2010-2011微2答案

1, $x=-\frac{y}{2}=-z$;

2, $8x-4y-4z-5=0$;

3, $\frac{1}{2}\sqrt{35}$;

4, $x^2+y^2-2x+2y-7=0$;

5, z;

6, $dz|_{(1,1)}=(1+2\ln 2)(dx-dy)$;

7, $(0,0)$处取得极大值0；

8, $\frac{1}{3}(\sqrt{2}-1)$;

9, π;

10, $\frac{\partial^2 z}{\partial u\partial v}=0$;

11, $\frac{11}{40}$;

12, $\frac{\sqrt{2}}{8}$;

13, $\frac{3}{2}(e-1)$;

14, (I)不存在；(II) $\sqrt{\cos^2\alpha+2\sin^2\alpha}$;

2012－2013春季学期《微积分（II）》A卷解答

1. $[(\vec{a}+\vec{b})\times(\vec{b}-\vec{c})]\cdot(\vec{c}-\vec{a})=(\vec{a}\times\vec{b})\cdot\vec{c}+(\vec{b}\times\vec{c})\cdot\vec{a}$

$=(\vec{a}\times\vec{b})\cdot\vec{c}+(\vec{a}\times\vec{b})\cdot\vec{c}=3+3=6.$

2. L_1的方向向量 $\vec{\tau}_1=\{-3,1,10\}$，L_2的方向向量 $\vec{\tau}_2=\{4,-1,2\}$

L的方向向量可取 $\vec{\tau}=\vec{\tau}_1\times\vec{\tau}_2=\{12,46,-1\}$，所以$L$

的方程为 $\dfrac{x+1}{12}=\dfrac{y+4}{46}=\dfrac{z-3}{-1}$.

还有其它方法可做，略.

3. $x^2+y^2=(1-t)^2+(2+t)^2=(1-\frac{z}{2})^2+(2+\frac{z}{2})^2$

$=5+z+\dfrac{z^2}{2}$.

化为 $x^2+y^2=\frac{1}{2}(z^2+2z+1)+\frac{9}{2}$

$=\frac{1}{2}(z+1)^2+\frac{9}{2}$，

即 $x^2+y^2-\frac{1}{2}(z+1)^2=\frac{9}{2}$，为单叶双曲面.

4. 经过直线 $L:\begin{cases}x-z+4=0\\x+5y+z=0\end{cases}$ 的平面方程（除 $x+5y+z=0$ 外）

可写成

$$x-z+4+\lambda(x+5y+z)=0,$$

即 $(1+\lambda)x+5\lambda y+(\lambda-1)z+4=0$.

它的法向量 $\vec{n}_1=\{(1+\lambda),5\lambda,(\lambda-1)\}$，而平面$P$的

法向量

$$\vec{n}_2=\{1,-4,-8\}.$$

$$\cos\frac{\pi}{4}=\left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|=\frac{|(1+\lambda)-20\lambda-8(\lambda-1)|}{\sqrt{(1+\lambda)^2+25\lambda^2+(\lambda-1)^2}\sqrt{1+16+64}},$$

即 $3\lambda^2+4\lambda=0$,

解得 $\lambda=0$, $\lambda=-\frac{4}{3}$.

得两个方程.

$x-z+4=0$

及 $x+20y+7z-12=0$.

又既然已得到两个解, 所以"除外"的那个方程不会是解. 解毕.

5. $f_x'(1,2)=f_x'(x,2)\big|_{x=1}=(4x^2+\arctan 2(x-1))'_{x=1}$

$=\left[-8x+\frac{2}{1+4(x-1)^2}\right]_{x=1}=\cancel{-4}$ -2

$f_y'(1,2)=f_y'(1,y)\big|_{y=2}=((y-4)^2)'_{y=2}=-4$.

6. $\left\{\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right\}_p=\{6x,4y,2z\}_p=\{6,4,2\}$.

