LASSO ADJUSTMENTS OF TREATMENT EFFECT ESTIMATES IN RANDOMIZED EXPERIMENTS

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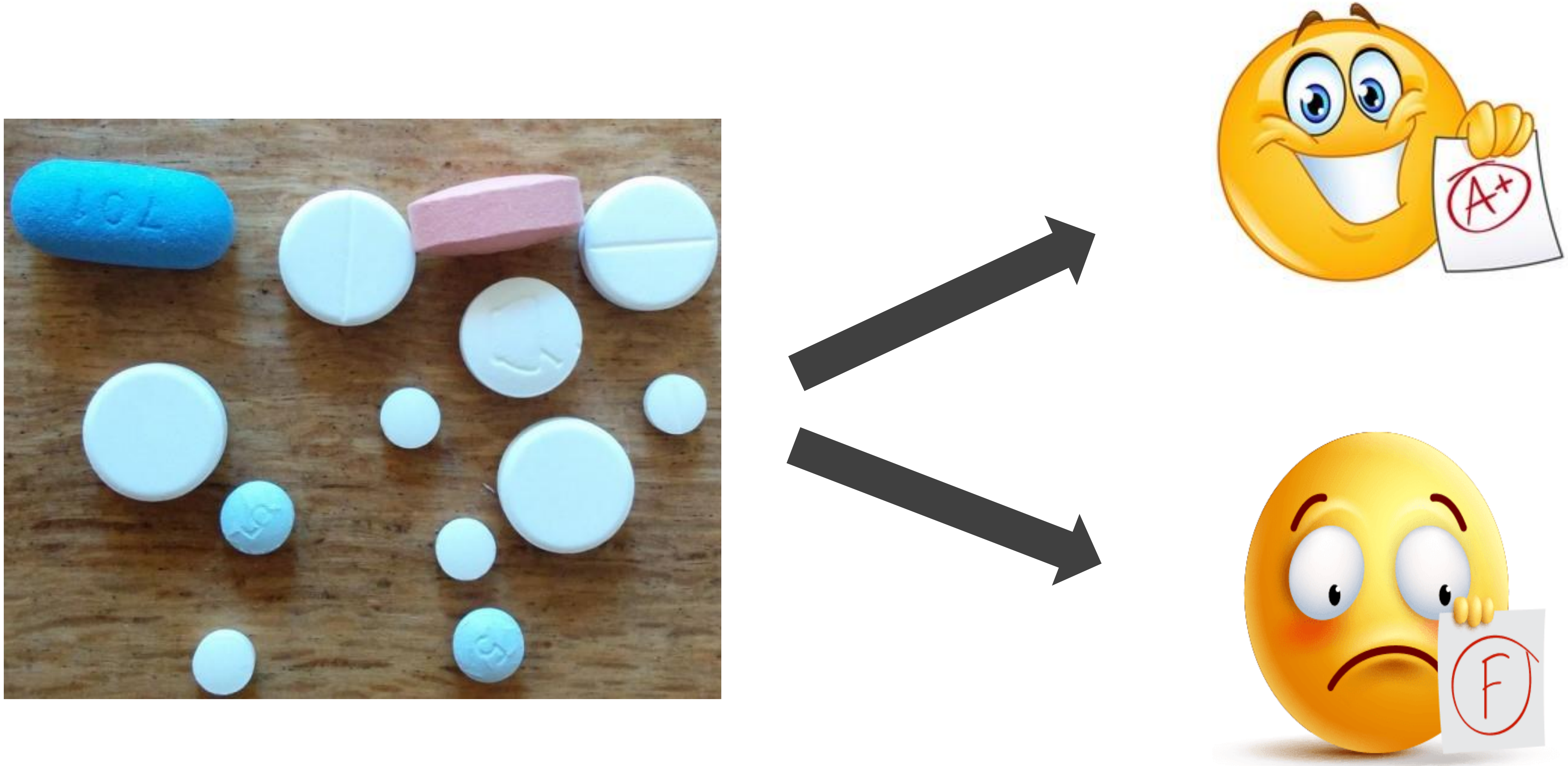
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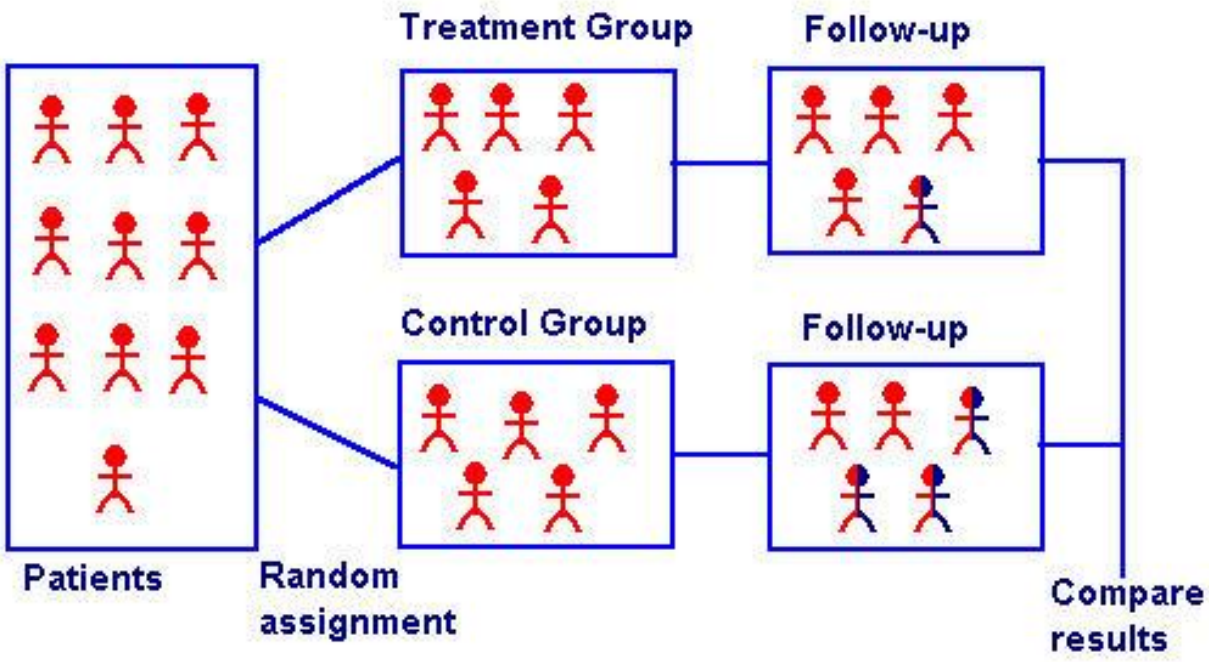

