WAVE EQUATION ANALYSIS (SMITH 1-D MODEL)

Project:	Sample Project	**Number:**	WE-2026-011
Prepared By:	S. OConnell, PE	**Date:**	2026-02-17
Company:	Geotech Associates		

1. INPUT PARAMETERS

Parameter	Symbol	Value	Unit
Analysis type	$Analysis$	`Bearing Graph`	
Hammer model	$Hammer$	`Delmag D30-32`	
Ram weight	W_{ram}	29.4	kN
Stroke	h	2.800	m
Rated energy	E_{rated}	82.3	kN-m
Hammer efficiency	η	0.80	
Impact velocity	v_{impact}	6.63	m/s
Hammer type	$Type$	`Diesel`	
Cushion stiffness	$k_{cushion}$	3,990,000,000	kN/m
Coefficient of restitution	COR	0.80	
Cushion thickness	$t_{cushion}$	75	mm
Helmet weight	W_{helmet}	5.0	kN
Pile length	L_{pile}	20.00	m
Pile area	A_{pile}	0.010000	m^2
Pile elastic modulus	E_{pile}	200,000,000,000	kPa
Number of segments	N_{seg}	20	
Pile impedance	Z	12650.7	kN-s/m

1. SMITH WAVE EQUATION MODEL

Smith 1-D Wave Equation Model

$Thepileismodeledasaseriesoflumpedmassesconnectedbylinearelasticsprings.Soilresistanceateachsegmentisanelast$

$$Model = \textbf{Smith(1960)—explicittimeintegration}$$

Smith (1960); FHWA GEC-12 Section 12.4

Ram Impact Velocity

$$v = \sqrt{2 \times E_{rated} \times \eta / m_{ram}}$$

$$v = \sqrt{2 \times 82.3 \times 0.80 / 2996.9}$$

$v_{impact} = \mathbf{6.629}$ m/s

FHWA GEC-12 Eq. 12-1

Stress Wave Speed

$$c = \sqrt{E/\rho}$$

$c = \mathbf{158,094}$ m/s

Smith (1960)

Courant Stability Condition

$$\Delta t \leq \Delta L / c (applied with safety factor 0.8)$$

$$\Delta t \leq 1.000 / 158,094$$

$\Delta t_{Courant} = \mathbf{0.0063}$ ms

WEAP87 Manual, Chapter 5

1. SOIL RESISTANCE MODEL

Static Soil Resistance (Elasto-Plastic)

$$R_{static} = (R_{ult} / Q) \times d, for d \leq Q (elastic) R_{static} = R_{ult}, for d > Q (plastic)$$

$$Q = quake (elastic limit displacement)$$

$Model = \mathbf{Smith elasto - plastic spring}$

Smith (1960); FHWA GEC-12 Table 12-3

Dynamic Soil Resistance (Smith Damping)

$$R_{total} = R_{static} + J \times R_{ultimate} \times v$$

$$J = Smithdampingfactor(s/m), v = pilesegmentvelocity$$

$Model = \textbf{Velocity} - \textbf{dependentviscousdamping}(\textbf{GRLWEAP}/\textbf{GEC} - \textbf{12})$

Smith (1960); FHWA GEC-12, Table 12-3

1. BEARING GRAPH RESULTS

Bearing Graph

$$ForeachassumedR_{ult,}simulateasingleblowandcompute : Set = permanentpilepenetrationperblowBlowcount = 1/$$

$N_{analyses} = \mathbf{10}$

FHWA GEC-12 Section 12.5

Bearing Graph Data

R_ult (kN)	Set (mm)	Blows/m	Max Comp. (kPa)	Max Tens. (kPa)
200	91.29	11	30,072,885	13,447,222
400	49.87	20	30,072,962	13,307,909
600	30.11	33	30,073,038	13,169,849
800	28.63	35	30,073,115	13,033,029
1,000	19.47	51	30,073,192	12,897,431
1,200	19.09	52	30,073,269	12,762,837
1,400	14.77	68	30,073,346	12,629,339
1,600	12.09	83	30,073,423	12,497,017
1,800	11.44	87	30,073,500	12,365,852
2,000	9.76	102	30,073,577	12,235,846