S的外法向量 $\vec{n}=\{2x,4y,6z\}_p=\{2,4,6\}$,

$\vec{n}^0=\frac{1}{2\sqrt{14}}\{2,4,6\}=\frac{1}{\sqrt{14}}\{1,2,3\}$

$\frac{\partial u}{\partial n}\Big|_p=\frac{1}{\sqrt{14}}\{6,4,2\}\cdot\{2,4,6\}=\cancel{\frac{40}{\sqrt{14}}}$ $\frac{20}{\sqrt{14}}$

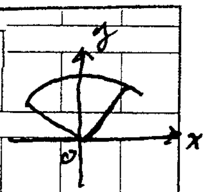
7. 由 $F(\frac{y}{x},\frac{z}{x})=0$,

$\frac{\partial z}{\partial x}=-\frac{F_x'}{F_z'}=-\frac{F_1'\cdot(-\frac{y}{x^2})+F_2'(-\frac{z}{x^2})}{F_1'\cdot 0+F_2'(\frac{1}{x})}=(\frac{y}{x}F_1'+\frac{z}{x}F_2')/F_2'$,

$\frac{\partial z}{\partial y}=-\frac{F_y'}{F_z'}=-\frac{F_1'(\frac{1}{x})}{F_1'\cdot 0+F_2'(\frac{1}{x})}=-\frac{F_1'}{F_2'}$.

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{yF_1'+zF_2'-yF_1'}{F_2'}=z$.

8. $D=\{(x,y)\mid |x|\le y\le\sqrt{2-x^2},\ -1\le x\le 1\}$，如图，



D 关于 y 轴对称，

$$\int_{-1}^{1}dx\int_{|x|}^{\sqrt{2-x^2}}(xy+1)\sin(x^2+y^2)\,dy$$

$$=\iint_D (xy+1)\sin(x^2+y^2)\,d\sigma=\iint_D \sin(x^2+y^2)\,d\sigma$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\theta\int_0^{\sqrt{2}} r\sin r^2\,dr=\frac{\pi}{4}(-\cos r^2)\Big|_0^{\sqrt{2}}=\frac{\pi}{4}(1-\cos 2).$$

9. $\frac{\partial u}{\partial x}=f'(r)\frac{\partial r}{\partial x}=\frac{x}{r}f'(r)$，$\frac{\partial^2 u}{\partial x^2}=\frac{1}{r}f'(r)-\frac{x^2}{r^3}f'(r)+\frac{x^2}{r^2}f''(r)$.

同理，$\frac{\partial^2 u}{\partial y^2}=\frac{1}{r}f'(r)-\frac{y^2}{r^3}f'(r)+\frac{y^2}{r^2}f''(r)$，

$\frac{\partial^2 u}{\partial z^2}=\frac{1}{r}f'(r)-\frac{z^2}{r^3}f'(r)+\frac{z^2}{r^2}f''(r)$，

$\therefore\ \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{3}{r}f'(r)-\frac{1}{r}f'(r)+f''(r)$

$=\frac{2}{r}f'(r)+f''(r)$.

10. $\frac{d}{dx}\varphi^3(x)=3\varphi^2(x)\varphi'(x)$，

$\varphi'(x)=f_1'(x,f(x,2x))+f_2'(x,f(x,2x))\frac{d}{dx}f(x,2x)$

$=f_1'(x,f(x,2x))+f_2'(x,f(x,2x))\big(f_1'(x,2x)+f_2'(x,2x)2\big)$.

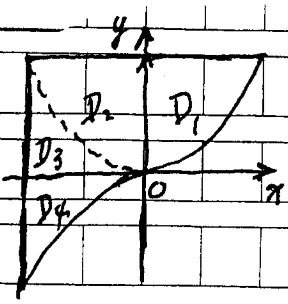
$\varphi'(1)=f_1'(1,f(1,2))+f_2'(1,f(1,2))\big(f_1'(1,2)+f_2'(1,2)2\big)$

$=f_1'(1,2)+f_2'(1,2)\big(f_1'(1,2)+2f_2'(1,2)\big)$

$=3+4(3+8)=47$.

