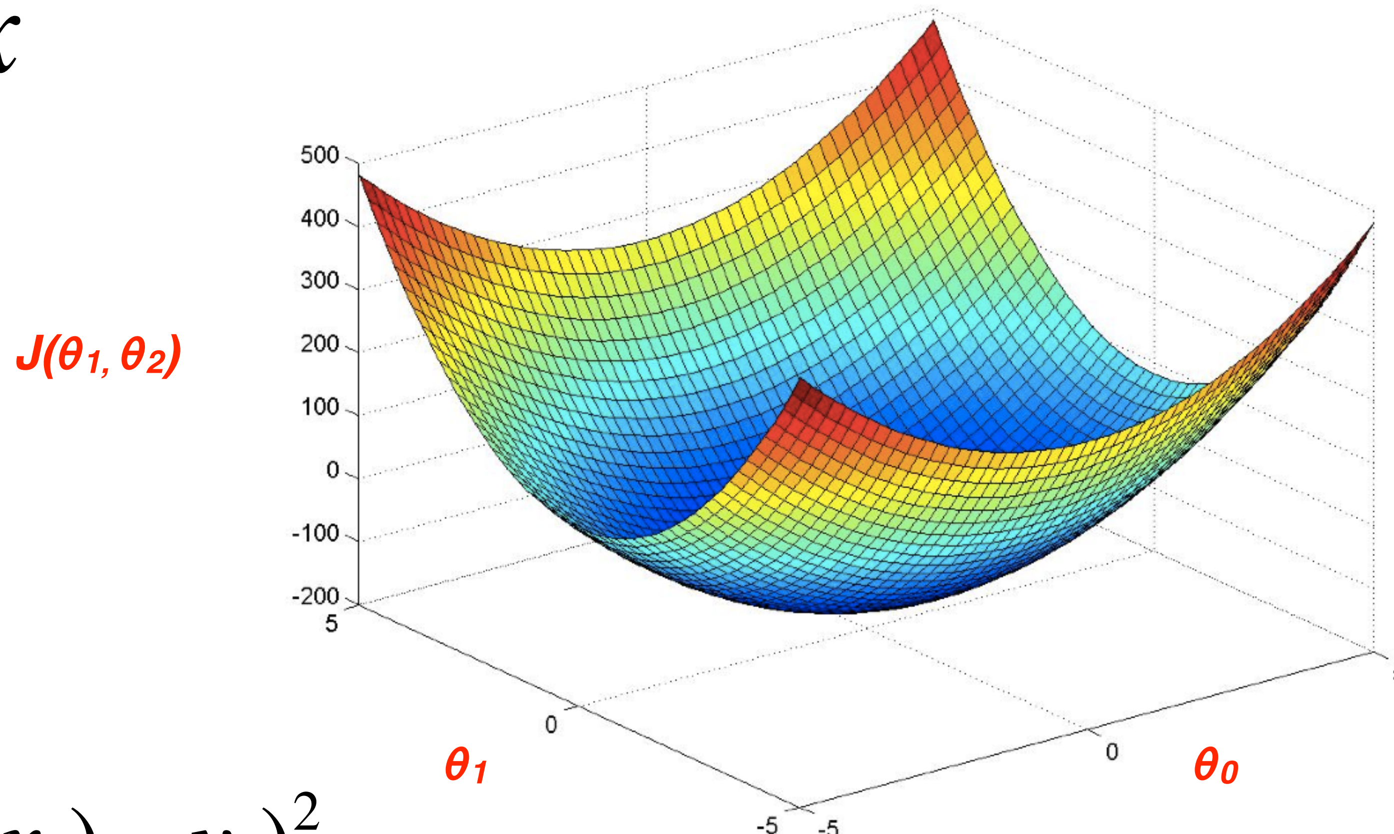
Linear Regression

- **Hypothesis**:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

- **Parameters:**

θ_0, θ_1



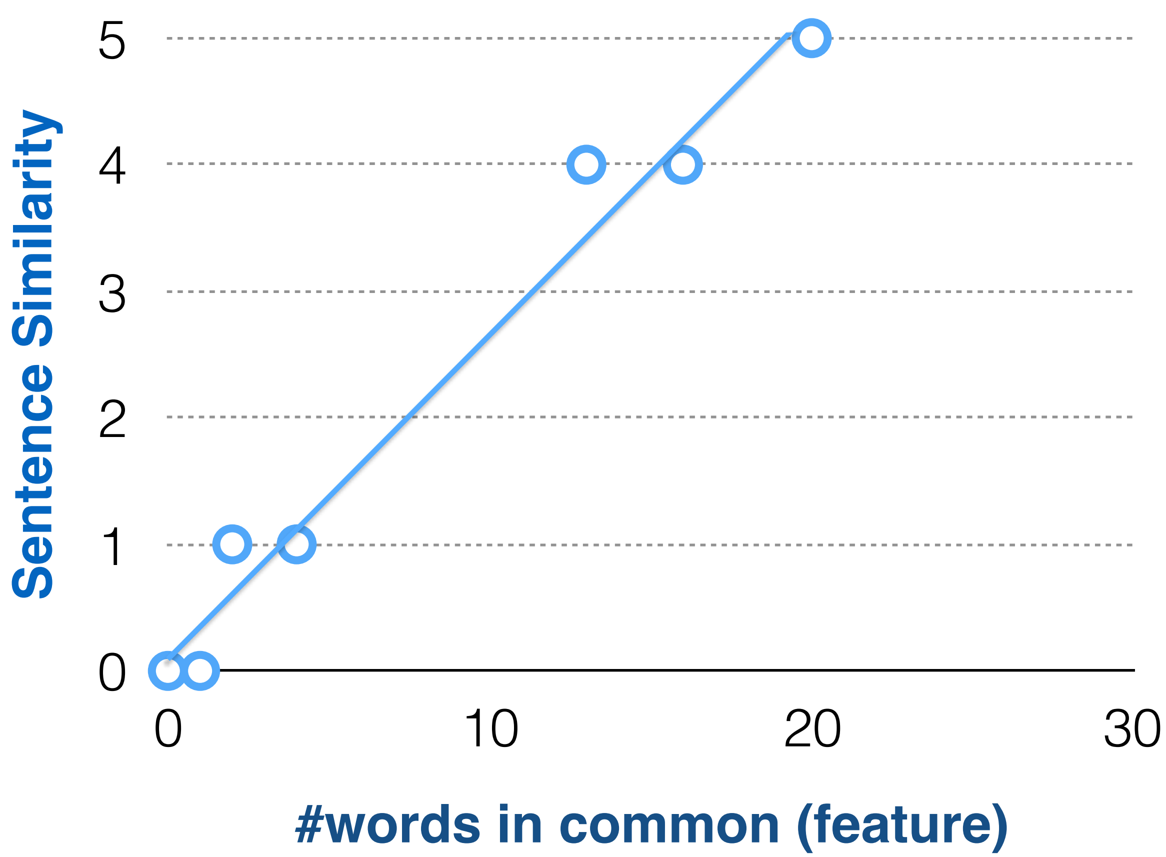
- **Cost Function:**

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

- **Goal:** $\underset{\theta_0, \theta_1}{\text{minimize}}\, J(\theta_0, \theta_1)$

(Recap) Linear Regression w/ one variable:

Cost Function



squared error function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

- **Idea:** choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for training examples (x, y)

$$\underset{\theta_0,\, \theta_1}{\text{minimize}}\, J(\theta_0, \theta_1)$$

(Recap)

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

}

simultaneous update
for j=0 and j=1

learning rate

(Recap) Linear Regression w/ one variable:

Gradient Descent

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i) \cdot x_i$$

}

simultaneous update ***θ_0***, ***θ_1***

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

(for convenience, define $x_0 = 1$)

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Linear Regression w/ multiple variables (features):

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$$h_\theta(x) = \theta^T x$$

Linear Regression w/ multiple variables (features):

Model Representation

- Hypothesis:

$$h_\theta(x) = \theta^T x$$

- Cost function: **# training examples**

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

(Recap)

Paraphrase Identification

obtain sentential paraphrases automatically

Mancini has been sacked by Manchester City

Mancini gets the boot from Man City

Yes!

WORLD OF JENKS IS ON AT 11

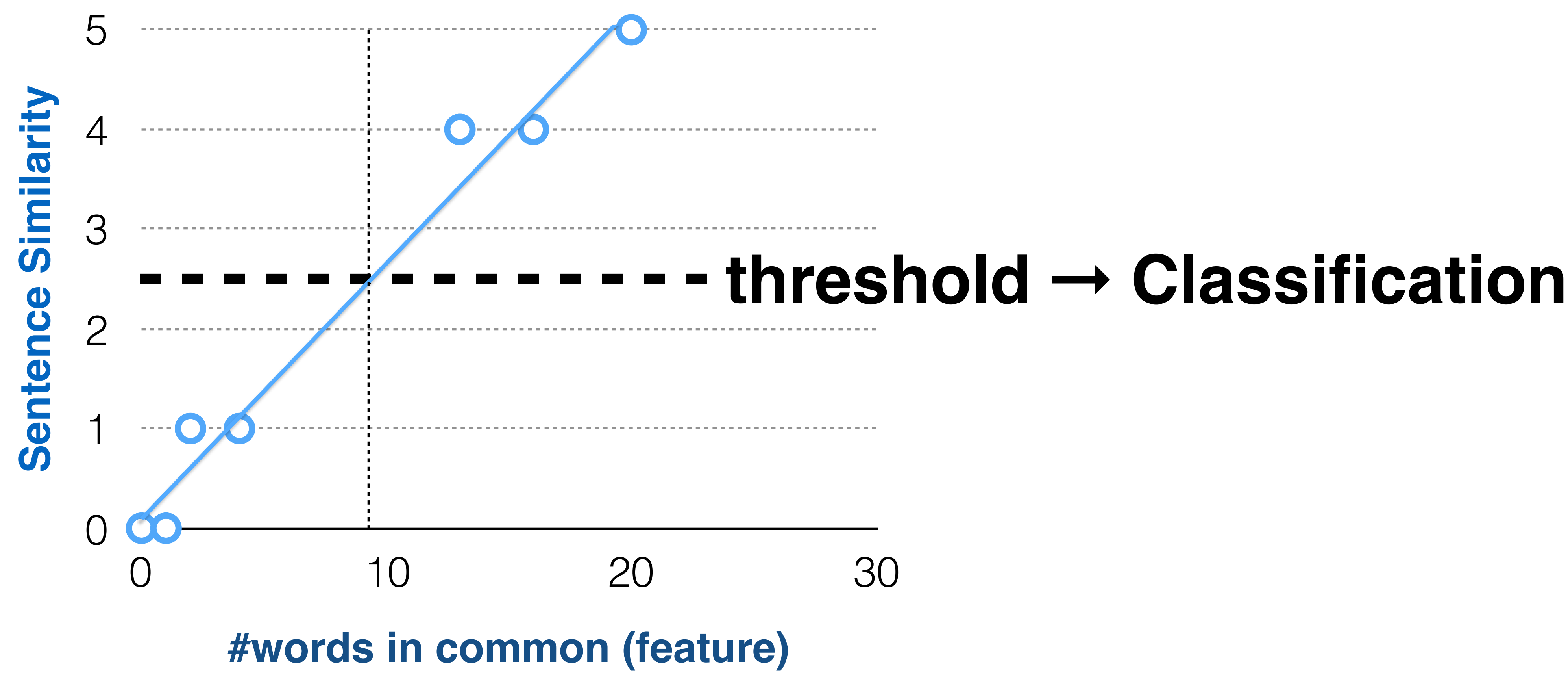
World of Jenks is my favorite show on tv

No!

Wei Xu, Alan Ritter, Chris Callison-Burch, Bill Dolan, Yangfeng Ji. “Extracting Lexically Divergent Paraphrases from Twitter” In TACL (2014)

(Recap)

Linear Regression



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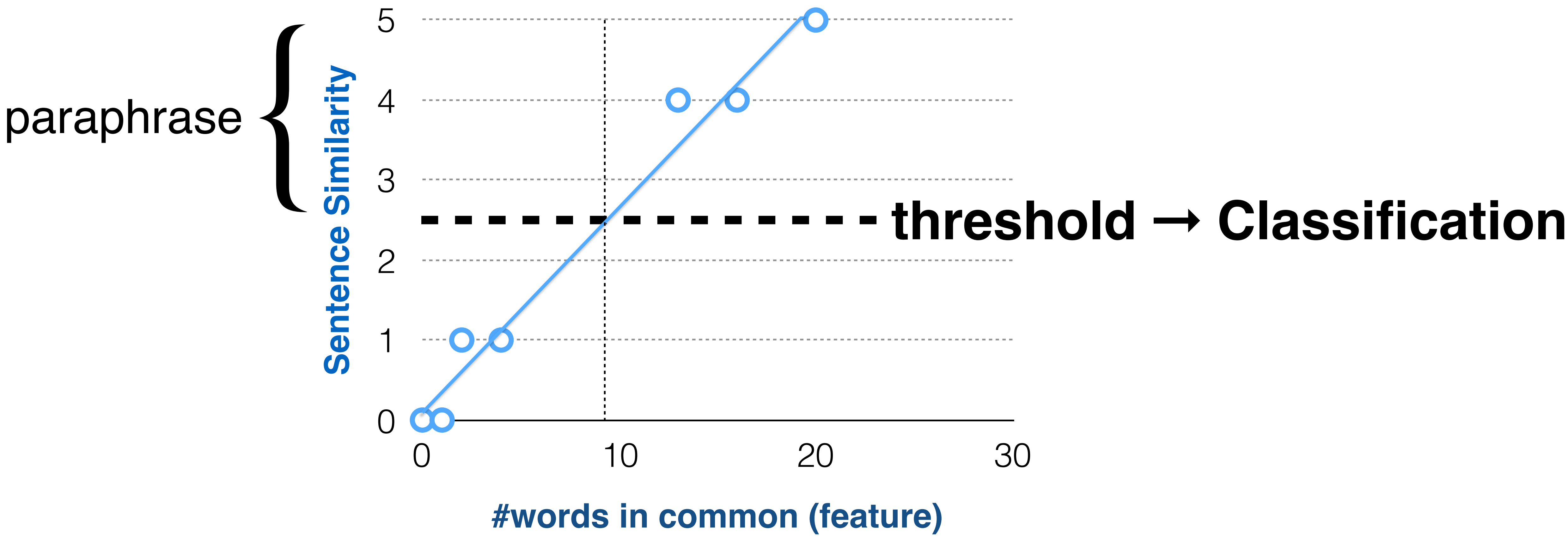
(Recap)

Linear Regression



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(Recap)

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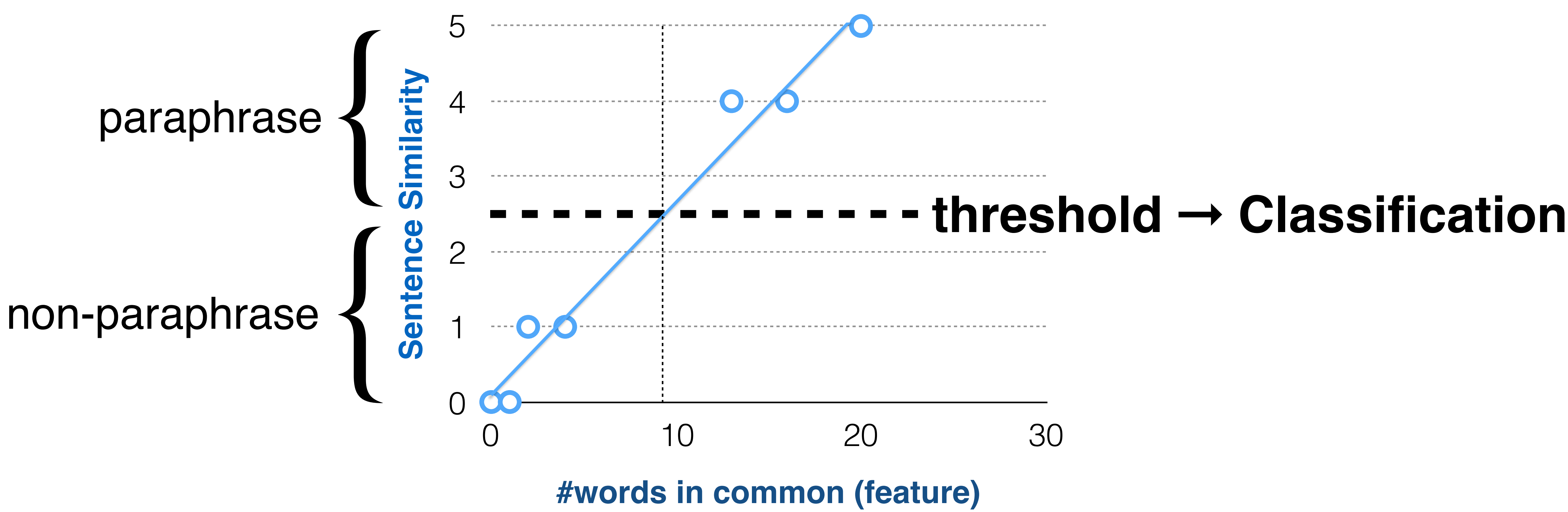

