ALGORITHMS & DATA STRUCTURES

20th Decemer 2021

PLAN FOR TODAY

- Bonus Exercise
- Final game

EXERCISE 12.1

BORUVKA

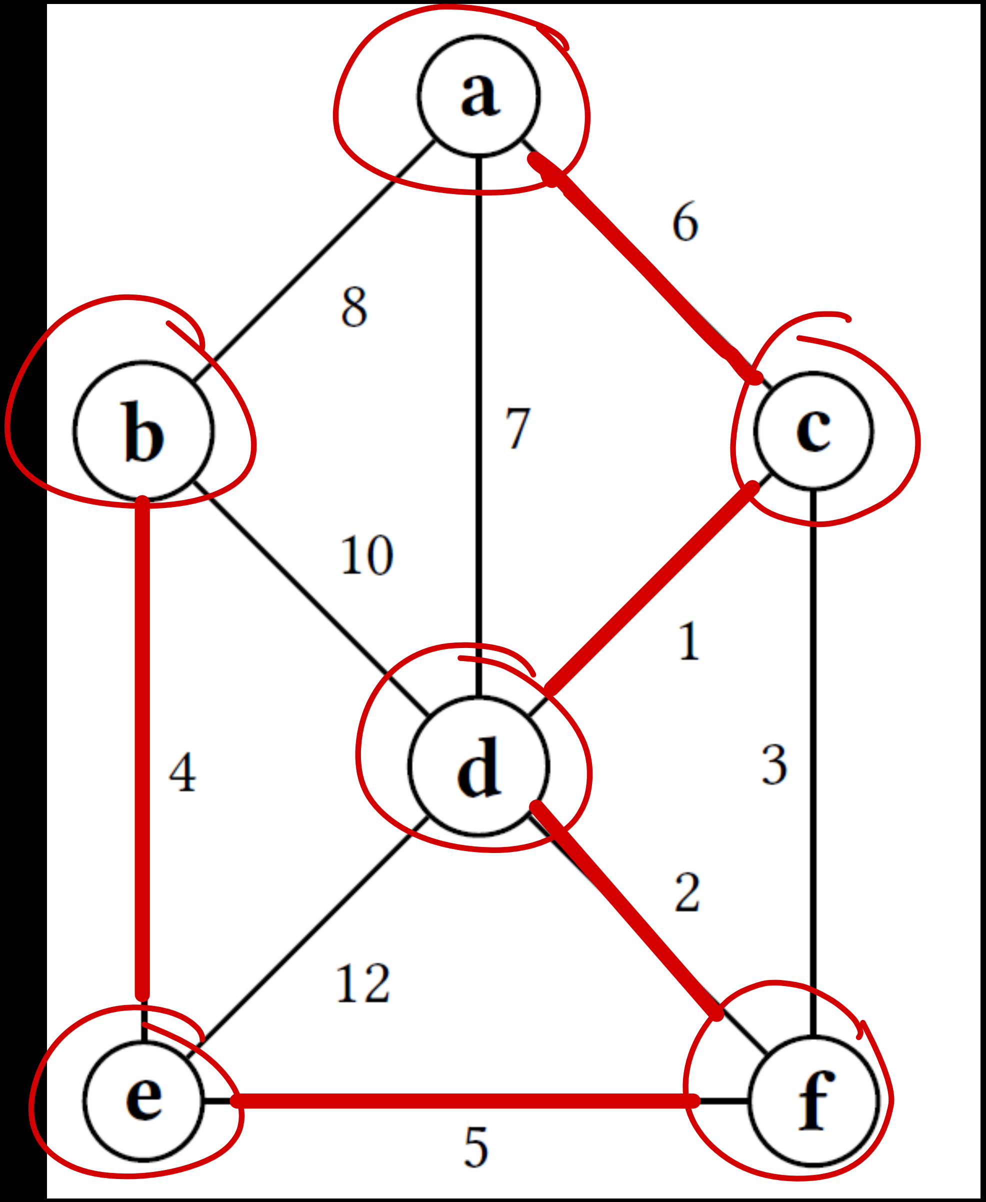
{a, c}

{b, e}

{c, d}

{d, f}

{e, f}