线

11. 方法一. 添虚线如图. 将D分成4块:



D_1, D_2, D_3, D_4

$$\iint_D (y^2+(xy)^{2013})dxdy$$

$$=\iint_{D_1}+\iint_{D_2}+\iint_{D_3}+\iint_{D_4}(y^2+(xy)^{2013})dxdy$$

由被积函数的奇偶性及积分区域的对称性,有

$$\iint_{D_1}+\iint_{D_2}(y^2+(xy)^{2013})dxdy=2\iint_{D_2}y^2dxdy,$$

$$\iint_{D_3}+\iint_{D_4}(y^2+(xy)^{2013})dxdy=2\iint_{D_3}y^2dxdy,$$

$$\therefore \iint_D (y^2+(xy)^{2013})dxdy=2\int_{-1}^{0}dx\int_0^1 y^2dy=\frac{2}{3}.$$

方法二. $\iint_D (y^2+(xy)^{2013})dxdy$

$$=\int_{-1}^{1}dx\int_{x^3}^{1}y^2dy+\int_{-1}^{1}x^{2013}dx\int_{x^3}^{1}y^{2013}dy$$

$$=\int_{-1}^{1}\left(\frac{1}{3}-\frac{1}{3}x^9\right)dx+\int_{-1}^{1}\left(\frac{x^{2013}}{2014}-\frac{x^{8055}}{2014}\right)dx=\frac{2}{3}.$$

12. $V=\iint_D y\sqrt{1+2x+y^2}\,dxdy=\int_0^4dx\int_0^x y\sqrt{1+2x+y^2}\,dy$

$$=\int_0^4\frac{1}{3}(1+2x+y^2)^{\frac{3}{2}}\Big|_{y=0}^{y=x}dx$$

$$=\frac{1}{3}\int_0^4[(1+x)^3-(1+2x)^{\frac{3}{2}}]dx$$

$$=\frac{1}{12}(1+x)^4\Big|_0^4-\frac{1}{15}(1+2x)^{\frac{5}{2}}\Big|_0^4=\frac{1221}{5}.$$

$\frac{538}{15}$

$20\times20=400$

第　　页

13. 设该长方体在第一卦限中且在抛物面上的一个顶点坐标为(x_0, y_0, z_0)，于是

$$V = 4x_0y_0(c - z_0),$$

且 $$\frac{z_0}{c} - \left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}\right) = 0.$$

方法一. 去掉下角0，考虑

$$F(x,y,z,\lambda) = 4xy(c-z) + \lambda\left(\frac{z}{c} - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right),$$

由拉格朗日乘数法，令

$$\frac{\partial F}{\partial x} = 0,\ \frac{\partial F}{\partial y} = 0,\ \frac{\partial F}{\partial z} = 0,\ \frac{\partial F}{\partial \lambda} = 0.$$

即

$$\begin{cases} 4y(c-z) - \frac{2\lambda}{a^2}x = 0, \\ 4x(c-z) - \frac{2\lambda}{b^2}y = 0, \\ -4xy + \frac{\lambda}{c} = 0, \\ \frac{z}{c} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0. \end{cases}$$

解得 $z_0 = \frac{c}{2}$, $x_0 = \frac{a}{2}$, $y_0 = \frac{b}{2}$. 在Ω中解唯一，且当点$(x_0, y_0, z_0) \to \Omega$的边界时，$V \to 0$. 故当$x_0 = \frac{a}{2}$, $y_0 = \frac{b}{2}$, $z_0 = \frac{c}{2}$时，V最大.

$$\max V = \frac{1}{2}abc.$$

方法二. 写成无条件的最值问题.

$$V = 4c\left[xy - \frac{x^3y}{a^2} - \frac{xy^3}{b^2}\right],$$

$$\begin{cases} \frac{\partial V}{\partial x} = 4c\left[y - \frac{3x^2y}{a^2} - \frac{y^3}{b^2}\right] \overset{令}{=} 0, \\ \frac{\partial V}{\partial y} = 4c\left[x - \frac{x^3}{a^2} - \frac{3xy^2}{b^2}\right] \overset{令}{=} 0. \end{cases}$$

解得 $x = \frac{a}{2}$, $y = \frac{b}{2}$. 于是 $z = \frac{c}{2}$.（下略）.

14. (1) 按定义,

$$f'_x(0,0)=\lim_{x\to 0}\frac{f(x,0)-f(0,0)}{x}=\lim_{x\to 0}\frac{0}{x^3}=0,$$

同理 $f'_y(0,0)=0$.

