Encoding Defensive Driving as a Dynamic Nash Game

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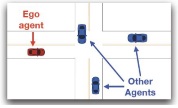
Introduction

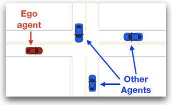
- **Interaction as a Differential Game:**
  - **Motivation**—Reliable decision making for autonomous navigation requires efficient coordination between prediction + planning modules
  - **Idea**—Construct an *N-player differential game* for the prediction + planning module
- **Notions of Robustness:**
  - **Hamilton-Jacobi-Isaacs PDE**—solution provides a Nash equilibrium in an adversarial reachability formulation
  - **Our setup**—Adversarial vs. Cooperative time intervals.



Adversarial Time Horizon

- **Setting:**
  - An ego agent attempts to navigate through an environment in which an arbitrary number of other agents are moving.
- **Defensive Driving:**

  - Key idea—Consider a time-varying cost $g(t, x, u_{1,N})$.
  - Adversarial phase: $t \in [0, T_{\text{adv}}]$—Ego agent anticipates that other agents will behave adversarially, e.g. will intend to induce collisions
  - Cooperative phase: $t \in [T_{\text{adv}}, T]$—Ego agent anticipates that other agents will behave cooperatively ("normally"), e.g. will intend to avoid collisions



Adversarial Time Horizon

- **Cost:**
  - Encoding defensive driving into the cost:

$$\tilde{J}_i := \int_0^{T_{\text{adv}}} g_{\text{adv},i}(x, u_{1:N})dt + \int_{T_{\text{adv}}}^{T} g_{\text{coop},i}(x, u_{1:N})dt\,.$$

  - Adversarial phase cost (Player i)—$g_{\text{adv},i}(x, u_{1,N})$
  - Normal phase cost for (Player i)—$g_{\text{coop},i}(x, u_{1,N})$
- **Constraints:**
  - Barrier function "cost" terms

Examples

- **Dynamics:** **(each agent)**
  - **Augmented Bicycle Dynamics:**

$$\begin{aligned} \dot{p}_{x,i} &= v_i \sin\theta_i, & \dot{v}_i &= a_i,\\ \dot{p}_{y,i} &= v_i \cos\theta_i, & \dot{\phi}_i &= \omega_i,\\ \dot{\theta}_i &= (v_i/L_i)\tan\phi_i, & \dot{a}_i &= j_i, \end{aligned}$$

  - State: $x := (p_{x,i}, p_{y,i}, \theta_i, v_i, \phi_i, a_i)_{i=1}^{N} \in \mathbb{R}^{6N}$

    x-position
    y-position
    Heading
    Linear velocity
    Angle (front wheel)
    Linear acceleration

  - Control (Player i): $u_i := (\omega_i, j_i)$

    Angular speed (front wheel)
    Linear jerk (front wheel)

  - Inter-axle distance (Player i): L_i

Examples

- **Costs:** **(each agent)**
  - **$g_{\text{adv},i}$ and $g_{\text{coop},i}$ are weighted sums of the following costs**

$$\begin{aligned} \text{Input:}\quad & u_i^T R_{ii} u_i\\ \text{Lane center:}\quad & \min_{p_\ell \in \ell_i} \|p_\ell - p_i\|\\ \text{Ideal speed:}\quad & (v_i - v_{\text{ref},i})^2\\ \text{Cooperative:}\quad & \mathbf{1}\{\|p_i - p_j\| < d_{\text{prox}}\} \cdot (d_{\text{prox}} - \|p_i - p_j\|)^2\\ \text{Adversarial:}\quad & \|p_i - p_j\|^2. \end{aligned}$$

  - Position (shorthand): $p_i := (p_{x,i}, p_{y,i})$
  - Lane centerline: ℓ_i
  - Reference velocity (Player i): $v_{\text{ref},i}$
  - Proximity Distance: d_{prox}
  - Note— Adversarial costs are present only during $[0, T_{\text{adv}}]$, while cooperative costs are present only during $[T_{\text{adv}}, T]$

Examples

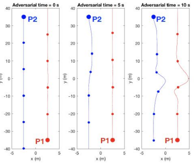
- **Constraint:** **(each agent)**
  - **Ego agent—Must also obey the following hard constraints:**
    (These constraints are encoded via barrier functions)

$$\begin{aligned} \text{Proximity :}\quad & \|p_i - p_j\| > d_{\text{prox}},\\ \text{lane :}\quad & \min_{p_\ell \in \ell_i} \|p_\ell - p_i\| < d_{\text{lane}},\\ \text{Speed range :}\quad & \underline{v}_i < v_i < \overline{v}_i. \end{aligned}$$

  - Lane half-width — vehicle half-width: d_{lane}
  - Speed limits: $(\underline{v}_i, \overline{v}_i)$
- Time Horizon—$T = 15\ s$
- Time Discretization—0.1 s

Examples

- **Example: Oncoming Vehicle**
- **Figure conventions:**
  - P1 - Ego trajectory
  - P2 - Other vehicle's trajectory
  - Small dots—3 s snapshots
  - Large dots—Initial positions
- **Key observations:**
  - In the ego's imagination, when $T_{\text{adv}} > 0$, the other vehicle swerve towards the ego vehicle
  - Ego vehicle swerves away to avoid the other vehicle



Examples

- **Example: Merging**
- **Figure conventions:**
  - P1—Ego trajectory
  - P2-P6—Adversaries' trajectories
  - Small dots—3 s snapshots
  - Large dots—Initial positions
- **Key observations:**
  - In the ego's imagination, when $T_{\text{adv}} > 0$, the vehicles in the adjacent lane (P5, P6) and on the ramp (P2, P4) initially swerve towards ego vehicle
  - The ego vehicle swerves away to avoid these other vehicle