unusual

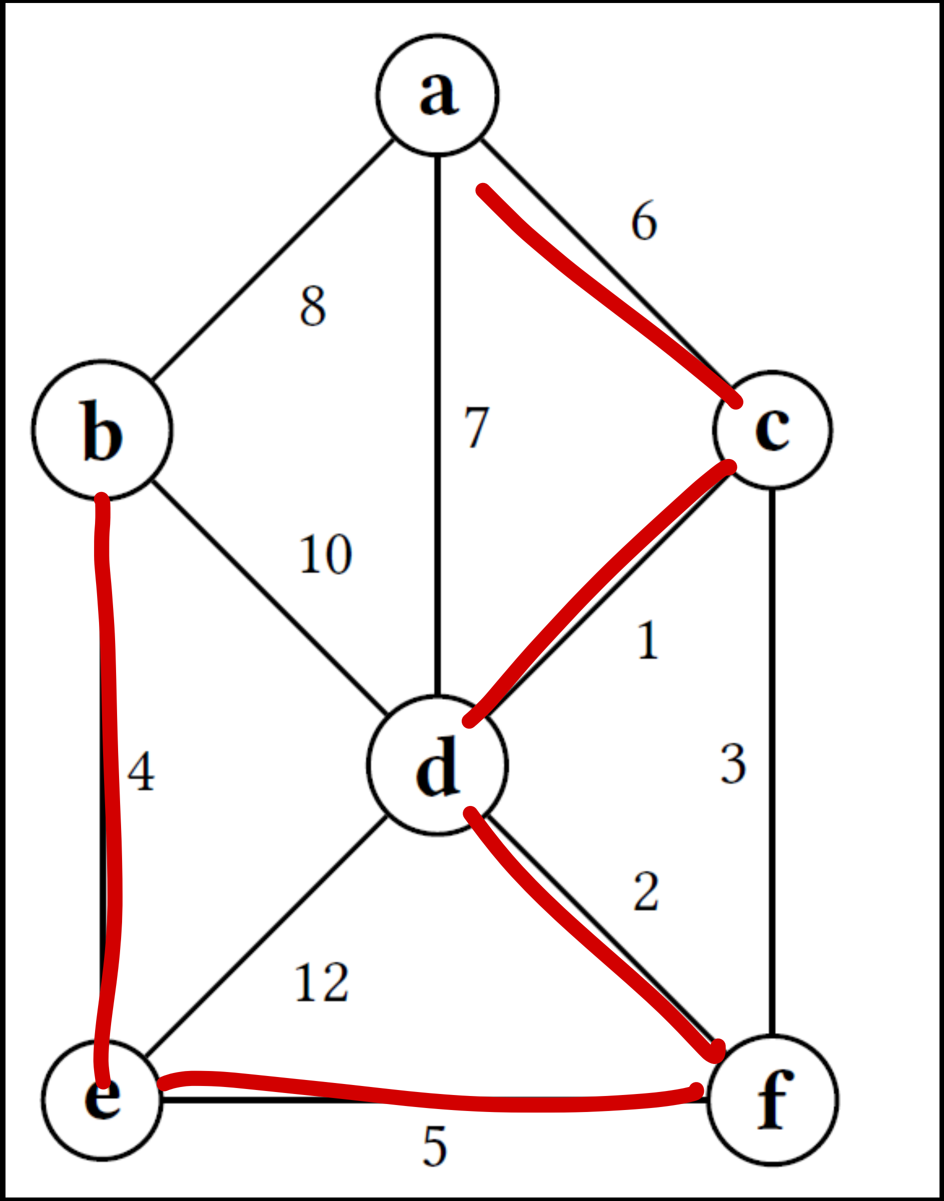
{c, d}

{d, f}

{b, e}

{c, f}

{a, c}



Prim

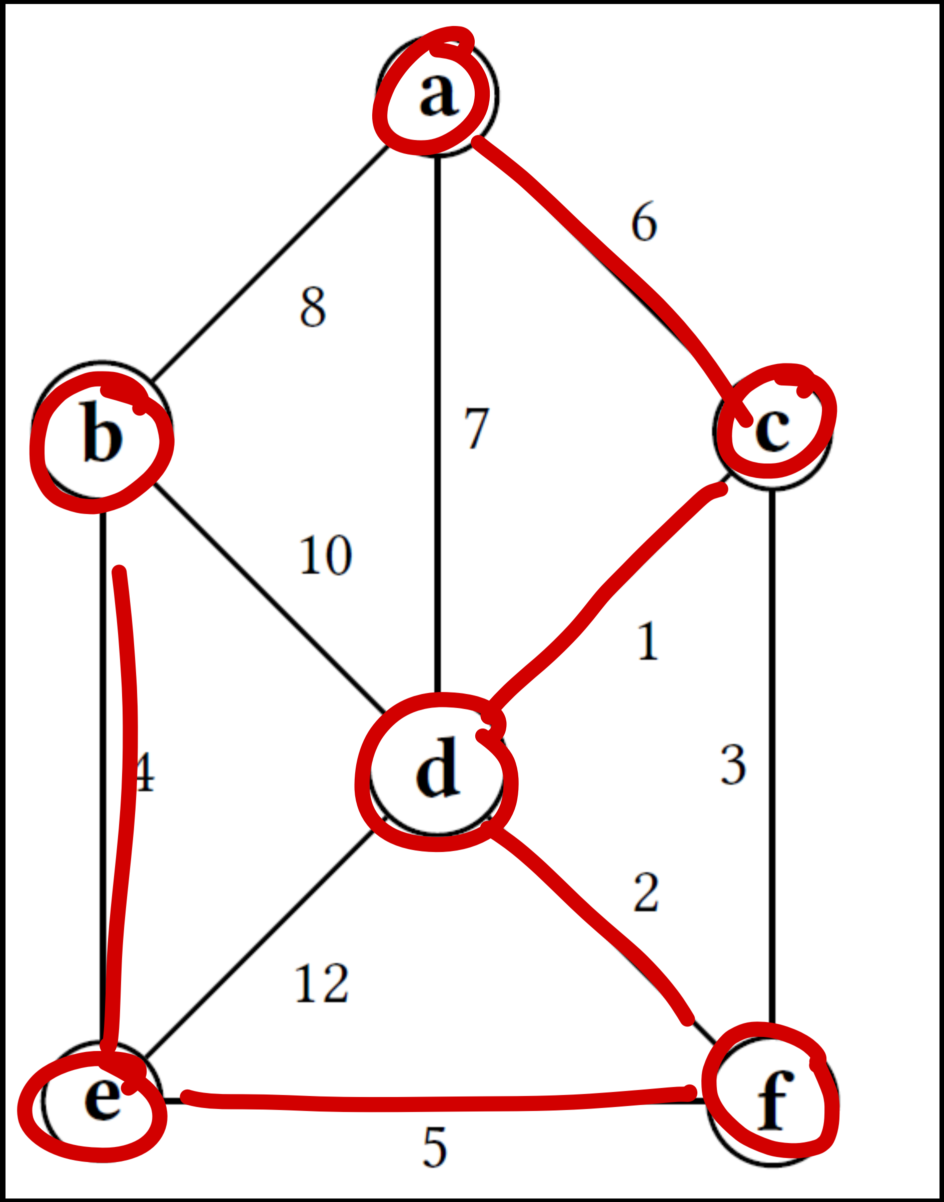
{a,c}

{d,f}

{e,f}

{b,e}

{a,c}



EXERCISE 12.2

Exercise 12.2 *Minimum Spanning Tree and Shortest Paths* ***(1 point).***

Let $G = (V, E)$ be a connected edge-weighted graph where all the weights are nonnegative and distinct. Let T be a minimum spanning tree of G.