(2) $\Delta f=f(0+\Delta x,0+\Delta y)-f(0,0)=\frac{(\Delta x)^2(\Delta y)^2}{((\Delta x)^2+(\Delta y)^2)^{3/2}}$

$f(x,y)$ 在点 $(0,0)$ 可微的充要条件是

$$\lim_{(\Delta x,\Delta y)\to(0,0)}\frac{\Delta f}{((\Delta x)^2+(\Delta y)^2)^{1/2}}=0,$$

即 $\lim_{(\Delta x,\Delta y)\to(0,0)}\frac{(\Delta x)^2(\Delta y)^2}{[(\Delta x)^2+(\Delta y)^2]^2}=0$

取 $\Delta y=k(\Delta x)$, 让 $\Delta x\to 0$,

$$\lim_{(\Delta x,\Delta y)\to(0,0)}\frac{(\Delta x)^2(\Delta y)^2}{[(\Delta x)^2+(\Delta y)^2]^2}=\frac{k^2}{(1+k^2)^2}$$ 随 k 而异

所以 $\lim_{(\Delta x,\Delta y)}\frac{(\Delta x)^2(\Delta y)^2}{[(\Delta x)^2+(\Delta y)^2]^2}$ 不存在,

所以 $f(x,y)$ 在点 $(0,0)$ 处不可微.

2012~2013夏季学期《微积分(Ⅲ)》A卷解答

1. 解 $a_{100}=\frac{1}{\pi}\int_{-\pi}^{\pi}\cos^{2013}x\cos 100x\,dx=\frac{2}{\pi}\int_{0}^{\pi}\cos^{2013}x\cos 100x\,dx$

$\xlongequal{x=\pi-t}\frac{2}{\pi}\int_{\pi}^{0}-\cos^{2013}t\cos 100t\cdot(-dt)=\frac{2}{\pi}\int_{\pi}^{0}\cos^{2013}t\cos 100t\,dt$,

$\therefore\ a_{100}=0$.

2. 解 $y'(x)=\sqrt{3+x^4}$, $dl=\sqrt{1+y'^2(x)}\,dx=\sqrt{4+x^4}\,dx$.

因为 $y(x)=\int_0^x\sqrt{1+t^4}\,dt$ 为 x 的奇函数，所以

$\int_l y\,dl=\int_{-1}^{1}y(x)\sqrt{4+x^4}\,dx=0$,

$\int_l |x|^3dl=2\int_0^1 x^3\sqrt{4+x^4}\,dx$

$=\frac{1}{3}(4+x^4)^{\frac{3}{2}}\Big|_0^1=\frac{1}{3}(5^{\frac{3}{2}}-8)$.

所以

$\int_l(y+|x|^3)\,dl=\frac{1}{3}(5^{\frac{3}{2}}-8)$.

3. 解 记 $D=\{(x,y)\mid 4x^2+y^2\le 8x\}$，由格林公式，

$\oint_l e^{y^2}dx+(x+y^2)dy=\iint\limits_D(1-2ye^{y^2})\,d\sigma$.

因为 $2ye^{y^2}$ 为 y 的奇函数，所以 $\iint\limits_D 2ye^{y^2}d\sigma=0$.

$\iint\limits_D(1-2ye^{y^2})\,d\sigma=\iint\limits_D d\sigma=D$ 的面积.

$4x^2+y^2-8x=4(x-1)^2+y^2-4=0$,

$\frac{(x-1)^2}{1}+\frac{y^2}{4}=1$,

是一个椭圆，长、短半轴分别为2及1，所以

D 的面积 $=2\pi$.

$\therefore\ \oint_l e^{y^2}dx+(x+y^2)dy=2\pi$.

4. 解 $P=\dfrac{x+2y}{x^2+4y^2}$, $Q=\dfrac{4y-2x}{x^2+4y^2}$,

$$\frac{\partial Q}{\partial x}=\frac{2x^2-8y^2-8xy}{(x^2+4y^2)^2},$$

$$\frac{\partial P}{\partial y}=\frac{2x^2-8y^2-8xy}{(x^2+4y^2)^2}.$$

所以在不包含点 $O(0,0)$ 在其内的单连通区域 D 内,该曲线积分与路径无关. 取椭圆

$$l_1:\ x^2+4y^2=\left(\frac{\pi}{2}\right)^2,\qquad y\geq 0.$$

或写成参数方程.

$$x=\frac{\pi}{2}\cos t,\ y=\frac{\pi}{4}\sin t,\ \text{从 } t=\pi \text{ 到 } t=0.$$

