SHOPPER: A PROBABILISTIC MODEL OF CONSUMER CHOICE WITH SUBSTITUTES AND COMPLEMENTS

Francisco J. R. Ruiz, David Blei and Susan Athey · 2020

By Soel Micheletti · Advanced Topics in Machine Learning and Data Science · May 2021



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COMPLEMENTS





Source: https://produkte.migros.ch

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Source: https://cb4.com





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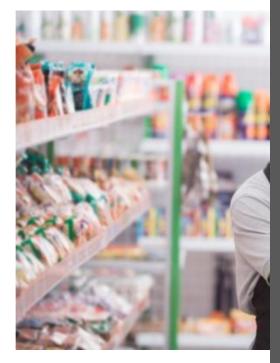


Source: https://cb4.com

- What products should be promoted? By how much?
- Should the price of certain products be changed?
- Insights about both products and customers
- Form personalized recommendations



Source; https://atlas-network.com



Source: https://cb4.com

"MAKE ACCURATE PREDICTIONS UNDER PRICE INTERVENTIONS"

Francisco J. Ruiz



Source; https://atlas-network.com

THE MODEL

OPTIMIZATION/ INFERENCE

1.The Model

2. Optimization/Inference

3. Results/ Discussion/ Openings

1.The Model

2. Optimization/Inference

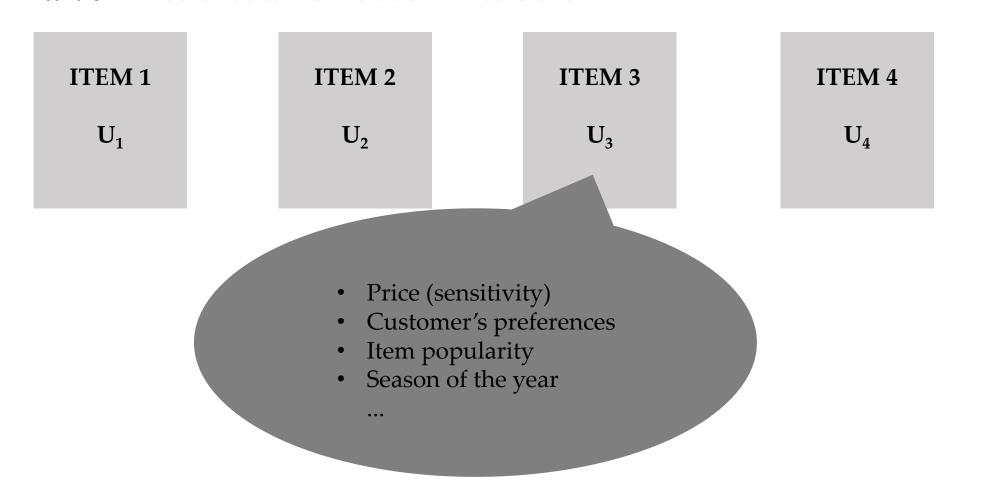
3. Results/ Discussion/ Openings

Intuition · Mathematical Formulation · Extensions

ITEM 1 ITEM 2 ITEM 3 ITEM 4 CHECK-OUT

Intuition · Mathematical Formulation · Extensions

Intuition · Mathematical Formulation · Extensions



CHECK-

OUT

 $\mathbf{U}_{\mathbf{C}}$

Intuition · Mathematical Formulation · Extensions

ITEM 1

 $\mathbf{U^*}_1$

ITEM 2

 U^*_2

ITEM 3

 U^*_3

ITEM 4

 $\mathbf{U^*}_4$

CHECK-OUT

U*_C

Intuition · Mathematical Formulation · Extensions

ITEM 1

 U^*_1

ITEM 2

 U^*_2

ITEM 3

 U^*_3

ITEM 4

 $\mathbf{U^*}_4$

CHECK-OUT

U*_C

Intuition • **Mathematical Formulation** • **Extensions**

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

$$\mathbf{y}_{t, i-1} = (y_{t1}, y_{t2}, ..., y_{t(i-1)})$$

Intuition · Mathematical Formulation · Extensions

$$\Pr[y_{ti} = c \mid y_{t,(i-1)}] = \frac{\exp\{\Psi(c, y_{t,i-1})\}}{\sum_{c' \neq y_{t,i-1}} \exp\{\Psi(c', y_{t,i-1})\}}$$