1. DRIVING STRESS SUMMARY

Maximum Compressive Driving Stress

$$MaxcompressivestressoverallR_{ult}values$$

$\sigma_{comp,\max} = \mathbf{30,073,577}$ kPa

= 30,073.6 MPa

Maximum Tensile Driving Stress

$$Max tensile stress over all R_{ult} values$$

$\sigma_{tens,\max} = 13,447,222$ kPa

= 13,447.2 MPa

Compressive Stress Ratio

$$\sigma_{comp} / E_{pile} \times 100(strain,$$

$$30,073,577 / 200,000,000,000 \times 100$$

$\varepsilon_{comp} = 0.0150$ %

For steel piles (fy = 250 MPa = 250,000 kPa): stress ratio = 12029.4% of fy. FHWA limit: 0.90 × fy.

1. REPRESENTATIVE BLOW DETAIL

Single Blow Detail (R_ult = 1,200 kN)

$$Explicit central - difference time integration$$

$n_{steps} = 9341$

Permanent set = 19.09 mm

Blow Simulation Summary

Quantity	Value	Unit
Ultimate resistance	1,200	kN
Permanent set	19.09	mm
Max compression stress	30,073,269	kPa
Max tension stress	12,762,837	kPa
Max compressive force	300,733	kN
Time steps computed	9341	

1. FIGURES

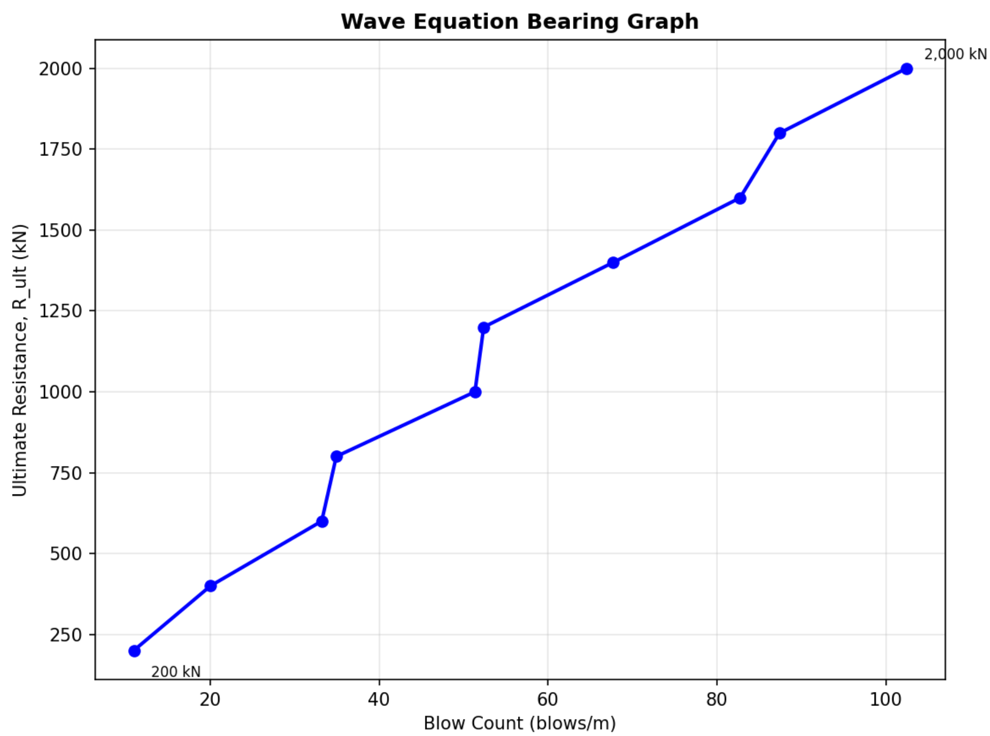

Figure 1: *
Figure 1: Wave equation bearing graph — ultimate resistance vs blow count. Range: 200 to 2,000 kN.

