Assignment 4

Hand in by May 19 to Søren (soren.wengel_mogensen@control.lth.se).

You need to solve Problem A or Problem B. You do not need to solve both. Hints are provided at the end of this document. The problems may be easier to solve after the lecture on May 15.

Problem A Let $V = \{1, 2, \ldots, n\}$. For a multivariate time series, $X_t = (X_t^1, \ldots, X_t^n)^T$, $t = 0, 1, 2, 3, \ldots$, and disjoint $A, B, C \subseteq V$, we say that X^A is *Granger noncausal* for X^B given X^C if for each t

$$X_t^B \perp\!\!\!\perp X_{p(t)}^A \mid X_{p(t)}^{B \cup C}$$

where $p(t)$ denotes $0, 1, 2, \ldots, t-1$.

We have seen that the global Markov property is useful in DAGs. In this problem, we will prove an analogous Markov property by arguing that a certain type of separation in the summary graph of X implies Granger noncausality.

Let $\mathcal{D} = (V, E)$ be a summary graph, and let $B \subseteq V$. We define $\mathcal{D}^B$ to be graph obtained from $\mathcal{D}$ be removing all edges with a tail at a node in B, i.e., all edges $k \to l$ such that $k \in B$. We say that B is *δ-separated* from A given C in $\mathcal{D}$ if there is no d-connecting path between any $i \in A$ and any $j \in B$ given C in $\mathcal{G}^B$.

Now to the task: Assume that $\mathcal{D} = (V, E)$ is the summary graph of the DAG representing the time series X. Prove that if B is δ-separated from A given C, then X^A is Granger noncausal for X^B given X^C.

Problem B In this problem, we will implement a simple algorithm to learn a summary graph from tests of Granger causality assuming full observation. Let $X_t = (X_t^1, \ldots, X_t^n)^T$ be a multivariate time series and let $V = \{1, \ldots, n\}$. Assuming full observation, we can for each $i, j \in V$ test

$$X_t^j \perp\!\!\!\perp X_{p(t)}^i \mid X_{p(t)}^{-i}$$

where $-i$ denotes $V \setminus \{i\}$. If we reject the independence, then we put in the edge $i \to j$ in the output graph. Looping over i and j, we construct the entire output graph.

Your task is to learn the summary graphs from the three data sets (granger1.csv, granger2.csv, granger3.csv) on the webpage (the graphs are not necessarily the same across data sets). Look for inspiration in Chapter 10 concerning the actual test, e.g., in (10.7) and (10.8). All tests require choosing a threshold for test values which can be somewhat arbitrary. You may also output a complete graph with test values assigned to edges to avoid this problem.

Implement a test and learn the summary graph using each of the three data sets. Describe your method and report your results.

Hints

Problem A: Prove first that if B is δ-separated from A given C in the summary graph, then X_t^B and $X_{p(t)}^A$ are d-separated given $X_{p(t)}^{B\cup C}$ in the causal DAG of the time series. Use the global Markov property in the DAG to reach the conclusion.

Problem B: Note that (10.7) and (10.8) are in the bivariate case. You need to generalize this to the multivariate case.