# Bringing State-Separating Proofs to EasyCrypt
# A Security Proof for Cryptobox

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# Cryptographic security

Security against computationally bounded adversaries

- Initially studied for primitives and simple schemes
- Extended to larger protocols built from such primitives

Approaches to formal verification:

- Focused on protocols (e.g. Cryptoverif)
  - Rely on strong assumptions or manual reasoning about primitives
- Focused on primitives (e.g. Certicrypt, EasyCrypt, CryptHOL)
  - Don't scale well to larger constructions

Challenge: combining formal reasoning at different levels

# State-separating proofs (SSP) [BDFKK18]

SSP addresses this challenge on paper:

- Modular reasoning at different levels of abstraction
- Primitives and bigger constructions are treated in a uniform way

How to take advantage of SSP?

- Formalize SSP (SSProve [AHR+21])
- Incorporate SSP reasoning style in existing tool (EasyCrypt)
  - Maintain existing proof support from EasyCrypt
  - Add flexibility of proof style

# Our contributions

Systematic mapping from SSP to EC concepts

- To take advantage of SSP reasoning style without actually formalising SSP
- SSP as guide for developing mechanized proofs

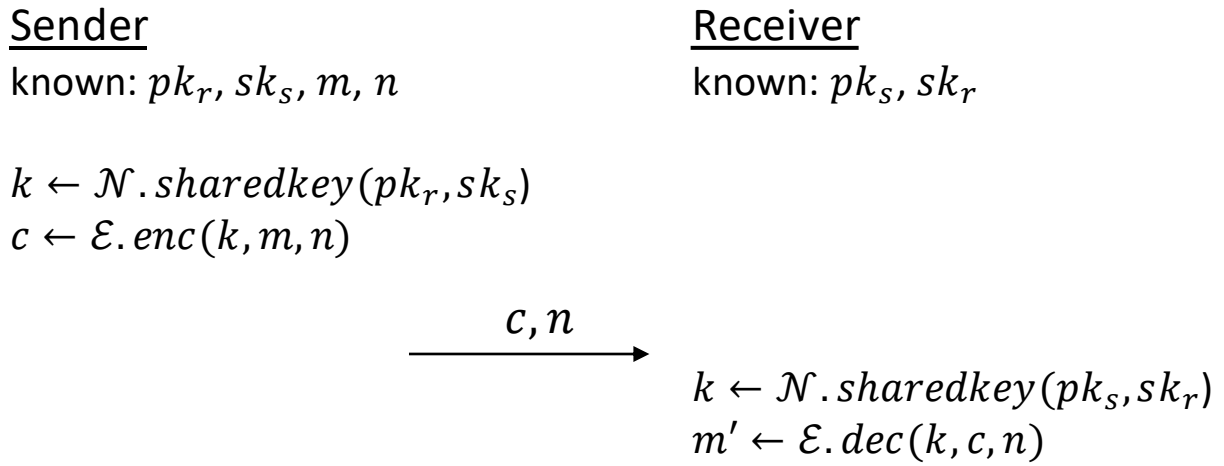
Case study in EasyCrypt: Cryptobox

- PKAE security of multiple concurrent instances in the presence of corruption
- First mechanized security proof of Cryptobox

# Cryptobox

# Cryptobox

- Default way of achieving "secure encryption from public keys", e.g. implemented in NaCl
- Cryptobox = NIKE + AE [DH76]




Here: Cryptobox security as nonce-based PKAE scheme

- Multi-instance setting with corruption
- Reduce to standard assumptions on NIKE and AE schemes

# State-separating proofs (SSP)

# Code-based game-playing [BR04/06]

Game = set of oracles with shared state

Security as distinguishing advantage of adversary with oracle access

For all PPT $\mathcal{A}$, $|\Pr[\mathcal{A}^{GPKAE_{\mathcal{P}}^{0}} = 1] - \Pr[\mathcal{A}^{GPKAE_{\mathcal{P}}^{1}} = 1]|$ negl.