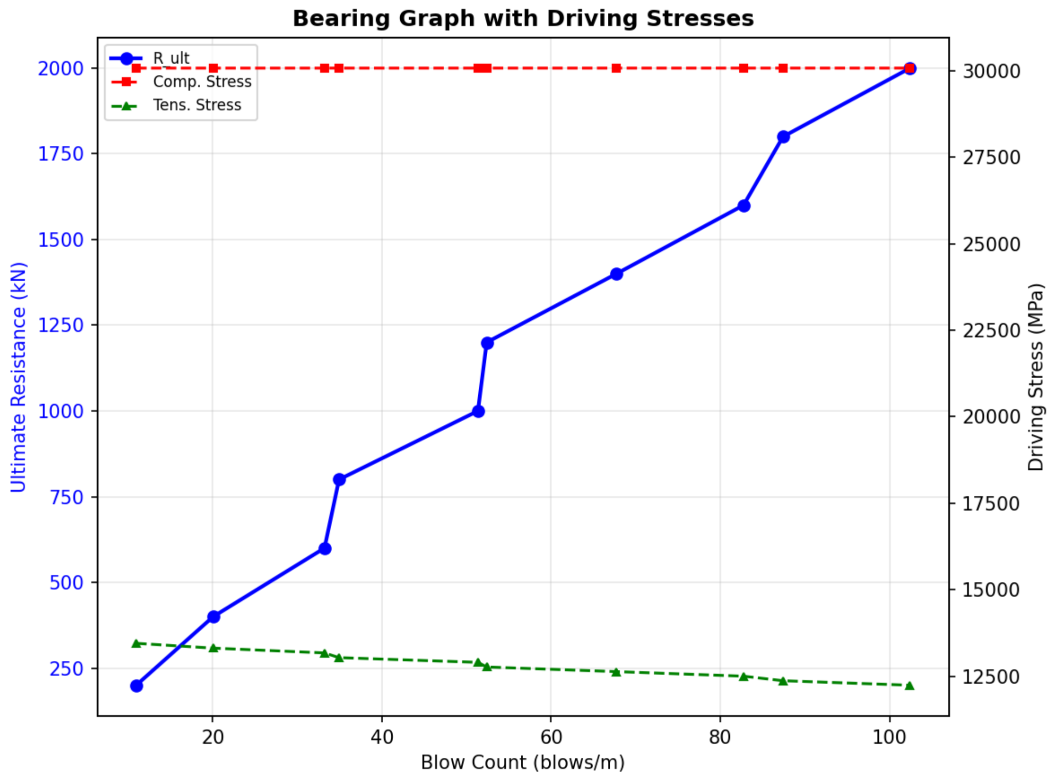

Figure 2: *
Figure 2: Bearing graph with overlaid maximum compressive and tensile driving stresses.