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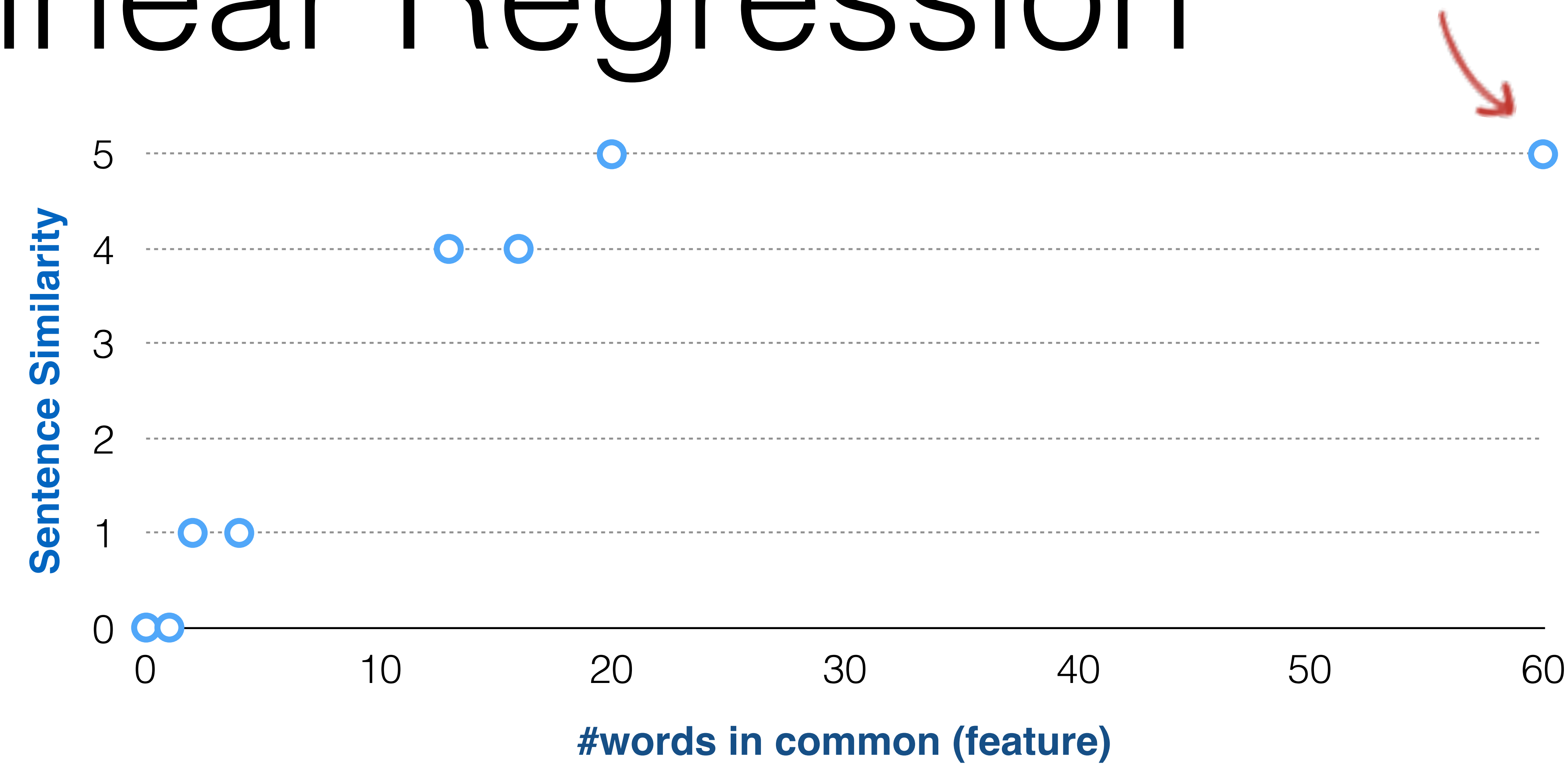
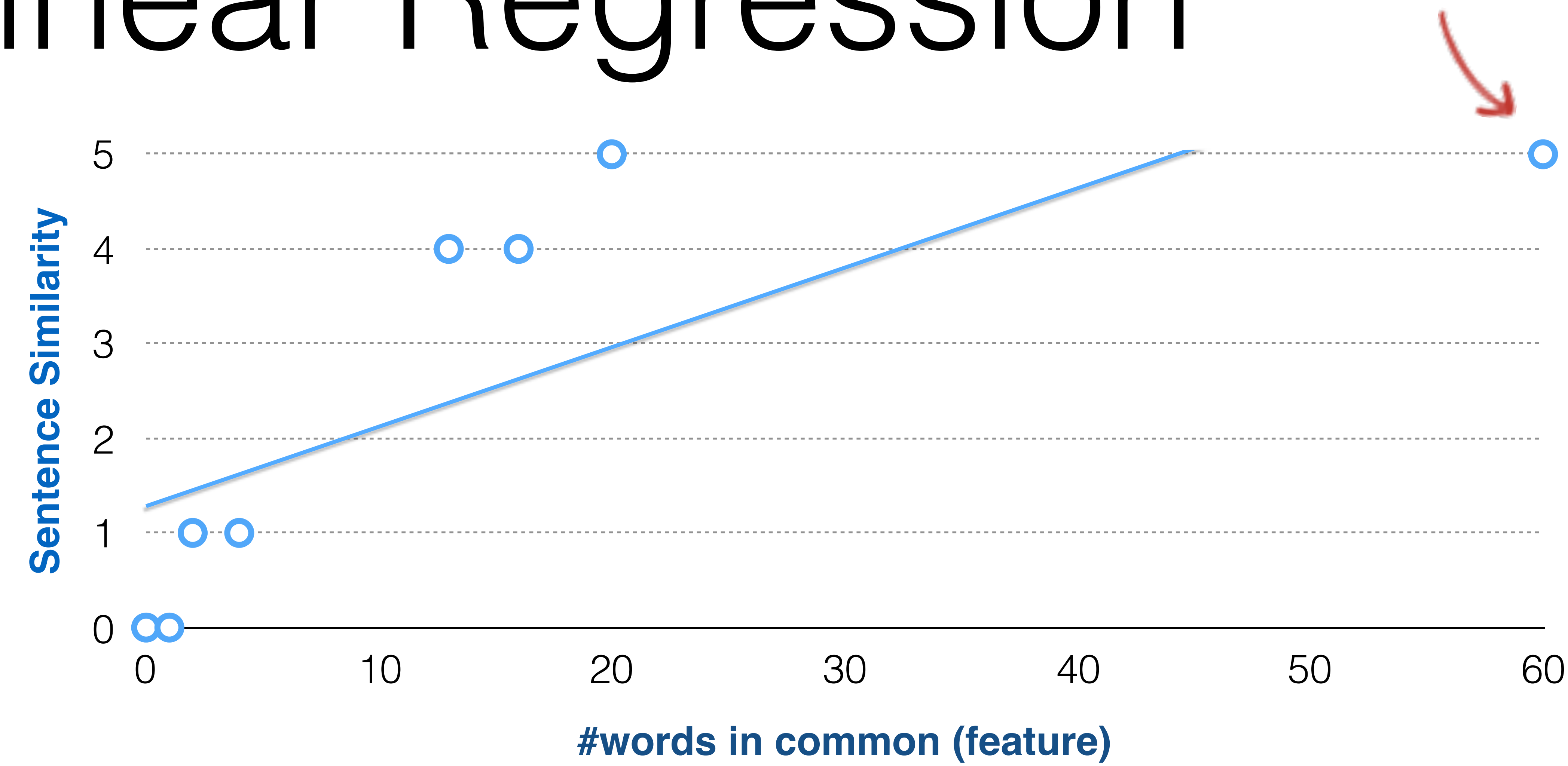
A problem in classification

Linear Regression



A problem in classification

Linear Regression



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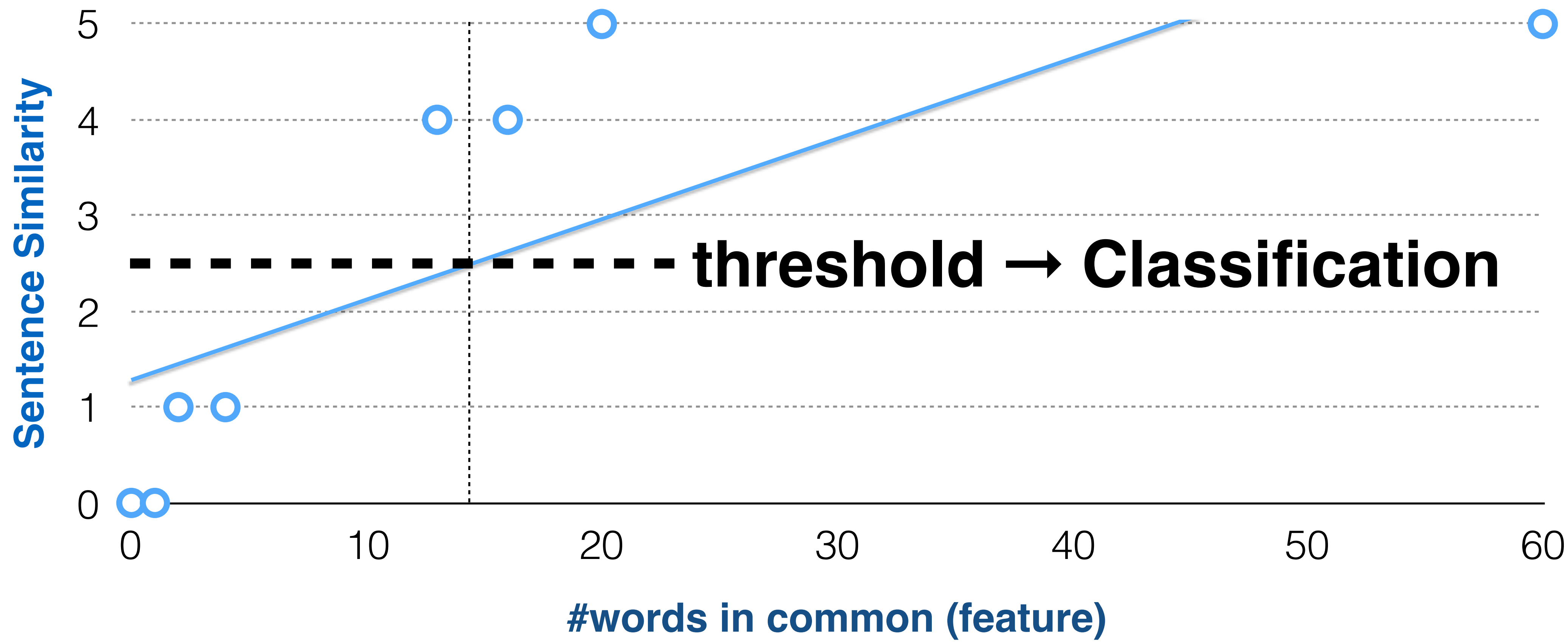

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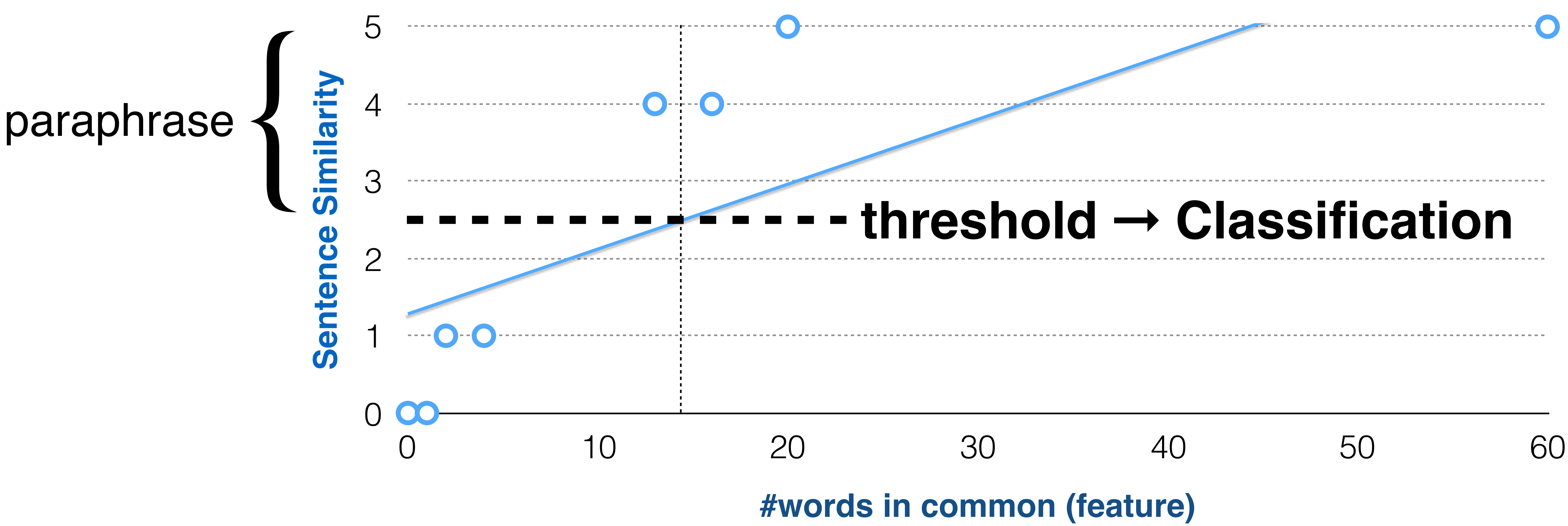

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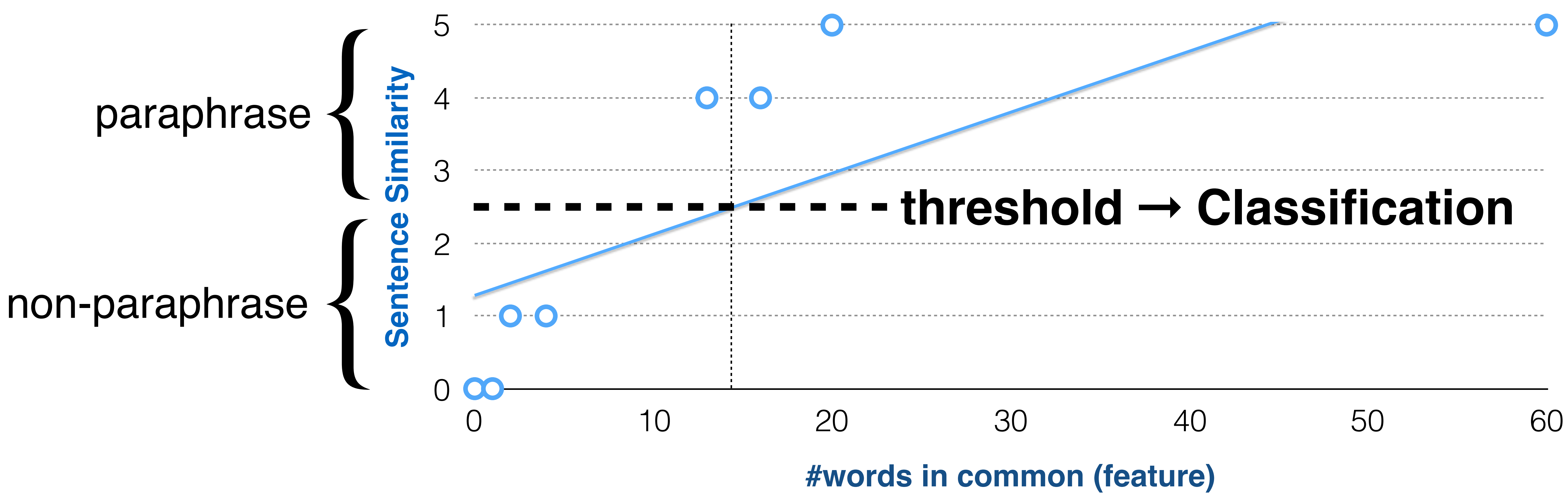