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Intuition · Mathematical Formulation · Extensions

$$\Psi(c, \mathbf{y}_{t,i-1}) = \Psi_{tc} + \rho_c^{\mathrm{T}} \left(\frac{1}{i-1} \sum_{j=1}^{i-1} \alpha_{y_{tj}}\right)$$

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Customer's preferences

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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Intuition · Mathematical Formulation · Extensions

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Interaction's coefficient

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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Intuition · Mathematical Formulation · Extensions

$$\Psi(c, \mathbf{y}_{t,i-1}) = \Psi_{tc} + \rho_c^{\mathrm{T}} \left(\frac{1}{\mathrm{i} - 1} \sum_{j=1}^{l-1} \alpha_{\mathbf{y}_{tj}}\right)$$
Latent representation of

the item

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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Intuition · Mathematical Formulation · Extensions

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Intuition • **Mathematical Formulation** • Extensions

$$\Psi_{tc} = \lambda_c$$

Item popularity

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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Intuition · Mathematical Formulation · Extensions

$$\Psi_{tc} = \lambda_c + \theta_u^{\mathrm{T}} \alpha_c$$

Customer's preferences

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Intuition · Mathematical Formulation · Extensions

$$\Psi_{tc} = \lambda_c + \theta_u^{\mathrm{T}} \alpha_c - \gamma_{u_t}^{T} \beta_c \log(r_{tc})$$
Price effects

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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Intuition • **Mathematical Formulation** • Extensions

$$\Psi_{tc} = \lambda_c + \theta_u^T \alpha_c - \gamma_{u_t}^T \beta_c \log(r_{tc}) + \delta_{w_t}^T \mu_c$$
Seasonal effects

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

 $\mathbf{y}_{t, i-1} = (y_{t1}, y_{t2}, ..., y_{t(i-1)})$

$$\Pr[y_{ti} = c | \mathbf{y}_{t,(i-1)}] = \frac{\exp\{\Psi(c, \mathbf{y}_{t,i-1})\}}{\sum_{c' \neq \mathbf{y}_{t,i-1}} \exp\{\Psi(c', \mathbf{y}_{t,i-1})\}}$$

$$\Psi(c, \mathbf{y}_{t,i-1}) = \Psi_{tc} + \rho_c^{\mathrm{T}}(\frac{1}{i-1}\sum_{j=1}^{i-1}\alpha_{y_{tj}})$$

Intuition · Mathematical Formulation · Extensions

We want to estimate the best parameters from the data. This allows us to:

- Answer conterfactual queries
- Decide whether two products are complements/ substitutes
- Gaining insights about customers and products

$$\mathbf{y}_{t} = (y_{t1}, y_{t2}, ..., y_{tn})$$

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$$\Psi(c, y_{t,i-1}) = \Psi_{tc} + \rho_c^{T} \left(\frac{1}{i-1} \sum_{j=1}^{i-1} \alpha_{y_{tj}}\right)$$

$$\Psi_{tc} = \lambda_c + \theta_u^{\mathrm{T}} \alpha_c - \gamma_{u_t}^{\mathrm{T}} \beta_c \log(r_{tc}) + \delta_{w_t}^{\mathrm{T}} \mu_c$$

Intuition · Mathematical Formulation · **Extensions**

Thinking ahead

- More realistic consumer behaviour
- Consider how the current choice influences the next k choices