Security proof: sequence of game hops
e.g. from $GPKAE_{\mathcal{P}}^{0}$ to $GPKAE_{\mathcal{P}}^{1}$

Game $GPKAE_{\mathcal{P}}^{b}$:

State: `PK, SK, ...`

`GEN():`
```
  ... // gen. honest key pair
```

`CSETPK(pk):`
```
  ... // register corrupt pk
```

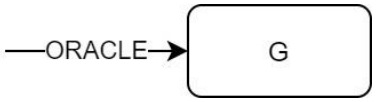
`PKENC(pks,pkr,m,n):`
```
  ... // behaviour depends on b
```

`PKDEC(pkr,pks,c,n):`
```
  ... // behaviour depends on b
```

# State-separating proofs (SSP) [BDFKK18]

Structure code-based games to achieve modularity and composition

- Game = set of oracles with state



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Structure code-based games to achieve modularity and composition

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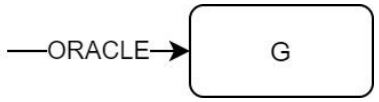
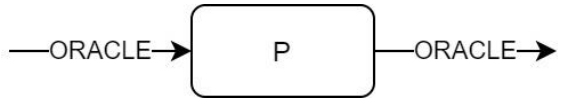
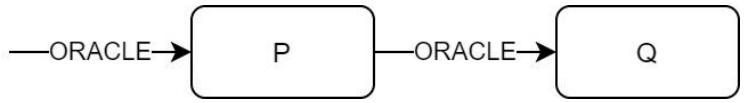


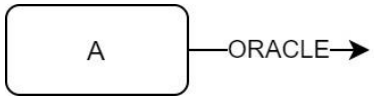

- **Package** = set of oracles with state that can call oracles




  - State separation between packages
  - Package composition (associative)

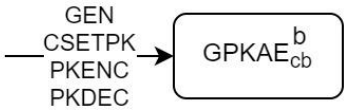



  - Adversary: special package



# Cryptobox and SSP

Games $GPKAE_{\mathcal{P}}^{b}$ for $\mathcal{P}$ = Cryptobox




<u>Sender</u>
known: $pk_r, sk_s, m, n$

$k \leftarrow \mathcal{N}.sharedkey(pk_r, sk_s)$
$c \leftarrow \mathcal{E}.enc(k, m, n)$

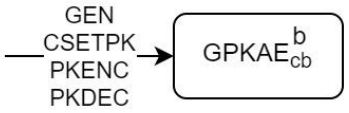
$c, n$ →

<u>Receiver</u>
known: $pk_s, sk_r$

$k \leftarrow \mathcal{N}.sharedkey(pk_s, sk_r)$
$m' \leftarrow \mathcal{E}.dec(k, c, n)$

# Cryptobox and SSP

Games $GPKAE_{\mathcal{P}}^{b}$ for $\mathcal{P}$ = Cryptobox

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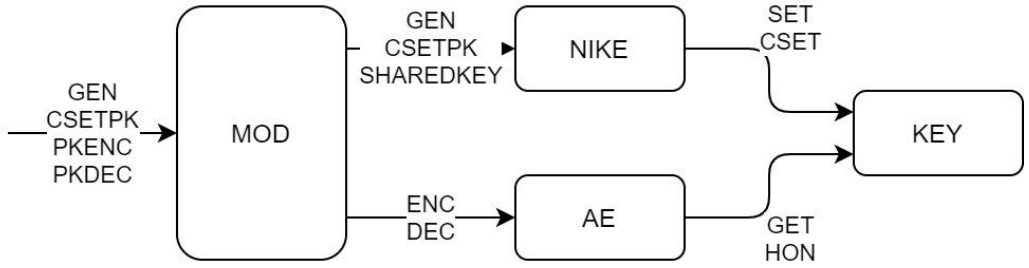
Alternative representation of PKAE game for proof