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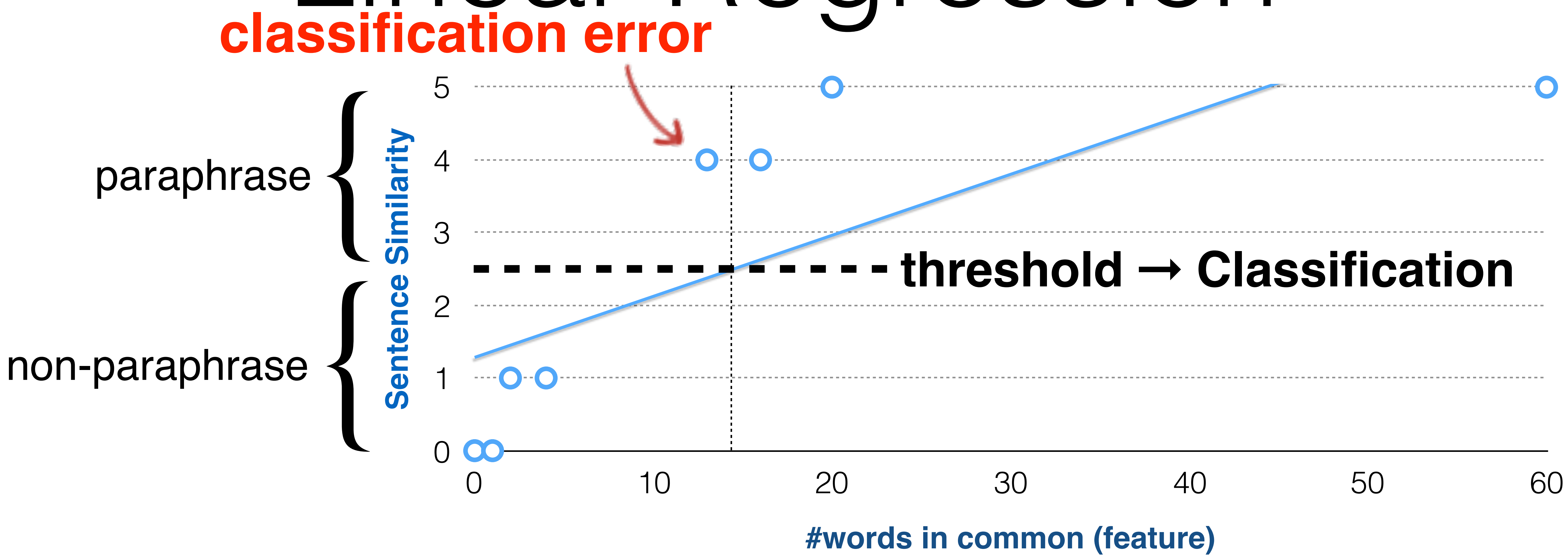
Linear Regression

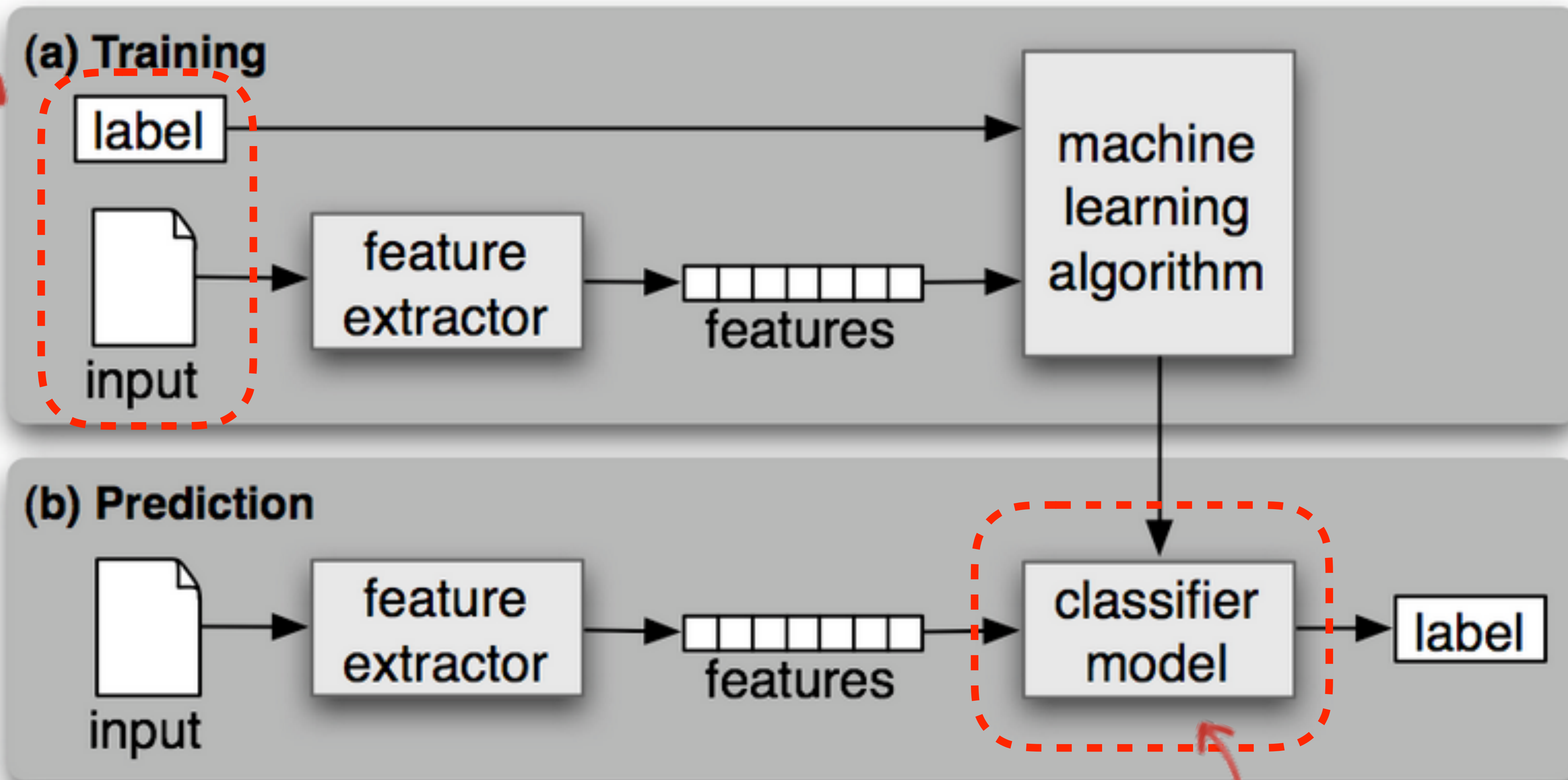


In practice, do not use linear regression for classification.

(Recap) Classification:

Supervised Machine Learning

training set



(also called) hypothesis

(Recap)

Logistic Regression

- One of the most useful **supervised machine learning algorithm** for classification!
- Generally high performance for a lot of problems.
- Much more robust than Naïve Bayes
(better performance on various datasets).

Hypothesis:

Linear → Logistic Regression

Classification: ***y = 0*** or ***y = 1***

- Linear Regression: $h_\theta(x)$ can be > 1 or < 0

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a classification (not regression) algorithm

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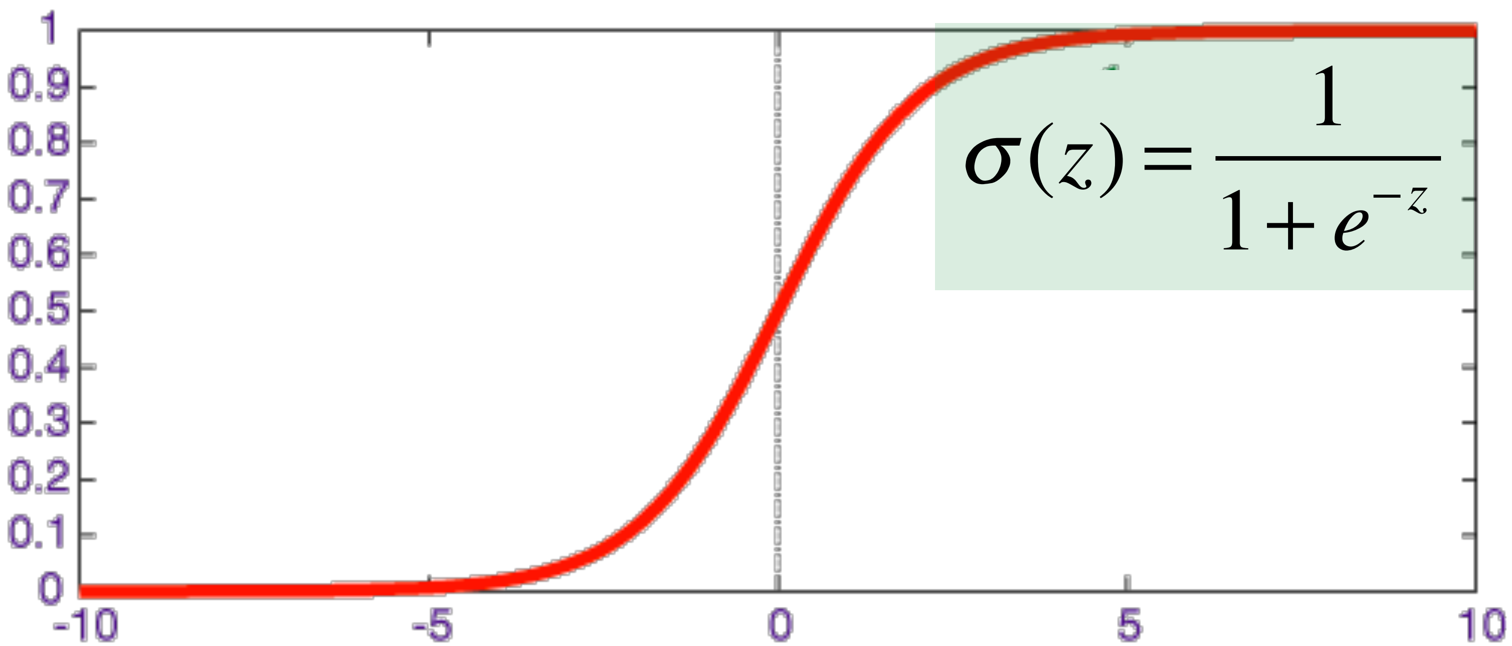
sigmoid (logistic) function



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- Linear Regression: $h_\theta(x) = \theta^T x$
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$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

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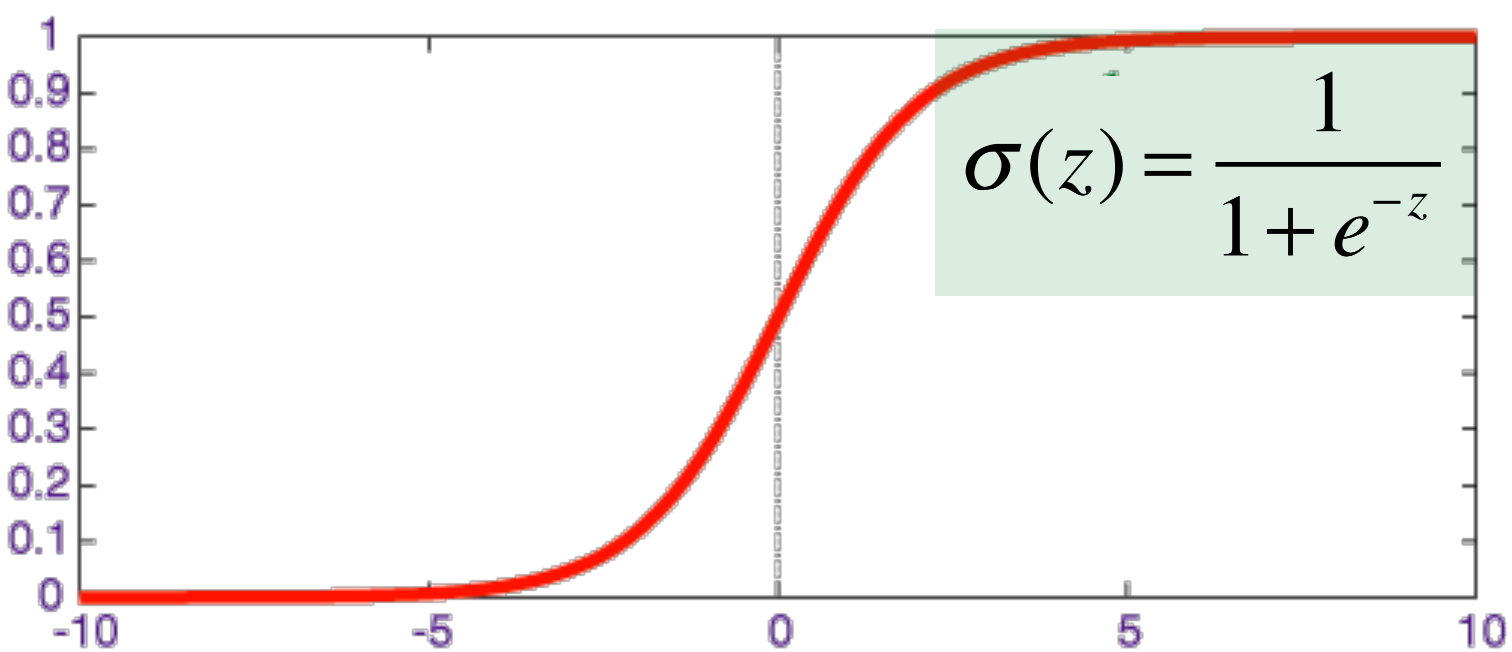

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Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

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$$If \quad x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \#words\_in\_common \end{bmatrix}, \; h_\theta(x) = 0.7$$

70% chance of the sentence pair being paraphrases

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$$h_\theta(x) = P(y = 1 \mid x; \theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

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Logistic Regression:

Interpretation of Hypothesis

- ***$h_\theta(x)$*** = estimated probability that ***$y = 1$*** on input

$$P(y=1 \mid x;\theta) + P(y=0 \mid x;\theta) = 1$$

$$h_\theta(x) = P(y=1 \mid x;\theta)$$

probability that *$y = 1$*, given x, parameterized by *θ*

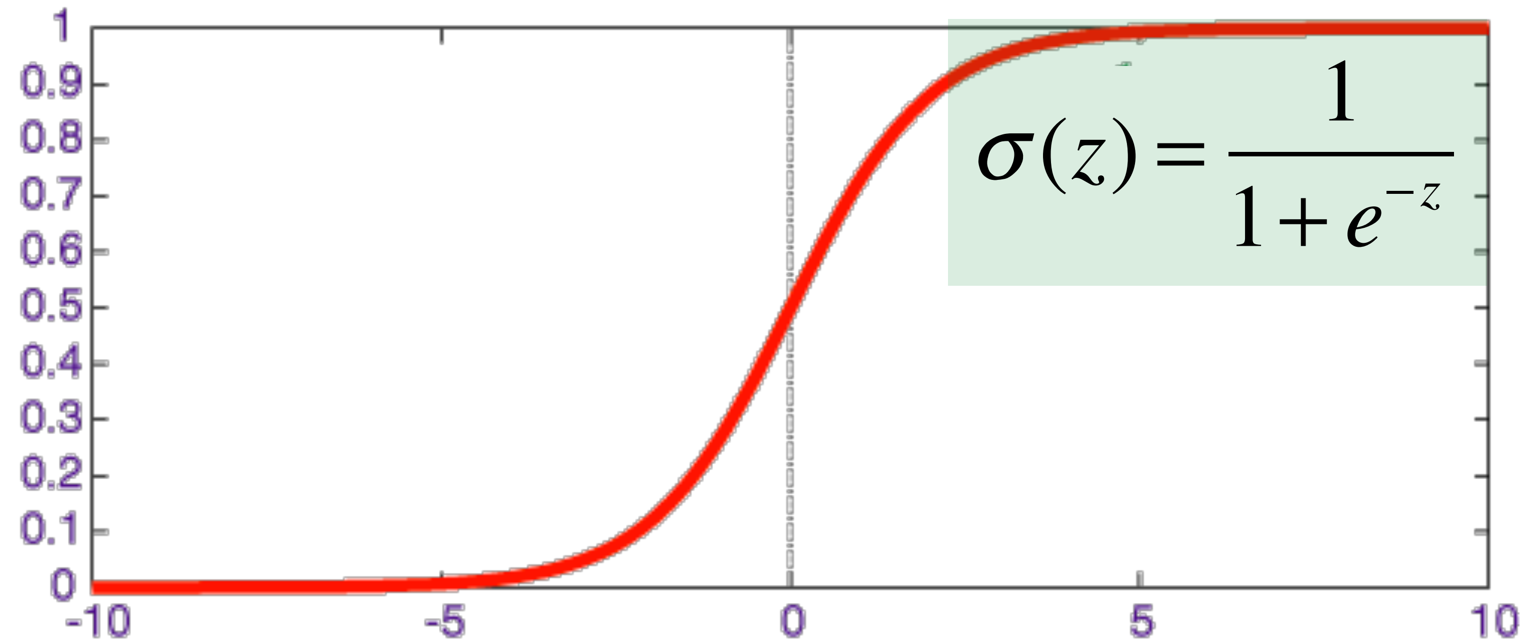
Decision Boundary

- Logistic Regression:

sigmoid (logistic) function

$$h_\theta(x) = \sigma\left(\theta^T x\right)$$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$