1.The Model

2. Optimization/Inference

3. Results/ Discussion/ Openings

Optimization

Given a dataset with observed characteristics $\mathbf{x} = \{u_t, w_t, r_t\}_{t=1}^N$ and shopping baskets $\widetilde{\mathbf{y}} = \{\widetilde{\mathbf{y}}_t\}_{t=1}^N$ we want to find the latent parameters $\ell = \{\alpha, \rho, \lambda, \theta, \gamma, \beta, \delta, \mu\}$ that maximize the MAP estimation.

$$\Pr[\ell \mid \widetilde{\boldsymbol{y}}, \boldsymbol{x}] = \frac{\Pr[\ell] \ \prod_{t} \Pr[\widetilde{\boldsymbol{y}}_{t} | \ell, \boldsymbol{x}_{t}]}{\Pr[\widetilde{\boldsymbol{y}} | \boldsymbol{x}]}$$

Optimization: the problem is intractable

Approximate $\Pr[\ell \mid \widetilde{\mathbf{y}}, \mathbf{x}]$ with a tractable approximating distribution $q(\ell, \nu)$ that minimizes the KL divergence. Or, equivalently, maxizing the ELBO

 $argmax_{\nu}\mathbb{E}_{q(\ell,\nu)}[\log \Pr[\ell,\widetilde{\boldsymbol{y}}|\boldsymbol{x}] - \log q(\ell,\nu)]$

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Approximate $\Pr[\ell \mid \widetilde{\mathbf{y}}, \mathbf{x}]$ with a tractable approximating distribution $q(\ell, \nu)$ that minimizes the KL divergence. Or, equivalently, maxizing the ELBO

$$argmax_{\nu}\mathbb{E}_{q(\ell,\nu)}[\log \Pr[\ell,\widetilde{\boldsymbol{y}}|\boldsymbol{x}] - \log q(\ell,\nu)]$$

This gives a tractable approximated distribution $q(\ell, \nu^*)$ that allows to estimate the final parameters ℓ .

Main Ideas to tackle intractable problems

Stochastic optimization addresses

- Large datasets (expensive gradients)
- Intractable expectations

Variational lower bounds on ELBO addresses

- Unordered baskets
- Large number of items

Plan

1.The Model

2. Optimization/Inference

3. Results/ Discussion/ Openings

Plan

1.The Mod

2. Optimi:

3. Results/

"LOOKING AT THE GOF IS NOT ENOUGH TO DO COUNTERFACTUALS"

Susan Athey

Dataset

- 6 mio. items
- 570 000 baskets
- 3200 customers
- 5600 unique items
- 97 weeks of data



Source: https://en.wikipedia.org

Quantitative Results

Log-likelihood

Model	All (540K)	Price ± 2.5% (231K)	Price ± 5% (139K)	Price ± 15% (25K)
B-Emb (Rudolph et al. (2016))	-5.119 (0.001)	-5.119 (0.002)	-5.148 (0.002)	-5.250 (0.006)
P-Emb (Rudolph et al. (2016))	-5.160(0.001)	-5.138(0.002)	-5.204(0.002)	-5.311(0.005)
HPF (Gopalan, Hofman and Blei (2015))	-4.914 (0.002)	-4.931 (0.002)	-4.994 (0.003)	-5.061 (0.009)
SHOPPER (I+U)	-4.744(0.002)	-4.743(0.003)	-4.776(0.003)	-4.82(0.01)
SHOPPER (I+U+S)	-4.730(0.002)	-4.778(0.003)	-4.801 (0.004)	-4.83(0.01)
SHOPPER (I+U+P)	-4.728(0.002)	-4.753(0.003)	-4.747 (0.004)	-4.69(0.01)
SHOPPER (I+U+P+S)	-4.724 (0.002)	-4.741 (0.003)	-4.774 (0.004)	-4.64 (0.01)