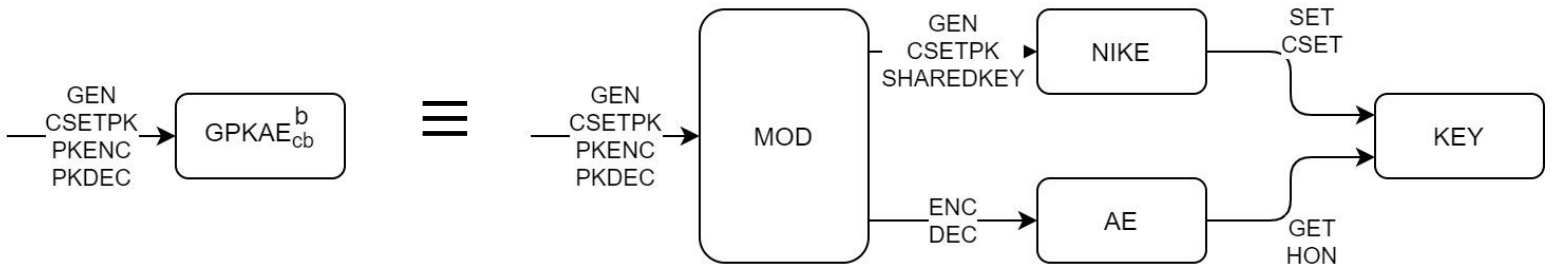
- Highlights (in)dependencies between building blocks: NIKE and AE
- Shared state: shared symmetric key



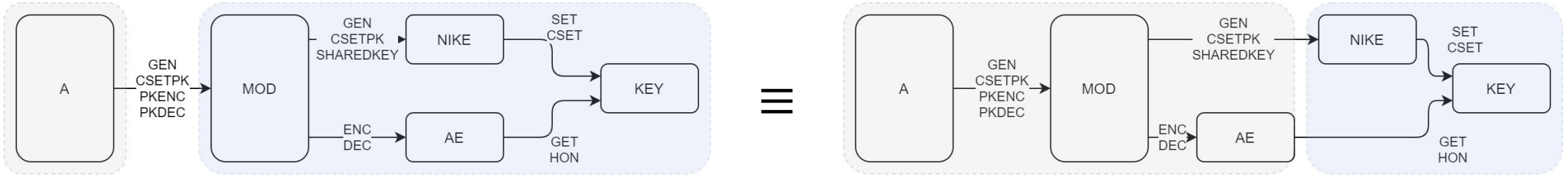
# Cryptobox and SSP

Types of proof steps:

- Code equivalence




- Reductions as shifting package boundaries
  - e.g. turn PKAE adversary A into NIKE adversary B




A → GPKAE B → GNIKE

# Mapping SSPs to EasyCrypt

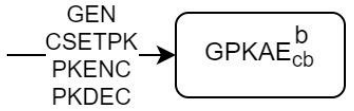
# EasyCrypt

- Interactive proof assistant for reasoning about probabilistic programs
- Module system
  - Modular reductions
  - Modular construction of adversaries

Goal: recreate SSP reasoning style

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
|---|---|
| Package | Module |




```
module type GPKAE = {... // proc. }

module GPKAE0 (P : PKAE) = {
  ...          // procedures
}
```


```
module NIKE (K : NIKE_in) = {
 proc gen() = {...}
 proc csetpk(pk : pkey) = {...}
 proc sharedkey(pk:pkey,sk:skey)
    = {...} // may query K's procedures
}
```

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
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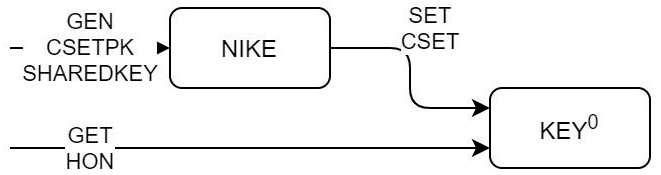
| Package | Module |
| Package composition | Module parameter (sequential), wiring module (parallel) |




```
module GNIKE (N : NIKE)
             (K : KEY) = {
 include N(K)[gen,csetpk,sharedkey]
 include K[get,hon]
}

module GNIKE0 = GNIKE(NIKE,K0)
```