于是

$$\int_l \frac{(x+2y)dx+(4y-2x)dy}{x^2+4y^2}$$

$$=\frac{4}{\pi^2}\int_\pi^0\left[\left(\frac{\pi}{2}\cos t+\frac{\pi}{2}\sin t\right)\left(-\frac{\pi}{2}\sin t\right)+(\pi\sin t-\pi\cos t)\frac{\pi}{4}\cos t\right]dt$$

$$=\int_\pi^0(-1)dt=\pi.$$

5. 解 添直线段 $\overline{BA}$, $\widehat{AB}\cup\overline{BA}$ 构成正向封闭曲线, 记为 l_1, l_1 围成的半圆区域记为 D.

$$\int_l=\int_{l\cup\overline{BA}}-\int_{\overline{BA}}$$

$$=\iint_D(\pi\varphi'(y)\cos\pi x-\pi\varphi'(y)\cos\pi x-\pi)d\sigma$$

$$-\int_3^1(\pi\varphi(x)\cos\pi x-\pi x+\varphi'(x)\sin\pi x-\pi)dx$$

$$=-\pi\cdot\frac{\pi}{2}(\sqrt{2})^2+\int_1^3(\pi\varphi(x)\cos\pi x+\varphi'(x)\sin\pi x)dx$$

$$-\int_1^3(\pi x+\pi)dx$$

$$=-\pi^2+\left[\varphi(x)\sin\pi x\right]_1^3-\left[\frac{\pi}{2}x^2+\pi x\right]_1^3$$

$$=-\pi^2-\left(\frac{9}{2}\pi+3\pi-\frac{\pi}{2}-\pi\right)=-\pi^2-6\pi.$$

6. 解 将 x 看成参数，L 的参数方程可写成

$$\begin{cases} x=x, \\ y=x \\ z=2x^2 \end{cases} \quad 0\le x\le 1.$$

$$\int_L y\,dl=\int_0^1 x\sqrt{1+1+(4x)^2}\,dx=\int_0^1 x\sqrt{2+16x^2}\,dx$$

$$=\sqrt{2}\int_0^1 x\sqrt{1+8x^2}\,dx=\cancel{\frac{1}{48}}(1+8x^2)^{\frac{3}{2}}\Big|_0^1$$

$$\cancel{=\frac{\sqrt{2}}{48}(27-1)=\frac{13}{24}\sqrt{2}}.$$

$$\frac{13\sqrt{2}}{12}$$

7. 解 $dS=\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}\,d\sigma=\frac{R\,d\sigma}{\sqrt{R^2-x^2}}$, $D=\{(x,y)\mid x^2+y^2\le a^2\}$,

$$I=\iint_D (x^2+y^2+R^2-x^2+2xy-2x\sqrt{R^2-x^2}-2y\sqrt{R^2-x^2})\,d\sigma$$

$$=R\iint_D (y^2+R^2)\,d\sigma=R\int_0^{2\pi}d\theta\int_0^a (r^2\sin^2\theta+R^2)r\,dr$$

$$=R\left[\int_0^{2\pi}\sin^2\theta\,d\theta\int_0^a r^3\,dr+2\pi R^2\int_0^a r\,dr\right]=\pi R a^2\left(\frac{a^2}{4}+R^2\right).$$

8. 解 用投影法，S 在 xOy 平面上的投影

$$D=\{(x,y)\mid x^2+y^2\le 1,\ x\ge 0,\ y\ge 0\}$$

$$\iint_S xyz\,dx\,dy=\iint_D xy\sqrt{1-x^2-y^2}\,dx\,dy$$

$$=\int_0^{\frac{\pi}{2}}d\theta\int_0^1 r^3\sqrt{1-r^2}\cos\theta\sin\theta\,dr$$

$$=\frac{1}{4}\int_0^1 r^2\sqrt{1-r^2}\,dr^2=\frac{1}{15}.$$

9. 解 添曲面片 $z=1$，$(x^2+y^2\le 1)$，方向向下，记为 S_1.

$$\iint_S (2xy^2+z)\,dy\,dz+z\,dx\,dy=\iint_{S\cup S_1}-\iint_{S_1}$$

$$=-\iiint_\Omega (2y^2+1)\,dv+\iint_D 1\,dx\,dy$$

其中 $D=\{(x,y) \mid x^2+y^2\le 1\}$, $\Omega=\{(x,y,z) \mid x^2+y^2\le z\le 1\}$.