Qualitative Results: Inference of Latent Factors

Mollusks	Organic vegetables	Granulated sugar	Cat food dry/moist
finfish all other—frozen	organic fruits	flour	cat food wet
crustacean nonshrimp	citrus	baking ingredients	cat litter & deodorant
shrimp family	cooking vegetables	brown sugar	pet supplies

Qualitative Results: Seasonal Effects

Halloween candy		Cherries		Turkey—frozen	
3.46	2006/10/25	3.07	2006/06/28	3.56	2005/11/16
3.34	2005/10/26	3.01	2006/07/12	3.30	2006/11/15
2.81	2005/10/19	2.85	2006/06/21	2.64	2005/11/23
			: :		
-1.28	2005/11/23	-3.59	2006/10/11	-1.25	2006/06/21
-1.31	2007/01/03	-3.89	2006/10/18	-1.29	2006/07/05
-1.33	2005/11/16	-4.54	2006/10/25	-1.30	2006/07/19

Qualitative Results: Complements and Substitutes

Complementarity metric:

$$C_{cc'} = \frac{1}{2} (\rho_c^T \alpha_c + \rho_{c'}^T \alpha_c)$$

Exchangeability metric:

$$E_{cc'} = \frac{1}{2} \Big(D_{KL}(p_{\cdot|c}||p_{\cdot|c'}) + D_{KL}(p_{\cdot|c'}||p_{\cdot|c}) \Big)$$

query	complementarity score	exchangeability score
mission tortilla taco 1	2.40 taco bell seasoning mix2.26 mcrmck seasoning mix2.24 lawrys seasoning mix	0.05 mission fajita0.07 mission tortilla taco 20.13 mission tortilla fluffy gordita
(private) hot dog buns	2.99 bp franks meat2.63 bp franks bun size2.37 bp franks beed bun length	0.11 ball park hot dog buns 0.13 (private) hot dog potato buns 1 0.15 (private) hot dog potato buns 2

• Econometrics + Machine Learning: https://www.youtube.com/watch?v=mjGPF5R5JOQ&t=347s

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- Counterfactual inference is used also on other scenarios: news consumption; what happens to the market if a shop closes...

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- Counterfactual inference is used also on other scenarios: news consumption; what happens to the market if a shop closes...
- The authors used the same ideas in a very interesting paper to answer counterfactual queries about restaurant locations:
 - https://arxiv.org/pdf/1801.07826.pdf

I. Empirical Model and Estimation

We model the consumer's choice of restaurant conditional on deciding to go out to lunch. We assume that the consumer selects the restaurant that maximizes utility, where the utility of user u for restaurant i on her t-th visit is

$$U_{uit} = \lambda_i + \theta_u^{\top} \alpha_i + \mu_i^{\top} \delta_{w_{ut}} - \gamma_u^{\top} \beta_i \cdot \log(d_{ui}) + \epsilon_{uit},$$

- Econometrics + Machine Learning:
 https://www.youtube.com/watch?v=mjGPF5R5JOQ&t=347s
- Counterfactual inference is used also on other scenarios: news consumption; what happens to the market if a shop closes...
- The authors used the same ideas in a very interesting paper to answer counterfactual queries about restaurant locations: https://arxiv.org/pdf/1801.07826.pdf
- Matrix factorization, embedding methods and Bayesian inference.

My Take

- + SHOPPER is good at counterfactuals (if training data is good enough)
- + Interpretability: extraction of valuable latent features
- +/- Practical paper, but there is still a gap to use the model on business decisions
- More details on the related work in the community would have been appreciated

Thank you for your attention.

References

- Counterfactual Inference for Consumer Choice Across Many Product Categories: https://arxiv.org/pdf/1906.02635.pdf
- Code Implementation SHOPPER: https://github.com/franrruiz/shopper-src

Backup Slides

How to use the model for Business Decisions

- Not clear how answering counterfactual queries about prices on a specific basket can help taking business decisions.
- Detect complements that are located physically far away in the store and put them together.
- We can detect the favorite products of a certain (class of) customer(s) and decide to promote them.
- Interpretation of the latent parameters not too useful here. Does this model generalize in more "interesting" scenarios?