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
|---|---|
| Package | Module |
| Package composition | Module parameter (sequential), wiring module (parallel) |
| Game | Fully instantiated module |
| Adversary | Abstract module |


```
module type A_pkae (G : GPKAE) = {
 proc run() : bool
}
```

Distinguishing advantage of `A : A_pkae`:

```
|Pr[ A(GPKAE0).run() @m : res ]
 - Pr[ A(GPKAE1).run() @m : res ]|
```

# Mapping SSPs to EasyCrypt

| **SSP** | **EasyCrypt** |
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| Implicit state separation | ? |

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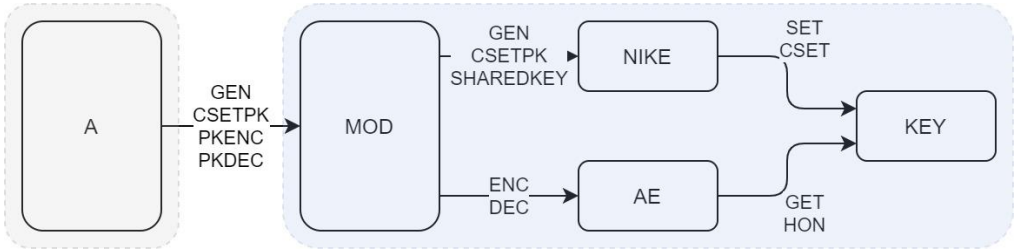


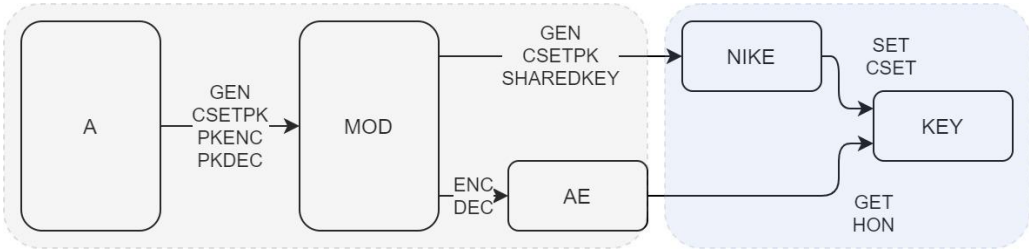

# Mapping SSPs to EasyCrypt

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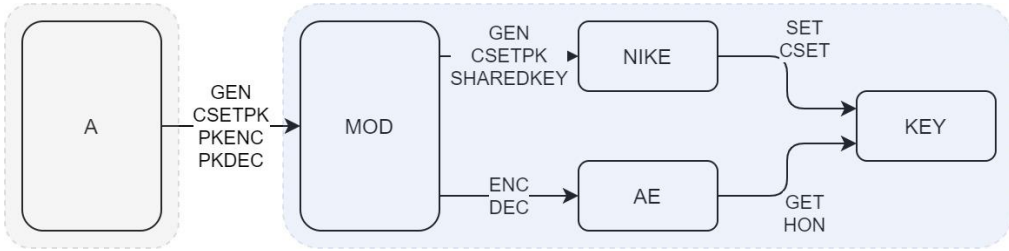


```
module GMOD (N : NIKE)
       (AE : AE) : GPKAE = {...}

module GMOD0 = GMOD(NIKE(K0),AE(K0))
```