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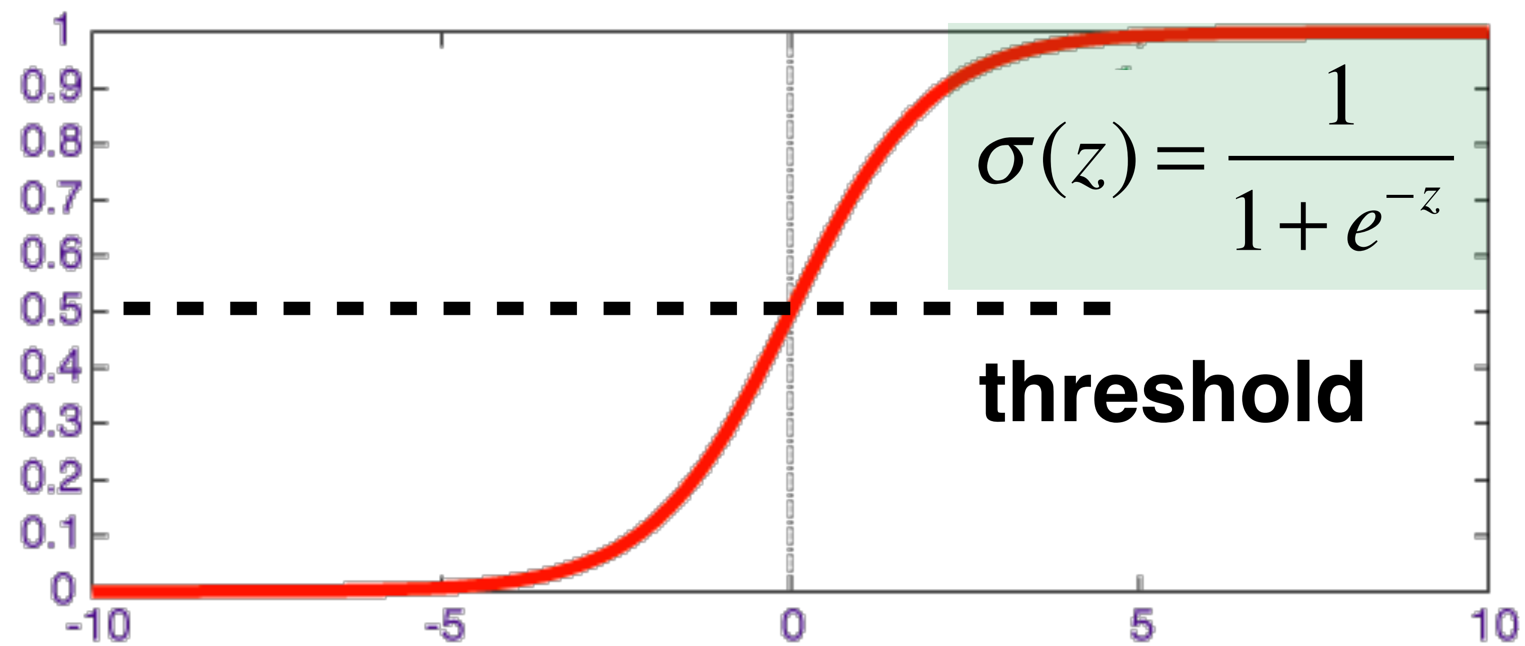

predict $\boldsymbol{y = 1}$ if $\boldsymbol{h_\theta(x) \geq 0.5}$

predict $\boldsymbol{y = 0}$ if $\boldsymbol{h_\theta(x) < 0.5}$

Logistic Regression:

Decision Boundary

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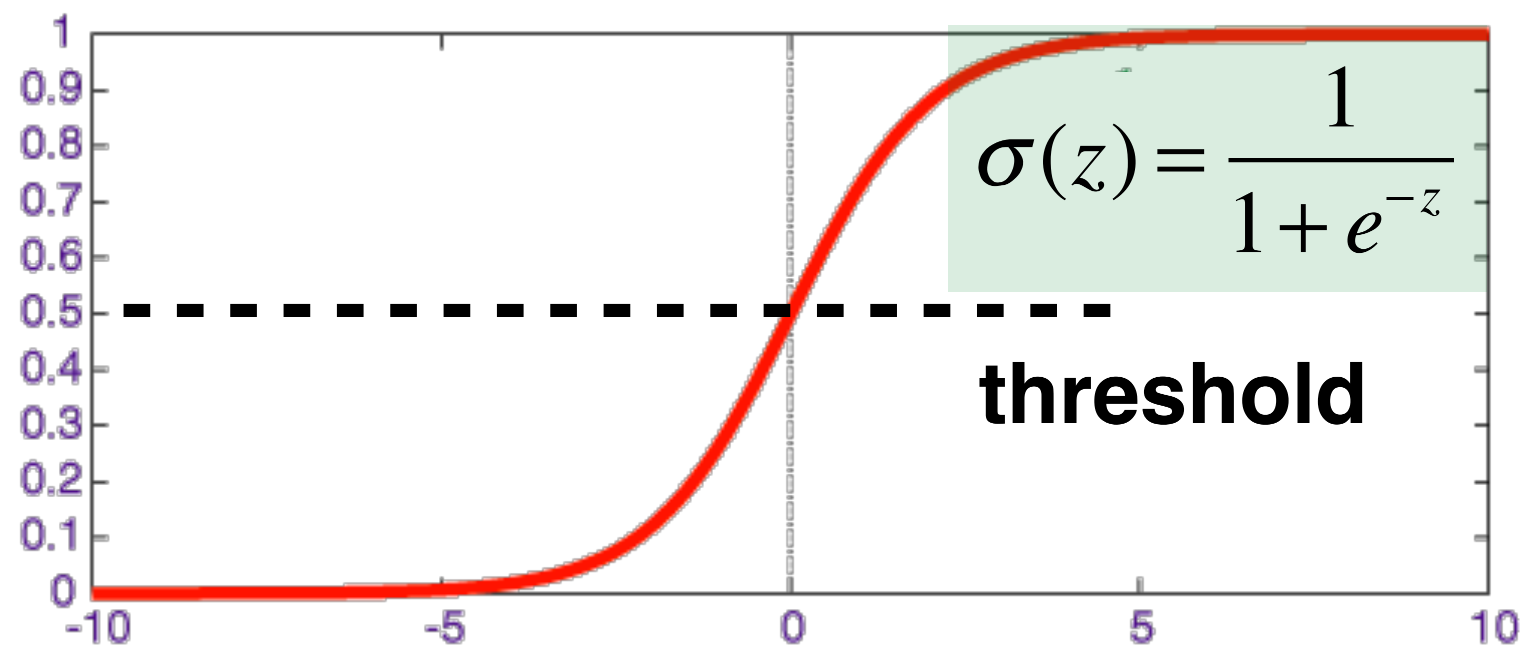
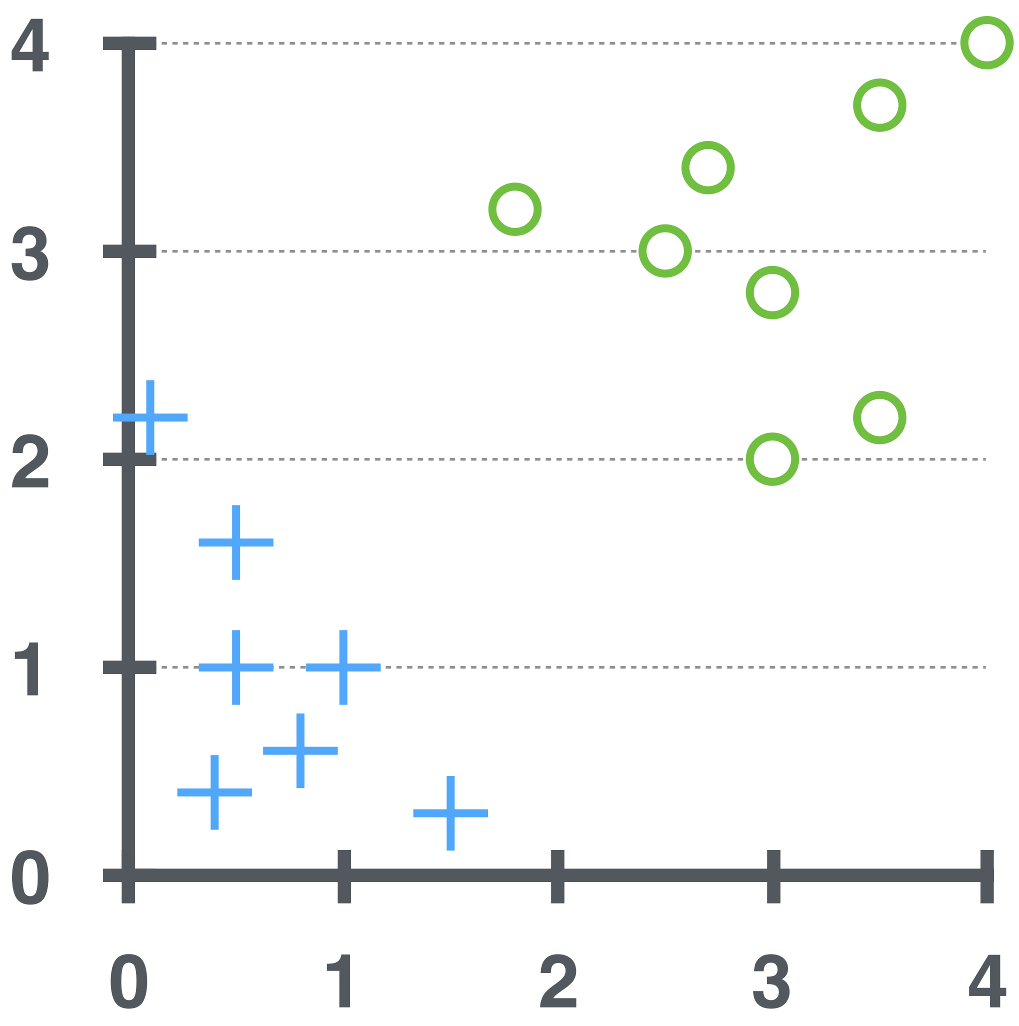


predict ***y = 1*** if ***$h_\theta(x) \geq 0.5$*** $\longleftarrow$ when ***$\theta^T x \geq 0$***

predict ***y = 0*** if ***$h_\theta(x) < 0.5$*** $\longleftarrow$ when ***$\theta^T x < 0$***

Logistic Regression:

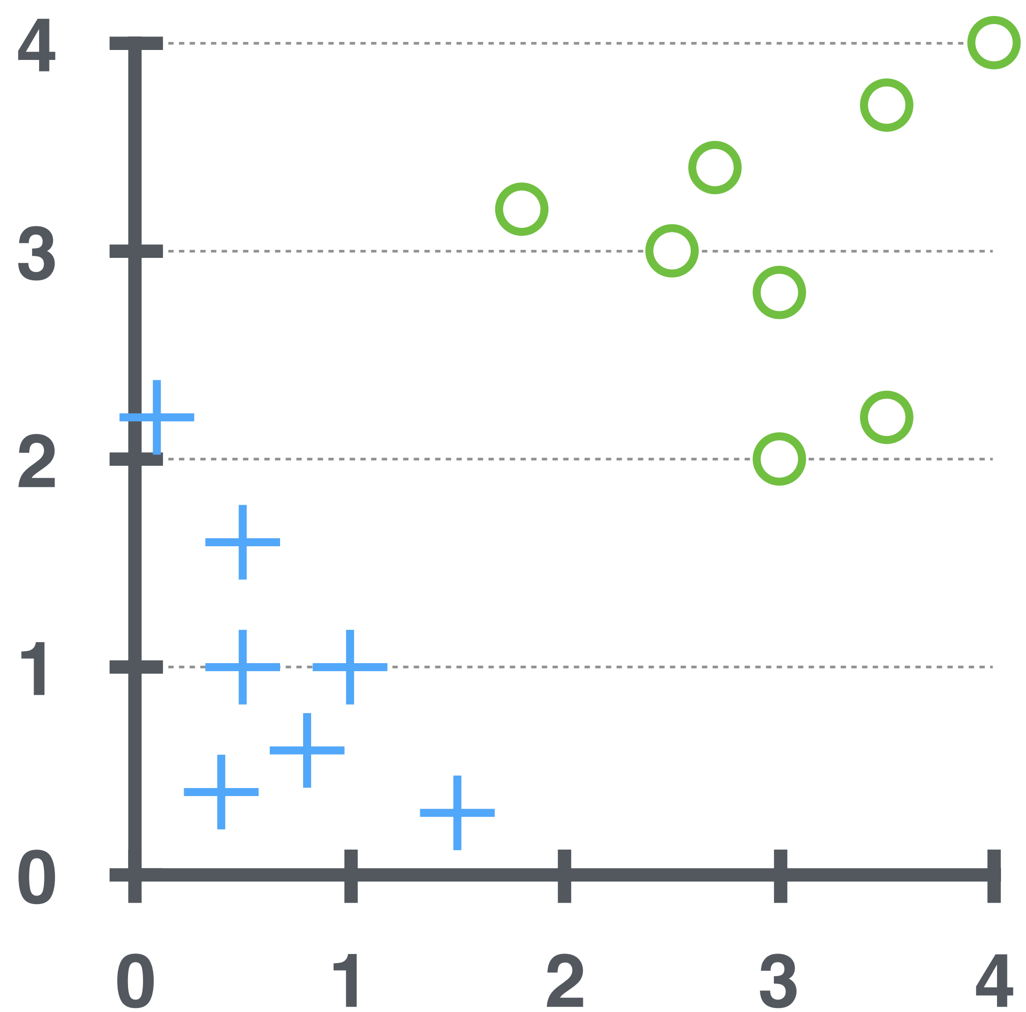
Decision Boundary



Logistic Regression:

Decision Boundary

$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$



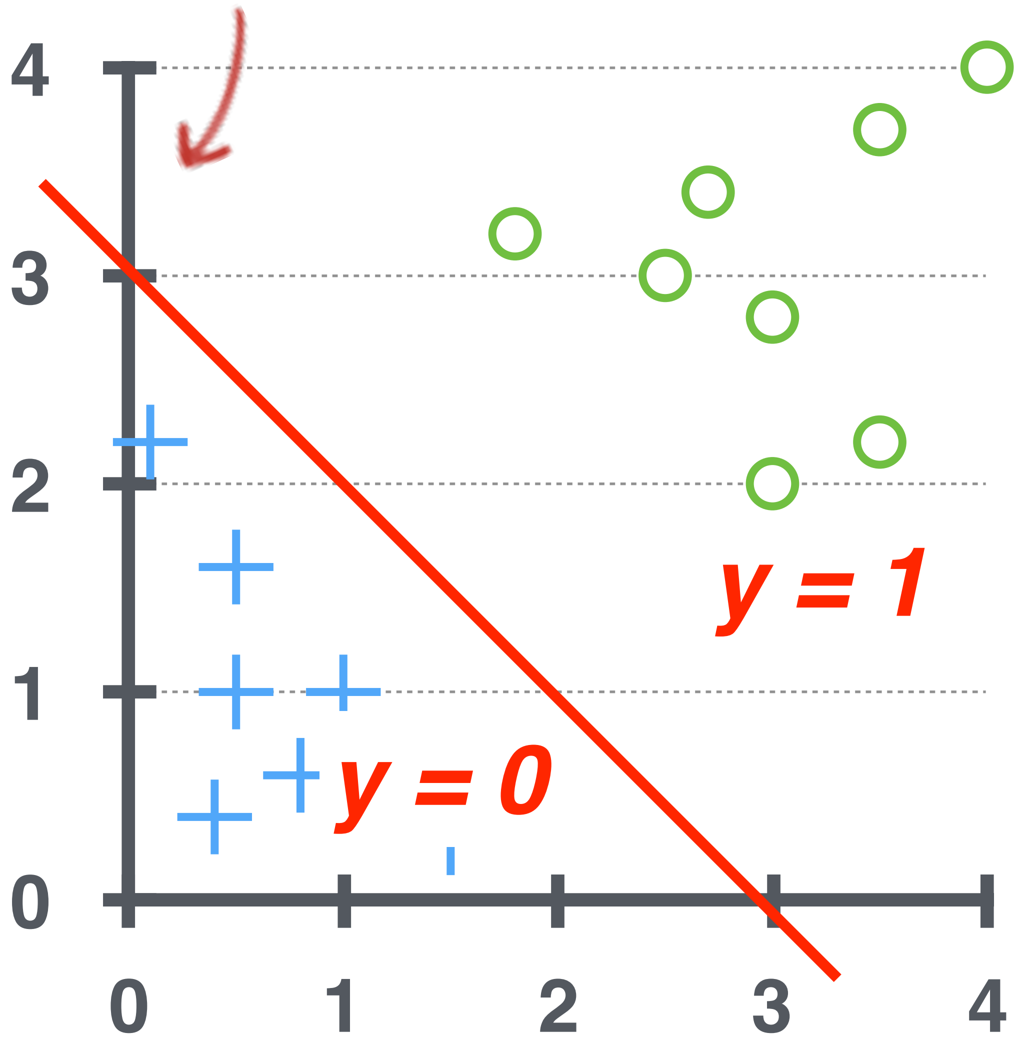
What if $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$?

predict ***y = 1*** if $\boldsymbol{\theta^T x \geq 0}$

Logistic Regression:

Decision Boundary

decision boundary



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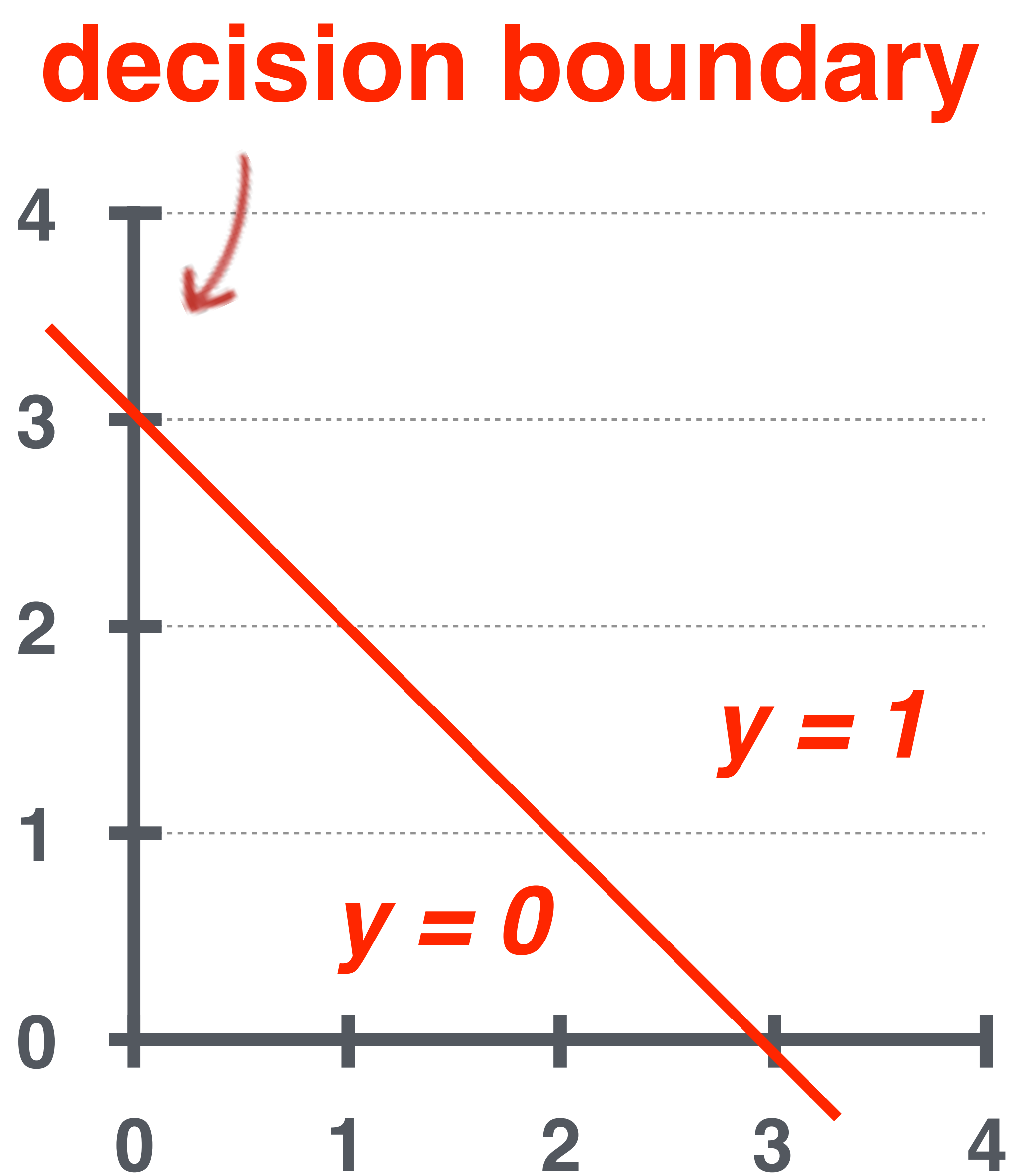
Logistic Regression:

Decision Boundary



$$h_\theta(x) = \sigma\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2\right)$$

a property of the hypothesis

a property of the parameters

~~a property of the dataset~~

Logistic Regression

- a training set of ***m*** hand-labeled sentence pairs

($x^{(1)}, y^{(1)}$), … , ($x^{(m)}, y^{(m)}$) ($y \in \{0, 1\}$)

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in R^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in R^{n+1}$$

Cost function:

Linear → Logistic Regression

- Linear Regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

squared error function

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this cost function is non-convex for logistic regression

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Linear → Logistic Regression

- Linear Regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

$$= \mathrm{Cost}\left(h_\theta(x), y\right)$$

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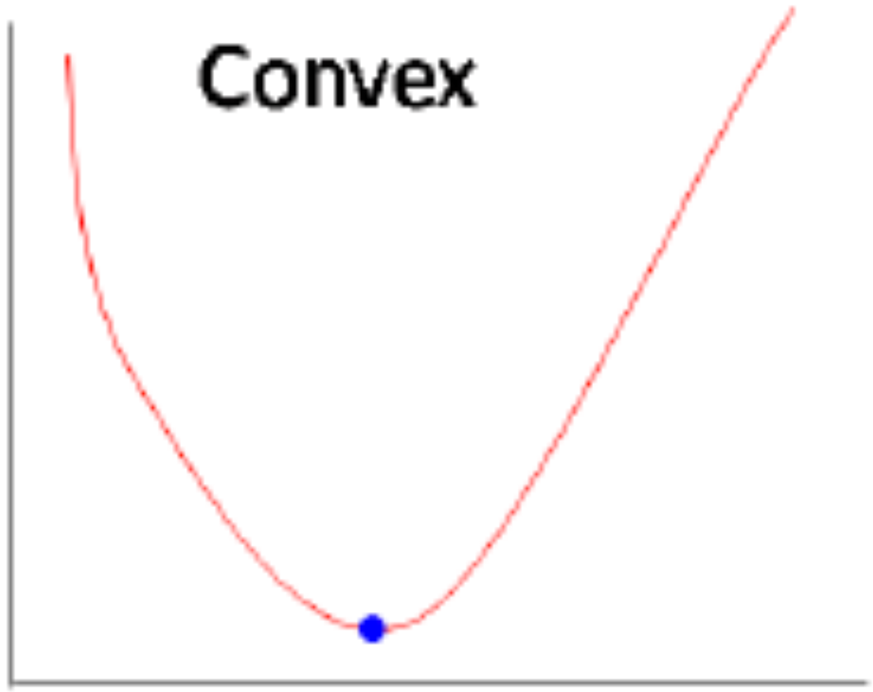
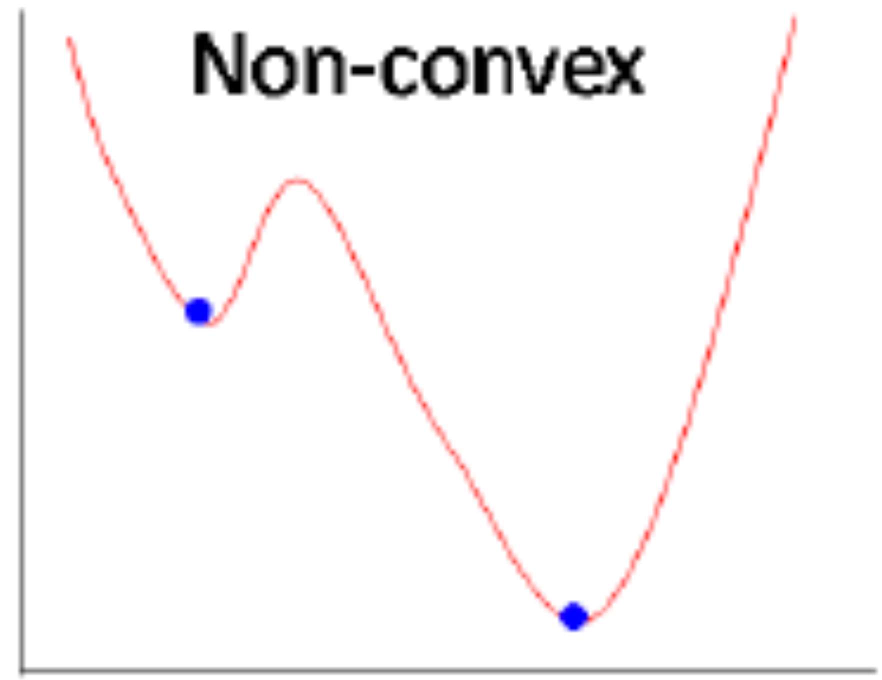
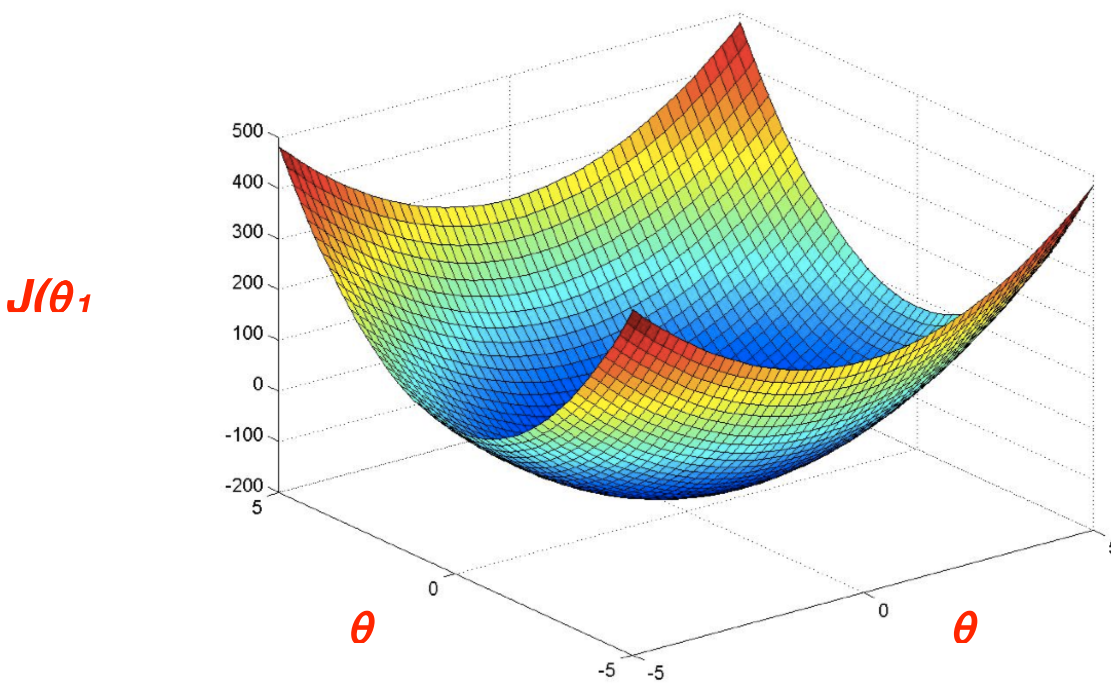
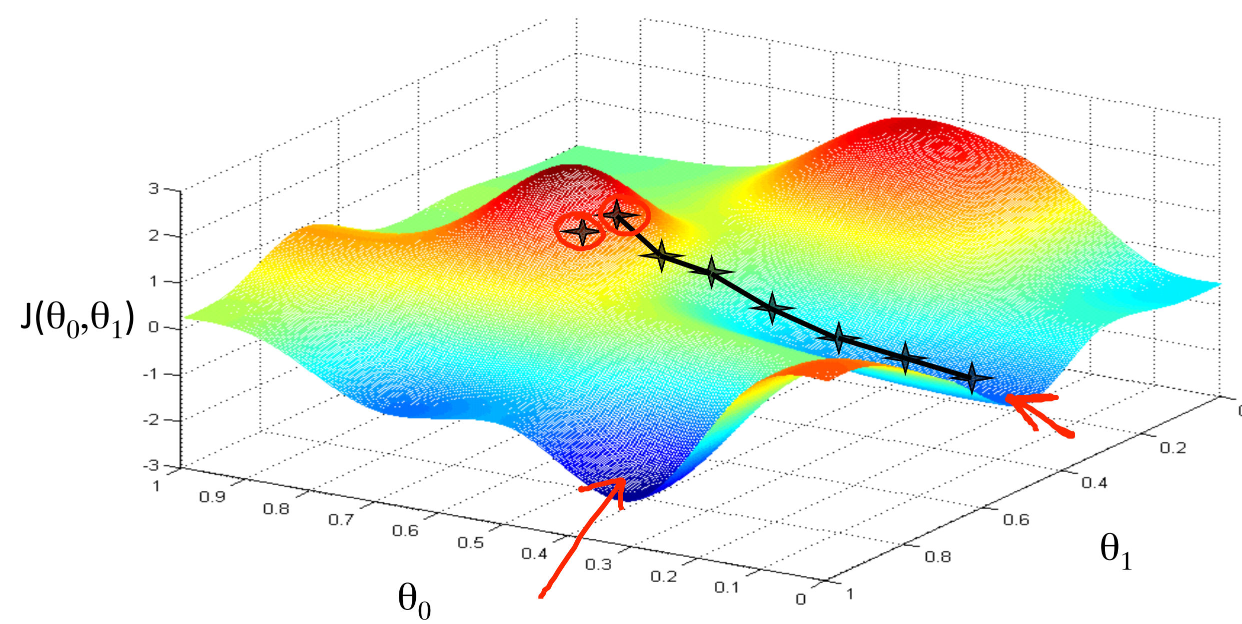
this cost function is non-convex for logistic regression