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Estimation of the Average Treatment Effect (ATE)

$$\widehat{ATE}_{unadj} = \bar{a}_A - \bar{b}_B$$

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High variance

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Take patients covariates into account to improve ATE estimation.

Plan

1. A LASSO based estimator

2. Theoretical results

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A LASSO based estimator

Potential outcome framework · Setting · Estimator

Before the treatment decision is made, there are two **potential outcomes** a_i and b_i.

In the course of the study, we observe only one of them, and the other becomes a **conterfactual outcome**.

The average causal effect over the population $\frac{1}{n}\sum_{i=1}^{n} a_i - b_i$

is unobservable, but can be estimated under certain assumptions (e.g. SUTVA).

A LASSO based estimator

Potential outcome framework · **Setting** · Estimator

For every patient we observe:

- T_i, an indicator random variable for the patient's assignment.
- $Y_i = T_i a_i + (1 - T_i) b_i$, i.e. the observed outcome for individual *i*. Note that T_i is the only source of randomness in the model, the potential outcomes are considered fixed (even if not all of them are observable).
- A p-dimensional vector x_i, denoting the covariates for individual *i*. We consider the case *p*>> *n*, where *n* is the number of individuals in the study.

$$\frac{1}{n}\sum_{i=1}^{n} a_i - b_i$$

$$\widehat{ATE}_{unadj} = \bar{a}_A - \bar{b}_B$$

A LASSO based estimator

Potential outcome framework · Setting · **Estimator**

$$\widehat{ATE}_{adj} = \left[\bar{a}_A - (\bar{x}_A - \bar{x})^T \hat{\beta}^{(a)}\right] - \left[\bar{b}_B - (\bar{x}_B - \bar{x})^T \hat{\beta}^{(b)}\right]$$

By "chance", let's assume that people in the treatment group have a better immune system (which is one of our covariate).

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A LASSO based estimator

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Overall, the term above is larger than the unadjusted estimator.

We hence conclude that our drug is less effective (higher ATE implies longer recovery), which is what we want.

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Where the beta coefficients are the coefficients of regressing the outcome of the respective group to their covariates.

- OLS: possible overfitting
- LASSO: variable selection
- LASSO-OLS combination: maximum performance (but attention to the post-selection problem!)

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Main result

Let's decompose the potential outcomes as follows

$$a_i = \bar{a} + (\mathbf{x}_i - \bar{\mathbf{x}})^T \boldsymbol{\beta}^{(a)} + e_i^{(a)},$$

$$b_i = \bar{b} + (\mathbf{x}_i - \bar{\mathbf{x}})^T \boldsymbol{\beta}^{(b)} + e_i^{(b)}.$$

Then, under certain conditions

$$\sqrt{n}\left(\widehat{ATE}_{\mathrm{Lasso}} - ATE\right) \xrightarrow{d} \mathcal{N}(0, \sigma^2),$$

where

$$\sigma^2 = \lim_{n\to\infty} \left[\frac{1-p_A}{p_A}\sigma^2_{e^{(a)}} + \frac{p_A}{1-p_A}\sigma^2_{e^{(b)}} + 2\sigma_{e^{(a)}e^{(b)}}\right].$$

Advantages of the LASSO estimator

- Lower variance than the unadjusted estimator, and the introduced bias vanishes quickly.
- Smaller MSE
- Narrower confidence intervals

Sufficient conditions: an intuition

Implications of oracle inequality

assume that $\phi_0^2 \geq L > 0$

$$\text{on } \mathcal{T}: \; \|X(\hat{\beta} - \beta^0\|_2^2/n + \lambda\|\hat{\beta} - \beta^0\|_1 \leq 4\lambda^2 s_0/\phi_0^2$$

if $\lambda(= 2\lambda_0) \asymp \sqrt{\log /n}$: as $p \geq n \to \infty$,

$$\|X(\hat{\beta} - \beta^0\|_2^2/n = O_P(s_0 \log(p)/n)$$
$$\|\hat{\beta} - \beta^0\|_1 = O_P(s_o\sqrt{\log(p)/n})$$

Slide from P. Buehlmann

Sufficient conditions: an intuition

- Similar conditions to the ones required for the "fast rate" of the LASSO
  - Model sparse enough
  - Cone invertibility condition (similar to the restricted eigenvalue condition)
  - Regularization parameters of a certain order

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- Not too "wild" error terms.

My takes from the paper

- Theoretical paper: LASSO has a long tradition, but they fine tune it to their use-case
- They provide sufficient conditions only
- Practical use-case: interesting, because they try to verify the assumptions


That's all Folks!