Let $v \in V$ be a fixed vertex and define T_v to be the tree of shortest paths that is obtained by applying Dijkstra's algorithm on G starting from the source v.

Is it possible that T and T_v do not have any edge in common ? If the answer is yes, provide an example showing that it is possible. Otherwise, prove that it is impossible

Solution: The answer is no: Since the edge-weights are nonnegative and distinct, there is a unique minimum spanning tree T. Therefore, T is the same as the tree that is obtained by applying Prim's algorithm starting from v.

Consider the edge $\{v, u\}$ that is incident to v and which has the minimum weight. The edge $\{v, u\}$ is the first edge to be added to T in Prim's algorithm. Similarly, (v, u) is a shortest path from v to u and $\{v, u\}$ is the first edge to be added to T_v is Dijkstra's algorithm. We conclude that T and T_v share the edge $\{v, u\}$.

EXERCISE 12.3

Exercise 12.3 *Constructing a Fiber Optic Network* ***(1 point)****.*

The government of Atlantis put you in charge of installing a fiber optic network that connects all its n cities. There are two technologies of fibre optic that you can use:

- Fibre 1.0: It is a good reliable technology that is relatively cheap. There is a list of pairs of cities between which it is possible to install a direct Fibre 1.0 link. Furthermore, for each such pair, there is a corresponding cost.
- Fibre 2.0: It is an emerging technology that it extremely good and can directly connect any two cities. However, its cost is too high and the government cannot afford a single Fibre 2.0 link.

Note that all direct links are two-directional. The installed network should connect all the cities of Atlantis: Between any two cities, there should be a connected path of direct links in the network that connects them.

A philanthropist volunteered to donate the cost of exactly k direct Fibre 2.0 links, and you can use them to connect any k pairs of cities. Your goal is to minimize the cost that is paid by the government for the Fibre 1.0 links that are needed to construct a connected network. Describe an algorithm that finds the network that costs the government the minimum amount of money.

Note that it is possible to construct a network connecting all the cities of Atlantis using only Fibre 1.0 links, but we would like to benefit from the k Fibre 2.0 links that were donated by the philantropist in order to minimize the cost that is paid by the government.

$G=(V,E)$

V: Städte

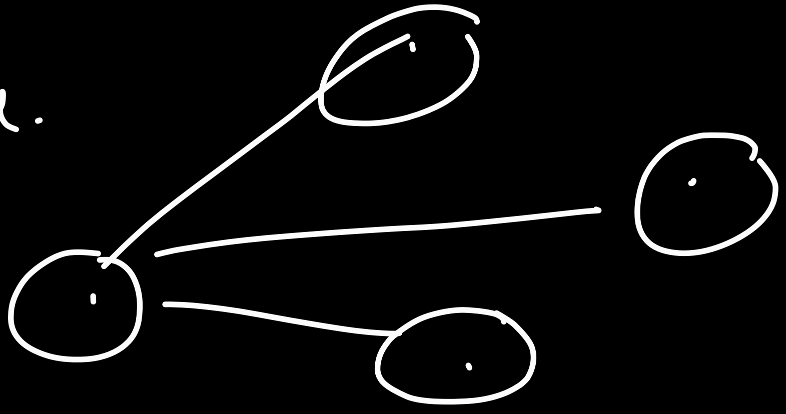
E: Fiber 1 Links

Wir können k Verbindungen für "free" hinzufügen (zwischen beliebige Städte").

$\Rightarrow$ Quasi-Kruskal

1. Wir hinzufügen die erste $n-1-k$ Kanten, die normales Kruskal hinzufügen würde

$k+1$



2. Hinzufüge k Fiber 2 Links, s.d. unseres $(k+1)$-Forest ein Baum wird.

IF YOU WANT TO PARTICIPATE, SEND ME AN E-MAIL WITH OBJECT: “PLAY”