$$\iiint\limits_{\Omega}(2y^2+1)\,dv=\iint\limits_{D}d\sigma\int_{x^2+y^2}^{1}(2y^2+1)\,dz$$

$$=\iint\limits_{D}(2y^2+1)(1-x^2-y^2)\,d\sigma$$

$$=\int_0^{2\pi}d\theta\int_0^1(2r^2\sin^2\theta+1)(1-r^2)\,r\,dr$$

$$=\int_0^{2\pi}d\theta\int_0^1(2r^3\sin^2\theta+r-2r^5\sin^2\theta-r^3)\,dr$$

$$=\int_0^{2\pi}\left(\frac{1}{2}\sin^2\theta-\frac{1}{3}\sin^2\theta+\frac{1}{4}\right)d\theta$$

$$=\frac{\pi}{2}-\frac{\pi}{3}+\frac{\pi}{2}=\frac{2\pi}{3}.$$

$$\iint\limits_{D}1\,dx\,dy=\pi.$$

$$I=\frac{\pi}{3}.$$

10. 解 (1) S 的法向量 $\vec{n}=\{1,1,1\}$, $\vec{n}^0=\frac{1}{\sqrt{3}}\{1,1,1\}$. 于是

$$I=\frac{1}{\sqrt{3}}\iint\limits_{S}(y+2z+z+3x+x+y)\,dS$$

$$=\frac{1}{\sqrt{3}}\iint\limits_{S}(4x+2y+3z)\,dS.$$

(2) 由 (1), 再由 $z=2-x-y$, $\frac{\partial z}{\partial x}=-1$, $\frac{\partial z}{\partial y}=-1$ 于是

$$I=\frac{1}{\sqrt{3}}\iint\limits_{D}(4x+2y+3(2-x-y))\sqrt{1+1+1}\,d\sigma$$

$$=\iint\limits_{D}(6-x-y)\,d\sigma$$

其中 $D=\{(x,y) \mid |x|+|y|\le 1\}$. 由于

$$\iint\limits_{D}x\,d\sigma=0,\quad \iint\limits_{D}y\,d\sigma=0,$$

$$\therefore I=\iint\limits_{D}6\,d\sigma=12.$$

11. 解 $\Omega_1=\{(x,y,z)\mid x^2+y^2+z^2\le 1 \text{ 且 } z\ge\sqrt{x^2+y^2}\}$,

$\Omega_2=\{(x,y,z)\mid x^2+y^2+z^2\ge 1 \text{ 且 } \sqrt{x^2+y^2}\le z\le 1\}$.

$$\iiint_\Omega \left|\sqrt{x^2+y^2+z^2}-1\right|dv$$

$$=\iiint_{\Omega_1}(1-\sqrt{x^2+y^2+z^2})\,dv+\iiint_{\Omega_2}(\sqrt{x^2+y^2+z^2}-1)\,dv$$

$$=\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{4}}d\varphi\int_0^1(1-\rho)\rho^2\sin\varphi\,d\rho+\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{4}}d\varphi\int_1^{\frac{1}{\cos\varphi}}(\rho-1)\rho^2\sin\varphi\,d\rho$$

$$=-\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{4}}\left(\frac16+\frac{1}{4\cos^4\varphi}-\frac{1}{3\cos^3\varphi}\right)d\cos\varphi=\frac{\pi}{6}(\sqrt2-1).$$

12. 解 (1) 由路无关定理知,

$$\frac{\partial}{\partial y}(xy(x+y)-f(x))\equiv\frac{\partial}{\partial x}(f'(x)+x^2y),$$

$$f''(x)=x^2,$$

$f(x)=\frac{1}{12}x^4+c_1x+c_2$. 再由 $f(0)=0$, $f'(0)=1$, 有

$$f(x)=\frac{x^4}{12}+x.$$

(2) 取 $(0,0)\to(x,0)\to(x,y)$ 的折线,

$$\int_{(0,0)}^{(x,y)}=-\int_0^x\left(\frac{x^4}{12}+x\right)dx+\int_0^y\left(\frac13x^3+1+x^2y\right)dy$$

$$=-\frac{1}{60}x^5-\frac12x^2+\frac13x^3y+y+\frac12x^2y^2.$$

13. 解 (1) 交换积分次序, 有

$$\int_0^1dz\int_0^zF(y)\,dy=\int_0^1dy\int_y^1F(y)\,dz=\int_0^1(1-y)F(y)\,dy.$$

(2) $\iiint_\Omega f(z)\,dv=\int_0^1dz\int_0^zdy\int_0^yf(x)\,dx$.