How to use the model for Business

Decisions

 Not clear how basket can help

- Detect complex
 put them toget
- We can detect to decide to prom
- Interpretation

REINFORCEMENT LEARNING APPROACH a specific

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er(s) and

this

model generalize in more "interesting" scenarios?

Why Thinking Ahead?

Illustration · Results

- Better description of consumer's behavior
- Choice: pick A with utility 1 or B with utility 2. If you picked 1 you can take an item with utility 10, if you picked B you can take an item with utility 5 → Greedily taking B in step 1 is suboptimal.

Toy Simulation: Performance Boost with Thinking Ahead

Illustration · Results

- Items: Coffee, Diapers, Ramen, Candy, Hot dogs, Hot Dog Buns, Taco Shells, Taco Seasoning
- Customers: 50% are new parents and never buy candy or ramen; 50% are students and nevery buy cofee of diapers
- Either you by Taco Shells AND Taco Seasoning or none. Same for Hot Dogs and Hot Dog Buns.
- About preferred items: if price is low buy with probability 0.95, otherwise 0.1. High price with probability 0.4.

Complementary 1	Complementary 2	Probability
LOW	LOW	0.5 for both
LOW	HIGH	0.85 for low, 0.15 for high
HIGH	LOW	0.85 for low, 0.15 for high
HIGH	HIGH	Does not happen

Toy Simulation: Performance Boost with Thinking Ahead

Illustration · Results

		stage 1: diapers	stage 2: hot dogs	stage 3: hot dog buns	stage 4: checkout
	diapers	0.31	0.00	0.00	0.00
	coffee (†)	0.03	0.02	0.05	0.21
non think-ahead	ramen	0.00	0.00	0.00	0.00
-ah	candy	0.00	0.00	0.00	0.00
ink	hot dogs	0.18	0.25	0.00	0.00
thi	hot dog buns	0.17	0.25	0.79	0.00
non	taco shells (†)	0.14	0.19	0.00	0.00
1	taco seasoning	0.17	0.24	0.00	0.00
	checkout	0.00	0.05	0.16	0.79
	diapers	0.37	0.00	0.00	0.00
	coffee (†)	0.02	0.02	0.07	0.10
þ	ramen	0.00	0.00	0.00	0.00
think-ahead	candy	0.00	0.00	0.00	0.00
(-a	hot dogs	0.24	0.34	0.00	0.00
lin	hot dog buns	0.24	0.42	0.85	0.00
÷	taco shells (†)	0.06	0.10	0.00	0.00
	taco seasoning	0.06	0.10	0.00	0.00
	checkout	0.00	0.02	0.08	0.90

Thinking Ahead on a Larger Dataset

Illustration · Results

	Three items	Entire baskets
Non think-ahead	-4.795 (0.005)	-4.96 (0.02)
Think-ahead	- 4.747 (0.004)	- 4.91 (0.02)

The Dataset

- 570 878 baskets rom a single grocery store
- 6 mio. purchases from 5590 unique items; 3206 different customers
- Span of 97 weeks
- Heterogeneous data (e.g. Prices change regularly)
- Training + validation + test split

Inference of Latent Features

cat food wet (excluding super premium) - cat food dry/moist (excluding super premium) -

cat litter & deodorant -

super premium dog food pet supplies dog food snacks (exluding super premium) dog food wet (excluding super premium)

wild bird food/treats/accessories

(a) Pet food and supplies.

facial tissue

bathroom tissue 1 bathroom tissue 2

paper towels

· refuse bags

· laundry detergent

specialty surface cleaners

· dish detergents

· general purpose cleaners

specific purpose cleaners

• bar soap

scouring/sponge

laundry pre-treatment

liquid hand soap.

(b) Cleaning and hygiene.

Qualitative Results: Inference of Latent Factors