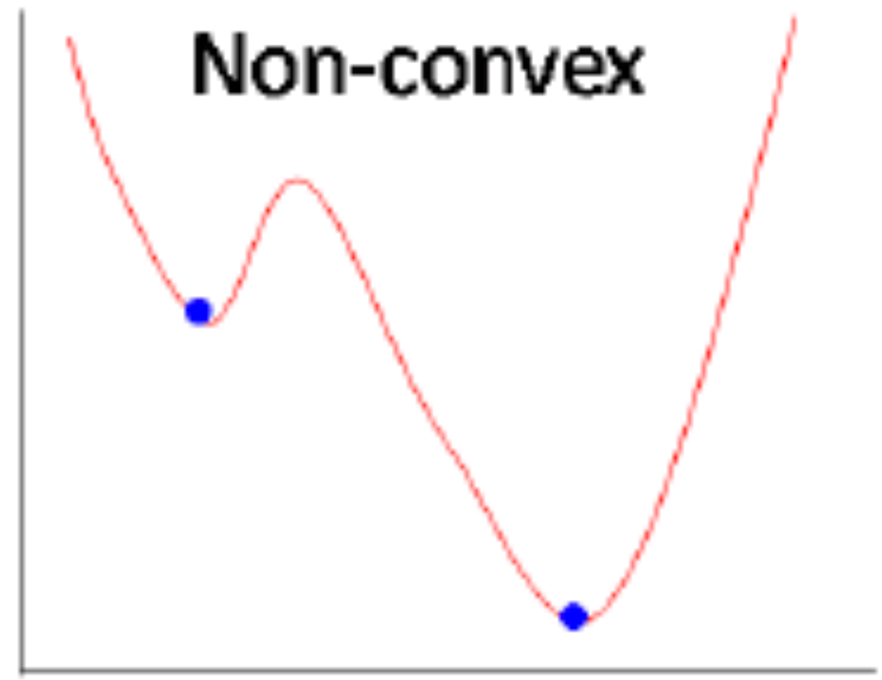

Convex

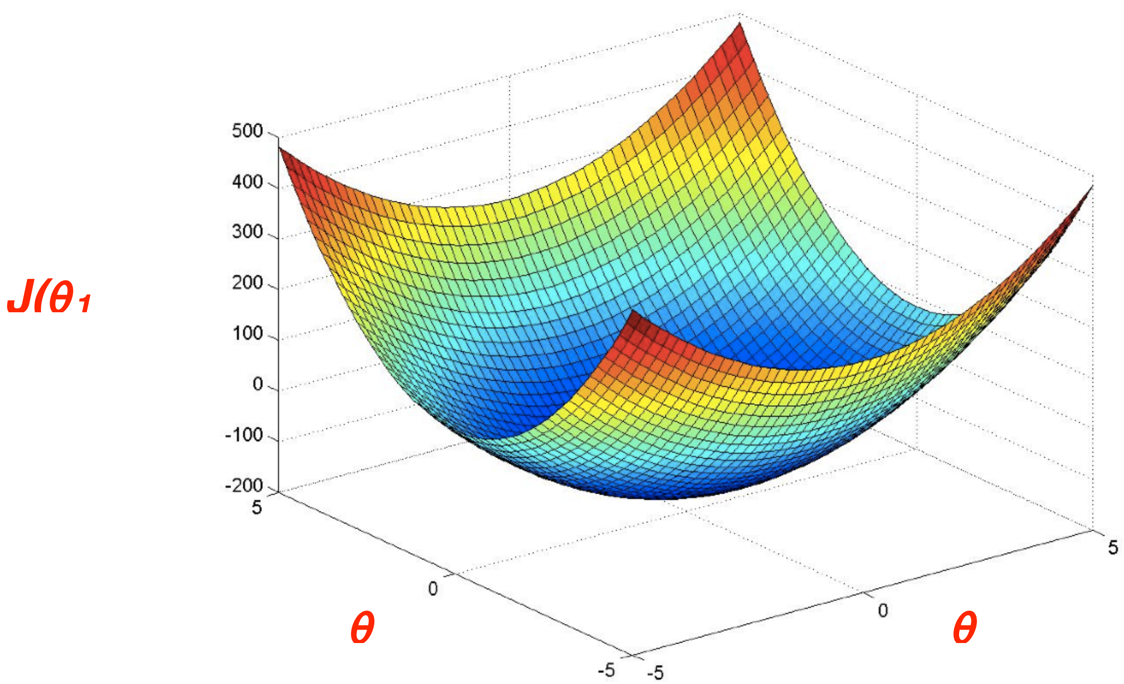

Non-convex

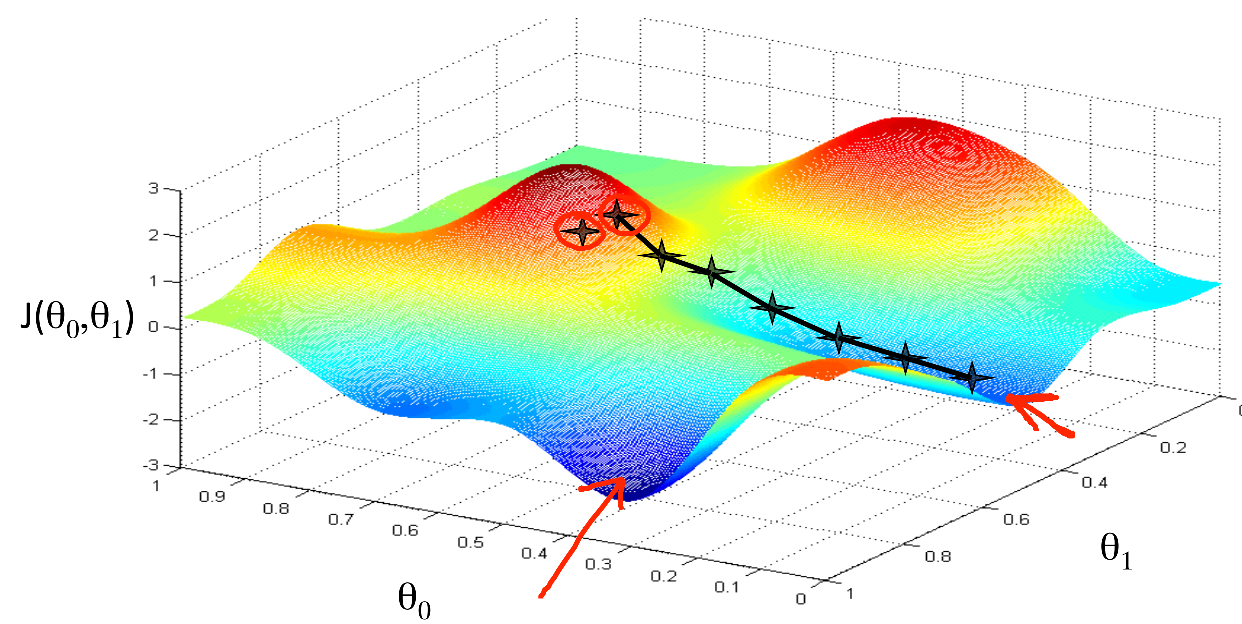

J(θ₁
θ
θ
500
400
300
200
100
0
-100
-200
5
0
-5
-5
0
5


J(θ0,θ1)
θ0
θ1
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

we want convex! easy gradient descent!







Logistic Regression:

Cost Function

$$\text{Cost}\big(h_\theta(x), y\big) = \begin{cases} -\log(h_\theta(x)) & \text{if } \; y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } \; y = 0 \end{cases}$$

remember that

$$0 \leq h_\theta(x) \;\; \leq 1$$

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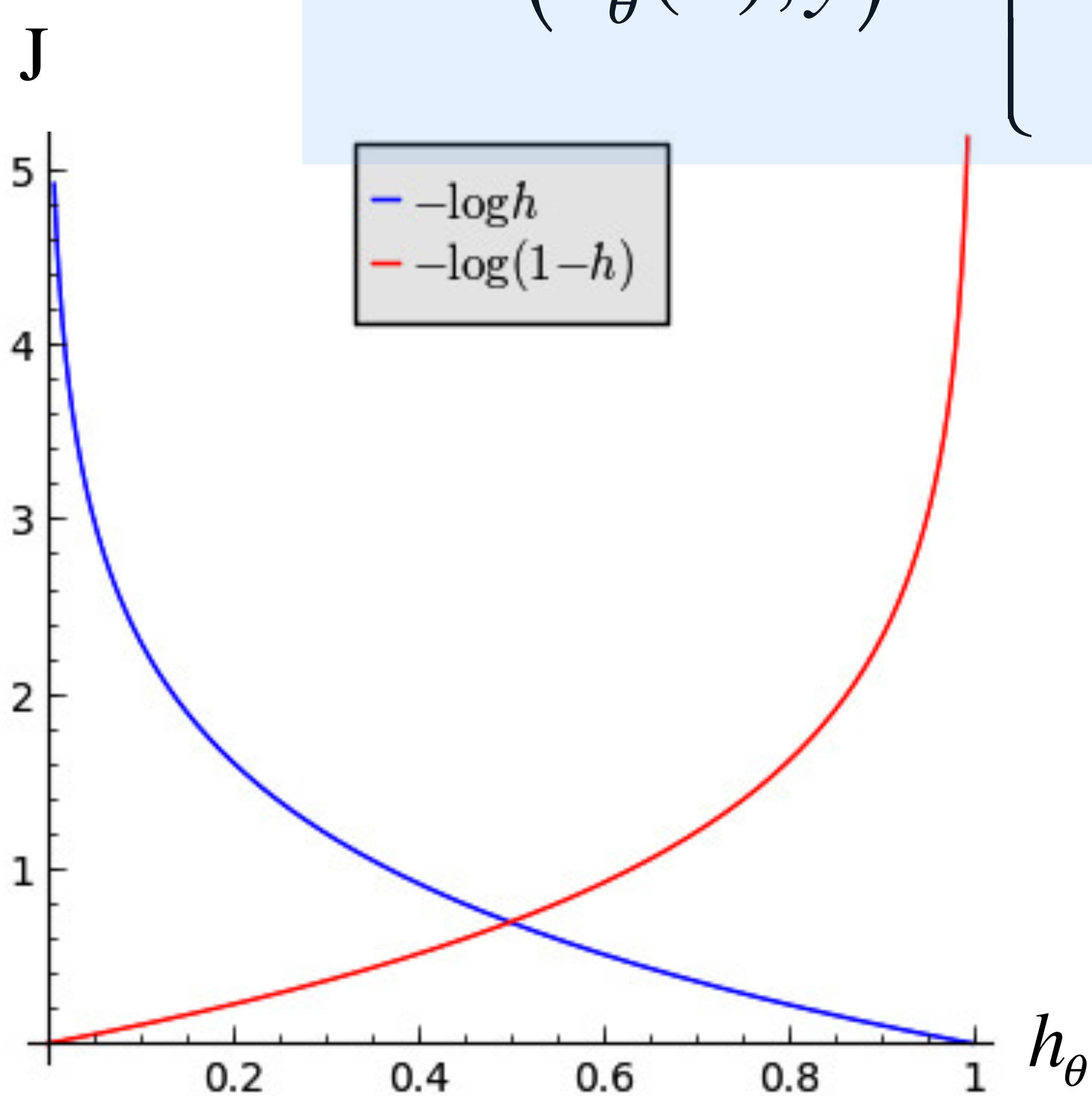

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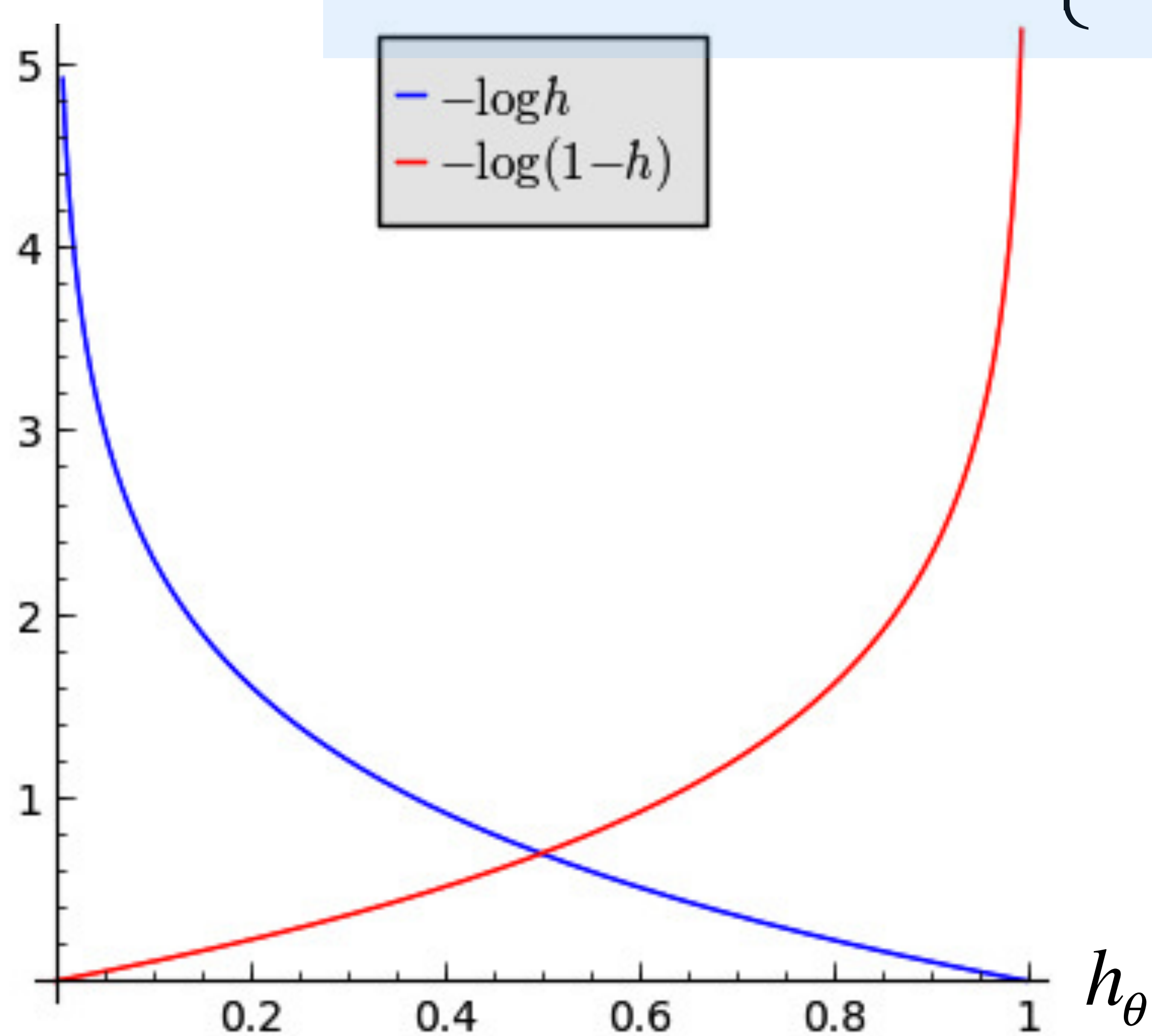