将 $\int_0^yf(x)dx$ 记为 $F(y)$, 由 (1),

$$\int_0^1dz\int_0^zdy\int_0^yf(x)\,dx=\int_0^1dz\int_0^zF(y)\,dy=\int_0^1(1-y)F(y)\,dy$$

$$=\int_0^1(1-y)\left(\int_0^yf(x)\,dx\right)dy=\int_0^1dy\int_0^y(1-y)f(x)\,dx$$

$$=\int_0^1dx\int_x^1(1-y)f(x)\,dy$$

$$=\frac12\int_0^1(1-x)^2f(x)\,dx.$$

浙大 2013-2014 学年微积分 2 答案

1.解：$|\vec{a}\times\vec{b}| = \sqrt{(|\vec{a}||\vec{b}|)^2-(a\cdot b)^2}$

$= \sqrt{4\times 2-4} = 2$

2.解：$\vec{AB} = (-4,0,1)$

$\vec{AC} = (0,3,1)$

$\vec{AD} = (2,-1.5,-1)$

$$(\vec{AB}\times\vec{AC})\cdot\vec{AD} = \begin{vmatrix} -4 & 0 & 1 \\ 0 & 3 & 1 \\ 2 & -1.5 & -1 \end{vmatrix} = \begin{vmatrix} -4 & 0 & 1 \\ 4 & 0 & -1 \\ 2 & -1.5 & -1 \end{vmatrix} = -1.5\times(-1)^{3+2}\times\begin{vmatrix} -4 & 1 \\ 4 & -1 \end{vmatrix} = 0$$

∴ 四点共面，平面的法向量 $\vec{n} = \vec{AB}\times\vec{AC} = (-3,4,-12)$

∴ 平面方程为 $-3(x-3)+4(y+1)-12z=0$，即 $-3x+4y-12z+13=0$

3.解：$\frac{\partial z}{\partial x} = (1+x^2y)^{xy^2}\left(y^2\ln(1+x^2y) + xy^2\frac{2xy}{1+x^2y}\right)$

$\frac{\partial z}{\partial y} = (1+x^2y)^{xy^2}\left(2xy\ln(1+x^2y) + xy^2\frac{x^2}{1+x^2y}\right)$

∴ $x\frac{\partial z}{\partial x} - 2y\frac{\partial z}{\partial y} = zxy(x-4y)\ln(1+x^2y)$.

4.解：$dz = 2dy + e^{2x-3z}(2dx-3dz)$

∴ $dz = \frac{2e^{2x-3z}dx+2dy}{1+3e^{2x-3z}}$

5.解：设切点 (x_0,y_0,z_0)

则切点处的切平面方程为：$z+z_0 = 2x_0x+2y_0y$

∴ $\frac{2x_0}{2} = \frac{2y_0}{-1} = \frac{-1}{-1}$

∴ $x_0=1$，$y_0=-\frac{1}{2}$，$z=1+\frac{1}{4}=\frac{5}{4}$

∴ 切平面方程为 $\frac{5}{4}+z = 2x-y$

6.解：$\ln x+\ln y+\ln z = 3\ln a$

$\frac{1}{x}+\frac{y'(x)}{y(x)}+\frac{z'(x)}{z(x)} = 0$

又 $2x+2yy'-2az' = 0$

$y(a)=a$，$z(a)=a$

∴ $\begin{cases} \frac{1+y'(a)+z'(a)}{a} = 0 \\ 2a+2ay'(a)-2az'(a) = 0 \end{cases} \Rightarrow \begin{cases} y'(a) = -1 \\ z'(a) = 0 \end{cases}$

切线方程为 $x-a-(y-a)=0$ 即 $x-y=0$

7. 解: $\vec{AB}=(4,-2,4)$

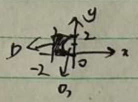
$\vec{AB}^0=\left(\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right)$

$\frac{\partial u}{\partial l^0}=\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)\Big|_A\cdot\vec{AB}^0$

$=(6,2,2)\cdot\left(\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right)$

$=\frac{14}{3}.$

8. 解:



$\iint\limits_D y\,d\sigma=\iint\limits_{D+D_1}y\,d\sigma-\iint\limits_{D_1}y\,d\sigma$

$=\int_{-2}^{0}dx\int_0^2 y\,dy-\int_{\frac{\pi}{2}}^{\pi}d\theta\int_0^{2\sin\theta}r\sin\theta\cdot r\,dr$

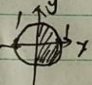
$=4-\int_{\frac{\pi}{2}}^{\pi}\frac{8}{3}\sin^4\theta\,d\theta$

$=4-\frac{8}{3}\int_0^{\frac{\pi}{2}}\sin^4\theta\,d\theta$

$=4-\frac{8}{3}\times\frac{\pi}{2}\times\frac{1}{2}\times\frac{3}{4}$

$=4-\frac{\pi}{2}$

9. 解: 积分曲区域关于x轴对称



$\iint\limits_D\frac{1+x+\sin(xy)}{1+x^2+y^2}d\sigma=\iint\limits_D\frac{1+x}{1+x^2+y^2}d\sigma$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\int_0^1\frac{1+r\cos\theta}{1+r^2}r\,dr$

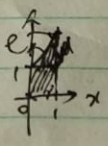
$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\left[\frac{1}{2}\ln(1+r^2)+r\cos\theta-\cos\theta\arctan r\right]\Big|_0^1$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1}{2}\ln2+\cos\theta-\frac{\pi}{4}\cos\theta\right)d\theta$

$=\frac{\pi}{2}\ln2+2\left(1-\frac{\pi}{4}\right)$

$=\frac{\pi}{2}\ln2+2-\frac{\pi}{2}$

10. 解: 原式 $=\int_0^1dx\int_0^{e^x}\sqrt{e^{2x}-y^2}\,dy$



$=\int_0^1\frac{\pi}{4}e^{2x}dx$

$=\frac{\pi}{8}e^{2x}\Big|_0^1=\frac{\pi}{8}(e^2-1)$

11. 解. $\frac{\partial g}{\partial x}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}$

$=y\frac{\partial f}{\partial u}+x\frac{\partial f}{\partial v}$

$\frac{\partial g}{\partial y}=x\frac{\partial f}{\partial u}-y\frac{\partial f}{\partial v}$

$\frac{\partial^2 g}{\partial x^2}=y\left(\frac{\partial^2 f}{\partial u^2}y+x\frac{\partial^2 f}{\partial u\partial v}\right)+\frac{\partial f}{\partial v}+x\left(\frac{\partial^2 f}{\partial v\partial u}y+x\frac{\partial^2 f}{\partial v^2}\right)$

$\frac{\partial^2 g}{\partial y^2}=x\left(\frac{\partial^2 f}{\partial u^2}x-\frac{\partial^2 f}{\partial u\partial v}y\right)-\frac{\partial f}{\partial v}$

$-y\left(\frac{\partial^2 f}{\partial v\partial u}x-\frac{\partial^2 f}{\partial v^2}y\right)$

$\therefore\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=x^2+y^2$

12. 解:

设 $L=u^2+\lambda(x^2+y^2-z)+u(x+y+z-4)$

则由
$$\begin{cases}L_x'=2x+2\lambda x+u=0\\L_y'=2y+2\lambda y+u=0\\L_z'=2z-\lambda+u=0\\L_\lambda'=x^2+y^2-z=0\\L_u'=x+y+z-4=0\end{cases}$$

得. $x=y=1$, $z=2$

$x=y=-2$, $z=8$

$\therefore$ 最大值. $u_{max}=\sqrt{(-2)^2+(-2)^2+8^2}$

$=6\sqrt{2}$

最小值 $u_{min}=\sqrt{1^2+1^2+2^2}=\sqrt{6}$

13. 解: $\frac{\partial u}{\partial x}=xy(x+y)-y$

$\frac{\partial u}{\partial y}=f(x)+x^2y$

$\frac{\partial^2 u}{\partial x\partial y}=x^2+2xy-1$

$\frac{\partial^2 u}{\partial y\partial x}=f'(x)+2xy$

$\therefore f'(x)=x^2-1$, $\therefore f(x)=\frac{x^3}{3}-x+C_1$

$u(x,y)=\frac{x^3}{3}y+\frac{x^2y^2}{2}-xy+C_1y+C_2.$