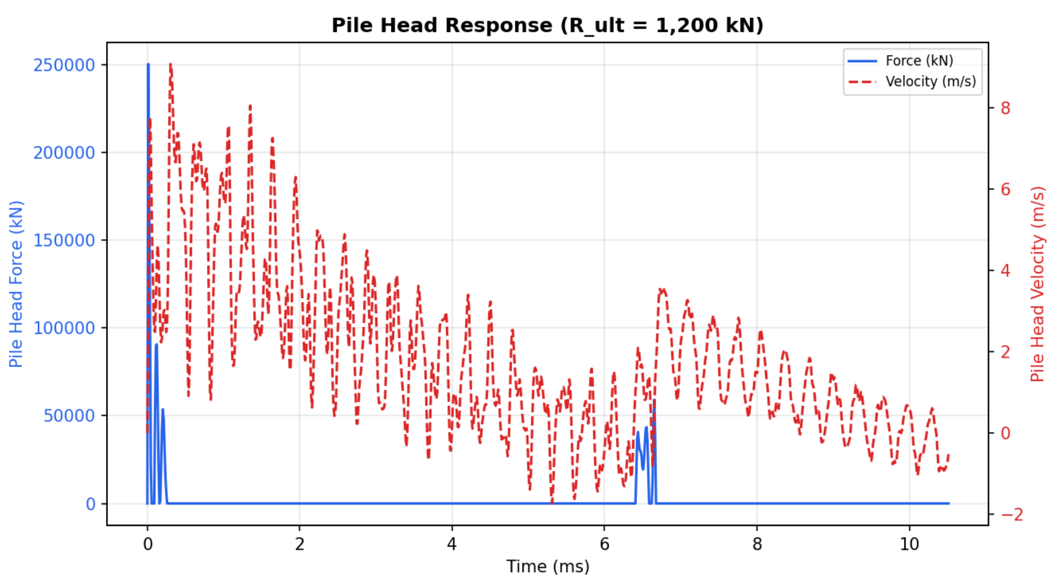

Figure 3: *
Figure 3: Force and velocity time histories at the pile head for R_ult = 1,200 kN. Permanent set = 19.09 mm.

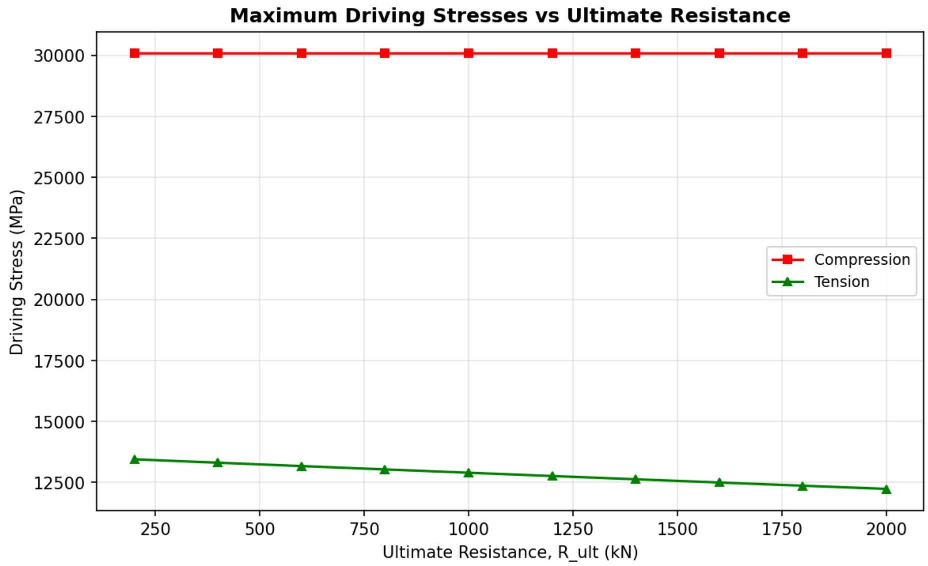

Figure 4: *
Figure 4: Maximum compressive and tensile driving stresses as a function of ultimate resistance.

1. REFERENCES

1. Smith, E.A.L. (1960). "Bearing Capacity of Piles Determined by Means of a Pile Driving Analysis by the Wave Equation." ASCE, Vol. 86, No. EM4.
2. Goble, G.G. & Rausche, F. (1976). "Wave Equation Analysis of Pile Driving — WEAP Program." FHWA IP-76-14.
3. FHWA GEC-12 (FHWA-NHI-16-009): Design and Construction of Driven Pile Foundations, Chapter 12 — Wave Equation Analysis.
4. WEAP87 Manual (FHWA, Goble & Rausche) — Hammer, Cushion, and Soil Models.