consider `A(GMOD0)`

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
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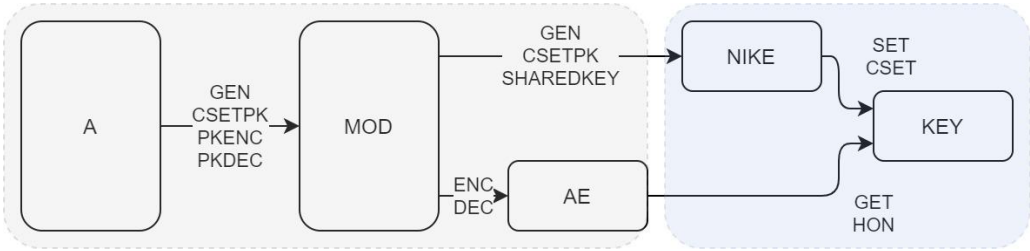


instantiate `GMOD` only partially:

```
module R (G : GNIKE) = GMOD(G,AE(K0))
```

then instantiate with a NIKE game

consider `A( R(GNIKE0) )`

NIKE adversary `B (G : GNIKE) = A(R(G))`

# Mapping SSPs to EasyCrypt

| **SSP** | **EasyCrypt** |
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| Package | Module |
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EasyCrypt: Designed for Halevi's style of code-based game-playing proofs [Halevi05]

- Experiment that initializes state, provides oracles and calls adversary
- Experiment can initialize state of other modules

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
|---|---|
| Package | Module |
| Package composition | Module parameter (sequential), wiring module (parallel) |
| Game | Fully instantiated module |
| Adversary | Abstract module |
| Implicit initialization | Explicit initialization or restrict initial memories |
| Implicit state separation | |
| Shifting package boundaries | Redefining module boundaries |

Options:

- Explicit initialization: composition issues
- Restricting initial memories: seems better for now

```
∀&m, GPKAE0.PK{m} = empty ∧ ... ⟹
|Pr[ A(GPKAE0).run() @m : res ]
 - Pr[ A(GPKAE1).run() @m : res ]|
```

(but: explicit initialization sometimes needed)

# Mapping SSPs to EasyCrypt

| SSP | EasyCrypt |
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Explicitly require separate memories of modules

```
∀(A <: Apkae{GPKAE0,GPKAE1})
∀&m, GPKAE0.PK{m} = empty ∧ ... ⟹
|Pr[ A(GPKAE0).run() @m : res ]
 - Pr[ A(GPKAE1).run() @m : res ]|
```

# Cryptobox security in EasyCrypt

# Cryptobox security proof

- PKAE multi-instance security notion with corruption
  - Extension of [An01] PKAE security from two honest parties to many parties with corruption
  - Variant of [BT16] AE security notion that adds corruption
- Cryptobox implementation
- Security of Cryptobox is reduced to (single-instance) security of NIKE and AE schemes

Not mentioned yet: statistical equivalence steps

- Bound probability of public key collisions
  - Cryptobox provides plausible deniability: no ID related to public keys

# What have we learned?

# Benefits of our approach

Why SSP?

- Local reasoning
- Reasoning at different levels of abstraction

Why EasyCrypt?

- Existing support from EasyCrypt: libraries, tactics, automation etc.
- Flexibility: switching between reasoning styles

SSP as tool for proof development

- Proof discipline
- Sketching proof outline (dependencies, shared state, ...)
- Visualizing proof steps

# Potential for improvement

State initialization

Assertions

- SSP: to enforce "good" adversary behaviour
- EasyCrypt: modeled as explicit control flow

Forward reasoning

- SSP: oracles close to the real-world interface of primitive
- In oracles: same oracle for honest and corrupt queries, distinguish through control flow
- In proof: case analysis and simplify execution path
- Forces forward reasoning when proving program equivalences, which goes against tool design
- Assertion modeling as control flow causes similar issue

# Conclusion

# SSPs to guide larger formalizations

Map SSP concepts to EasyCrypt constructs

- Informal yet systematic
- Preserves key SSP features (local and modular reasoning)
- Can be combined with traditional reasoning style
- Identified friction points and potential for improvement

New example: Cryptobox

- PKAE security proof in multi-instance setting with corruption

Code: https://gitlab.com/fdupress/ec-cryptobox/