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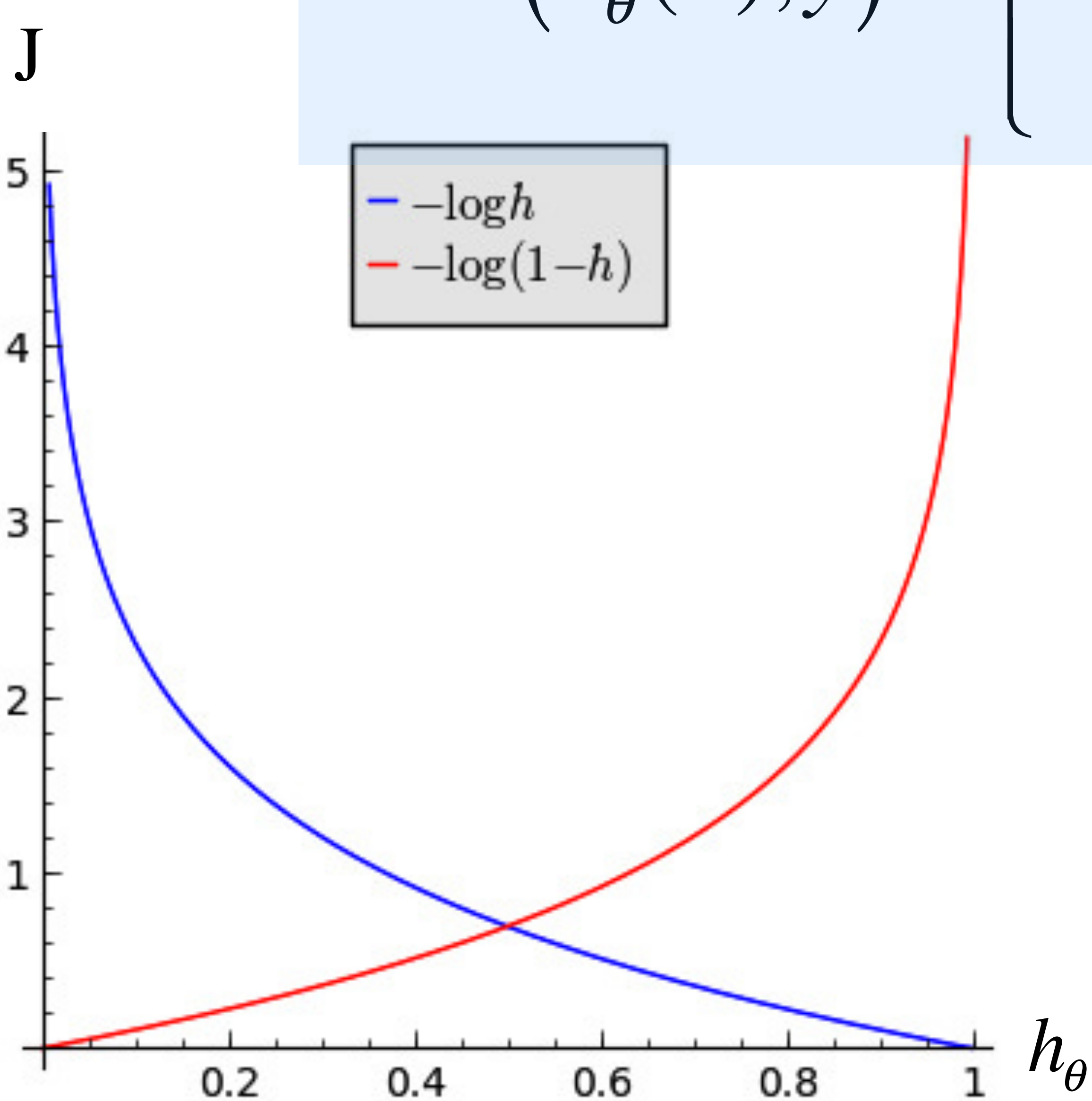


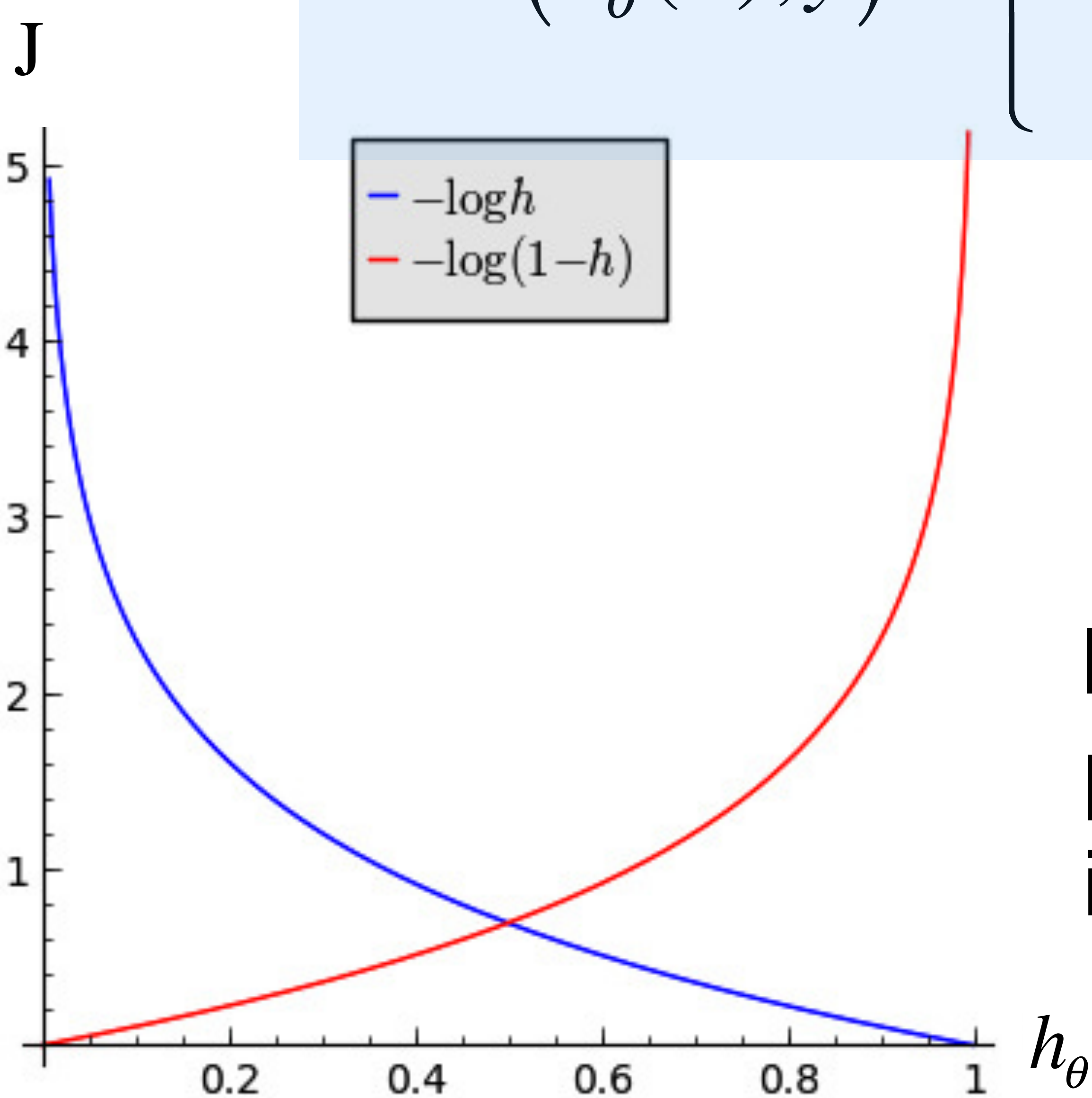
Cost = 0 if ***y = 1***, ***h_θ(x) = 1***

But as ***$h_\theta(x) \to 0$*** , ***Cost $\to \infty$***

Logistic Regression:

Cost Function

$$\mathrm{Cost}\big(h_\theta(x),y\big)=\begin{cases}-\log(h_\theta(x)) & \text{if } y=1\\ -\log(1-h_\theta(x)) & \text{if } y=0\end{cases}$$



Cost = 0 if ***y = 1***, ***h_θ(x) = 1***

But as ***h_θ(x) → 0***, ***Cost → ∞***

Intuition:
penalize learning algorithm
if ***h_θ(x) = 0*** (predict **P(y=1|x;θ) = 0**),
but ***y = 1***

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$\mathrm{Cost}\left(h_\theta(x), y\right) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

remember that $y = 0$ or 1 always

Logistic Regression:

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

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remember that $y = 0$ or 1 always

the same

cross entropy loss:

$$\mathrm{Cost}\left(h_\theta(x), y\right) = -y\log(h_\theta(x)) - \left(1 - y\right)\log(1 - h_\theta(x))$$

Learn more: https://en.wikipedia.org/wiki/Cross_entropy

Logistic Regression

- **Cost Function:**

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}\left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$= -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + \left(1 - y^{(i)}\right)\log(1 - h_\theta(x^{(i)}))$$

- **Goal:**

learn parameters θ to $\underset{\theta}{\mathrm{minimize}}\, J(\theta)$

- **Hypothesis (to make a prediction):** $h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$

P(y=1|x;θ)

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

simultaneous update for all θ_j

learning rate

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

simultaneous update for all θ_j

}

learning rate

 # training examples

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

Logistic Regression:

Gradient Descent

simultaneous update for all θ_j

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

This look the same as linear regression!!???

Logistic Regression:

Gradient Descent

repeat until convergence {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

}

simultaneous update for all θ_j

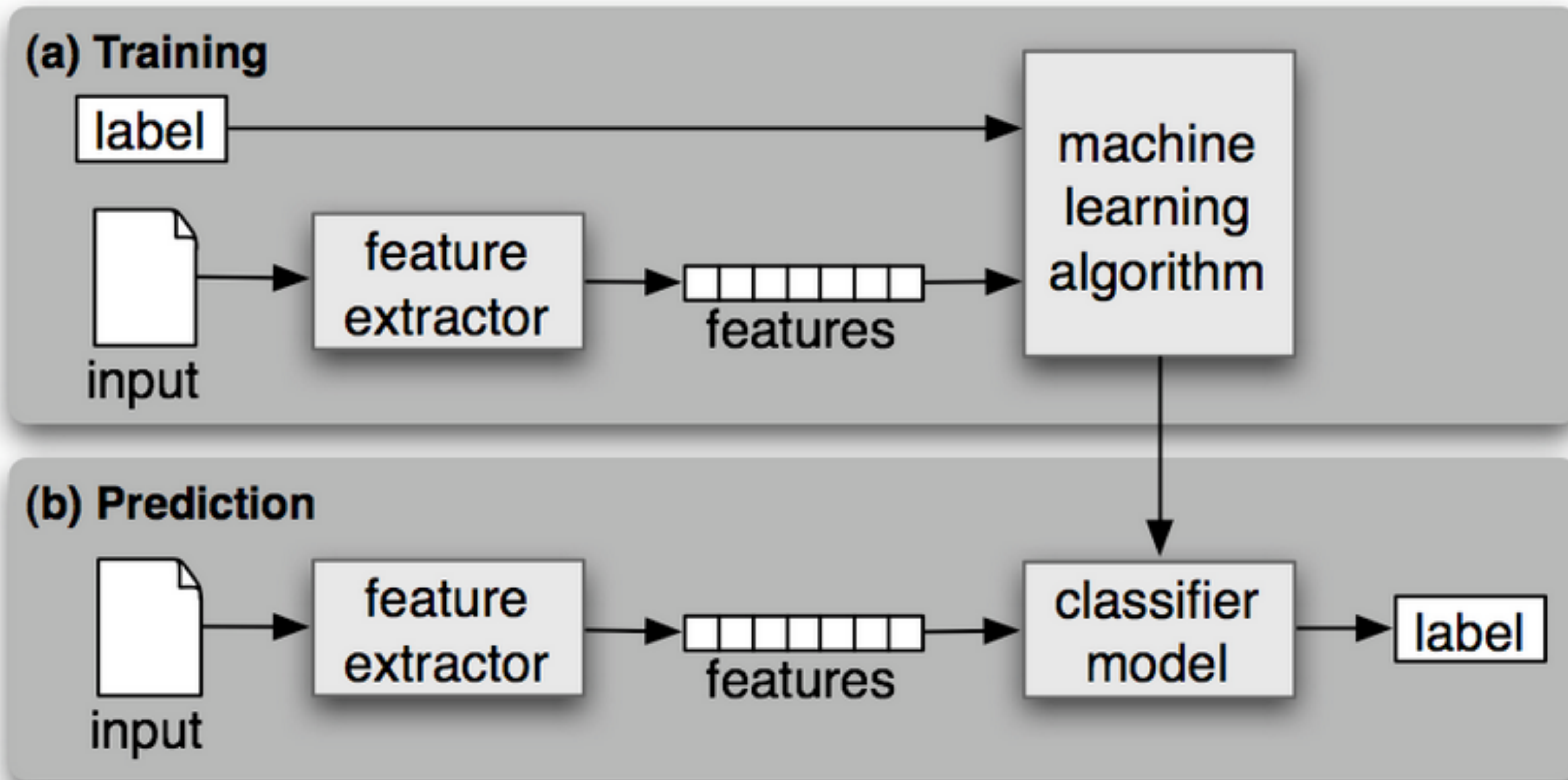
$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$h_\theta(x^{(i)}) = \frac{1}{1+e^{-\theta^T x}}$$

using different hypothesis from linear regression

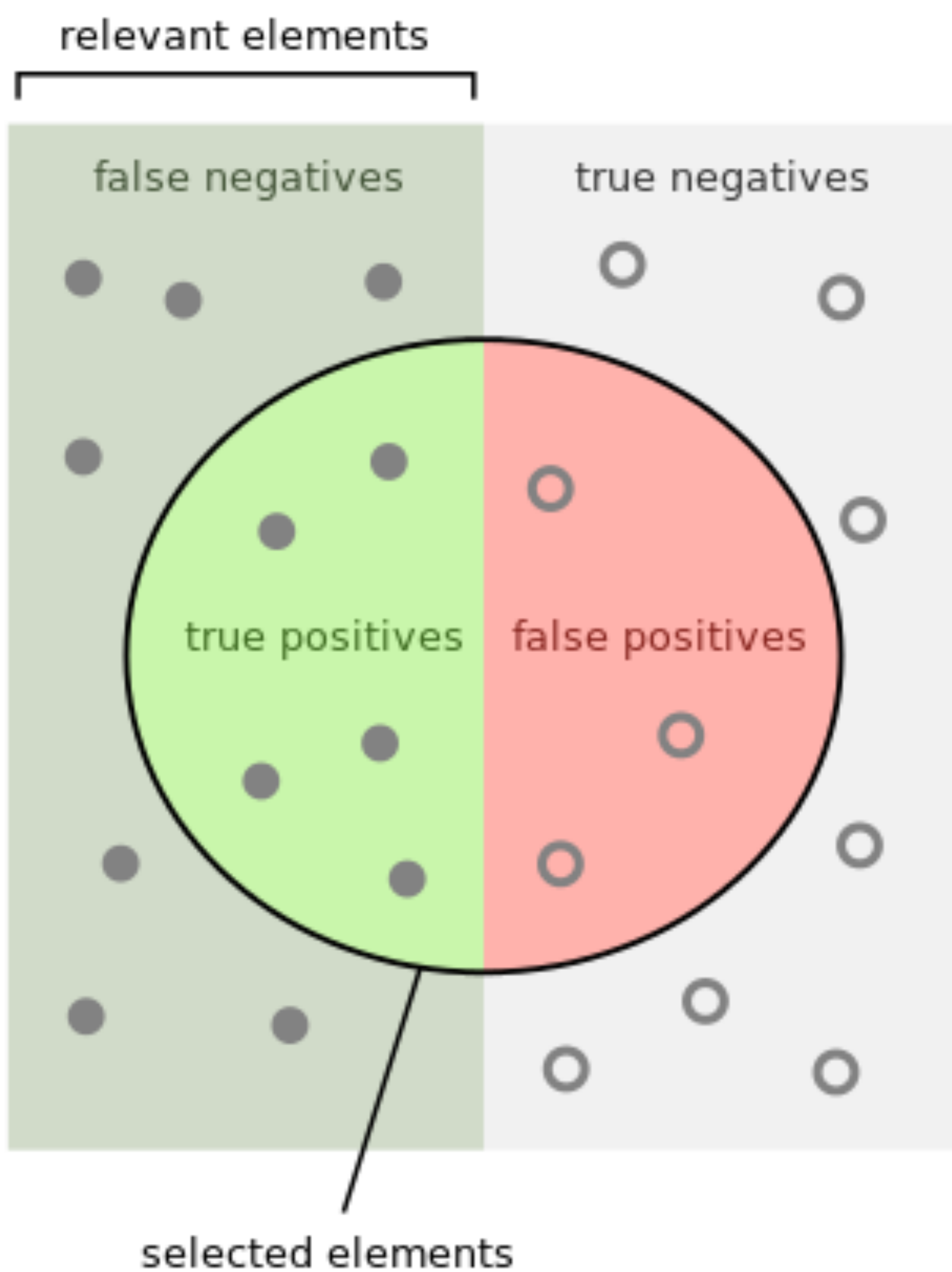
[Recap] Classification Method:

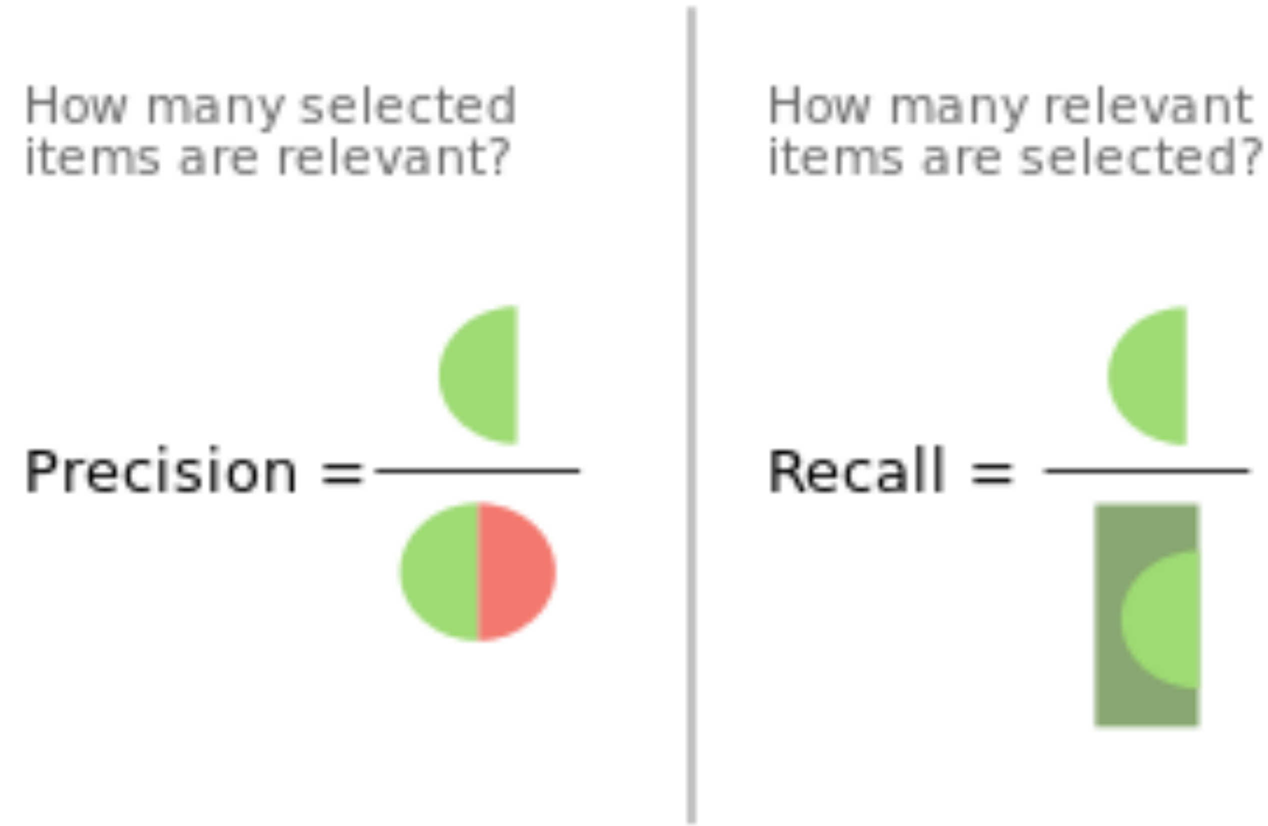
Supervised Machine Learning



Source: NLTK Book

Classification Evaluation



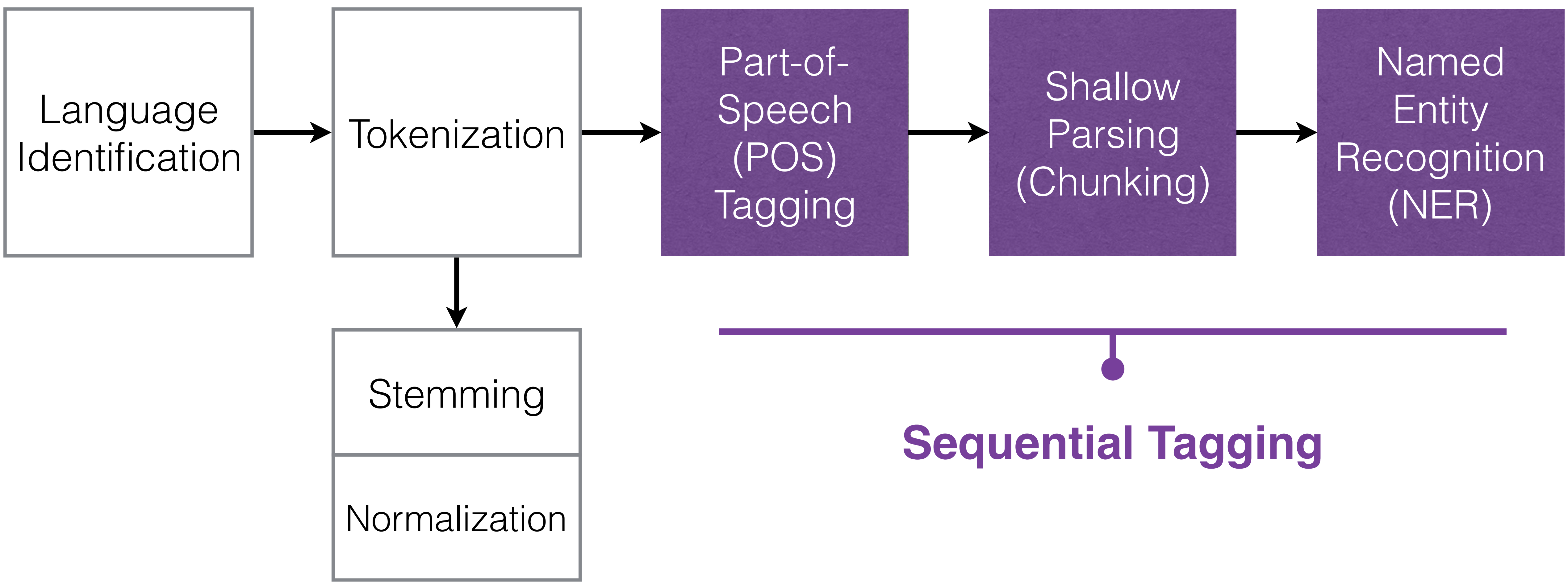


F-measure:

$$F_1 = \frac{2 \cdot precision \cdot recall}{precision + recall}$$

Source: Wikipedia

NLP Pipeline (next)



Part-of-Speech (POS) Tagging

Cant	MD
wait	VB
for	IN
the	DT
ravens	NNP
game	NN
tomorrow	NN
…	:
go	VB
ray	NNP
rice	NNP
!!!!!!!	.

Chunking

Cant	VP
wait	
for	PP
the	NP
ravens	
game	
tomorrow	NP
…	
go	VP
ray	NP
rice	
!!!!!!!	

Named Entity Recognition(NER)

Cant	
wait	
for	
the	
ravens	ORG
game	
tomorrow	
…	
go	
ray	PER
rice	PER
!!!!!!!	.

ORG: organization

PER: person

LOC: location

IO tag encoding

Cant	VP	VP
wait		VP
for	PP	PP
the		NP
ravens	NP	NP
game		NP
tomorrow	NP	NP
...		O
go	VP	VP
ray	NP	NP
rice		NP
!!!!!!!		O

IO tag encoding

Cant	VP	VP	B-VP
wait		VP	I-VP
for	PP	PP	B-PP
the	NP	NP	B-NP
ravens		NP	I-NP
game		NP	I-NP
tomorrow	NP	NP	B-NP
…		O	O
go	VP	VP	B-VP
ray	NP	NP	B-VP
rice		NP	I-VP
!!!!!!!		O	O

I: Inside

O: outside

B: Begin

BIO allows separation of adjacent chunks/entities

Classification Method:

Supervised Machine Learning

- Naïve Bayes
- Logistic Regression
- Support Vector Machines (SVM)
- …
- Hidden Markov Model (HMM)
- Conditional Random Fields (CRF)
- …

sequential models

Classification Method:

Sequential Supervised Learning

- Input:
    - rather than just individual examples ***(w_1 = the, c_1 = DT)***
    - a training set consists of ***m*** sequences of labeled examples ***$(x_1, y_1), \ldots, (x_m, y_m)$***

      x_1 = <the back door> and y_1= <DT JJ NN>

- Output:
    - a learned classifier to predict label sequences ***$\gamma: x \rightarrow y$***

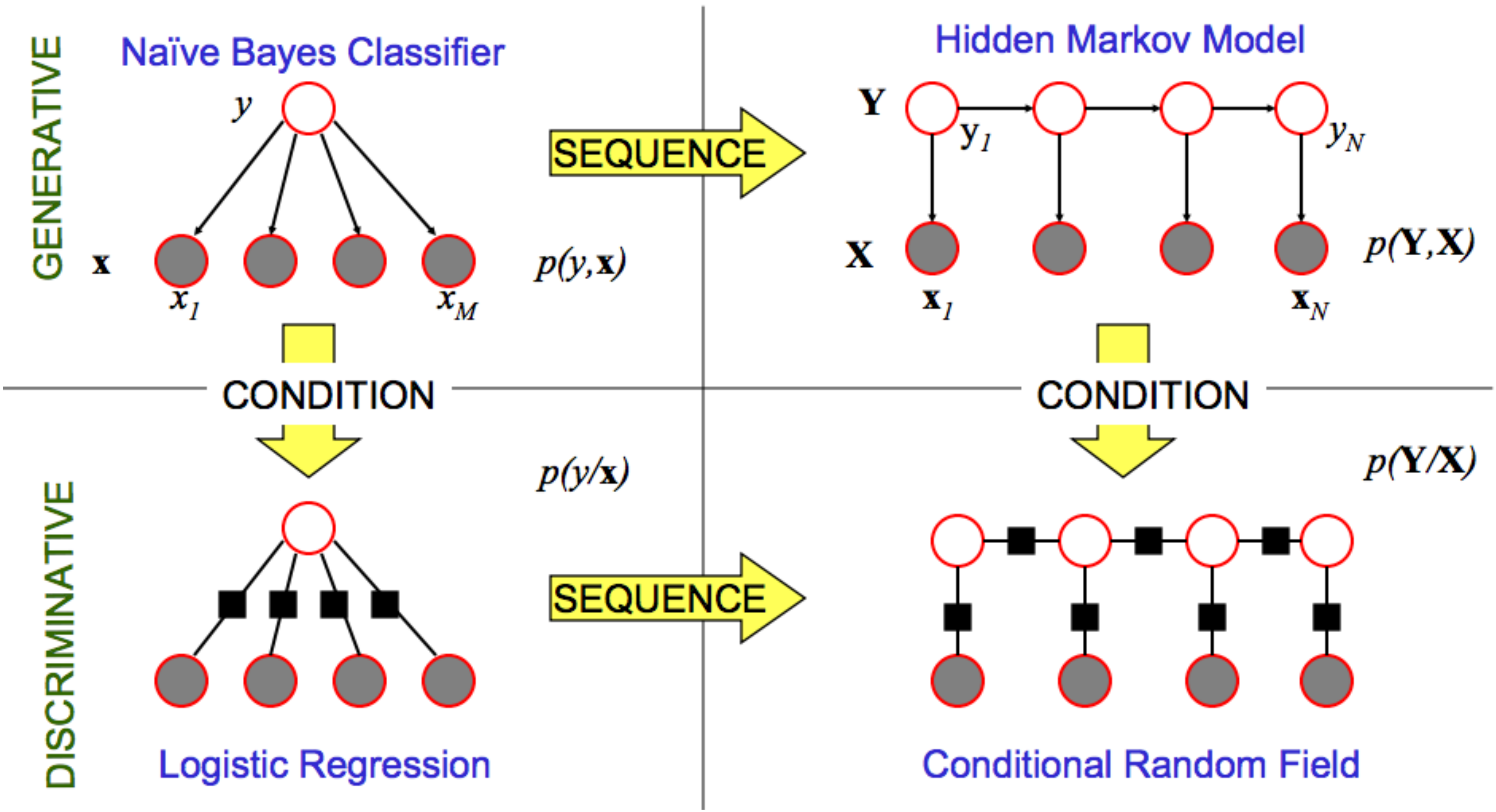
Features for Sequential Tagging

- Words:
  - current words
  - previous/next word(s) — context
- Other linguistic information:
  - word substrings
  - word shapes
  - POS tags
- Contextual Labels
  - previous (and perhaps next) labels

word shapes

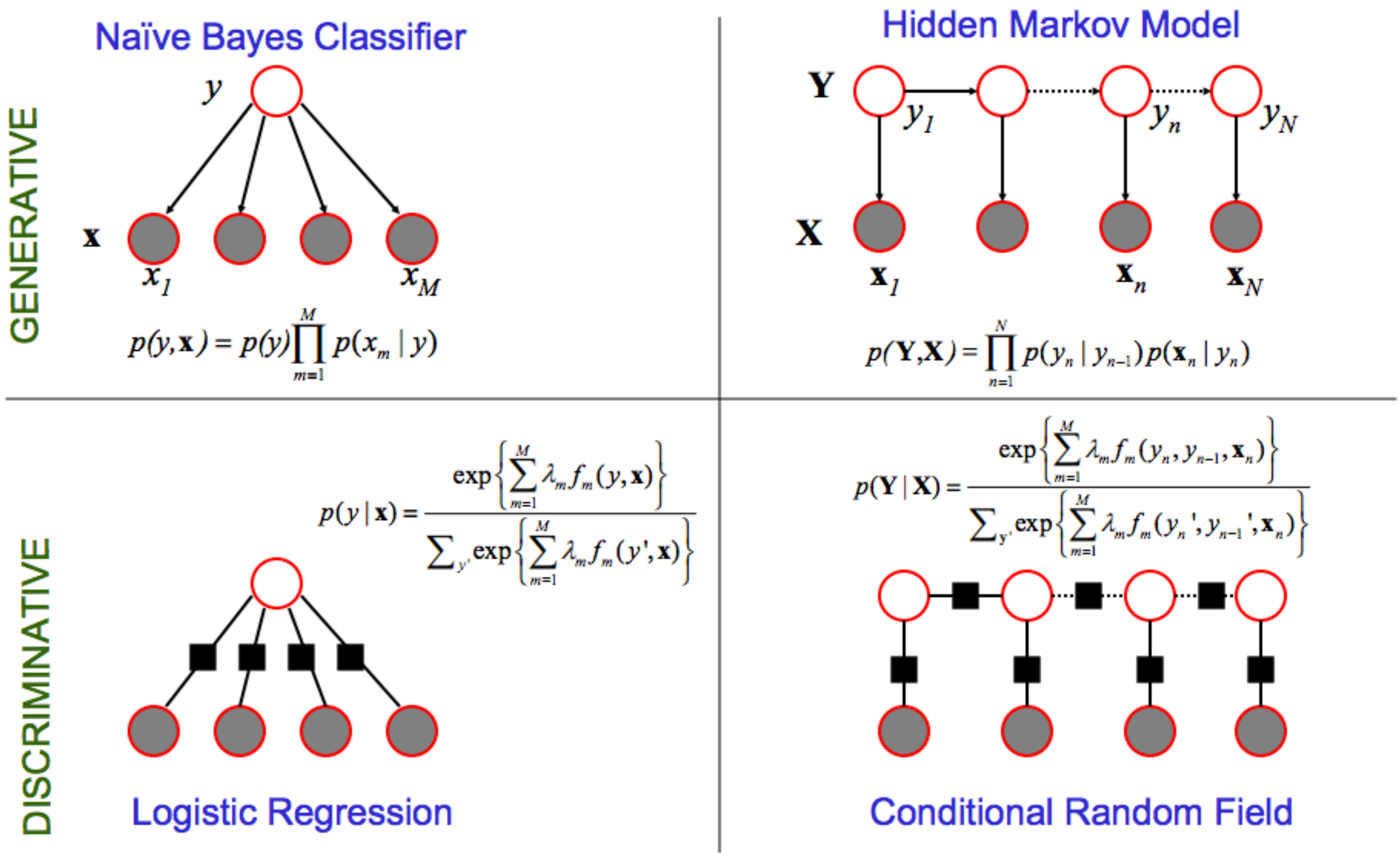
Varicella-zoster	Xx-xxx
mRNA	xXXX
CPA1	XXXd

Probabilistic Graphical Models



Source: Andrew McCallum, Sargur Srihari

Probabilistic Graphical Models



Source: Andrew McCallum, Sargur Srihari

Probabilistic Graphical Models